Consumable Workbooks Many of the worksheets contained in the Chapter Resource Masters are available as consumable workbooks in both English and Spanish.

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Study Guide and Intervention Workbook</td>
<td>0-07-890848-5</td>
<td>978-0-07-890848-4</td>
</tr>
<tr>
<td>Homework Practice Workbook</td>
<td>0-07-890849-3</td>
<td>978-0-07-890849-1</td>
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Spanish Version

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Homework Practice Workbook</td>
<td>0-07-890853-1</td>
<td>978-0-07-890853-8</td>
</tr>
</tbody>
</table>

Answers for Workbooks The answers for Chapter 6 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

StudentWorks Plus™ This CD-ROM includes the entire Student Edition test along with the English workbooks listed above.

TeacherWorks Plus™ All of the materials found in this booklet are included for viewing, printing, and editing in this CD-ROM.


These masters contain a Spanish version of Chapter 6 Test Form 2A and Form 2C.
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- A24–A34
Teacher’s Guide to Using the
Chapter 6 Resource Masters

The Chapter 6 Resource Masters includes the core materials needed for Chapter 6. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing on the TeacherWorks Plus™ CD-ROM.

Chapter Resources

**Student-Built Glossary** (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to recording definitions and/or examples for each term. You may suggest that student highlight or star the terms with which they are not familiar. Give to students before beginning Lesson 6-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

**Anticipation Guide** (pages 3–4) This master presented in both English and Spanish is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

**Study Guide and Intervention** These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

**Skills Practice** This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

**Practice** This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

**Word Problem Practice** This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

**Enrichment** These activities may extend the concepts of the lesson, offer a historical or multicultural look at the concepts, or widen students’ perspectives on the mathematics they are learning. They are written for use with all levels of students.

**Graphing Calculator or Spreadsheet Activities** These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.
Assessment Options

The assessment masters in the *Chapter 6 Resource Masters* offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

**Student Recording Sheet** This master corresponds with the standardized test practice at the end of the chapter.

**Extended-Response Rubric** This master provides information for teachers and students on how to assess performance on open-ended questions.

**Quizzes** Four free-response quizzes offer assessment at appropriate intervals in the chapter.

**Mid-Chapter Test** This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

**Vocabulary Test** This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students’ knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

**Leveled Chapter Tests**
- *Form 1* contains multiple-choice questions and is intended for use with below grade level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Form 3* is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

**Extended-Response Test** Performance assessment tasks are suitable for all students. Samples answers and a scoring rubric are included for evaluation.

**Standardized Test Practice** These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

**Answers**
- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages.
- Full-size answer keys are provided for the assessment masters.
This is an alphabetical list of the key vocabulary terms you will learn in Chapter 6. As you study the chapter, complete each term’s definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>base</td>
<td></td>
<td></td>
</tr>
<tr>
<td>diagonal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>isosceles trapezoid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>legs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>midsegment of a trapezoid</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(continued on the next page)
<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>parallelogram</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rectangle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rhombus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>square</td>
<td></td>
<td></td>
</tr>
<tr>
<td>trapezoid</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Anticipation Guide
## Quadrilaterals

### Step 1

**Before you begin Chapter 6**
- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>STEP 1 A, D, or NS</th>
<th>Statement</th>
<th>STEP 2 A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>A triangle has no diagonals.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>A diagonal of a polygon is a segment joining the midpoints of two sides of the polygon.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>The sum of the measures of the angles in a polygon can be determined by subtracting 2 from the number of sides and multiplying the result by 180.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>For a quadrilateral to be a parallelogram it must have two pairs of parallel sides.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>The diagonals of a parallelogram are congruent.</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>If you know that one pair of opposite sides of a quadrilateral is both parallel and congruent, then you know the quadrilateral is a parallelogram.</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>If a quadrilateral is a rectangle, then all four angles are congruent.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>The diagonals of a rhombus are congruent.</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>The properties of a rhombus are not true for a square.</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>A trapezoid has only one pair of parallel sides.</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>The median of a trapezoid is perpendicular to the bases.</td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>An isosceles trapezoid has exactly one pair of congruent sides.</td>
<td></td>
</tr>
</tbody>
</table>

### Step 2

**After you complete Chapter 6**
- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.
## Ejercicios preparatorios

### Cuadriláteros

#### Paso 1
**Antes de comenzar el Capítulo 6**

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a).

<table>
<thead>
<tr>
<th>PASO 1</th>
<th>Enunciado</th>
<th>PASO 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, D o NS</td>
<td>1. Un triángulo no tiene diagonales.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. La diagonal de un polígono es un segmento que une los puntos medios de dos lados del polígono.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. La suma de las medidas de los ángulos de un polígono puede determinarse restando 2 del número de lados y multiplicando el resultado por 180.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. Para que un cuadrilátero sea un paralelogramo debe tener dos pares de lados paralelos.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5. Las diagonales de un paralelogramo son congruentes.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6. Si sabes que un par de lados opuestos de un cuadrilátero son paralelos y congruentes, entonces sabes que el cuadrilátero es un paralelogramo.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7. Si un cuadrilátero es un rectángulo, entonces todos los cuatro lados son congruentes.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8. Las diagonales de un rombo son congruentes.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9. Las propiedades de un rombo no son verdaderas para un cuadrado.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10. Un trapecio sólo tiene dos lados paralelos.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>11. La mediana de un trapecio es perpendicular a las bases.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>12. Un trapecio isósceles tiene exactamente un par de lados congruentes.</td>
<td></td>
</tr>
</tbody>
</table>

#### Paso 2
**Después de completar el Capítulo 6**

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.
Lesson 6-1

Study Guide and Intervention

Angles of Polygons

Polygon Interior Angles Sum  The segments that connect the nonconsecutive vertices of a polygon are called diagonals. Drawing all of the diagonals from one vertex of an \( n \)-gon separates the polygon into \( n - 2 \) triangles. The sum of the measures of the interior angles of the polygon can be found by adding the measures of the interior angles of those \( n - 2 \) triangles.

| Polygon Interior Angle Sum Theorem | The sum of the interior angle measures of an \( n \)-sided convex polygon is \( (n - 2) \cdot 180 \). |

**Example 1**  A convex polygon has 13 sides. Find the sum of the measures of the interior angles.

\[
(n - 2) \cdot 180 = (13 - 2) \cdot 180 \\
= 11 \cdot 180 \\
= 1980
\]

**Example 2**  The measure of an interior angle of a regular polygon is 120. Find the number of sides.

The number of sides is \( n \), so the sum of the measures of the interior angles is \( 120n \).

\[
120n = (n - 2) \cdot 180 \\
120n = 180n - 360 \\
-60n = -360 \\
\[ n = 6 \]

Exercises

Find the sum of the measures of the interior angles of each convex polygon.

1. decagon  
2. 16-gon  
3. 30-gon  
4. octagon  
5. 12-gon  
6. 35-gon

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

7. 150  
8. 160  
9. 175  
10. 165  
11. 144  
12. 135

13. Find the value of \( x \).
Angles of Polygons

Polygon Exterior Angles Sum There is a simple relationship among the exterior angles of a convex polygon.

| Polygon Exterior Angle Sum Theorem | The sum of the exterior angle measures of a convex polygon, one angle at each vertex, is 360. |

Example 1 Find the sum of the measures of the exterior angles, one at each vertex, of a convex 27-gon.

For any convex polygon, the sum of the measures of its exterior angles, one at each vertex, is 360.

Example 2 Find the measure of each exterior angle of regular hexagon \(ABCDEF\).

The sum of the measures of the exterior angles is 360 and a hexagon has 6 angles. If \(n\) is the measure of each exterior angle, then

\[6n = 360\]

\[n = 60\]

The measure of each exterior angle of a regular hexagon is 60.

Exercises

Find the sum of the measures of the exterior angles of each convex polygon.

1. decagon
2. 16-gon
3. 36-gon

Find the measure of each exterior angle for each regular polygon.

4. 12-gon
5. hexagon
6. 20-gon

7. 40-gon
8. heptagon
9. 12-gon

10. 24-gon
11. dodecagon
12. octagon

Chapter 6
Skills Practice

Angles of Polygons

Find the sum of the measures of the interior angles of each convex polygon.

1. nonagon
2. heptagon
3. decagon

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

4. 108
5. 120
6. 150

Find the measure of each interior angle.

7. \( A \quad B \)
   \( D \quad C \)
   \( (2x - 15)^\circ \quad x^\circ \)
   \( x^\circ \quad (2x - 15)^\circ \)

8. \( L \quad M \)
   \( P \quad N \)
   \( (2x + 20)^\circ \quad (3x - 10)^\circ \)
   \( (2x)^\circ \quad (2x - 10)^\circ \)

9. \( S \quad T \quad U \)
   \( W \)
   \( (2x + 16)^\circ \)
   \( (x + 14)^\circ \)
   \( (2x + 16)^\circ \)
   \( (x + 14)^\circ \)

10. \( D \quad E \quad F \quad G \quad H \)
    \( I \)
    \( (7x)^\circ \quad (7x)^\circ \)
    \( (4x)^\circ \quad (4x)^\circ \)
    \( (7x)^\circ \quad (7x)^\circ \)

Find the measures of each interior angle of each regular polygon.

11. quadrilateral
12. pentagon
13. dodecagon

Find the measures of each exterior angle of each regular polygon.

14. octagon
15. nonagon
16. 12-gon
6-1 Practice

Angles of Polygons

Find the sum of the measures of the interior angles of each convex polygon.

1. 11-gon  
2. 14-gon  
3. 17-gon

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

4. 144  
5. 156  
6. 160

Find the measure of each interior angle.

7. \[ (2x + 15)^\circ \] \[ (3x - 20)^\circ \]
8. \[ (6x - 4)^\circ \] \[ (2x + 8)^\circ \]

Find the measures of an exterior angle and an interior angle given the number of sides of each regular polygon. Round to the nearest tenth, if necessary.

9. 16  
10. 24  
11. 30

12. 14  
13. 22  
14. 40

15. CRYSTALLOGRAPHY Crystals are classified according to seven crystal systems. The basis of the classification is the shapes of the faces of the crystal. Turquoise belongs to the triclinic system. Each of the six faces of turquoise is in the shape of a parallelogram. Find the sum of the measures of the interior angles of one such face.
6-1 Word Problem Practice

Angles of Polygons

1. ARCHITECTURE In the Uffizi gallery in Florence, Italy, there is a room built by Buontalenti called the Tribune (La Tribuna in Italian). This room is shaped like a regular octagon. What angle do consecutive walls of the Tribune make with each other?

2. BOXES Jasmine is designing boxes she will use to ship her jewelry. She wants to shape the box like a regular polygon. In order for the boxes to pack tightly, she decides to use a regular polygon that has the property that the measure of its interior angles is half the measure of its exterior angles. What regular polygon should she use?

3. THEATER A theater floor plan is shown in the figure. The upper five sides are part of a regular dodecagon. Find $m\angle 1$.

4. ARCHEOLOGY Archeologists unearthed parts of two adjacent walls of an ancient castle.

Before it was unearthed, they knew from ancient texts that the castle was shaped like a regular polygon, but nobody knew how many sides it had. Some said 6, others 8, and some even said 100. From the information in the figure, how many sides did the castle really have?

5. POLYGON PATH In Ms. Ricketts’ math class, students made a “polygon path” that consists of regular polygons of 3, 4, 5, and 6 sides joined together as shown.

a. Find $m\angle 2$ and $m\angle 5$.

b. Find $m\angle 3$ and $m\angle 4$.

c. What is $m\angle 1$?
Central Angles of Regular Polygons
You have learned about the interior and exterior angles of a polygon. Regular polygons also have **central angles**. A central angle is measured from the center of the polygon.

The center of a polygon is the point equidistant from all of the vertices of the polygon, just as the center of a circle is the point equidistant from all of the points on the circle. The central angle is the angle drawn with the vertex at the center of the circle and the sides of angle drawn through consecutive vertices of the polygon. One of the central angles that can be drawn in this regular hexagon is \( \angle APB \).

You may remember from making circle graphs that there are 360° around the center of a circle.

1. By using logic or by drawing sketches, find the measure of the central angle of each regular polygon.

![Diagram of a regular hexagon with central angle \( \angle APB \)]

2. Make a conjecture about how the measure of a central angle of a regular polygon relates to the measures of the interior angles and exterior angles of a regular polygon.

3. **CHALLENGE** In obtuse \( \triangle ABC \), \( BC \) is the longest side. \( AC \) is also a side of a 21-sided regular polygon. \( AB \) is also a side of a 28-sided regular polygon. The 21-sided regular polygon and the 28-sided regular polygon have the same center point \( P \). Find \( n \) if \( BC \) is a side of a \( n \)-sided regular polygon that has center point \( P \).
   
   \( \text{(Hint: Sketch a circle with center } P \text{ and place points } A, B, \text{ and } C \text{ on the circle.)} \)
Lesson 6-2

Study Guide and Intervention

Parallelograms

Sides and Angles of Parallelograms A quadrilateral with both pairs of opposite sides parallel is a parallelogram. Here are four important properties of parallelograms.

<table>
<thead>
<tr>
<th>Property</th>
<th>If PQRS is a parallelogram, then</th>
</tr>
</thead>
<tbody>
<tr>
<td>If a quadrilateral is a parallelogram, then its opposite sides are congruent.</td>
<td>( PQ \equiv SR ) and ( PS \equiv QR )</td>
</tr>
<tr>
<td>If a quadrilateral is a parallelogram, then its opposite angles are congruent.</td>
<td>( \angle P \equiv \angle R ) and ( \angle S \equiv \angle Q )</td>
</tr>
<tr>
<td>If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.</td>
<td>( \angle P ) and ( \angle S ) are supplementary; ( \angle S ) and ( \angle R ) are supplementary; ( \angle R ) and ( \angle Q ) are supplementary; ( \angle Q ) and ( \angle P ) are supplementary.</td>
</tr>
<tr>
<td>If a parallelogram has one right angle, then it has four right angles.</td>
<td>If ( m\angle P = 90 ), then ( m\angle Q = 90 ), ( m\angle R = 90 ), and ( m\angle S = 90 ).</td>
</tr>
</tbody>
</table>

Example If \( ABCD \) is a parallelogram, find the value of each variable.

\( AB \) and \( CD \) are opposite sides, so \( AB \equiv CD \).

\[ 2a = 34 \]
\[ a = 17 \]

\( \angle A \) and \( \angle C \) are opposite angles, so \( \angle A \equiv \angle C \).

\[ 8b = 112 \]
\[ b = 14 \]

Exercises

Find the value of each variable.

1. \[ \begin{array}{c}
\text{3x°} \\
\text{4y°}
\end{array} \]

2. \[ \begin{array}{c}
\text{8y°} \\
\text{6x°}
\end{array} \]

3. \[ \begin{array}{c}
\text{6x} \\
\text{3y} \\
\text{12}
\end{array} \]

4. \[ \begin{array}{c}
\text{3y°} \\
\text{6x°} \\
\text{12x°}
\end{array} \]

5. \[ \begin{array}{c}
\text{55°} \\
\text{5x°} \\
\text{60°} \\
\text{2y°}
\end{array} \]

6. \[ \begin{array}{c}
\text{2y°} \\
\text{30x} \\
\text{150}
\end{array} \]
**Parallelograms**

Two important properties of parallelograms deal with their diagonals.

<table>
<thead>
<tr>
<th>If (ABCD) is a parallelogram, then</th>
</tr>
</thead>
<tbody>
<tr>
<td>If a quadrilateral is a parallelogram, then its diagonals bisect each other.</td>
</tr>
<tr>
<td>If a quadrilateral is a parallelogram, then each diagonal separates the parallelogram into two congruent triangles.</td>
</tr>
</tbody>
</table>

**Example**

Find the value of \(x\) and \(y\) in parallelogram \(ABCD\).

The diagonals bisect each other, so \(AE = CE\) and \(DE = BE\).

\[
6x = 24 \quad 4y = 18
\]

\[
x = 4 \quad y = 4.5
\]

**Exercises**

Find the value of each variable.

1. \[
\begin{array}{c}
3x \\
4y \\
12 \\
8
\end{array}
\]

2. \[
\begin{array}{c}
28 \\
4x \\
12 \\
2y
\end{array}
\]

3. \[
\begin{array}{c}
60^\circ \\
2x \\
4y
\end{array}
\]

4. \[
\begin{array}{c}
30^\circ \\
10 \\
y \\
3x
\end{array}
\]

5. \[
\begin{array}{c}
12 \\
3x \\
2y
\end{array}
\]

6. \[
\begin{array}{c}
4 \\
y \\
17
\end{array}
\]

**COORDINATE GEOMETRY** Find the coordinates of the intersection of the diagonals of \(\square ABCD\) with the given vertices.

7. \(A(3, 6), B(5, 8), C(3, -2),\) and \(D(1, -4)\)

8. \(A(-4, 3), B(2, 3), C(-1, -2),\) and \(D(-7, -2)\)

9. **PROOF** Write a paragraph proof of the following.

   **Given:** \(\square ABCD\)
   **Prove:** \(\triangle AED \cong \triangle BEC\)
**Skills Practice**

**Parallelograms**

**ALGEBRA** Find the value of each variable.

1. \[ R \]

   \[ 2b - 1 \]

   \[ 2a \]

   \[ S \]

   \[ b + 3 \]

   \[ U \]

   \[ 3a - 5 \]

   \[ T \]

   \[ 1. \]

2. \[ J \]

   \[ x^\circ \]

   \[ 44^\circ \]

   \[ K \]

   \[ y^\circ \]

   \[ M \]

   \[ y \]

   \[ L \]

   \[ 2. \]

3. \[ G \]

   \[ 26 \]

   \[ 19 \]

   \[ F \]

   \[ x - 2 \]

   \[ y + 1 \]

   \[ D \]

   \[ E \]

   \[ 3. \]

4. \[ W \]

   \[ 9b + 8 \]

   \[ Z \]

   \[ a + 14 \]

   \[ X \]

   \[ 10b + 1 \]

   \[ Y \]

   \[ 4. \]

5. \[ D \]

   \[ (x + 8)^\circ \]

   \[ A \]

   \[ (y + 9)^\circ \]

   \[ C \]

   \[ (3x)^\circ \]

   \[ B \]

   \[ 5. \]

6. \[ O \]

   \[ x + 5 \]

   \[ P \]

   \[ x + 10 \]

   \[ A - 3 \]

   \[ 3y - 1 \]

   \[ N \]

   \[ Q \]

   \[ 6. \]

**COORDINATE GEOMETRY** Find the coordinates of the intersection of the diagonals of \( \square HJKL \) with the given vertices.

7. \( H(1, 1), J(2, 3), K(6, 3), L(5, 1) \)

8. \( H(-1, 4), J(3, 3), K(3, -2), L(-1, -1) \)

9. **PROOF** Write a paragraph proof of the theorem *Consecutive angles in a parallelogram are supplementary.*
**6-2 Practice**

**Parallelograms**

**ALGEBRA** Find the value of each variable.

1. \[ b+1 \]

2. \[ (y+10)^\circ \]

3. \[ \frac{x+2}{5} = 15 \]

4. \[ \frac{3y-4}{5} = 12 \]

**ALGEBRA** Use \( \square RSTU \) to find each measure or value.

5. \( m \angle RST = \) ________

6. \( m \angle STU = \) ________

7. \( m \angle TUR = \) ________

8. \( b = \) ________

**COORDINATE GEOMETRY** Find the coordinates of the intersection of the diagonals of \( \square PRYZ \) with the given vertices.

9. \( P(2, 5), R(3, 3), Y(-2, -3), Z(-3, -1) \)

10. \( P(2, 3), R(1, -2), Y(-5, -7), Z(-4, -2) \)

11. **PROOF** Write a paragraph proof of the following.

   **Given:** \( \square PRST \) and \( \square PQVU \)

   **Prove:** \( \angle V \cong \angle S \)

12. **CONSTRUCTION** Mr. Rodriguez used the parallelogram at the right to design a herringbone pattern for a paving stone. He will use the paving stone for a sidewalk. If \( m \angle 1 \) is 130, find \( m \angle 2, m \angle 3, \) and \( m \angle 4. \)
Word Problem Practice

Parallelograms

1. WALKWAY A walkway is made by adjoining four parallelograms as shown.

Are the end segments \( a \) and \( e \) parallel to each other? Explain.

2. DISTANCE Four friends live at the four corners of a block shaped like a parallelogram. Gracie lives 3 miles away from Kenny. How far apart do Teresa and Travis live from each other?

3. SOCCER Four soccer players are located at the corners of a parallelogram. Two of the players in opposite corners are the goalies. In order for goalie A to be able to see the three others, she must be able to see a certain angle \( x \) in her field of vision.

What angle does the other goalie have to be able to see in order to keep an eye on the other three players?

4. VENN DIAGRAMS Make a Venn diagram showing the relationship between squares, rectangles, and parallelograms.

5. SKYSCRAPERS On vacation, Tony’s family took a helicopter tour of the city. The pilot said the newest building in the city was the building with this top view. He told Tony that the exterior angle by the front entrance is 72°. Tony wanted to know more about the building, so he drew this diagram and used his geometry skills to learn a few more things. The front entrance is next to vertex \( B \).

a. What are the measures of the four angles of the parallelogram?

b. How many pairs of congruent triangles are there in the figure? What are they?
Diagonals of Parallelograms

In some drawings the diagonal of a parallelogram appears to be the angle bisector of both opposite angles. When might that be true?

1. **Given**: Parallelogram $PQRS$ with diagonal $PR$. $PR$ is an angle bisector of $\angle QPS$ and $\angle QRS$.

   What type of parallelogram is $PQRS$? Justify your answer.

2. **Given**: Parallelogram $WPRK$ with angle bisector $KD$, $DP = 5$, and $WD = 7$.

   Find $WK$ and $KR$.

3. Refer to Exercise 2. Write a statement about parallelogram $WPRK$ and angle bisector $KD$.

4. **Given**: Parallelogram $ABCD$ with diagonal $BD$ and angle bisector $BP$. $PD = 5$, $BP = 6$, and $CP = 6$.

   The perimeter of triangle $PCD$ is 15. Find $AB$ and $BC$. 
Conditions for Parallelograms There are many ways to establish that a quadrilateral is a parallelogram.

<table>
<thead>
<tr>
<th>If:</th>
<th>If:</th>
</tr>
</thead>
<tbody>
<tr>
<td>both pairs of opposite sides are parallel,</td>
<td>$AB \parallel DC$ and $AD \parallel BC$,</td>
</tr>
<tr>
<td>both pairs of opposite sides are congruent,</td>
<td>$AB \cong DC$ and $AD \cong BC$,</td>
</tr>
<tr>
<td>both pairs of opposite angles are congruent,</td>
<td>$\angle ABC \cong \angle ADC$ and $\angle DAB \cong \angle BCD$,</td>
</tr>
<tr>
<td>the diagonals bisect each other,</td>
<td>$AE \cong CE$ and $DE \cong BE$,</td>
</tr>
<tr>
<td>one pair of opposite sides is congruent and parallel,</td>
<td>$AB \parallel CD$ and $AB \cong CD$, or $AD \parallel BC$ and $AD \cong BC$,</td>
</tr>
<tr>
<td>then: the figure is a parallelogram.</td>
<td>then: $ABCD$ is a parallelogram.</td>
</tr>
</tbody>
</table>

**Example** Find $x$ and $y$ so that $FGHJ$ is a parallelogram.

$FGHJ$ is a parallelogram if the lengths of the opposite sides are equal.

$$
4x - 2y = 2 \\
4(2) - 2y = 2 \\
8 - 2y = 2 \\
-2y = -6 \\
y = 3
$$

**Exercises**

Find $x$ and $y$ so that the quadrilateral is a parallelogram.

1. \[ 2x - 2 = 8 \]
2. \[ 5y^\circ = 55^\circ \]
3. \[ 5y^\circ = 25^\circ \]
4. \[ 9x^\circ = 6y \]
5. \[ (x + y)^\circ = 30^\circ \]
6. \[ 6y^\circ = 3x^\circ \]
Study Guide and Intervention
(continued)

Tests for Parallelograms

Parallelograms on the Coordinate Plane On the coordinate plane, the Distance,
Slope, and Midpoint Formulas can be used to test if a quadrilateral is a parallelogram.

Example Determine whether \(ABCD\) is a parallelogram.

The vertices are \(A(-2, 3), B(3, 2), C(2, -1),\) and \(D(-3, 0)\).

**Method 1:** Use the Slope Formula, \(m = \frac{y_2 - y_1}{x_2 - x_1}\).

- slope of \(\overline{AD} = \frac{3 - 0}{-2 - (-3)} = \frac{3}{1} = 3\)
- slope of \(\overline{BC} = \frac{2 - (-1)}{3 - 2} = \frac{3}{1} = 3\)
- slope of \(\overline{AB} = \frac{2 - 3}{3 - (-2)} = \frac{1}{5}\)
- slope of \(\overline{CD} = \frac{-1 - 0}{2 - (-3)} = \frac{1}{5}\)

Since opposite sides have the same slope, \(\overline{AB} || \overline{CD}\) and \(\overline{AD} || \overline{BC}\). Therefore, \(ABCD\) is a parallelogram by definition.

**Method 2:** Use the Distance Formula, \(d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\).

- \(AB = \sqrt{(-2 - 3)^2 + (3 - 2)^2} = \sqrt{25 + 1}\) or \(\sqrt{26}\)
- \(CD = \sqrt{(2 - (-3))^2 + (-1 - 0)^2} = \sqrt{25 + 1}\) or \(\sqrt{26}\)
- \(AD = \sqrt{(-2 - (-3))^2 + (3 - 0)^2} = \sqrt{1 + 9}\) or \(\sqrt{10}\)
- \(BC = \sqrt{(3 - 2)^2 + (2 - (-1))^2} = \sqrt{1 + 9}\) or \(\sqrt{10}\)

Since both pairs of opposite sides have the same length, \(\overline{AB} \cong \overline{CD}\) and \(\overline{AD} \cong \overline{BC}\). Therefore, \(ABCD\) is a parallelogram by Theorem 6.9.

Exercises

Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

1. \(A(0, 0), B(1, 3), C(5, 3), D(4, 0)\); Slope Formula

2. \(D(-1, 1), E(2, 4), F(6, 4), G(3, 1)\); Slope Formula

3. \(R(-1, 0), S(3, 0), T(2, -3), U(-3, -2)\); Distance Formula

4. \(A(-3, 2), B(-1, 4), C(2, 1), D(0, -1)\); Distance and Slope Formulas

5. \(S(-2, 4), T(-1, -1), U(3, -4), V(2, 1)\); Distance and Slope Formulas

6. \(F(3, 3), G(1, 2), H(-3, 1), I(-1, 4)\); Midpoint Formula

7. A parallelogram has vertices \(R(-2, -1), S(2, 1),\) and \(T(0, -3)\). Find all possible coordinates for the fourth vertex.
6-3 Skills Practice
Tests for Parallelograms

Determine whether each quadrilateral is a parallelogram. Justify your answer.

1. 

2. 

3. 

4. 

COORDINATE GEOMETRY
Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

5. P(0, 0), Q(3, 4), S(7, 4), Y(4, 0); Slope Formula

6. S(−2, 1), R(1, 3), T(2, 0), Z(−1, −2); Distance and Slope Formulas

7. W(2, 5), R(3, 3), Y(−2, −3), N(−3, 1); Midpoint Formula

ALGEBRA
Find x and y so that each quadrilateral is a parallelogram.

8. 

9. 

10. 

11. 


6-3 Practice

Tests for Parallelograms

Determine whether each quadrilateral is a parallelogram. Justify your answer.

1.  

2.  

3.  

4.  

COORDINATE GEOMETRY  Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

5.  \(P(-5, 1), S(-2, 2), F(-1, -3), T(2, -2); \) Slope Formula

6.  \(R(-2, 5), O(1, 3), M(-3, -4), Y(-6, -2); \) Distance and Slope Formulas

ALGEBRA  Find \(x\) and \(y\) so that the quadrilateral is a parallelogram.

7.  

8.  

9.  

10.  

11.  TILE DESIGN  The pattern shown in the figure is to consist of congruent parallelograms. How can the designer be certain that the shapes are parallelograms?
1. **BALANCING** Nikia, Madison, Angela, and Shelby are balancing themselves on an “X”-shaped floating object. To balance themselves, they want to make themselves the vertices of a parallelogram.

In order to achieve this, do all four of them have to be the same distance from the center of the object? Explain.

2. **COMPASSES** Two compass needles placed side by side on a table are both 2 inches long and point due north. Do they form the sides of a parallelogram?

3. **FORMATION** Four jets are flying in formation. Three of the jets are shown in the graph. If the four jets are located at the vertices of a parallelogram, what are the three possible locations of the missing jet?

4. **STREET LAMPS** When a coordinate plane is placed over the Harrisville town map, the four street lamps in the center are located as shown. Do the four lamps form the vertices of a parallelogram? Explain.

5. **PICTURE FRAME** Aaron is making a wooden picture frame in the shape of a parallelogram. He has two pieces of wood that are 3 feet long and two that are 4 feet long.

   a. If he connects the pieces of wood at their ends to each other, in what order must he connect them to make a parallelogram?

   b. How many different parallelograms could he make with these four lengths of wood?

   c. Explain something Aaron might do to specify precisely the shape of the parallelogram.
Tests for Parallelograms

By definition, a quadrilateral is a parallelogram if and only if both pairs of opposite sides are parallel. What conditions other than both pairs of opposite sides parallel will guarantee that a quadrilateral is a parallelogram? In this activity, several possibilities will be investigated by drawing quadrilaterals to satisfy certain conditions. Remember that any test that seems to work is not guaranteed to work unless it can be formally proven.

Complete.

1. Draw a quadrilateral with one pair of opposite sides congruent. Must it be a parallelogram?

2. Draw a quadrilateral with both pairs of opposite sides congruent. Must it be a parallelogram?

3. Draw a quadrilateral with one pair of opposite sides parallel and the other pair of opposite sides congruent. Must it be a parallelogram?

4. Draw a quadrilateral with one pair of opposite sides both parallel and congruent. Must it be a parallelogram?

5. Draw a quadrilateral with one pair of opposite angles congruent. Must it be a parallelogram?

6. Draw a quadrilateral with both pairs of opposite angles congruent. Must it be a parallelogram?

7. Draw a quadrilateral with one pair of opposite sides parallel and one pair of opposite angles congruent. Must it be a parallelogram?
Rectangles

Properties of Rectangles  A rectangle is a quadrilateral with four right angles. Here are the properties of rectangles.

A rectangle has all the properties of a parallelogram.

- Opposite sides are parallel.
- Opposite angles are congruent.
- Opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Also:

- All four angles are right angles.  \( \angle UTS, \angle TSR, \angle SRU, \text{ and } \angle RUT \) are right angles.
- The diagonals are congruent.  \( TR \cong US \)

Example 1  Quadrilateral \( RUTS \) above is a rectangle. If \( US = 6x + 3 \) and \( RT = 7x - 2 \), find \( x \).

The diagonals of a rectangle are congruent, so \( US = RT \).

\[ 6x + 3 = 7x - 2 \]
\[ 3 = x - 2 \]
\[ 5 = x \]

Example 2  Quadrilateral \( RUTS \) above is a rectangle. If \( m\angle STR = 8x + 3 \) and \( m\angle UTR = 16x - 9 \), find \( m\angle STR \).

\( \angle UTS \) is a right angle, so  
\[ m\angle STR + m\angle UTR = 90. \]

\[ 8x + 3 + 16x - 9 = 90 \]
\[ 24x - 6 = 90 \]
\[ 24x = 96 \]
\[ x = 4 \]

\[ m\angle STR = 8x + 3 = 8(4) + 3 \text{ or } 35 \]

Exercises

Quadrilateral \( ABCD \) is a rectangle.

1. If \( AE = 36 \) and \( CE = 2x - 4 \), find \( x \).

2. If \( BE = 6y + 2 \) and \( CE = 4y + 6 \), find \( y \).

3. If \( BC = 24 \) and \( AD = 5y - 1 \), find \( y \).

4. If \( m\angle BEA = 62 \), find \( m\angle BAC \).

5. If \( m\angle AED = 12x \) and \( m\angle BEC = 10x + 20 \), find \( m\angle AED \).

6. If \( BD = 8y - 4 \) and \( AC = 7y + 3 \), find \( BD \).

7. If \( m\angle DBC = 10x \) and \( m\angle ACB = 4x^2 - 6 \), find \( m\angle ACB \).

8. If \( AB = 6y \) and \( BC = 8y \), find \( BD \) in terms of \( y \).
6-4 Study Guide and Intervention (continued)

Rectangles

Prove that Parallelograms Are Rectangles The diagonals of a rectangle are congruent, and the converse is also true.

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

In the coordinate plane you can use the Distance Formula, the Slope Formula, and properties of diagonals to show that a figure is a rectangle.

Example Quadrilateral ABCD has vertices A(−3, 0), B(−2, 3), C(4, 1), and D(3, −2). Determine whether ABCD is a rectangle.

Method 1: Use the Slope Formula.

slope of \( \overline{AB} \) = \( \frac{3 - 0}{-2 - (-3)} \) = \( \frac{3}{1} \) or 3 

slope of \( \overline{CD} \) = \( \frac{2 - 1}{3 - 4} \) = \( -\frac{1}{3} \) or 3

Opposite sides are parallel, so the figure is a parallelogram. Consecutive sides are perpendicular, so \( \text{ABCD} \) is a rectangle.

Method 2: Use the Distance Formula.

\( AB = \sqrt{(-3 - (-2))^2 + (0 - 3)^2} \) or \( \sqrt{10} \) 

\( CD = \sqrt{(4 - 3)^2 + (1 - (-2))^2} \) or \( \sqrt{10} \) 

Opposite sides are congruent, thus \( \text{ABCD} \) is a parallelogram.

\( AC = \sqrt{(-3 - 4)^2 + (0 - 1)^2} \) or \( \sqrt{50} \) 

\( BD = \sqrt{(-2 - 3)^2 + (3 - (-2))^2} \) or \( \sqrt{50} \)

\( \text{ABCD} \) is a parallelogram with congruent diagonals, so \( \text{ABCD} \) is a rectangle.

Exercises

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

1. A(−3, 1), B(−3, 3), C(3, 3), D(3, 1); Distance Formula

2. A(−3, 0), B(−2, 3), C(4, 5), D(3, 2); Slope Formula

3. A(−3, 0), B(−2, 2), C(3, 0), D(2, −2); Distance Formula

4. A(−1, 0), B(0, 2), C(4, 0), D(3, −2); Distance Formula
6-4 Skills Practice
Rectangles

ALGEBRA Quadrilateral $ABCD$ is a rectangle.

1. If $AC = 2x + 13$ and $DB = 4x - 1$, find $DB$.

2. If $AC = x + 3$ and $DB = 3x - 19$, find $AC$.

3. If $AE = 3x + 3$ and $EC = 5x - 15$, find $AC$.

4. If $DE = 6x - 7$ and $AE = 4x + 9$, find $DB$.

5. If $m \angle DAC = 2x + 4$ and $m \angle BAC = 3x + 1$, find $m \angle BAC$.

6. If $m \angle BDC = 7x + 1$ and $m \angle ADB = 9x - 7$, find $m \angle BDC$.

7. If $m \angle ABD = 7x - 31$ and $m \angle CDB = 4x + 5$, find $m \angle ABD$.

8. If $m \angle BAC = x + 3$ and $m \angle CAD = x + 15$, find $m \angle BAC$.

9. PROOF: Write a two-column proof.

   Given: $RSTV$ is a rectangle and $U$ is the midpoint of $VT$.
   Prove: $\triangle RUV \cong \triangle SUT$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RU = SU$</td>
<td>Side-Side-Side (SSS) Congruence</td>
</tr>
<tr>
<td>$\angle RUV = \angle SUT$</td>
<td>Angle-Angle-Side (AAS) Congruence</td>
</tr>
<tr>
<td>$\angle UV = \angle ST$</td>
<td>Angle-Side-Angle (ASA) Congruence</td>
</tr>
</tbody>
</table>

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

10. $P(-3, -2), Q(-4, 2), R(2, 4), S(3, 0)$; Slope Formula

11. $J(-6, 3), K(0, 6), L(2, 2), M(-4, -1)$; Distance Formula

12. $T(4, 1), U(3, -1), X(-3, 2), Y(-2, 4)$; Distance Formula
6-4 Practice

Rectangles

ALGEBRA Quadrilateral $RSTU$ is a rectangle.

1. If $UZ = x + 21$ and $ZS = 3x - 15$, find $US$.

2. If $RZ = 3x + 8$ and $ZS = 6x - 28$, find $UZ$.

3. If $RT = 5x + 8$ and $RZ = 4x + 1$, find $ZT$.

4. If $m\angle SUT = 3x + 6$ and $m\angle RUS = 5x - 4$, find $m\angle SUT$.

5. If $m\angle SRT = x + 9$ and $m\angle UTR = 2x - 44$, find $m\angle UTR$.

6. If $m\angle RSU = x + 41$ and $m\angle TUS = 3x + 9$, find $m\angle RSU$.

Quadrilateral $GHJK$ is a rectangle. Find each measure if $m\angle 1 = 37$.

7. $m\angle 2$

8. $m\angle 3$

9. $m\angle 4$

10. $m\angle 5$

11. $m\angle 6$

12. $m\angle 7$

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

13. $B(-4, 3), G(-2, 4), H(1, -2), L(-1, -3)$; Slope Formula

14. $N(-4, 5), O(6, 0), P(3, -6), Q(-7, -1)$; Distance Formula

15. $C(0, 5), D(4, 7), E(5, 4), F(1, 2)$; Slope Formula

16. LANDSCAPING Huntington Park officials approved a rectangular plot of land for a Japanese Zen garden. Is it sufficient to know that opposite sides of the garden plot are congruent and parallel to determine that the garden plot is rectangular? Explain.
6-4 Word Problem Practice

Rectangles

1. FRAMES Jalen makes the rectangular frame shown.

![Diagram of a rectangle]

In order to make sure that it is a rectangle, Jalen measures the distances BD and AC. How should these two distances compare if the frame is a rectangle?

2. BOOKSHELVES A bookshelf consists of two vertical planks with five horizontal shelves. Are each of the four sections for books rectangles? Explain.

3. LANDSCAPING A landscaper is marking off the corners of a rectangular plot of land. Three of the corners are in place as shown.

![Diagram of a coordinate grid]

What are the coordinates of the fourth corner?

4. SWIMMING POOLS Antonio is designing a swimming pool on a coordinate grid. Is it a rectangle? Explain.

![Diagram of a swimming pool]

5. PATTERNS Veronica made the pattern shown out of 7 rectangles with four equal sides. The side length of each rectangle is written inside the rectangle.

![Diagram of a pattern]

a. How many rectangles can be formed using the lines in this figure?

b. If Veronica wanted to extend her pattern by adding another rectangle with 4 equal sides to make a larger rectangle, what are the possible side lengths of rectangles that she can add?
Constant Perimeter

Douglas wants to fence a rectangular region of his back yard for his dog. He bought 200 feet of fence.

1. Complete the table to show the dimensions of five different rectangular pens that would use the entire 200 feet of fence. Then find the area of each rectangular pen.

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>Length</th>
<th>Width</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>45</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Do all five of the rectangular pens have the same area? If not, which one has the larger area?

3. Write a rule for finding the dimensions of a rectangle with the largest possible area for a given perimeter.

4. Let \( x \) represent the length of a rectangle and \( y \) the width. Write the formula for all rectangles with a perimeter of 200. Then graph this relationship on the coordinate plane at the right.

Julio read that a dog the size of his new pet, Bennie, should have at least 100 square feet in his pen. Before going to the store to buy fence, Julio made a table to determine the dimensions for Bennie's rectangular pen.

5. Complete the table to find five possible dimensions of a rectangular fenced area of 100 square feet.

<table>
<thead>
<tr>
<th>Area</th>
<th>Length</th>
<th>Width</th>
<th>How much fence to buy</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Julio wants to save money by purchasing the least number feet of fencing to enclose the 100 square feet. What will be the dimensions of the completed pen?

7. Write a rule for finding the dimensions of a rectangle with the least possible perimeter for a given area.

8. For length \( x \) and width \( y \), write a formula for the area of a rectangle with an area of 100 square feet. Then graph the formula.
6-4 Graphing Calculator Activity

TI-Nspire: Exploring Rectangles

A quadrilateral with four right angles is a rectangle. The TI-Nspire can be used to explore some of the characteristics of a rectangle. Use the following steps to draw a rectangle.

**Step 1** Set up the calculator in the correct mode.
- Choose *Graphs & Geometry* from the Home Menu.
- From the View menu, choose 4: *Hide Axis*

**Step 2** Draw the rectangle
- From the 8: *Shapes* menu choose 3: *Rectangle*.
- Click once to define the corner of the rectangle. Then move and click again. The side of the rectangle is now defined. Move perpendicularly to draw the rectangle. Click to anchor the shape.

**Step 3** Measure the lengths of the sides of the rectangle.
- From the 7: *Measurement* menu choose 1: *Length* (Note that when you scroll over the rectangle, the value now shown is the perimeter of the rectangle.)
- Select each endpoint of a segment of the rectangle. Then click or press Enter to anchor the length of the segment in the work area.
- Repeat for the other sides of the rectangle.

**Exercises**

1. What appears to be true about the opposite sides of the rectangle?

2. Draw the diagonals of the rectangle using 5: *Segment* from the 6: *Points and Lines* Menu. Click on two opposite vertices to draw the diagonal. Repeat to draw the other diagonal.
   a. Measure each diagonal using the measurement tool. What do you observe?
   b. What is true about the triangles formed by the sides of the rectangle and a diagonal? Justify your conclusion.

3. Press Clear three times and select Yes to clear the screen. Repeat the steps and draw another rectangle. Do the relationships that you found for the first rectangle you drew hold true for this rectangle?
6-4 Geometer’s Sketchpad Activity

Exploring Rectangles

A quadrilateral with four right angles is a rectangle. The Geometer’s Sketchpad is a useful tool for exploring some of the characteristics of a rectangle. Use the following steps to draw a rectangle.

**Step 1**  Use the Line tool to draw a line anywhere on the screen.

**Step 2**  Use the Point tool to draw a point that is not on the line. To draw a line perpendicular to the first line you drew, select the first line and the point. Then choose **Perpendicular Line** from the **Construct** menu.

**Step 3**  Use the Point tool to draw a point that is not on either of the lines you have drawn. Repeat the procedure in Step 2 to draw lines perpendicular to the two lines you have drawn.

A rectangle is formed by the segments whose endpoints are the points of intersection of the lines.

**Exercises**

Use the measuring capabilities of The Geometer's Sketchpad to explore the characteristics of a rectangle.

1. What appears to be true about the opposite sides of the rectangle that you drew? Make a conjecture and then measure each side to check your conjecture.

2. Draw the diagonals of the rectangle by using the Selection Arrow tool to choose two opposite vertices. Then choose **Segment** from the **Construct** menu to draw the diagonal. Repeat to draw the other diagonal.
   
   **a.** Measure each diagonal. What do you observe?

   **b.** What is true about the triangles formed by the sides of the rectangle and a diagonal? Justify your conclusion.

3. Choose **New Sketch** from the **File** menu and follow steps 1–3 to draw another rectangle. Do the relationships you found for the first rectangle you drew hold true for this rectangle also?
Rhombi and Squares

Properties of Rhombi and Squares A **rhombus** is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.

<table>
<thead>
<tr>
<th>The diagonals are perpendicular.</th>
<th>$MH \perp RO$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Each diagonal bisects a pair of opposite angles.</td>
<td>$MH$ bisects $\angle RMO$ and $\angle RHO$. $RO$ bisects $\angle MRH$ and $\angle MOH$.</td>
</tr>
</tbody>
</table>

A **square** is a parallelogram with four congruent sides and four congruent angles. A square is both a rectangle and a rhombus; therefore, all properties of parallelograms, rectangles, and rhombi apply to squares.

**Example** In rhombus $ABCD$, $m\angle BAC = 32$. Find the measure of each numbered angle.

$ABCD$ is a rhombus, so the diagonals are perpendicular and $\triangle ABE$ is a right triangle. Thus $m\angle 4 = 90$ and $m\angle 1 = 90 - 32$ or 58. The diagonals in a rhombus bisect the vertex angles, so $m\angle 1 = m\angle 2$. Thus, $m\angle 2 = 58$.

A rhombus is a parallelogram, so the opposite sides are parallel. $\angle BAC$ and $\angle 3$ are alternate interior angles for parallel lines, so $m\angle 3 = 32$.

**Exercises** Quadrilateral $ABCD$ is a rhombus. Find each value or measure.

1. If $m\angle ABD = 60$, find $m\angle BDC$.
2. If $AE = 8$, find $AC$.
3. If $AB = 26$ and $BD = 20$, find $AE$.
4. Find $m\angle CEB$.
5. If $m\angle CBD = 58$, find $m\angle ACB$.
6. If $AE = 3x - 1$ and $AC = 16$, find $x$.
7. If $m\angle CDB = 6y$ and $m\angle ACB = 2y + 10$, find $y$.
8. If $AD = 2x + 4$ and $CD = 4x - 4$, find $x$. 
Rhombi and Squares

Conditions for Rhombi and Squares  The theorems below can help you prove that a parallelogram is a rectangle, rhombus, or square.

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.

If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.

If a quadrilateral is both a rectangle and a rhombus, then it is a square.

Determine whether parallelogram $ABCD$ with vertices $A(-3, -3), B(1, 1), C(5, -3), D(1, -7)$ is a rhombus, a rectangle, or a square.

$AC = \sqrt{(-3 - 5)^2 + ((-3 - (-3))^2} = \sqrt{64} = 8$

$BD = \sqrt{(1-1)^2 + (-7-1)^2} = \sqrt{64} = 8$

The diagonals are the same length; the figure is a rectangle.

Slope of $\overline{AC} = \frac{-3 - (-3)}{-3 - 5} = \frac{0}{-8} = 0$  The line is horizontal.

Slope of $\overline{BD} = \frac{1 - (-7)}{1-1} = \frac{8}{0} = undefined$  The line is vertical.

Since a horizontal and vertical line are perpendicular, the diagonals are perpendicular. Parallelogram $ABCD$ is a square which is also a rhombus and a rectangle.

Exercises

Given each set of vertices, determine whether $\square ABCD$ is a rhombus, rectangle, or square. List all that apply. Explain.

1. $A(0, 2), B(2, 4), C(4, 2), D(2, 0)$  
2. $A(-2, 1), B(-1, 3), C(3, 1), D(2, -1)$

3. $A(-2, -1), B(0, 2), C(2, -1), D(0, -4)$  
4. $A(-3, 0), B(-1, 3), C(5, -1), D(3, -4)$

5. PROOF  Write a two-column proof.
   Given: Parallelogram $RSTU. RS \cong ST$
   Prove: $RSTU$ is a rhombus.
6-5 Skills Practice

Rhombi and Squares

ALGEBRA Quadrilateral $DKLM$ is a rhombus.

1. If $DK = 8$, find $KL$.

2. If $m \angle DML = 82$ find $m \angle DKM$.

3. If $m \angle KAL = 2x - 8$, find $x$.

4. If $DA = 4x$ and $AL = 5x - 3$, find $DL$.

5. If $DA = 4x$ and $AL = 5x - 3$, find $AD$.

6. If $DM = 5y + 2$ and $DK = 3y + 6$, find $KL$.

7. PROOF Write a two-column proof.
   Given: $RSTU$ is a parallelogram.
   $RX \cong TX \cong SX \cong UX$
   Prove: $RSTU$ is a rectangle.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RX \cong TX \cong SX \cong UX$</td>
<td>Given</td>
</tr>
<tr>
<td>$RX \cong TX \cong SX \cong UX$</td>
<td>$RSTU$ is a parallelogram</td>
</tr>
<tr>
<td>$RX \cong TX \cong SX \cong UX$</td>
<td>$RSTU$ is a rectangle</td>
</tr>
</tbody>
</table>

COORDINATE GEOMETRY Given each set of vertices, determine whether $\square QRST$ is a rhombus, a rectangle, or a square. List all that apply. Explain.

8. $Q(3, 5), R(3, 1), S(-1, 1), T(-1, 5)$

9. $Q(-5, 12), R(5, 12), S(-1, 4), T(-11, 4)$

10. $Q(-6, -1), R(4, -6), S(2, 5), T(-8, 10)$

11. $Q(2, -4), R(-6, -8), S(-10, 2), T(-2, 6)$
**6-5 Practice**

**Rhombi and Squares**

PRYZ is a rhombus. If RK = 5, RY = 13 and \(m\angle YRZ = 67\), find each measure.

1. \(KY\)
2. \(PK\)
3. \(m\angle YKZ\)
4. \(m\angle PZR\)

MNPQ is a rhombus. If \(PQ = 3\sqrt{2}\) and \(AP = 3\), find each measure.

5. \(AQ\)
6. \(m\angle APQ\)
7. \(m\angle MNP\)
8. \(PM\)

**COORDINATE GEOMETRY** Given each set of vertices, determine whether \(\square BEFG\) is a rhombus, a rectangle, or a square. List all that apply. Explain.

9. \(B(-9, 1), E(2, 3), F(12, -2), G(1, -4)\)

10. \(B(1, 3), E(7, -3), F(1, -9), G(-5, -3)\)

11. \(B(-4, -5), E(1, -5), F(-2, -1), G(-7, -1)\)

12. **TESELLATIONS** The figure is an example of a tessellation. Use a ruler or protractor to measure the shapes and then name the quadrilaterals used to form the figure.
1. TRAY RACKS A tray rack looks like a parallelogram from the side. The levels for the trays are evenly spaced.

What two labeled points form a rhombus with base $AA'$?

2. SLICING Charles cuts a rhombus along both diagonals. He ends up with four congruent triangles. Classify these triangles as acute, obtuse, or right.

3. WINDOWS The edges of a window are drawn in the coordinate plane.

Determine whether the window is a square or a rhombus.

4. SQUARES Mackenzie cut a square along its diagonals to get four congruent right triangles. She then joined two of them along their long sides. Show that the resulting shape is a square.

5. DESIGN Tatianna made the design shown. She used 32 congruent rhombi to create the flower-like design at each corner.

   a. What are the angles of the corner rhombi?

   b. What kinds of quadrilaterals are the dotted and checkered figures?
Creating Pythagorean Puzzles

By drawing two squares and cutting them in a certain way, you can make a puzzle that demonstrates the Pythagorean Theorem. A sample puzzle is shown. You can create your own puzzle by following the instructions below.

1. Carefully construct a square and label the length of a side as $a$. Then construct a smaller square to the right of it and label the length of a side as $b$, as shown in the figure above. The bases should be adjacent and collinear.

2. Mark a point $X$ that is $b$ units from the left edge of the larger square. Then draw the segments from the upper left corner of the larger square to point $X$, and from point $X$ to the upper right corner of the smaller square.

3. Cut out and rearrange your five pieces to form a larger square. Draw a diagram to show your answer.

4. Verify that the length of each side is equal to $\sqrt{a^2 + b^2}$. 

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Properties of Trapezoids  A trapezoid is a quadrilateral with exactly one pair of parallel sides. The midsegment or median of a trapezoid is the segment that connects the midpoints of the legs of the trapezoid. Its measure is equal to one-half the sum of the lengths of the bases. If the legs are congruent, the trapezoid is an isosceles trapezoid. In an isosceles trapezoid both pairs of base angles are congruent and the diagonals are congruent.

Example  The vertices of $ABCD$ are $A(-3, -1), B(-1, 3), C(2, 3),$ and $D(4, -1)$. Show that $ABCD$ is a trapezoid and determine whether it is an isosceles trapezoid.

slope of $AB = \frac{3 - (-1)}{-1 - (-3)} = \frac{4}{2} = 2$

slope of $AD = \frac{-1 - (-1)}{4 - (-3)} = \frac{0}{7} = 0$

slope of $BC = \frac{3 - 3}{2 - (-1)} = \frac{0}{3} = 0$

slope of $CD = \frac{-1 - 3}{4 - 2} = \frac{-4}{2} = -2$

$AB = \sqrt{(-3 - (-1))^2 + (-1 - 3)^2} = \sqrt{4 + 16} = 2\sqrt{5}$

$CD = \sqrt{(2 - 4)^2 + (3 - (-1))^2} = \sqrt{4 + 16} = 2\sqrt{5}$

Exactly two sides are parallel, $\overline{AD}$ and $\overline{BC}$, so $ABCD$ is a trapezoid. $AB = CD$, so $ABCD$ is an isosceles trapezoid.

Exercises

Find each measure.

1. $m \angle D$

![Diagram of a trapezoid with $A(-3, -1), B(-1, 3), C(2, 3), D(4, -1)$]

2. $m \angle L$

![Diagram of a trapezoid with $J(1, 3), K(3, 1), L(3, -2), M(-2, 3)$]

COORDINATE GEOMETRY For each quadrilateral with the given vertices, verify that the quadrilateral is a trapezoid and determine whether the figure is an isosceles trapezoid.

3. $A(-1, 1), B(3, 2), C(1, -2), D(-2, -1)$

4. $J(1, 3), K(3, 1), L(3, -2), M(-2, 3)$

For trapezoid $HJKL$, $M$ and $N$ are the midpoints of the legs.

5. If $HJ = 32$ and $LK = 60$, find $MN$.

6. If $HJ = 18$ and $MN = 28$, find $LK$. 

![Diagram of a trapezoid with $HJKL$, $M$ and $N$ as midpoints]
6-6 Study Guide and Intervention (continued)

Trapezoids and Kites

Properties of Kites  A kite is a quadrilateral with exactly two pairs of consecutive congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.

The diagonals of a kite are perpendicular.
For kite $RMNP$, $\overline{MP} \perp \overline{RN}$

In a kite, exactly one pair of opposite angles is congruent.
For kite $RMNP$, $\angle M \cong \angle P$

Example 1: If $WXYZ$ is a kite, find $m\angle Z$.

The measures of $\angle Y$ and $\angle W$ are not congruent, so $\angle X \not\cong \angle Z$.
$m\angle X + m\angle Y + m\angle Z + m\angle W = 360$
$m\angle X + 60 + m\angle Z + 80 = 360$
$m\angle X + m\angle Z = 220$
$m\angle X = 110, m\angle Z = 110$

Example 2: If $ABCD$ is a kite, find $BC$.

The diagonals of a kite are perpendicular. Use the Pythagorean Theorem to find the missing length.
$BP^2 + PC^2 = BC^2$
$5^2 + 12^2 = BC^2$
$25 + 144 = BC^2$
$169 = BC^2$
$13 = BC$

Exercises

If $GHJK$ is a kite, find each measure.

1. Find $m\angle JRK$.

2. If $RJ = 3$ and $RK = 10$, find $JK$.

3. If $m\angle GHJ = 90$ and $m\angle GJK = 110$, find $m\angle HGK$.

4. If $HJ = 7$, find $HG$.

5. If $HG = 7$ and $GR = 5$, find $HR$.

6. If $m\angle GHJ = 52$ and $m\angle GJK = 95$, find $m\angle HGK$. 

Chapter 6  38  Glencoe Geometry
6-6 Skills Practice

Trapezoids and Kites

ALGEBRA Find each measure.

1. $m\angle S$

   \[ Q \begin{array}{c} 14 \\ 14 \\ 83^\circ \end{array} \hspace{1cm} R \]

2. $m\angle M$

   \[ M \begin{array}{c} 21 \\ 21 \\ 142^\circ \end{array} \hspace{1cm} R \]

3. $m\angle D$

   \[ A \begin{array}{c} 36^\circ \\ 70^\circ \\ \parallel \end{array} \hspace{1cm} B \]

4. $RH$

   \[ R \begin{array}{c} 12 \\ 12 \\ \parallel \end{array} \hspace{1cm} S \]

ALGEBRA For trapezoid $HJKL$, $T$ and $S$ are midpoints of the legs.

5. If $HJ = 14$ and $LK = 42$, find $TS$.

6. If $LK = 19$ and $TS = 15$, find $HJ$.

7. If $HJ = 7$ and $TS = 10$, find $LK$.

8. If $KL = 17$ and $JH = 9$, find $ST$.

COORDINATE GEOMETRY $EFGH$ is a quadrilateral with vertices $E(1, 3)$, $F(5, 0)$, $G(8, -5)$, $H(-4, 4)$.

9. Verify that $EFGH$ is a trapezoid.

10. Determine whether $EFGH$ is an isosceles trapezoid. Explain.
6-6 Practice

Trapezoids and Kites

Find each measure.

1. \( m \angle T \)

2. \( m \angle Y \)

3. \( m \angle Q \)

4. \( BC \)

ALGEBRA For trapezoid \( FEDC \), \( V \) and \( Y \) are midpoints of the legs.

5. If \( FE = 18 \) and \( VY = 28 \), find \( CD \).

6. If \( m \angle F = 140 \) and \( m \angle E = 125 \), find \( m \angle D \).

COORDINATE GEOMETRY \( RSTU \) is a quadrilateral with vertices \( R(−3, −3) \), \( S(5, 1) \), \( T(10, −2) \), \( U(−4, −9) \).

7. Verify that \( RSTU \) is a trapezoid.

8. Determine whether \( RSTU \) is an isosceles trapezoid. Explain.

9. CONSTRUCTION A set of stairs leading to the entrance of a building is designed in the shape of an isosceles trapezoid with the longer base at the bottom of the stairs and the shorter base at the top. If the bottom of the stairs is 21 feet wide and the top is 14 feet wide, find the width of the stairs halfway to the top.

10. DESK TOPS A carpenter needs to replace several trapezoid-shaped desktops in a classroom. The carpenter knows the lengths of both bases of the desktop. What other measurements, if any, does the carpenter need?
1. **PERSPECTIVE** Artists use different techniques to make things appear to be 3-dimensional when drawing in two dimensions. Kevin drew the walls of a room. In real life, all of the walls are rectangles. In what shape did he draw the side walls to make them appear 3-dimensional?

2. **PLAZA** In order to give the feeling of spaciousness, an architect decides to make a plaza in the shape of a kite. Three of the four corners of the plaza are shown on the coordinate plane. If the fourth corner is in the first quadrant, what are its coordinates?

3. **AIRPORTS** A simplified drawing of the reef runway complex at Honolulu International Airport is shown below.

   How many trapezoids are there in this image?

4. **LIGHTING** A light outside a room shines through the door and illuminates a trapezoidal region $ABCD$ on the floor.

   Under what circumstances would trapezoid $ABCD$ be isosceles?

5. **RISERS** A riser is designed to elevate a speaker. The riser consists of 4 trapezoidal sections that can be stacked one on top of the other to produce trapezoids of varying heights.

   All of the stages have the same height. If all four stages are used, the width of the top of the riser is 10 feet.

   **a.** If only the bottom two stages are used, what is the width of the top of the resulting riser?

   **b.** What would be the width of the riser if the bottom three stages are used?
Enrichment

Quadrilaterals in Construction

Quadrilaterals are often used in construction work.

1. The diagram at the right represents a roof frame and shows many quadrilaterals. Find the following shapes in the diagram and shade in their edges.
   a. isosceles triangle
   b. scalene triangle
   c. rectangle
   d. rhombus
   e. trapezoid (not isosceles)
   f. isosceles trapezoid

2. The figure at the right represents a window. The wooden part between the panes of glass is 3 inches wide. The frame around the outer edge is 9 inches wide. The outside measurements of the frame are 60 inches by 81 inches. The height of the top and bottom panes is the same. The top three panes are the same size.
   a. How wide is the bottom pane of glass?
   b. How wide is each top pane of glass?
   c. How high is each pane of glass?

3. Each edge of this box has been reinforced with a piece of tape. The box is 10 inches high, 20 inches wide, and 12 inches deep. What is the length of the tape that has been used?
Multiple Choice

Read each question. Then fill in the correct answer.

1. Ø Ø Ø Ø
2. Ø Ø Ø Ø
3. Ø Ø Ø Ø
4. Ø Ø Ø Ø
5. Ø Ø Ø Ø
6. Ø Ø Ø Ø
7. Ø Ø Ø Ø

Short Response/Gridded Response

Record your answer in the blank.

For gridded response questions, also enter your answer in the grid by writing each number or symbol in a box. Then fill in the corresponding circle for that number or symbol.

8. _____________ (grid in)
9. _____________
10. _____________
11. _____________
12. _____________ (grid in)
13. _____________

Extended Response

Record your answers for Question 14 on the back of this paper.
General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended-response questions require the student to show work.

- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is not considered a fully correct response.

- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 14 Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>Specific Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Students correctly determine the quadrilateral in part a is a parallelogram since opposite sides are congruent. Students correctly determine the quadrilateral in part b does not contain sufficient information to prove it is a parallelogram. The two horizontal sides must also be congruent. Students correctly determine that the quadrilateral in part c is a parallelogram since both pairs of opposite angles are congruent.</td>
</tr>
<tr>
<td>3</td>
<td>A generally correct solution, but may contain minor flaws in reasoning or computation.</td>
</tr>
<tr>
<td>2</td>
<td>A partially correct interpretation and/or solution to the problem.</td>
</tr>
<tr>
<td>1</td>
<td>A correct solution with no evidence or explanation.</td>
</tr>
<tr>
<td>0</td>
<td>An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.</td>
</tr>
</tbody>
</table>
**Chapter 6 Quiz 1**  
*(Lessons 6-1 and 6-2)*

1. Find the sum of the measures of the interior angles of a convex 70-gon.  
2. The measure of each interior angle of a regular polygon is 172. Find the number of sides in the polygon.  
3. The measure of each exterior angle of a regular polygon is 18. Find the number of sides in the polygon.  
4. Given parallelogram $ABCD$ with $C(5, 4)$, find the coordinates of $A$ if the diagonals $AC$ and $BD$ intersect at $(2, 7)$.  
5. **MULTIPLE CHOICE** Find $m\angle 1$ in parallelogram $ABCD$.  
   - A  64  
   - B  58  
   - C  46  
   - D  36

**Chapter 6 Quiz 2**  
*(Lesson 6-3)*

1. Determine whether this quadrilateral is a parallelogram. Justify your answer.  

**For Questions 2-4, write true or false.**  
2. A quadrilateral with two pairs of parallel sides is always a parallelogram.  
3. The diagonals of a parallelogram are always perpendicular.  
4. The slope of $AB$ and $CD$ is $\frac{3}{5}$ and the slope of $BC$ and $AD$ is $-\frac{5}{3}$. $ABCD$ is a parallelogram.  
5. Refer to parallelogram $ABCD$. If $AB = 8$ cm, what is the perimeter of the parallelogram?
Chapter 6 Quiz 3
(Lessons 6-4 and 6-5)

1. MULTIPLE CHOICE \( RSTV \) is a rhombus. Which of the following statements is NOT true?
   A \( RV \equiv TS \)
   B \( RV \perp TS \)
   C \( RS \parallel TV \)
   D \( \angle R \equiv \angle T \)

For Questions 2 and 3, refer to trapezoid \( MNPQ \).

2. Find \( m\angle M \).

3. Find \( m\angle Q \).

4. True or false. A quadrilateral that is a rectangle and a rhombus is a square.

5. \( \square ABCD \) has vertices \( A(4, 0), B(0, 4), C(-4, 0), \) and \( D(0, -4) \).
   Determine whether \( ABCD \) is a rectangle, rhombus, or square.
   List all that apply.

Chapter 6 Quiz 4
(Lesson 6-6)

For Questions 1 and 2, refer to kite \( DEFC \).

1. If \( m\angle DCF = 34 \) and \( m\angle DEF = 90 \), find \( m\angle CDE \).

2. If \( DR = 5 \) and \( RE = 5 \), find \( FE \).

For Questions 3 and 4, refer to trapezoid \( NPQM \) where \( X \) and \( Y \) are midpoints of the sides.

3. If \( MQ = 15 \) and \( XY = 10 \), find \( NP \).

4. If \( NP = 13 \) and \( MQ = 18 \), find \( XY \).

5. If \( CDEF \) is a trapezoid with vertices \( C(0, 2), D(2, 4), E(7, 3), \) and \( F(1, -3) \), how can you prove that it is an isosceles trapezoid?
Chapter 6 Mid-Chapter Test
(Lessons 6-1 through 6-3)

Part I Write the letter for the correct answer in the blank at the right of each question.

1. Find the measure of each exterior angle of a regular 56-gon. Round to the nearest tenth.
   A 3.2 B 6.4 C 173.6 D 9720 1. ___

2. Given $BE = 2x + 6$ and $ED = 5x - 12$ in parallelogram $ABCD$, find $BD$.
   F 6 H 18 G 12 J 36 2. ___

3. If the slope of $PQ$ is $\frac{2}{3}$ and the slope of $QR$ is $-\frac{1}{2}$, find the slope of $SR$ so that $PQRS$ is a parallelogram.
   A $\frac{2}{3}$ B $\frac{3}{2}$ C $-\frac{1}{2}$ D 2 3. ___

4. Find $m\angle W$ in parallelogram $RSTW$.
   F 17 H 55 G 33 J 125 4. ___

5. Find the sum of the measures of the interior angles of a convex 48-gon.
   A 172.5 B 360 C 8280 D 8640 5. ___

Part II

6. Find $x$.

7. $ABCD$ is a parallelogram with $m\angle A = 138$. Find $m\angle B$.

8. Determine whether $ABCD$ is a parallelogram if $AB = 6$, $BC = 12$, $CD = 6$, and $DA = 12$. Justify your answer.

9. In parallelogram $MLKJ$, find $m\angle MLK$ and $m\angle LKJ$.

10. $XYZW$ is a quadrilateral with vertices $W(1, -4)$, $X(-4, 2)$, $Y(1, -1)$, and $Z(-2, -3)$. Determine if the quadrilateral is a parallelogram. Use slope to justify your answer.

10. ________________
Choose from the terms above to complete each sentence.

1. A quadrilateral with only one pair of opposite sides parallel and the other pair of opposite sides congruent is a(n) _______.

2. A quadrilateral with two pairs of opposite sides parallel is a(n) _______.

3. A quadrilateral with only one pair of opposite sides parallel is a(n) _______.

4. A quadrilateral that is both a rectangle and a rhombus is a(n) _______.

5. A quadrilateral with four congruent sides is a(n) _______.

Write whether each sentence is true or false. If false, replace the underlined word or number to make a true sentence.

6. A quadrilateral with four right angles is a trapezoid.

7. A quadrilateral with two pairs of congruent consecutive sides is a kite.

Choose the correct term to complete each sentence.

8. Segments that join opposite vertices in a quadrilateral are called (medians, diagonals).

9. The segment joining the midpoints of the nonparallel sides of a trapezoid is called the (median, diagonal).

Define each term in your own words.

10. base angles of an isosceles trapezoid

11. legs of a trapezoid
Write the letter for the correct answer in the blank at the right of each question.

1. Find the sum of the measures of the interior angles of a convex 30-gon.
   A 5400  B 5040  C 360  D 168  1. ____

2. Find the sum of the measures of the exterior angles of a convex 21-gon.
   F 21  G 180  H 360  J 3420  2. ____

3. If the measure of each interior angle of a regular polygon is 108, find the
   measure of each exterior angle.
   A 18  B 72  C 90  D 108  3. ____

4. For parallelogram $ABCD$, find the value of $x$.
   F 4  G 10.25  H 16  J 21.5  4. ____

5. Which of the following is a property of a parallelogram?
   A The diagonals are congruent.  C The diagonals are perpendicular.
   B The diagonals bisect the angles.  D The diagonals bisect each other.
   5. ____

6. Find the values of $x$ and $y$ so that $ABCD$ will be
   a parallelogram.
   F $x = 6$, $y = 42$  G $x = 6$, $y = 22$  H $x = 20$, $y = 42$  J $x = 20$, $y = 22$  6. ____

7. Find the value of $x$ so that this quadrilateral is
   a parallelogram.
   A 44  B 46  C 90  D 134  7. ____

8. Parallelogram $ABCD$ has vertices $A(0, 0)$, $B(2, 4)$, and $C(10, 4)$. Find the
   coordinates of $D$.
   F $D(8, 0)$  G $D(10, 0)$  H $D(0, 4)$  J $D(10, 8)$  8. ____

9. Which of the following is a property of all rectangles?
   A four congruent sides  C diagonals are perpendicular
   B diagonals bisect the angles  D four right angles  9. ____

10. $ABCD$ is a rectangle with diagonals $AC$ and $BD$. If $AC = 2x + 10$ and
    $BD = 56$, find the value of $x$.
    F 23  G 33  H 78  J 122  10. ____

11. $ABCD$ is a rectangle with $B(-5, 0)$, $C(7, 0)$ and $D(7, 3)$. Find the coordinates
    of $A$.
    A $A(-5, 7)$  B $A(3, 5)$  C $A(-5, 3)$  D $A(7, -3)$  11. ____
12. For rhombus \(ABCD\), find \(m\angle 1\).
   - F 45
   - G 60
   - H 90
   - J 120
   \[\text{12. } \underline{\hspace{2cm}}\]

13. Find \(m\angle PRS\) in square \(PQRS\).
   - A 30
   - B 45
   - C 60
   - D 90
   \[\text{13. } \underline{\hspace{2cm}}\]

14. Choose a pair of base angles of trapezoid \(ABCD\).
   - F \(\angle A, \angle C\)
   - G \(\angle B, \angle D\)
   - H \(\angle A, \angle D\)
   - J \(\angle D, \angle C\)
   \[\text{14. } \underline{\hspace{2cm}}\]

15. In trapezoid \(DEFG\), find \(m\angle D\).
   - A 44
   - B 72
   - C 108
   - D 136
   \[\text{15. } \underline{\hspace{2cm}}\]

16. The hood of Olivia’s car is the shape of a trapezoid. The base bordering the windshield measures 30 inches and the base at the front of the car measures 24 inches. What is the width of the median of the hood?
   - F 25 in.
   - G 27 in.
   - H 28 in.
   - J 29 in.
   \[\text{16. } \underline{\hspace{2cm}}\]

17. The length of one base of a trapezoid is 44, the median is 36, and the other base is \(2x + 10\). Find the value of \(x\).
   - A 9
   - B 17
   - C 21
   - D 40
   \[\text{17. } \underline{\hspace{2cm}}\]

18. Given trapezoid \(ABCD\) with median \(\overline{EF}\), which of the following is true?
   - F \(EF = \frac{1}{2} AD\)
   - G \(AE = FD\)
   - H \(AB = EF\)
   - J \(EF = \frac{BC + AD}{2}\)
   \[\text{18. } \underline{\hspace{2cm}}\]

19. \(PQRS\) is a kite. Find \(m\angle S\).
   - A 100
   - B 160
   - C 200
   - D 360
   \[\text{19. } \underline{\hspace{2cm}}\]

20. \(JKLM\) is a kite, find \(JM\).
   - F \(\sqrt{29}\)
   - G \(\sqrt{89}\)
   - H \(\sqrt{13}\)
   - J 11
   \[\text{20. } \underline{\hspace{2cm}}\]

Bonus Find \(x\) and \(m\angle WYZ\) in rhombus \(XYZW\).

B: \(\underline{\hspace{2cm}}\)
Write the letter for the correct answer in the blank at the right of each question.

1. Find the sum of the measures of the interior angles of a convex 45-gon.
   - A 8100
   - B 7740
   - C 360
   - D 172
   1. ___________

2. Find the value of \( x \).
   - F 30
   - H 102
   - G 66
   - J 138
   2. ___________

3. Find the sum of the measures of the exterior angles of a convex 39-gon.
   - A 39
   - B 90
   - C 180
   - D 360
   3. ___________

4. Which of the following is a property of a parallelogram?
   - F Each pair of opposite sides is congruent.
   - G Only one pair of opposite angles is congruent.
   - H Each pair of opposite angles is supplementary.
   - J There are four right angles.
   4. ___________

5. For parallelogram \( ABCD \), find \( m\angle 1 \).
   - A 60
   - B 54
   - C 36
   - D 18
   5. ___________

6. \( ABCD \) is a parallelogram with diagonals intersecting at \( E \). If \( AE = 3x + 12 \) and \( EC = 27 \), find the value of \( x \).
   - F 5
   - G 17
   - H 27
   - J 47
   6. ___________

7. Find the values of \( x \) and \( y \) so that this quadrilateral is a parallelogram.
   - A \( x = 13 \), \( y = 24 \)
   - B \( x = 13 \), \( y = 6 \)
   - C \( x = 7 \), \( y = 24 \)
   - D \( x = 7 \), \( y = 6 \)
   7. ___________

8. Find the value of \( x \) so that this quadrilateral is a parallelogram.
   - F 12
   - G 24
   - H 36
   - J 132
   8. ___________

9. Parallelogram \( ABCD \) has vertices \( A(8, 2) \), \( B(6, -4) \), and \( C(-5, -4) \). Find the coordinates of \( D \).
   - A \( D(-5, 2) \)
   - B \( D(-3, 2) \)
   - C \( D(-2, 2) \)
   - D \( D(-4, 8) \)
   9. ___________

10. \( ABCD \) is a rectangle. If \( AC = 5x + 2 \) and \( BD = x + 22 \), find the value of \( x \).
    - F 5
    - G 6
    - H 11
    - J 26
   10. ___________

11. Which of the following is true for all rectangles?
    - A The diagonals are perpendicular.
    - B The diagonals bisect the angles.
    - C The consecutive sides are congruent.
    - D The consecutive sides are perpendicular.
   11. ___________
12. \(ABCD\) is a rectangle with \(B(-4, 6), C(-4, 2),\) and \(D(10, 2).\) Find the coordinates of \(A.\)
   - F: \(A(6, 4)\)
   - G: \(A(10, 4)\)
   - H: \(A(2, 6)\)
   - J: \(A(10, 6)\)

13. For rhombus \(GHJK,\) find \(m \angle 1.\)
   - A: 22
   - C: 68
   - B: 44
   - D: 90

14. The diagonals of square \(ABCD\) intersect at \(E.\) If \(AE = 2x + 6\) and \(BD = 6x - 10,\) find \(AC.\)
   - F: 11
   - G: 28
   - H: 56
   - J: 90

15. \(ABCD\) is an isosceles trapezoid with \(A(10, -1), B(8, 3),\) and \(C(-1, 3).\) Find the coordinates of \(D.\)
   - A: \(D(-3, -1)\)
   - B: \(D(-10, -11)\)
   - C: \(D(-1, 8)\)
   - D: \(D(-3, 3)\)

16. For isosceles trapezoid \(MNOP,\) find \(m \angle MNP.\)
   - F: 44
   - G: 64
   - H: 80
   - J: 116

17. The length of one base of a trapezoid is 19 inches and the length of the median is 16 inches. Find the length of the other base.
   - A: 35 in.
   - B: 19 in.
   - C: 17.5 in.
   - D: 13 in.

18. Judith built a fence to surround her property. On a coordinate plane, the four corners of the fence are located at \((-16, 1), (-6, 5), (4, 1),\) and \((-6, -3).\) Which of the following most accurately describes the shape of Judith’s fence?
   - F: square
   - H: rhombus
   - G: rectangle
   - J: trapezoid

19. For kite \(PQRS,\) find \(m \angle S.\)
   - A: 248
   - B: 68
   - C: 112
   - D: 124

20. \(ABCD\) is a parallelogram with coordinates \(A(4, 2), B(4, -1), C(-2, -1),\) and \(D(-2, 2).\) To prove that \(ABCD\) is a rectangle, you would plot the parallelogram on a coordinate plane and then find which of the following?
   - F: measures of the angles
   - H: slopes of the diagonals
   - G: lengths of the diagonals
   - J: midpoints of the diagonals

**Bonus** Find the possible value(s) of \(x\) in rectangle \(JKLM.\)
Write the letter for the correct answer in the blank at the right of each question.

1. Find the sum of the measures of the interior angles of a convex 50-gon.
   A 9000   B 8640   C 360   D 172.8

2. Find the value of x.
   F 16   H 50
   G 34   J 70

3. Find the sum of the measures of the exterior angles of a convex 65-gon.
   A 5.54   B 90   C 180   D 360

4. Which of the following is a property of all parallelograms?
   F Each pair of opposite angles is congruent.
   G Only one pair of opposite sides is congruent.
   H Each pair of opposite angles is supplementary.
   J There are four right angles.

5. For parallelogram ABCD, find m\(\angle 1\).
   A 19   C 52
   B 38   D 56

6. ABCD is a parallelogram with diagonals intersecting at E. If AE = 4x - 8 and EC = 36, find the value of x.
   F 7   G 11   H 15.5   J 38

7. Find the values of the values of x and y so that the quadrilateral is a parallelogram.
   A x = 27, y = 90   C x = 13, y = 90
   B x = 27, y = 40   D x = 13, y = 40

8. Find the value of x so that the quadrilateral is a parallelogram.
   F 7\(\frac{1}{3}\)   H 12
   G 8   J 66

9. ABCD is a parallelogram with A(5, 4), B(−1, −2), and C(8, −2). Find the coordinates of D.
   A D(−5, 4)   B D(8, 2)   C D(14, 4)   D D(4, 1)

10. ABCD is a rectangle. If AB = 7x - 6 and CD = 5x + 30, find the value of x.
    F 5\(\frac{1}{3}\)   G 12   H 13   J 18

11. Which of the following is true for all rectangles?
    A The diagonals are perpendicular.
    B The consecutive angles are supplementary.
    C The opposite sides are supplementary.
    D The opposite angles are complementary.
12. **ABCD** is a rectangle with \(B(-7, 3), C(5, 3),\) and \(D(5, -8).\) Find the coordinates of \(A.\)
   \[F \ A(-8, -7) \quad G \ A(-7, -8) \quad H \ A(-5, -3) \quad J \ A(-8, -5)\]

13. For rhombus **GHJK**, find \(m \angle 1.\)
   \[A \ 90 \quad B \ 64 \quad C \ 52 \quad D \ 38\]

14. The diagonals of square **ABCD** intersect at \(E.\) If \(AE = 3x - 4\) and \(BD = 10x - 48,\) find \(AC.\)
   \[F \ 90 \quad G \ 52 \quad H \ 26 \quad J \ 10\]

15. **ABCD** is an isosceles trapezoid with \(A(0, -1), B(-3, 3),\) and \(D(6, -1).\) Find the coordinates of \(C.\)
   \[A \ C(6, 1) \quad B \ C(9, 4) \quad C \ C(2, 3) \quad D \ C(8, 3)\]

16. For isosceles trapezoid **MNOP**, find \(m \angle MNP.\)
   \[F \ 42 \quad G \ 70 \quad H \ 82 \quad J \ 98\]

17. The length of one base of a trapezoid is 19 meters and the length of the median is 23 meters. Find the length of the other base.
   \[A \ 15 \text{ m} \quad B \ 21 \text{ m} \quad C \ 27 \text{ m} \quad D \ 42 \text{ m}\]

18. On a coordinate plane, the four corners of Ronald’s garden are located at \((0, 2), (4, 6), (8, 2),\) and \((4, -2).\) Which of the following most accurately describes the shape of Ronald’s garden?
   \[F \ \text{square} \quad G \ \text{rectangle} \quad H \ \text{rhombus} \quad J \ \text{trapezoid}\]

19. For kite **WXYZ**, find \(m \angle W.\)
   \[A \ 106 \quad B \ 148 \quad C \ 212 \quad D \ 360\]

20. **ABCD** is a parallelogram with coordinates \(A(4, 2), B(3, -1), C(-1, -1),\) and \(D(-1, 2).\) To prove that **ABCD** is a rhombus, you would plot the parallelogram on a coordinate plane and then find which of the following?
   \[F \ \text{measures of the angles} \quad G \ \text{lengths of the diagonals} \quad H \ \text{slopes of the diagonals} \quad J \ \text{midpoints of the diagonals}\]

**Bonus**
The sum of the measures of the interior angles of a convex polygon is ten times the sum of the measures of its exterior angles. Find the number of sides of the polygon.
   \[B:\]
1. What is the sum of the interior angles of an octagonal box?

2. A convex pentagon has interior angles with measures $(5x - 12)\degree$, $(2x + 100)\degree$, $(4x + 16)\degree$, $(6x + 15)\degree$, and $(3x + 41)\degree$. Find the value of $x$.

3. If the measure of each interior angle of a regular polygon is 171, find the number of sides in the polygon.

4. In parallelogram $ABCD$, $m\angle 1 = x + 12$, and $m\angle 2 = 6x - 18$. Find $m\angle 1$.

5. Find the measure of each exterior angle of a regular 45-gon.

6. In parallelogram $ABCD$, $m\angle A = 58$. Find $m\angle B$.

7. Find the coordinates of the intersection of the diagonals of parallelogram $XYZW$ with vertices $X(2, 2)$, $Y(3, 6)$, $Z(10, 6)$, and $W(9, 2)$.

8. Determine whether $ABCD$ is a parallelogram. Justify your answer.

9. Determine whether the quadrilateral with vertices $A(5, 7)$, $B(1, -2)$, $C(-6, -3)$, and $D(2, 5)$ is a parallelogram. Use the slope formula.

10. For quadrilateral $ABCD$, the slope of $AB$ is $\frac{1}{4}$, the slope of $BC$ is $-\frac{2}{3}$, and the slope of $CD$ is $\frac{1}{4}$. Find the slope of $DA$ so that $ABCD$ will be a parallelogram.

11. Given rectangle $ABCD$, find the value of $x$.

12. $ABCD$ is a parallelogram and $\overline{AC} \cong \overline{BD}$. Determine whether $ABCD$ is a rectangle. Justify your answer.

13. $ABCD$ is a rhombus with diagonals intersecting at $E$. If $m\angle ABC$ is three times $m\angle BAD$, find $m\angle EBC$.  

14. **TUVW** is a square with **U** (10, 2), **V** (8, 8), and **W** (2, 6). Find the coordinates of **T**.

15. For isosceles trapezoid **MNOP**, find \( m\angle MNQ \).

16. **ABCD** is a quadrilateral with vertices **A** (8, 3), **B** (6, 7), **C** (−1, 5), and **D** (−6, −1). Determine whether **ABCD** is a trapezoid. Justify your answer.

17. The length of the median of trapezoid **EFGH** is 13 feet. If the bases have lengths \( 2x + 4 \) and \( 10x - 50 \), find \( x \).

18. **ABCD** is a kite, if **RC** = 10, and **BD** = 48, find **CD**.

**For Questions 19–25, write true or false.**

19. A rectangle is always a parallelogram.

20. The diagonals of a rhombus are always perpendicular.

21. The diagonals of a square always bisect each other.

22. A trapezoid always has two congruent sides.

23. The median of a trapezoid is always parallel to the bases.

24. A kite has exactly two congruent angles.

25. If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rectangle.

**Bonus** In parallelogram **ABCD**, \( AB = 2x - 7 \), \( BC = x + 3y \), \( CD = x + y \), and \( AD = 2x - y - 1 \). Find the values of \( x \) and \( y \).

**B:** ______________
1. Bruce is building a tabletop in the shape of an octagon. Find the sum of the external angles of the tabletop.

2. A convex octagon has interior angles with measures \((x + 55)°, (3x + 20)°, 4x°, (4x - 10)°, (6x - 55)°, (3x + 52)°, 3x°, \) and \((2x + 30)°\). Find the value of \(x\).

3. If the measure of each interior angle of a regular polygon is 176 find the number of sides in the polygon.

4. In parallelogram \(ABCD\), \(m\angle1 = x + 25\), and \(m\angle2 = 2x\). Find \(m\angle2\).

5. Find the measure of each exterior angle of a regular 100-gon.

6. In parallelogram \(ABCD\), \(m\angleA = 63\). Find \(m\angleB\).

7. Find the coordinates of the intersection of the diagonals of parallelogram \(XYZW\) with vertices \(X(3, 0), Y(3, 8), Z(-2, 6), \) and \(W(-2, -2)\).

8. Determine whether this quadrilateral is a parallelogram. Justify your answer.

9. Determine whether a quadrilateral with vertices \(A(5, 7), B(1, -1), C(-6, -3),\) and \(D(-2, 5)\) is a parallelogram. Use the slope formula.

10. If the slope of \(AB\) is \(\frac{1}{2}\), the slope of \(BC\) is \(-4\), and the slope of \(CD\) is \(\frac{1}{2}\), find the slope of \(DA\) so that \(ABCD\) is a parallelogram.

11. For rectangle \(ABCD\), find the value of \(x\).

12. \(ABCD\) is a parallelogram and \(m\angleA = 90\). Determine whether \(ABCD\) is a rectangle. Justify your answer.
13. **ABCD** is a rhombus with diagonals intersecting at **E**. If \( m \angle ABC \) is four times \( m \angle BAD \), find \( m \angle EBC \).

14. **PQRS** is a square with \( Q(-2, 8), R(5, 7), \) and \( S(4, 0) \). Find the coordinates of **P**.

15. For isosceles trapezoid **MNOP**, find \( m \angle MNQ \).

16. **ABCD** is a quadrilateral with \( A(8, 21), B(10, 27), C(26, 26), \) and \( D(18, 2) \). Determine whether **ABCD** is a trapezoid. Justify your answer.

17. The length of the median of trapezoid **EFGH** is 17 centimeters. If the bases have lengths \( 2x + 4 \) and \( 8x - 50 \), find the value of \( x \).

18. For kite **ABCD**, if \( RA = 15 \), and \( BD = 16 \), find \( AD \).

**For Questions 19–25, write true or false.**

19. A parallelogram always has four right angles.

20. The diagonals of a rhombus always bisect the angles.

21. A rhombus is always a square.

22. A rectangle is always a square.

23. The diagonals of an isosceles trapezoid are always congruent.

24. The median of a trapezoid always bisects the angles.

25. The diagonals of a kite are always perpendicular.

**Bonus** The measure of each interior angle of a regular polygon is 24 more than 38 times the measure of each exterior angle. Find the number of sides of the polygon.

**B:**
1. The sum of the interior angles of an animal pen is 900°. How many sides does the pen have?  
2. A convex hexagon has interior angles with measures $x^\circ$, $(5x - 103)^\circ$, $(2x + 60)^\circ$, $(7x - 31)^\circ$, $(6x - 6)^\circ$, and $(9x - 100)^\circ$. Find the value of $x$ and the measure of each angle.  
3. Find the measure of each exterior angle of a regular $2x$-gon.  
4. For parallelogram $ABCD$, find $\angle 1$.  
5. $ABCD$ is a parallelogram with diagonals that intersect each other at $E$. If $AE = x^2$ and $EC = 6x - 8$, find all possible values of $AC$.  
6. Determine whether the quadrilateral is a parallelogram. Justify your answer.  
7. For quadrilateral $ABCD$, the slope of $AB$ is $\frac{2}{3}$ and the slope of $BC$ is $-2$. Find the slopes of $CD$ and $DA$ so that $ABCD$ will be a parallelogram.  
8. In rectangle $ABCD$, find $\angle 1$.  
9. The diagonals of rhombus $ABCD$ intersect at $E$. If $\angle BAE = \frac{2}{3}(\angle ABE)$, find $\angle BCD$.  
10. The diagonals of square $ABCD$ intersect at $E$. If $AE = 2$, find the perimeter of $ABCD$.  
11. For isosceles trapezoid $ABCD$, find $AE$.  
12. Points $G$ and $H$ are midpoints of $AF$ and $DE$ in regular hexagon $ABCDEF$. If $AB = 6$ find $GH$.  
13. The vertices of trapezoid $ABCD$ are $A(10, -1)$, $B(6, 6)$, $C(-2, 6)$, and $D(-8, -1)$. Find the length of the median.
14. Determine whether the quadrilateral \(ABCD\) with vertices \(A(0, -1), B(-4, -3), C(-5, 1), D(1, 7)\) is a kite. Justify your answer.

15. Determine whether the quadrilateral \(ABCD\) with vertices \(A(6, 2), B(2, 10), C(-6, 6), D(-2, -2)\) is a rectangle. Justify your answer.

16. Determine whether quadrilateral \(ABCD\) with vertices \(A(1, 6), B(7, 6), C(2, -3), D(-4, -3)\) is a parallelogram. Use the distance formula.

17. Find the value of \(x\) in kite \(EFGH\).

For Questions 18 and 19, complete the two-column proof by supplying the missing information for each corresponding location.

Given: \(ABCD\) is a parallelogram.

\(BQ \cong DS, PA \cong RC\)

Prove: \(PQRS\) is a parallelogram.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (ABCD) is a (\square).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. (AD \cong CB)</td>
<td>2. (Question 18)</td>
</tr>
<tr>
<td>3. (PA \cong RC)</td>
<td>3. Given</td>
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<tr>
<td>5. (AB \cong CD)</td>
<td>5. Opp. sides of a (\square) are (\cong).</td>
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<td>6. (BQ \cong DS)</td>
<td>6. Given</td>
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<td>7. (AQ \cong CS)</td>
<td>7. Seg. Sub. Prop.</td>
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<tr>
<td>8. (\angle B \cong \angle D, \angle A \cong \angle C)</td>
<td>8. Opp. (\angle) of a (\square) are (\cong).</td>
</tr>
<tr>
<td>9. (\triangle QBR \cong \triangle SDP, \triangle PAQ \cong \triangle RCS)</td>
<td>9. SAS</td>
</tr>
<tr>
<td>10. (QP \cong RS, QR \cong PS)</td>
<td>10. CPCTC</td>
</tr>
<tr>
<td>11. (PQRS) is a parallelogram.</td>
<td>11. (Question 19)</td>
</tr>
<tr>
<td>20. In isosceles trapezoid (ABCD), (AE = 2x + 5), (EC = 3x - 12), and (BD = 4x + 20). Find the value of (x).</td>
<td>20.</td>
</tr>
</tbody>
</table>

**Bonus** If three of the interior angles of a convex hexagon each measure 140, a fourth angle measures 84, and the measure of the fifth angle is 3 times the measure of the sixth angle, find the measure of the sixth angle.

**B:** _____________
Chapter 6 Extended-Response Test

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. a. Draw a regular convex polygon and a convex polygon that is not regular, each with the same number of sides.

b. Label the measures of each exterior angle on your figures.

c. Find the sum of the exterior angles for each figure. What conjecture can be made?

2. Draw a rectangle. Connect the midpoints of the consecutive sides. What type of quadrilateral is formed? How do you know?

3. Draw an example to show why one pair of opposite sides congruent and the other pair of opposite sides parallel is not sufficient to form a parallelogram.

4. a. Name a property that is true for a square and not always true for a rectangle.

b. Name a property that is true for a square and not always true for a rhombus.

c. Name a property that is true for a rectangle and not always true for a parallelogram.
1. Find the coordinates of X if V(0.5, 5) is the midpoint of UX with U(15, 21). (Lesson 1-3)
   A (−14, −11)  C (0, 0)
   B (7.75, 22.5)  D (15.5, −5)  1. 0 0 0 0

2. Which of the following are possible measures for vertical angles G and H? (Lesson 2-8)
   F m∠G = 125 and m∠H = 55
   G m∠G = 125 and m∠H = 125
   H m∠G = 55 and m∠H = 45
   J m∠G = 55 and m∠H = 152.5  2. 0 0 0 0

3. Determine which lines are parallel. (Lesson 3-5)
   A NS || PT  C QR || ST
   B NP || ST  D NP || QR  3. 0 0 0 0

4. Find the coordinates of B, the midpoint of AC, if A(2a, b) and C(0, 2b). (Lesson 4-8)
   F (2a, 2b)  G (a, b)  H \left(\frac{3}{2} a, b\right)  J \left\{\frac{3}{2} a, b\right\}  4. 0 0 0 0

5. If RV is an angle bisector, find m∠UVT. (Lesson 5-1)
   A 10  C 68
   B 34  D 136  5. 0 0 0 0

6. Find the slope of the line that passes through points A(−7, 14) and B(5, −2). (Lesson 3-3)
   F −\frac{4}{3}  G −\frac{3}{4}  H \frac{3}{4}  J \frac{4}{3}  6. 0 0 0 0

7. Which statement ensures that quadrilateral QRST is a parallelogram? (Lesson 6-3)
   A ∠Q ≅ ∠S  C QT || RS
   B QR ≅ TS and QR || TS  D m∠Q + m∠S = 180  7. 0 0 0 0
8. What is the equation of the line that contains \((-12, 9)\) and is perpendicular to the line \(y = \frac{2}{3}x + 5\)? (Lesson 3-4)

- \(F\) \(y = -\frac{3}{2}x - 9\)
- \(H\) \(y = -\frac{2}{3}x - 1\)
- \(G\) \(y = \frac{3}{2}x - 1\)
- \(J\) \(y = \frac{2}{3}x + 17\)

9. Which of the following theorems can be used to prove \(\triangle ABC \cong \triangle DEC\)? (Lesson 4-5)

- \(A\) SSS
- \(B\) SAS
- \(C\) AAS
- \(D\) ASA

10. What is the value of \(x\)? (Lesson 6-6)

- \(F\) 2
- \(H\) 5.5
- \(G\) 4
- \(J\) 7

11. For \(\triangle ABC\), \(AB = 6\) and \(BC = 17\). Which of the following is a possible length for \(AC\)? (Lesson 5-3)

- \(A\) 5
- \(B\) 9
- \(C\) 13
- \(D\) 24

12. What is \(m \angle T\) in kite \(STVW\)?

- \(F\) 100
- \(H\) 95
- \(G\) 130
- \(J\) 260

Part 2: Gridded Response

**Instructions:** Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

13. If \(\triangle UVW\) is an isosceles triangle, \(UV \cong WU\), \(UV = 16b - 40\), \(VW = 6b\), and \(WU = 10b + 2\), find the value of \(b\). (Lesson 4-1)

14. Find the sum of the measures of the interior angles for a convex heptagon. (Lesson 6-1)
15. A polygon has six congruent sides. Lines containing two of its sides contain points in its interior. Name the polygon by its number of sides, and then classify it as convex or concave and regular or irregular. (Lesson 1-6)

16. If $RT \cong QM$ and $RT = 88.9$ centimeters, find $QM$. (Lesson 2-7)

17. Which segment is the shortest segment from $D$ to $JM$? (Lesson 5-2)

18. If $\triangle ABC \cong \triangle WXY$, $AB = 72$, $BC = 65$, $CA = 13$, $XY = 7x - 12$, and $WX = 19y + 34$, find the values of $x$ and $y$. (Lesson 4-3)

19. Freda bought two bells for just over $90 before tax. State the assumption you would make to write an indirect proof to show that at least one of the bells costs more than $45$. (Lesson 5-4)

20. The area of the base of a cylinder is $5$ cm$^2$ and the height of the cylinder is $8$ cm. Find the volume of the cylinder. (Lesson 1-7)

21. $JKLM$ is a kite. Complete each statement. (Lesson 6-6)
   a. $MJ \cong ____$
   b. $MK \perp ____$
   c. $m\angle L = m\angle ____$
1. Use a protractor to classify \( \triangle UVW \), \( \triangle UWX \), and \( \triangle XWY \) as acute, equiangular, obtuse, or right.

2. In the figure, \( \angle 1 \cong \angle 2 \). Find the measures of the numbered angles.

3. Name the corresponding congruent sides for \( \triangle AFP \cong \triangle STX \).

4. Determine whether \( \triangle ABC \cong \triangle PQR \) given \( A(2, -7), B(5, 3), C(-4, 6), P(8, -1), Q(11, 9), \) and \( R(2, 12) \).

5. In the figure, \( \overline{LK} \) bisects \( \angle JKM \) and \( \angle KLJ \cong \angle KLM \). Determine which theorem or postulate can be used to prove that \( \triangle JKL \cong \triangle MKL \).

6. Triangle \( ABC \) is isosceles with \( AB = BC \). Name a pair of congruent angles in this triangle.

7. For kite \( WXYZ \), find \( m\angle Z \).

For Questions 8 and 9, refer to the figure.

8. Find the value of \( a \) and \( m\angle ZWT \) if \( \overline{ZW} \) is an altitude of \( \triangle XYZ \), \( m\angle ZWT = 3a + 5 \), and \( m\angle TWY = 5a + 13 \).

9. Determine which angle has the greatest measure: \( \angle YWZ \), \( \angle WZY \), or \( \angle ZYW \).

10. Mr. Ramirez bought a stove and a dishwasher for just over $1206. State the assumption you would make to start an indirect proof to show that at least one of the appliances cost more than $603.
11. Determine whether 128 feet, 136 feet, and 245 feet can be the
   lengths of the sides of a triangle.

12. Write an inequality to
describe the possible
   values of $x$.

13. The measure of an interior angle of a regular polygon is 140.
   Find the number of sides in the polygon.

14. For parallelogram $JKMH$, find
   $m\angle JHK$, $m\angle HMK$, and the value of $x$.

15. Determine whether the vertices of quadrilateral $DEFG$ form a
    parallelogram given $D(-3, 5), E(3, 6), F(-1, 0)$, and $G(6, 1)$.

16. For rectangle $WXYZ$ with diagonals $\overline{WY}$ and $\overline{XZ}$,
    $WY = 3d + 4$ and $XZ = 4d - 1$, find the value of $d$.

17. If $m\angle BEC = 9z + 45$ in rhombus
    $ABCD$, find the value of $z$.

18. In trapezoid $HJLK$, $M$ and $N$ are
    midpoints of the legs. Find $KL$.

19. Prove that quadrilateral $PQRS$ is
    NOT a parallelogram.
**Step 1** Before you begin Chapter 6

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

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**Step 2** After you complete Chapter 6

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

---

**Example 1** A convex polygon has 13 sides. Find the sum of the measures of the interior angles.

\[(n - 2) \cdot 180 = (13 - 2) \cdot 180\]
\[= 11 \cdot 180\]
\[= 1980\]

**Example 2** The measure of an interior angle of a regular polygon is 120. Find the number of sides.

\[120n = (n - 2) \cdot 180\]
\[120n = 180n - 360\]
\[-60n = -360\]
\[n = 6\]

**Exercises**

Find the sum of the measures of the interior angles of each convex polygon.

1. decagon 2. 15-gon 3. 30-gon
   1440 2520 5040
4. octagon 5. 12-gon 6. 35-gon
   1080 1800 5940

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

7. 150 8. 160 9. 175
   12 18 72
10. 165 11. 144 12. 135
   24 10 8

13. Find the value of x.
   \[\angle A = 10^\circ\]
   \[\angle B = 50^\circ\]
   \[\angle C = 30^\circ\]
   \[\angle D = 10^\circ\]
   \[\angle E = ?\]

---

**Answers (Anticipation Guide and Lesson 6-1)**

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Chapter 6

NAME ___________________________ DATE ______ PERIOD ______

**6 Anticipation Guide**

**Quadrilaterals**

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Chapter 6

6-1 Skills Practice

Angles of Polygons

Find the sum of the measures of the interior angles of each convex polygon.

1. nonagon 2. heptagon 3. decagon

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

4. 108 5. 130 6. 150

Find the measure of each interior angle.

7. \( m\angle A = 115, m\angle B = 65, \) \( m\angle C = 115, m\angle D = 65 \)

8. \( m\angle L = 100, m\angle M = 110, \) \( m\angle N = 70, m\angle P = 80 \)

9. \( m\angle S = 116, m\angle T = 116, \) \( m\angle W = 64, m\angle U = 64 \)

10. \( m\angle D = 140, m\angle E = 140, \) \( m\angle F = 80, m\angle P = 80, \) \( m\angle H = 140, m\angle G = 140 \)

Find the measures of each interior angle of each regular polygon.

11. quadrilateral 12. pentagon 13. dodecagon

14. octagon 15. nonagon 16. 12-gon

Find the measures of each exterior angle of each regular polygon.

17. \( 45 \) 18. \( 45 \) 19. \( 40 \) 20. \( 30 \)
6-1 Practice
Angles of Polygons
Find the sum of the measures of the interior angles of each convex polygon.

1. 11-gon
2. 14-gon
3. 17-gon

\[
\begin{align*}
1620 \\
2160 \\
2700 \\
\end{align*}
\]

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

4. 144°
5. 156°
6. 160°

Find the measure of each interior angle.

7. 
8.

\[
\begin{align*}
&\text{Find the measure of each interior angle.} \\
&m\angle R = 128, m\angle S = 52 \\
&m\angle T = 128, m\angle U = 52 \\
\end{align*}
\]

Find the measures of an exterior angle and an interior angle given the number of sides of each regular polygon. Round to the nearest tenth, if necessary.

9. 16
10. 24
11. 30

\[
\begin{align*}
&157.5, 22.5 \\
&165, 15 \\
&168, 12 \\
\end{align*}
\]

12. 14
13. 22
14. 40

\[
\begin{align*}
&154.3, 25.7 \\
&163.6, 16.4 \\
&171, 9 \\
\end{align*}
\]

15. CRYSTALLOGRAPHY Crystals are classified according to seven crystal systems. The basis of the classification is the shapes of the faces of the crystal. Turquoise belongs to the triclinic system. Each of the six faces of turquoise is in the shape of parallelogram. Find the sum of the measures of the interior angles of one such face.

\[
360
\]

6-1 Word Problem Practice
Angles of Polygons
1. ARCHITECTURE In the Uffizi gallery in Florence, Italy, there is a room built by Buontalenti called the Tribune (La Tribuna in Italian). This room is shaped like a regular octagon.

What angle do consecutive walls of the Tribune make with each other?

135°

2. BOXES Jasmine is designing boxes she will use to ship her jewelry. She wants to shape the box like a regular polygon. In order for the boxes to pack tightly, she decides to use a regular polygon that has the property that the measure of its interior angles is half the measure of its exterior angles. What regular polygon should she use?

an equilateral triangle

3. THEATER A theater floor plan is shown in the figure. The upper five sides are part of a regular dodecagon.

a. Find \( m\angle 2 \) and \( m\angle 5 \).

90 and 60

b. Find \( m\angle 3 \) and \( m\angle 4 \).

162 and 132

c. What is \( m\angle 1 \)?

96

4. ARCHEOLOGY Archeologists unearthed parts of two adjacent walls of an ancient castle.

Before it was unearthed, they knew from ancient texts that the castle was shaped like a regular polygon, but nobody knew how many sides it had. Some said 6, others 8, and some even said 100. From the information in the figure, how many sides did the castle really have?

15

5. POLYGON PATH In Ms. Rickets’ math class, students made a “polygon path” that consists of regular polygons of 3, 4, 5, and 6 sides joined together as shown.

a. Find \( m\angle 2 \) and \( m\angle 5 \).

90 and 60

b. Find \( m\angle 3 \) and \( m\angle 4 \).

162 and 132

c. What is \( m\angle 1 \)?

96

Find \( m\angle 1 \).

120

\[
\begin{align*}
\angle 1 \\
\angle 2 \\
\angle 3 \\
\angle 4 \\
\angle 5 \\
\end{align*}
\]

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Central Angles of Regular Polygons

You have learned about the interior and exterior angles of a polygon. Regular polygons also have central angles. A central angle is measured from the center of the polygon.

The center of a polygon is the point equidistant from all of the vertices of the polygon, just as the center of a circle is the point equidistant from all of the points on the circle. The central angle is the angle drawn with the vertex at the center of the circle and the sides of angle drawn through consecutive vertices of the polygon. One of the central angles that can be drawn in this regular hexagon is \(\angle APB\).

You may remember from making circle graphs that there are 360° around the center of a circle.

1. By using logic or by drawing sketches, find the measure of the central angle of each regular polygon.

| 120 | 90 | 72 | 60 | about 51.43 | 45 |

2. Make a conjecture about how the measure of a central angle of a regular polygon relates to the measures of the interior angles and exterior angles of a regular polygon.

The measure of the exterior angle equals the measure of the central angle. The central angle is supplementary to interior angle.

3. CHALLENGE In obtuse \(\triangle ABC\), \(BC\) is the longest side. \(AC\) is also a side of a 21-sided regular polygon. \(AB\) is also a side of a 28-sided regular polygon. The 21-sided regular polygon and the 28-sided regular polygon have the same center point \(P\). Find \(n\) if \(BC\) is a side of a \(n\)-sided regular polygon that has center point \(P\).

**Hint:** Sketch a circle with center \(P\) and place points \(A\), \(B\), and \(C\) on the circle.

\(n = 12\)

**Example**

If \(ABCD\) is a parallelogram, find the value of each variable.

\(AB\) and \(CD\) are opposite sides, so \(\overline{AB} \cong \overline{CD}\).

\(2x = 34\)
\(x = 17\)
\(\angle A\) and \(\angle C\) are opposite angles, so \(\angle A \cong \angle C\).

\(8b = 112\)
\(b = 14\)

**Exercises**

Find the value of each variable.

1. \(x = 30\); \(y = 22.5\)
2. \(x = 15\); \(y = 11\)
3. \(x = 2\); \(y = 4\)
4. \(x = 10\); \(y = 40\)
5. \(x = 13\); \(y = 32.5\)
6. \(x = 5\); \(y = 180\)
Skills Practice

Parallelograms

1. If \( \text{ABCD} \) is a parallelogram, then its diagonals bisect each other. \( \triangle ABC \) and \( \triangle CDA \) are congruent.

2. Diagonals of a parallelogram bisect each other.

3. \( \triangle ABC \) and \( \triangle CDA \) are congruent.

4. \( \triangle ABC \) and \( \triangle CDA \) are congruent.

5. \( \triangle ABC \) and \( \triangle CDA \) are congruent.

6. \( \triangle ABC \) and \( \triangle CDA \) are congruent.

7. \( \triangle ABC \) and \( \triangle CDA \) are congruent.

8. \( \triangle ABC \) and \( \triangle CDA \) are congruent.

9. \( \triangle ABC \) and \( \triangle CDA \) are congruent.

COORDINATE GEOMETRY

Write a paragraph proof of the following.

Prove: \( \text{ABCD} \) is a parallelogram.

Given: \( \text{ABCD} \) is a parallelogram.

Prove: \( \text{ABCD} \) is a parallelogram.

Proof: We are given \( \text{ABCD} \) is a parallelogram.

Because opposite sides of a parallelogram are congruent, therefore, \( \text{AB} \parallel \text{CD} \) and \( \text{AD} \parallel \text{BC} \).

Diagonals of parallelograms bisect each other. \( \triangle AEC \) and \( \triangle DEC \) are congruent.

\( \angle AEB \) and \( \angle CEB \) are supplementary. \( \angle AEB \) and \( \angle CEB \) are supplementary.

\( \angle AEB \) and \( \angle CEB \) are supplementary.

\( \angle AEB \) and \( \angle CEB \) are supplementary.

\( \angle AEB \) and \( \angle CEB \) are supplementary.

\( \angle AEB \) and \( \angle CEB \) are supplementary.
6-2 Practice

Parallelograms

ALGEBRA Find the value of each variable.

1. \(a = 3, b = 1\)

2. \(x = 30^\circ, y = 50^\circ\)

3. \(x = 18, y = 9\)

4. \(x = 2, y = 4.5\)

ALGEBRA Use \(\triangle RSTU\) to find each measure or value.

5. \(m\angle RST = 125^\circ\)

6. \(m\angle STU = 55^\circ\)

7. \(m\angle TUR = 125^\circ\)

8. \(a = 6\)

COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of \(\square PRYZ\) with the given vertices.

9. \(P(2, 5), R(3, 3), Y(-2, -3), Z(-3, -1)\)

10. \(P(2, 3), R(1, -2), Y(-5, -7), Z(-4, -2)\)

\((0, 1), (-1.5, -2)\)

11. PROOF Write a paragraph proof of the following.

Given: \(\square PRST\) and \(\square PQVU\)

Prove: \(\angle V \cong \angle S\)

Proof: We are given \(\square PRST\) and \(\square PQVU\). Since opposite angles of a parallelogram are congruent, \(\angle P \cong \angle V\) and \(\angle P \cong \angle S\). Since congruence of angles is transitive, \(\angle V \cong \angle S\) by the Transitive Property of Congruence.

12. CONSTRUCTION Mr. Rodriguez used the parallelogram at the right to design a herringbone pattern for a paving stone. He will use the paving stone for a sidewalk. If \(m\angle 1 = 130^\circ\), find \(m\angle 2, m\angle 3,\) and \(m\angle 4\).

50, 130, 50

Chapter 6  14

Glencoe Geometry

6-2 Word Problem Practice

Parallelograms

1. WALKWAY A walkway is made by adjoining four parallelograms as shown.

Are the end segments \(a\) and \(e\) parallel to each other? Explain.

Yes. Opposite sides of a parallelogram are parallel and the parallel property is transitive.

2. DISTANCE Four friends live at the four corners of a block shaped like a parallelogram. Gracie lives 3 miles away from Kenny. How far apart do Teresa and Travis live from each other?

3 mi

3. SOCCER Four soccer players are located at the corners of a parallelogram. Two of the players in opposite corners are the goalies. In order for goalie \(A\) to be able to see the three others, she must be able to see a certain angle \(x\) in her field of vision.

What angle does the other goalie have to be able to see in order to keep an eye on the other three players?

He or she also has to see angle \(x\).

4. VENN DIAGRAMS Make a Venn diagram showing the relationship between squares, rectangles, and parallelograms.

5. SKYSCRAPERS On vacation, Tony’s family took a helicopter tour of the city. The pilot said the newest building in the city was the building with the top view. He told Tony that the exterior angle by the front entrance is 72°. Tony wanted to know more about the building, so he drew this diagram and used his geometry skills to learn a few more things.

a. What are the measures of the four angles of the parallelogram?

72, 72, 108, 108

b. How many pairs of congruent triangles are there in the figure?

What are they?

4 pairs;

\(\triangle ABE \cong \triangle DCE\),

\(\triangle ABC \cong \triangle DBC\),

\(\triangle ACE \cong \triangle DBE\), and

\(\triangle ABD \cong \triangle DCA\)
Diagonals of Parallelograms

In some drawings, the diagonal of a parallelogram appears to be the angle bisector of both opposite angles. When might that be true?

1. Given: Parallelogram $PQRS$ with diagonal $PR$. $PR$ is an angle bisector of $\angle QPS$ and $\angle QRS$.
   What type of parallelogram is $PQRS$? Justify your answer.
   - $\angle QPS \cong \angle QRS$ opposite angles of a parallelogram
   - $\angle QPR \cong \angle SPR$ definition of angle bisector
   - $\angle QRP \cong \angle SRP$ definition of angle bisector
   - $\angle QPR \cong \angle SPR$ Alternate Interior Angles
   - $\angle QRP \cong \angle SRP$ Transitive Property
   - $SP \cong SR$ Isosceles Triangle Theorem
   So all sides are $\cong$ and $QPRS$ is a rhombus.

   Find $WK$ and $KR$.
   - $WK = 7$, $KR = 12$

3. Refer to Exercise 2. Write a statement about parallelogram $WPRK$ and angle bisector $KD$.
   Sample answer: Parallelogram $WPRK$ is not a rhombus and $KD$ is not a diagonal.

   The perimeter of triangle $PCD$ is 15.
   Find $AB$ and $BC$.
   - $AB = 4$, $BC = 9$

Exercises

Find $x$ and $y$ so that the quadrilateral is a parallelogram.

1. $2x - 2 = 6$, $y = 4$
2. $5x = 5$, $y = 25$
3. $x = 31$, $y = 5$
4. $x = 5$, $y = 3$
5. $x = 15$, $y = 9$
6. $x = 30$, $y = 15$
Study Guide and Intervention (continued)

Tests for Parallelograms

Parallelograms on the Coordinate Plane. On the coordinate plane, the Distance, Slope, and Midpoint Formulas can be used to test if a quadrilateral is a parallelogram.

EXAMPLE

Determine whether \( ABCD \) is a parallelogram.

The vertices are \( A(-2, 3), B(3, 2), C(2, -1), \) and \( D(-3, 0) \).

**Method 1:** Use the Slope Formula, \( m = \frac{y_2 - y_1}{x_2 - x_1} \).

\[
\text{slope of } \overline{AB} = \frac{-2 - 3}{3 - (-2)} = \frac{-5}{5} = -1
\]

\[
\text{slope of } \overline{BC} = \frac{-1 - 2}{2 - 3} = \frac{-3}{-1} = 3
\]

\[
\text{slope of } \overline{CD} = \frac{0 - (-1)}{-3 - 2} = \frac{1}{-5} = -\frac{1}{5}
\]

\[
\text{slope of } \overline{DA} = \frac{-1 - 0}{-3 - (-2)} = \frac{-1}{-1} = 1
\]

Since opposite sides have the same slope, \( \overline{AB} || \overline{CD} \) and \( \overline{AD} || \overline{BC} \). Therefore, \( ABCD \) is a parallelogram by definition.

**Method 2:** Use the Distance Formula, \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \).

\[
\text{Distance of } \overline{AB} = \sqrt{(-2 - 3)^2 + (3 - 2)^2} = \sqrt{25 + 1} = \sqrt{26}
\]

\[
\text{Distance of } \overline{CD} = \sqrt{(2 - 3)^2 + (2 - (-1))^2} = \sqrt{1 + 9} = \sqrt{10}
\]

\[
\text{Distance of } \overline{DA} = \sqrt{(-3 - 2)^2 + (0 - (-1))^2} = \sqrt{25 + 1} = \sqrt{26}
\]

\[
\text{Distance of } \overline{BC} = \sqrt{(2 - 3)^2 + (-1 - 0)^2} = \sqrt{1 + 9} = \sqrt{10}
\]

Since both pairs of opposite sides have the same length, \( \overline{AB} \equiv \overline{CD} \) and \( \overline{AD} \equiv \overline{BC} \). Therefore, \( ABCD \) is a parallelogram by Theorem 6.9.

**Exercises**

Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

**See students’ work**

1. \( A(0, 0), B(1, 3), C(3, 3), D(4, 0) \); Slope Formula
   - yes

2. \( D(-1, 1), E(2, 4), F(6, 4), G(3, 1) \); Slope Formula
   - yes

3. \( R(-1, 0), S(3, 0), T(-2, -3), U(-3, -2) \); Distance Formula
   - no

4. \( A(-3, 2), B(-1, 4), C(2, 1), D(0, -1) \); Distance and Slope Formulas
   - yes

5. \( S(-2, 4), T(-1, -1), U(3, -4), V(2, 1) \); Distance and Slope Formulas
   - yes

6. \( F(3, 3), G(1, 2), H(-3, -1), I(-1, 4) \); Midpoint Formula
   - no

7. A parallelogram has vertices \( R(-2, -1), S(2, 1), \) and \( T(0, -3) \). Find all possible coordinates for the fourth vertex. \((4, -1), (0, 3), \) or \((-4, -5)\)

**Answers**

1. yes
2. yes; a pair of opposite sides is parallel and congruent.
3. No; none of the tests for parallelograms is fulfilled.
4. Yes; both pairs of opposite sides are congruent.

**COORDINATE GEOMETRY** Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

**See students’ graphs.**

5. \( P(0, 0), Q(3, 4), R(-7, 4), S(-4, 0) \); Slope Formula
   - yes; the slope of \( \overline{PQ} \) and \( \overline{RS} \) are equal and the slope of \( \overline{PR} \) and \( \overline{QS} \) is \( \perp \) to \( \overline{RS} \) and \( \overline{PR} \) parallel.

6. \( S(-2, 1), T(1, 3), U(7, 0), V(-1, -2) \); Distance and Slope Formulas
   - yes; \( SR = ZT \) and the slopes of \( \overline{SR} \) and \( \overline{ZT} \) are equal, so one pair of opposite sides is parallel and congruent.

7. \( W(2, 5), X(3, 3), Y(-2, -3), Z(-3, 1) \); Midpoint Formula
   - No; the midpoints of the diagonals are not the same point.

**ALGEBRA** Find \( x \) and \( y \) so that each quadrilateral is a parallelogram.

8. \( x = 24, y = 19 \)

9. \( x = 3, y = 14 \)

10. \( x = 45, y = 20 \)

11. \( x = 17, y = 9 \)
### 6-3 Practice

#### Tests for Parallelograms

Determine whether each quadrilateral is a parallelogram. Justify your answer.

1. **Yes; the diagonals bisect each other.**

2. **No; none of the tests for parallelograms is fulfilled.**

3. **Yes; both pairs of opposite angles are congruent.**

4. **Yes; the lengths of the opposite sides are congruent.**

#### Coordinate Geometry

Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

See students' work.

5. \( P(-5, 1), S(-2, 2), R(-1, -3), T(2, -2); \) Slope Formula

6. \( R(-2, 5), O(1, 3), M(-3, -4), Y(-6, -2); \) Distance and Slope Formulas

#### Algebra

Find \( x \) and \( y \) so that the quadrilateral is a parallelogram.

7. \( \frac{5x + 20}{7x - 11} = \frac{9y - 15}{y - 5} \)

   \( x = 20, \ y = 12 \)

8. \( \frac{-6x + 6}{3x + 6} = \frac{4x - 8 + 2}{2x - 4} \)

   \( x = -6, \ y = 13 \)

9. \( \frac{x + 3}{7y - 7} = \frac{-4x + 6}{4x - 6} \)

   \( x = -3, \ y = 2 \)

10. \( \frac{x - 2}{y + 3} = \frac{4x + 5}{2x - 4} \)

    \( x = -2, \ y = -5 \)

11. **TILE DESIGN** The pattern shown in the figure is to consist of congruent parallelograms. How can the designer be certain that the shapes are parallelograms?

    **Sample answer:** Confirm that both pairs of opposite \( \Delta \) are \( \cong \).

### 6-3 Word Problem Practice

#### Tests for Parallelograms

1. **BALANCING** Nikia, Madison, Angela, and Shelby are balancing themselves on an “X”-shaped floating object. To balance themselves, they want to make themselves the vertices of a parallelogram.

   In order to achieve this, do all four of them have to be the same distance from the center of the object? Explain.

   **Yes; the lengths of the opposite sides are congruent.**

2. **COMPASSES** Two compass needles placed side by side on a table are both 2 inches long and point due north. Do they form the sides of a parallelogram?

   **Yes**

3. **FORMATION** Four jets are flying in formation. Three of the jets are shown in the graph. If the four jets are located at the vertices of a parallelogram, what are the three possible locations of the missing jet?

   Sample answer: Confirm that both pairs of opposite \( \Delta \) are \( \cong \).

4. **STREET LAMPS** When a coordinate plane is placed over the Harrisville town map, the four street lamps in the center are located as shown. Do the four lamps form the vertices of a parallelogram? Explain.

   **Yes; the lengths of the opposite sides are congruent.**

5. **PICTURE FRAME** Aaron is making a wooden picture frame in the shape of a parallelogram. He has two pieces of wood that are 3 feet long and two that are 4 feet long.

   a. If he connects the pieces of wood at their ends to each other, in what order must he connect them to make a parallelogram?

      **He must alternate the lengths 3, 4, 3 or 4, 3, 4.**

   b. How many different parallelograms could he make with these four lengths of wood?

      **Infinite many.**

   c. Explain something Aaron might do to specify precisely the shape of the parallelogram.

      **Sample answer:** He could specify the length of a diagonal.
Chapter 6

6-3 Enrichment

Tests for Parallelograms

By definition, a quadrilateral is a parallelogram if and only if both pairs of opposite sides are parallel. What conditions other than both pairs of opposite sides parallel will guarantee that a quadrilateral is a parallelogram? In this activity, several possibilities will be investigated by drawing quadrilaterals to satisfy certain conditions. Remember that any test that seems to work is not guaranteed to work unless it can be formally proven.

Complete.

1. Draw a quadrilateral with one pair of opposite sides congruent. Must it be a parallelogram? no

2. Draw a quadrilateral with both pairs of opposite sides congruent. Must it be a parallelogram? yes

3. Draw a quadrilateral with one pair of opposite sides parallel and the other pair of opposite sides congruent. Must it be a parallelogram? no

4. Draw a quadrilateral with one pair of opposite sides both parallel and congruent. Must it be a parallelogram? yes

5. Draw a quadrilateral with one pair of opposite angles congruent. Must it be a parallelogram? no

6. Draw a quadrilateral with both pairs of opposite angles congruent. Must it be a parallelogram? yes

7. Draw a quadrilateral with one pair of opposite sides parallel and one pair of opposite angles congruent. Must it be a parallelogram? yes

6-4 Study Guide and Intervention

Rectangles

Properties of Rectangles

A rectangle is a quadrilateral with four right angles. Here are the properties of rectangles.

• Opposite sides are parallel.
• Opposite angles are congruent.
• Opposite sides are congruent.
• Consecutive angles are supplementary.
• The diagonals bisect each other.

Also:
• All four angles are right angles.
• \( \angle UTS, \angle TSR, \angle SRT, \) and \( \angle RUT \) are right angles.
• The diagonals are congruent.

Example 1

Quadrilateral RUTS above is a rectangle. If \( US = 6x + 3 \) and \( RT = 7x - 2 \), find \( x \).

The diagonals of a rectangle are congruent, so \( US = RT \).

\[ 6x + 3 = 7x - 2 \]
\[ 3 = x \]
\[ 5 = x \]

Example 2

Quadrilateral RUTS above is a rectangle. If \( m\angle STR = 8x + 3 \) and \( m\angle UTR = 16x - 9 \), find \( m\angle STR \).

\( \angle UTS \) is a right angle, so \( m\angle STR + m\angle UTR = 90 \).

\[ 8x + 3 + 16x - 9 = 90 \]
\[ 24x - 6 = 90 \]
\[ 24x = 96 \]
\[ x = 4 \]

\( m\angle STR = 8x + 3 = 8(4) + 3 \) or 35

Exercises

Quadrilateral \( ABCD \) is a rectangle.

1. If \( AB = 36 \) and \( CE = 2x - 4 \), find \( x \). 20
2. If \( BE = 6y + 2 \) and \( CE = 4y + 6 \), find \( y \). 2
3. If \( BC = 24 \) and \( AD = 5y - 1 \), find \( y \). 5
4. If \( m\angle BEA = 62 \), find \( m\angle BAC \). 59
5. If \( m\angle AED = 12x \) and \( m\angle BDC = 10x + 20 \), find \( m\angle AED \). 120
6. If \( BD = 8y - 4 \) and \( AC = 7y + 3 \), find \( BD \). 52
7. If \( m\angle DBC = 10x \) and \( m\angle ACB = 4x^2 - 6 \), find \( m\angle ACB \). 30
8. If \( AB = 6y \) and \( BC = 8y \), find \( BD \) in terms of \( y \). 10y
6-4 Study Guide and Intervention (continued)

Rectangles

Prove that Parallelograms Are Rectangles The diagonals of a rectangle are congruent, and the converse is also true.

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

In the coordinate plane, you can use the Distance Formula, the Slope Formula, and properties of diagonals to show that a figure is a rectangle.

Example Quadrilateral ABCD has vertices A(−3, 0), B(−2, 3), C(4, 1), and D(3, −2). Determine whether ABCD is a rectangle.

Method 1: Use the Distance Formula.

\[ AB = \sqrt{(-3 - (-2))^2 + (0 - 3)^2} = 5 \]
\[ BC = \sqrt{(-2 - 4)^2 + (3 - 1)^2} = 4 \]
\[ CD = \sqrt{(4 - 3)^2 + (1 - 3)^2} = 2 \]
\[ DA = \sqrt{(-3 - 3)^2 + (0 - (-2))^2} = 5 \]

Opposite sides are congruent, thus ABCD is a parallelogram.

Method 2: Use the Slope Formula.

\[ \text{slope of AB} = \frac{-3 - 0}{-2 - (-3)} = \frac{3}{1} \quad \text{slope of AD} = \frac{-3 - 3}{-3 - 3} = 1 \]
\[ \text{slope of BC} = \frac{3 - 1}{4 - (-2)} = \frac{2}{3} \quad \text{slope of CD} = \frac{-2 - 3}{3 - 4} = 1 \]

Opposite sides are parallel, so the figure is a parallelogram. Consecutive sides are perpendicular, so ABCD is a rectangle.

Exercises

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

See students’ work.

1. A(−3, 1), B(−3, 3), C(3, 3), D(3, 1); Distance Formula
   Yes; AB = 2, BC = 6, CD = 2, DA = 6, AC = 4\sqrt{2}, BD = 4\sqrt{2}, opposite sides and diagonals are congruent.
2. A(−3, 0), B(−2, 3), C(4, 5), D(3, 2); Distance Formula
   No; slope of AB = 3, slope of BC = 1, slopes show that two consecutive sides are not perpendicular.
3. A(−3, 0), B(−2, 2), C(3, 0), D(2, −2); Distance Formula
   No; AC = 6, BD = 2\sqrt{2}, diagonals are not congruent.
4. A(1, 0), B(0, 2), C(4, 0), D(3, −2); Distance Formula
   Yes; AB = \sqrt{5}, BC = \sqrt{20}, CD = \sqrt{5}, DA = \sqrt{20}, AC = 5, BD = 5, opposite sides and diagonals are congruent.

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

See students’ work.

1. RSTV is a rectangle.
2. \(\angle V \cong \angle T\)
3. \(\angle V \equiv \angle T\)
4. U is the midpoint of V\(\overline{T}\).
5. UV \equiv TU
6. VR \equiv TS
7. \(\triangle RUV \equiv \triangle SUT\)

Chapter 6 24 Glencoe Geometry

6-4 Skills Practice

Rectangles

ALGEBRA Quadrilateral ABCD is a rectangle.

1. If AC = 2x + 13 and DB = 4x − 1, find DB.
2. If AC = x + 3 and DB = 3x − 19, find AC.
3. If AE = 3x + 3 and EC = 5x − 15, find AC.
4. If DE = 6x − 7 and AE = 4x + 9, find DB.
5. If m\(\angle DAC\) = 2x + 4 and m\(\angle BAC\) = 3x + 1, find m\(\angle BAC\).
6. If m\(\angle BDC\) = 7x − 1 and m\(\angle ADB\) = 9x − 3, find m\(\angle BDC\).
7. If m\(\angle ABD\) = 7x − 31 and m\(\angle CDB\) = 4x + 5, find m\(\angle ABD\).
8. If m\(\angle BAC\) = x + 3 and m\(\angle CAD\) = x + 15, find m\(\angle BAC\).

PROOF: Write a two-column proof.

Given: RSTV is a rectangle and U is the midpoint of V\(\overline{T}\).

Prove: \(\triangle RUV \cong \triangle SUT\)

Proof

Statements

1. RSTV is a rectangle.
2. \(\angle V\) and \(\angle T\) are right angles.
3. \(\angle V \cong \angle T\)
4. U is the midpoint of V\(\overline{T}\).
5. UV \equiv TU
6. VR \equiv TS
7. \(\triangle RUV \cong \triangle SUT\)

Reasons

1. Given
2. Definition of rectangle
3. All rt \(\angle\)s are \(\cong\)
4. Given
5. Definition of midpoint
6. Opp sides of \(\square\) are congruent
7. SAS

Answers (Lesson 6-4)

Chapter 6 25 Glencoe Geometry
6-4 Practice

Rectangles

ALGEBRA Quadrilateral $RSTU$ is a rectangle.

1. If $UZ = x + 21$ and $ZS = 3x - 15$, find $US$. 78
2. If $RZ = 3x + 8$ and $ZS = 6x - 28$, find $UZ$. 44
3. If $RT = 5x + 8$ and $RZ = 4x + 1$, find $ZT$. 9
4. If $m\angle SUT = 3x + 6$ and $m\angle RUS = 5x - 4$, find $m\angle SUT$. 39
5. If $m\angle SRT = x + 9$ and $m\angle UTR = 2x - 44$, find $m\angle UTR$. 62
6. If $m\angle BSU = x + 41$ and $m\angle TUS = 3x + 9$, find $m\angle BSU$. 57

Quadrilateral $GHJK$ is a rectangle. Find each measure if $m\angle 1 = 37$.

7. $m\angle 2$ 53
8. $m\angle 3$ 37
9. $m\angle 4$ 37
10. $m\angle 5$ 53
11. $m\angle 6$ 106
12. $m\angle 7$ 74

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

See students' work

13. $B(-4, 3), G(-2, 4), H(1, -2), L(-1, -3)$; Slope Formula
   Yes; sample answer: Opposite sides are parallel and consecutive sides are perpendicular.

14. $N(-4, 5), O(6, 0), P(3, -6), Q(-7, -13)$; Distance Formula
   Yes; sample answer: Opposite sides are congruent and diagonals are congruent.

15. $O(0, 5), D(4, 7), E(5, 4), F(1, 2)$; Slope Formula
   No; sample answer: Diagonals are not congruent.

16. LANDSCAPING Huntington Park officials approved a rectangular plot of land for a Japanese Zen garden. Is it sufficient to know that opposite sides of the garden plot are congruent and parallel to determine that the garden plot is rectangular? Explain.
   No; if you only know that opposite sides are congruent and parallel, the most you can conclude is that the plot is a parallelogram.

6-4 Word Problem Practice

Rectangles

1. FRAMES Jalen makes the rectangular frame shown.

   In order to make sure that it is a rectangle, Jalen measures the distances $BD$ and $AC$. How should these two distances compare if the frame is a rectangle? They should be equal.

2. BOOKSHELVES A bookshelf consists of two vertical planks with five horizontal shelves. Are each of the four sections for books rectangles? Explain.
   Yes; each of the four angles of each rectangle is created by a straight horizontal line and a straight vertical line, so each has four $90^\circ$ angles.

3. LANDSCAPING A landscaper is marking off the corners of a rectangular plot of land. Three of the corners are in place as shown.

   What are the coordinates of the fourth corner? $(-6, 3)$

4. SWIMMING POOLS Antonio is designing a swimming pool on a coordinate grid. Is it a rectangle? Explain.
   Yes. Sample answer: The slope of the long sides is $3$ and the slope of the short sides is $-\frac{1}{3}$, so each pair of sides is parallel. Also, the long and short sides have slopes that are perpendicular to each other, making the four corners right angles.

5. PATTERNS Veronica made the pattern shown out of $7$ rectangles with four equal sides. The side length of each rectangle is written inside the rectangle.

   a. How many rectangles can be formed using the lines in this figure? 11
   b. If Veronica wanted to extend her pattern by adding another rectangle with $4$ equal sides to make a larger rectangle, what are the possible side lengths of rectangles that she can add? 8 and 13 units
6-4 Enrichment

Constant Perimeter
Douglas wants to fence a rectangular region of his backyard for his dog. He bought 200 feet of fence.

1. Complete the table to show the dimensions of five different rectangular pens that would use the entire 200 feet of fence. Then find the area of each rectangular pen.

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>Length</th>
<th>Width</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>80</td>
<td>20</td>
<td>1600</td>
</tr>
<tr>
<td>200</td>
<td>70</td>
<td>30</td>
<td>2100</td>
</tr>
<tr>
<td>200</td>
<td>60</td>
<td>40</td>
<td>2400</td>
</tr>
<tr>
<td>200</td>
<td>50</td>
<td>50</td>
<td>2500</td>
</tr>
<tr>
<td>200</td>
<td>45</td>
<td>55</td>
<td>2475</td>
</tr>
</tbody>
</table>

2. Do all five of the rectangular pens have the same area? If not, which one has the larger area?

No, the 50 x 50 pen has the largest area.

3. Write a rule for finding the dimensions of a rectangle with the largest possible area for a given perimeter.

The rectangle with the largest area for a given perimeter is a square.

4. Let x represent the length of a rectangle and y the width. Write the formula for all rectangles with a perimeter of 200. Then graph this relationship on the coordinate plane at the right.

2x + 2y = 200 or x + y = 100

Julio read that a dog the size of his new pet, Bennie, should have at least 100 square feet in his pen. Before going to the store to buy fence, Julio made a table to determine the dimensions for Bennie’s rectangular pen.

5. Complete the table to find five possible dimensions of a rectangular fenced area of 100 square feet.

<table>
<thead>
<tr>
<th>Area</th>
<th>Length</th>
<th>Width</th>
<th>How much fence to buy</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1</td>
<td>100</td>
<td>202</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
<td>50</td>
<td>104</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>25</td>
<td>58</td>
</tr>
<tr>
<td>100</td>
<td>5</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>100</td>
<td>10</td>
<td>10</td>
<td>40</td>
</tr>
</tbody>
</table>

6. Julio wants to save money by purchasing the least number of feet of fencing to enclose the 100 square feet. What will be the dimensions of the completed pen?

10 ft x 10 ft

7. Write a rule for finding the dimensions of a rectangle with the least possible perimeter for a given area.

The rectangle with the least possible perimeter for a given area is a square.

8. For length x and width y, write a formula for the area of a rectangle with an area of 100 square feet. Then graph the formula.

xy = 100

A13

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Chapter 6

Glencoe Geometry
**6-4 Geometer's Sketchpad Activity**

**Exploring Rectangles**

A quadrilateral with four right angles is a rectangle. The Geometer's Sketchpad is a useful tool for exploring some of the characteristics of a rectangle. Use the following steps to draw a rectangle.

**Step 1** Use the Line tool to draw a line anywhere on the screen.

**Step 2** Use the Point tool to draw a point that is not on the line. To draw a line perpendicular to the first line you drew, select the first line and the point. Then choose **Perpendicular Line** from the **Construct** menu.

**Step 3** Use the Point tool to draw a point that is not on either of the lines you have drawn. Repeat the procedure in Step 2 to draw lines perpendicular to the two lines you have drawn.

A rectangle is formed by the segments whose endpoints are the points of intersection of the lines.

**Exercises**

Use the measuring capabilities of The Geometer’s Sketchpad to explore the characteristics of a rectangle.

1. What appears to be true about the opposite sides of the rectangle that you drew? Make a conjecture and then measure each side to check your conjecture.

   The opposite sides of a rectangle are congruent.

2. Draw the diagonals of the rectangle by using the Selection Arrow tool to choose two opposite vertices. Then choose **Segment** from the **Construct** menu to draw the diagonal. Repeat to draw the other diagonal.
   a. Measure each diagonal. What do you observe?

   The diagonals of a rectangle are congruent.

   b. What is true about the triangles formed by the sides of the rectangle and a diagonal? Justify your conclusion.

   The triangles are congruent by SSS.

3. Choose **New Sketch** from the **File** menu and follow steps 1–3 to draw another rectangle. Do the relationships you found for the first rectangle you drew hold true for this rectangle also?

   Yes, the opposite sides are congruent and the diagonals are congruent.

---

**6-5 Study Guide and Intervention**

**Rhombi and Squares**

**Properties of Rhombi and Squares** A rhombus is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.

- The diagonals are perpendicular. \( MH \perp RO \)
- Each diagonal bisects a pair of opposite angles.
- \( MH \) bisects \( \angle RMO \) and \( \angle RHO \).
- \( RO \) bisects \( \angle RMO \) and \( \angle RHO \).

A square is a parallelogram with four congruent sides and four congruent angles. A square is both a rectangle and a rhombus; therefore, all properties of parallelograms, rectangles, and rhombi apply to squares.

**Example**

In rhombus \( ABCD \), \( m \angle BAC = 32 \). Find the measure of each numbered angle.

\( ABCD \) is a rhombus, so the diagonals are perpendicular and \( \triangle ABE \) is a right triangle. Thus \( m \angle 4 = 90 \) and \( m \angle 1 = 90 - 32 \) or 58. The diagonals in a rhombus bisect the vertex angles, so \( m \angle 1 = m \angle 2 \). Thus, \( m \angle 2 = 58 \).

A rhombus is a parallelogram, so the opposite sides are parallel. \( \angle BAC \) and \( \angle CDB \) are alternate interior angles for parallel lines, so \( m \angle 3 = 32 \).

**Exercises**

Quadrilateral \( ABCD \) is a rhombus. Find each value or measure.

1. If \( m \angle ABD = 60 \), find \( m \angle BDC \).
2. If \( AE = 8 \), find \( AC \).
3. If \( AB = 26 \) and \( BD = 20 \), find \( AE \).
4. Find \( m \angle CEB \).
5. If \( m \angle CDB = 58 \), find \( m \angle ACB \).
6. If \( AE = 3x - 1 \) and \( AC = 16 \), find \( x \).
7. If \( m \angle CDB = 6y \) and \( m \angle ACB = 2y + 10 \), find \( y \).
8. If \( AD = 2x + 4 \) and \( CD = 4x - 4 \), find \( x \).
Conditions for Rhombi and Squares The theorems below can help you prove that a parallelogram is a rectangle, rhombus, or square.

1. If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.
2. If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.
3. If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.
4. If a parallelogram is both a rectangle and a rhombus, then it is a square.

5. Definition of segment congruence

6. Definition of angle congruence

7. If a quadrilateral is both a rectangle and a rhombus, then it is a square.

Example Determine whether parallelogram ABCD with vertices A(−3, −3), B(1, 1), C(3, −3), D(1, −7) is a rhombus, a rectangle, or a square.

AC = \sqrt{(-3 - 3)^2 + (-3 - (-3))^2} = \sqrt{64} = 8
BD = \sqrt{(1-1)^2 + (-7-1)^2} = \sqrt{64} = 8

The diagonals are the same length; the figure is a rectangle.

Slope of AC = \frac{-3 - (-3)}{-3 - 5} = \frac{0}{-8} = 0

Slope of BD = \frac{1-(-7)}{1-1} = \frac{8}{0} = \text{undefined}

The line is vertical.

Since a horizontal and vertical line are perpendicular, the diagonals are perpendicular. Parallelogram ABCD is a square which is also a rhombus and a rectangle.

Exercises
Given each set of vertices, determine whether \( \square ABCD \) is a rhombus, rectangle, or square. List all that apply. Explain.

1. A(0, 2), B(2, 4), C(4, 2), D(2, 0)
   - Rectangles, rhombus, square; the four sides are \( \parallel \) and consecutive sides are \( \perp \).

2. A(−2, 1), B(−1, 3), C(3, 1), D(2, −1)
   - Rectangle; consecutive sides are \( \perp \).

3. A(−2, −1), B(0, 2), C(2, −1), D(0, −4)
   - Rhombus; the four sides are \( \parallel \) and consecutive sides are not \( \perp \).

4. A(−3, 0), B(−1, 3), C(5, −1), D(3, −4)
   - Rectangle; consecutive sides are \( \perp \).

5. PROOF Write a two-column proof.
   - Given: Parallelogram RSTU, RS \( \equiv \) ST
   - Prove: RSTU is a rhombus.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. RSTU is a parallelogram.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. RS ( \equiv ) UT, RU ( \equiv ) ST</td>
<td>2. Definition of a parallelogram</td>
</tr>
<tr>
<td>3. UT ( \equiv ) RS ( \equiv ) ST ( \equiv ) RU</td>
<td>3. Substitution</td>
</tr>
<tr>
<td>4. RSTU is a rhombus</td>
<td>4. Definition of a rhombus</td>
</tr>
</tbody>
</table>

COORDINATE GEOMETRY Given each set of vertices, determine whether \( \square QRST \) is a rectangle, a rectangle, or a square. List all that apply. Explain.

8. Q(3, 5), R(3, 1), S(−1, 1), T(−1, 5)
   - Rhombus; all sides are congruent and the diagonals are perpendicular and congruent.

9. Q(−5, 12), R(5, 12), S(−1, 4), T(−11, 4)
   - Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.

10. Q(−6, −1), R(4, −6), S(2, 5), T(−8, 10)
   - Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.

   None; opposite sides are congruent, but the diagonals are neither congruent nor perpendicular.
6-5 Practice

Rhombi and Squares

PRYZ is a rhombus. If RK = 5, RY = 13 and m∠YRZ = 67°, find each measure.

1. KY 12
2. PK 12
3. m∠YKZ 90°
4. m∠ZPR 67°

MNPQ is a rhombus. If PQ = 3√2 and AP = 3, find each measure.

5. AQ 3
6. m∠APQ 45°
7. m∠MNP 90°
8. PM 6

COORDINATE GEOMETRY Given each set of vertices, determine whether □BEFG is a rhombus, a rectangle, or a square. List all that apply. Explain.

9. B(−9, 1), E(2, 3), F(12, −2), G(1, −4)
   Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.

10. B(1, 3), E(7, −3), F(1, −9), G(−5, −3)
    Rhombus, rectangle, square; all sides are congruent and the diagonals are perpendicular and congruent.

11. B(−4, −5), E(1, −5), F(−2, −1), G(−7, −1)
    Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.

12. TESSELLATIONS The figure is an example of a tessellation. Use a ruler or protractor to measure the shapes and then name the quadrilaterals used to form the figure.
    The figure consists of 6 congruent rhombi.

---

6-5 Word Problem Practice

Rhombi and Squares

1. TRAY RACKS A tray rack looks like a parallelogram from the side. The levels for the trays are evenly spaced.

   What two labeled points form a rhombus with base AA? F and F'.
   2. SLICING Charles cuts a rhombus along both diagonals. He ends up with four congruent triangles. Classify these triangles as acute, obtuse, or right.

   right triangles

   3. WINDOWS The edges of a window are drawn in the coordinate plane.

   Determine whether the window is a square or a rhombus. rhombus

   Sample answer: The diagonals of a square create four congruent right isosceles triangles. When two congruent right isosceles triangles are joined along their hypotenuses, the result is a quadrilateral with 4 equal sides and 4 right angle corners (because 45° + 45° = 90°), making it a square.

   a. What are the angles of the corner rhombi? 45°, 135°, 45°, 135°

   b. What kinds of quadrilaterals are the dotted and checkered figures? They are all squares.

   Sample answer: The diagonals of a square create four congruent right isosceles triangles. When two congruent right isosceles triangles are joined along their hypotenuses, the result is a quadrilateral with 4 equal sides and 4 right angle corners (because 45° + 45° = 90°), making it a square.

   a. What are the angles of the corner rhombi? 45°, 135°, 45°, 135°

   b. What kinds of quadrilaterals are the dotted and checkered figures? They are all squares.
Creating Pythagorean Puzzles

By drawing two squares and cutting them in a certain way, you can make a puzzle that demonstrates the Pythagorean Theorem. A sample puzzle is shown. You can create your own puzzle by following the instructions below.

1. Carefully construct a square and label the length of a side as $a$. Then construct a smaller square to the right of it and label the length of a side as $b$, as shown in the figure above. The bases should be adjacent and collinear.

2. Mark a point X that is $b$ units from the left edge of the larger square. Then draw the segments from the upper left corner of the larger square to point $X$, and from point $X$ to the upper right corner of the smaller square.

3. Cut out and rearrange your five pieces to form a larger square. Draw a diagram to show your answer.

4. Verify that the length of each side is equal to $\sqrt{a^2 + b^2}$.

See students’ work. A sample answer is shown.

**Enrichment**

**Study Guide and Intervention**

**Trapezoids and Kites**

**Properties of Trapezoids** A trapezoid is a quadrilateral with exactly one pair of parallel sides. The midsegment or median of a trapezoid is the segment that connects the midpoints of the legs of the trapezoid. Its measure is equal to one-half the sum of the lengths of the bases. If the legs are congruent, the trapezoid is an isosceles trapezoid. In an isosceles trapezoid both pairs of base angles are congruent and the diagonals are congruent.

**Example**

The vertices of $ABCD$ are $A(-3, -1), B(-1, 3), C(2, 3)$, and $D(4, -1)$. Show that $ABCD$ is a trapezoid and determine whether it is an isosceles trapezoid.

- slope of $AB = \frac{-1 - (-3)}{-1 - (-3)} = \frac{4}{2} = 2$
- $AB = \sqrt{(-3 - (-1))^2 + (-1 - 3)^2} = \sqrt{4 + 16} = 2\sqrt{5}$
- slope of $AD = \frac{-1 - 3}{4 - (-3)} = \frac{-4}{7} = -\frac{4}{7}$
- $CD = \sqrt{(2 - (-3))^2 + (3 - (-1))^2} = \sqrt{25 + 16} = 5\sqrt{5}$

Exactly two sides are parallel, $\overline{AB}$ and $\overline{CD}$, so $ABCD$ is a trapezoid. $AB = CD$, so $ABCD$ is an isosceles trapezoid.

**Exercises**

Find each measure.

1. $m \angle D = 55$

2. $m \angle L = 140$

COORDINATE GEOMETRY For each quadrilateral with the given vertices, verify that the quadrilateral is a trapezoid and determine whether the figure is an isosceles trapezoid.

3. $A(-1, 1), B(3, 2), C(1, -2), D(-2, -1)$

4. $J(1, 3), K(3, 1), L(-2, -2), M(-2, 3)$

$AD \parallel BC, AB \parallel CD$, $ABCD$ is a trapezoid but not an isosceles trapezoid because $\overline{AB} = \sqrt{17}$, $CD = \sqrt{10}$.

For trapezoid $HJKL$, $M$ and $N$ are the midpoints of the legs.

5. If $HJ = 32$ and $LK = 60$, find $MN$. 46

6. If $HJ = 18$ and $MN = 28$, find $LK$. 38
**6-6 Study Guide and Intervention (continued)**

**Trapezoids and Kites**

**Properties of Kites** A kite is a quadrilateral with exactly two pairs of consecutive congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.

The diagonals of a kite are perpendicular.

For kite $RMNP$, $MP \perp RN$

In a kite, exactly one pair of opposite angles is congruent.

For kite $RMNP$, $\angle M \cong \angle P$

**Example 1** If $WXYZ$ is a kite, find $\angle Z$.

The measures of $\angle Y$ and $\angle W$ are not congruent, so $\angle X \cong \angle Z$.

$m \angle X + m \angle Y + m \angle Z + m \angle W = 360$

$m \angle X + 60 + m \angle Z + 80 = 360$

$m \angle X + m \angle Z = 220$

$m \angle X = 110, m \angle Z = 110$

**Example 2** If $ABCD$ is a kite, find $BC$.

The diagonals of a kite are perpendicular. Use the Pythagorean Theorem to find the missing length.

$BP^2 + PC^2 = BC^2$

$5^2 + 12^2 = BC^2$

$169 = BC^2$

$13 = BC$

**Exercises**

If $GHJK$ is a kite, find each measure.

1. Find $m \angle JHK$. $90$

2. If $BJ = 3$ and $RK = 10$, find $JK$. $\sqrt{109}$

3. If $m \angle GHJ = 90$ and $m \angle GJK = 110$, find $m \angle HJ$. $80$

4. If $HJ = 7$, find $HG$. $7$

5. If $HG = 7$ and $GR = 5$, find $HR$. $\sqrt{24} = 2\sqrt{6}$

6. If $m \angle GHJ = 52$ and $m \angle GJK = 95$, find $m \angle HJ$. $106.5$

---

**6-6 Skills Practice**

**Trapezoids and Kites**

**ALGEBRA** Find each measure.

1. $m \angle S = 117$

2. $m \angle M = 38$

3. $m \angle D = 127$

4. $RH = 4\sqrt{34}$

**Exercises**

If trapezoid $HJKL$, $T$ and $S$ are midpoints of the legs.

5. If $HJ = 14$ and $LK = 42$, find $TS$. $28$

6. If $LK = 19$ and $TS = 15$, find $HJ$. $11$

7. If $HJ = 7$ and $TS = 10$, find $LK$. $13$

8. If $KL = 17$ and $JH = 9$, find $ST$. $13$

**COORDINATE GEOMETRY** $EFGH$ is a quadrilateral with vertices $E(1, 3)$, $F(5, 0)$, $G(8, -5)$, $H(-4, 4)$.

9. Verify that $EFGH$ is a trapezoid.

$EF \parallel GH$, but $HE \parallel FG$

10. Determine whether $EFGH$ is an isosceles trapezoid. Explain.

not isosceles; $EH = \sqrt{26}$ and $FG = \sqrt{34}$
### Practice

**Trapezoids and Kites**

Find each measure.

1. \( \angle T \) = 60°
2. \( \angle Y \) = 112°
3. \( \angle Q \) = 101°
4. \( BC = \sqrt{65} \)

#### ALGEBRA

For trapezoid \( FEDC \), \( V \) and \( Y \) are midpoints of the legs.

5. If \( FE = 18 \) and \( VY = 28 \), find \( CD \).
6. If \( m \angle F = 140 \) and \( m \angle E = 125 \), find \( m \angle D \).

#### COORDINATE GEOMETRY

\( RSTU \) is a quadrilateral with vertices \( R(−3, −3) \), \( S(5, 1) \), \( T(10, −2) \), \( U(−4, −9) \).

7. Verify that \( RSTU \) is a trapezoid. \( RS \parallel TU \)
8. Determine whether \( RSTU \) is an isosceles trapezoid. Explain.
   - not isosceles; \( RU = \sqrt{37} \) and \( ST = \sqrt{34} \)
9. **CONSTRUCTION** A set of stairs leading to the entrance of a building is designed in the shape of an isosceles trapezoid with the longer base at the bottom of the stairs and the shorter base at the top. If the bottom of the stairs is 21 feet wide and the top is 14 feet wide, find the width of the stairs halfway to the top. 17.5 ft
10. **DESK TOPS** A carpenter needs to replace several trapezoid-shaped desktops in a classroom. The carpenter knows the lengths of both bases of the desktop. What other measurements, if any, does the carpenter need?
   - Sample answer: the measures of the base angles

### Word Problem Practice

**Trapezoids and Kites**

1. **PERSPECTIVE** Artists use different techniques to make things appear to be 3-dimensional when drawing in two dimensions. Kevin drew the walls of a room. In real life, all of the walls are rectangles. In what shape did he draw the side walls to make them appear 3-dimensional?

2. **PLAZA** In order to give the feeling of spaciousness, an architect decides to make a plaza in the shape of a kite. Three of the four corners of the plaza are shown on the coordinate plane. If the fourth corner is in the first quadrant, what are its coordinates?

3. **AIRPORTS** A simplified drawing of the reef runway complex at Honolulu International Airport is shown below.

   How many trapezoids are there in this image? 5

4. **LIGHTING** A light outside a room shines straight through the door and illuminates a trapezoidal region \( ABCD \) on the floor.

   Under what circumstances would trapezoid \( ABCD \) be isosceles?

   When the light source is an equal distance from \( C \) and \( D \), shining straight through the door.

5. **RISERS** A riser is designed to elevate a speaker. The riser consists of 4 trapezoidal sections that can be stacked one on top of the other to produce trapezoids of varying heights.

   All of the stages have the same height. If all four stages are used, the width of the top of the riser is 10 feet.

   a. If only the bottom two stages are used, what is the width of the top of the resulting riser? 15 ft
   b. What would be the width of the riser if the bottom three stages are used? 12.5 ft
Quadrilaterals in Construction

Quadrilaterals are often used in construction work.

1. The diagram at the right represents a roof frame and shows many quadrilaterals. Find the following shapes in the diagram and shade in their edges. See students’ work.
   a. isosceles triangle
   b. scalene triangle
   c. rectangle
   d. rhombus
   e. trapezoid (not isosceles)
   f. isosceles trapezoid

2. The figure at the right represents a window. The wooden part between the panes of glass is 3 inches wide. The frame around the outer edge is 9 inches wide. The outside measurements of the frame are 60 inches by 81 inches. The height of the top and bottom panes is the same. The top three panes are the same size.
   a. How wide is the bottom pane of glass? 42 in.
   b. How wide is each top pane of glass? 12 in.
   c. How high is each pane of glass? 30 in.

3. Each edge of this box has been reinforced with a piece of tape. The box is 10 inches high, 20 inches wide, and 12 inches deep. What is the length of the tape that has been used? 168 in.
### Chapter 6 Assessment Answer Key

#### Quiz 1 (Lessons 6-1 and 6-2)
**Page 45**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1.</td>
<td>12,240</td>
</tr>
<tr>
<td>2.</td>
<td>45</td>
</tr>
<tr>
<td>3.</td>
<td>20</td>
</tr>
<tr>
<td>4.</td>
<td>(−1, 10)</td>
</tr>
<tr>
<td>5.</td>
<td>A</td>
</tr>
</tbody>
</table>

#### Quiz 2 (Lesson 6-3)
**Page 45**

No; none of the tests for $\cong$s are fulfilled.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>true</td>
</tr>
<tr>
<td>2.</td>
<td>false</td>
</tr>
<tr>
<td>3.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>true</td>
</tr>
<tr>
<td>5.</td>
<td>28 cm</td>
</tr>
</tbody>
</table>

#### Quiz 3 (Lessons 6-4 and 6-5)
**Page 46**

<p>| | |</p>
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>B</td>
</tr>
<tr>
<td>2.</td>
<td>65</td>
</tr>
<tr>
<td>3.</td>
<td>115</td>
</tr>
<tr>
<td>4.</td>
<td>true</td>
</tr>
<tr>
<td>5.</td>
<td>rectangle, rhombus, square</td>
</tr>
</tbody>
</table>

#### Quiz 4 (Lesson 6-6)
**Page 46**

<p>| | |</p>
<table>
<thead>
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<tbody>
<tr>
<td>1.</td>
<td>118</td>
</tr>
<tr>
<td>2.</td>
<td>$\sqrt{50} = 5\sqrt{2}$</td>
</tr>
<tr>
<td>3.</td>
<td>5</td>
</tr>
<tr>
<td>4.</td>
<td>15.5</td>
</tr>
</tbody>
</table>

Use the distance formula to show $CF = DE.$

#### Mid-Chapter Test
**Page 47**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>B</td>
</tr>
<tr>
<td>2.</td>
<td>J</td>
</tr>
<tr>
<td>3.</td>
<td>A</td>
</tr>
<tr>
<td>4.</td>
<td>G</td>
</tr>
<tr>
<td>5.</td>
<td>C</td>
</tr>
<tr>
<td>6.</td>
<td>50</td>
</tr>
<tr>
<td>7.</td>
<td>42</td>
</tr>
<tr>
<td>8.</td>
<td>$\cong$ to each other</td>
</tr>
<tr>
<td>9.</td>
<td>30, 150</td>
</tr>
<tr>
<td>10.</td>
<td>No; slope $\overline{XY} = -\frac{3}{5}$ and slope of $\overline{WZ} = -\frac{1}{3}$, so opposite sides are not parallel.</td>
</tr>
</tbody>
</table>
Chapter 6 Assessment Answer Key

Vocabulary Test
Page 48

1. isosceles trapezoid
2. parallelogram
3. trapezoid
4. square
5. rhombus
6. false, rectangle
7. false; rhombus
8. diagonals
9. median
Sample answer: angles formed by the base and one of the legs of the trapezoid
10. the nonparallel sides of a trapezoid
11. C

Form 1
Page 49
Page 50

1. B
2. H
3. B
4. H
5. D
6. F
7. B
8. F
9. D
10. F
11. C
12. H
13. B
14. J
15. A
16. G
17. A
18. J
19. A
20. G

example: $x = 7, \quad m\angle WYZ = 41$
<table>
<thead>
<tr>
<th>Form 2A</th>
<th>Form 2B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Page 51</td>
<td>Page 51</td>
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<tr>
<td>Page 52</td>
<td>Page 52</td>
</tr>
<tr>
<td>2. G</td>
<td>12. J</td>
</tr>
<tr>
<td>3. D</td>
<td>2. J</td>
</tr>
<tr>
<td>4. F</td>
<td>3. D</td>
</tr>
<tr>
<td>5. C</td>
<td>15. D</td>
</tr>
<tr>
<td>6. F</td>
<td>4. F</td>
</tr>
<tr>
<td>8. H</td>
<td>5. B</td>
</tr>
<tr>
<td>10. F</td>
<td>18. F</td>
</tr>
<tr>
<td>20. G</td>
<td>19. A</td>
</tr>
<tr>
<td>B: 7 or -4</td>
<td>B: 22</td>
</tr>
</tbody>
</table>
1. 1080
2. 19
3. 40
4. 18
5. 8
6. 122
7. (6, 4)
   Yes; \( \overline{AB} \) and \( \overline{CD} \) are \( \parallel \) and \( \cong \).
8. No; the slopes are \( \frac{9}{4}, \frac{1}{7}, 1, \) and \( \frac{2}{3} \).
   Thus, \( ABCD \) does not have \( \parallel \) sides.
9. \( -\frac{2}{3} \)
10. 22
11. Yes; if the diagonals of a \( \square \) are \( \cong \), then the \( \square \) is a rectangle.
12. 67.5
13. True
14. (4, 0)
15. 31
16. Yes; \( ABCD \) has only one pair of opposite sides \( \parallel \), \( \overline{BC} \) and \( \overline{AD} \).
17. 6
18. 26
19. True
20. True
21. True
22. False
23. True
24. True
25. False
B: \( x = 9, y = 2 \)
Chapter 6 Assessment Answer Key

Form 2D

Page 57

1. 360

2. 38

3. 90

4. 50

5. 3.6

6. 117

7. \(\left(\frac{1}{2}, 3\right)\)

Yes; Both pairs of opp. sides are \(\cong\).

8. 

9. \(ABCD\) has two pairs of \(\parallel\) sides, \(AB \parallel CD\) and \(BD \parallel DA\); it is a \(\square\).

10. 

11. 8

One rt. \(\angle\) means that the other \(\triangle\) will be rt. \(\triangle\). If all 4 \(\triangle\) are rt. \(\triangle\), the \(\square\) is a rectangle.

12. 

Page 58

13. 72

14. \((-3, 1)\)

15. 16

16. Yes; \(ABCD\) has only one pair of opp. sides \(\parallel\), \(AB\) and \(BD\).

17. 

18. 17

19. false

20. true

21. false

22. false

23. true

24. false

25. true

B: 90
Chapter 6 Assessment Answer Key

Form 3
Page 59

Page 60

No; two pairs of congruent consecutive sides do not exist.

CD = \sqrt{72}, DA = \sqrt{65},

14. \ AB = \sqrt{20}, BC = \sqrt{17}

Yes; \ AB \perp BC,

15. \ BC \perp CD, CD \perp AD,

so opp \triangle are \cong, and all \triangle are rt. \triangle.

16. \ ABCD has 2 pairs of opp. sides \cong, \ AB

\cong CD and BC \cong DA,

so \ ABCD is a \square.

17. \ 105

Opp. sides of a \□ are \cong.

18. \ □ are \cong.

If both pairs of opp. sides of a quad. are \cong, then the quad. is a \square.

19. \ □

20. \ 27

B: \ 54
# Chapter 6 Assessment Answer Key

## Extended-Response Test, Page 61

### Scoring Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>General Description</th>
<th>Specific Criteria</th>
</tr>
</thead>
</table>
| 4     | Superior            | • Shows thorough understanding of the concepts of angles of polygons, properties of parallelograms, rectangles, rhombi, squares, and trapezoids.  
• Uses appropriate strategies to solve problems.  
• Computations are correct.  
• Written explanations are exemplary.  
• Graphs and figures are accurate and appropriate.  
• Goes beyond requirements of some or all problems. |
| 3     | Satisfactory        | • Shows an understanding of the concepts of angles of polygons, properties of parallelograms, rectangles, rhombi, squares, and trapezoids.  
• Uses appropriate strategies to solve problems.  
• Computations are mostly correct.  
• Written explanations are effective.  
• Graphs and figures are mostly accurate and appropriate.  
• Satisfies all requirements of problems. |
| 2     | Nearly Satisfactory | • Shows an understanding of most of the concepts of angles of polygons, properties of parallelograms, rectangles, rhombi, squares, and trapezoids.  
• May not use appropriate strategies to solve problems.  
• Computations are mostly correct.  
• Written explanations are satisfactory.  
• Graphs and figures are mostly accurate.  
• Satisfies the requirements of most of the problems. |
| 1     | Nearly Unsatisfactory | • Final computation is correct.  
• No written explanations or work shown to substantiate the final computation.  
• Graphs and figures may be accurate but lack detail or explanation.  
• Satisfies minimal requirements of some of the problems. |
| 0     | Unsatisfactory      | • Shows little or no understanding of most of the concepts of angles of polygons, properties of parallelograms, rectangles, rhombi, squares, and trapezoids.  
• Does not use appropriate strategies to solve problems.  
• Computations are incorrect.  
• Written explanations are unsatisfactory.  
• Graphs and figures are inaccurate or inappropriate.  
• Does not satisfy requirements of problems.  
• No answer may be given. |
Chapter 6 Assessment Answer Key

Extended-Response Test, Page 61

Sample Answers

In addition to the scoring rubric found on page A30, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. a. Any type of convex polygon can be drawn as long as one is regular and one is not regular and both have the same number of sides.

   ![Diagram of a polygon with angles labeled: 90°, 90°, 120°, 110°, 70°, 60°.]

   b. Check to be sure that the exterior angles have been properly drawn and accurately measured.

   c. \(4(90) = 360; 120 + 70 + 60 + 110 = 360\); The sum of the exterior angles of each figure should be 360°. The sum of the exterior angles of both the regular convex polygon and the irregular convex polygon is 360°.

2. The student should draw a rectangle and join the midpoints of consecutive sides. The figure formed inside is a rhombus. Since all four small triangles can be proved to be congruent by SAS, the four sides of the interior quadrilateral are congruent by CPCTC, making it a rhombus.

   ![Diagram of a rhombus formed inside a rectangle by joining midpoints.]

3. The student should draw an isosceles trapezoid with one pair of opposite sides parallel and the other pair of opposite sides congruent, as in the figure below.

   ![Diagram of an isosceles trapezoid.]

4. a. Possible properties:
   A square has four congruent sides and a rectangle may not.
   A square has perpendicular diagonals and a rectangle may not.
   The diagonals of a square bisect the angles and those in a rectangle may not.

   b. Possible properties:
   A square has four right angles and a rhombus may not.
   The diagonals of a square are congruent and those of a rhombus may not be.

   c. Possible properties:
   A rectangle has four right angles and a parallelogram may not.
   The diagonals of a rectangle are congruent and those of a parallelogram may not be.
Chapter 6 Assessment Answer Key

Standardized Test Practice

Page 62

1. ● ● ● ●

2. ○ ● ○ ○

3. ○ ○ ○ ●

4. ○ ○ ● ○

5. ○ ○ ● ○

6. ● ○ ○ ○

7. ○ ● ○ ●

8. ○ ○ ○ ●

9. ○ ○ ○ ●

10. ○ ○ ○ ●

11. ○ ○ ● ●

12. ○ ● ○ ○

13. 

14. 

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Chapter 6 Assessment Answer Key

Standardized Test Practice (continued)

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15. hexagon; concave; irregular

16. 88.9 cm

17. $DL$

18. $x = 11, y = 2$

Assume that neither bell cost more than $45.

19. 

20. $40 \text{ cm}^3$

21. a. $\overline{ML}$
   
   b. $\overline{JL}$
   
   c. $J$
Chapter 6 Assessment Answer Key

Unit 2 Test

Page 65

1. **right; obtuse; acute**

   \( \angle 1 = 25; \)
   \( \angle 2 = 25; \)
   \( \angle 3 = 130 \)

2. \( AB \cong ST, \)
   \( FP \cong TX, \)
   \( PA \cong XS \)

3. **yes**

4. **ASA Postulate**

5. \( \angle A \cong \angle C \)

6. \( 5 \)

7. **111**

8. \( a = 9; \)
   \( \angle ZWT = 32 \)

9. \( \angle YWZ \)

   Neither appliance cost more than $603.

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11. **yes**

12. \( \frac{7}{2} < x < \frac{31}{2} \)

13. **9**

   \( \angle JHK = 52; \)
   \( \angle HMK = 108, \)
   and \( x = 8 \)

   No; opp. side are not \( \parallel \).

15. **not \( \parallel \).**

16. **5**

17. **5**

18. **11**

   In a parallelogram, opposite sides are congruent. Using the distance formula,
   \( PQ = \sqrt{20}, \)
   \( PS = \sqrt{20}, QR = \sqrt{8}, \)
   \( RS = \sqrt{8}. \)