Name____Period___________Date_____________

Test 4.1 Conic Sections (10.2 to 10.4) ______/81

Parabola	Ellipse	Hyperbola
$(x-h)^2 = 4p(y-k)$	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
$(y-k)^2 = 4p(x-h)$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
	$c^2 = a^2 - b^2$ $\quad e = \frac{c}{a}$	$c^2 = a^2 + b^2$ $\quad e = \frac{c}{a}$

<u>Hyperbolas, ellipses, and parabolas…oh my!</u>

1. (7 pts total) Find the standard form of the equation of the parabola with the following characteristics. Sketch the graph.
Vertex: (3, -1)
Focus: (3, 1)

Sketch of graph. Include vertex, focus and directrix (3 pts)

2. (7pts total) Find the standard form of the equation of a hyperbola with the given characteristics. Sketch the graph.
Vertices: (0, -1) and (8, -1)
Foci: (-1, -1) and (9, -1)

Sketch of graph. Include vertices, foci, and asymptotes. (3 pts)

3. (11 pts) Determine whether the following equation represents a circle or an ellipse. Find the center, foci, vertices, and eccentricity.

$16x^2 + 16y^2 - 64x + 32y + 55 = 0$

Center:______________

Foci:___________ and ___________

Vertices: __________ and __________

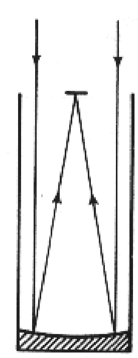
and _________ and ________

Eccentricity:________________

4. Classify the graph of the equation as a circle, ellipse, hyperbola, or parabola. Find the standard form of each and state how you know.

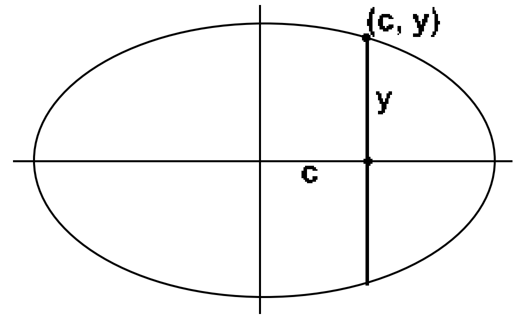
$x^2 - 4x - 8y + 28 = 0$	$x^2 - 4y^2 - 2x + 16y - 20 = 0$
Type of conic and why (2 pts):	Type of conic and why (2 pts):
Work for conversion to vertex form (4 pts)	Work for conversion to vertex form (4 pts)
Vertex Form (2pts)	Vertex Form (2pts)

5. (5 pts) The Hale telescope at the Mount Palomar Observatory has a parabolic mirror that is 200 inches wide. The shape of the mirror allows all light from the stars to be collected at one point called the prime focus. The mirror is only 3.79 inches deep at its center. Find the focal length.



A hint of proofiness

6. (9 pts) A line segment through a focus of an ellipse with endpoints on the ellipse and perpendicular to the major axis is called a **latus rectum** of the ellipse. This means an ellipse has two latus recta! (eww) The length of these lines helps in finding additional points on an ellipse. Show that the length of one latus rectum is equal to $\frac{2b^2}{a}$ by completing the chart below.



Algebraic Step	Justification/Description of step
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	Start with the equation of an ellipse
$\frac{\cancel{a^2}b^2\, x^2}{\cancel{a^2}} + \frac{a^2\cancel{b^2}\, y^2}{\cancel{b^2}} = a^2b^2 \cdot 1$	Multiply by a^2b^2 to remove the denominators
	Result after multiplying by a^2b^2
	Solve for y^2
	Factor out b^2
	The value of *x* in this situation is *c*, so replace *x* with *c*
$y^2 = \frac{b^2(b^2)}{a^2}$	
	Simplify the numerator
	The length is 2 times the value of *y*

True or False

7. (5 pts) A circle has no foci. Why or why not?	8. (5 pts) If the vertex and focus of a parabola are on a horizontal line, then the directrix of the parabola is vertical. Why or why not?

To infinity and beyond!

9. (8pts) On the ellipse word problem handout, we plotted the path of Haley's Comet, which moves similar to a planet's orbit. Like planetary orbits, Halley's Comet has an elliptical orbit with the sun as a focus. We found the path of Haley's Comet can be modeled by the following equation if the center is at the origin: $\frac{x^2}{321.84}+\frac{y^2}{20.89}=1$

The eccentricity of the Earth's orbit is 0.0167, and the earth has a major axis length of 2 astronomical units. Compare the path of Haley's Comet to the path of Earth's orbit by graphing the path of both orbits on the same coordinate plane. **Place the center of Earth's orbit at the origin.** Haley's comet does not have the same center as the Earth, but they do have the same focus which is the sun. Don't forget to label the location of the sun and the coordinates of all vertices.

Bonus (You can only receive credit if all other questions have been answered)(5pts) The first artificial satellite to orbit Earth was Sputnik I which was launched by the former Soviet Union in 1957. The highest point from the Earth's surface was 947 km, and its lowest point was 228 km. The center of the Earth was the focus of the elliptical orbit, and the radius of the Earth is approximately 6378 km. How circular is the orbit? Justify your answer mathematically.