Indirect Proof/ Proof by Contradiction

How to prove what your name is...

An indirect proof, also known as a proof by contradiction, is typically used in Geometry to prove statements that are fairly obvious. For example, suppose you wanted to prove your first name. That is really tricky because it is so obvious, how would you do it? Well a two column proof is not really an option because there are really no facts to start with.... I suppose you could produce your birth certificate. We all know how difficult that can be to track down. Here is what you could do. You could pretend for a minute that your name is not your name, then you could demonstrate why this isn't true (Like say "well everyone calls me that," "My mom calls me that so that's not true" etc.). So if the opposite isn't true then the original must be... confused? Let's try another one...

Prove that any triangle has three sides.

First: Assume that a triangle does not have three sides.

Second: Demonstrate that step one is false. That's easy! The definition of a triangle is that it has three sides. So it's false.

So what you have done is prove that the opposite is false, because you did this so skillfully it means that the original statement "any triangle has three sides" must be true.

Let's try one more...

Prove: Perpendicular lines intersect at right angles.

Step 1. Perpendicular lines don't intersect at right angles.

Step 2. This violates the definition of perpendicular angles.

It seems too easy, right? Sometimes you have to be more creative. Let's do one more...

Prove: A triangle can have at most one right angle.

Step1. A triangle can have two right angles. (This is the opposite because two is more than one.)

Step 2. If it does then 90+90=180. If there is a third angle then this violates the triangle sum theorem which says that all three angles of a triangle add to 180. So, the statement is false.

Sometimes you have to show why something is false instead of, or in addition to, using a defintion as you can see.

Sometimes it is hard to tell exactly how to write the opposite. Here are some guidelines. If it says a statement with no quantity like, "Only two points define a line." Write "it doesn't" or "they don't." If it says "exactly some number," write some other number or more than or less than. If it says "at most," write one more than the number. If it says "at least," write one less.

Are you ready to try some on your own?

Let's practice! (whoever you are....)

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Choose the choice that is the first step in each proof by contradiction.

1. Exactly 3 points define a plane.

A. A plane contains at least 3 points.
B. A Plane contains at least one line.
C. A plane is a two dimensional surface.
D. Two points define a plane.

2. A rhombus has 2 pairs of congruent sides.

A. A rhombus is a quadrilateral.
B. A rhombus has only one pair of congruent sides.
C. A plane is a two dimensional surface.
D. Two points define a plane.

3. An octagon has 8 sides.

A. An octagon is not a polygon.
B. An octagon is a polygon.
C. An octagon doesn't have 8 sides.
D. An octagon is always regular.

4. A triangle with one right angle is a right triangle.

A. A triangle with one right angle is not a right triangle.
B. A triangle with one obtuse angle is an obtuse triangle.
C. Right triangles are always isoceles.
D. Right triangles are all congruent.

5. An equilateral triangle has exactly 3 congruent sides.

A. An equilateral triangle only has two congruent angles.
B. All equilateral triangles are similar.
C. An equilateral triangle has 3 congruent angles.
D. An equilateral triangle has only two congruent sides.

6. A square has exactly 4 right angles.

A. A square has exactly 4 congruent sides.
B. A square is a quadrilateral.
C. A square has only 3 right angles.
D. A square is also a rhombus.

7. All of the radii in a circle are the same length.

A. The radii in a circle are different lengths.
B. The diameter of a circle is twice the radius.
C. A circle has only one radius.
D. A diameter of a circle is half the radius.

8. Two congruent triangles have the same area.

A. No two triangles have the same area.
B. The area of a triangle is one half base times height.
C. Triangles are congruent if their areas are equal.
D. Two congruent triangles have different areas.

9. All circles with the same radius are congruent.

A. Circles with the same radius are not congruent.
B. All circles are congruent.
C. All circles are similar.
D. Congruent shapes are also similar.

10. A scalene triangle has three sides of different lengths.

A. Scalene triangles are congruent.
B. Scalene triangles have two or more sides that are the same length.
C. All triangles are scalene.
D. The sum of the interior angles or a triangle is 180°.

Bubble the correct answer choice from each item above.

#1.	#2.	#3.	#4.	#5.	#6.	#7.	#8.	#9.	#10.
○A.	○A.	○A.	○A.	○A.	○A.	○A.	○A.	○A.	○A.
○B.	○B.	○B.	○B.	○B.	○B.	○B.	○B.	○B.	○B.
○C.	○C.	○C.	○C.	○C.	○C.	○C.	○C.	○C.	○C.
○D.	○D.	○D.	○D.	○D.	○D.	○D.	○D.	○D.	○D.

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Choose the choice that is the second step in each proof by contradiction.

1. An equilateral triangle has exactly 3 congruent sides.

   Step 1: An equilateral triangle has 2 or less congruent sides.

   A. The definition of an equilateral triangle is a triangle with 3 congruent sides.
   B. All equilateral triangles are similar.
   C. An equilateral triangle has 3 congruent angles.
   D. The sum of the measures of a triangle is 180°.

2. If two sides of a triangle are congruent then the triangle is isosceles.

   Step 1: A triangle with two congruent sides is not isosceles .

   A. Isosceles triangles have two congruent angles.
   B. The definition of an isosceles triangle is that it has two conguent sides.
   C. An isosceles triangle is regular.
   D. Isosceles triangles are not equilateral.

3. A triangle can contain at most 1 obtuse angle.

   Step 1: A triangle has 2 obtuse angles.

   A. A right triangle has one right angle.
   B. The sum of the angles in a triangle is 180°.
   C. The longest side of a triangle is opposite the largest angle.
   D. The sum of two sides of a triangle must be greater than the third side.

4. The three lengths 4in, 5in, and 2in make a triangle.

   Step 1: The three lengths 4in, 5in, and 2in don't make a triangle.

   A. A triangle has 3 sides.
   B. The sum of the angles in a triangle is 180°.
   C. The longest side of a triangle must be greater than the sum of the two shorter sides.
   D. The sum of two sides of a triangle must be greater than the third side.

5. Parallelogram ABCD has two pairs of parallel sides.

   Step 1: Parallelogram ABCD has only one pair of parallel lines.

   A. A parallelogram has exactly two pairs of parallel lines.
   B. A parallelogram is a quadrilateral.
   C. A parallelogram has 4 congruent sides.
   D. A parallelogram is also a trapezoid.

6. Triangle ABC is isosceles if AB≅BC.

   Step 1: Triangle ABC is not isosceles if AB≅BC.

   A. All isosceles triangles are similar.
   B. The sum of the angles in a triangle is 180°.
   C. An isosceles triangle has two pairs of congruent angles.
   D. An isosceles triangle has two pairs of congruent sides.

Bubble the correct answer choice from each item above.

#1.	#2.	#3.	#4.	#5.	#6.
○A.	○A.	○A.	○A.	○A.	○A.
○B.	○B.	○B.	○B.	○B.	○B.
○C.	○C.	○C.	○C.	○C.	○C.
○D.	○D.	○D.	○D.	○D.	○D.

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Write just the first step in a proof by contradiction for each one of these...

1. An angle is obtuse if it is greater than 90°.

2. A right angle measures exactly 90°.

3. A square has exactly four sides.

4. Parallel lines in a plane never intersect.

5. Two angles that are supplementary add to 180°.

6. A polygon has at least 3 sides.

7. A school is a place you can go to learn things.

8. A parallelogram has exactly two pairs of parallel sides.

9. Lines in a plane intersect in only one point.

10. The area of a triangle is one half its base.

Prove each of these by contradiction...

11. A triangle has exactly 3 sides.

12. Lines in a plane intersect in only one point.

13. A trapezoid has exactly one pair of parallel sides.

14. An acute angle measures less than 90°.

15. Complementary angles add to 90°.

16. A triangle has at most one obtuse angle.

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