Planning the Unit

Unit 4

In prior units students have looked at linear, exponential, and quadratic functions, and matrices, and sequences and series. This unit extends student understanding of polynomial functions and their operations, and introduces students to counting methods, combinations, permutations and binomial probability.

Academic Vocabulary

Blackline masters for use in developing students' vocabulary skills are located at the back of this Teacher's Edition. Encourage students to explore the meanings of the academic vocabulary words in this unit, using graphic organizers and class discussions to help students understand the key concepts related to the terms. Encourage students to place their vocabulary organizers in their math notebooks and to revisit these pages to make notes as their understanding of concepts increases.

Embedded Assessments

The three Embedded Assessments for this unit follow Activities 4.2, 4.4, and 4.7.

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AP/College Readiness

Unit 4 expands on students' understanding of polynomial functions and their graphs, and introduces students to counting principals and binomial probability by:

- Providing contextual situations dealing with polynomial functions as required in the prerequisite topic list for AP Calculus.
- Giving students the opportunity to look at polynomial functions graphically, numerically, algebraically, and verbally, both in and out of contextual situations.
- Beginning to lay the foundation of understanding of local extrema, and methods and tools that will be used at the calculus level to find them.
- Building students' facility with manipulation and computational competence, while extending the conceptual understanding of broad concepts.
- Using technology to solve problems relating to functions and their attributes, and make and verify conjectures about function behavior.

Embedded Assessment 1 This Test is Square

- Polynomial functions

Embedded Assessment 2 Sketch Artist

- Factoring polynomials
- Graphing polynomials

Embedded Assessment 3 The Wedding

- Permutations
- Combinations
- Probability
- Binomial expansion
- Binomial Probability

Suggested Pacing

The following table provides suggestions for pacing either a 45-minute period or a block schedule class of 90 minutes. Space is left for you to write your own pacing guidelines based on your experiences in using the materials.

	45-Minute Period	90-Minute Period	Comments on Pacing
Unit Overview	$\frac{1}{2}$	$\frac{1}{4}$	
Activity 4.1	3	$1\frac{1}{2}$	
Activity 4.2	3	$1\frac{1}{2}$	
Embedded Assessment 1	1	$\frac{1}{2}$	
Activity 4.3	2	1	
Activity 4.4	3	$1\frac{1}{2}$	
Embedded Assessment 2	1	$\frac{1}{2}$	
Activity 4.5	4	2	
Activity 4.6	2	1	
Activity 4.7	2	1	
Embedded Assessment 3	1	$\frac{1}{2}$	
Total	$22\frac{1}{2}$	$11\frac{1}{4}$	

Unit Practice

Practice Problems for each activity in the unit appear at the end of the unit.

Math Standards Review

To help accustom students to the formats and types of questions they may encounter on high stakes tests, additional problems are provided at the end of the unit. These problems are constructed for multiple choice, short response, extended response, and gridded responses.

Unit 4

Polynomials

Unit Overview

In this unit you will study polynomials, beginning with operations and factoring and then investigating intercepts, end behavior, and relative maximums. You will also study permutations, combinations, and binomial probability.

Unit 4 Academic Vocabulary

Add these words and others you encounter in this unit to your vocabulary notebook.

- combination
- end behavior
- extrema
- factorial
- permutation
- polynomial function
- probability distribution

Essential Questions

- How do polynomial functions help to model real-world behavior?
- How is probability used in real-world settings?

EMBEDDED ASSESSMENTS

This unit has three embedded assessments, following Activities 4.2, 4.4, and 4.7. The first two will give you an opportunity to demonstrate your understanding of polynomial functions and the third assessment focuses on your understanding of permutations and combinations.

UNIT 4 OVERVIEW

Unit Overview

Ask students to read the unit overview, define polynomial, and review operations they have studied related to binomial and trinomial expressions. Have students give an example of finding a probability.

Essential Questions

Read the essential questions with students. Remind to review these questions periodically as they complete the activities in the unit.

Materials

- Graphing calculators

Academic Vocabulary

Encourage students to use the mathematics vocabulary that they been acquiring throughout the year whenever they are asked to discuss or write about a mathematical idea.

Embedded Assessments

There are three embedded assessments in this unit with evaluation rubrics. You may want to review skills needed for the assessment with students prior to the beginning of their work.

UNIT 4 GETTING READY

You may wish to assign some or all of these exercises to gauge students' readiness for Unit 4 topics.

Prerequisite Skills

- Rectangular prisms (Item 1)
- Operations with complex numbers (Item 8)
- Combining like terms (Item 2)
- Factoring (Items 3, 4)
  - GCF
  - Difference of squares
  - Trinomials
- Simple probability (Items 5, 6, 7)

Answer Key

1. SA = 220 square units; V = 200 cubic units

2. $4x^2 - x - 2$

3. $x^2(3x + 7y)(3x - 7y)$

4. $(2x + 1)(x - 5)$

5a. $\frac{11}{36}$ **b.** $\frac{1}{36}$

c. $\frac{25}{36}$

6. Check students' drawings. $\frac{1}{4}$ of the spinner should be red, $\frac{1}{4}$ should be blue, and $\frac{1}{2}$ should be yellow

7a. $\frac{3}{4}$ **b.** $\frac{1}{16}$

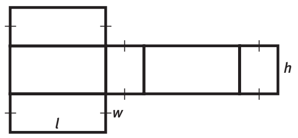
8. sum: $6 + 3i$
product: 26

UNIT 4

Getting Ready

Write your answers on notebook paper or grid paper. Show your work.

1. Find the surface area and volume of a rectangular prism formed by the net below. The length is 10 units, the width is 4 units, and the height is 5 units.



2. Simplify $(2x^2 + 3x + 7) - (4x - 2x^2 + 9)$.

3. Factor $9x^4 - 49x^2y^2$.

4. Factor $2x^2 - 9x - 5$.

5. Two number cubes are tossed at the same time. Find each probability.

a. Exactly one cube shows a 3.
b. Both cubes show a 3.
c. Neither cube shows a 3.

6. A game spinner is 25% red, 25% blue, and 50% yellow. Draw a spinner that matches that description.

7. Using the spinner described in Item 6, find each probability.

a. spinning once and not landing on blue
b. spinning twice and landing on red both times

8. Find the sum and product of $(2 - 3i)$ and $(4 + 6i)$.

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200 SpringBoard® Mathematics with Meaning™ Algebra 2

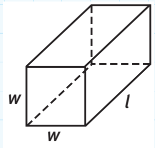
Introduction to Polynomials
Postal Service

ACTIVITY 4.1

SUGGESTED LEARNING STRATEGIES: Shared Reading, Create Representations, Think/Pair/Share

My Notes

The United States Postal Service will not accept rectangular packages if the perimeter of one end of the package plus the length of the package is greater than 130 in. Consider a rectangular package with square ends as shown in the figure.



1. Assume that the perimeter of one end of the package plus the length of the package equals the maximum 130 in. Complete the table with some possible measurements for the length and width of the package. Then find the corresponding volume of each package.
Some possible values are shown in the table below.

Width (in.)	Length (in.)	Volume (in.^{3})
10	90	9000
15	70	15,750
20	50	20,000
25	30	18,750
30	10	9000

2. Give an estimate for the largest possible volume of an acceptable United States Postal Service rectangular package with square ends.
Answers will vary. Estimates should be between 20,000 and 22,000 cubic inches.

CONNECT TO AP

In calculus, you must be able to model a written description of a physical situation with a function.

3. Use the package described in Item 1.

a. Write an expression for l, the length of the package, in terms of w, the width of the square ends of the package.
$l = 130 - 4w$

b. Write the volume of the package V as a function of w, the width of the square ends of the package.
$V = (130 - 4w)w^2$

ACTIVITY 4.1 Investigative

Introduction to Polynomials

Activity Focus
- Introduction to polynomial functions
- Cubic functions
- Intercepts
- Relative maximum and minimum
- End behavior

Materials
- Graphing calculators

Chunking the Activity

#1	#6–7	#11–12
#2–3	#8	#13–14
#4–5	#9–10	

First Paragraph Shared Reading

1 Create Representations This is an entry-level Item designed to help students get comfortable with the context.

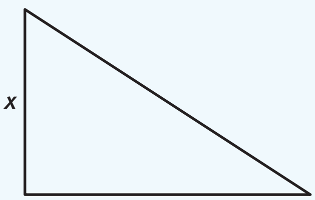
2 Debriefing The purpose of this Item is to have students realize that there will be a maximum value. The accuracy of their estimates may vary and any reasonable estimate should be accepted.

3 Create Representations, Think/Pair/Share The volume of the box is given by $V = lw^2$, so this function, which can be expressed as $V = -4w^3 + 130w^2$, is an example of a cubic function.

Connect to AP

In this activity, students are following up on the process learned in Activity 3.1 where they used numerical data to develop a function. By the time they get to calculus, students should be prepared to write functions from physical situations with relative ease. You can provide additional practice with different geometric situations. For example, give students a right triangle labeled as shown and tell students that the sum of the lengths of the legs is 10. Make sure to ask why the horizontal leg is labeled $10 - x$. Then ask students to write a function for the area of the triangle. [$A(x) = \frac{1}{2}x(10 - x)$]

Then assign a similar problem without any variable labels on the diagram. Finally, assign a similar problem, giving only a written description.



ACTIVITY 4.1 *Continued*

4 Naturally the width of the box must be greater than 0 inches. If the maximum 130 inches is divided by 4, the sides of the end of the box, or width, would be 32.5 inches. However, the length of the box must also be included in the 130 inches, so the width must be less than 32.5 inches. This Item is designed to have students focus only on the positive first quadrant values of the function at this time. Students will explore the function in greater detail later in this activity.

5 Create Representations, Debriefing

6 Note Taking Students can use the graph and trace to the maximum point or they can use the calculator's Max or Min capabilities to find the values. If students are unfamiliar with the graphing calculator, walk them through the *Mini-Lesson: Finding the Maximum Using a Graphing Calculator* below. You may need to adapt the steps for the particular calculator you are using. For some students, writing this process in their notes will be helpful as they progress through the course.

7 Quickwrite

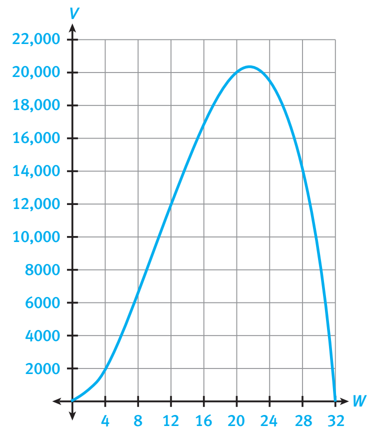
SUGGESTED LEARNING STRATEGIES: Create Representations, Note-taking, Quickwrite

My Notes

4. Consider the smallest and largest possible values for w that makes sense for the function you wrote in Item 3b. Give the domain of the function as a model of the volume of the postal package.

$0 < w < 32.5$

5. Sketch a graph of the function in Item 3(b) over the domain that you found in Item 4. Include the scale on each axis.



6. Use a graphing calculator to find the coordinates of the maximum point of the function that you graphed in Item 5.

(21.667, 20,342.593)

7. What information do the coordinates of the maximum point of the function found in Item 6 provide with respect to an acceptable United States Postal Service package with square ends?

The coordinates indicate that a package of the type described will have a maximum volume of 20,342.593 in^3 and that the width of the box will be 21.667 inches.

TECHNOLOGY TIP

Graphing calculators will allow you to find the maximum and minimum of functions in the graphing window.

CONNECT TO AP

In calculus, you will learn about the derivative of a function, which can be used to find the maximum and minimum values of a function.

MINI-LESSON: Finding the Maximum Using a Graphing Calculator

- Press the **2nd** button and **Trace**. Select **4: maximum.**
- Move the cursor to the left of the maximum value and press ENTER for the **Left Bound?**
- Move the cursor to the right of the maximum value and press ENTER for the **Right Bound?**
- Press ENTER again for the **GUESS?**

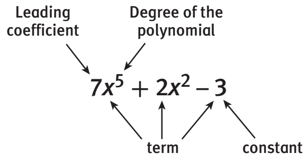
The calculator will then give the point at which the maximum occurs on the chosen interval.

Introduction to Polynomials
Postal Service

SUGGESTED LEARNING STRATEGIES: Marking the Text, Note-taking, Vocabulary Organizer, Interactive Word Wall, Create Representations

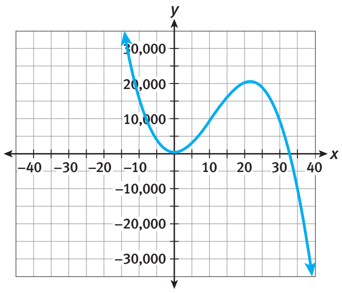
When using a function to model a given situation, such as the acceptable United States Postal Service package, you may be looking at only a portion of the entire domain of the function. Moving beyond the specific situation, you can examine the entire domain of the *polynomial function*.

A **polynomial function** in one variable is a function that can be written in the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$, where n is a nonnegative integer, the coefficients $a_0, a_1, \ldots a_n$ are real numbers, and $a_n \neq 0$. n is the **degree of the polynomial function**.



8. Write a polynomial function $f(x)$ defined over the set of real numbers such that it has the same function rule as $V(w)$ the rule you found in Item 3b. Sketch a graph of the function.

$f(x) = -4x^3 + 130x^2$



My Notes

ACADEMIC VOCABULARY

polynomial function

MATH TERMS

Polynomial functions are named by their **degree.** Here is a list of some common polynomial functions.

Degree	Name
0	Constant
1	Linear
2	Quadratic
3	Cubic
4	Quartic

ACTIVITY 4.1 *Continued*

Paragraphs Marking the Text, Note-taking, Vocabulary Organizer, Interactive Word Wall

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 206, #1–4

UNIT 4 PRACTICE
p. 259, #1–4

8 Create Representations, Debriefing Students move beyond the context of the acceptable rectangular package and examine the graph of the cubic function in all four quadrants.

ACTIVITY 4.1 *Continued*

9 Vocabulary Organizer, Interactive Word Wall, Think/Pair/Share Students can find the relative maximum and minimum values by using the graphing calculator capabilities.

10 Students can find the answer by either factoring this cubic function or using a graphing calculator. The function is simple enough to factor and provides a discussion that prepares students for Activity 4.3.

This would be an appropriate time to investigate the number of x-intercepts that a cubic function may have. This function has two x-intercepts. Ask students to use their graphing calculators to explore whether a cubic function can have fewer or more x-intercepts.

Paragraph Vocabulary Organizer, Interactive Word Wall The symbols $-\infty$ and ∞ have a special meaning in the context of limits. Therefore, remind students that, in this instance, they should remember that the phrase *approaches positive infinity* means "increases without bound," and that *approaches negative infinity* means "decreases without bound."

11-12 Create Representations, Discussion Group These Items are the first opportunity for students to look at end behavior of polynomial functions. They will investigate this concept in more detail in the next Item and in the activities that follow.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Interactive Word Wall, Think/Pair/Share, Create Representations, Discussion Group

My Notes

MATH TERMS

A function value $f(a)$ is called a **relative maximum** of f if there is an interval around a where for any x in that interval $f(a) \geq f(x)$. A function value $f(a)$ is called a **relative minimum** of f if there is an interval around a where for any x in that interval $f(a) \leq f(x)$.

ACADEMIC VOCABULARY

end behavior

MATH TIP

Recall that the phrase "approaches positive infinity ∞" means "increases without bound," and that "approaches negative infinity $-\infty$" means "decreases without bound."

9. Name any **relative maximum** values and **relative minimum** values of the function $f(x)$ in Item 8.

The relative minimum is zero. The relative maximum is about 20,342.593.

10. Name any x- or y-intercepts of the function $f(x) = -4x^3 + 130x^2$.

The x-intercepts are 0 and 32.5. The y-intercept is 0.

When looking at the **end behavior** of a graph, you determine what happens to the graph on the extreme right and left ends of the x-axis. That is, you look to see what happens to y as x approaches $-\infty$ and ∞.

11. Examine the end behavior of $f(x) = -4x^3 + 130x^2$.

a. As x goes to ∞, what behavior does the function have?

as $x \to \infty$, $y \to -\infty$

b. How is the function behaving as x approaches $-\infty$?

as $x \to -\infty$, $y \to \infty$

12. Examine the end behavior of $f(x) = 3x^2 - 6$.

a. As x goes to ∞, what behavior does the function have?

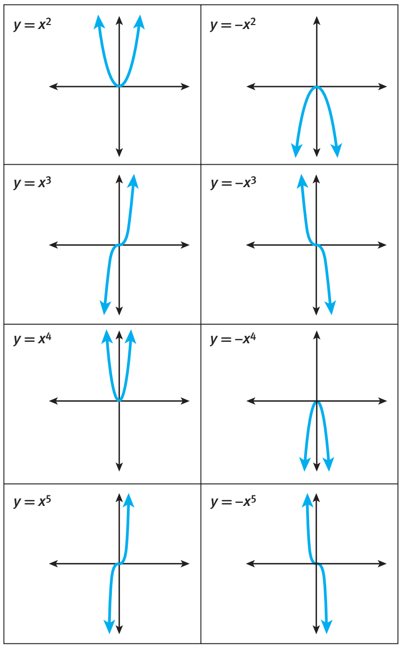
as $x \to \infty$, $y \to \infty$

b. How is the function behaving as x approaches $-\infty$?

as $x \to -\infty$, $y \to \infty$

SUGGESTED LEARNING STRATEGIES: Create Representations, Think/Pair/Share

13. Use a graphing calculator to examine the *end behavior* of polynomial functions in general. Sketch each given function on the axes below.



My Notes

ACTIVITY 4.1 *Continued*

13 Create Representations, Think/Pair/Share This Item is designed to look at only the end behavior of the functions given. The concept of leading coefficient effects can be discussed here, but will be analyzed in greater detail in Activity 4.4.

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14 Quickwrite, Group Presentation, Debriefing

A discussion of the topic of even and odd functions would be appropriate now. See *Mini-Lesson: Even and Odd Functions* below.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 206, #5–8

UNIT 4 PRACTICE
p. 259, #5–7

CHECK YOUR UNDERSTANDING

1. yes, $f(x) = 3x^5 - x^3 + 5x - 2$; 5th degree; 3
2. yes, $f(x) = -8x^4 - \frac{2}{3}x^3 - 2x + 7$; 4th degree; -8
3. No
4. No
5. 141
6. As $x \to \pm\infty$, $f(x) \to \infty$
7. As $x \to \infty$, $f(x) \to -\infty$, and as $x \to -\infty$, $f(x) \to \infty$
8. Answers will vary.

SUGGESTED LEARNING STRATEGIES: Quickwrite, Group Presentation

My Notes

14. Make a conjecture about the end behavior of polynomial functions. Explain your reasoning.

Polynomial functions that have even degrees have end behavior in the same direction, and polynomial functions with odd degrees have end behavior in the opposite direction.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

For Items 1–4, decide if each function is a polynomial. If it is, write the function in standard form, and then state the degree and leading coefficient.

1. $f(x) = 5x - x^3 + 3x^5 - 2$
2. $f(x) = -\frac{2}{3}x^3 - 8x^4 - 2x + 7$
3. $f(x) = 4^x + 2x^2 + x + 5$
4. $f(x) = -5x^3 + x^6 + \frac{2}{x}$
5. Given $f(x) = 3x^3 + 5x^2 + 4x + 3$, find $f(3)$.

Describe the end behavior of each function.

6. $f(x) = x^6 - 2x^3 + 3x^2 + 2$
7. $f(x) = -x^3 + 7x^2 - 11$
8. MATHEMATICAL REFLECTION Which new concept in this investigation has been easiest for you to understand? Which one has been most difficult?

MINI-LESSON: Even and Odd Functions

In even functions, $f(-x) = f(x)$. You get the same function when you substitute the opposite value. Even functions are symmetric around the y-axis. For example:

$$f(x) = 3x^2 + 1$$
$$f(-x) = 3(-x)^2 + 1 = 3x^2 + 1 = f(x)$$

In odd functions, $f(-x) = -f(x)$. You get the opposite function when you substitute the opposite value. Odd functions are symmetric around the origin. For example:

$$f(x) = 2x^3 + 3x$$
$$f(-x) = 2(-x)^3 + 3(-x) = -2x^3 - 3x = -f(x)$$

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Polynomial Operations
Polly's Pasta

ACTIVITY 4.2

SUGGESTED LEARNING STRATEGIES: Close Reading, Discussion Group, Create Representations, Think/Pair/Share, Self/Peer Revision

My Notes

Polly's Pasta and Pizza Supply sells wholesale goods to local restaurants. They keep track of revenue from kitchen supplies and food products. The function K models revenue from kitchen supplies and the function F models revenue from food product sales for one year in dollars, where t represents the number of the month (1–12) on the last day of the month.

$$K(t) = 15t^3 - 312t^2 + 1600t + 1100$$
$$F(t) = 36t^3 - 720t^2 + 3800t - 1600$$

MATH TIP

Some companies run their business on a fiscal year from July to June. Others, like Polly's Pasta, start the business year in January, so $t = 1$ represents January.

1. What kind of functions are these revenue functions?

Answers may vary. Sample answers: cubic functions; polynomial functions.

2. How much did Polly make from kitchen supplies in March? How much did she make from selling food products in August?

kitchen supplies \$3497

food products \$1152

3. In which month was her revenue from kitchen supplies the greatest? The least?

March is the maximum, October is the minimum.

4. In which month was her revenue from food products the greatest? The least?

April is the maximum, October is the minimum.

5. What was her total revenue from both kitchen supplies and food products in January? Explain how you arrived at you answer.

By adding the values for $K(1) = 2403$, and $F(1) = 1516$, the total is \$3919.

6. Complete the table for each given value of t.

t	$K(t)$	$F(t)$	$S(t) = K(t) + F(t)$
1	2403	1516	3919
2	3172	3408	6580
3	3497	4292	7789
4	3468	4384	7852
5	3175	3900	7075

ACTIVITY 4.2 Guided

Polynomial Operations

Activity Focus

- Polynomial operations including long division and synthetic division

Materials

- Graphing calculators

Chunking the Activity

#1–2	#10
#3–4	#11
#5	Try These B–#12
#6	#13
#7	Example 2
#8–9	#14–TryThese C
Example 1–Try These A	Example 3
	#15–Try These D

TEACHER TO TEACHER This guided exploration introduces students to polynomial operations, initially set in context.

The first group of items recalls vocabulary learned in Activity 4.1 and includes adding polynomials. Although the context has a discrete domain, students should be comfortable with adding non-discrete functions as well. This keeps the calculations simple and allows students exploration through tables, graphs, and analytic methods.

First Paragraph Close Reading

1 Discussion Group

3-4 Create Representations, Think/Pair/Share, Self/Peer Revision Students can find solutions in several ways, either through a table, graph, or analytic methods. Choose groups that represent the spectrum of solution methods to share with the entire class.

6 Create Representations

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ACTIVITY 4.2 *Continued*

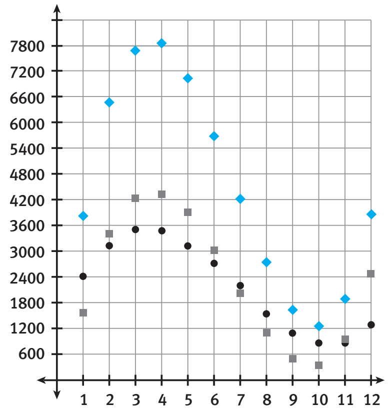
7 Create Representations, Quickwrite, Group Presentation This Item is designed to help students see the connection between the graphic representation of the sum of two polynomial functions and the process of adding two functions algebraically. The round dots represent the values of the function $K(t)$. The square dots represent the values of the function $F(t)$. Students should be able to use the graph to create the points for $S(t)$. If students do not see how to find the points representing the function $S(t)$ graphically, help them to make the connection by adding the value of one function to the value of the other at every point.

8 Quickwrite This Item uses the context to introduce the concept of polynomial subtraction.

My Notes

SUGGESTED LEARNING STRATEGIES: Create Representations, Quickwrite, Group Presentation

7. The graph below shows $K(t)$ and $F(t)$. Graph $S(t) = K(t) + F(t)$, and explain how you used the graph to find the values of $S(t)$.



Each value for $S(t)$ is the sum of the values for $K(t)$ and $F(t)$ at the given t.

Patty's monthly costs are represented by the function
$$C(t) = 5t^3 - 110t^2 + 600t + 1000.$$

8. Profit equals total revenue minus total costs. How much profit did Patty earn in December? Explain how you found your solution.

To find the value of \$2820 for profit, subtract $C(12)$ from $S(12)$.

SUGGESTED LEARNING STRATEGIES: Create Representations, Note-taking

My Notes

9. Complete the table for each value of t.

t	$S(t)$	$C(t)$	$P(t) = S(t) - C(t)$
8	2764	1320	1444
9	1687	1135	552
10	1300	1000	300
11	1909	945	964
12	3820	1000	2820

To add and subtract polynomials, add or subtract the coefficients of like terms.

EXAMPLE 1

a. Add $(3x^3 + 2x^2 - 5x + 7) + (4x^2 + 2x - 3)$.

Step 1: *Group like terms* $(3x^3) + (2x^2 + 4x^2) + (-5x + 2x) + (7 - 3)$
Step 2: *Combine like terms.* $3x^3 + 6x^2 - 3x + 4$
Solution: $3x^3 + 6x^2 - 3x + 4$

b. Subtract $(2x^3 + 8x^2 + x + 10) - (5x^2 - 4x + 6)$.

Step 1: *Distribute the negative.* $2x^3 + 8x^2 + x + 10 - 5x^2 + 4x - 6$
Step 2: *Group like terms.* $2x^3 + (8x^2 - 5x^2) + (x + 4x) + (10 - 6)$
Step 3: *Combine like terms.* $2x^3 + 3x^2 + 5x + 4$
Solution: $2x^3 + 3x^2 + 5x + 4$

TRY THESE A

Find each sum or difference. Write your answers in the My Notes space. Show your work.

a. $(2x^4 - 3x + 8) + (3x^3 + 5x^2 - 2x + 7)$
$2x^4 + 3x^3 + 5x^2 - 5x + 15$

b. $(4x - 2x^3 + 7 - 9x^2) + (8x^2 - 6x - 7)$
$-2x^3 - x^2 - 2x$

c. $(3x^2 + 8x^3 - 9x) - (2x^3 + 3x - 4x^2 - 1)$
$6x^3 + 7x^2 - 12x + 1$

ACTIVITY 4.2 *Continued*

9 Create Representations, Debriefing

EXAMPLE 1 Note Taking

TRY THESE A These Items are useful as formative assessment. For those students having difficulty with addition and subtraction of polynomials, some interventions may be necessary. Use vertical addition and subtraction to focus attention on grouping and then combining like terms. Use additional practice and one-on-one assistance as needed.

10 Think/Pair/Share, Debriefing This Item ties the sums and differences of polynomials back to the context. This connection can also be analyzed using the tables and graphing features of the calculators to help further the concept of adding and subtracting polynomials with tables or graphs.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 214, #1–4

UNIT 4 PRACTICE
p. 259, #8–11

11 Activating Prior Knowledge, Think/Pair/Share This Item extends the concept of the distributive property to polynomials with more than two terms. *Mini-Lesson: Use a Graphic Organizer to Multiply Polynomials* can help students keep track of partial products.

12 Quickwrite

My Notes

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Activating Prior Knowledge, Quickwrite

10. The **standard form of a polynomial** is $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$, where a is a real number and $a_n \neq 0$. Use what you learned about how to add and subtract polynomials to write $S(t)$ from Item 6 and $P(t)$ from Item 8 in standard form.

$S(t) = 51t^3 - 1032t^2 + 5400t - 500$
$P(t) = 46t^3 - 922t^2 + 4800t - 1500$

11. The steps to multiply $(x + 3)(4x^2 + 6x + 7)$ are shown below. Use appropriate math terminology to describe what occurs in each step.

$x(4x^2 + 6x + 7) + 3(4x^2 + 6x + 7)$	Use distributive property.
$(4x^3 + 6x^2 + 7x) + (12x^2 + 18x + 21)$	Multiply.
$4x^3 + 6x^2 + 12x^2 + 18x + 7x + 21$	Use associative property to rearrange terms.
$4x^3 + 18x^2 + 25x + 21$	Combine like terms.

TRY THESE **B**

Find each product. Write your answers in the My Notes space. Show your work.

a. $(x + 5)(x^2 + 4x - 5)$
$x^3 + 9x^2 + 15x - 25$

b. $(2x^2 + 3x - 8)(2x - 3)$
$4x^3 - 25x + 24$

c. $(x^2 - x + 2)(x^2 + 3x - 1)$
$x^4 + 2x^3 - 2x^2 + 7x - 2$

d. $(x^2 - 1)(x^3 + 4x)$
$x^5 + 3x^3 - 4x$

12. When multiplying polynomials, how are the degrees of the factors related to the degree of the product?
The sum of the degrees of the factors is equal to the degree of the product.

MINI-LESSON: Use a Graphic Organizer to Multiply Polynomials

Find the product $(x + 2)(3x^3 + 5x^2 + 7x)$.

Use a graphic organizer to multiply the polynomials. Fill in the boxes by multiplying the terms.

	$3x^3$	$5x^2$	$7x$
x	$3x^4$	$5x^3$	$7x^2$
2	$6x^3$	$10x^2$	$14x$

Combine like terms: $3x^4 + 5x^3 + 6x^3 + 7x^2 + 10x^2 + 14x$
$= 3x^4 + 11x^3 + 17x^2 + 14x$

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SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Note-taking

Polynomial long division has a similar algorithm to numerical long division.

13. Use long division to find the quotient $\frac{592}{46}$. Write your answer in the My Notes space. $12\frac{20}{23}$

EXAMPLE 2

Divide $x^3 - 7x^2 + 14$ by $x - 5$, using long division.

Step 1: *Set up the division problem with the divisor and dividend written in descending order of degree. Include zero coefficients for any missing terms.*

$$x - 5\overline{)x^3 - 7x^2 + 0x + 14}$$

Step 2: *Divide the first term of the dividend [x^3] by the first term of the divisor [x].*

$$\begin{array}{r} x^2 \\ x - 5\overline{)x^3 - 7x^2 + 0x + 14} \end{array}$$

Step 3: *Multiply the result [x^2] by the divisor [$x^2(x - 5) = x^3 - 5x^2$].*

$$\begin{array}{r} x^2 \\ x - 5\overline{)x^3 - 7x^2 + 0x + 14} \\ x^3 - 5x^2 \end{array}$$

Step 4: *Subtract to get a new result [$-2x^2 + 0x + 14$].*

$$\begin{array}{r} x^2 \\ x - 5\overline{)x^3 - 7x^2 + 0x + 14} \\ \underline{-(x^3 - 5x^2)} \\ -2x^2 + 0x + 14 \end{array}$$

Step 5: *Repeat the steps.*

$$\begin{array}{r} x^2 - 2x - 10 \\ x - 5\overline{)x^3 - 7x^2 + 0x + 14} \\ \underline{-(x^3 - 5x^2)} \\ -2x^2 + 0x + 14 \\ \underline{-(-2x^2 + 10x)} \\ -10x + 14 \\ \underline{-(-10x + 50)} \\ -36 \end{array}$$

Solution: $\frac{x^3 - 7x^2 + 14}{x - 5} = x^2 - 2x - 10 - \frac{36}{x - 5}$

My Notes

MATH TIP

When the division process is complete, the degree of the remainder will be less than the degree of the divisor.

ACTIVITY 4.2 *Continued*

13 **Activating Prior Knowledge** The process of long division of integers is similar to that of long division of polynomials. Working through this problem will help students see the similarities in the division algorithm and make the process more comfortable.

EXAMPLE 2 **Note Taking** Remind students that in long division of polynomials, during the subtraction process, the difference of the first terms should be zero. If it is not, then students need to look at the term they are multiplying to identify the error.

Differentiating Instruction

If students are having difficulty with the Example, you may want to relate the process to a division involving integers, such as $7\overline{)2059}$.

14 **Create Representations, Discussion Group** Students have the opportunity to work though this problem using Example 2 as support. Give students an opportunity to work in groups and monitor their progress. Students will benefit from guided practice with long division before working on the Try These C exercises independently.

TEACHER TO TEACHER It may be helpful for students to look at the end behavior of the quotient they found in Item 14 and explain their reasoning on how to find it. This will be a good prelude to investigating rational functions and end behavior in Unit 5. It relates to the concept of limits and asymptotes while not developing those concepts explicitly. This will give students an advantage when they begin to look at a more complex analysis of these functions.

My Notes

SUGGESTED LEARNING STRATEGIES: Create Representations

When a polynomial function $f(x)$ is divided by another polynomial function $d(x)$, the outcome is a new quotient function consisting of a polynomial $p(x)$ plus a remainder function $r(x)$.

$$\frac{f(x)}{d(x)} = p(x) + \frac{r(x)}{d(x)}$$

14. Follow the steps from Example 2 to find the quotient of $\frac{x^3 - x^2 + 4x + 6}{x + 2}$.

$x + 2\overline{)x^3 - x^2 + 4x + 6}$

$x^2 - 3x + 10 - \frac{14}{(x + 2)}$

TRY THESE C

Use long division to find each quotient. Write your answers in the My Notes space. Show your work.

a. $(x^2 + 5x - 3) \div (x - 5)$

$x + 10 + \frac{47}{x - 5}$

b. $\frac{4x^4 + 12x^3 + 7x^2 + x + 6}{-2x + 3}$

$-2x^3 - 9x^2 - 17x - 26 + \frac{84}{-2x + 3}$

c. $\frac{-4x^3 - 8x^2 + 32x}{x^2 + 2x - 8}$

$-4x$

d. $(x^3 - 9) \div (x + 3)$

$x^2 - 3x + 9 - \frac{36}{x + 3}$

SUGGESTED LEARNING STRATEGIES: Note-taking, Discussion Group

My Notes

Synthetic division is another method of polynomial division that is useful when the divisor has the form $x - k$.

EXAMPLE 3

Divide $x^4 - 13x^2 + 32$ by $x - 3$.

Step 1: *Set up the division problem using only coefficients for the dividend and only the constant for the divisor. Include zero coefficients for any missing terms* [x^3 *and* x].

$$3 \,|\; 10 \quad -13 \quad 0 \quad 32$$

Step 2: *Bring down the leading coefficient* [1].

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -13 & 0 & 32 \\ & \downarrow & & & & \\ \hline & 1 & & & & \end{array}$$

Step 3: *Multiply the coefficient* [1] *by the divisor* [3]. *Write the product* [$1 \cdot 3 = 3$] *under the second coefficient* [0] *and add* [$0 + 3 = 3$].

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -13 & 0 & 32 \\ & & 3 & & & \\ \hline & 1 & 3 & & & \end{array}$$

Step 4: *Repeat this process until there are no more coefficients.*

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -13 & 0 & 32 \\ & & 3 & 9 & -12 & -36 \\ \hline & 1 & 3 & -4 & -12 & -4 \end{array}$$

Step 5: *The numbers in the bottom row become the coefficients of the quotient. The number in the last column is the remainder. Write it over the divisor.*

$$x^3 + 3x^2 - 4x - 12 - \frac{4}{x - 3}$$

Solution: $x^3 + 3x^2 - 4x - 12 - \frac{4}{x - 3}$

MATH TIP

In synthetic division, the quotient is always one degree less than the dividend.

15. Use synthetic division to divide $\frac{x^3 - x^2 + 4x + 6}{x + 2}$.

$x^2 - 3x + 10 - \frac{14}{x + 2}$

ACTIVITY 4.2 *Continued*

EXAMPLE 3 Note Taking

Differentiating Instruction

Have students look at the value of $f(3)$ in the polynomial in Example 3. Have them make a conjecture about the relationship between the remainder when they divide a polynomial by $(x - 3)$ and the value of $f(3)$.

TEACHER TO TEACHER $f(x) = 2x^3 + 4x^2 + 3x + 6$ is equivalent to the nested form $f(x) = ((2x + 4)x + 3)x + 6$. Substituting into the nested form is the same as doing synthetic division.

15 Discussion Group This Item is another opportunity for students to practice a new process before working on subsequent problems independently.

Connect to Math History

William Horner (1786–1837) is associated with a method of finding the zeros of a function called Horner's algorithm. It is more commonly referred to as *synthetic division* or *synthetic substitution*.

TRY THESE D Have students work independently on these Items, after having them work in groups on the guided problem in Item 15.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 214, #5–11

UNIT 4 PRACTICE
p. 259, #12–17

CHECK YOUR UNDERSTANDING

1. $8x^2 - 3$
2. $-x^2 - 7x + 4$
3. $4x^2 - 9x - 2$
4. $-2x^2 - 6x - 21$
5. $3x^3 + 21x^2 + 24x$
6. $2x^4 - 15x^3 + 28x^2 - 9x + 18$
7. $x - 7 + \frac{11}{x + 1}$
8. $5x^2 - x - 2 + \frac{16x + 2}{x^2 + 3x + 1}$
9. $x - 4 + \frac{20}{x + 4}$
10. $3x^2 + 2x + 20 + \frac{58}{x - 4}$
11. Answers may vary based on students comfort level with the various operations.

My Notes

TRY THESE D

Use synthetic division to find each quotient.

a. $\frac{x^3 + 3x^2 - 10x - 24}{x + 4}$

$x^2 - x - 6$

b. $\frac{-5x^5 - 2x^4 + 32x^2 - 48x + 32}{x - 2}$

$-5x^4 - 12x^3 - 24x^2 - 16x - 80 - \frac{128}{x - 2}$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

Find each sum or difference.

1. $(3x^2 - 4) + (5x^2 + 1)$
2. $(x^2 - 6x + 5) - (2x^2 + x + 1)$
3. $(4x^2 - 12x + 9) + (3x - 11)$
4. $(6x^2 - 13x + 4) - (8x^2 - 7x + 25)$

Find each product.

5. $3x(x^2 + 7x + 8)$
6. $(x - 3)(2x^3 - 9x^2 + x - 6)$

Find each quotient, using long division.

7. $\frac{x^2 - 6x + 4}{x + 1}$
8. $(5x^4 + 14x^3 + 9x) \div (x^2 + 3x + 1)$

Find each quotient, using synthetic division.

9. $(x^2 + 4) \div (x + 4)$
10. $\frac{3x^3 - 10x^2 + 12x - 22}{x - 4}$
11. MATHEMATICAL REFLECTION Which operations on polynomials have been easy for you to understand? Which have been more difficult?

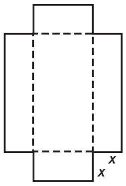
Polynomial Operations

THIS TEST IS SQUARE

Embedded Assessment 1

Use after Activity 4.2.

Congruent squares of length x are cut from the corners of a 10 in. by 15 in. piece of cardboard to create a box without a lid.



1. Write an expression for each:

a. height of the box

b. length of the box

c. width of the box

2. Write a function $V(x)$ for the volume of the box in terms of x.

3. What are the possible values of x?

4. Use a graphing calculator to determine the value of x that gives the maximum volume.

Use these functions for Items 5–8.

$f(x) = x + 4$ $\quad$ $g(x) = -x^2 + 9x + 20$

$p(x) = x^2 - x - 6$ $\quad$ $h(x) = x^3 + 11x^2 + 38x + 40$

5. Find $g(x) + p(x)$.

6. Find $f(x) \cdot p(x)$.

7. Find $\frac{p(x)}{f(x)}$, using long division.

8. Find $\frac{h(x)}{f(x)}$, using synthetic division.

9. Write a function of degree 5 or higher that has this end behavior:

- as x goes to ∞, y approaches ∞
- as x goes to $-\infty$, y approaches ∞

Embedded Assessment 1

Assessment Focus

- Relative maximum
- End behavior
- Polynomial operations

Materials

- Graphing calculators

1-4 These Items build on an understanding of volume as students look at a maximization problem similar to those done at the AP Calculus level. The Items are scaffolded to bring them to the level of an Algebra 2 student.

Answer Key

1a. $h = x$

b. $l = 15 - 2x$

c. $w = 10 - 2x$

2. $V(x) = x(15 - 2x)(10 - 2x)$

3. $0 < x < 5$

4. $x \approx 1.962$

3-4 Due to the nature of Item 1, students who are unable to come up with the function in Item 2 will not be able to answer Items 3 and 4. This should be taken into account when grading those problems.

5. $g(x) + p(x) = 8x + 14$

6. $f(x) \cdot p(x) = x^3 + 3x^2 - 10x - 24$

7. $x - 5 + \frac{14}{x + 4}$

8. $x^2 + 7x + 10$

9. Answers may vary but the answer should be a polynomial with even degree and positive leading coefficient.

Embedded Assessment 1

TEACHER TO TEACHER You may wish to read through the rubric with students and discuss the differences in the expectation levels. Make sure students understand the meanings of any terms used.

Embedded Assessment 1

Use after Activity 4.2.

Polynomial Operations

THIS TEST IS SQUARE

	Exemplary	Proficient	Emerging
Math Knowledge #5, 6, 7, 8	The student: • Finds correct expressions for $g(x) + p(x)$ and $f(x) \times p(x)$. (5, 6) • Uses long division to find the correct quotient for $p(x)/f(x)$ and uses synthetic division to find the correct quotient for $h(x)/f(x)$. (7, 8)	The student: • Finds only one of the correct expressions. • Uses only one of the methods to find both correct quotients.	The student: • Finds neither of the correct expressions. • Attempts, unsuccessfully, to find the quotients.
Problem Solving #3, 4, 9	The student: • Gives the correct interval for the possible values of *x*. (3) • Determines the correct value of *x* that gives the maximum value. (4) • Writes a correct fifth or higher degree function with both of the given characteristics. (9)	The student: • Provides all of the correct integer values of *x*. • Writes a function of degree five or higher with one of the given characteristics.	The student: • Provides at least two correct integer values for *x*. • Does not determine the correct value of *x*. • Writes a function of degree five or higher with neither of the given characteristics.
Representations #1a, b, c; 2	The student: • Writes correct expressions for the height, length, and width of the box. (1a, b, c) • Writes a correct function for the volume of the box. (2)	The student: • Writes correct expressions for only two of the dimensions of the box. • Writes a function for the volume of the box that is correct for the incorrect dimensions given in question 1	The student: • Writes a correct expression for only one of the dimensions of the box. • Writes an incorrect function.

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216 SpringBoard® Mathematics with Meaning™ Algebra 2

Factors of Polynomials
Factoring For Experts

ACTIVITY 4.3

SUGGESTED LEARNING STRATEGIES: Shared Reading, Activating Prior Knowledge, Discussion Group, Note-taking

My Notes

When you factor a polynomial, you rewrite the original polynomial as a product of two or more polynomial factors.

1. State the common factor of the terms in the polynomial $4x^3 + 2x^2 - 6x$. Then factor the polynomial.
$2x$, $2x(2x^2 + x - 3)$

2. Consider the expression $x^2(x - 3) + 2x(x - 3) + 3(x - 3)$.

a. How many terms does it have?
3

b. What factor do all the terms have in common?
$x - 3$

3. Factor $x^2(x - 3) + 2x(x - 3) + 3(x - 3)$.
$(x - 3)(x^2 + 2x + 3)$

Some quadratic trinomials, $ax^2 + bx + c$, can be factored into two binomial factors.

EXAMPLE 1

Factor $2x^2 + 7x - 4$.

Step 1:	*Find the product of* a *and* c.	$2(-4) = 8$
Step 2:	*Find the factors of* ac *that have a sum of* b, 7.	$8 + (-1) = 7$
Step 3:	*Rewrite the polynomial, separating the linear term.*	$2x^2 + 8x - 1x - 4$
Step 4:	*Group the first two terms and the last two terms.*	$(2x^2 + 8x) + (-x - 4)$
Step 5:	*Factor each group separately.*	$2x(x + 4) - 1(x + 4)$
Step 6:	*Factor out the binomial.*	$(x + 4)(2x - 1)$

Solution: $(x + 4)(2x - 1)$

MATH TIP

Check your answer to a factoring problem by multiplying the factors together to get the original polynomial.

ACTIVITY 4.3 Directed

Factors of Polynomials

Activity Focus

- Factoring polynomials

Materials

- No special materials are needed.

Chunking the Activity

#1–3
Example 1–Try These A
Example 2–Try These B
#4–Try These C
#5–Try These D
Example 3–Try These E
#7
Example 4–Try These F

First Paragraph Shared Reading

1-3 Activating Prior Knowledge, Discussion Group These Items will help students recall the concept of a factor, and then extend it to polynomials beyond binomial factors. Students may need to spend some time on these items to succeed. Have students who have factored correctly share their work. You may want to give an example with a trinomial factor as well.

EXAMPLE 1 Activating Prior Knowledge, Note Taking This demonstrates factoring a quadratic trinomial with a leading coefficient again. In Activity 3.2, students factored quadratic trinomials. This example shows factoring a quadratic trinomial by using factoring by grouping. It can help to make the connection back to the box method used in Activity 3.2 as well.

TRY THESE **A** Activating Prior Knowledge

EXAMPLE 2 Note Taking Point out how polynomials in the two parts of the example are the same, except that the middle two terms are transposed. Point out that the answers are the same except that the order of the factors is different. To verify the answers, students can multiply the factors.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Note-taking

My Notes

TRY THESE **A**

a. Use Example 1 as a guide to factor $6x^2 + 19x + 10$.

$a \cdot c = 60 = 6 \cdot 10$

Factors with a sum of 19: 15, 4: $15 + 4 = 19$; $15 \cdot 4 = 60$

Separate linear term: $6x^2 + 15x + 4x + 10$

Factor each group: $3x(2x + 5) + 2(2x + 5)$

$(2x + 5)(3x + 2)$

Factor each trinomial. Write your answers in the My Notes space. Show your work.

b. $3x^2 - 8x - 3$ $(x - 3)(3x + 1)$ **c.** $2x^2 + 7x + 6$ $(x + 2)(2x + 3)$

Some higher-degree polynomials can also be *factored by grouping.*

EXAMPLE 2

a. Factor $3x^3 + 9x^2 + 4x + 12$ by grouping.

Step 1: *Group the terms.* $(3x^3 + 9x^2) + (4x + 12)$

Step 2: *Factor each group separately.* $3x^2(x + 3) + 4(x + 3)$

Step 3: *Factor out the binomial.* $(x + 3)(3x^2 + 4)$

Solution: $(x + 3)(3x^2 + 4)$

b. Factor $3x^3 + 4x + 9x^2 + 12$ by grouping.

Step 1: *Group the terms.* $(3x^3 + 4x) + (9x^2 + 12)$

Step 2: *Factor each group separately.* $x(3x^2 + 4) + 3(3x^2 + 4)$

Step 3: *Factor out the binomial.* $(3x^2 + 4)(x + 3)$

Solution: $(3x^2 + 4)(x + 3)$

TRY THESE **B**

Factor by grouping. Write your answers in the My Notes space. Show your work.

a. $2x^3 + 10x^2 - 3x - 15$
$(x + 5)(2x^2 - 3)$

b. $4x^3 + 3x^2 + 4x + 3$
$(x^2 + 1)(4x + 3)$

218 SpringBoard® Mathematics with Meaning™ Algebra 2

SUGGESTED LEARNING STRATEGIES: Marking the Text, Shared Reading, Look for a Pattern, Identify a Subtask, Simplify a Problem, Activating Prior Knowledge

A difference of two squares can be factored by using a specific pattern, $a^2 - b^2 = (a + b)(a - b)$. A *difference of two cubes* and a *sum of two cubes* also have a factoring pattern.

Difference of Cubes	Sum of Cubes
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

4. What patterns do you notice in the formulas that appear above?
Answers will vary. Sample answer: For a difference of cubes, the cube root of the first term minus the cube root of the second term is multiplied by the quantity of the cube root of the first term squared plus the product of cube root of each term plus the cube root of the second term squared. For a sum of cubes, the cube root of the first term plus the cube root of the second term is multiplied by the quantity of the cube root of the first term squared minus the product of cube root of each term plus the cube root of the second term squared.

TRY THESE C

Factor each difference or sum of cubes.

a. $x^3 - 8$
$(x - 2)(x^2 + 2x + 4)$

b. $x^3 + 27$
$(x + 3)(x^2 - 3x + 9)$

c. $8x^3 - 64$
$(2x - 4)(4x^2 + 8x + 16)$

d. $27 + 125x^3$
$(3 + 5x)(9 - 15x + 25x^2)$

Some higher-degree polynomials can be factored by using the same patterns or formulas that you used when factoring quadratic binomials or trinomials.

5. Use the difference of squares formula $a^2 - b^2 = (a + b)(a - b)$ to factor $16x^4 - 25$. (It may help to write each term as a square.)
$(4x^2 + 5)(4x^2 - 5)$

My Notes

ACTIVITY 4.3 *Continued*

First Paragraph and Displayed Text Marking the Text, Shared Reading

4 Look for a Pattern, Group Presentation, Debriefing The patterns of factoring a difference of cubes and a sum of cubes help students to factor cubic functions quickly. This will help in graphing functions and solving for roots later in this unit and through Calculus.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 222, #1–2

UNIT 4 PRACTICE
p. 259, #18–19

5 Identify a Subtask, Simplify a Problem, Activating Prior Knowledge This Item, along with Item 6, is an opportunity for students to see quadratic methods of factoring used with higher degree polynomials or substitution.

6 Quickwrite, Think/Pair/Share, Debriefing Students should recognize that the trinomial was factored by using a formula for a quadratic trinomial. This may be difficult for some students to grasp. Having other students share their work may help those that are having difficulty.

TRY THESE D Identify a Subtask, Simplify a Problem These Items will help students see other types of quadratics that they may encounter. Help students make connections with the familiar patterns of factoring they dealt with in Unit 3. If students need more practice, use *Mini-Lesson: Factoring Higher-Level Polynomials*.

Paragraph Vocabulary Organizer In higher-level courses, the statement of the theorem is "Every polynomial $P(x)$ of degree $n \geq 1$ with complex coefficients has at least one root, which is a complex number (real or imaginary)." In Algebra 2, students do not work with imaginary coefficients.

EXAMPLE 3 Note Taking

Factors of Polynomials
Factoring For Experts

SUGGESTED LEARNING STRATEGIES: Quickwrite, Think/Pair/Share, Identify a Subtask, Simplify the Problem, Vocabulary Organizer, Note-taking

My Notes

6. Explain the steps used to factor $2x^5 + 6x^3 - 8x$.

$2x^5 + 6x^3 - 8x$	Original expression
$= 2x(x^4 + 3x^2 - 4)$	Factor out the GCF.
$= 2x(x^2 + 4)(x^2 - 1)$	Factor the trinomial using a quadratic pattern.
$= 2x(x^2 + 4)(x + 1)(x - 1)$	Factor a difference of 2 squares.

TRY THESE D

Use the formulas for quadratic trinomials to factor each expression.

a. $x^4 + x^2 - 20$
$(x^2 + 5)(x + 2)(x - 2)$

b. $16x^4 - 81$
$(4x^2 + 9)(4x^2 - 9) = (4x^2 + 9)(2x + 3)(2x - 3)$

c. $(x - 2)^4 + 10(x - 2)^2 + 9$
$((x - 2)^2 + 9)((x - 2)^2 + 1)$

MATH TERMS

Let $p(x)$ be a polynomial function of degree n, where $n > 0$. The **Fundamental Theorem of Algebra** states that $p(x) = 0$ has at least one zero in the complex number system.

As a consequence of the **Fundamental Theorem of Algebra**, a polynomial $p(x)$ of degree $n \geq 0$ has exactly n linear factors, counting multiple factors.

EXAMPLE 3

Find the zeros of $f(x) = 3x^3 + 2x^2 + 6x + 4$.

Step 1: Set the function equal to 0. $3x^3 + 2x^2 + 6x + 4 = 0$

Step 2: *Look for a factor common to all terms, use the quadratic trinomial formulas, or factor by grouping, as was done here.* $(3x^3 + 6x) + (2x^2 + 4) = 0$

Step 3: *Factor each group separately.* $3x(x^2 + 2) + 2(x^2 + 2) = 0$

Step 4: *Factor out the binomial to write the factors.* $(x^2 + 2)(3x + 2) = 0$

Step 5: *Use the Zero Product Property to solve for x.* $x^2 + 2 = 0$, $x = \pm i\sqrt{2}$; $3x + 2 = 0$, $x = -\frac{2}{3}$

Solution: $x = \pm i\sqrt{2};\ x = -\frac{2}{3}$

MINI-LESSON: Factoring Higher-Level Polynomials

Assign these problems if students need more practice.

Use the formulas for quadratic trinomials to factor each expression.

1. $x^4 + x^2 - 6$
$(x^2 + 3)(x^2 - 2)$

2. $625 - x^4$
$(5 + x)(5 - x)(25 + x^2)$

3. $(x + 3)^4 + 10(x + 3)^2 + 24$
$((x + 3)^2 + 6)((x + 3)^2 + 4)$

4. $x^7 - 2x^4 - 48x$
$x(x^3 - 8)(x^3 + 6)$

5. $2x^6 - 50x^2$
$(2x^2)(x^2 - 5)(x^2 + 5)$

6. $5x^5 - 20x^3 - 60x$
$5x(x^2 - 6)(x^2 + 2)$

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Group Presentation, Vocabulary Organizer, Note-taking, Work Backward

My Notes

TRY THESE E

Find the zeros of the functions by factoring and using the Zero Product Property.

a. $f(x) = x^3 + 9x$ $x = 0, x = \pm 3i$

b. $g(x) = x^4 - 16$ $x = \pm 2i, x = \pm 2$

c. $h(x) = (x - 2)^2 + 4(x - 2) + 4$ $x = 0$

d. $k(x) = x^3 - 3x^2 - 15x + 125$ $x = -5, x = 4 \pm 3i$

e. $p(x) = x^3 - 64$ $x = 4, x = -2 \pm 2i\sqrt{3}$

f. $w(x) = x^3 + 216$ $x = -6, x = 3 \pm 3i\sqrt{3}$

7. Create a flow chart, other organizational scheme, or set of directions for factoring polynomials.
Answers will vary. Check students' work.

It is possible to find a polynomial function, given its zeros. The **Complex Conjugate Root Theorem** states that if $a + bi$, $b \neq 0$, is a zero of a polynomial function with real coefficients, the conjugate $a - bi$ is also a zero of the function.

EXAMPLE 4

Find a polynomial function of 4th degree that has zeros 1, −1, and $1 + 2i$.

Step 1: *Use the Complex Conjugate Root Theorem to find all zeros.* $x = 1, x = -1, x = 1 + 2i, x = 1 - 2i$

Step 2: *Write the factors.* $f(x) = (x - 1)(x + 1)(x - (1 + 2i))(x - (1 - 2i))$

Step 3: *Multiply using the fact that* $(a - b)(a + b) = a^2 - b^2$. $f(x) = (x^2 - 1)(x^2 - 2x + 5)$

Step 4: *Multiply out the factors to get the polynomial function.* $f(x) = x^4 - 2x^3 + 4x^2 + 2x - 5$

Solution: $f(x) = x^4 - 2x^3 + 4x^2 + 2x - 5$

ACTIVITY 4.3 *Continued*

TRY THESE E Students will need to use the Quadratic Formula to solve some of these problems. It may be necessary to remind some students of the formula. Each problem will factor initially, but may not be completely factorable over the integers.

7 Graphic Organizer, Group Presentation, Debriefing Students will have many different ways to illustrate the answer.

Paragraph Vocabulary Organizer

TEACHER TO TEACHER You can use a connection to the Quadratic Formula to illustrate the concept of the Complex Conjugate Root Theorem. The Quadratic Formula always yields two solutions, one adding the square root term and one subtracting it. If the formula yields complex solutions, they are of the form $a \pm bi$, where a and b are real numbers.

EXAMPLE 4 Note Taking, Work Backward Note that there are other polynomials with these roots.

TRY THESE F Work Backward
Notice in Part (c) the mention of a double root. Be sure that students write the factor $(x + 4)$ twice.

TEACHER TO TEACHER A root b of a polynomial is called a double root, or a root of multiplicity 2, of the polynomial if $(x - b)$ appears as a factor of the polynomial exactly twice. In general, if r is a zero of a polynomial and $(x - r)$ appears as a factor of the polynomial exactly k times, then the zero r is called a "zero of multiplicity k."

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 222, #3–7

UNIT 4 PRACTICE
p. 259, #20–22

CHECK YOUR UNDERSTANDING

1a. $(8x^2 + 1)(x - 8)$
b. $(2x^2 - 5)(6x + 1)$
2a. $(5x + 6)(25x^2 - 30x + 36)$
b. $(x^2 - 3)(x^4 + 3x^2 + 9)$
3a. $(x^2 - 11)(x^2 - 3)$
b. $(3x + 5)(3x - 5)(9x^2 + 25)$
c. $(x^2 + 5)(x^2 + 12)$
4a. $x = 0, x = \pm 3i$
b. $x = 1, x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$
c. $x = \pm 3, x = \frac{6}{5}$
5. B
6a. $x^3 + 2x^2 - 25x - 50$
b. $x^4 + 16x^2 - 225$
7. Answers may vary. Sample answer: There will be less time spent factoring polynomials, using other methods.

Factors of Polynomials
Factoring For Experts

SUGGESTED LEARNING STRATEGIES: Work Backward

My Notes

TRY THESE F

Write a polynomial function of nth degree that has the given real or complex roots. Write your answers on a separate sheet of notebook paper. Show your work.

a. $n = 3; x = -2, x = 3i$ $x^3 + 2x^2 + 9x + 18$

b. $n = 4; x = 3, x = -3, x = 1 + 2i$ $x^4 - 2x^3 - 4x^2 + 18x - 45$

c. $n = 4; x = 2, x = -5$, and $x = -4$ is a double root
$x^4 + 11x^3 + 30x^2 - 32x - 160$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Factor by grouping.
a. $8x^3 - 64x^2 + x - 8$
b. $12x^3 + 2x^2 - 30x - 5$

2. Factor each difference or sum of cubes.
a. $125x^3 + 216$
b. $x^6 - 27$

3. Use the formulas for factoring quadratic trinomials to factor each expression.
a. $x^4 - 14x^2 + 33$
b. $81x^4 - 625$
c. $x^4 + 17x^2 + 60$

4. Find the zeros of the functions by factoring and using the Zero Product Property.
a. $f(x) = 2x^4 + 18x^2$
b. $g(x) = 3x^3 - 3$
c. $h(x) = 5x^3 - 6x^2 - 45x + 54$

5. The table of values shows coordinate pairs on the graph of $f(x)$. Which of the following could be $f(x)$?
a. $x(x + 1)(x - 1)$
b. $(x - 1)(x + 1)(x - 3)$
c. $(x + 1)^2(x + 3)$
d. $(x + 1)(x - 2)^2$
e. $x(x - 1)(x + 3)$

x	$f(x)$
-1	0
0	3
1	0
2	-3

6. Write a polynomial function of nth degree that has the given real or complex roots.
a. $n = 3; \quad x = -2, x = 5, x = -5$
b. $n = 4; \quad x = -3, x = 3, x = 5i$

7. MATHEMATICAL REFLECTION How do you think memorizing the factoring patterns for the sum and difference of cubes and a difference of squares will benefit you as you progress in mathematics?

Graphs of Polynomials
Graphing Polynomials

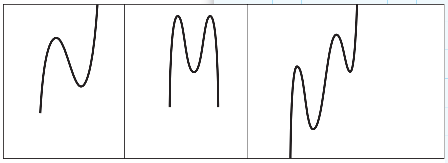
ACTIVITY 4.4

SUGGESTED LEARNING STRATEGIES: Quickwrite, Look for a Pattern, Group Presentation, Create Representations, Think/Pair/Share

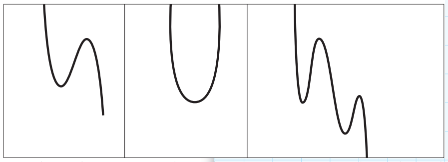
My Notes

1. Each graph to the right shows a polynomial of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, where $a_n \neq 0$. Use each graph to make a conjecture about how the leading coefficient and degree affect the end behavior of the function.

For even degree functions with a positive leading coefficient, as $x \to \pm\infty$, $y \to +\infty$. For even degree functions with a negative leading coefficient, as $x \to \pm\infty$, $y \to -\infty$. For odd degree functions with a positive leading coefficient, as $x \to +\infty$, $y \to +\infty$ and as $x \to -\infty$, $y \to -\infty$. For odd degree functions with a negative leading coefficient, as $x \to +\infty$, $y \to -\infty$ and as $x \to -\infty$, $y \to +\infty$.



$y = 2x^3 - 4x^2 + 1$ $y = -3x^4 + 8x^2 + 1$ $y = 2x^5 + 4x^4 - 5x^3 - 8x^2 + 5x$



$y = -2x^3 - 4x^2 + 1$ $y = 3x^4 - 8x^2 + 1$ $y = -2x^5 - 4x^4 + 5x^3 + 8x^2 - 5x$

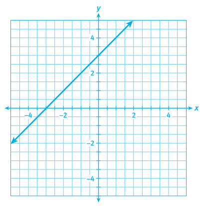
MATH TIP

If $(x - a)$ is a factor of a polynomial $f(x)$, then a is an x-intercept of the graph $f(x)$.

2. Use what you know about end behavior and zeros of a function to sketch a graph of each function in the My Notes section.
See on the right.

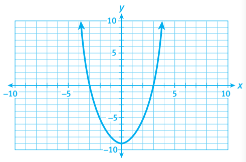
a. $f(x) = x + 3$

a.



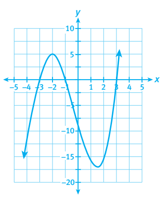
b. $g(x) = x^2 - 9 = (x + 3)(x - 3)$

b.



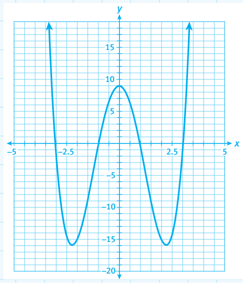
c. $h(x) = x^3 + x^2 - 9x - 9 =$
$(x + 3)(x - 3)(x + 1)$

c.



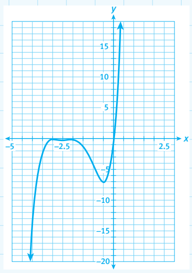
d. $k(x) = x^4 - 10x^2 + 9 =$
$(x + 3)(x - 3)(x + 1)(x - 1)$

d.



e. $p(x) = x^5 + 10x^4 + 37x^3 + 60x^2 + 36x =$
$x(x + 2)^2(x + 3)^2$

e.



ACTIVITY 4.4 Investigative

Graphs of Polynomials

Activity Focus
- Graphing polynomials

Materials
- Graphing calculators

Chunking the Activity

#1	Example 2
#2	Example 3–4
#3–6	Example 5
#7–8	#10
#9	#11
Example 1	

1 Quickwrite, Look for a Pattern, Group Presentation, Debriefing Students generalize the behavior of graphs, based on the leading coefficient of the polynomial. They should have some familiarity with this concept from Activity 4.1.

2 Create Representations, Think/Pair/Share Students will analyze and graph functions using end behavior and the zeros. It is also acceptable to have students use a graphing calculator and then explain the patterns they notice instead of graphing the functions by hand.

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Unit 4 • Polynomials 223

First Paragraph Vocabulary Organizer, Marking the Text, Item the Text

3-6 Guide students through the process of graphing factorable polynomial functions. These Items can be done with or without technology. Finding the extrema at this stage is not an easy process.

6 Create Representations, Debriefing This Item gives students the *x*-coordinate to help them locate a close approximation.

7 Create Representations, Think/Pair/Share If students are going to graph this function by hand, it is sufficient to have them simply sketch how the function could be behaving rather than determine the exact value of the extrema. If students use a graphing calculator, they can also use them for finding the extrema.

8 Create Representations, Think/Pair/Share

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 230, #1–4

UNIT 4 PRACTICE
p. 260, #23–26

Connect to AP

Students will learn to use the first derivative of a polynomial function to algebraically determine the coordinates of the extrema. The second derivative of a polynomial function can be used to determine a function's concavity. The point where a function changes concavity is called an inflection point.

My Notes

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Marking the Text, Question the Text, Create Representations, Think/Pair/Share

ACADEMIC VOCABULARY

Maxima and *minima* are known as **extrema.** They are the greatest value (the maximum) or the least value (the minimum) of a function. When these values occur at a point within a given interval, they are called *relative extrema*. When they occur on the entire domain of the function, they are called *global extrema*.

CONNECT TO AP

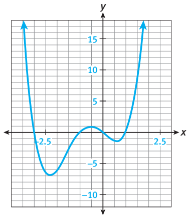
In calculus, you will use the first derivative of a polynomial function to algebraically determine the coordinates of the extrema.

Polynomial functions are **continuous functions**, meaning that their graphs have no gaps or breaks. Their graphs are smooth, unbroken curves with no sharp turns. Graphs of polynomial functions with degree n have n zeros, as you saw in the Fundamental Theorem of Algebra. They also have at most $n - 1$ **relative extrema** (*maximum* or *minimum* points).

3. Find the x-intercepts of $f(x) = x^4 + 3x^3 - x^2 - 3x$.
$x = -1, 1, -3, 0$

4. Find the y-intercept of $f(x)$.
$y = 0$

5. How can the zeros of a polynomial function help you identify where the relative extrema will occur?
For a polynomial relative extrema occur between the zeros of the polynomial function.

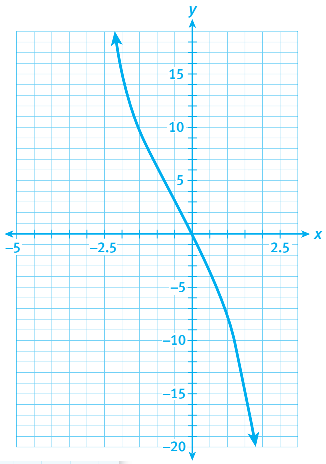


6. The relative extrema occur at approximately $x = 0.6$, $x = -0.5$, and $x = -2.3$. Find the approximate values of the extrema and graph $f(x) = x^4 + 3x^3 - x^2 - 3x$.
(0.6, −1.382), (−0.5, 0.938), (−2.3, −6.907)

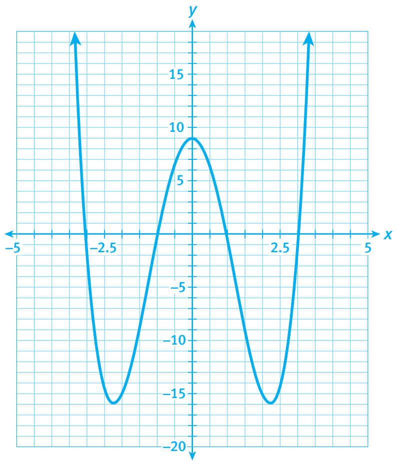
7. Sketch a graph of $f(x) = -x^3 - x^2 - 6x$ in the My Notes section.

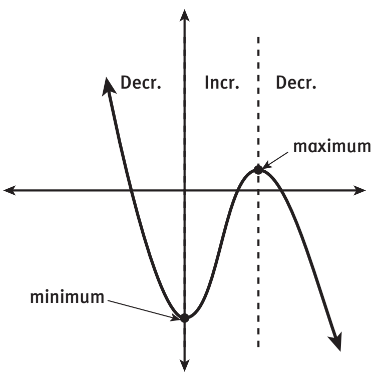
8. Sketch a graph of $f(x) = x^4 - 10x^2 + 9$ below.

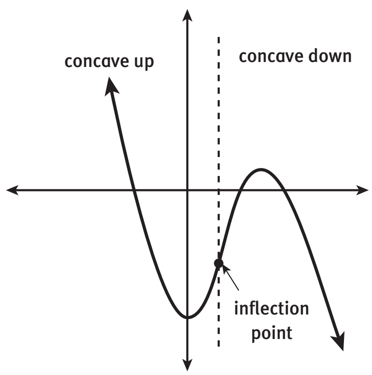
7.



8.







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Graphs of Polynomials
Graphing Polynomials

SUGGESTED LEARNING STRATEGIES: Shared Reading, Vocabulary Organizer, Predict and Confirm, Quickwrite

My Notes

The function $f(x) = x^3 - 2x^2 - 5x + 6$ is not factorable using the tools that you have. However, to graph a function of this form without a calculator, the following tools will be helpful.

The Rational Root Theorem	Finds possible rational roots.
Descartes' Rule of Signs	Finds the possible number of real roots.
The Remainder Theorem	Determines if a value is a zero.
The Factor Theorem	Another way to determine if a value is a zero.

The Rational Root Theorem

If a polynomial function $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$, $a_n \neq 0$, has integer coefficients, then every rational root of $f(x) = 0$ has the form $\frac{p}{q}$, where p is a factor of a_0, and q is a factor of a_n.

The Rational Root Theorem determines the possible rational roots of the polynomial.

9. Consider the quadratic equation $2x^2 + 9x - 3 = 0$.

a. Make a list of the only possible rational roots to this equation.
$\pm 3, \pm\frac{3}{2}, \pm\frac{1}{2}, \pm 1$

b. Explain why you think these are the only possible rational roots.
According to the Rational Root Theorem, every rational root has the form $\frac{p}{q}$.

c. Does your list of rational roots satisfy the equation?
no

d. What can you conclude from Part c?
The Rational Root Theorem gives possible rational roots, but not always actual roots.

e. Verify your conclusion in Part c by finding the roots of the quadratic by using the Quadratic Formula.
$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-9 \pm \sqrt{105}}{4}$; **the roots are irrational**

First Paragraph and Theorem
Shared Reading, Vocabulary Organizer The four theorems listed will be introduced and used during the rest of the activity.

9 Predict and Confirm, Quickwrite This Item allows students to explore the special case of quadratic polynomials.

MINI-LESSON: Finding Relative Extrema Using a Graphing Calculator

- Press the **2nd** button and **Trace**. Select 4: max, or 3 min.
- Move the cursor to the left of the maximum value and press ENTER for the Left Bound?
- Move the cursor to the right of the maximum value and press ENTER for the Right Bound?
- Press ENTER again for the GUESS?

The calculator will then give the point at which the maximum occurs on the chosen interval.

TEACHER TO TEACHER Examples 1 through 5 guide students through the process of graphing a polynomial function by hand, using the same function in each example: $f(x) = x^3 - 2x^2 - 5x + 6$. The flow is designed to have students find the possible rational roots using the Rational Root Theorem, and then use Descartes' Rule of Signs to try to eliminate a few possibilities for the zeros. Next they can use the Factor or the Remainder theorems to help find the zeros. Remind students that they can factor or use the Quadratic Formula when they have divided enough factors out of the original polynomial to have a quadratic factor.

EXAMPLE 1 Note Taking Make sure students understand that the example shows all *possible* roots, not the actual zeros of the function.

Paragraph Shared Reading

Descartes' Rule of Signs
Vocabulary Organizer

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Quickwrite, Note-taking, Shared Reading, Vocabulary Organizer

My Notes

EXAMPLE 1

Find all the possible rational zeros of $f(x) = x^3 - 2x^2 - 5x + 6$.

Step 1:	*Find the factors q of the leading coefficient 1 and the factors p of the constant term 6.*	q could equal ±1 p could equal $\pm1, \pm2, \pm3, \pm6$
Step 2:	*Write all combinations of* $\frac{p}{q}$. *Then simplify.*	$\frac{\pm1, \pm2, \pm3, \pm6}{\pm1}$

Solution: $\pm1, \pm2, \pm3, \pm6$

The Rational Root Theorem can yield a large number of possible roots. To help eliminate some possibilities, you can use Descartes' Rule of Signs. While Descartes' rule does not tell you the value of the roots, it does tell you the maximum number of positive and negative real roots.

Descartes' Rule of Signs

If $f(x)$ is a polynomial function with real coefficients and a nonzero constant term arranged in descending powers of the variable, then

- the number of positive real roots of $f(x) = 0$ equals the number of variations in sign of the terms of $f(x)$, or is less than this number by an even integer.
- the number of negative real roots of $f(x) = 0$ equals the number of variations in sign of the terms of $f(-x)$, or is less than this number by an even integer.

SUGGESTED LEARNING STRATEGIES: Note-taking, Marking the Text, Vocabulary Organizer

My Notes

EXAMPLE 2

Find the number of positive and negative roots of $f(x) = x^3 - 2x^2 - 5x + 6$.

Step 1: *Determine the sign changes in* $f(x)$: $f(x) = x^3 - 2x^2 - 5x + 6$

There are 2 sign changes:

- one between the 1st and 2nd terms when the sign goes from positive to negative
- one between the 3rd and 4th terms when the sign goes from negative to positive

So there are either 2 or 0 positive real roots.

Step 2: *Determine the sign changes in* $f(-x)$: $f(-x) = -x^3 - 2x^2 + 5x + 6$

There is 1 sign change:

- between the 2nd and the 3rd terms when the sign goes from negative to positive

So there is 1 negative real root.

Solution: There are either 2 or 0 positive real roots and 1 negative real root.

You have found all the possible rational roots and the number of positive and negative real roots of a polynomial. The theorems below help you to find the zeros of the function. The Remainder Theorem tells if the factor is a zero, or another point on the polynomial. The Factor Theorem gives another way to test if a possible root is a zero.

The Remainder Theorem

If a polynomial $P(x)$ is divided by $(x - k)$ where k is a constant, then the remainder r is $P(k)$.

The Factor Theorem

A polynomial $P(x)$ has a factor $(x - k)$ if and only if $P(k) = 0$.

ACTIVITY 4.4 *Continued*

EXAMPLE 2 Note Taking Again, students will not find the actual roots in this example. However, they do find out something about the *types* of roots.

Paragraph Marking the Text

Theorems Vocabulary Organizer

EXAMPLES 3–4 Note Taking
These examples show two methods for finding the real zeros of the function.

My Notes

SUGGESTED LEARNING STRATEGIES: Note-taking

EXAMPLE 3

Use synthetic division to find the zeros and factor $f(x) = x^3 - 2x^2 - 5x + 6$.

From Examples 1 and 2, you know the possible rational zeros are $\pm1, \pm2, \pm3, \pm6$. You also know that the polynomial has either 2 or 0 positive real roots and 1 negative real root.

Step 1: *Divide* $(x^3 - 2x^2 - 5x + 6)$ *by* $(x + 1)$.

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -5 & 6 \\ & & -1 & 3 & 2 \\ \hline & 1 & -3 & -2 & 8 \end{array}$$

So you have found a point $(-1, 8)$.

Step 2: *Continue this process, finding either points on the polynomial and/or zeros for each of the possible roots.*
Divide $(x^3 - 2x^2 - 5x + 6)$ by $(x - 1)$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ & & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

So you have found a point $(1, 0)$ and a factor, $f(x) = (x - 1)(x^2 - x - 6)$.

Step 3: *As soon as you have a quadratic factor remaining after the division process, you can factor the quadratic factor by inspection, if possible, or use the Quadratic Formula.*

Solution: $f(x) = (x - 1)(x + 2)(x - 3)$; the real zeros are 1, −2, and 3.

Using the Factor Theorem, follow a similar process to find the real zeros.

EXAMPLE 4

Use the Factor Theorem to find the real zeros of $f(x) = x^3 - 2x^2 - 5x + 6$. Again, you know the possible rational roots are $\pm1, \pm2, \pm3, \pm6$.

Step 1: *Test* $(x + 1)$: $f(-1) = (-1)^3 - 2(-1)^2 - 5(-1) + 6 = 8$
So you have a point $(-1, 8)$.

Step 2: *Test* $(x - 1)$: $f(1) = (1)^3 - 2(1)^2 - 5(1) + 6 = 0$
So you have a zero at $x = 1$.

Step 3: *Test* $(x - 2)$: $f(2) = (2)^3 - 2(2)^2 - 5(2) + 6 = -4$

Step 4: *Continue to test rational zeros or use division to simplify the polynomial and factor or use the quadratic formula to find the real zeros.*

Solution: The real zeros are 1, −2, and 3.

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228 SpringBoard® Mathematics with Meaning™ Algebra 2

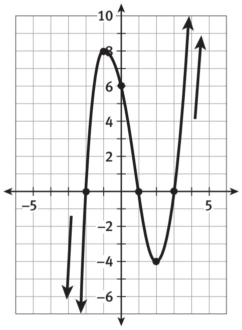
SUGGESTED LEARNING STRATEGIES: Note-taking, Create Representations

My Notes

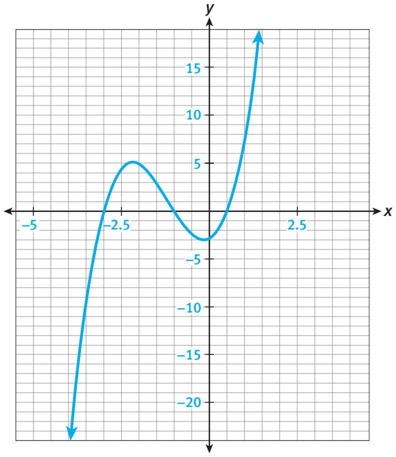
EXAMPLE 5

Graph $f(x) = x^3 - 2x^2 - 5x + 6$, using the information you have found so far, including the y-intercept and the end behavior of the function.

x	y
−1	8
1	0
−2	0
0	6
3	0
2	−4



10. Follow the examples above to graph $f(x) = 2x^3 + 7x^2 + 2x - 3$.



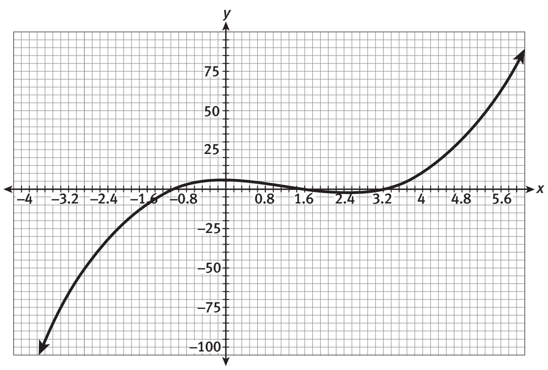
ACTIVITY 4.4 *Continued*

EXAMPLE 5 Note Taking

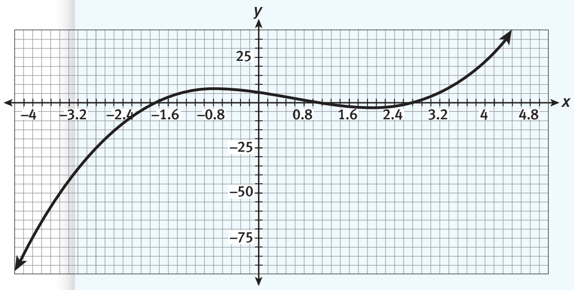
10 Create Representations, Debriefing This Item gives students the opportunity to practice the process they just observed. It also allows for formative assessment on how well students understood the process. Some students may need extra support through this Item. Use your questioning skills to lead students through the process without telling them how to do it a second time.

CHECK YOUR UNDERSTANDING

8.



6.



First Paragraph Note Taking

11 Simplify the Problem, Identify a Subtask, Create Representations This Item gives students an introduction to the concept of solving polynomial inequalities. It is not designed to be an intensive investigation. Give students an example of the process or have them attempt to work through the directions with support.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 230, #5–10

UNIT 4 PRACTICE
p. 260, #27–31

CHECK YOUR UNDERSTANDING

1. as $x \to -\infty, y \to \infty$, as $x \to \infty, y \to -\infty$
2. as $x \to -\infty, y \to \infty$, as $x \to \infty, y \to \infty$
3. See below right.
4. See below right
5. $x = \pm 1, \pm 5$
6. See page 229.
7. There are 2 or 0 possible positive real zeros and one negative real zero.
8. See page 229.
9. $x < -\sqrt{2}$ and $0 < x < \sqrt{2}$
10. Answers may vary. Students' answers may include the following: the ease or difficulty of setting a good viewing window; the possible distortions of graphs on a calculator screen; the advantages or disadvantages of the ZOOM and TRACE functions.

My Notes

SUGGESTED LEARNING STRATEGIES: Note-taking, Simplify the Problem, Identify a Subtask, Create Representations

To solve a **polynomial inequality** by graphing, use the fact that a polynomial can only change signs at its zeros.

Step 1: Write the polynomial inequality with one side equal to zero.
Step 2: Graph the inequality and determine the zeros.
Step 3: Find the intervals where the conditions of the inequality are met.

11. Solve the polynomial inequality $x^4 - 13x^2 + 6 < -30$ by graphing on a graphing calculator or by hand.

Check students' graphs.
$-3 < x < -2$, and $2 < x < 3$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper or grid paper. Show your work.

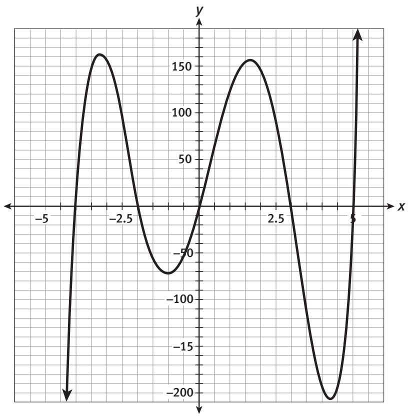
Determine the end behavior of each function.

1. $y = -3x^5 - 4x^3 + 5x + 7$
2. $y = 5x^{12} + 43x^8 - 14x^5 + 12x^2 + 8x$

Use what you know about end behavior and zeros to graph each function.

3. $y = x^5 - 2x^4 - 25x^3 + 26x^2 + 120x$
   $= x(x - 5)(x - 3)(x + 2)(x + 4)$
4. $y = x^5 + 9x^4 + 16x^3 - 60x^2 - 224x - 192$
   $= (x - 3)(x + 2)^2(x + 4)^2$
5. Determine all the possible rational zeros of $f(x) = x^3 - 2x^2 - 4x + 5$.
6. Graph $f(x) = x^3 - 2x^2 - 4x + 5$.
7. Determine the possible number of positive and negative real zeros for $h(x) = x^3 - 4x^2 + x + 5$.
8. Graph $h(x) = x^3 - 4x^2 + x + 5$.
9. Solve the inequality $x^3 - 2x < 0$.
10. **MATHEMATICAL REFLECTION** Write a paragraph arguing for or against the use of graphing calculators in graphing and understanding polynomial functions.

3.



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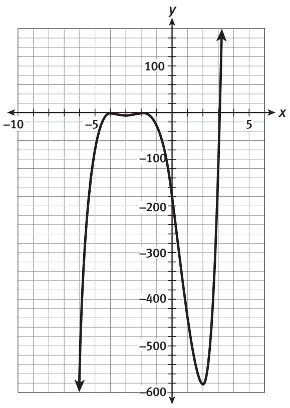


Factoring and Graphing Polynomials

SKETCH ARTIST

1. Factor $f(x) = x^3 + 3x^2 - x - 3$. Then find the zeros and y-intercept. Sketch a graph of the function.

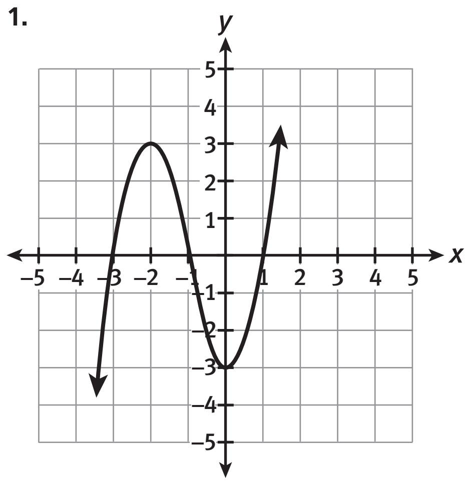
2. Find two different ways to show that $g(x) = -x^3 + 27$ has only one x-intercept. Use a sketch of the graph as one method, if necessary.

3. List all the characteristics of the graph for this polynomial function that you would expect to see, based on what you have learned thus far.

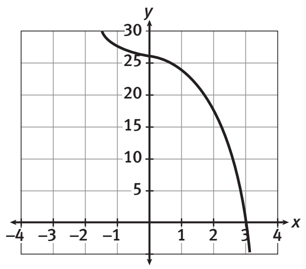
$$f(x) = (x + 3)(x - 3)(x + 2)(x - 1)(x + 2i)(x - 2i)$$

4. Find a polynomial function of 4th degree that has the zeros 2, −2, and $1 - 3i$. Then write it in standard form.

1.



2.



Embedded Assessment 2

Assessment Focus

- Factoring polynomials
- Graphing polynomials

Materials

- No special materials are needed.

Answer Key

1. $(x + 1)(x - 1)(x + 3)$; zeros: $x = -3$, 1, and −1; y-intercept: $y = -3$.

2. Answers may vary, but two options are sketching the graph (see below left) and factoring the equation.Sample answer: The factors are $(3 - x)(9 + 3x + x^2)$, so $x = 3$ is the solution associated with the first factor, and the second factor has no real solutions.

3 Make sure students have done similar problems in the Activity 4.4 Check for Understanding and Practice problems to help them understand the information that is being requested in this Item. If students do not have some experience in knowing what the expectations are for the question, they may not be successful due to the open-ended way it is presented.

3. There are zeros at $x = 3, -3, -2$, and 1; the y-intercept is 72. It is a 6th degree polynomial that has end behavior as $x \to \infty$, $y \to \infty$, and $x \to -\infty$, $y \to \infty$.

4. Answers may vary. Sample answer:
$x^4 - 2x^3 + 6x^2 + 8x - 40$

Embedded Assessment 2

TEACHER TO TEACHER You may wish to read through the rubric with students and discuss the differences in the expectation levels. Make sure students understand the meanings of any terms used.

Embedded Assessment 2

Use after Activity 4.4.

Factoring and Graphing Polynomials

SKETCH ARTIST

	Exemplary	Proficient	Emerging
Math Knowledge #1	The student factors $f(x)$ correctly. (1)	The student writes only one correct factor.	The student writes no correct factors.
Problem Solving #1, 2, 3, 4	The student: • Finds correct values for the three zeros and the y-intercept of $f(x)$. (1) • Uses two different correct ways to show that $g(x)$ has only one x-intercept. (2) • Lists the correct zeros, y-intercept, degree, and end-behavior of $f(x)$. (3) • Writes a correct polynomial function in standard form. (4)	The student: • Finds only three of the correct values for the zeros and y-intercept. • Uses only one way to show that $g(x)$ has only one x-intercept. • Lists at least four of the characteristics of $f(x)$ correctly. • Writes a correct polynomial function, but not in standard form.	The student: • Finds only one or two of the correct values. • Is not successful in showing that $g(x)$ has only one x-intercept. • Lists at least two of the characteristics of $f(x)$ correctly. • Writes an incorrect 4^{th} degree polynomial.
Representations #1	The student sketches a correct graph of $f(x)$. (1)	The student sketches a partially correct graph of $f(x)$.	The student sketches a graph that has no correct features.

Counting Methods
Let Me Count the Ways

ACTIVITY 4.5

SUGGESTED LEARNING STRATEGIES: Role Play, Graphic Organizer, Simplify the Problem

Sandwich Shop offers a combo meal that includes a choice of four sandwiches, three sides, and five drinks. The *Sandwich Shop* menu is shown at the right.

1. How many different combo meals consisting of one sandwich, one side dish, and one drink are offered at *Sandwich Shop*? Explain how you arrived at your answer.
$4 \cdot 3 \cdot 5 = 60$ different combo meals; Explanations will vary.

The *Gold Diner* also offers a combo meal consisting of eight main dishes, four side dishes, and six drinks.

Gold Diner Menu		
Main Courses	**Side Dishes**	**Drinks**
Fiesta Chicken	Salad	Grapefruit Juice
Grilled Fish	Soup	Orange Juice
Chicken Broccoli Pasta	Applesauce	Milk
Pork Chops	Steamed Vegetables	Bottled Water
Roasted Turkey		Lemonade
Vegetable Lasagna		Iced Tea
Broiled Shrimp		
BBQ Ribs		

2. How many combo meals consisting of one main course, one side dish, and one drink are offered at *Gold Diner*? Explain how you arrived at your answer.
$8 \cdot 4 \cdot 6 = 192$ different combo meals; Explanations will vary.

3. Every day you eat at *Sandwich Shop* or *Gold Diner* and order a different combo meal. How many days will it take you to order all the possible combo meals at each restaurant? Explain your reasoning.
$60 + 192 = 252$ days for 252 different meal choices; Add the number of possible combos for both restaurants.

My Notes

***Sandwich Shop* Menu**
Sandwiches
Veggie Wrap
Tuna Fish
Turkey
Chicken Breast
Side Dishes
Salad
Soup
Mixed Fruit
Drinks
Milk
Iced Tea
Apple Juice
Orange Juice
Bottled Water

ACTIVITY 4.5 Investigative

Counting Methods

Activity Focus
- Patterns/pattern recognition
- Organizing lists for counting purposes
- Fundamental Counting Principle
- Factorial, $n!$
- Permutation $_nP_r$ of n things taken r at a time
- Combination $_nC_r$ of n things taken r at a time

Materials
- Graphing calculators

Chunking the Activity

#1	#9–10	#16
#2	#11–12	#17
#3	#13	#18
#4–5	#14	#19
#6–8	#15	#20

1 Role Play, Graphic Organizer There are 4 main course items, 3 side dish items, and 5 drink choices. Assign students to each item sold at the *Sandwich Shop*. Line up the main courses, the side dishes, and the drink choices next to each other. Have one of the main courses shake hands with a side dish, and a side dish shake hands with each of the drinks. Then have the same main course shake hands with the next side dish, and so on. Have students count how many handshakes occur. This is a physical model of a tree diagram. It helps students to see the large number of handshakes and to make the association with the Fundamental Counting Principle.

2 Graphic Organizer Students read the table to determine the number of choices for each food type.

3 Simplify the Problem

4 Simplify the Problem Make sure students notice that since there was no overlap in the main courses, the sum of the number of main courses for each restaurant gives the total number of main courses for the new restaurant. However, since there is overlap in the side dishes and drinks, the total number offered in the new restaurant is not the sum of what was offered in the original two restaurants.

5 Quickwrite, Think/Pair/Share, Debriefing

Paragraph Vocabulary Organizer, Interactive Word Wall

6 Graphic Organizer, Vocabulary Organizer This Item is designed to help students get a feel for factorial notation and to build the base understanding of a permutation of n items taken n at a time. Later they will see n items taken r at a time. This Item can be referred back to, so students can see a purpose for the definition of $0! = 1$, the connections to the Fundamental Counting Principle, and the formula for finding the number of permutations of n objects taken r at a time.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 242, #1–2

UNIT 4 PRACTICE
p. 260, #32–33

My Notes

SUGGESTED LEARNING STRATEGIES: Simplify the Problem, Quickwrite, Think/Pair/Share, Vocabulary Organizer, Interactive Word Wall, Graphic Organizer

4. *Sandwich Shop* and *Gold Diner* are going to merge into one restaurant, so a customer will be able to order a combo meal from a combined list of all the choices. How many different combo meals can be ordered at the new restaurant? Explain your reasoning.
There are $12 \cdot 5 \cdot 7 = 420$ different combo meals; Explanations will vary.

5. Explain why the answer in Item 3 is different from the answer in Item 4.
Once the restaurants merged their menus, the new choices resulted in meals that were not possible in the individual menus.

MATH TERMS

Fundamental Counting Principle:

If there are p ways to make the first choice, q ways to make the second choice, r ways to make the third choice, and so on, then the product $p \cdot q \cdot r \cdot \ldots$ is the total number of ways a sequence of choices can be made.

The **Fundamental Counting Principle** is a useful way to count outcomes, especially in situations where it is impractical or even impossible to list them all.

6. A class has 4 students.

a. Use the boxes below to represent the seats for these 4 students. Write in each box the number of students that the teacher will choose from as she assigns each seat, beginning with Seat 1.

Seat 1	Seat 2	Seat 3	Seat 4
4	3	2	1

b. Use the seating diagram above and the Fundamental Counting Principle to determine the total number of ways that the teacher can assign the seats.
24

ACADEMIC VOCABULARY

A **factorial** is the product of a natural number, n, and all natural numbers less than n, written as $n!$.

$n! = n(n-1)(n-2) \cdot \ldots \cdot 2 \cdot 1.$

Zero factorial is defined as 1, or $0! = 1$.

c. Write your answer in **factorial** notation.
$4! = 4 \cdot 3 \cdot 2 \cdot 1$

SUGGESTED LEARNING STRATEGIES: Simplify the Problem, Graphic Organizer, Discussion Group

My Notes

7. A class has 20 students and 5 rows of 4 seats.

a. Write in each box the number of students that the teacher will choose from as she assigns each seat in the first row.

	Seat 1	Seat 2	Seat 3	Seat 4
Row 1	20	19	18	17

b. In how many ways can the teacher assign 4 of the 20 students to the seats in the first row?
There are 20 • 19 • 18 • 17 = 116,280 ways to assign the seats.

8. Complete the diagram below for Row 2 as you did in Item 7. Then use it and the Fundamental Counting Principle to find the number of ways that the teacher can assign 4 students to the seats in the second row, after assigning 4 students to the seats in the first row.

	Seat 1	Seat 2	Seat 3	Seat 4
Row 2	16	15	14	13

16 • 15 • 14 • 13 = 43,680

9. Consider the other rows of seats in the classroom.

a. Now that the teacher has seated eight students in the first 2 rows, in how many ways can the teacher seat the next 4 students in the seats in the third row?
12 • 11 • 10 • 9 = 11,880

b. Now that the teacher has seated 12 students in the first 3 rows, in how many ways can the teacher seat the next 4 students in the seats in the fourth row?
8 • 7 • 6 • 5 = 1680

c. Now that the teacher has seated 16 students in the first 4 rows, in how many ways can the teacher seat the next 4 students in the seats in the fifth row?
4 • 3 • 2 • 1 = 24

CONNECT TO AP

In AP Statistics, counting methods such as permutations and combinations are used when solving probability problems in which the sample space is very large and it is not feasible to write the entire sample space.

ACTIVITY 4.5 *Continued*

7 Simplify the Problem, Graphic Organizer

8 Simplify the Problem, Graphic Organizer

9 Simplify the Problem, Discussion Group Students may need a graphic organizer to help them. They can fill in all the numbers in the seats and see how the multiplication will work.

Connect to AP

Students learn counting methods including permutations and combinations in this unit. Students can use these counting techniques to solve the types of probability problems they will encounter in AP Statistics. The activities at the end of this unit will help students understand why the formulas work the way they do and increase the likelihood that they will be able to apply them in novel situations like those they will encounter on the Advanced Placement examinations.

10 Look for a Pattern, Debriefing

Paragraphs in Middle of Page
Vocabulary Organizer, Note Taking, Marking the Text, Interactive Word Wall

11 Think/Pair/Share

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Vocabulary Organizer, Note-taking, Marking the Text, Interactive Word Wall, Think/Pair/Share

My Notes

10. In how many ways can the teacher seat all 20 students in the 20 seats?
$20 \cdot 19 \cdot 18 \cdot 17 \cdot \ldots \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 20!$

Placing students in seats is an example in which order is important. One seating arrangement for the first row of seats is shown below.

	Seat 1	Seat 2	Seat 3	Seat 4
Row 1	Al	Jo	Ty	Le

A different arrangement for the first row of seats is shown below.

	Seat 1	Seat 2	Seat 3	Seat 4
Row 1	Al	Le	Ty	Jo

ACADEMIC VOCABULARY

permutation

In mathematics, an ordered arrangement of items is called a **permutation** of the set of items. A permutation of n distinct things taken r at a time is expressed by the **permutation notation** ${}_nP_r$.

WRITING MATH

You write the *permutation notation* for 30 things taken 5 at a time as ${}_{30}P_5$. A contextual example of this is finding the total number of ways to seat students from a class of 30 students in a single row of 5 seats.

11. Use permutation notation ${}_nP_r$ to express the number of ways that the teacher can assign students to each row as described in Items 6–8. For example, the number of ways seat assignments can be made for Row 1 is expressed as ${}_{20}P_4$.

a. Row 2 ${}_{16}P_4$

b. Row 3 ${}_{12}P_4$

c. Row 4 ${}_{8}P_4$

d. Row 5 ${}_{4}P_4$

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236 SpringBoard® Mathematics with Meaning™ Algebra 2

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Look for a Pattern, Quickwrite, Simplify a Problem, Group Presentation

My Notes

The class arrangement of seats has 5 rows of 4 seats. Sometimes counting can be carried out in different ways. For example, looking at the seats in the classroom from another perspective, the classroom has 4 columns, and each column has 5 seats.

12. Use permutation notation to express the number of seating choices the teacher can make from the class of 20 students for the first column of seats.

$_{20}P_5$

13. Does $_{20}P_4 \cdot {}_{16}P_4 \cdot {}_{12}P_4 \cdot {}_{8}P_4 \cdot {}_{4}P_4 = {}_{20}P_5 \cdot {}_{15}P_5 \cdot {}_{10}P_5 \cdot {}_{5}P_5$? Explain why or why not.

Yes, because writing out all the factors in $_{20}P_4 \cdot {}_{16}P_4 \cdot {}_{12}P_4 \cdot {}_{8}P_4 \cdot {}_{4}P_4$ and $_{20}P_5 \cdot {}_{15}P_5 \cdot {}_{10}P_5 \cdot {}_{5}P_5$ shows $20 \cdot 19 \cdot 18 \cdot 17 \cdot \ldots \cdot 4 \cdot 3 \cdot 2 \cdot 1$ for each product. Thus, the equation is true.

14. The general formula for a permutation of n distinct things taken r at a time is

$$_nP_r = n(n-1)(n-2)\cdot \ldots \cdot (n-r+1)$$

a. Verify that the above formula gives the correct value for $_{20}P_4$ and $_{10}P_5$.

The value of $_{20}P_4 = 20(20-1)(20-2)(20-4+1) = 20 \cdot 19 \cdot 18 \cdot 17$, and the value of $_{10}P_5 = 10(10-1)(10-2)\ldots(10-5+1) = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6$.

b. Verify that $_nP_r = \dfrac{n!}{(n-r)!}$.

$$_nP_r = \frac{n!}{(n-r)!}$$

$$= \frac{n(n-1)(n-2)(n-3)\ldots(n-r+1)(n-r)(n-r-1)(n-r-2)\ldots(1)}{(n-r)(n-r-1)(n-r-2)\ldots(1)}$$

$$= n(n-1)(n-2)(n-3)\ldots(n-r+1)$$

ACTIVITY 4.5 *Continued*

12 Think/Pair/Share

13 Look for a Pattern, Quickwrite

14 Simplify the Problem, Group Presentation, Debriefing

ACTIVITY 4.5 *Continued*

Formula Box Note Taking

15 Create Representations, Debriefing In part (a), students should list the arrangements, such as AMY, AYM, without using the formula or counting principle. In part (b), students are asked to verify using the Fundamental Counting Principle, and in part (c), students use the permutation formula for a different name, FRANK.

My Notes

SUGGESTED LEARNING STRATEGIES: Note-taking, Create Representations

Permutation Formula

The number of n items chosen r at a time is

$${}_nP_r = n(n-1)(n-2) \cdot \ldots \cdot (n-r+1) = {}_nP_r = \frac{n!}{(n-r)!}$$

15. A teacher asks the class to find the number of ways that the letters in their names, all in uppercase, can be placed into different arrangements, whether or not these arrangements spell a word.

a. List all the possible arrangements for the name AMY.
AMY, AYM, MAY, MYA, YAM, and YMA

b. Use the Fundamental Counting Principle to verify that the list in Part a is complete.
$3 \cdot 2 \cdot 1 = 6$

c. Use permutations to find the number of arrangements of the letters in the name FRANK.
$5! = 120$

SUGGESTED LEARNING STRATEGIES: Create Representations, Quickwrite, Discussion Group

My Notes

16. PIPPI is also a student in this class.

a. List all the possible arrangements for the name PIPPI.
The distinguishable letter arrangements of PIPPI are listed below.

PPPII	IIPPP
PPIPI	IPIPP
PIPPI	IPPIP
PPIIP	IPPPI
PIIPP	PIPIP

b. What is different about the letters in PIPPI's name as compared to the letters in FRANK's name?
The name PIPPI has three P's and two I's, whereas the name FRANK contains letters that are all different.

c. Explain why your answer to Part b will make a difference in the total number of arrangements of the letters in PIPPI's name as compared to the letters in FRANK's name.
When the three P's and the two I's in PIPPI are permuted, some permutations result in the same arrangement of the letters in PIPPI. With different letters in the name FRANK, every permutation gives rise to a different arrangement.

d. For PIPPI's name, suppose that the three P's are labeled P_1, P_2, and P_3, and the two I's are labeled I_1 and I_2. How many different arrangements are there for the P's and how many different arrangements are there for the I's?
There are 3! or six different arrangements of P's and 2! or two different arrangements of I's.

e. For PIPPI's name, suppose that the P's are labeled P_1, P_2, and P_3 and the I's are labeled I_1 and I_2 to keep track of the P's and I's when the letters are arranged differently. How many arrangements of PIPPI's name will result?
5!, or 120

ACTIVITY 4.5 *Continued*

16 a Create Representations

16 c Quickwrite

16 d Discussion Group

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16f Quickwrite, Debriefing

16g Look for a Pattern

Paragraphs Vocabulary Organizer, Note Taking, Interactive Word Wall

17 Think/Pair/Share, Self/Peer Revision

18 Quickwrite

SUGGESTED LEARNING STRATEGIES: Quickwrite, Look for a Pattern, Vocabulary Organizer, Note-taking, Interactive Word Wall, Think/Pair/Share, Self/Peer Revision

My Notes

16. (*continued*)

f. Let N be the number of ways that the letters in PIPPI can be arranged into distinguishable arrangements. Use the results of Part d and the Fundamental Counting Principle to explain what $3! \cdot 2! \cdot N$ equals.

If N denotes the number of different arrangements of PIPPI, then $3! \cdot 2! \cdot N = 5!$.

g. Determine the value of N in Part f. How does this value compare to the answer in Part a?

The value of N is $N = \frac{5!}{3! \cdot 2!} = 10$, which is the same answer obtained in Part a.

MATH TERMS

The number of **distinguishable permutations** of n items is $P = \frac{n!}{p!q!r!\ldots}$, when the n items include p copies of one item, q copies of another item, r copies of a third item, and so on.

When letters in a word, as in the name PIPPI, are rearranged, some arrangements are the same because identical letters have been interchanged or permuted. These permutations are called **indistinguishable permutations**.

Unique arrangements of items are **distinguishable permutations**.

17. Give the number of distinguishable permutations of the letters in each name. Show how you have used the general rule in the box at the left to set up and count the distinguishable permutations in each name.

PIPPI

The total number of distinguishable permutations of PIPPI is $\frac{5!}{3!2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 10$.

BELLE

The total number of distinguishable permutations of BELLE is $\frac{5!}{2!2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 30$.

BABBETTE

The total number of distinguishable permutations of BABBETTE is $\frac{8!}{3!2!2!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 1680$.

18. How many different 10-digit numbers can be formed by rearranging the digits of the number 3,644,644,622? Show your work in the My Notes space.

The number of different 10-digit numbers that can be obtained by rearranging the digits of the number 3,644,644,622 is $\frac{10!}{4!3!2!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 12{,}600$.

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240 SpringBoard® Mathematics with Meaning™ Algebra 2

SUGGESTED LEARNING STRATEGIES: Shared Reading, Graphic Organizer, Create Representations, Quickwrite, Group Presentation

My Notes

19. A class of 20 students is electing class officers. The teacher will select a nominating committee of 4 students from the class. The committee will then determine the candidates for the election.

a. Sally, Clarence, Manuel, and Tisha were selected. Who could the teacher have selected first, second, third, and fourth? Use the boxes below to give two possible orders that the teacher could have had for selecting the nominating committee members.
Answers will vary.

1st Selection	2nd Selection	3rd Selection	4th Selection
Sally	Clarence	Manuel	Tisha

1st Selection	2nd Selection	3rd Selection	4th Selection
Manuel	Sally	Tisha	Clarence

b. How many arrangements could the teacher have made? Show your work to explain your reasoning
4! or 24

c. Explain why the order in which a teacher selects the committee members is *not* important.
The order is *not* important, because the committee members do not change as the order changes. It is committee membership, rather than the order of selection, that counts.

d. Let N be the number of ways that the teacher can make the nominating committee selections. Explain what $4! \cdot N$ equals.
$4! \cdot N = 20 \cdot 19 \cdot 18 \cdot 17 = {}_{20}P_4$.

e. Determine the value of N in Part d.
N is given by $\frac{20 \cdot 19 \cdot 18 \cdot 17}{4!} = \frac{{}_{20}P_4}{4!} = 4845$.

ACTIVITY 4.5 *Continued*

19 Shared Reading You can use a modified group presentation and have different groups assigned different parts of the Items. If you assign the presentations randomly, after a certain time given to the students, this will encourage each group to work through the entire problem and be prepared for each section.

19 a Graphic Organizer, Create Representations

19 b Quickwrite

19 c Quickwrite

19 d Quickwrite, Group Presentation

Paragraphs Vocabulary Organizer, Note Taking, Interactive Word Wall

20 Think/Pair/Share, Self/Peer Revision, Debriefing Make sure students follow up this Item by sharing their answers. Then guide the class to the formal definition: ${}_nC_r = \frac{n!}{r!(n-r)!}$. Students should put this into their notebooks.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 242, #3–7

UNIT 4 PRACTICE
p. 260, #34–37

CHECK YOUR UNDERSTANDING

1. 24
2. 840
3. ${}_5P_3 = \frac{5!}{(5-3)!} = 60$
4. 10
5. ${}_{28}C_{14} = \frac{28!}{14!(28-14)!}$
6. ${}_{28}C_{14} = \frac{28!}{14!(28-14)!}$ $= 40{,}116{,}600$
7. Answers may vary. Sample answer: Students may state that they know how to compute using counting methods but may still be unsure as to which situations call for which kind of counting method.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Note-taking, Interactive Word Wall, Think/Pair/Share, Self/Peer Revision

My Notes

MATH TIP

Recall that a permutation is defined as ${}_nP_r = \frac{n!}{(n-r)!}$.

In mathematics, collections of items without regard to order are called **combinations**. The number of combinations of n distinct things taken r at a time is denoted by ${}_nC_r$.

In Item 19e, the value of N is a combination of 20 things taken 4 at a time and can be represented by ${}_{20}C_4$. In terms of the notation for combinations and permutations, this means that $4! \cdot {}_{20}C_4 = {}_{20}P_4$.

20. Write a formula, similar to the one for permutations, for ${}_nC_r$, the number of combinations of n things taken r at a time, in terms of n and r.

A combination is defined as ${}_nC_r = \frac{n!}{r!(n-r)!}$.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. In how many ways can the letters in the word MATH be arranged without any of the letters being repeated?
2. Find $\frac{10!}{6! \cdot 3!}$.

A committee of 3 people is selected from a group of 5 people.

3. Use permutation notation to express the number of ways the committee can be selected.
4. Find the number of committees that can be selected.

The student council is collecting movies on DVD to send to troops overseas. Allie has 28 movies on DVD. She has decided to donate half of her movies to the student council collection.

5. Express the number of ways that Allie could choose the movies to be donated by using notation for combinations.
6. Use a calculator to find the number of combinations.
7. MATHEMATICAL REFLECTION What did you learn from doing this investigation? What questions do you still have?

Combinations and Permutations
Pick It or Skip It

ACTIVITY 4.6

SUGGESTED LEARNING STRATEGIES: Shared Reading, Marking the Text, Activating Prior Knowledge, Group Presentation

My Notes

Games of chance are used by some states to raise money for state services to benefit people. There are many types of games of chance. Examine a few games to see what the chances of winning each game really are.

Deuce: The player must choose 2 different letters from the alphabet. To win, the player must match the first and second letters drawn in the game in the correct order.

1. How many different ways can the 2 letters be chosen?

$_{26}P_2 = 26 \cdot 25 = 650$

2. If you play one time, what is the *probability* of winning at Deuce?

$\frac{1}{650}$

MATH TIP

If all outcomes in a finite sample space are equally likely to occur, then the *probability* of an event *A* is the ratio of the number of outcomes in event *A* to the total number of outcomes in the sample space. $P(A) = \frac{\text{Outcomes in event } A}{\text{Outcomes in sample space}}$

Pick-em: The player chooses 3 different numbers from 0 to 9, for example, 0-7-8. If the same 3 numbers are drawn in the game, in any order, the player wins.

3. In how many ways can the 3 numbers be selected?

$_{10}C_3 = \frac{10!}{7!3!} = 120$

4. If you play one time, what is the probability of winning Pick-em?

$\frac{1}{120}$

5. In **Straight**, the player chooses 4 numbers from 0 to 9. To win, the player must match all 4 numbers drawn in the game in the correct order. If you play one time, what is the probability of winning?

$\frac{1}{_{10}P_4} = \frac{1}{5040}$

ACTIVITY 4.6 Investigative

Combinations and Permutations

Activity Focus

- Probability with combinations and permutations
- Binomial Theorem

Materials

- Graphing calculators

Chunking the Activity

#1–2	#8–11	#22
#3–4	#12–15	#23–27
#5	#16–17	
#6–7	#18–21	

First Paragraph Shared Reading

Second Paragraph Marking the Text

1-2 Activating Prior Knowledge, Debriefing These Items are designed to make the connection between what students learned in Activity 4.5 and their earlier studies in probability. It may be helpful to review some simple probability concepts such as how to find a probability.

Third Paragraph Marking the Text

3-4 Activating Prior Knowledge, Group Presentation, Debriefing Students can use either permutations or combinations to solve these problems. Having them share while exploring both possibilities will be advantageous.

5 Marking the Text

6-15 These Items will guide students through finding the probability of winning the Pick It game. The process includes more than one factor in the numerator, so this may be new for some students.

First Paragraph Shared Reading

7 It is important that students see that there are 3 that match and there are 2 that do not match. This will be critical to determining the number of chances to win the game.

SUGGESTED LEARNING STRATEGIES: Shared Reading

My Notes

ACADEMIC VOCABULARY

The number of **combinations** of n distinct things taken r at a time can be written as $_nC_r = \frac{n!}{r!(n-r)!}$.
An alternative notation is $\binom{n}{r} = \frac{n!}{r!(n-r)!}$.

Pick It: The player chooses 5 numbers out of 20. The order does not matter. If the player matches exactly 3 of the numbers drawn in the game, the player wins.

6. How many possible **combinations** of numbers can be drawn?

$_{20}C_5 = \frac{20!}{5!(20-5)!} = 15{,}504$

7. The numbers selected were 1, 4, 12, 16, and 19. Write 6 different possible winning tickets. What do the tickets you wrote have in common?

Answers will vary. Sample answers: 1, 4, 12, 18, 20 and 1, 3, 12, 16, 14. Three numbers match the numbers selected.

8. How many ways are there to match 3 numbers out of the 5 selected by the game?

$\binom{5}{3} = 10$

9. How many numbers out of 20 were not selected in the drawing?

15

10. How many ways are there to match the 2 numbers not selected in the drawing out of the number you gave as your answer to Item 9?

$\binom{15}{2} = 105$

11. What is the probability of winning this game if you play once?

$\frac{\binom{5}{3}\binom{15}{2}}{\binom{20}{5}} = \frac{1050}{15{,}504} = \frac{175}{2584} = 0.068$

SUGGESTED LEARNING STRATEGIES: Simplify the Problem, Think/Pair/Share, Quickwrite, Group Presentation, Look For a Pattern

My Notes

12. You can win a better prize by matching exactly 4 of the numbers. Determine the probability of winning this prize.

$$\frac{\binom{5}{4}\binom{15}{1}}{\binom{20}{5}} = \frac{75}{15{,}504} = 0.00484$$

13. How does the probability in Item 11 compare to that of Item 12?

The probability of getting 3 numbers correct is 14 times as great as the probability of getting 4.

14. You can win the best prize by matching exactly 5 of the numbers. What is the probability of winning the best prize? Explain.

$$\frac{\binom{5}{5}\binom{15}{0}}{\binom{20}{5}} = \frac{1}{15{,}504} = 0.00006449$$

15. What is the probability of winning the Pick-It game if "pick 3 or more" is considered a win?

$$\frac{\binom{5}{3}\binom{15}{2} + \binom{5}{4}\binom{15}{1} + \binom{5}{5}\binom{15}{0}}{\binom{20}{5}} = \frac{1050 + 75 + 1}{15{,}504} = \frac{1126}{15{,}504} = 0.0726$$

16. Find the values of each ${}_nC_r$ and place them in a triangular pattern similar to the one given.

$$\binom{0}{0}$$
$$\binom{1}{0}\ \binom{1}{1}$$
$$\binom{2}{0}\ \binom{2}{1}\ \binom{2}{2}$$
$$\binom{3}{0}\ \binom{3}{1}\ \binom{3}{2}\ \binom{3}{3}$$
$$\binom{4}{0}\ \binom{4}{1}\ \binom{4}{2}\ \binom{4}{3}\ \binom{4}{4}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

ACTIVITY 4.6 *Continued*

12 Simplify the Problem, Think/Pair/Share Students will now use what they learned in Items 7–11 to find the probability for matching 4 numbers.

13 Quickwrite

14 Quickwrite

15 Group Presentation, Debriefing To find the probability of winning by matching 3, 4, or 5 numbers, students should add the probabilities. For those students who want to multiply, have them compare their answers for the probabilities they found for 3, 4, or 5. They should see that it is not possible for the probability of all three opportunities to win combined to be less than that of getting one of the numbers.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 248, #1–3

UNIT 4 PRACTICE
p. 260, #38–42

16 Look for a Pattern, Think/Pair/Share In this Item, students create Pascal's Triangle. They will learn later in this Activity that the rows of the triangle correspond to the coefficients in the Binomial Theorem.

17 Look For a Pattern, Quickwrite Discussion Group Students may find the answer by looking at many patterns—sums of numbers in the previous row or patterns on diagonals. Have students share their methods.

18 Group Presentation, Discussion Group By expanding the binomials, students have the opportunity to find the relationships between Pascal's Triangle and the binomial expansion. Assign a few expansions to each group and have them share with the class to save time. Then have students discuss the patterns that they see.

19-21 Look For a Pattern, Quickwrite, Discussion Group, Debriefing Students develop a number of patterns that relate the expansions to the degree, the number of terms, and the coefficients. It is desirable for students to also recognize that the sum of the powers of each term of $(a + b)^n$ is 2^n. Although this is not explicitly stated here, it should be brought out in a discussion of patterns.

Connect to Math History

The triangle known as Pascal's Triangle was produced in *Traité du triangle arithmétique* (Treatise on arithmetical triangle) in 1665 where he collected several known results and used them to solve probability theory problems. In Iran it is known as the Khayyam triangle for Omar Khayyam (1048–1123) who also understood its relationship to the Binomial Theorem. In China it is called Yang Hui's triangle and in Italy, it is referred to as Tartaglia's triangle, named for the Italian algebraist Niccolò Fontana Tartaglia (1500–1557), who lived a century before Pascal.

SUGGESTED LEARNING STRATEGIES: Look For a Pattern, Quickwrite, Discussion Group

My Notes

The triangular pattern that you created is called Pascal's Triangle. It has many interesting patterns.

17. Write the numbers that will fill in the next row. How did you determine what the numbers would be?

1 5 10 10 5 1; Answers will vary.

18. Expand each binomial.

$(a + b)^0 =$ 1

$(a + b)^1 =$ $a + b$

$(a + b)^2 =$ $a^2 + 2ab + b^2$

$(a + b)^3 =$ $a^3 + 3a^2b + 3ab^2 + b^3$

$(a + b)^4 =$ $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

19. How do the coefficients of the expanded binomials relate to the numbers in Pascal's Triangle?

The coefficients in the rows equal the numbers in the matching row of Pascal's Triangle.

20. What patterns do you notice in the exponents of the expanded binomials in Item 18?

The degree of a starts with the value of the exponent and decreases to zero, whereas the degree of b starts with zero and increases to the value of the exponent. The sum of the powers in any term is equal to the exponent.

21. How does the number of terms in the expansion of $(a + b)^n$ relate to the degree n?

There are $n + 1$ terms.

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246 SpringBoard® Mathematics with Meaning™ Algebra 2

SUGGESTED LEARNING STRATEGIES: Note-taking, Vocabulary Organizer, Create Representations, Think/Pair/Share, Simplify the Problem, Discussion Group

My Notes

The Binomial Theorem

For any positive n, the binomial expansion is:

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \dots \binom{n}{n}a^0 b^n.$$

22. Write the Binomial Theorem using summation notation and $\binom{n}{k}$ to represent the combination.

$$(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$

To find the r^{th} term of any binomial expansion $(a+b)^n$, use the expression $\binom{n}{r-1}a^{n-(r-1)}b^{r-1}$.

23. Find the coefficient of the 4^{th} term in the expansion of $(x-3)^8$.

$\binom{8}{3}(-3)^3 = \frac{8!}{5! \cdot 3!}(-27) = -1512$

24. Find the coefficient of the 6^{th} term in the expansion of $(x+2)^{11}$.

$\binom{11}{5}(2)^5 = 14{,}784$

25. Find the 7^{th} term in the expansion of $(x+4)^9$.

$\binom{9}{6}x^3 4^6 = 344{,}064x^3$

ACTIVITY 4.6 *Continued*

Theorem Box Note Taking, Vocabulary Organizer

22 Create Representations, Think/Pair/Share, Note Taking Students use what they know about sigma notation to rewrite the Binomial Theorem.

23-25 Simplify the Problem, Discussion Group, Debriefing Students should take notes on the Binomial Theorem and finding the r^{th} term of a binomial expansion. Have them work in their groups to verify that they can find coefficients and terms for binomials of the form $(a+b)^n$.

ACTIVITY 4.6 *Continued*

26-27 Create Representations, Discussion Group, Debriefing
Have students work through the problems in their groups to verify that they can complete expansions for binomials of the form $(a + b)^n$.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 248, #4–7

UNIT 4 PRACTICE
p. 260, #43–45

CHECK YOUR UNDERSTANDING

1. $\frac{1}{10!} = \frac{1}{3628800}$
2. $\frac{\binom{15}{3}\binom{25}{2}}{\binom{40}{5}} = \frac{136500}{658008} \approx 0.207$
3. $\frac{\binom{25}{5}\binom{15}{2} + \binom{25}{6}\binom{15}{1} + \binom{25}{7}\binom{15}{0}}{\binom{40}{7}}$
$= \frac{10{,}575{,}400}{18643560} \approx 0.567$
4. $\frac{6!}{4! \cdot 2!} = 15$
5. $109{,}824x^7$
6. $x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$
7. Answers will vary.

My Notes

SUGGESTED LEARNING STRATEGIES: Create Representations, Discussion Group

26. Use the Binomial Theorem to write the binomial expansion of $(x + 4)^7$.

$\binom{7}{0}x^7 4^0 + \binom{7}{1}x^6 4^1 + \binom{7}{2}x^5 4^2 + \binom{7}{3}x^4 4^3 + \binom{7}{4}x^3 4^4 + \binom{7}{5}x^2 4^5 + \binom{7}{6}x^1 4^6 + \binom{7}{7}x^0 4^7$

$= x^7 + 28x^6 + 336x^5 + 2240x^4 + 8960x^3 + 21{,}504x^2 + 28{,}672x + 16{,}384$

27. Use the Binomial Theorem to write the binomial expansion of $(x - 4)^7$.

$x^7 - 28x^6 + 336x^5 - 2240x^4 + 8960x^3 - 21{,}504x^2 + 28{,}672x - 16{,}384$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

You selected 10 songs for a playlist on your MP3 player. The MP3 player is set to play the songs at random. The player will play all 10 songs without repeating any one song.

1. What is the probability that the songs will be played in the exact order that they are listed in the playlist?

A jar contains 40 marbles. There are 15 red and 25 yellow marbles.

2. What is the probability that if you draw 5 marbles from the jar without replacement, 3 are red?
3. What is the probability that if you draw 7 marbles from the jar without replacement, at least 5 are yellow?
4. Find the coefficient of the 5th term in the expansion of $(x + 1)^6$.
5. Find the 7th term in the expansion of $(x - 2)^{13}$.
6. Use the Binomial Theorem to write the binomial expansion of $(x + y)^5$.
7. MATHEMATICAL REFLECTION What did you learn from doing this investigation? What questions do you still have?

Binomial Probability
Are You My Type?

ACTIVITY 4.7

SUGGESTED LEARNING STRATEGIES: Marking the Text, Activating Prior Knowledge, Close Reading

My Notes

Janet and Bob both have Type A blood. Each carries the dominant gene for the Type A antigen and the recessive gene for the Type O antigen. A Punnett Square that represents the possible gene combinations for their children is shown below.

	A	O
A	AA	AO
O	AO	OO

A gene combination of AA or AO represents a child with Type A blood. A gene combination of OO represents a child with Type O blood. Both Bob and Janet are curious about the probabilities involving the blood types of their 8 children.

1. What is the probability that a child of Janet and Bob will be Type O?

$P(\text{type O}) = \frac{1}{4} = 0.25$

2. What is the probability that a child of Janet and Bob will be Type A?

$P(\text{Type A}) = \frac{3}{4} = 0.75$

3. What is the sum of the two probabilities in Items 1 and 2?

$0.25 + 0.75 = 1$

The probability experiment described above is an example of a **binomial experiment**. A binomial experiment has several important characteristics:

- The situation involves a fixed number of trials.
- Each trial has only two possible outcomes. For the sake of convenience, one of these outcomes is labeled a *success*, while the other outcome is labeled a *failure*.
- The trials are **independent**, meaning that the outcome of one trial does not affect the probability of success in subsequent trials.
- The probability of success remains the same for each trial.

CONNECT TO SCIENCE

Antigens are antibody-producing proteins found on the surface of red blood cells. The type of antigen, A, B, or O, on the surface of a person's red blood cells determines that person's blood type.

ACTIVITY 4.7 Investigative

Binomial Probability

Activity Focus
- Binomial experiments
- Binomial probability formula
- Binomial probability distribution

Materials
- Calculators

Chunking the Activity

#1–3	#9–12	#21
#4	#13–14	#22–23
#5–7	#15–19	#24
#8	#20	

Paragraphs 1 and 2 Marking the Text

1-2 Activating Prior Knowledge
Many students in Algebra 2 may have some familiarity with genetics and Punnett Squares. Have volunteers present basic information regarding genetics and dominant/recessive genes so that all have access to the context. The basic probability derived from the Punnett square is addressed in these Items.

3 Recognizing that the sum of the two probabilities in Items 1 and 2 equals one infers that these are the only two possibilities in each trial. This is a necessary condition for binomial probability, so classroom discussion with probing questions is essential so that students are aware of this fact.

Paragraph 3 Close Reading, Note Taking In the description of a binomial experiment, some students may be misled by the terms *success* and *failure*. Make certain that students understand that these are merely labels of convenience and convention in probability.

Note that probabilities may be expressed as fractions, decimals, or percents. For the sake of convenience, fractions are used throughout this text, but students are free to use alternate representations.

ACTIVITY 4.7 *Continued*

4 Quickwrite Students may have difficulty in associating the reality of having a child with a theoretical trial in a probability experiment. Make sure that students can identify all four of the necessary components of the binomial experiment in the context of the situation when responding to this Item.

Paragraph 1 Shared Reading

5 Quickwrite, Discussion Group A familiar yet simple probability experiment of tossing a coin can facilitate student access to the concept of binomial probability. Again, all four components of a binomial experiment are necessary in the students' responses.

SUGGESTED LEARNING STRATEGIES: Quickwrite, Shared Reading, Discussion Group

My Notes

MATH TIP

The probability of successive independent events *A*, *B*, *C*, ... occurring is
$P(A \text{ and } B \text{ and } C \text{ and} \ldots) = P(A) \cdot P(B) \cdot P(C) \cdot \ldots$

4. Explain how finding the probability that 3 out of 8 of Bob and Janet's children will have Type O blood is a binomial experiment.
Student answers may vary, but must include the following components:
- The number of trials (children) is eight, a fixed number.
- There are only two outcomes, Type A and Type O.
- All trials (children) are independent.
- The probability for each trial is the same.

To determine the probabilities in a binomial experiment like the one described above, it is helpful to consider a simple probability experiment consisting of tossing a fair coin 3 times.

5. Explain how tossing a fair coin 3 times is a binomial experiment.
Student responses may vary, but must include the following statements.
- Three coin tosses represent the fixed number of trials of a binomial experiment.
- Only two types of outcomes exist: heads or tails.
- One coin toss does not affect the other tosses; therefore, the trials are independent.
- The probability of success is $\frac{1}{2}$ for all trials.

6. Find the probability of obtaining a head, a head, and then a tail, in that order, when tossing a coin 3 times in the following two ways.

a. List all of the outcomes in the sample space.
HHH HHT HTH HTT TTT TTH THT THH

b. Apply the probability rules for successive independent events.
$P(HHT) = P(H) \cdot P(H) \cdot P(T) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

SUGGESTED LEARNING STRATEGIES: Discussion Group, Shared Reading, Quickwrite

My Notes

7. Consider the question, "What is the probability of obtaining exactly 2 heads from 3 coin tosses?" How does this question differ from the situation in Item 6?

The difference between "heads, heads, then tails" and "exactly two heads" is that the *order does not matter* in the second instance.

8. Find the probability of obtaining exactly 2 heads from 3 tosses of a coin in the following two ways.

a. List all of the outcomes in the sample space.

HHH HHT HTH HTT TTT TTH THT THH

b. Apply the appropriate probability rules.

$P(HHT) + P(HTH) + P(THH) = \frac{3}{8}$

MATH TIP

The probability of *A* or *B* occurring if *A* and *B* are mutually exclusive events is $P(A \text{ or } B) = P(A) + P(B)$.

The probability of *A* or *B* occurring if *A* and *B* are not mutually exclusive events is $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

A fast food restaurant wants to increase customer interest in a new chicken sandwich. They are offering one scratch-off card with each purchase of a chicken sandwich. Each scratch-off card has a 20% chance of being a winning card, and a customer has collected three cards from previous purchases.

9. Explain why this situation represents a binomial experiment.

- **Three scratch-off cards represent a fixed number of trials.**
- **Each scratch-off card has only two possible outcomes—winning or losing.**
- **The outcome of one scratch-off card does not affect the outcome of the other cards.**
- **The probability of success for each scratch-off card is 0.20.**

10. What is the probability of having the first scratch-off card be a winner, the second card be a winner, and the third card be a loser? Explain your reasoning.

$\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{4}{5} = \frac{4}{125} = 0.032$; **they are independent events**

ACTIVITY 4.7 *Continued*

7 Discussion Group By inspecting the sample space listed in Item 6(a), students should recognize that there are multiple ways of obtaining exactly two heads—HHT, HTH, THH. The difference that students will identify is the order of the coin tosses.

8 Students should recognize that the sample space is the same as in Item 6a, but now a successful outcome is defined differently (HHT, HTH, THH). In part (b), students will apply the addition rule for the probability of mutually exclusive events to arrive at the probability $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$.

9 Quickwrite Given the scratch-off card scenario, students identify the components of a binomial experiment once again.

10 Quickwrite After recognizing and using the multiplication rule for probabilities of successive independent events in Item 6b, students should realize that they need to multiply: $\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)\left(\frac{4}{5}\right) = \frac{4}{125}$.

11 **Quickwrite** Students again are asked to distinguish between WWL and WWL, WLW, and LWW. It is likely that students will need to write the different ways of having exactly two winning cards in order to internalize the fact that order does not matter. A tree diagram (with probabilities on each branch) may be helpful.

12 Paralleling Item 8b, students will add probabilities of mutually exclusive events to find the probability of exactly 2 winning cards out of 3: $\frac{4}{125} + \frac{4}{125} + \frac{4}{125} = \frac{12}{125}$.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 256, #1–3

UNIT 4 PRACTICE
p. 261, #46–48

13 It is vital that students realize the difference between probability questions in which order does matter and those in which order does not matter. This Item asks students to reflect on previous Items where order does not matter.

14 **Activating Prior Knowledge** Students are clued in to the correct answer by the phrases *counting method* and *order is not important*. If students are unable to recognize these characteristics of a combination scenario, then additional practice may be necessary to review such topics.

15 **Discussion Group, Group Presentation** The scratch-off card scenario is extended to include 5 cards. When students are asked to find the number of ways to have 2 winners, students will recognize that combinations are necessary.

SUGGESTED LEARNING STRATEGIES: Quickwrite, Activating Prior Knowledge, Discussion Group, Group Presentation

My Notes

11. Consider the question, "What is the probability that a customer has exactly 2 winning scratch-off cards out of 3 cards?" How does this question differ from the situation in Item 10?
The difference between "win, win, lose" and "exactly two winners" is that *order does not matter* in the second situation. Exactly two winners could be WWL, WLW, or LWW.

12. What is the probability that a customer has exactly 2 winning scratch-off cards out of 3 cards?
$\frac{4}{125} + \frac{4}{125} + \frac{4}{125} = \frac{12}{125} = 0.096$

13. In probability experiments, sometimes the order in which the events occur is important. At other times it is not. Which two of the previous situations are examples of a probability experiment where the order in which successive events occur is *not* important?
In Item 8 and Item 12, order does not matter.

14. What counting method can you use to determine the total number of possibilities in probability experiments where the order is not important?
combinations

In the fast food situation, suppose that a customer has collected 5 scratch-off cards and wants to know the probability that exactly 2 cards will be winners.

15. How many different ways can a customer have 2 winning cards out of the 5 scratch-off cards? Use the counting method you identified in Item 14.
$\binom{5}{2} = \frac{5!}{2!(5-2)!} = 10$

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252 SpringBoard® Mathematics with Meaning™ Algebra 2

SUGGESTED LEARNING STRATEGIES: Discussion Group, Group Presentation

16. List all of the different outcomes for having 2 winning cards out of the 5 scratch-off cards. One outcome is shown below. Does the number of outcomes in your list agree with your answer to Item 15?

WWLLL	WLWLL	WLLWL	WLLLW	LWWLL
LWLWL	LWLLW	LLWWL	LLWLW	LLLWW

Yes.

17. Find the probability of each of the possible outcomes listed in Item 16.

$\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right) = \left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3 = \frac{64}{3125} = 0.02048$

18. Use your answers to Items 15 and 17 to find the probability of a customer having exactly 2 winning cards out of 5 scratch-off cards.

$\binom{5}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3 = \frac{640}{3125} = 0.2048$

19. In Item 18, you found the probability of a customer having 2 winning cards out of 5 scratch-off cards. What are all the possible numbers of winning cards that a customer can have with 5 scratch-off cards?

The possible number of successes in the scratch-off card situation are 0, 1, 2, 3, 4, and 5. This means that out of 5 scratch-off cards, there could be zero winners, one winner, two winners, and so on.

My Notes

ACTIVITY 4.7 *Continued*

16 Discussion Group, Group Presentation Students may have difficulty listing all 10 outcomes. A group presentation may be an appropriate method for identifying all outcomes.

17 Discussion Group, Group Presentation Regardless of the outcome chosen, the factors of $\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)$, in any order, will yield a probability of $\frac{64}{3125}$.

18 Discussion Group, Group Presentation Combining the concepts of the previous three Items (the number of ways of having 2 winning tickets and the probability of each of those outcomes), students will multiply the probability by the number of favorable outcomes. In doing so, students have unwittingly used the formula for binomial probabilities.

19 Discussion Group, Group Presentation In order to construct the probability distribution, all possible numbers of winning tickets must be identified. Students are likely to miss the zero possibility.

20 Create Representations, Vocabulary Organizer In part (a), students will most likely duplicate their efforts from Items 17 and 18, this time for 3 winning tickets. This may lead students to believe that a generalization exists for the process. Part (b) is an opportunity to construct the complete probability distribution for this situation. Students can check to make sure that their distribution is correct by adding all the probabilities to see that the sum is 1.

21 The repetitive nature of completing the table with different discrete random variables in Item 20(b) should motivate students to look for a generalization of the process. This may yield the correct formula for some students, and the opportunity to share and compare responses will be valuable. If no students arrive at the correct formula, then take the opportunity to walk students through the process, making connections to previous Items that comprise the different components of the formula. Group presentation and whole class debriefing can assist in making the appropriate conclusions.

SUGGESTED LEARNING STRATEGIES: Create Representations, Vocabulary Organizer

My Notes

MATH TERMS

A *discrete random* variable may take on only a countable number of distinct values such as 0, 1, 2, 3, 4, ...

ACADEMIC VOCABULARY

A **probability distribution** describes the *values* and *probabilities* associated with a random event. The values must cover all of the possible outcomes of the event and the sum of all the probabilities must be exactly one.

20. In this situation, the number of winning scratch-off cards is called a **discrete random variable**, *X*. A **probability distribution of *X*** lists the possible number of successes *X* and their associated probabilities, *P*(*X*).

a. Find the probability of a customer having exactly 3 winning cards out of a total of 5 scratch-off cards. Show your method.

$P(3) = \binom{5}{3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2 = \frac{160}{3125} = 0.0512$

b. Complete the table below to create the probability distribution of *x* winning cards out of 5 scratch-off cards.

X (number of winning cards)	0	1	2	3	4	5
P(*X*) (the probability of exactly *X* winning cards out of 5 scratch-off cards)	$\frac{1024}{3125}$	$\frac{1280}{3125} = \frac{256}{625}$	$\frac{640}{3125} = \frac{128}{625}$	$\frac{160}{3125} = \frac{32}{625}$	$\frac{20}{3125} = \frac{4}{625}$	$\frac{1}{3125}$

21. Consider a binomial experiment with a probability of success equal to *p*. The notation *P*(*k*) represents the probability of *k* successes in *n* trials. Write an expression below that gives the value of *P*(*k*), for *n* trials.

$P(k) = \binom{n}{k}p^k(1-p)^{n-k}$

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254 SpringBoard® Mathematics with Meaning™ Algebra 2

SUGGESTED LEARNING STRATEGIES: Identify a Subtask, Quickwrite

My Notes

22. Remember that Bob and Janet have 8 children. Determine the following probabilities. Write your answers in the My Notes space. Show your work.

a. They have 3 children with Type O blood.
$P(3) = \binom{8}{3}(0.25)^3(0.75)^5 = \frac{13,608}{65,536} \approx 0.2076$

b. They have 5 children with Type O blood.
$P(5) = \binom{8}{5}(0.25)^5(0.75)^3 = \frac{1512}{65,536} \approx 0.0231$

c. They have 7 children with Type O blood.
$P(7) = \binom{8}{7}(0.25)^7(0.75)^1 = \frac{24}{65,536} \approx 0.00037$

23. Recall that the probability of a child of Bob and Janet having Type O blood is 25%.

a. Which probability from Item 22 was the largest?
The probability of having 3 children with Type O blood was greatest.

b. Find 25% of 8 and use this value to explain why your answer in Part a is reasonable.
25% of 8 is 2, and of the outcomes listed, 3 is closest to it. Since 3 is closest to 2, the expected value, one would expect that probability to be greater than that of 5 or 7.

c. Without calculating the probabilities, which of the following would have the largest probability among 8 children: 1 with Type O blood, 2 with Type O blood, or 3 with Type O blood. Explain your reasoning.
Two children with Type O blood is the greatest probability since it is the expected value.

24. Elisha and Ismael have 6 children. Elisha carries the genes for Type A antigens and Type B antigens. Ismael carries the genes for Type O antigens only. Given that the genes for Type O antigens are recessive to genes for Type A and Type B antigens, what is the probability that 3 of their children have Type B blood? Use the Punnett Square at the right to help answer this question.
$\binom{6}{3}(0.5)^3(0.5)^3 = \frac{20}{64} = 0.3125$

	A	B
O	AO	BO
O	AO	BO

ACTIVITY 4.7 *Continued*

22 Identify a Subtask Returning to the blood-type scenario, students are asked to find three specific probabilities. This problem gives students the opportunity to practice using the binomial probability formula in a different setting.

23 Identify a Subtask The purpose of this Item is to make a connection between expected value (25% of 8) and binomial probability. The probabilities closest to the expected value will be the greatest, and students are given the opportunity to address this concept in part (c). To reinforce the concept, students may wish to identify the binomial probabilities involved to verify their response.

24 Quickwrite This Item serves as a culminating activity in which students have an opportunity to practice. Some students may need assistance in completing the Punnett Square and finding the appropriate probabilities.

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Unit 4 • Polynomials 255

ACTIVITY 4.7 *Continued*

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 256, #4–6

UNIT 4 PRACTICE
p. 261, #49–50

CHECK YOUR UNDERSTANDING

1. Student responses may vary, but must include the following statements:
   - Six coin tosses represent the fixed number of trials of a binomial experiment.
   - Only two types of outcomes exist: heads or tails.
   - One coin toss does not affect the other tosses; therefore, the trials are independent.
   - The probability of success is $\frac{1}{2}$ for all trials.
2. ${}_6C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2 = \frac{15}{64} \approx 0.234$
3. $P(4) + P(5) + P(6) = \frac{15+6+1}{64} \approx 0.344$
4. ${}_8C_4\,(0.7)^4(0.3)^4 \approx 0.136$
5. $P(4) + P(5) + P(6) + P(7) + P(8) \approx 0.942$
6. Answers may vary. Students may indicate surprise at how binomials are related to probability and how many applications there are.

CHECK YOUR UNDERSTANDING

Write your answers on a separate sheet of notebook paper. Show your work.

1. Explain how tossing a fair coin 6 times is a binomial experiment.
2. Find the probability of getting 4 heads in 6 tosses of a fair coin.
3. Find the probability of getting 4 or more heads in 6 tosses of a fair coin.

An archer shoots 8 arrows at a target. Assume that each of her shots are independent and that each have the probability of hitting the bull's-eye of 0.7.

4. What is the probability that she hits the bull's-eye exactly 4 times?
5. What is the probability that she hits the bull's-eye at least 4 times?
6. MATHEMATICAL REFLECTION What did you learn from doing this investigation? What questions do you still have?

Combinations, Permutations, and Probability

THE WEDDING

Brad and Janet are getting married. They have a wedding party of 5, with 3 bridesmaids and 2 groomsmen.

1. In how many different ways can 4 of the 5 members of the wedding party line up for a photo?

2. If 3 from the group of 5 wedding party members are chosen at random for another picture, in how many ways can this be done?

Brad and Janet have family members plus 25 guests coming to the wedding. They plan on seating the family in the front row, but they will seat the rest of the guests randomly.

3. What is the probability that Janet's 2 best friends will be selected to sit in the first two available seats in the second row?

4. Brad has 7 close friends. What is the probability that 2 of his close friends sit in the first 2 available seats in the second row?

5. Use the Binomial Theorem to expand $(x + 3)^5$.

6. There are 2 types of wedding favors, a white candle and a black candle, being given to the guests. Each guest is equally likely to get either candle. If 10 people are given wedding favors, what is the probability that 7 people will receive black candles?

Embedded Assessment 3

Assessment Focus

- Permutations
- Combinations
- Probability
- Binomial expansion
- Binomial probability

Materials

- Graphing calculators

TEACHER TO TEACHER Students will work through the mathematical skills in the context of a wedding party. Due to the nature of the assessment, it is necessary for students to use a graphing calculator. If, however, they have one capable of binomial expansion, you may want to make Item 5 calculator-inactive.

Answer Key

1. 120

2. 10

3. $\frac{1}{300} = 0.00\overline{3}$

4. $\frac{21}{300} = 0.07$

5. $x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243$

6. $\frac{120}{1024} \approx 0.117$

Embedded Assessment 3

TEACHER TO TEACHER You may wish to read through the rubric with students and discuss the differences in the expectation levels. Make sure students understand the meanings of any terms used.

Embedded Assessment 3

Use after Activity 4.7.

Combinations, Permutations, and Probability

THE WEDDING

	Exemplary	Proficient	Emerging
Math Knowledge #5	The student correctly uses the Binomial Theorem. (5)	The student finds the correct product, but does not use the Binomial Theorem. OR The expansion is partially correct, using the Binomial Theorem.	The student gives an incorrect expansion.
Problem Solving #1, 2, 3, 4, 6	The student: • Gives the correct number of different ways. (1, 2) • States the correct probability. (3, 4, 6)	The student: • Gives the correct number of different ways for one of question 1 or 2, but not both. • States the correct probability for two of the three questions.	The student: • Gives the correct number of different ways for neither question. • States the correct probability at least one of the three questions.

ACTIVITY 4.1

Decide if each function is a polynomial. If it is, write the function in standard form. Then state the degree and leading coefficient.

1. $f(x) = 7x^2 - 9x^3 + 3x^7 - 2$
2. $f(x) = 2x^3 + x - 5^x + 9$
3. $f(x) = x^4 + x + 5 - \frac{1}{4}x^3$
4. $f(x) = -0.32x^3 + 0.08x^4 + 5x^{-1} - 3$

Describe the end behavior of each function.

5. $f(x) = -4x^4 + 5x^3 + 2x^2 - 6$
6. $f(x) = x^{13} + 7x^{12} - 13x^5 + 12x^2 - 6$
7. A cylindrical can is being designed for a new product. The height of the can plus twice its radius must be 45 cm.
   a. Find an equation that represents the volume of the can, given the radius.
   b. Find the radius that yields the maximum volume.
   c. Find the maximum volume of the can.

ACTIVITY 4.2

Find each sum or difference.

8. $(4x^3 + 14) + (5x^2 + x)$
9. $(2x^2 - x + 1) - (x^2 + 5x + 9)$
10. $(5x^2 - x + 10) + (12x - 1)$
11. $(7x^2 - 11x + 5) - (12x^2 + 8x + 19)$

Find each product.

12. $5x^2(4x^2 + 3x - 9)$
13. $(x + 2)(3x^3 - 8x^2 + 2x - 7)$

Find each quotient, using long division.

14. $\frac{x^4}{(x+1)^3}$
15. $(2x^3 - 3x^2 + 4x - 7) \div (x - 2)$

Find each quotient, using synthetic division.

16. $(2x^3 - 4x^2 - 15x + 4) \div (x + 3)$
17. $\frac{x^3 - x^2 - 14x + 11}{x - 2}$

ACTIVITY 4.3

18. Factor by grouping.
   a. $25x^3 + 5x^2 + 30x + 6$
   b. $28x^3 + 16x^2 - 21x - 12$
19. Use the pattern of a difference or a sum of cubes to factor each expression.
   a. $125x^9 + y^3$
   b. $x^3 - 216y^6$
20. Factor, using quadratic patterns.
   a. $x^4 - 7x^2 + 6$
   b. $x^4 - 4x^2 + 3$
   c. $x^6 - 100$
21. Find the zeros of each function by factoring and using the Zero Product Property.
   a. $f(x) = x^3 - 1331$
   b. $g(x) = -4x^3 + 20x^2 + 56x$
   c. $h(x) = 3x^3 - 36x^2 + 108x$
22. Write a polynomial function of n^{th} degree, given real or complex roots.
   a. $n = 4; x = -3, x = 2i, x = 4$
   b. $n = 3; x = -2, x = 1 + 2i$

UNIT 4 PRACTICE

Activity 4.1

1. yes, $f(x) = 3x^7 - 9x^3 + 7x^2 - 2$; degree 7, coefficient 3
2. no
3. yes, $f(x) = x^4 - \frac{1}{4}x^3 + x + 5$ degree 4, coefficient 1
4. no
5. As $x \to \pm\infty$, $f(x) \to -\infty$
6. As $x \to \infty$, $f(x) \to \infty$, and as $x \to -\infty$, $f(x) \to -\infty$

7a. $V = -2\pi r^3 + 45\pi r^2$

b. 15 cm

c. about 10,603 cm^3

Activity 4.2

8. $4x^3 + 5x^2 + x + 14$
9. $x^2 - 6x - 8$
10. $5x^2 + 11x + 9$
11. $-5x^2 - 19x - 14$
12. $20x^4 + 15x^3 - 45x^2$
13. $3x^4 - 2x^3 - 14x^2 - 3x - 14$
14. $x - 3 + \frac{6x^2 + 8x + 3}{x^3 + 3x^2 + 3x + 1}$
15. $2x^2 + x + 6 + \frac{5}{x - 2}$
16. $2x^2 - 10x + 15 - \frac{41}{x + 3}$
17. $x^2 + x - 12 - \frac{13}{x - 2}$

Activity 4.3

18a. $(5x^2 + 6)(5x + 1)$

b. $(4x^2 - 3)(7x + 4)$

19a. $(5x^3 + y)(25x^6 - 5x^3y + y^2)$

b. $(x - 6y^2)(x^2 + 6xy^2 + 36y^4)$

20a. $(x^2 - 6)(x + 1)(x - 1)$

b. $(x^2 - 3)(x + 1)(x - 1)$

c. $(x^3 - 10)(x^3 + 10)$

21a. $x = 11, x = -\frac{11}{2} \pm \frac{11\sqrt{3}}{2}i$

b. $x = 0, x = 7, x = -2$

c. $x = 0, x = 6$

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Unit 4 • Polynomials 259

22a. $x^4 - x^3 - 8x^2 - 4x - 48$

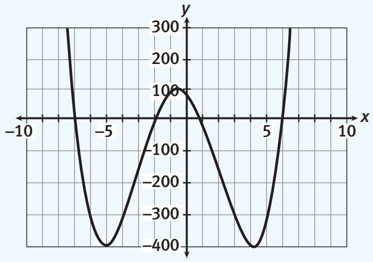
b. $x^3 + x + 10$

Activity 4.4

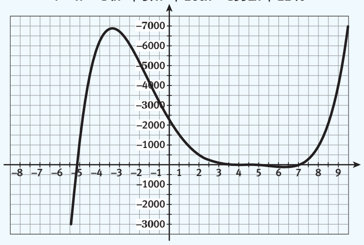
23. As $x \to -\infty$, $y \to -\infty$, as $x \to \infty$, $y \to \infty$

24. As $x \to -\infty$, $y \to \infty$, as $x \to \infty$, $y \to -\infty$

25.

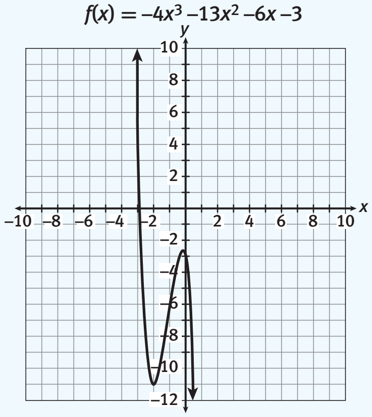


26. $Y = x^5 - 14x^4 + 37x^3 + 260x^2 - 1552x + 2240$



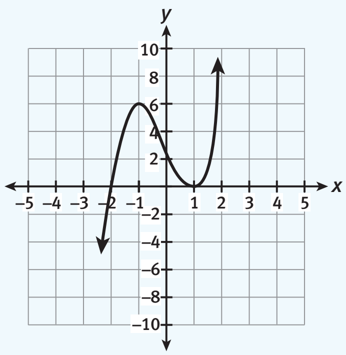
27. $\frac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 4}$, or $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 3, \pm\frac{3}{2}, \pm\frac{3}{4}$

28. $f(x) = -4x^3 - 13x^2 - 6x - 3$



29. 2 or 0 positive real zeros and 1 negative real zero

30.



UNIT 4 Practice

ACTIVITY 4.4

Determine the end behavior of each function.

23. $y = 4x^7 - 2x^3 + 8x + 6$

24. $y = -3x^{11} + 4x^9 - x^4 + 10x^3 + 9$

Use what you know about end behavior and zeros to graph each function.

25. $y = x^4 + 2x^3 - 43x^2 - 44x + 84$
$= (x - 1)(x - 6)(x + 2)(x + 7)$

26. $y = x^5 - 14x^4 + 37x^3 + 260x^2 - 1552x - 2240$
$= (x - 7)(x + 5)(x - 4)^3$

27. Determine all the possible rational zeros of $f(x) = -4x^3 - 13x^2 - 6x - 3$.

28. Graph $f(x) = -4x^3 - 13x^2 - 6x - 3$.

29. Determine the possible number of positive and negative real zeros for $h(x) = 2x^3 + x^2 - 5x + 2$.

30. Graph $h(x) = 2x^3 + x^2 - 5x + 2$.

31. Solve the inequality $-x^4 + 20x^2 - 32 \geq 32$.

ACTIVITY 4.5

32. In how many ways can the numbers 1, 2, 3, 4, and 5 be arranged without any of the numbers being repeated?

33. Find $\frac{8!}{4! \cdot 3!}$.

A basketball team of 5 players is being selected from a group of 20 players.

34. Use permutation notation to express the number of ways that the team can be selected.

35. Find the number of team configurations that can be selected.

36. Give the number of distinguishable permutations of the name JEANNETTE.

37. A pizzeria offers a vegetarian pizza with a choice of any three different vegetable toppings from a list of eight. How many different vegetarian pizzas can be ordered? Show your work.

ACTIVITY 4.6

Cards are drawn at random from a standard deck of 52 cards, without replacement.

38. If two cards are drawn, what is the probability that both cards are jacks?

39. If two cards are drawn, what is the probability that both cards are hearts?

40. If four cards are drawn, what is the probability that two cards are hearts?

Use this information for Items 41–42. A jar contains 50 marbles. Twenty are red, 10 are yellow and 20 are green.

41. What is the probability that if you draw 8 marbles from the jar without replacement, 5 are green?

42. What is the probability that if you draw 8 marbles from the jar without replacement, 6 are yellow?

43. Find the coefficient of the 3rd term in the expansion of $(x^2 + 2)^5$.

44. Find the 6th term in the expansion of $(4x - 3)^{10}$.

45. Use the Binomial Theorem to write the binomial expansion of $(3x - y)^5$.

31. $-4 \leq x \leq -2$, and $2 \leq x \leq 4$

Activity 4.5

32. 120

33. 280

34. ${}_{20}P_5 = \frac{20!}{(20-5)!} = 1{,}860{,}480$

35. $\frac{20!}{5!(20-5)!} = 15{,}504$

36. 15,120

37. ${}_8C_3 = \frac{8!}{3!5!} = 56$

ACTIVITY 4-7

Seventy-five percent of a population are children. A sample of 16 people is selected with replacement from the population.

46. Explain how this situation represents a binomial experiment.

47. Find the probability that 5 people selected from the sample are adults.

48. Find the probability that 7 people selected from the sample are adults.

49. What is the most likely number of adults in the sample?

50. What is the probability of getting exactly the number you found in Item 49?

UNIT 4 PRACTICE *Continued*

Activity 4.6

38. $\frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652} \approx 0.00452$

39. $\frac{13}{52} \cdot \frac{12}{51} = \frac{156}{2652} \approx 0.059$

40. $\frac{{}_{13}C_2 \cdot {}_{39}C_2}{{}_5C_4} = \frac{57{,}798}{270{,}725} \approx 0.213$

41. $\frac{{}_{20}C_5 \cdot {}_{30}C_3}{{}_{50}C_8} = \frac{62{,}946{,}240}{536{,}878{,}650}$
≈ 0.117

42. $\frac{{}_{10}C_6 \cdot {}_{40}C_2}{{}_{50}C_8} = \frac{163{,}800}{536{,}878{,}650}$
≈ 0.000303

43. 40

44. $252(4x)^5(-3)^5$
$= -62{,}705{,}664x^5$

45. $243x^5 - 405x^4y + 270x^3y^2 - 90x^2y^3 + 15xy^4 - y^5$

Activity 4.7

46. Student responses may vary, but must include the following statements.

- 16 people being chosen represent the fixed number of trials of a binomial experiment.
- Only two types of outcomes exist: Adult or child.
- Selecting one person does not affect selecting another; therefore, the trials are independent.
- The probability of success is 75% for children and 25% for adults for all trials.

47. ${}_{16}C_5\,(0.25)^5(0.75)^{11} \approx 0.18$

48. ${}_{16}C_7\,(0.25)^7(0.75)^9 \approx 0.052$

49. Selecting 4 adults is the most likely.

50. ${}_{16}C_4\,(0.25)^4(0.75)^{12} \approx 0.225$

Reflection

Student Reflection

Discuss the essential questions with students. Have them share how their understanding of the questions has changed through studying the concepts in the unit.

Review the academic vocabulary. You may want students to revisit the graphic organizers they have completed for academic vocabulary terms and add other notes about their understanding of terms.

Encourage students to evaluate their own learning and to recognize the strategies that work best for them. Help them identify key concepts in the unit and to set goals for addressing their weaknesses and acquiring effective learning strategies.

Teacher Reflection

1. Of the key concepts in the unit, did any present special challenges for students?
2. How will you adjust your future instruction for students/activities?
3. Which strategies were most effective for facilitating student learning?
4. When you teach this unit again, what will you do differently?

UNIT 4

Reflection

An important aspect of growing as a learner is to take the time to reflect on your learning. It is important to think about where you started, what you have accomplished, what helped you learn, and how you will apply your new knowledge in the future. Use notebook paper to record your thinking on the following topics and to identify evidence of your learning.

Essential Questions

1. Review the mathematical concepts and your work in this unit before you write thoughtful responses to the questions below. Support your responses with specific examples from concepts and activities in the unit.
   - How do polynomial functions help to model real-world behavior?
   - How is probability used in real-world settings?

Academic Vocabulary

2. Look at the following academic vocabulary words:

   - combination
   - end behavior
   - extrema
   - factorial
   - permutation
   - polynomial function
   - probability distribution

   Choose three words and explain your understanding of each word and why each is important in your study of math.

Self-Evaluation

3. Look through the activities and Embedded Assessments in this unit. Use a table similar to the one below to list three major concepts in this unit and to rate your understanding of each.

Unit Concepts	Is Your Understanding Strong (S) or Weak (W)?
Concept 1	
Concept 2	
Concept 3	

   a. What will you do to address each weakness?
   b. What strategies or class activities were particularly helpful in learning the concepts you identified as strengths? Give examples to explain.

4. How do the concepts you learned in this unit relate to other math concepts and to the use of mathematics in the real world?

Math Standards Review

Unit 4

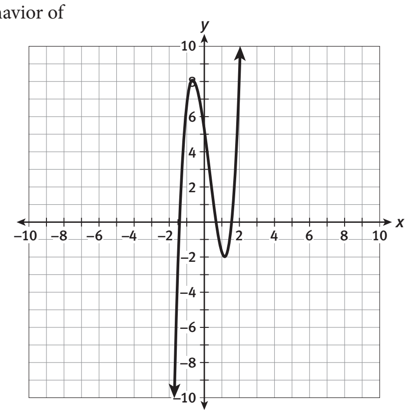
1. Given the graph of $f(x) = 4x^3 - 3x^2 - 8x + 5$, which statement about the end behavior of $f(x) = 4x^3 - 3x^2 - 8x + 5$ is true?

A. $f(x) \to +\infty$ as $x \to 0$

B. $f(x) \to +\infty$ as $x \to -\infty$

C. $f(x) \to -\infty$ as $x \to +\infty$

D. $f(x) \to -\infty$ as $x \to -\infty$

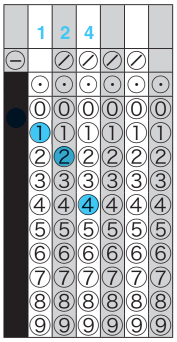


1. Ⓐ Ⓑ Ⓒ Ⓓ

2. What is the remainder for the following?

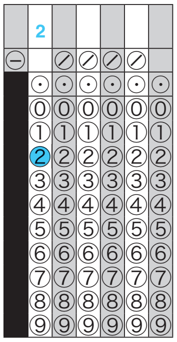
$(5x^3 - 2x^2 + 7) \div (x - 3)$

2.



3. Given the function $f(x) = x^3 - 2x^2 - x + 2$, what is the **sum** of the zeros of the function?

3.



UNIT 4 Math Standards Review

These two pages provide practice with three standardized test question formats that are used in many national and state high-stakes tests:

- Multiple choice
- Gridded response
- Extended response

These items also provide practice with the mathematics content of this unit.

1 Multiple choice

- Graph polynomial functions
- End behavior

2 Gridded response

- Dividing polynomials

3 Gridded response

- Zeros of a polynomial function
- Theorems of polynomial behavior

4 Extended response

- Using polynomial equations to solve real-world problems
- Characteristics of polynomial functions
- Characteristics of graphs of polynomial functions

UNIT 4 Math Standards Review

TEACHER TO TEACHER You might read through the extended-response item with students and discuss your expectation levels. Make sure students understand the meanings of any terms used.

Math Standards Review

Unit 4 (continued)

Read
Solve
Explain

4. An object is projected vertically upward from ground level with a velocity of 352 feet per second. The height ***h*** after ***t*** seconds is given by the function below:

$$h(t) = -16t^2 + 352t$$

a. Find when the object reaches the maximum height and determine that height. Show work to support your answer.

Answer and Explain

a. After 11 seconds, the object will reach its maximum height.

b. The maximum height is 1936 ft.

b. Give the interval(s) of time over which the height is increasing and the interval(s) of time over which it is decreasing.

Answer and Explain

The object is increasing in height over the interval from 0 to 11 seconds; it is decreasing in height over the interval from 11 seconds to 22 seconds.

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264 SpringBoard® Mathematics with Meaning™ Algebra 2