Planning the Unit

Unit 7

The study of conic sections provides students with the opportunity to make many connections between algebra and geometry. Students are engaged in creating conic sections based on their definitions. They learn how to identify and apply conics in real-world settings. They make connections between the graphs of a conic, the standard-form equation, and the general second-degree equation of the conic section.

Academic Vocabulary

Blackline masters for use in developing students' vocabulary skills are located at the back of this Teacher's Edition. Encourage students to explore the meanings of the academic vocabulary words in this unit, using graphic organizers and class discussions to help students understand the key concepts related to the terms. Encourage students to place their vocabulary organizers in their Math notebooks and to revisit these pages to make notes as their understanding of concepts increases.

Embedded Assessments

The Embedded Assessment for this unit follows Activity 7.5.

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AP/College Readiness

Unit 7 prepares students to use conic sections as models in a variety of problems encountered AP Calculus and promotes expertise in a variety of process skills students need to be successful in AP courses by:

- Making the connections between algebraic and graphical representations of relations and functions explicit to students.
- Modeling a written or graphical description of a physical situation using implicitly defined relations and models.
- Using technology to explore relationships, make conjectures, and support conclusions.
- Emphasizing mathematical models in the coordinate plane to prepare students for differential and integral calculus applications of conic sections.
- Allowing students to communicate their mathematical knowledge verbally and in writing.

Embedded Assessment 1 **Working with Us**

- Identifying equations as the equation of a particular conic
- Graphing conic sections
- Writing the equations of conic sections.

Suggested Pacing

The following table provides suggestions for pacing either a 45-minute period or a block schedule class of 90 minutes. Space is left for you to write your own pacing guidelines based on your experiences in using the materials.

	45-Minute Period	90-Minute Period	Comments on Pacing
Unit Overview	$\frac{1}{2}$	$\frac{1}{4}$	
Activity 7.1	1	$\frac{1}{2}$	
Activity 7.2	2	1	
Activity 7.3	2	1	
Activity 7.4	2	1	
Activity 7.5	2	1	
Embedded Assessment 1	1	$\frac{1}{2}$	
Total	**$10\frac{1}{2}$**	**$5\frac{1}{4}$**	

Unit Practice

Practice Problems appear at the end of the unit.

Math Standards Review

To help accustom students to the formats and types of questions they may encounter on high stakes tests, additional problems are provided at the end of the unit. These problems are constructed for multiple choice, short response, extended response, and gridded responses.

Conic Sections

Unit 7

Unit Overview

In this unit you will investigate the curves formed when a plane intersects a cone. You will graph these curves known as the conic sections and you will identify conic sections by their equations.

Academic Vocabulary

Add these words and others you encounter in this unit to your vocabulary notebook.

- conic section
- ellipse
- hyperbola
- quadratic relation
- standard form

Essential Questions

- How are the algebraic representations of the conic sections similar and how are they different?
- How do the conic sections model real world phenomena?

EMBEDDED ASSESSMENTS

This unit has one embedded assessment, following Activity 7.5. It will give you the opportunity to demonstrate your ability to recognize and graph circles, ellipses, parabolas and hyperbolas.

Embedded Assessment 1	
Conic Sections	p. 409

UNIT 7 OVERVIEW

Unit Overview

Ask students to read the unit overview and mark the text to identify key phrases that indicate what they will learn in this unit.

Essential Questions

Read the essential questions with students. Encourage them to investigate real-world applications of the conic sections as they study each type of conic.

Academic Vocabulary

Read through the vocabulary list with students. Assess prior knowledge by asking students if they can define any of the terms. Encourage students to explore these words in depth using a graphic organizer and to add the words to their math notebooks.

Embedded Assessment

There is one embedded assessment in this unit, with an evaluation rubric. You may want to review skills needed for the assessment with students prior to the beginning of their work.

UNIT 7 GETTING READY

You may wish to assign some or all of these exercises to gauge students' readiness for Unit 7 topics.

Prerequisite Skills

- Graphing (Items 1, 2, 7)
- Writing the equation of lines (Item 4)
- Distance formula (Item 5)
- Simplifying radicals (Item 8)
- Simplifying polynomials (Item 6)
- Completing the square (Item 3)

Answer Key

1a. line with slope of $\left(-\frac{5}{3}\right)$ passing through (6, 0) and (3, 5)

b. vertical line, 9 units to the right of the *y*-axis

c. horizontal line, 2 units above the *x*-axis

2. Answers may vary. Sample answer: Pick several points on the graph and substitute their values into each equation to see which ones satisfy the equation. Or, pick two or three values of *x*, solve for *y* and see if the ordered pairs are points on the graph.

3.
$$x^2 + 6x - 11 = 0$$
$$x^2 + 6x = 11$$
$$x^2 + 6x + 9 = 11 + 9$$
$$(x + 3)^2 = 20$$

4. $y = \pm\frac{1}{2}x + 3$

5. $\sqrt{58}$

6. $3x^2 - 30x + 75$

7. Sample answers: (−5, −1) and (−3, −1); (−6, 5) and (−2, 5)

8a. $10\sqrt{2}$

b. $7\sqrt{3}$

UNIT 7

Getting Ready

Write your answers on notebook paper. Show your work.

1. Describe the geometric representation of each equation.

a. $5x + 3y = 30$

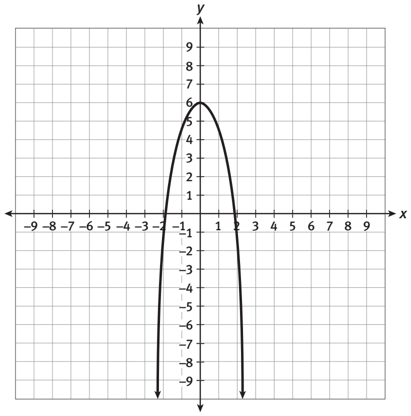
b. $x = 9$

c. $2(x - 2y) = 2x + y - 10$

2. Given the four quadratic equations below, explain how you could determine which equation is represented by the graph displayed.

$y = 2x^2 + 6$ $\quad$ $y = 2x^2 - 6$

$y = -6 - 2x^2$ $\quad$ $y = 6 - 2x^2$



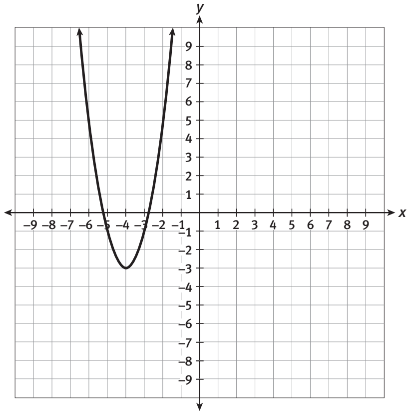
3. Model the process for completing the square using the equation $x^2 + 6x - 11 = 0$.

4. Write the equations of the diagonals of a rectangle that has vertices (−2, 4), (2, 4), (−2, 2), and (2, 2).

5. Find the distance between (−5, 3) and (2, 6).

6. Simplify $3(x - 5)^2$.

7. Identify 2 pairs of points that are symmetric about the line of symmetry in the parabola below.



8. Simplify.

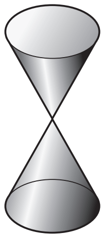
a. $\sqrt{200}$ $\quad$ **b.** $\sqrt{147}$

The Conic Sections
It's How You Slice It

ACTIVITY 7.1

SUGGESTED LEARNING STRATEGIES: Marking the Text, Visualization, Use Manipulatives, Summarize/Paraphrase/Retell, Discussion Group, Group Presentation

In the 3rd century BCE the Greek mathematician Apollonius wrote an eight volume text, *Conic Sections*, detailing curves formed by the intersection of a plane and a double cone. Nearly two millennia later Johannes Kepler used one of these intersections to model the path planets follow when orbiting the sun. René Descartes also studied the work of Apollonius, discovering that the coordinate system he created, the Cartesian Plane, could be applied to the conic sections and each could be represented by a quadratic relation.



Follow the instructions for the figures your teacher has assigned.

Figure One

Materials:
Piece of plain paper
Index card
Scissors

Instructions:

1. In the center of a plain piece of paper, place a point and label it *C*.
2. Using one corner of an index card as a right angle cut the index card to form a right triangle.
3. Label the vertex of the right angle of the triangle *Q* and the vertices of the acute angles P_1 and P_2.
4. Place P_1 on *C* and mark the point on the paper where P_2 falls.
5. Repeat step four 25–30 times keeping P_1 on *C* and moving P_2 to different locations on the paper.
6. Join the points formed by P_2 with a smooth curve to form a closed geometric figure.
7. Using the definitions of the conic sections in the My Notes section, identify the figure you created, sketch the figure in the space above these instructions, and, near the figure, write its name.

The figure is a circle.

1. a. How would the resulting figure change if P_2 were placed on *C* and the mark was made where P_1 falls?

Answers may vary. Sample answer: You would create a circle with the same radius as the original circle.

b. Explain how the work you did to create your figure models the definition of the curve you created.

Answers may vary. Sample answer: All the points drawn were the distance P_1P_2 from the center, *C*. The segment P_1P_2 has a constant length. A circle is the set of points equally distant from a fixed point.

My Notes

ACADEMIC VOCABULARY

conic sections

MATH TERMS

A **circle** is the set of all points in a plane that are equidistant from a fixed point.

ACADEMIC VOCABULARY

An **ellipse** is the set of all points in a plane such that the sum of the distances from each point to two fixed points is a constant.

ACADEMIC VOCABULARY

A **hyperbola** is the set of all points in a plane such that the absolute value of the differences from each point to two fixed points is constant.

MATH TERMS

A **parabola** is the set of points in a plane that are equidistant from a fixed point and a fixed line.

ACTIVITY 7.1 Investigative

The Conic Sections

Activity Focus

- Creating conic sections
- Relating models to the definitions of the conic sections

Materials

- Plain paper
- Index cards
- Scissors
- String
- Tape or tacks
- Patty paper or waxed paper
- Compass
- Straightedge

Chunking the Activity

#1–4 #5–7

TEACHER TO TEACHER Have students create, individually or in groups, one or more of the conic sections described in Figures One–Four.

There are several ways to assign students to this work.

(1) You may form groups of four students and assign each student in the group a different figure to create.

(2) You may have each student create two of the figures.

(3) You may group students by figure and have them discuss the instructions among themselves as they create the figure.

1-4 Marking the Text, Visualize, Use Manipulatives, Summarize/Paraphrase/Retell, Discussion Group, Group Presentation, Debriefing Figures One–Four each generate a different conic. The circle and the ellipse are the easiest for students to create.

1-4 (*continued*) Debrief the activity by having students share the figure(s) they created and discuss how their work modeled the definition of the conic section.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Visualization, Use Manipulatives, Summarize/Paraphrase/Retell, Discussion Group, Group Presentation

My Notes

Figure Two

Materials:

Piece of plain paper

Piece of string between 3 and 8 inches long

Tape or tacks

Instructions:

1. Draw a line on the paper.
2. Place two points on the line and label them F_1 and F_2.
3. Using tape or tacks secure one end of the string to F_1 and the other end of the string to F_2.
4. Use a pencil to pull the string tight.
5. With the tip of the pencil on the paper and keeping the string tight, move the pencil until a closed geometric figure is formed.
6. Using the definitions of the conic sections in the My Notes section on page 379, identify the figure you created, sketch the figure in the space above these instructions, and, near the figure, write its name.
   The figure is an ellipse.

2. a. What would happen if F_1 and F_2 were closer to each other?
   Answers may vary. Sample answer: As F_1 and F_2 get closer to each other the ellipse becomes more circular.

b. Explain how the work you did to create your figure models the definition of the curve you created.

2.b. Answers may vary. Sample answer: By putting the ends of the string on the two fixed points and pulling it out to a point not on the line, you are creating two segments of string. As the pencil moves, one segment of string becomes longer as the other becomes shorter. The sum of the segments is always the length of the string and is therefore is a constant.

Figure Three

Materials:

Piece of plain paper, waxed paper or patty paper

Instructions:

1. Label the top of one side of the paper *A*. Then turn the paper over as you would turn the page of a book and label the top of the other side of the paper *B*.
2. Place a point on side *A* about a third of the way down the page and in the middle. Label the point *F*.
3. On side B, place 25 points along the bottom edge of the page. The points should be evenly spaced out across the bottom of the page.
4. Fold the paper so that one point on the bottom falls on point *F* and crease the paper.
5. Repeat Step 4 for each point on the bottom of side *B*.
6. With a pencil trace the smooth curve formed by these folds.
7. Using the definitions of the conic sections in the My Notes section on page 379, identify the figure you created, sketch the figure in the space above these instructions, and, near the figure, write its name.
   The figure is a parabola.

3. Explain how the work you did to create your figure models the definition of the curve you created.

3. Answers may vary. Sample answer: When a point on the bottom of the paper is placed on the point *F* there is a point, *P*, on the "fold line" that is the same distance from *F* as it is from the point on the bottom of the piece of paper. When all the points, *P*, are connected they form a parabola.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Visualization, Use Manipulatives, Summarize/Paraphrase/Retell, Discussion Group, Group Presentation

My Notes

Figure Four
Materials:
Piece of plain paper
Compass and straight edge

Instructions:
1. Draw a line, l, across the center of a piece of plain paper.
2. Place two points on the line and label them F_1 and F_2.
3. Fold F_1 onto F_2 to find the midpoint of $\overline{F_1F_2}$ and mark the midpoint C.
4. Pick a length, x, that is less than the length of $\overline{F_1F_2}$ and greater than the length of $\overline{F_1C}$ or $\overline{CF_2}$.
5. Place the point of a compass on F_1 and using the compass, mark a point x units from F_1 on $\overline{F_1F_2}$.
6. Place the point of a compass on F_2 and using the compass, mark a point x units from F_2 on $\overline{F_1F_2}$.
7. Label the points identified in steps 5 and 6 V_1 and V_2.
8. Pick two numbers, a and b, so that $|a - b| = x$.
9. Assign a convenient unit of length for a and b. Set the pencil point and the compass point a units apart. Place the point of a compass on F_1 and draw an arc extending above and below line, l.
10. Move the point of the compass to F_2 and draw an arc of radius a extending above and below line, l.
11. Set the pencil point and the compass point b units apart. Place the point of a compass on F_1 and draw an arc of radius b extending above and below line, l.
12. Move the point of the compass to F_2 and draw an arc of radius b extending above and below line, l.
13. Place a point where the arcs of radius a intersect the arcs of radius b. You should have 4 points.
14. Repeat steps 8 through 13 with 3 additional values of a and b.
15. With a pencil connect the points to form two smooth curves.
16. Using the definitions of the conic sections in the My Notes section on page 379, identify the figure you created, sketch the figure in the space above these instructions, and, near the figure, write its name.

The figure is a hyperbola.

4. Explain the work you did to create your figure models the definition of the curve you created.

Answers may vary. Sample answer: For each point at the arc intersections, the absolute value of the difference between the distance to F_1 and the distance to F_2 is always x.

First Paragraph and Visual Display Vocabulary Organizer Be sure students understand the parts of the double cone as they will need these terms to describe various conic sections.

5-7 Think/Pair/Share, Visualization

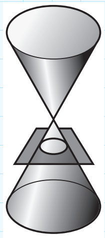
Suggested Assignment

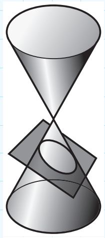
There are no Check Your Understanding or Practice problems designated for this activity. For homework after the activity is completed, you may want to have students investigate a "real-life" application of the conic(s) they created.

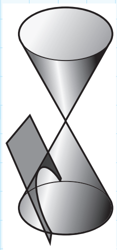
The Conic Sections

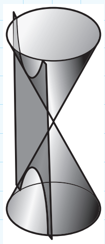
It's How You Slice It

My Notes


circle

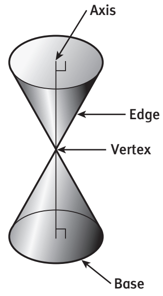

ellipse


parabola


hyperbola

SUGGESTED LEARNING STRATEGIES: Visualization, Think/Pair/Share

The four conic sections you have created are known as non-degenerate conic sections. A point, a line, and a pair of intersecting line are known as **degenerate conics**.



The figures to the left illustrate a plane intersecting a double cone. Label each conic section as an ellipse, circle, parabola or hyperbola.

5. Describe the way in which a plane intersects the cone to form each of the conic sections.
Answers may vary. Sample answers:
Circle: The plane is perpendicular to the axis of the cone and parallel to the base of the cone.
Ellipse: The plane intersects only one cone. It is not perpendicular to the axis, not parallel to the edge or base, and not parallel to the axis of the cone.
Parabola: The plane intersects only one cone and is parallel to the edge of the cone.
Hyperbola: The plane intersects both cones, but not at the vertex and is perpendicular to the bases.

6. How would a plane intersect the double cone to form a point?
The plane would intersect the double cone at the vertex of the cones and at no other point.

7. How would a plane intersect the double cone to form a line?
Plane would be tangent to the edge of the cones.

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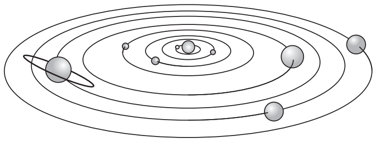
Ellipses and Circles
Round and Round We Go

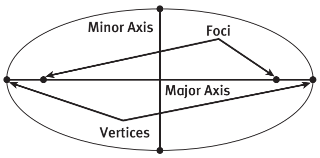
ACTIVITY 7.2

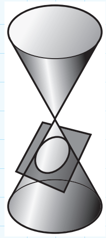
SUGGESTED LEARNING STRATEGIES: Shared Reading, Interactive Word Wall, Vocabulary Organizer, Marking the Text, Look for a pattern, Guess and Check



Prior to the 17th century, astronomers believed the orbit of the planets around the sun was circular. In the early 17th century, Johannes Kepler discovered that the orbital path was elliptical and the sun was not at the center of the orbit, but at one of the two foci.

An **ellipse** is the set of all points in a plane such that the sum of the distances from each point to two fixed points, called **foci**, is a constant. The **center** of an ellipse is the midpoint of the segment which has the foci as its endpoints. The **major** (longer) **axis** of an ellipse contains the foci and the center and has endpoints on the ellipse, the **vertices**. The **minor axis** of the ellipse is the line segment perpendicular to the major axis which passes through the center of the ellipse and has endpoints on the ellipse.





1. Match the graphs in the table on the following page with the corresponding equations from the list of equations given below by writing the equation in the appropriately headed column.

$\frac{x^2}{16} + \frac{y^2}{81} = 1$ $\qquad \frac{(x-2)^2}{9} + \frac{y^2}{25} = 1$

$\frac{x^2}{100} + \frac{y^2}{49} = 1$ $\qquad \frac{(x+3)^2}{4} + \frac{(y-1)^2}{36} = 1$

$\frac{(x+1)^2}{64} + \frac{(y+4)^2}{9} = 1$

2. For each equation and graph, find the coordinates of the center point, the length of the major axis and the length of the minor axis to complete the chart.

ACTIVITY 7.2 Guided

Ellipses and Circles

Activity Focus
- Ellipses
- Circles

Materials
- No additional materials

Chunking the Activity

#1–2	#8	#11–14
#3–5	#9	#15–17
#6–7	#10	

TEACHER TO TEACHER This activity allows students to explore the standard form of an ellipse. Spend some time discussing the vocabulary of the ellipse.

First Paragraph Shared Reading

Second Paragraph Vocabulary Organizer, Marking the Text, Interactive Word Wall

1-2 Look for a Pattern, Guess and Check, Debriefing To help students complete the chart on the next page, list these equations on the board. This will allow students to see the equations without having to flip back and forth between pages.

MINI-LESSON:

You may want to have students algebraically derive the standard form of the equation for an ellipse. See the mini-lesson. By having students experience this derivation, students are given the opportunity of seeing why this is the algebraic form of the ellipse with center at (0, 0). This derivation also shows students that the length of the major axis equals the constant sum to (x, y) on the ellipse from the foci $(-c, 0)$ and $(c, 0)$. They also see the relationship between a, b, and c as $a^2 = b^2 + c^2$.

(*continued on next page*)

MINI-LESSON: Algebraic Derivation of the Standard Form

Have students start with two points $(-c, 0)$ and $(c, 0)$, and express the sum of the distances from (x, y), a point on an ellipse, to these two points using the distance formula.

$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$$

Rewrite as

$$\sqrt{(x-c)^2 + y^2} = 2a - \sqrt{(x+c)^2 + y^2}$$

Square both sides and solve for the square root. $a\sqrt{(x+c)^2 + y^2} = a^2 + cx$

Square again and simplify. $a^2x^2 + 2a^2cx + a^2c^2 + a^2y^2 = a^4 + 2a^2cx + c^2x^2$

$$(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$$

Let $b^2 = a^2 - c^2$ $\qquad b^2x^2 + a^2y^2 = a^2b^2$

This can be written as $\qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

For some students, you may want to use specific numbers for a, b, and c. For example, if $a = 5$ and $c = 4$, we have $\sqrt{(x+4)^2 + y^2} + \sqrt{(x-4)^2 + y^2} = 10$

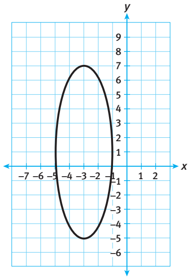
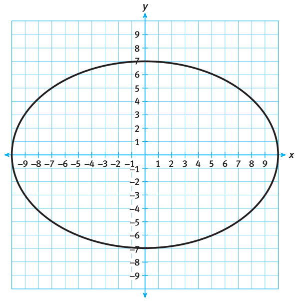
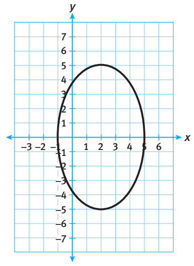
Following the steps in the Mini-Lesson this becomes $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

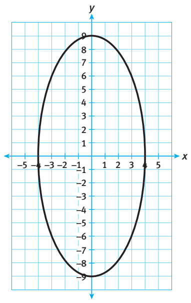
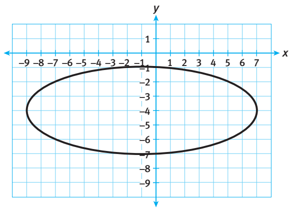
2 Look for a Pattern, Guess and Check, Debriefing These items are designed to have students use their knowledge of transformations of functions to match the graphs.

For students that are still having trouble identifying transformations, it is possible for them to use guess and check to determine which graph matches the functions. Students should be allowed to struggle with and explore this item with little support.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Guess and Check

My Notes

Graph			
Equation	$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{36} = 1$	$\frac{x^2}{100} + \frac{y^2}{49} = 1$	$\frac{(x-2)^2}{9} + \frac{y^2}{25} = 1$
Coordinates of Center of Ellipse	(−3, 1)	(0, 0)	(2, 0)
Length of Major Axis	12	20	10
Length of Minor Axis	4	14	6

Graph		
Equation	$\frac{x^2}{16} + \frac{y^2}{81} = 1$	$\frac{(x+1)^2}{64} + \frac{(y+4)^2}{9} = 1$
Coordinates of Center of Ellipse	(0, 0)	(−1, −4)
Length of major Axis	18	16
Length of Minor Axis	8	6

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384 SpringBoard® Mathematics with Meaning™ Algebra 2

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Quickwrite, Note taking, Vocabulary Organizer, Interactive Word Wall, Think/Pair/Share, Create Representations

My Notes

3. How are the denominators of the equations related to the major and minor axes of an ellipse?
Answers may vary. Sample answers: The larger number is equal to the square of $\frac{1}{2}$ the length of the major axis. The square root of the smaller number is $\frac{1}{2}$ the length of the minor axis.

4. How are numerators of the equations related to the center of the ellipse?
Answers may vary. Sample answer: The coordinates of the center are the opposite of the number being added to x and y in the numerators.

5. How can you determine the orientation of the major axis from the form of the equation of the ellipse?
Answers may vary. Sample answer: If the larger denominator is in the x term the major axis is horizontal and parallel to the x-axis. If the larger denominator is in the y-term the major axis is vertical and parallel to the y-axis.

6. If $a > b$, the **standard form of an ellipse** is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

a. Where is the center of the ellipse located? (h, k)

b. How is the major axis oriented in the coordinate plane? parallel to x-axis

c. How long is the major axis and what are the coordinates of the endpoints? Length is $2a$. Coordinates of the endpoints are $(h \pm a, k)$

d. How long is the minor axis? $2b$

ACADEMIC VOCABULARY

standard form of the equation of an ellipse

7. If $a > b$, the **standard form of an ellipse** is $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$.

a. Where is the center of the ellipse located? (h, k)

b. What direction is the major axis? parallel to y-axis

c. How long is the major axis and what are the coordinates of the endpoints? Length is $2a$. Coordinates of the endpoints are $(h, k \pm a)$

d. How long is the minor axis? $2b$

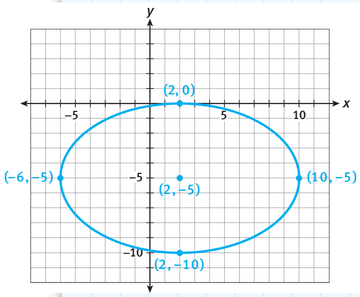
8. Using what you found in Items 6 and 7, find the following information for the ellipse $\frac{(x-2)^2}{64} + \frac{(y+5)^2}{25} = 1$.

a. the coordinates of the center
$(2, -5)$

b. the length and coordinates of the endpoints of the major axis
Length is 16. End points are $(10, -5)$ and $(-6, -5)$.

c. the length and coordinates of the endpoints of the minor axis
Length is 10. Endpoints are $(2, -10)$ and $(2, 0)$.

d. In the My Notes section, graph the ellipse and label the center and endpoints of the axes.



MINI-LESSON: Scaffolding for Standard Form of an Ellipse

1. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is an ellipse centered at the origin. If $a > b$, explain how to find the lengths of the major and minor axes and tell the coordinates of the endpoints of the major axis.
2. $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ is not centered at the origin. If $a > b$, find the lengths of the major and minor axes and find the coordinates of the endpoints of the major axis.

(*For Mini-Lesson answers, see next page.*)

3 Look for a Pattern, Quickwrite, Think/Pair/Share This item, along with the next two, helps students verbalize the understanding they gleaned from exploring the table.

4 Look for a Pattern, Quickwrite, Think/Pair/Share Students should make some connection to the numerators of the fractions $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ and state the center is located at (h, k). Since they have not seen the standard form of an ellipse, their verbalization of this may not be precise at this point. Items 5 and 6 will solidify the language.

5 Look for a Pattern, Quickwrite, Think/Pair/Share Students should say something along the lines of "If the larger denominator is in the x term the major axis is horizontal. If the larger denominator is in the y-term the major axis is vertical."

6-7 Look for a Pattern, Note Taking, Vocabulary Organizer, Interactive Word Wall, Group Presentation, Debriefing Students should formalize the standard form and how it relates to the graph of an ellipse. If students need more scaffolding, do the *Mini-Lesson*.

8 Create Representations This item acts as another check to verify student understanding of the relationship between the equation and the graph of an ellipse. If students need more practice, use these equations.

a. $\frac{x^2}{16} + \frac{(y+4)^2}{36} = 1$

b. $\frac{(x+2)^2}{25} + \frac{(y-3)^2}{9} = 1$

MINI-LESSON (*answers*)

1. Since $a > b$, the length of the major axis is equal to twice the square root of a^2, or $2a$. The minor axis is equal to twice the square root of b^2, or $2b$. The endpoints of the major axis are on the x-axis at $(-a, 0)$, and $(a, 0)$.
2. Since $a > b$, the length of the major axis is $2a$ and the length of the minor axis if $2b$. The major axis is parallel to the y-axis and has endpoints $(h, k \pm a)$.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 390, #1–2

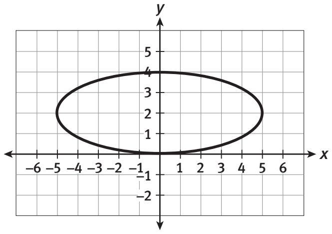
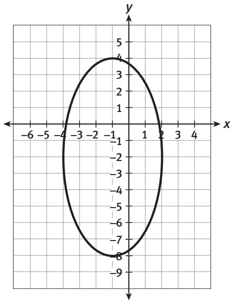
UNIT 7 PRACTICE
p. 411, #1–2

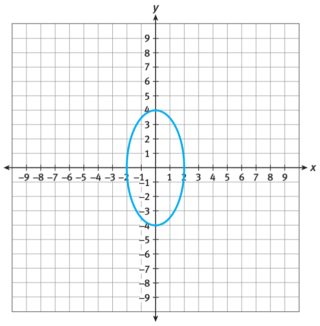
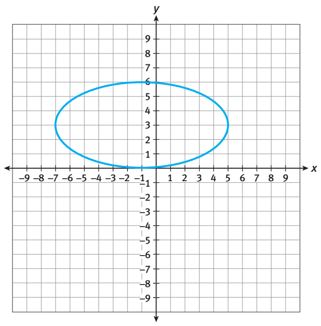
9 **Create Representations, Work Backward, Think/Pair/Share** This item is designed to build the students' capacity to translate between graphical, analytic, and numeric representations of an ellipse. Students should once again be allowed to struggle with the item.

SUGGESTED LEARNING STRATEGIES: Create Representations, Work Backward/Think/Part/Share

My Notes

9. Complete the table below using the information given.

Ellipse		
Center	(0, 2)	(–1, –2)
Length and orientation of major axis	10 units horizontal	12 units vertical
Length and Orientation of Minor Axis	4 units vertical	6 units horizontal
Equation of Ellipse	$\frac{x^2}{25} + \frac{(y-2)^2}{4} = 1$	$\frac{(x+1)^2}{9} + \frac{(y+2)^2}{36} = 1$

Graph		
Center	(0, 0)	(–1, 3)
Length and orientation of major axis	8 units vertical	12 units horizontal
Length and Orientation of Minor Axis	4 units horizontal	6 units vertical
Equation of Ellipse	$\frac{x^2}{4} + \frac{y^2}{16} = 1$	$\frac{(x+1)^2}{36} + \frac{(y-3)^2}{9} = 1$

386 SpringBoard® Mathematics with Meaning™ Algebra 2

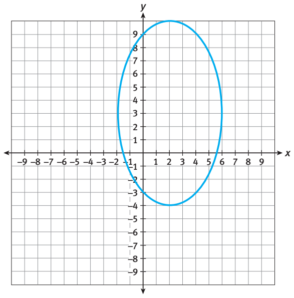
SUGGESTED LEARNING STRATEGIES: Create Representations, Work Backward, Vocabulary Organizer, Interactive Word Wall, Note Taking, Quickwrite

My Notes

10. Use the information below. Write the equation and then graph the ellipse described.

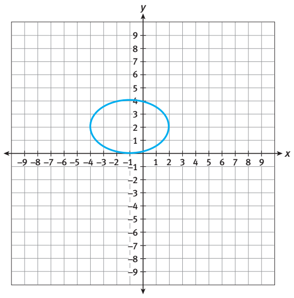
a. length of vertical major axis: 14
length of minor axis: 8
center: (2, 3)

$\frac{(x-2)^2}{16} + \frac{(y-3)^2}{49} = 1$



b. endpoints of major axis: (2, 2) and (−4, 2)
endpoints of minor axis: (−1, 0) and (−1, 4)

$\frac{(x+1)^2}{9} + \frac{(y-2)^2}{4} = 1$



CONNECT TO AP

In calculus, you will have to quickly recognize a particular conic section from its equation and produce its sketch.

The foci of an ellipse are located on the major axis c units from the center. The values a, b, and c are related by the equation $c^2 = a^2 - b^2$.

The **eccentricity** of a conic section is $\frac{c}{a}$. The eccentricity of a conic section or an **orbit's eccentricity** indicates the roundness or flatness of the shape.

11. Give the coordinates of the foci of each ellipse.

a. $\frac{x^2}{81} + \frac{y^2}{25} = 1$ $(-2\sqrt{14}, 0), (2\sqrt{14}, 0)$

b. $\frac{(x+2)^2}{4} + \frac{y^2}{25} = 1$ $(-2, \sqrt{21}), (-2, -\sqrt{21})$

ACTIVITY 7.2 *Continued*

10 Create Representations, Work Backward, Discussion Group This item allows for formative assessment on student understanding. Walk around the room and question students to help them see solutions and gain clarity.

TEACHER TO TEACHER Eccentricity may be interpreted as a measure of how much this shape deviates from a circle. Under standard assumptions, **eccentricity** $\left(\frac{c}{a}\right)$ is defined for all circular, elliptical, parabolic and hyperbolic orbits and may take the following values.

- circular orbits: $\left(\frac{c}{a}\right) = 0$
- elliptic orbits: $0 < \left(\frac{c}{a}\right) < 1$
- parabolic trajectories: $\left(\frac{c}{a}\right) = 1$
- hyperbolic trajectories: $\left(\frac{c}{a}\right) > 1$

Paragraph Vocabulary Organizer, Marking the Text, Note Taking, Interactive Word Wall

Connect to AP

When students study calculus, they will have to quickly recognize a particular conic section from its equation and produce a sketch. Once that is done, they can perform a variety of calculations including finding the area of the conic section, the equation of a line tangent to the curve at a point, or the volume of a solid formed by rotating a portion of the curve about a horizontal or vertical line.

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ACTIVITY 7.2 *Continued*

12 Create Representations

13 Quickwrite, Think/Pair/Share The eccentricity $\left(\frac{c}{a}\right)$ for the ellipse satisfies $0 < \left(\frac{c}{a}\right) < 1$ and gives a comparative measure of the "flatness" of the ellipse. Students can be asked to discuss what it means about the shape of the ellipse when the eccentricity is close to 0 and when it is close to 1.

14 Create Representations, Quickwrite, Think/Pair/Share, Debriefing While a circle is technically not an ellipse, this item allows students to work with what they know about an ellipse and eccentricity to develop the equation of a circle.

TEACHER TO TEACHER The eccentricity of an ellipse, $\frac{c}{a}$, is a number between 0 and 1. Knowing that every ellipse must obey the relationship $c^2 = a^2 - b^2$ can help you understand eccentricity geometrically.

$$c^2 = a^2 - b^2$$

$$\frac{c^2}{a^2} = \frac{a^2 - b^2}{a^2}$$

$$\frac{c^2}{a^2} = 1 - \frac{b^2}{a^2}$$

$$\frac{c}{a} = \sqrt{1 - \frac{b^2}{a^2}}$$

This expresses eccentricity in terms of a and b. If a and b are close in value, an ellipse is close in shape to a circle. Then $\frac{c}{a} = \sqrt{1 - \frac{b^2}{a^2}}$ will be close to zero (since $\frac{b^2}{a^2}$ will be close to one). If b is relatively small compared to a, an ellipse will have a long, thin shape. Then $\frac{c}{a} = \sqrt{1 - \frac{b^2}{a^2}}$ will be close to one (since $\frac{b^2}{a^2}$ will be close to zero). For example, in Item 12a eccentricity $= \frac{\sqrt{99}}{10} \approx 0.995$ and in Item 12b eccentricity $= \frac{3}{5} = 0.6$.

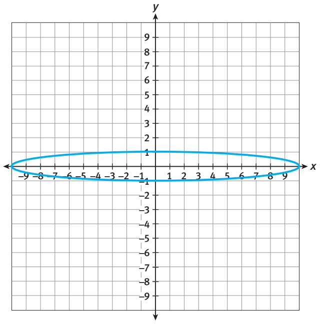
SUGGESTED LEARNING STRATEGIES: Create Representations, Quickwrite, Think/Pair/Share

My Notes

12. Graph each ellipse. Determine the eccentricity.

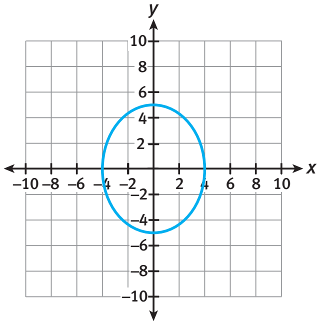
a. $\frac{x^2}{100} + \frac{y^2}{1} = 1$

eccentricity ≈ 0.995



b. $\frac{x^2}{16} + \frac{y^2}{25} = 1$

eccentricity = 0.6



13. What does the eccentricity tell you about the graph of an ellipse?

Answers may vary. Sample answer: A narrow ellipse has an eccentricity close to 1. A wide ellipse has an eccentricity close to 0.

14. Consider the equation for an ellipse in which $a = b$.

a. Give a verbal, visual, and symbolic representation of the conic.

Answers may vary. Sample answer: It is a circle; ; $(x - h)^2 + (y - k)^2 = a^2$

b. What is the eccentricity of the ellipse?

0

c. What does the eccentricity tell you about the major and minor axes of the ellipse?

They are the same length.

d. What does the eccentricity tell you about foci of the ellipse?

They are in the center.

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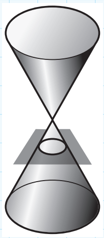
SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Interactive Word Wall, Marking the Text, Note taking, Activating Prior Knowledge, Think/Pair/Share, Create Representations

My Notes

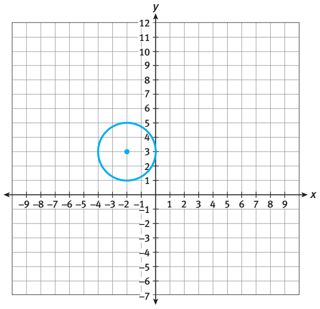


ACADEMIC VOCABULARY

standard form of the equation of a circle

A **circle** is the set of all points in a plane that are equidistant from a fixed point the center. The **standard form of the equation of a circle** is $(x - h)^2 + (y - k)^2 = r^2$ where the center is (h, k) and the radius is r.

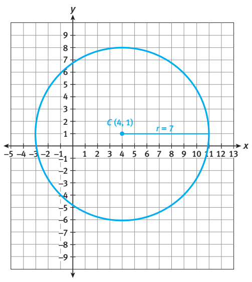
15. Write $\frac{(x+2)^2}{4} + \frac{(y-3)^2}{4} = 1$ in the standard form of a circle. Identify the center and radius and then graph the circle.



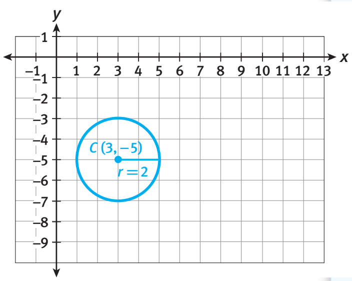
$(x + 2)^2 + (y - 3)^2 = 4$; center: $(-2, 3)$; radius: 2

16. Graph each circle and label the center and radius.

a. $(x - 4)^2 + (y - 1)^2 = 49$



b. $(x - 3)^2 + (y + 5)^2 = 4$



17. Write the equation of each circle.

a. center $(-5, 2)$, radius 7
$(x + 5)^2 + (y - 2)^2 = 49$

b. center $(1, 1)$ and passing through $(4, 5)$
$(x - 1)^2 + (y - 1)^2 = 25$

ACTIVITY 7.2 *Continued*

Paragraph Vocabulary Organizer, Interactive Word Wall, Marking the Text, Note Taking

15 Activate Prior Knowledge, Create Representations Students are given the opportunity to make a connection between the equation of an ellipse in standard form and that of a circle.

16 Create Representations, Think/Pair/Share Have students share their graphs on white boards. They can then do any self or peer editing on their paper.

17 Create Representations, Think/Pair/Share, Debriefing

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Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 390, #3–6

UNIT 7 PRACTICE
p. 411, #3–4

CHECK YOUR UNDERSTANDING

1a. $(x - 1)^2 + (y + 2)^2 = 16$

b. $\frac{x^2}{9} + \frac{(y - 3)^2}{25} = 1$

c. $\frac{(x + 4)^2}{1} + \frac{(y + 1)^2}{49} = 1$

2. a. and b. See below right.

3. $\frac{x^2}{169} + \frac{y^2}{25} = 1$

4. $(x + 6)^2 + (y - 2)^2 = 25$

5. $(h, k \pm a)$

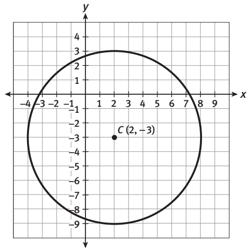
6. Answers may vary. Possible answers: Circles are a limit for ellipses. They are like ellipses whose axes are congruent; or, circles are like ellipses whose eccentricity is zero.

Answers may be compared graphically as well.

SUGGESTED LEARNING STRATEGIES: Create Representations

My Notes

c.



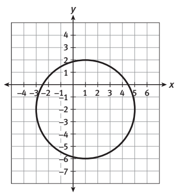
$(x - 2)^2 + (y + 3)^2 = 36$

CHECK YOUR UNDERSTANDING

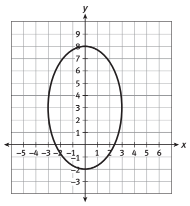
Write your answers on notebook paper or grid paper. Show your work.

1. Write the equation of each graph.

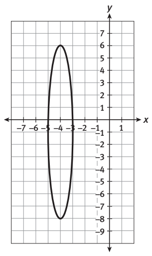
a.



b.



c.



2. Graph each equation. Label the center and endpoints of the major and minor axes.

a. $\frac{x^2}{81} + \frac{y^2}{16} = 1$

b. $\frac{(x + 5)^2}{121} + \frac{(y + 3)^2}{49} = 1$

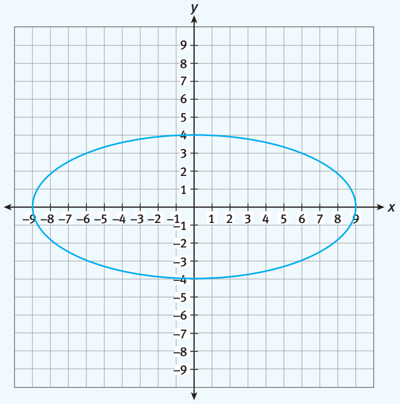
3. Write the equation of an ellipse that has the endpoints of the major axis at (13, 0) and (−13, 0) and endpoints of the minor axis at (0, 5) and (0, −5).

4. Write the equation of a circle that has center (−6, 2) and a diameter of length 10.

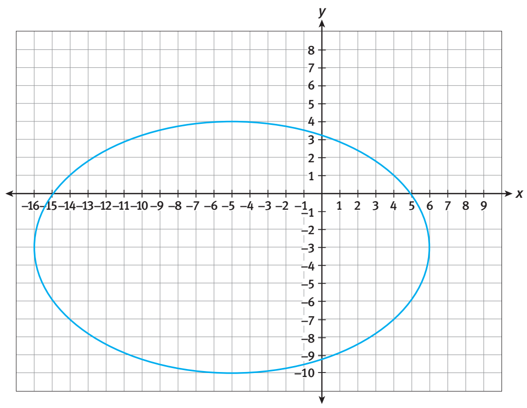
5. If $a > b$, what are the endpoints of the major axis of the ellipse $\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$?

6. MATHEMATICAL REFLECTION How are circles and ellipses related?

2a.



b.

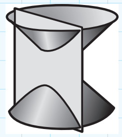


Hyperbolas
What's the Difference?

ACTIVITY **7.3**

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Vocabulary Organizer, Quickwrite, Close Reading, Graphic Organizer

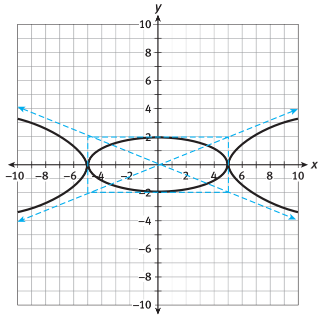
My Notes



Recall the definitions of ellipse and hyperbola:

An *ellipse* is the set of all points in a plane such that the sum of their distances to two fixed points is a constant. A *hyperbola* is the set of all points in a plane such that the absolute value of the difference of their distances to two fixed points, the *foci,* is a constant.

The ellipse $4x^2 + 25y^2 = 100$ and the hyperbola $4x^2 - 25y^2 = 100$ are graphed on the right.



1. Tell the coordinates of the center and the endpoints of the major and minor axes of the ellipse.

Center: (0, 0); Endpoints of major axis: (–5, 0), (5, 0); Endpoints of minor axis: (0, 2), (0, –2)

MATH TERMS

transverse axis
conjugate axis
asymptote

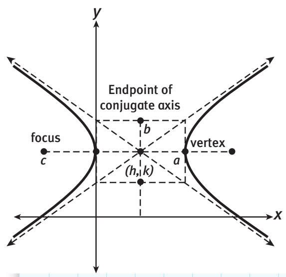
2. a. Using dashed line segments draw an auxiliary rectangle with vertices (5, 2), (5, –2), (–5, 2), and (–5, –2). Also using dashed lines, draw two diagonal lines that pass through the center and vertices of the rectangle and extend to the edges of the grid.

b. What relationships do the rectangle and lines have to the ellipse and hyperbola?

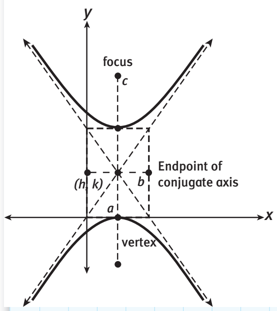
Answers may vary. Sample answer: The lines go through the center of the ellipse and the branches of the hyperbola approach the lines. The ellipse is enclosed within the rectangle. The ellipse and the hyperbola have their vertices on the rectangle.

c. Why are dashed lines used when sketching the rectangle and diagonals of the rectangle?

Answers may vary. Sample answer: Since the lines are only used to help draw the relation and are not part of it, they should not be drawn as solid lines.



The **transverse axis** of a hyperbola has endpoints on the hyperbola. The *center* of a hyperbola is the midpoint of the transverse axis. The foci are on the line containing the transverse axis. The **conjugate axis** of the hyperbola is the line segment perpendicular to the transverse axis passing through the center of the hyperbola. The hyperbola has **asymptotes**, lines which the branches of the hyperbola approach. The asymptotes contain the center of the hyperbola and pass through the vertices of the auxiliary rectangle.



ACTIVITY 7.3 Guided

Hyperbolas

Activity Focus

- Hyperbolas

Materials

- No additional materials

Chunking the Activity

#1–2	#6
#3	#7
#4–5	#8
Ex. 1–TT A	#9

First Paragraph Activating Prior Knowledge, Vocabulary Organizer, Interactive Word Wall

1 Activating Prior Knowledge This item connects students back to what they have learned about ellipses and connects to hyperbolas.

2 Quickwrite Students need to recognize that the tools that are used to help graph relations are not part of the relation itself.

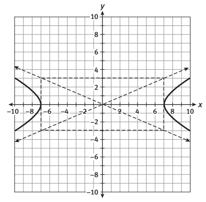
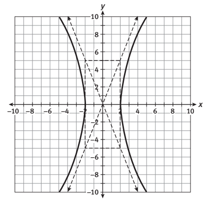
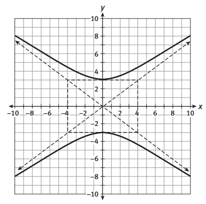
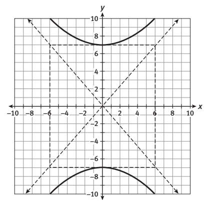
Second Paragraph Close Reading, Graphic Organizer, Vocabulary Organizer, Interactive Word Wall It may be helpful to use a graphic organizer to arrange the vocabulary. Students should draw a picture of a hyperbola, and place terms where they belong. This will help to make connections to the concepts they are pulling from the reading and make the ideas more concrete.

3 Think/Pair/Share, Self/Peer Revision This table is similar to those students completed in Activity 7.2. It gives students a chance to explore multiple quadratic relations that represent hyperbolas. They can use their understanding of quadratic relations or guess and check to determine the information. Students should once again be allowed to struggle with little assistance from you. Have groups share answers that you saw as you walked around the room and allow time for students to correct any errors they may have had before continuing to the next two questions.

My Notes

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Self/Peer Revision, Guess and Check

3. Complete the table below. The first row has been done for you using the hyperbola $\frac{x^2}{49} - \frac{y^2}{9} = 1$ as an example.

Graph	Equation	Length, Endpoints, and Orientation of Transverse Axis	Length, Endpoints, and Orientation of Conjugate Axis	Equations of Asymptotes
	$\frac{x^2}{49} - \frac{y^2}{9} = 1$	14 units (7, 0), (−7, 0) horizontal	6 units (0, 3), (0, −3) vertical	$y^2 = \frac{9}{49}x^2$ $y = \pm\frac{3}{7}x$
	$\frac{x^2}{4} - \frac{y^2}{25} = 1$	4 units (2, 0), (−2, 0) horizontal	10 units (0, 5), (0, −5) vertical	$y = \pm\frac{5}{2}x$
	$\frac{y^2}{9} - \frac{x^2}{16} = 1$	6 units (0, 3), (0, −3) vertical	8 units (4, 0), (−4, 0) horizontal	$y = \pm\frac{3}{4}x$
	$\frac{y^2}{49} - \frac{x^2}{36} = 1$	14 units (0, 7), (0, −7) vertical	12 units (6, 0), (−6, 0) horizontal	$y = \pm\frac{7}{6}x$

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392 SpringBoard® Mathematics with Meaning™ Algebra 2

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Quickwrite, Notetaking, Vocabulary Organizer, Create Representations

My Notes

4. How do the equations of the asymptotes relate to the equation of the hyperbola?
Answer may vary. Sample answer: The equations of the asymptotes are found by setting the quadratic terms equal to each other and solving for y.

5. How can the direction in which the branches of the hyperbola open be determined by the equation?
Answer may vary. Sample answer: The branches open towards the axis that is in the positive term of the hyperbola.

ACADEMIC VOCABULARY

Standard form of the equation of a hyperbola

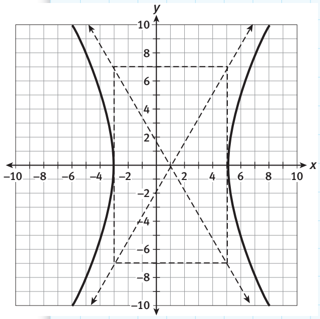
The **standard form of a hyperbola** is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, when the transverse axis is horizontal. The standard form of a hyperbola is $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ when the transverse axis is vertical. The endpoints of the transverse axis are the **vertices** of the branches, and are located a units from the center of the hyperbola that is located at the point (h, k). The equations of the asymptotes are found by setting the quadratic terms equal to each other and solving for y.

EXAMPLE 1

Sketch the hyperbola $\frac{(x-1)^2}{16} - \frac{y^2}{49} = 1$. Tell the coordinates of the center and the vertices, and give the equations of the asymptotes

- The positive term is $\frac{(x-1)^2}{16}$, so the transverse axis is horizontal.
- Since a^2 is 16, then $a = 4$ and the transverse axis is 8 units long.
- The center is (1, 0).
- The vertices on the transverse axis are 4 units from the center: (−3, 0) and (5, 0).
- Setting $\frac{(x-1)^2}{16} = \frac{y^2}{49}$ and solving for y gives the equations of the asymptotes.

$$y^2 = \frac{49(x-1)^2}{16} \rightarrow y = \pm\frac{7(x-1)}{4}$$



ACTIVITY 7.3 *Continued*

4 Look for a Pattern, Quickwrite Students look for patterns to determine the relationship between the asymptotes and the equation of the hyperbola. Having the opportunity to verbalize the process will connect students to a different learning modality and enhance the learning process.

5 Look for a Pattern, Quickwrite, Debriefing This item allows students to make a conjecture about how the hyperbola opens. Having students come up with the concept will help them retain the idea.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 398, #1–5

UNIT 7 PRACTICE
p. 412, #5–9

Boxed Text Note Taking, Vocabulary Organizer

EXAMPLE 1 Note Taking, Create Representations Walk students through the example. Have students verbalize how each of the important tools (central rectangle, asymptotes) for graphing hyperbolas is found and used.

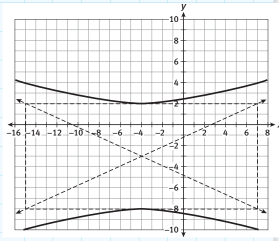
EXAMPLE 2 Note Taking, Create Representations Walk students through the example. Have students compare this example to the previous one and again discuss the important tools used for graphing.

TRY THESE A Create Representations Use these questions as an opportunity to formatively assess students. Verify that they are opening the branches in the correct direction and they are able to find the equations of the asymptotes. Use questioning techniques to bring the students to understanding.

6 Identify a Subtask This item gives students an opportunity to work with a hyperbola that is in standard form and centered at the origin with less scaffolding than in the beginning of the activity.

SUGGESTED LEARNING STRATEGIES: Notetaking, Create Representations, Identify a Subtask

My Notes



EXAMPLE 2

Sketch the hyperbola $\frac{(y+3)^2}{25} - \frac{(x+4)^2}{121} = 1$. Tell the coordinates of the center and the vertices, and give the equations of the asymptotes.

- The positive term is $\frac{(y+3)^2}{25}$, so the transverse axis is vertical.
- Since a^2 is 25, then $a = 5$ and the transverse axis is 10 units long.
- The center is $(-4, -3)$.
- The vertices on the transverse axis are 5 units from the center: $(-4, 2)$ and $(-4, -8)$.
- Setting $\frac{(y+3)^2}{25} = \frac{(x+4)^2}{121}$ and solving for y gives the equations of the asymptotes.

$$(y+3)^2 = \frac{25(x+4)^2}{121} \rightarrow (y+3) = \pm\frac{5(x+4)}{11} \rightarrow y = -3 \pm \frac{5(x+4)}{11}$$

TRY THESE A

Write your answers on notebook or grid paper. Show your work. Sketch each hyperbola. Tell the coordinates of the center, label the vertices and give the equations of the asymptotes.

a. $\frac{x^2}{100} - \frac{y^2}{49} = 1$

center: (0, 0); equations of asymptotes: $y = \pm\frac{7}{10}x$

b. $\frac{y^2}{9} - \frac{x^2}{64} = 1$

center: (0, 0); equations of the asymptotes: $y = \pm\frac{3}{8}x$

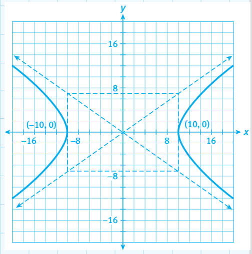
c. $\frac{x^2}{16} - \frac{(y+4)^2}{36} = 1$

center: (0, −4); equations of the asymptotes: $y = -4 \pm \frac{3}{2}x$

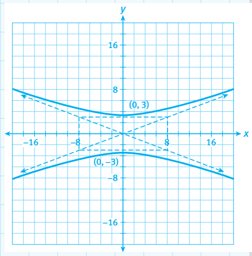
d. $\frac{(x+2)^2}{25} - \frac{(y-3)^2}{9} = 1$

center: (−2, 3); equations of the asymptotes: $y = 3 \pm \frac{3}{5}(x+2)$

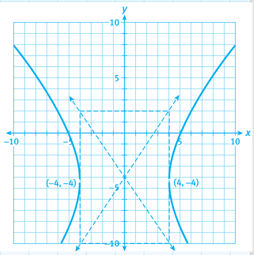
a.



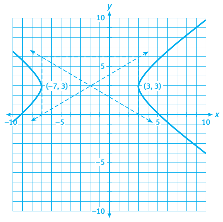
b.



c.



d.



6. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a hyperbola centered at the origin. Find each item.

a. the direction of the transverse axis

horizontal

b. the length and endpoints of the transverse axis

length: $2a$; endpoints: $(a, 0)$ and $(-a, 0)$

c. the length of the conjugate axis

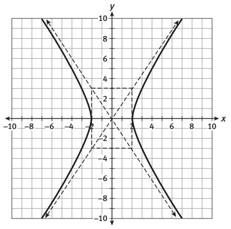
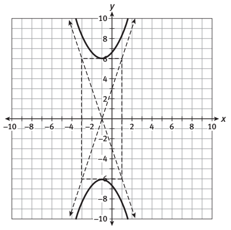
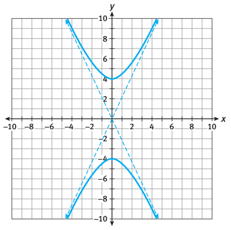
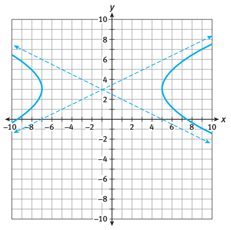
length: $2b$

d. the equation of the asymptotes $y = \pm\frac{b}{a}x$

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Self/Peer Revision, Create Representations

My Notes

7. Complete the table below using the information given.

Hyperbola	Center	Length and Orientation of Transverse Axis	Length and Orientation of Conjugate Axis	Equation of Hyperbola
	(0, 0)	4 horizontal	6 vertical	$\frac{x^2}{4} - \frac{y^2}{9} = 1$
	(−1, 0)	12 vertical	4 horizontal	$\frac{y^2}{36} - \frac{(x+1)^2}{4} = 1$
	(0, 0)	8 units vertical	4 units horizontal	$\frac{y^2}{16} - \frac{x^2}{4} = 1$
	(−1, 3)	12 units horizontal	6 units vertical	$\frac{(x+1)^2}{36} - \frac{(y-3)^2}{9} = 1$

7 Think/Pair/Share, Self/Peer Revision, Create Representations This item is designed to build the students' capacity to translate between graphical, analytic, and numeric representations of a hyperbola. Students should once again be allowed to struggle with the question. Make sure they are aware that they need to label axes. Students should share answers and any student that needs to make corrections should do so before moving on.

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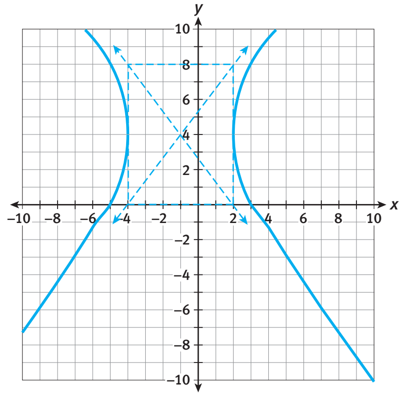
8 Create Representations, Quickwrite This item is similar to Item 7, but here students are given different types of information and have to both write the equation and create the graph.

SUGGESTED LEARNING STRATEGIES: Create Representations

My Notes

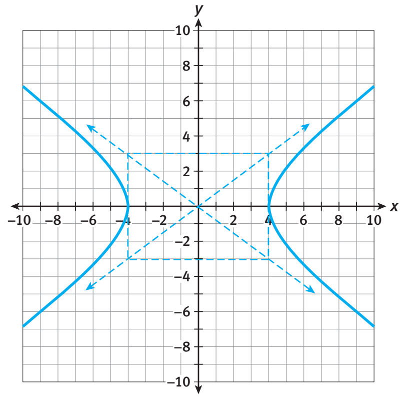
8. Write the equation and graph the hyperbola described.

a. center (−1, 4), transverse axis 6 units, vertical conjugate axis 8 units



$$\frac{(x+1)^2}{9} - \frac{(y-4)^2}{16} = 1$$

b. asymptotes $y = \pm\frac{3}{4}x$, vertices (4, 0), (−4, 0)



$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

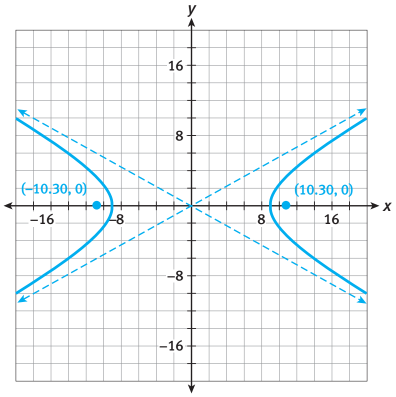
SUGGESTED LEARNING STRATEGIES: Create Representations

My Notes

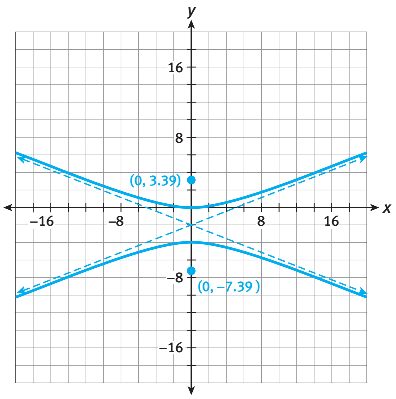
The **foci** of a hyperbola are located on the transverse axis c units from the center. The values a, b, and c are related by the equation $c^2 = a^2 + b^2$.

9. Graph each hyperbola and label the foci with their coordinates.

a. $\frac{x^2}{81} - \frac{y^2}{25} = 1$



b. $\frac{(y+2)^2}{4} - \frac{x^2}{25} = 1$



8. Answers may vary. Sample answer: The branches of the hyperbola approach, but never reach the asymptotes. They indicate the limiting edge of the hyperbola when it gets graphed. They also contain the center of the hyperbola.

ACTIVITY 7.3 *Continued*

9 Create Representations

This item allows for formative assessment. Students must complete similar actions to those done in Questions 7 and 8, but with less scaffolding.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 398, #6–8

UNIT 7 PRACTICE
p. 412, #10–14

CHECK YOUR UNDERSTANDING

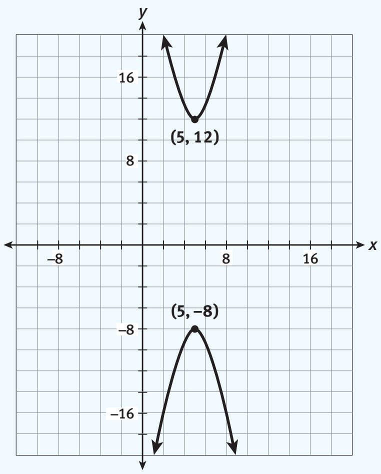
(*continued from page 398*)

5a. (5, 2)

b. vertical

c. $y = 2 \pm 5(x - 5)$

d.



6a. (0, 0)

b. vertical, 4 units

c. $y = \pm \frac{1}{2}x$

d. $\frac{y^2}{4} - \frac{x^2}{16} = 1$

7a. (−6, 0)

b. horizontal, 4 units

c. $y = \pm 4(x + 6)$

d. $\frac{(x+6)^2}{4} - \frac{y^2}{64} = 1$

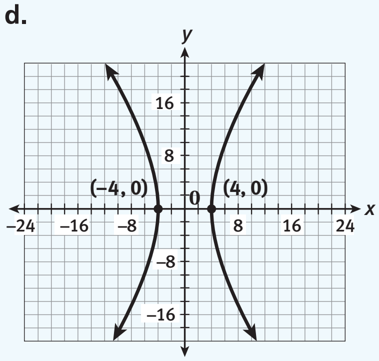
CHECK YOUR UNDERSTANDING

1a. (0, 0)

b. horizontal

c. $y = \pm \frac{7}{4}x$

d.

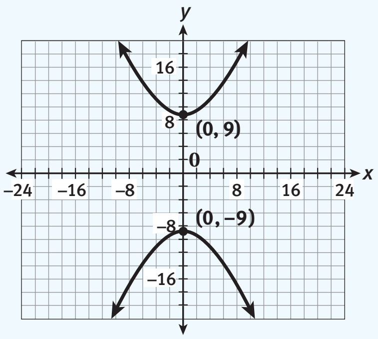


2a. (0, 0)

b. vertical

c. $y = \pm \frac{9}{5}x$

d.

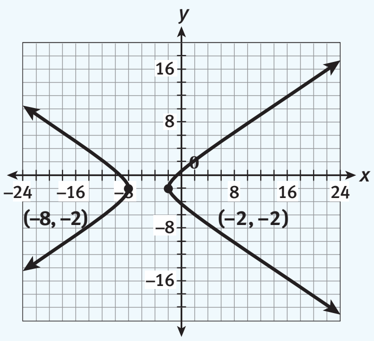


3a. (−5, −2)

b. horizontal

c. $y = -2 \pm \frac{2}{3}(x + 5)$

d.



4a. (2, −3)

b. horizontal

c. $y = -3 \pm 8(x - 2)$

ACTIVITY 7.3 *continued*

Hyperbolas
What's the Difference?

CHECK YOUR UNDERSTANDING

Write your answers on notebook or grid paper. Show your work.

For each hyperbola in Questions 1–5:

a. Give the coordinates of the center.

b. Tell the direction of the transverse axis.

c. Write the equations of the asymptotes.

d. Sketch the hyperbola and label the endpoints of the transverse axis.

1. $\frac{x^2}{16} - \frac{y^2}{49} = 1$

2. $\frac{y^2}{81} - \frac{x^2}{25} = 1$

3. $\frac{(x+5)^2}{9} - \frac{(y+2)^2}{4} = 1$

4. $\frac{(x-2)^2}{1} - \frac{(y+3)^2}{64} = 1$

5. $\frac{(y-2)^2}{100} - \frac{(x-5)^2}{4} = 1$

For each hyperbola in Questions 6–7:

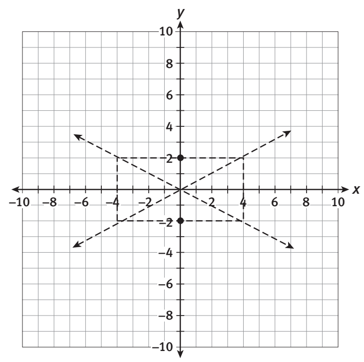
a. Give the coordinates of the center.

b. Tell the direction and length of the transverse axis.

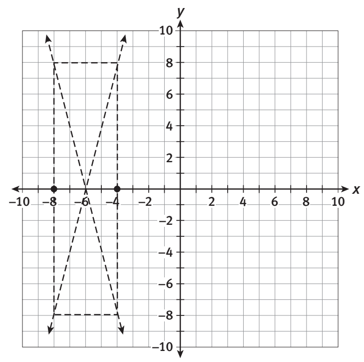
c. Write the equations of the asymptotes.

d. Write the equation of the hyperbola.

6.



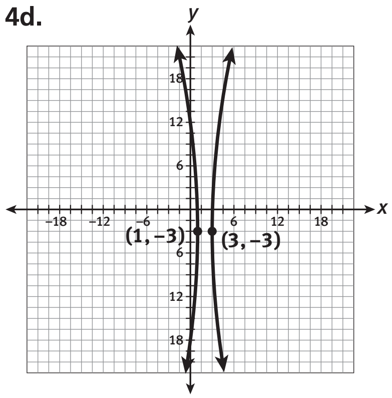
7.



8. MATHEMATICAL REFLECTION How do the asymptotes of a hyperbola help you graph the hyperbola?

398 SpringBoard® Mathematics with Meaning™ Algebra 2

4d.



5–8: See page 397.

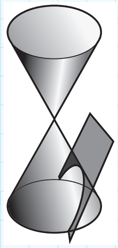
Parabolas
A Parabola on the Roof

ACTIVITY
7.4

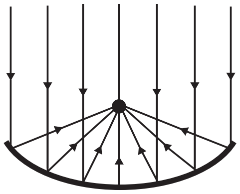
SUGGESTED LEARNING STRATEGIES: Shared Reading, Questioning the Text, Marking the Text, Vocabulary Organizer, Create Representations

In previous units you learned about quadratic functions. The graph of a quadratic function is a parabola, one of the conic sections you have studied in this unit. In this activity, you will learn more about geometric properties of parabolas, their applications in real world settings, and how to recognize and graph them.

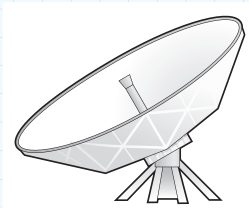
Many people have a parabola on the roof of their homes. The satellite television dishes used to detect television signals are parabolic reflectors. The reason these dishes are shaped like a parabola is due to the following geometric property of a parabola.



When any line parallel to the axis of a parabola hits its surface, the line is reflected through the focus.

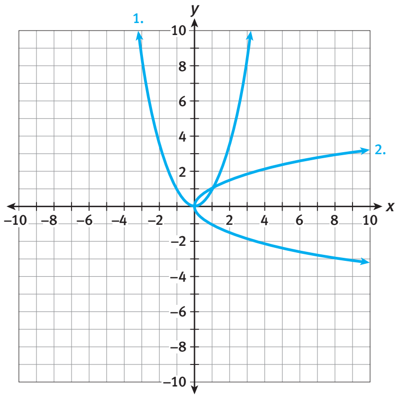


In a satellite dish, the device collects satellite signals over the surface area of the dish. The overall signal is amplified when the individual signals are all reflected to the focus point, where the actual antenna is located at 0.



A **parabola** is the set of points in a plane that are equidistant from a fixed point and a fixed line. The fixed point is called the **focus** and the fixed line is called the **directrix**.

1. Graph $y = x^2$.



2. Form the inverse relation by exchanging x and y and use your knowledge of the properties of inverses to graph this relation on the graph in Item 1. The inverse relation is $x = y^2$.

TEACHER TO TEACHER There are many applications of parabolic surfaces including solar collectors and parabolic mirrors. You may want students to investigate other real-world uses of the parabolic shape.

ACTIVITY 7.4 Guided

Parabolas

Activity Focus
- Properties of parabolas
- Graphing parabolas given the equation in standard form
- Standard form of a parabola

Materials
- Graphing calculator (optional)

Chunking the Activity

#1–2	#8–9
#3–6	Ex. 1–TT A
#7	#10

Differentiating Instruction

Students may need a review of sketching parabolas using transformations prior to beginning this activity. The terms *focus* and *directrix* are introduced on the first page of this unit. Refer students back to the paper folding they did in Activity 7.1 and have them label the focus point and the directrix.

First and Second Paragraphs Shared Reading, Questioning the Text

Third and Fourth Paragraphs Marking the Text

Fifth Paragraph Interactive Word Wall, Vocabulary Organizer

1 Create Representations Make sure students are using key points such as (–2, 4), (–1, 1), (0, 0), (1, 1), and (2, 4) when they make their graphs.

2 Create Representations, Debriefing For groups that struggle with this question, ask them if they recall from a previous unit how to form the inverse of a relation. They can also graphically form the inverse by exchanging the *x*- and *y*-coordinates of the points on the original function.

3 Create Representations
Students will write the inverse relation by exchanging *x* and *y* and then graph the pair of parabolas. Students should be familiar with graphing using transformations. When they graph the horizontal parabolas, make sure they are translating the graphs in the correct direction. As a quick check, the pairs of parabolas should be symmetric with respect to the line $y = x$.

4 Create Representations
Students review the concept of axis of symmetry and understand that some parabolas have a horizontal axis of symmetry.

5 Group Presentation
Students review *vertex* and see that, in terms of conic sections, the vertex is not always a maximum or minimum value of *y*.

6 Quickwrite, Debriefing
This is a critical question. Some students might say that "$x =$" parabolas are horizontal while "$y =$" parabolas are vertical. Ask them about the orientation of a parabola like $x + y^2 = 4$. When you debrief this question, check that students make a connection between the clues given in equations and the orientation of their graphs. Encourage students to examine the equations $y = x^2$ and $x = y^2$ in a thoughtful manner.

7 Think/Pair/Share, Create Representations, Debriefing
This question provides students with independent practice. Use Think/Pair/Share to encourage group members to attempt the problems individually before consulting with their group members.

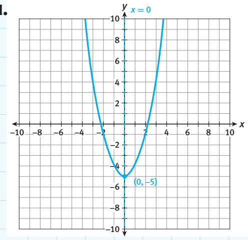
SUGGESTED LEARNING STRATEGIES: Create Representations, Group Presentation, Quickwrite, Think/Pair/Share

My Notes

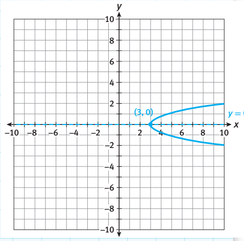
MATH TIP

You can use key points and transformations when graphing vertical or horizontal parabolas.

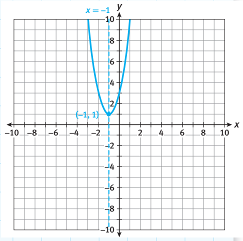
7a.



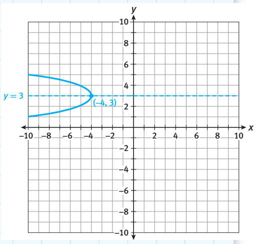
7b.



7c.

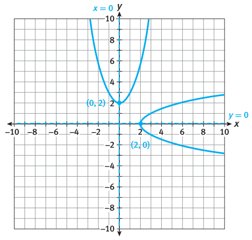


7d.

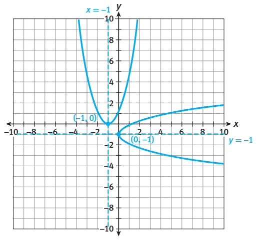


3. For each parabola, write the inverse relation and then sketch the original parabola and its inverse.

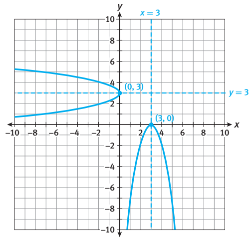
a. $y = x^2 + 2$
The inverse relation is $x = y^2 + 2$.



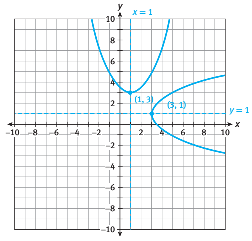
b. $y = (x + 1)^2$
The inverse relation is $x = (y + 1)^2$.



c. $y = -2(x - 3)^2$
The inverse relation is $x = -2(y - 3)^2$.



d. $y = \frac{1}{2}(x - 1)^2 + 3$
The inverse relation is $x = \frac{1}{2}(y - 1)^2 + 3$.



The inverse relations you graphed in Items 2 and 3 are parabolas with a horizontal axis of symmetry.

4. Sketch and label the axis of symmetry for each graph in Item 3.

5. Label the coordinates of the vertex for each parabola in Item 3.

6. How can you determine whether or not a parabola has a vertical or horizontal axis of symmetry?
If the *y* variable has a degree of 2 then the parabola has a horizontal axis of symmetry. If the *x* variable has a degree of 2 then the parabola has a vertical axis of symmetry.

7. Sketch the graph of each parabola, labeling the vertex coordinates and the axis of symmetry. Use the My Notes section of your book.

a. $y = x^2 - 5$ **b.** $x = 2y^2 + 3$

c. $y - 1 = 2(x + 1)^2$ **d.** $x + 4 = -(y - 3)^2$

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 402, #1

UNIT 7 PRACTICE
p. 413, #15

SUGGESTED LEARNING STRATEGIES: Note-taking, Visualization, Look for a Pattern, Create Representations, Identify a Subtask

My Notes

Standard Form of a Parabola

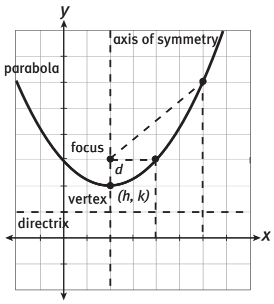
Vertical Axis of Symmetry

$$y - k = \frac{1}{4d}(x - h)^2$$

Horizontal Axis of Symmetry

$$x - h = \frac{1}{4d}(y - k)^2$$

where (h, k) is the vertex and d is the distance from the vertex to the focus.



To find the coordinates of the focus, you add or subtract d to either h or k depending on the orientation of the parabola.

8. For the vertical parabola, what are the coordinates of the focus?
The coordinates of the focus are $(h, k + d)$.

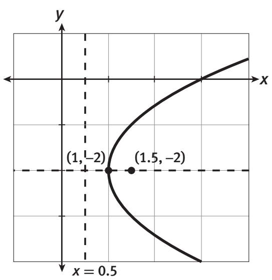
Recall that all points on a parabola are equidistant from the focus and the directrix, including the vertex. To find the equation of the directrix, you subtract d from h or k depending on the orientation of the parabola.

9. For the vertical parabola, what is the equation of the directrix?
The equation of the directrix is $y = k - d$.

EXAMPLE 1

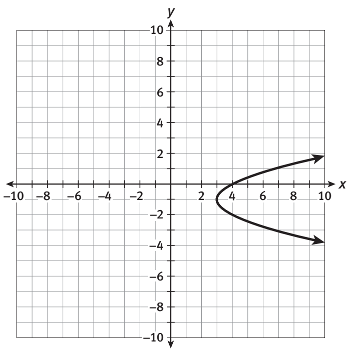
Graph the parabola $x - 1 = \frac{1}{2}(y + 2)^2$. Find the equation of the axis of symmetry, the directrix and the coordinates of the vertex and focus.

- horizontal orientation
- vertex: $(1, -2)$
- axis of symmetry: $y = -2$
- Solve $\frac{1}{4d} = \frac{1}{2}$ to find d.
- $d = \frac{1}{2}$
- Add d to the x-coordinate of the vertex. Focus: $(1.5, -2)$
- Subtract d from the x-coordinate of the vertex.
- Directrix is $x = \frac{1}{2}$

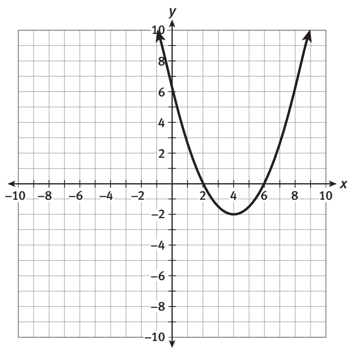


CHECK YOUR UNDERSTANDING (continued from page 402)

3a. vertex $(3, -1)$, axis of symmetry $y = -1$, focus $(3.25, 1)$, directrix $x = 2.75$



3b. vertex $(4, -2)$, axis of symmetry $x = 4$, focus $(4, -1.5)$, directrix $y = -2.5$



ACTIVITY 7.4 *Continued*

TEACHER TO TEACHER Today's lesson focuses on the geometric properties of a parabola and how to identify the vertex, axis of symmetry, focus, and directrix from a standard form equation.

Boxed Text Note Taking, Visualize Make sure to connect the diagram to the equations presented. The dotted lines show that each point on a parabola is equidistant from the focus and directrix.

If needed, provide a similar generic diagram of a horizontal parabola. Have students label the vertex, focus, axis of symmetry, and directrix.

8 Look for a Pattern, Create Representations The x-coordinate stays the same as the vertex and the y-coordinate changes.

9 Look for a Pattern, Create Representations, Debriefing Check to see that students are writing an equation in "$y =$" form, not just an expression for the directrix.

EXAMPLE 1 Identify a Subtask, Note Taking, Create Representations, Debriefing Students should take notes on this example and record the standard forms in their notebooks. The coefficients on these problems were selected so students could find the value of d fairly quickly.

4. Answers may vary. Sample answer: The parabola is horizontal if it is a function of y. The parabola is a vertical parabola if it is a function of x. Only one variable in the equation of a parabola has a squared term.

TRY THESE A Identify a Subtask, Create Representations, Group Presentation

10 Marking the Text, Visualize, Create Representations, Debriefing All students really need to do is figure out the value of d. You might extend this question by having them graph the parabola. If they scale the x- and y-axes equally, then they will see that the curvature of a satellite dish is fairly flat.

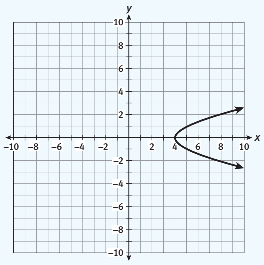
Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 402, #2–4

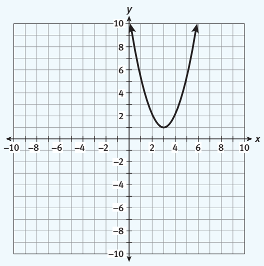
UNIT 7 PRACTICE
p. 413, #16–17

CHECK YOUR UNDERSTANDING

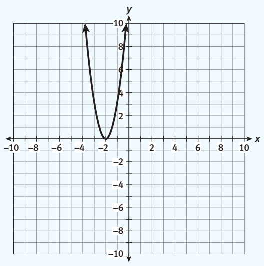
1a. vertex (4, 0), axis of symmetry $y = 0$



b. vertex (3, 1), axis of symmetry $x = 3$

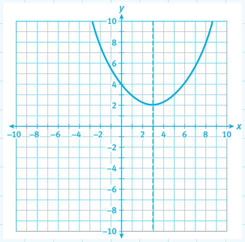


c. vertex (−2, 0), axis of symmetry $x = -2$

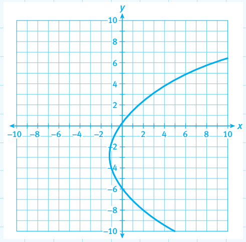


My Notes

a.



b.



SUGGESTED LEARNING STRATEGIES: Identify a Subtask, Create Representations, Group Presentation, Marking the Text, Visualize, Debriefing

TRY THESE A

Graph the parabola. Find the equation of the axis of symmetry, the directrix, and the coordinates of the vertex and focus.

a. $y - 2 = \frac{1}{4}(x - 3)^2$

Axis of symmetry: $x = 3$; directrix: $y = 1$; vertex: (3, 2); focus: (3, 3).

b. $x + 1 = \frac{1}{8}(y + 3)^2$

Axis of symmetry: $y = -3$; directrix: $x = -3$; vertex: (−1, −3); focus: (1, −3)

10. The curvature of a satellite dish is modeled by the parabola

$$y = \frac{1}{64}x^2 + 1$$

where x is measured in inches. If the tip of the antenna needs to be located at the focus of the parabola, then how long should the antenna be?

Solve $\frac{1}{4d} = \frac{1}{64}$, $d = 16$ inches. The antenna should be 16 inches long.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper or grid paper. Show your work.

1. Graph the parabola. State the vertex and axis of symmetry.

a. $x = y^2 + 4$

b. $y = (x - 3)^2 + 1$

c. $y = 4(x + 2)^2$

d. $x + 3y^2 = 1$

2. State the coordinates of the focus and the directrix equation for each parabola.

a. $y = x^2$

b. $x = -\frac{1}{2}y^2$

c. $y - 2 = \frac{1}{12}(x + 1)^2$

d. $x = \frac{1}{16}(y - 2)^2$

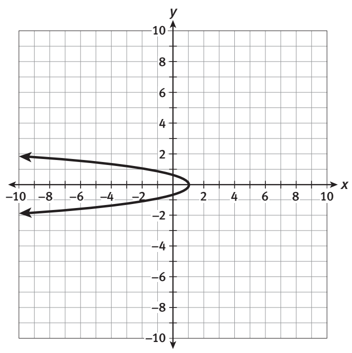
3. Graph the parabola. State the vertex, axis of symmetry, focus and directrix.

a. $x - 3 = (y + 1)^2$

b. $y + 2 = \frac{1}{2}(x - 4)^2$

4. MATHEMATICAL REFLECTION How can you determine whether a parabola has a horizontal or vertical orientation? How does the equation of a parabola differ from the other conic sections you have studied?

1d. vertex (1, 0); axis of symmetry $y = 0$



2a. (0, 0.25), $y = -0.25$

b. (−0.5, 0), $x = 0.5$

c. (−1, 5), $y = -1$

d. (4, 2), $x = -4$

3–4. See p. 401.

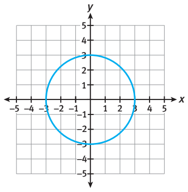
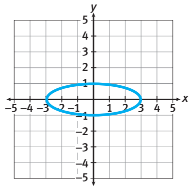
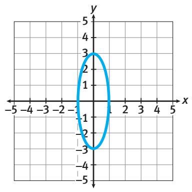
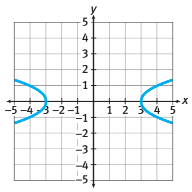
Identifying Conic Sections
How Can You Tell?

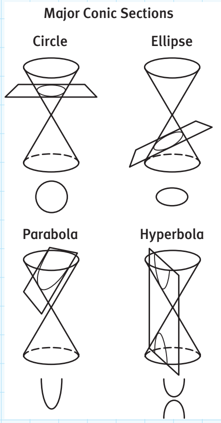
ACTIVITY 7.5

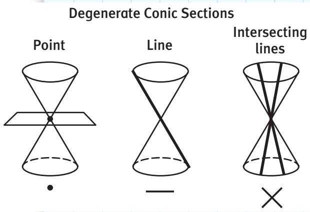
SUGGESTED LEARNING STRATEGIES: Shared Reading, Marking the Text, Activating Prior Knowledge, Create Representations, Think/Pair/Share

As you have been graphing and identifying geometric properties of the conic sections, you have generally been using the standard form of the relation. Each of the conic sections can also be represented by the general form $Ax^2 + Cy^2 + Dx + Ey + F = 0$, where A, C, D, E, and F are constants. The values of A, C, D, E, and F determine the conic and its properties.

1. Complete the chart below by sketching and identifying the conic section and stating the values of A and C.

Equation Conic Section Values of A and C	Graph
a. $x^2 + y^2 - 9 = 0$ Conic: Circle $A = 1$ $C = 1$	
b. $x^2 + 9y^2 - 9 = 0$ Conic: Ellipse $A = 1$ $C = 9$	
c. $9x^2 + y^2 - 9 = 0$ Conic: Ellipse $A = 9$ $C = 1$	
d. $x^2 - 9y^2 - 9 = 0$ Conic: Hyperbola $A = 1$ $C = -9$	





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ACTIVITY 7.5 Guided

Identifying Conic Sections

Activity Focus
- Identifying conic sections
- Completing the square to convert from general to standard form of a conic section
- Graphing conic sections in general form

Materials
- No special materials are needed.

Chunking the Activity

#1–3	Example 2
Try These A	Example 3
#4	Example 4
Example 1–Try These B	Example 5
	Try These C

First Paragraph Shared Reading, Marking the Text

1 Activating Prior Knowledge, Create Representations, Think/Pair/Share, Debriefing Students explore how the general form of quadratic relations relates to the different conic sections and look for patterns in identifying the conic section, given the equation in general form. Allow students to explore these equations with little support. Use questioning techniques to lead students to make the connections between standard and general forms. Have students share in their groups and do self and peer editing as appropriate. Then have students discuss how to determine the conic section, given the equation in general form.

1 (*continued*) Activating Prior Knowledge, Create Representations, Think/Pair/Share, Debriefing These parts of the Item continue the work described in the teacher note on the previous page.

2 Look for a Pattern, Notetaking, Group Presentation Students formalize the methodology for determining the conic section from the general form of the quadratic relation. Have groups share their work identifying the conic, and then bring the entire class together to formalize the ideas in their notes.

TEACHER TO TEACHER Students may notice that the *Bxy* term of the general form of a quadratic relation is not included in the equation shown in Item 2. Explain that the *Bxy* term is used indicate a rotation of a graph and its shape. Since none of the graphs in this unit are rotated, the value of *B* can be considered to be zero.

Paragraph Vocabulary Organizer, Interactive Word Wall Students may not be familiar with the term *degenerate form*. Discuss the definition, using visuals to solidify the concept in their minds. Use the vocabulary organizer to help students verbalize and clarify the concept.

3 Look for a Pattern, Think/Pair/Share, Debriefing Follow a similar pattern as was used in Item 2 to have students articulate how to identify the relationship between the quadratic relations and the degenerate forms. In place of full group presentations, have a few students share their thoughts so that multiple thought processes are heard.

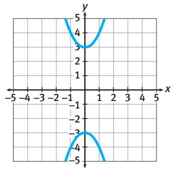
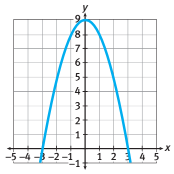
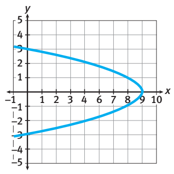
My Notes

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Create Representations, Think/Pair/Share, Look for a Pattern, Note-taking, Group Presentation, Vocabulary Organizer

ACADEMIC VOCABULARY

A **quadratic relation** has the general form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

1. (*continued*)

Equation Conic Section Values of *A* and *C*	Graph
e. $y^2 - 9x^2 - 9 = 0$ Conic: Hyperbola $A = -9$ $C = 1$	
f. $x^2 + y - 9 = 0$ Conic: Parabola $A = 1$ $C = 0$	
g. $y^2 + x - 9 = 0$ Conic: Parabola $A = 0$ $C = 1$	

MATH TERMS

A degenerate state is a limiting case in which an object changes its nature so that it belongs to another, usually simpler description. For example, the point is a degenerate case of the circle as the radius approaches 0, and the circle is a degenerate form of an ellipse as the eccentricity approaches 0. The **degenerate conic sections** are the point, the line, and two intersecting lines.

2. Compare and contrast the values of *A* and *C*. Make conjectures that complete the statement.

The graph of $Ax^2 + Cy^2 + Dx + Ey + F = 0$ is

a. a circle if $A = C$

b. an ellipse if $A \neq C$ and $AC > 0$

c. a hyperbola if $AC < 0$

d. a parabola if $A = 0$ or $C = 0$

The **degenerate conic sections** are also represented by the equation $Ax^2 + Cy^2 + Dx + Ey + F = 0$.

3. What values of the coefficients would produce

a. a line?
$A = 0$ and $C = 0$

b. a point?
$A = C$ and $D, E, F = 0$

SUGGESTED LEARNING STRATEGIES: Shared Reading, Vocabulary Organizer, Interactive Word Wall, Look for a Pattern, Quickwrite, Think/Pair/Share, Marking the Text, Activating Prior Knowledge, Note-taking

My Notes

TRY THESE A

Identify each equation as a *circle, ellipse, hyperbola, line,* or *parabola*.

a. $x^2 - 9y^2 + 10x + 54y - 47 = 0$ hyperbola
b. $x^2 + y^2 = 100$ circle
c. $y^2 - 6y - x + 3 = 0$ parabola
d. $9x^2 + 4y^2 - 54x + 16y - 479 = 0$ ellipse
e. $x^2 + 4y - 36 = 0$ parabola
f. $9y - 3x - 12 = 0$ line
g. $y^2 - 4x^2 + 32x + 4y - 96 = 0$ hyperbola
h. $9x^2 + 25y^2 = 225$ ellipse

In Item 1(a), the values of *D* and *E* were zero. The **quadratic relations** below represent the graphs of four different circles, some of which have *C* and *D* coefficients.

Quadratic Relation	Center	Radius
$x^2 + y^2 = 16$	(0, 0)	4
$x^2 + y^2 + 6x = 7$	(−3, 0)	4
$x^2 + y^2 - 4y = 12$	(0, 2)	4
$x^2 + y^2 + 6x - 4y = 3$	(−3, 2)	4

4. Make several conjectures about the relationship between the coefficients of the terms of each quadratic relation and the center of the circle it represents.
Answers will vary; Sample answer: Coordinates of center are (opposite of $\frac{1}{2}$ coefficient on *x*, opposite of $\frac{1}{2}$ coefficient on *y*).

Because graphing and identifying the geometric characteristics of a conic section is most easily done from the standard form of the relation, it is important to be able to write the general form in the standard form.

To find the center and radius of a circle given its general form, complete the square on each variable to write the equation in the form $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle and the radius is *r*.

MATH TIP

Since conic sections are second degree and are not always functions, they are also known as **quadratic relations.**

CONNECT TO AP

On AP Calculus exams, you may use a graphic calculator to graph a function, solve an equation, and perform other computations without having to show any additional work.

EXAMPLE 1

Find the center and radius of $x^2 + y^2 + 8x - 10y - 8 = 0$.

- $(x^2 + 8x) + (y^2 - 10y) = 8$
  $\frac{1}{2}(8) = 4; (4)^2 = 16$ $\qquad \frac{1}{2}(-10) = (-5); (-5)^2 = 25$
- $(x^2 + 8x + 16) + (y^2 - 10y + 25) = 8 + 16 + 25$
- $(x + 4)^2 + (y - 5)^2 = 49$

Center: (−4, 5); radius: 7

MATH TIP

To complete the square:

- Group like variables together and isolate the constant.
- Take one-half the coefficient on the linear term(s), square the result(s).
- Add the square(s) to both sides of the equation.
- Factor and simplify.

Connect to AP

Calculators are required for certain questions on both the AP Statistics and AP Calculus Examinations. On the AP Calculus examination, calculators can be used to graph a function, solve an equation, compute a numerical integral and compute a numerical derivative without showing any additional work. Students can confirm their work to identify the type of conic section by solving the given equation for *y* and graphing it on their calculator.

ACTIVITY 7.5 *Continued*

TRY THESE A These Items will provide assessment for concepts covered in Items 2 and 3. Work individually with those students who are still having trouble with the concepts.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 408, #1

UNIT 7 PRACTICE
p. 413, #18–22

First Paragraph and Boxed Text Shared Reading, Vocabulary Organizer, Interactive Word Wall

4 Look for a Pattern, Quickwrite, Think/Pair/Share

Second and Third Paragraphs Shared Reading, Marking the Text

EXAMPLE 1 Activating Prior Knowledge, Note Taking
Students should have some familiarity with completing the square. Connect back to prior learning and reinforce understanding by having a volunteer work through completing the square on a quadratic.

TECHNOLOGY TIP

To graph a conic section on a calculator, solve the equation for *y*. Then, the two branches of the conic section can be entered into two separate equations. Sometimes the screen resolution prevents the two halves from appearing connected to one another. Make sure students understand that the curves do not have any gaps, even if it appears that way on the calculator screen.

ACTIVITY 7.5 *Continued*

TRY THESE B Create Representations Assist students as needed with completing the square.

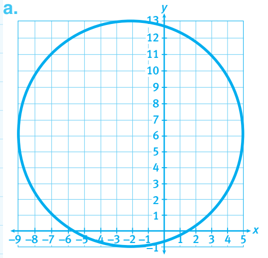
EXAMPLES 2–5 Note Taking, Discussion Group Using this set of examples, consider doing a jigsaw with students. Assign a group of students to each example and have them become the expert on it. Circulate among the groups to make sure that no one has questions. Then have members of each group form new groups that include an expert for each example. Students will then lead a discussion group and share how to complete the square, given each conic section.

Identifying Conic Sections
How Can You Tell?

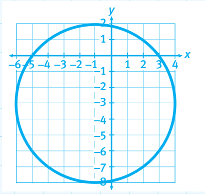
SUGGESTED LEARNING STRATEGIES: Create Representations, Note-taking, Discussion Group

My Notes

a.



b.



TRY THESE B

Write each circle in standard form. Find the center and radius. Then graph the circle in the My Notes space.

a. $x^2 + y^2 + 4x - 12y = 9$ $(x + 2)^2 + (y - 6)^2 = 49$, center: $(-2, 6)$, radius: 7

b. $x^2 + y^2 + 2x + 6y - 15 = 0$ $(x + 1)^2 + (y + 3)^2 = 25$, center: $(-1, -3)$, radius: 5

The general form of any conic can be written in standard form by completing the square.

EXAMPLE 2

Write $x^2 + 25y^2 + 6x - 100y + 9 = 0$ in standard form. Identify the conic and center.

- $x^2 + 6x + 25y^2 - 100y = -9$
  $x^2 + 6x + 25(y^2 - 4y) = -9$
- $\frac{1}{2}(6) = 3; (3)^2 = 9$ $\qquad \frac{1}{2}(-4) = -2; (-2)^2 = 4$
- $(x^2 + 6x + 9) + 25(y^2 - 4y + 4) = -9 + 9 + 100$
- $(x + 3)^2 + 25(y - 2)^2 = 100$

$$\frac{(x+3)^2}{100} + \frac{25(y-2)^2}{100} = 1$$
$$\frac{(x+3)^2}{100} + \frac{(y-2)^2}{4} = 1$$

ellipse; center: $(-3, 2)$

EXAMPLE 3

Write $y^2 - 4x^2 - 8x - 18y + 13 = 0$ in standard form. Identify the conic and center.

- $y^2 - 18y - 4(x^2 + 2x) = -13$
- $\frac{1}{2}(-18) = -9; (-9)^2 = 81$ $\qquad \frac{1}{2}(2) = 1; (1)^2 = 1$
- $(y^2 - 18y + 81) - 4(x^2 + 2x + 1) = -13 + 81 - 4$
- $(y - 9)^2 - 4(x + 1)^2 = 64$

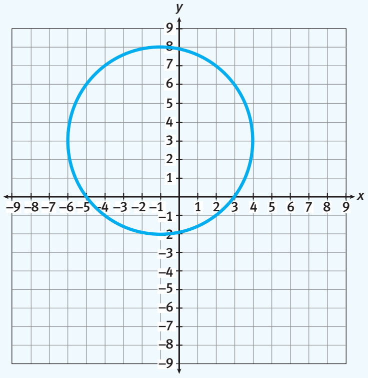
$$\frac{(y-9)^2}{64} - \frac{4(x+1)^2}{64} = 1$$
$$\frac{(y-9)^2}{64} - \frac{(x+1)^2}{16} = 1$$

hyperbola; center: $(-1, 9)$

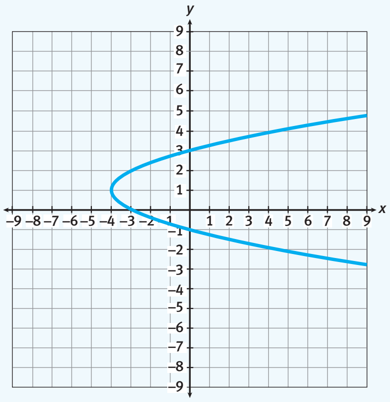
CHECK YOUR UNDERSTANDING

(*continued from page 407*)

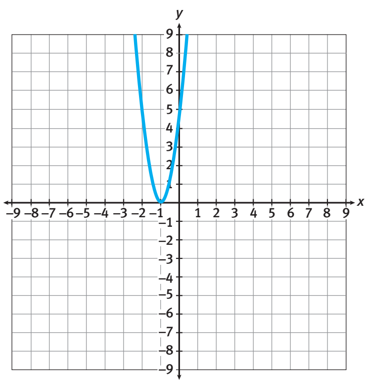
1g.



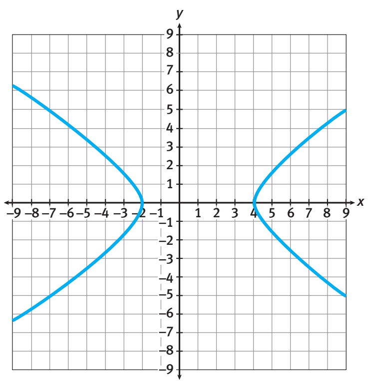
1h.



1i.



1j.



SUGGESTED LEARNING STRATEGIES: Note-taking, Discussion Group

My Notes

EXAMPLE 4

Write $9x^2 + 4y^2 + 36x - 8y + 4 = 0$ in standard form. Identify the conic and center.

- $9x^2 + 36x + 4y^2 - 8y = -4$
  $9(x^2 + 4x) + 4(y^2 - 2y) = -4$
- $\frac{1}{2}(4) = 2;\ 2^2 = 4$ $\qquad \frac{1}{2}(-2) = -1;\ (-1)^2 = 1$
- $9(x^2 + 4x + 4) + 4(y^2 - 2y + 1) = -4 + 36 + 4$
- $9(x + 2)^2 + 4(y - 1)^2 = 36$
  $\frac{9(x+2)^2}{36} + \frac{4(y-1)^2}{36} = 1$
  $\frac{(x+2)^2}{4} + \frac{(y-1)^2}{9} = 1$
  ellipse; center: $(-2, 1)$

EXAMPLE 5

Write $2y^2 - x - 8y = 0$ in standard form. Identify the conic and vertex.

- $2y^2 - 8y - x = 0$
  $2(y^2 - 4y) - x = 0$
- $\frac{1}{2}(4) = 2;\ 2^2 = 4$
- $2(y^2 - 4y + 4) - x = 0 + 8$
- $2(y - 2)^2 - x = 8$
  $x + 8 = 2(y - 2)^2$
  parabola; vertex: $(-8, 2)$

TRY THESE C

Write each equation in standard form. Identify the conic section and its geometric characteristics. Write your answers in the My Notes space.

a. $x^2 - y^2 + 8x + 6y - 18 = 0$
$\frac{(x+4)^2}{25} - \frac{(y-3)^2}{25} = 1$, Hyperbola, center: $(-4, 3)$, asymptotes: $y = 3 \pm (x + 4)$, Transverse axis is horizontal with vertices $(1, 3)$, $(-9, 3)$

b. $4x^2 + y^2 + 16x - 6y + 9 = 0$
$\frac{(x+2)^2}{4} + \frac{(y-3)^2}{16} = 1$, Ellipse, Center: $(-2, 3)$, Major axis is vertical, and 8 units in length, with vertices $(-2, 7)$, $(-2, -1)$

c. $x^2 - 6x - y + 4 = 0$
$y = (x - 3)^2 - 5$, Parabola, vertex $(3, -5)$; axis of symmetry $x = 3$

ACTIVITY 7.5 *Continued*

EXAMPLES 4–5 Note Taking, Discussion Group See the note on the previous page, describing a way to involve all students and the groups with these examples.

TRY THESE C After students have discussed the examples, use these problems for assessment. The experts for each of the examples can become the resource for those students who have questions related to their conic section.

Suggested Assignment

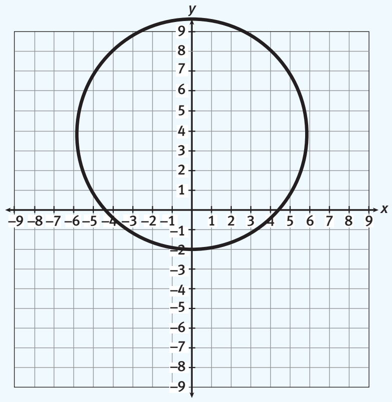
CHECK YOUR UNDERSTANDING
p. 408, #2

UNIT 7 PRACTICE
p. 413, #23–27

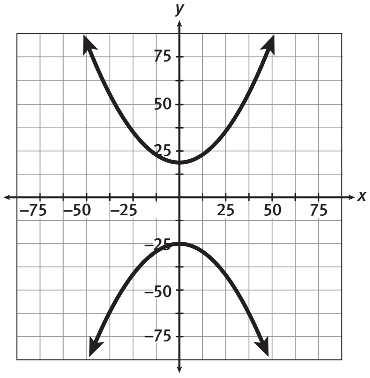
CHECK YOUR UNDERSTANDING

(*continued from page 408*)

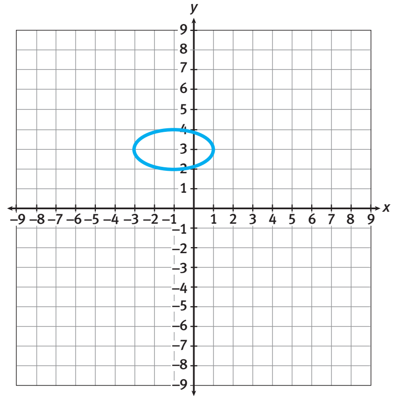
1d.



1e.



1f.



CHECK YOUR UNDERSTANDING

1a. parabola; $y + 3 = 2(x - 2)^2$

b. hyperbola; $\frac{y^2}{25} - \frac{x^2}{4} = 1$

c. ellipse; $\frac{(x-5)^2}{4} + \frac{(y+3)^2}{16} = 1$

d. circle; $x^2 + (y - 4)^2 = 36$

e. hyperbola; $\frac{(y+3)^2}{400} - \frac{(x-1)^2}{\frac{400}{3}} = 1$

f. ellipse; $\frac{(x+1)^2}{4} + (y - 3)^2 = 1$

g. circle; $(x + 1)^2 + (y - 3)^2 = 25$

h. parabola; $x + 4 = (y - 1)^2$

i. parabola; $y = 6(x + 1)^2$

j. hyperbola; $\frac{(x-1)^2}{9} - \frac{y^2}{4} = 1$

Graphs for 1a–j are on pages 406 and 407.

2. Answers may vary. Sample answer: Identifying certain characteristics of conic sections is easier when the equation is in a particular form.

ACTIVITY 7.5 *continued*

Identifying Conic Sections
How Can You Tell?

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper or grid paper. Show your work.

1. For Parts (a)–(j) below, identify each equation as representing a *circle*, *ellipse*, *hyperbola*, or *parabola*. Write each equation in standard form. Graph each relation.

a. $2x^2 - 8x - y + 5 = 0$

b. $4y^2 - 25x^2 = 100$

c. $4x^2 + y^2 - 40x + 6y = -93$

d. $x^2 + y^2 - 8y - 20 = 0$

e. $y^2 - 3x^2 + 6x + 6y - 394 = 0$

f. $x^2 + 4y^2 + 2x - 24y + 33 = 0$

g. $x^2 + y^2 + 2x - 6y - 15 = 0$

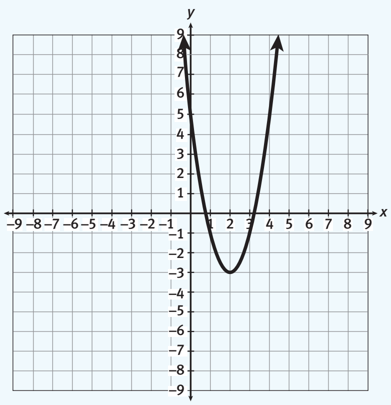
h. $y^2 - x - 2y - 3 = 0$

i. $6x^2 + 12x - y + 6 = 0$

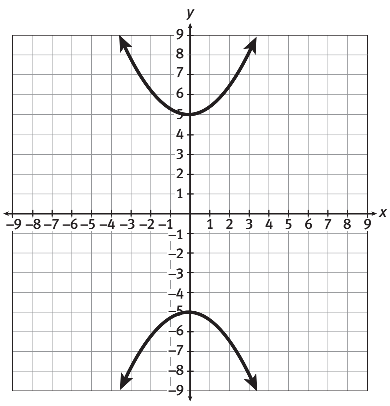
j. $4x^2 - 9y^2 - 8x - 32 = 0$

2. **MATHEMATICAL REFLECTION** Why is it useful to be able to change the form of the equation for a conic section?

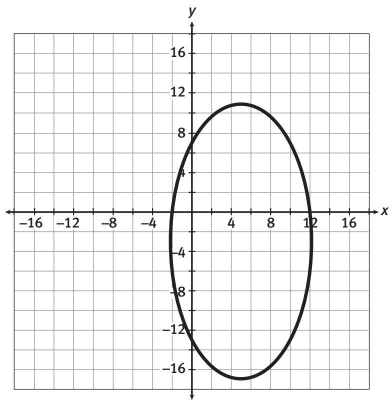
1a.



b.



c.



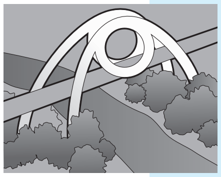
Conic Sections

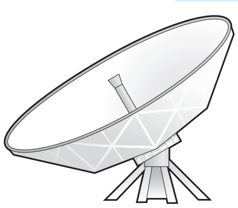
WORKING WITH US

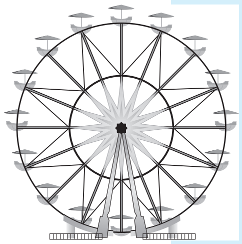
Embedded Assessment 1

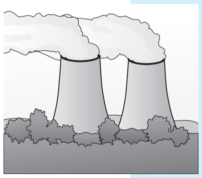
Use after Activity 7.5.

When studying astronomy we learn that stars, planets and comets have orbital paths that are circular, elliptical, parabolic and hyperbolic. Applications of the conic sections also occur in everyday life; such as machine gears, telescopes, headlights, radar, sound waves, navigation, roller coasters, hyperbolic cooling towers and suspension bridges.









State whether each equation represents a *circle*, *ellipse*, *hyperbola*, or *parabola*.

1. $x^2 + y^2 + 2x - 8 = 0$

2. $x^2 - 9y = 0$

3. $25y^2 - 9x^2 - 50y - 200 = 0$

4. $x^2 - 2x - y + 1 = 0$

5. $4x^2 + 3y^2 + 32x - 6y + 67 = 0$

Sketch the graph of each equation.

6. $2y^2 + x - 12y + 10 = 0$

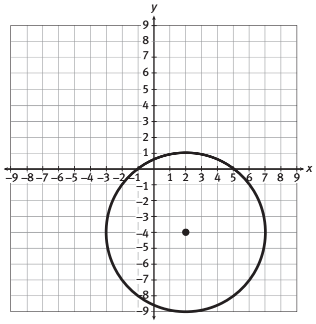
7. $x^2 + y^2 - 10x - 4y - 20 = 0$

8. $9x^2 + 36y^2 - 216y = 0$

9. $16x^2 - 9y^2 - 144 = 0$

Give the standard equation of each graph.

10.



11. a parabola with vertex (4, 1), axis of symmetry $y = 1$ and passing through the point (3, 3)

12. an ellipse with vertices of the major axis at (10, 2) and (−8, 2) and minor axis of length 6

Embedded Assessment 1

Assessment Focus

- Identifying equations as the equation of a particular conic section
- Graphing conic sections
- Writing the equations of conic sections

Materials

- Graph paper

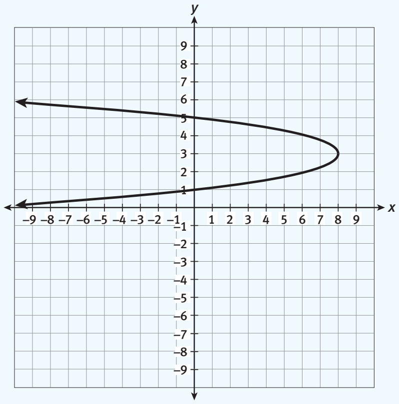
1-5 Students identify the conic section represented by each equation.

Answer Key

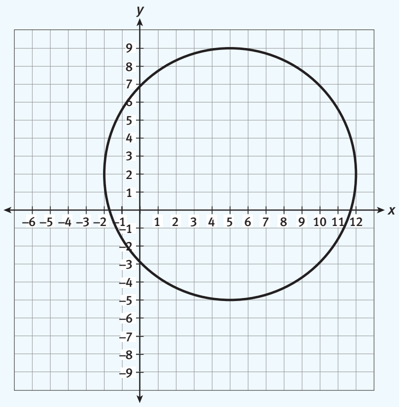
1. circle
2. parabola
3. hyperbola
4. parabola
5. degenerate conic

6-9 Students graph conic sections.

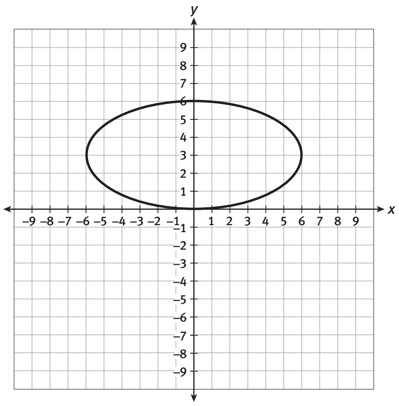
6.



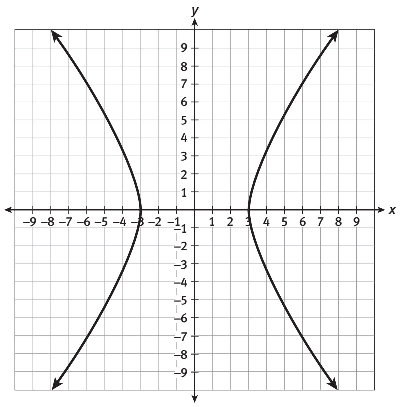
7.



8.



9.



Embedded Assessment 1

Answer Key (*Continued*)

10–12 Students write the equation of conic sections when given the graph or information about the conic.

10. $(x - 2)^2 + (y + 4)^2 = 25$

11. $x = -\frac{1}{4}(y - 1)^2 + 4$

12. $\frac{(x - 1)^2}{81} + \frac{(y - 2)^2}{9} = 1$

TEACHER TO TEACHER You may wish to read through the rubric with students and discuss the differences in the expectation levels. Make sure students understand the meanings of any terms used.

Embedded Assessment 1

Use after Activity 7.5

Conic Sections

WORKING WITH US

	Exemplary	Proficient	Emerging
Math Knowledge	The student: • Correctly identifies the equations. (1–5) • Recognizes the conic section that the equation represents. (6–9)	The student: • Correctly identifies only three or four of the equations. • Recognizes only two or three of the equations.	The student: • Correctly identifies only one or two of the equations. • Recognizes only one of the equations.
Problem Solving	The student gives the correct standard equations. (10–12)	The student gives only two correct standard equations.	The student gives only one correct standard equation.
Representations	The student graphs the equations correctly. (6–9)	The student graphs only two or three of the equations correctly.	The student graphs only one of the equations correctly.

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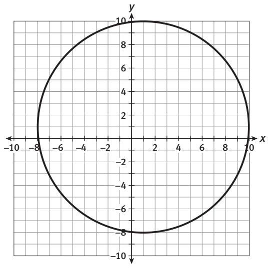
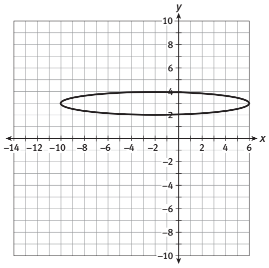
ACTIVITY 7.2

1. Write the equation of each ellipse and circle in standard form.

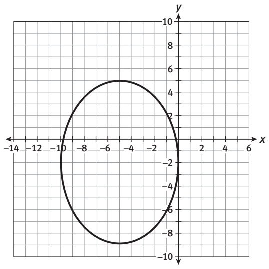
a.



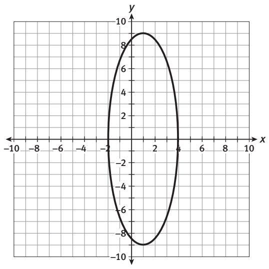
b.



c.



d.



2. Graph each equation. Label key points on each graph.

a. $\frac{x^2}{25} + \frac{y^2}{81} = 1$

b. $\frac{(x-2)^2}{16} + \frac{(y-3)^2}{49} = 1$

c. $\frac{(x+3)^2}{16} + (y-7)^2 = 1$

3. Write the equation for each figure.

a. a circle with center $(-3, 4)$ and radius 6

b. a circle whose diameter has endpoints $(3, 5)$ and $(11, 5)$

c. an ellipse centered at the origin and having a major axis of 10 units and a vertical minor axis of 6 units

d. an ellipse with center $(3, 4)$, a horizontal minor axis 10 units long and a major axis 20 units long.

4. Given an ellipse with a major axis with endpoint $(-6, 0)$ and foci $(-4, 0)$ and $(4, 0)$.

a. Explain how to find the center.

b. Explain how to find the endpoints and the length of the minor axis.

c. Write the equation of the ellipse.

UNIT 7 PRACTICE

Activity 7.2

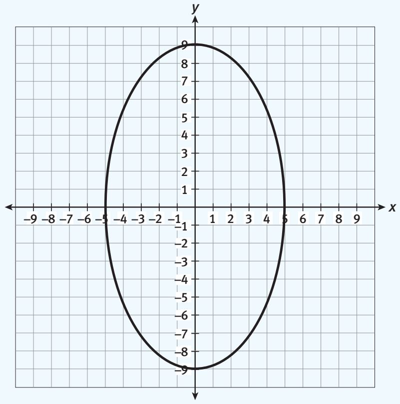
1a. $(x-1)^2 + (y-1)^2 = 81$

b. $\frac{(x+2)^2}{64} + (y-3)^2 = 1$

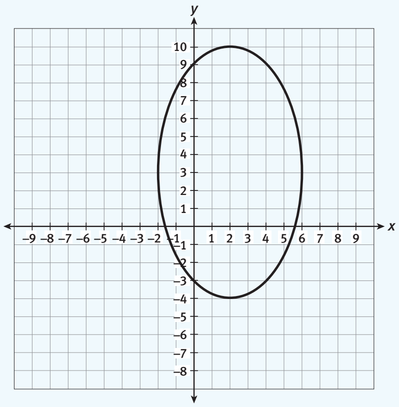
c. $\frac{(x+5)^2}{25} + \frac{(y+2)^2}{49} = 1$

d. $\frac{(x-1)^2}{9} + \frac{y^2}{81} = 1$

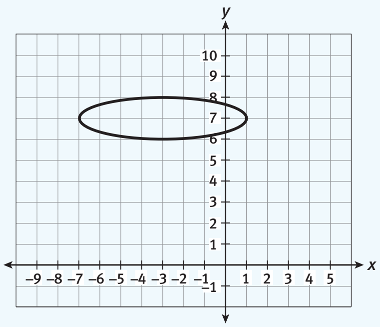
2a.



b.



c.



3a. $(x+3)^2 + (y-4)^2 = 36$

b. $(x-7)^2 + (y-5)^2 = 16$

c. $\frac{x^2}{25} + \frac{y^2}{9} = 1$

d. $\frac{(x-3)^2}{25} + \frac{(y-4)^2}{100} = 1$

4a. The center will occur at the midpoint of the line segment from $F_1(-4, 0)$ to $F_2(4, 0)$, or $(0, 0)$.

b. The major and minor axis are related by the equation $c^2 = a^2 - b^2$, therefore plugging in $c = 4$ and $a = 6$ we find $b \approx 4.472$, or 4.5. So the endpoints of the minor axis would be about $(0, \pm 4.5)$ and the length is about 9.

c. $\frac{x^2}{36} + \frac{y^2}{20} = 1$

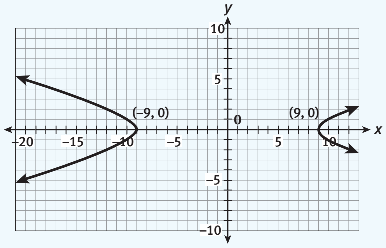
Activity 7.3

5a. (0, 0)

b. horizontal

c. $y = \pm\frac{2}{9}x$

d.

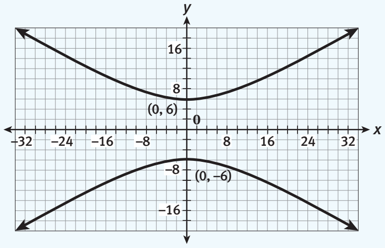


6a. (0, 0)

b. vertical

c. $y = \pm\frac{3}{5}x$

d.

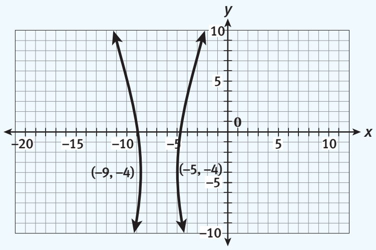


7a. (−7, −4)

b. horizontal

c. $y = -4 \pm 4(x + 7)$

d.

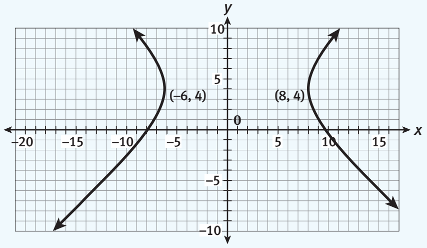


8a. (1, 4)

b. horizontal

c. $y = 4 \pm \frac{6}{7}(x - 1)$

d.



UNIT 7 **Practice**

ACTIVITY 7.3

For each hyperbola in Questions 5–9:

a. Give the coordinates of the center.
b. Tell the direction of the transverse axis.
c. Tell the equations of the asymptotes.
d. Sketch the hyperbola and label the endpoints of the transverse axis.

5. $\frac{x^2}{81} - \frac{y^2}{4} = 1$

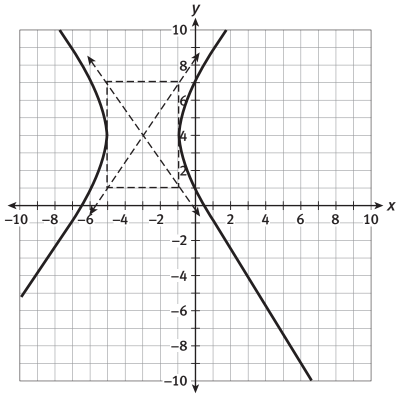
6. $\frac{y^2}{36} - \frac{x^2}{100} = 1$

7. $\frac{(x+7)^2}{4} - \frac{(y+4)^2}{64} = 1$

8. $\frac{(x-1)^2}{49} - \frac{(y-4)^2}{36} = 1$

9. $\frac{(y+3)^2}{121} - \frac{(x-3)^2}{9} = 1$

10. Label the coordinates of the center, the vertices and the foci of the hyperbola below.



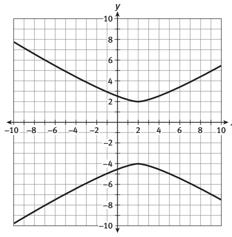
Match each equation below with the correct graph.

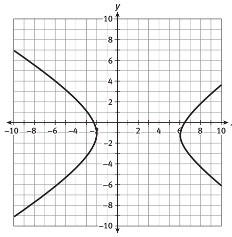
11. $\frac{(x+2)^2}{16} - \frac{(y-1)^2}{9} = 1$

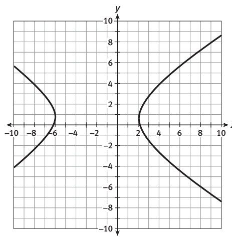
12. $\frac{(x-2)^2}{16} - \frac{(y+1)^2}{9} = 1$

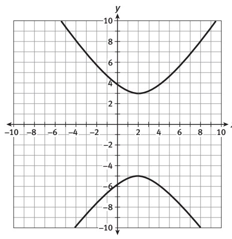
13. $\frac{(y+1)^2}{16} - \frac{(x-2)^2}{9} = 1$

14. $\frac{(y+1)^2}{9} - \frac{(x-2)^2}{16} = 1$

A.


B.


C.


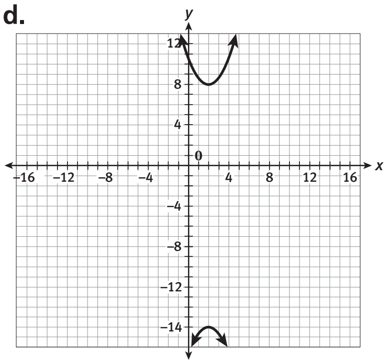
D.


9a. (3, −3)

b. vertical

c. $y = -3 \pm \frac{11}{3}(x - 3)$

d.



10. Center: (−3, 4); Vertices: (−5, 4), (−1, 4); Foci: $(-3 - \sqrt{13}, 4)$, $(-3 + \sqrt{13}, 4)$

11. C

12. B

13. D

14. A

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ACTIVITY 7.4

15. Graph the parabola, state the vertex and axis of symmetry.

a. $y = -3x^2 + 5$

b. $x - 3 = -(y + 2)^2$

16. State the coordinates of the focus and the directrix equation for each parabola.

a. $y = \frac{1}{8}x^2 - 4$

b. $x + 4 = \frac{1}{20}(y - 1)^2$

17. Graph the parabola

$$x + 5 = -\frac{1}{4}(y - 2)^2$$

State the vertex, axis of symmetry, focus, and directrix.

ACTIVITY 7.5

For each equation in questions 18–25:

a. Identify each equation as representing a *circle*, *ellipse*, *hyperbola*, or *parabola*.

b. Write each equation in standard form.

c. Graph each relation.

18. $x^2 - 4y^2 - 8x + 16y - 36 = 0$

19. $3y^2 - x - 6y - 5 = 0$

20. $16y^2 + 25x^2 = 400$

21. $x^2 + y^2 - 16x + 6y - 27 = 0$

22. $25x^2 + 9y^2 - 50x - 54y - 119 = 0$

23. $x^2 - 2y - 6 = 0$

24. $9x^2 + 4y^2 + 8y - 140 = 0$

25. $x^2 - y^2 + 6y - 10 = 0$

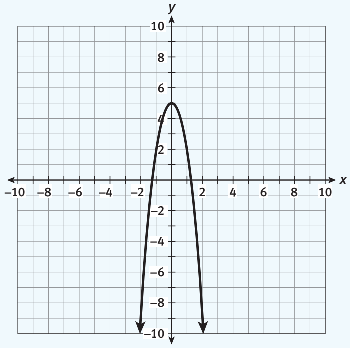
26. $2x^2 - y + 16x + 28 = 0$

27. $x^2 - 4x + y + 3 = 0$

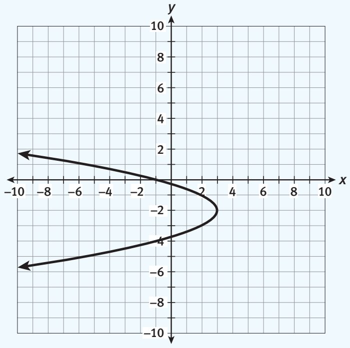
UNIT 7 PRACTICE *Continued*

Activity 7.4

15a. vertex (0, 5), axis of symmetry $x = 0$



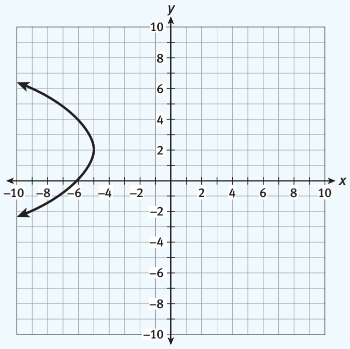
b. vertex (3, −2), axis of symmetry $y = -2$



16a. focus (0, −2), directrix $y = -6$

b. focus (1, 1), directrix $x = -9$

17. vertex (−5, 2), axis of symmetry $y = 2$, focus (−6, 2), directrix $x = -4$



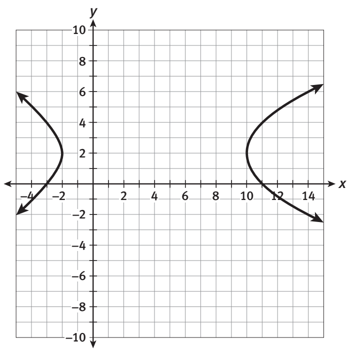
Activity 7.5

18a. hyperbola

b. $\frac{(x-4)^2}{16} - \frac{(y-2)^2}{4} = 1$

20–27. See Additional Answers, pp. 436–437.

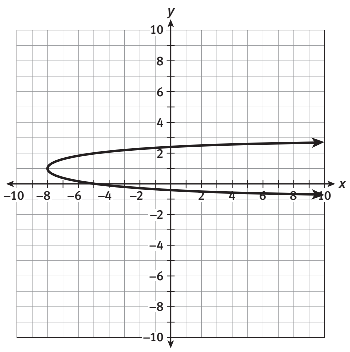
18c.



19a. parabola

b. $x + 8 = 3(y - 1)^2$

c.



Reflection

Student Reflection

Discuss the essential questions with students. Have them share how their understanding of the questions has changed through studying the concepts in the unit.

Review the academic vocabulary. You may want students to revisit the graphic organizers they have completed for academic vocabulary terms and add other notes about their understanding of terms.

Encourage students to evaluate their own learning and to recognize the strategies that work best for them. Help them identify key concepts in the unit and to set goals for addressing their weaknesses and acquiring effective learning strategies.

Teacher Reflection

1. Of the key concepts in the unit, did any present special challenges for students?
2. How will you adjust your future instruction for students/activities?
3. Which strategies were most effective for facilitating student learning?
4. When you teach this unit again, what will you do differently?

UNIT 7

Reflection

An important aspect of growing as a learner is to take the time to reflect on your learning. It is important to think about where you started, what you have accomplished, what helped you learn, and how you will apply your new knowledge in the future. Use notebook paper to record your thinking about the following topics and to identify evidence of your learning.

Essential Questions

1. Review the mathematical concepts and your work in this unit before you write thoughtful responses to the questions below. Support your responses with specific examples from concepts and activities in the unit.

   1 How are the algebraic representations of the conic sections similar and how are they different?

   2 How do the conic sections model real world phenomena?

Academic Vocabulary

2. Look at the following academic vocabulary words:

   - conic section
   - hyperbola
   - standard form
   - ellipse
   - quadratic relation

   Choose three words and explain your understanding of each word and why each is important in your study of math.

Self-Evaluation

3. Look through the activities and Embedded Assessment in this unit. Use a table similar to the one below to list three major concepts in this unit and to rate your understanding of each.

Unit Concepts	Is Your Understanding Strong (S) or Weak (W)?
Concept 1	
Concept 2	
Concept 3	

   a. What will you do to address each weakness?

   b. What strategies or class activities were particularly helpful in learning the concepts you identified as strengths? Give examples to explain.

4. How do the concepts you learned in this unit relate to other math concepts and to the use of mathematics in the real world?

Additional Notes

Math Standards Review

Unit 7

1. The hyperbola $\frac{y^2}{25} - \frac{x^2}{9} = 1$ has foci at the points

A. $(\sqrt{34}, 0), (-\sqrt{34}, 0)$

B. $(0, \sqrt{34}), (0, -\sqrt{34})$

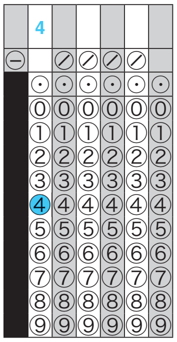
C. $(5, 3), (5, -3)$

D. $(-5, 3), (-5, -3)$

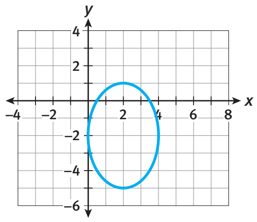
1. Ⓐ Ⓑ Ⓒ Ⓓ

2. A circle has the equation $(x - 3)^2 + (y + 2)^2 = 16$. What is the radius of the circle?

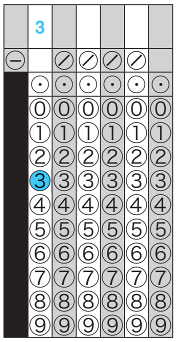
2.



3. Given the equation and graph of the ellipse $\frac{(x-2)^2}{4} + \frac{(y+2)^2}{a^2} = 1$, what is the value of a?



3.



UNIT 7 Math Standards Review

These two pages provide practice with standardized test question formats that are used in many national and state high-stakes tests:

- Multiple choice
- Gridded response
- Constructed response

These items also provide practice with the mathematics content of this unit.

1 Multiple choice
- Find the foci of a hyperbola from its equation

2 Gridded response
- Find the radius of a circle from its equation

3 Gridded response
- Ellipses

4 Constructed response
- Parabolas

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UNIT 7 Math Standards Review

TEACHER TO TEACHER You might read through the extended-response item with students and discuss your expectation levels. Make sure students understand the meanings of any terms used.

Math Standards Review

Unit 7 (continued)

Read
Solve
Explain

4. Given the equation of the parabola $y = -\frac{1}{4}x^2 - \frac{1}{4}x + \frac{39}{16}$

Part A: Write the equation in standard form.

Answer and Explain

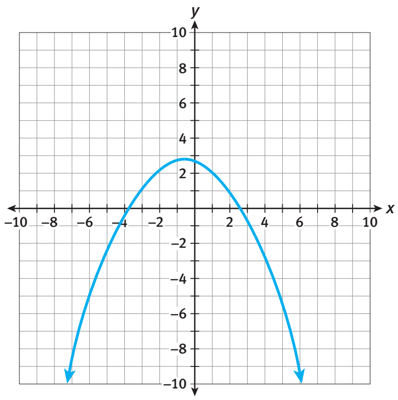
$y - \frac{5}{2} = -\frac{1}{4}\left(x + \frac{1}{2}\right)^2$

Part B: Find the vertex, focus, and directrix.

Answer and Explain

Vertex: $\left(-\frac{1}{2}, \frac{5}{2}\right)$; Focus: $\left(-\frac{1}{2}, \frac{3}{2}\right)$; Directrix: $y = \frac{7}{2}$

Part C: Sketch a graph.



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