CHAPTER 12

Extending Transformational Geometry

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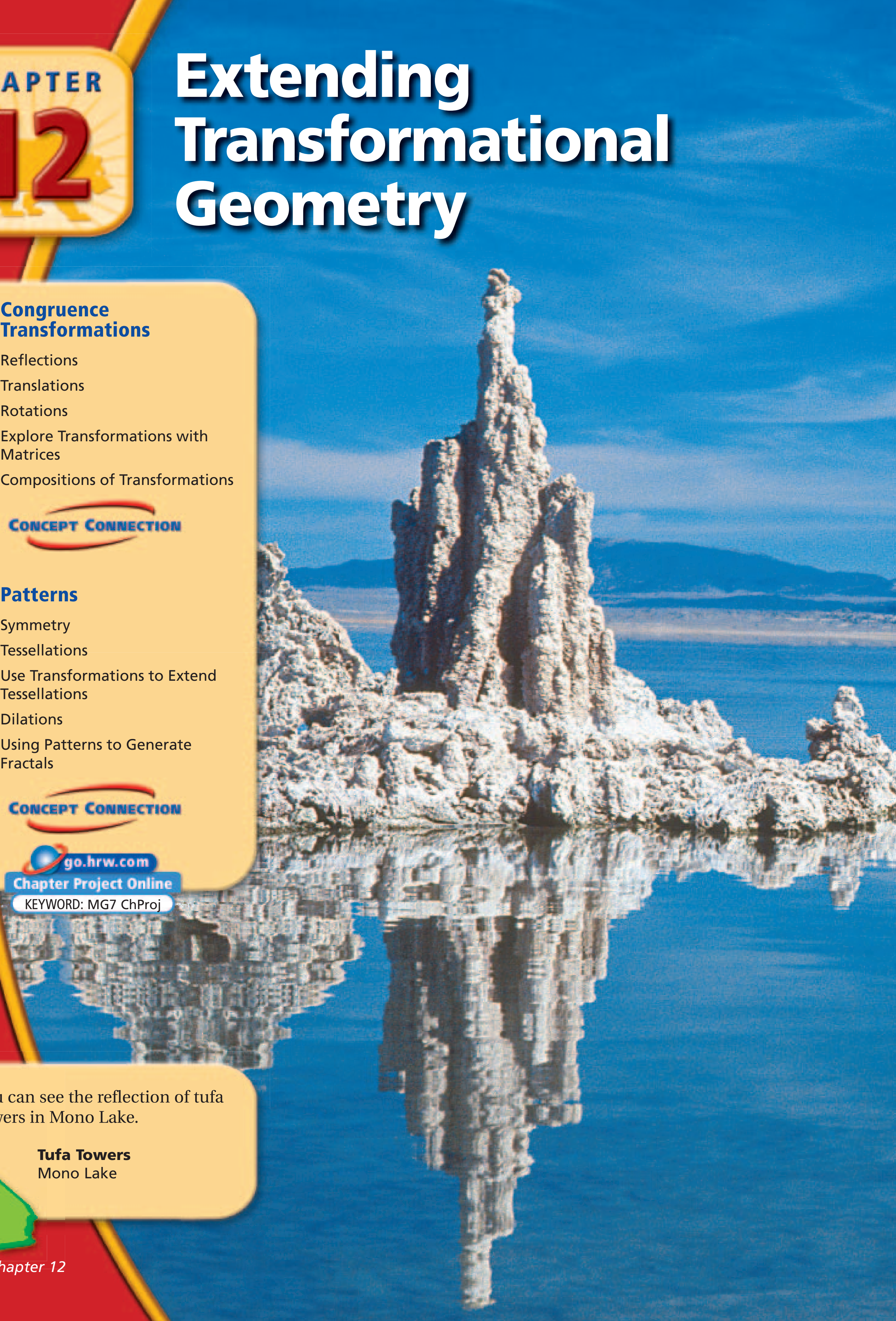
Chapter Project Online
KEYWORD: MG7 ChProj



You can see the reflection of tufa towers in Mono Lake.

Tufa Towers
Mono Lake

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. image
2. preimage
3. transformation
4. vector

A. a mapping of a figure from its original position to a new position

B. a ray that divides an angle into two congruent angles

C. a shape that undergoes a transformation

D. a quantity that has both a size and a direction

E. the shape that results from a transformation of a figure

Ordered Pairs

Graph each ordered pair.

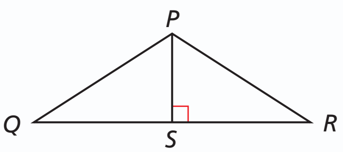
5. $(0, 4)$ **6.** $(-3, 2)$ **7.** $(4, 3)$

8. $(3, -1)$ **9.** $(-1, -3)$ **10.** $(-2, 0)$

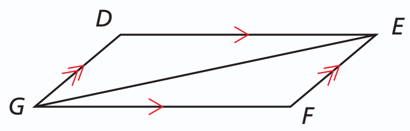
Congruent Figures

Can you conclude that the given triangles are congruent? If so, explain why.

11. $\triangle PQS$ and $\triangle PRS$



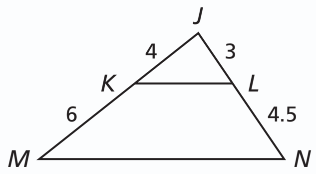
12. $\triangle DEG$ and $\triangle FGE$



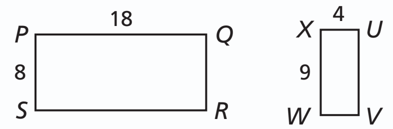
Identify Similar Figures

Can you conclude that the given figures are similar? If so, explain why.

13. $\triangle JKL$ and $\triangle JMN$



14. rectangle $PQRS$ and rectangle $UVWX$



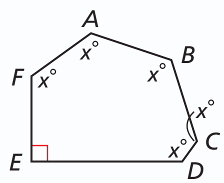
Angles in Polygons

15. Find the measure of each interior angle of a regular octagon.

16. Find the sum of the interior angle measures of a convex pentagon.

17. Find the measure of each exterior angle of a regular hexagon.

18. Find the value of x in hexagon $ABCDEF$.



CHAPTER 12

Unpacking the Standards

The information below "unpacks" the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

California Standard	Academic Vocabulary	Chapter Concept
8.0 **Students know,** derive, **and solve problems involving the perimeter,** circumference, **area,** volume, lateral area, and surface area **of common geometric figures.** (Lesson 12-7)	**common geometric figures** figures formed with straight lines and/or simple shapes, for example, rectangles, squares, and circles	You learn how to identify and draw dilations of figures. You also find the perimeters and areas of the image and preimage of the figures.
11.0 Students determine how changes in dimensions affect the perimeter, area, and volume **of common geometric figures** and solids. (Lesson 12-7)	**determine** find out **dimensions** sizes of objects	You find the scale factor of a dilation. Then you determine the effect on the perimeter and area of the image after the measurements of the preimage have been multiplied by a scale factor.
22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections. (Lessons 12-1, 12-2, 12-3, 12-4, 12-5, 12-6) (Labs 12-3, 12-6)	**effect** outcome **rigid motions** movements of a figure that do not change its shape or size	You identify and draw reflections, translations, and rotations of two- and three-dimensional objects. You learn that the image of a figure is congruent to the preimage after one or more of these transformations.

Standards 1.0 and 16.0 are also covered in this chapter. To see these standards unpacked, go to Chapter 1, p. 4.

Reading and Writing Math

Study Strategy: Prepare for Your Final Exam

Math is a cumulative subject, so your final exam will probably cover all of the material you have learned since the beginning of the course. Preparation is essential for you to be successful on your final exam. It may help you to make a study timeline like the one below.

2 weeks before the final:

- Look at previous exams and homework to determine areas I need to focus on; rework problems that were incorrect or incomplete.
- Make a list of all formulas, postulates, and theorems I need to know for the final.
- Create a practice exam using problems from the book that are similar to problems from each exam.

1 week before the final:

- Take the practice exam and check it. For each problem I miss, find two or three similar ones and work those.
- Work with a friend in the class to quiz each other on formulas, postulates, and theorems from my list.

1 day before the final:

- Make sure I have pencils, calculator (check batteries!), ruler, compass, and protractor.

Try This

1. Create a timeline that you will use to study for your final exam.

12-1 Reflections

Objective
Identify and draw reflections.

Vocabulary
isometry

Who uses this?
Trail designers use reflections to find shortest paths. (See Example 3.)

An **isometry** is a transformation that does not change the shape or size of a figure. Reflections, translations, and rotations are all isometries. Isometries are also called *congruence transformations* or *rigid motions*.

Recall that a reflection is a transformation that moves a figure (the preimage) by flipping it across a line. The reflected figure is called the image. A reflection is an isometry, so the image is always congruent to the preimage.

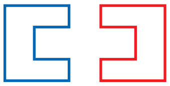
Remember!
To review basic transformations, see Lesson 1-7, pages 50–55.

California Standards
22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, **including** rotations, translations, and **reflections.**

EXAMPLE 1 Identifying Reflections

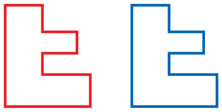
Tell whether each transformation appears to be a reflection. Explain.

A



Yes; the image appears to be flipped across a line.

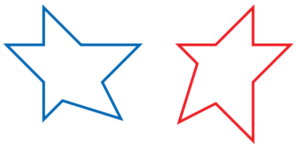
B



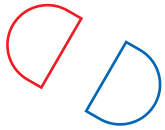
No; the figure does not appear to be flipped.

CHECK IT OUT! **Tell whether each transformation appears to be a reflection.**

1a.

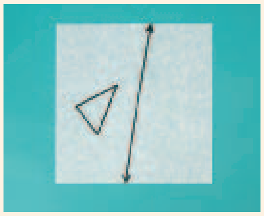


1b.



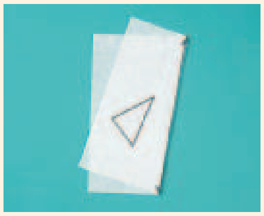
Construction Reflect a Figure Using Patty Paper

1



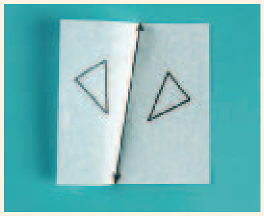
Draw a triangle and a line of reflection on a piece of patty paper.

2



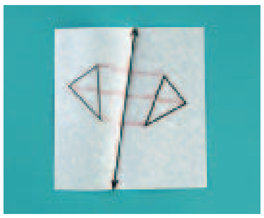
Fold the patty paper back along the line of reflection.

3



Trace the triangle. Then unfold the paper.

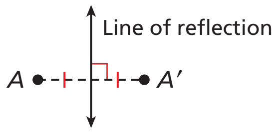
Draw a segment from each vertex of the preimage to the corresponding vertex of the image. Your construction should show that the line of reflection is the perpendicular bisector of every segment connecting a point and its image.



Reflections

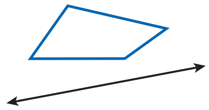
Know it! Note

A reflection is a transformation across a line, called the line of reflection, so that the line of reflection is the perpendicular bisector of each segment joining each point and its image.



EXAMPLE 2 **Drawing Reflections**

Math Builders
For more on reflections, see the Transformation Builder on page MB2.

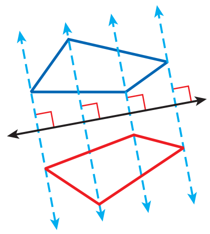


Copy the quadrilateral and the line of reflection. Draw the reflection of the quadrilateral across the line.

Step 1 Through each vertex draw a line perpendicular to the line of reflection.

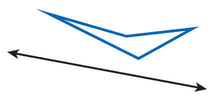
Step 2 Measure the distance from each vertex to the line of reflection. Locate the image of each vertex on the opposite side of the line of reflection and the same distance from it.

Step 3 Connect the images of the vertices.



CHECK IT OUT!

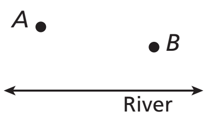
2. Copy the quadrilateral and the line of reflection. Draw the reflection of the quadrilateral across the line.



EXAMPLE 3 ***Problem-Solving Application***

PROBLEM SOLVING

A trail designer is planning two trails that connect campsites *A* and *B* to a point on the river. He wants the total length of the trails to be as short as possible. Where should the trail meet the river?



1 Understand the Problem

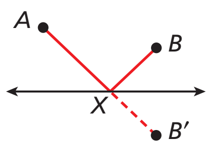
The problem asks you to locate point X on the river so that $AX + XB$ has the least value possible.

2 Make a Plan

Let B' be the reflection of point B across the river. For any point X on the river, $\overline{XB'} \cong \overline{XB}$, so $AX + XB = AX + XB'$. $AX + XB'$ is least when A, X, and B' are collinear.

3 Solve

Reflect B across the river to locate B'. Draw $\overline{AB'}$ and locate X at the intersection of $\overline{AB'}$ and the river.



4 Look Back

To verify your answer, choose several possible locations for X and measure the total length of the trails for each location.

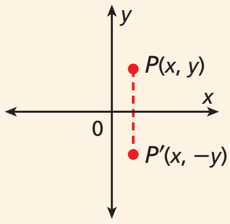
CHECK IT OUT!

3. What if...? If A and B were the same distance from the river, what would be true about $\overline{AX}$ and $\overline{BX}$?

Know it! Note

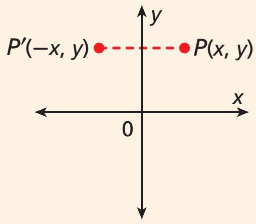
Reflections in the Coordinate Plane

ACROSS THE x-AXIS



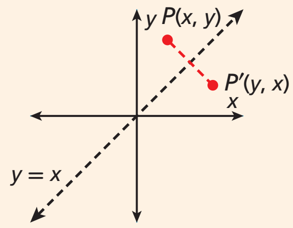
$(x, y) \rightarrow (x, -y)$

ACROSS THE y-AXIS



$(x, y) \rightarrow (-x, y)$

ACROSS THE LINE $y = x$



$(x, y) \rightarrow (y, x)$

EXAMPLE 4 Drawing Reflections in the Coordinate Plane

Reflect the figure with the given vertices across the given line.

A $M(1, 2), N(1, 4), P(3, 3)$; y-axis

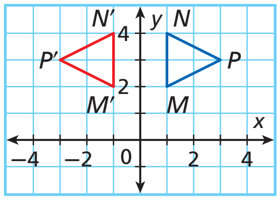
The reflection of (x, y) is $(-x, y)$.

$M(1, 2) \rightarrow M'(-1, 2)$

$N(1, 4) \rightarrow N'(-1, 4)$

$P(3, 3) \rightarrow P'(-3, 3)$

Graph the preimage and image.



B $D(2, 0), E(2, 2), F(5, 2), G(5, 1)$; $y = x$

The reflection of (x, y) is (y, x).

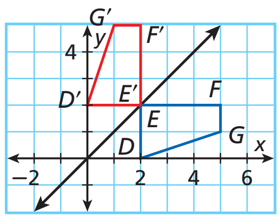
$D(2, 0) \rightarrow D'(0, 2)$

$E(2, 2) \rightarrow E'(2, 2)$

$F(5, 2) \rightarrow F'(2, 5)$

$G(5, 1) \rightarrow G'(1, 5)$

Graph the preimage and image.



CHECK IT OUT!

4. Reflect the rectangle with vertices $S(3, 4)$, $T(3, 1)$, $U(-2, 1)$, and $V(-2, 4)$ across the x-axis.

THINK AND DISCUSS

1. Acute scalene $\triangle ABC$ is reflected across $\overline{BC}$. Classify quadrilateral $ABA'C$. Explain your reasoning.

2. Point A' is a *reflection* of point A across line ℓ. What is the relationship of ℓ to $\overline{AA'}$?

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer.

Line of Reflection	Image of (a, b)	Example
x-axis		
y-axis		
$y = x$		

12-1 Exercises

California Standards
2.0, 8.0, 11.0, 16.0, 19.0, 22.0, 7MG3.2

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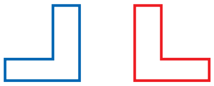
Parent Resources Online
KEYWORD: MG7 Parent

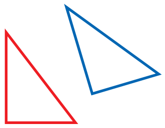
GUIDED PRACTICE

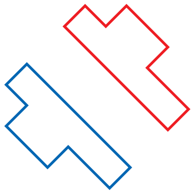
1. **Vocabulary** If a transformation is an *isometry*, how would you describe the relationship between the preimage and the image?

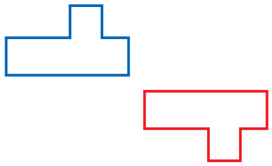
SEE EXAMPLE 1
p. 824

Tell whether each transformation appears to be a reflection.

2. 

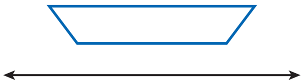
3. 

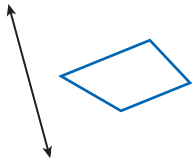
4. 

5. 

SEE EXAMPLE 2
p. 825

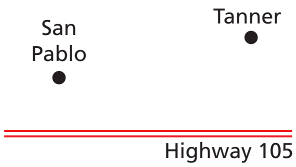
Multi-Step **Copy each figure and the line of reflection. Draw the reflection of the figure across the line.**

6. 

7. 

SEE EXAMPLE 3
p. 825

8. **City Planning** The towns of San Pablo and Tanner are located on the same side of Highway 105. Two access roads are planned that connect the towns to a point *P* on the highway. Draw a diagram that shows where point *P* should be located in order to make the total length of the access roads as short as possible.



SEE EXAMPLE 4
p. 826

Reflect the figure with the given vertices across the given line.

9. $A(-2, 1)$, $B(2, 3)$, $C(5, 2)$; x-axis
10. $R(0, -1)$, $S(2, 2)$, $T(3, 0)$; y-axis
11. $M(2, 1)$, $N(3, 1)$, $P(2, -1)$, $Q(1, -1)$; $y = x$
12. $A(-2, 2)$, $B(-1, 3)$, $C(1, 2)$, $D(-2, -2)$; $y = x$

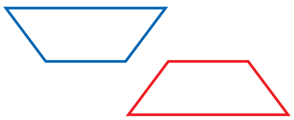
PRACTICE AND PROBLEM SOLVING

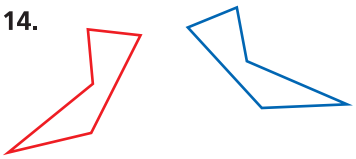
Independent Practice

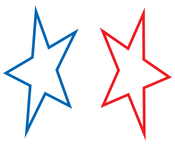
For Exercises	See Example
13–16	1
17–18	2
19	3
20–23	4

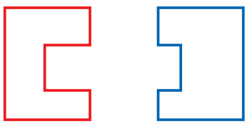
Extra Practice
Skills Practice p. S26
Application Practice p. S39

Tell whether each transformation appears to be a reflection.

13. 

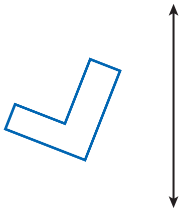
14. 

15. 

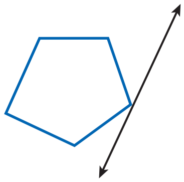
16. 

Multi-Step Copy each figure and the line of reflection. Draw the reflection of the figure across the line.

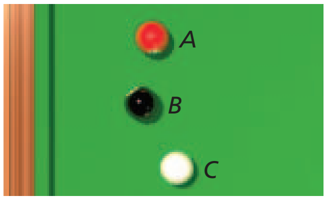
17.



18.



19. **Recreation** Cara is playing pool. She wants to hit the ball at point *A* without hitting the ball at point *B*. She has to bounce the cue ball, located at point *C*, off the side rail and into her ball. Draw a diagram that shows the exact point along the rail that Cara should aim for.



Reflect the figure with the given vertices across the given line.

20. $A(-3, 2)$, $B(0, 2)$, $C(-2, 0)$; y-axis

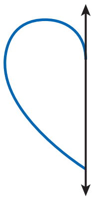
21. $M(-4, -1)$, $N(-1, -1)$, $P(-2, -2)$; $y = x$

22. $J(1, 2)$, $K(-2, -1)$, $L(3, -1)$; x-axis

23. $S(-1, 1)$, $T(1, 4)$, $U(3, 2)$, $V(1, -3)$; $y = x$

Copy each figure. Then complete the figure by drawing the reflection image across the line.

24.

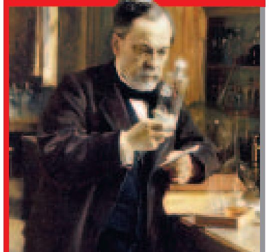


25.

26.

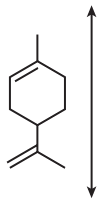
LINK

Chemistry



Louis Pasteur (1822–1895) is best known for the pasteurization process, which kills germs in milk. He discovered chemical chirality when he observed that two salt crystals were mirror images of each other.

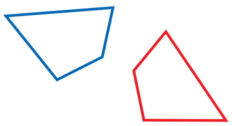
27. **Chemistry** In chemistry, *chiral* molecules are mirror images of each other. Although they have similar structures, chiral molecules can have very different properties. For example, the compound R-(+)-limonene smells like oranges, while its mirror image, S-(−)-limonene, smells like lemons. Use the figure and the given line of reflection to draw S-(−)-limonene.



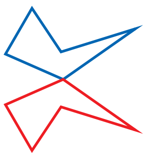
R-(+)-limonene

Each figure shows a preimage and image under a reflection. Copy the figure and draw the line of reflection.

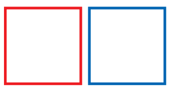
28.



29.



30.



Use arrow notation to describe the mapping of each point when it is reflected across the given line.

31. $(5, 2)$; x-axis

32. $(-3, -7)$; y-axis

33. $(0, 12)$; x-axis

34. $(-3, -6)$; $y = x$

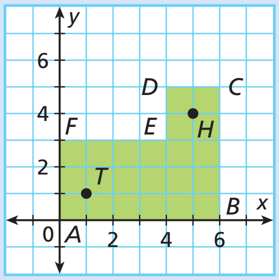
35. $(0, -5)$; $y = x$

36. $(4, 4)$; $y = x$

Concept Connection

37. This problem will prepare you for the Concept Connection on page 854.

The figure shows one hole of a miniature golf course.



a. Is it possible to hit the ball in a straight line from the tee T to the hole H?

b. Find the coordinates of H', the reflection of H across $\overline{BC}$.

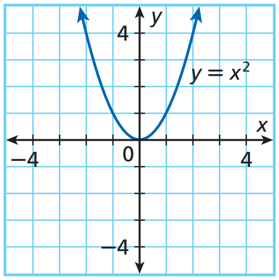
c. The point at which a player should aim in order to make a hole in one is the intersection of $\overline{TH'}$ and $\overline{BC}$. What are the coordinates of this point?

38. **Critical Thinking** Sketch the next figure in the sequence below.

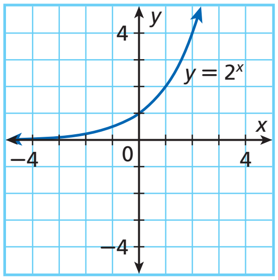
39. **Critical Thinking** Under a reflection in the coordinate plane, the point $(3, 5)$ is mapped to the point $(5, 3)$. What is the line of reflection? Is this the only possible line of reflection? Explain.

Draw the reflection of the graph of each function across the given line.

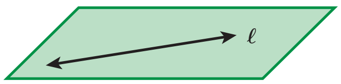
40. x-axis



41. y-axis



42. **Write About It** Imagine reflecting all the points in a plane across line ℓ. Which points remain fixed under this transformation? That is, for which points is the image the same as the preimage? Explain.



Construction **Use the construction of a line perpendicular to a given line through a given point (see page 179) and the construction of a segment congruent to a given segment (see page 14) to construct the reflection of each figure across a line.**

43. a point **44.** a segment **45.** a triangle

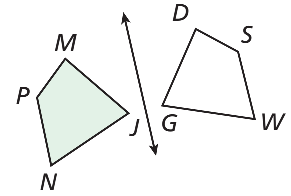
Standardized Test Prep

46. Daryl is using a coordinate plane to plan a garden. He draws a flower bed with vertices $(3, 1)$, $(3, 4)$, $(-2, 4)$, and $(-2, 1)$. Then he creates a second flower bed by reflecting the first one across the x-axis. Which of these is a vertex of the second flower bed?

Ⓐ $(-2, -4)$ Ⓒ $(2, 1)$

Ⓑ $(-3, 1)$ Ⓓ $(-3, -4)$

47. In the reflection shown, the shaded figure is the preimage. Which of these represents the mapping?



Ⓕ *MJNP* → *DSWG*　　Ⓗ *JMPN* → *GWSD*

Ⓖ *DGWS* → *MJNP*　　Ⓙ *PMJN* → *SDGW*

48. What is the image of the point (–3, 4) when it is reflected across the *y*-axis?

Ⓐ (4, –3)　　Ⓒ (3, 4)

Ⓑ (–3, –4)　　Ⓓ (–4, –3)

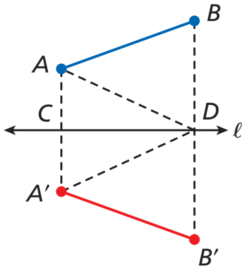
CHALLENGE AND EXTEND

Find the coordinates of the image when each point is reflected across the given line.

49. $(4, 2)$; $y = 3$　　**50.** $(-3, 2)$; $x = 1$　　**51.** $(3, 1)$; $y = x + 2$

52. Prove that the reflection image of a segment is congruent to the preimage.

Given: $\overline{A'B'}$ is the reflection image of $\overline{AB}$ across line ℓ.
Prove: $\overline{AB} \cong \overline{A'B'}$
Plan: Draw auxiliary lines $\overline{AA'}$ and $\overline{BB'}$ as shown. First prove that $\triangle ACD \cong \triangle A'CD$. Then use CPCTC to conclude that $\angle CDA \cong \angle CDA'$. Therefore $\angle ADB \cong \angle A'DB'$, which makes it possible to prove that $\triangle ADB \cong \triangle A'DB'$. Finally use CPCTC to conclude that $\overline{AB} \cong \overline{A'B'}$.



Once you have proved that the reflection image of a segment is congruent to the preimage, how could you prove the following? Write a plan for each proof.

53. If $\overline{A'B'}$ is the reflection of $\overline{AB}$, then $AB = A'B'$.

54. If $\angle A'B'C'$ is the reflection of $\angle ABC$, then $m\angle ABC = m\angle A'B'C'$.

55. The reflection $\triangle A'B'C'$ is congruent to the preimage $\triangle ABC$.

56. If point *C* is between points *A* and *B*, then the reflection *C′* is between *A′* and *B′*.

57. If points *A*, *B*, and *C* are collinear, then the reflections *A′*, *B′*, and *C′* are collinear.

SPIRAL REVIEW

A jar contains 2 red marbles, 6 yellow marbles, and 4 green marbles. One marble is drawn and replaced, and then a second marble is drawn. Find the probability of each outcome. *(Previous course)*

58. Both marbles are green.

59. Neither marble is red.

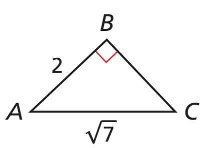
60. The first marble is yellow, and the second is green.

The width of a rectangular field is 60 m, and the length is 105 m. Use each of the following scales to find the perimeter of a scale drawing of the field. *(Lesson 7-5)*

61. 1 cm : 30 m　　**62.** 1.5 cm : 15 m　　**63.** 1 cm : 25 m

Find each unknown measure. Round side lengths to the nearest hundredth and angle measures to the nearest degree. *(Lesson 8-3)*

64. *BC*　　**65.** $m\angle A$　　**66.** $m\angle C$



12-2 Translations

Objective
Identify and draw translations.

Who uses this?
Marching band directors use translations to plan their bands' field shows. (See Example 4.)

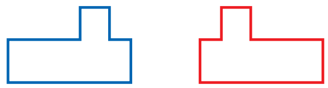
California Standards
22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, **including** rotations, **translations,** and reflections.

A translation is a transformation where all the points of a figure are moved the same distance in the same direction. A translation is an isometry, so the image of a translated figure is congruent to the preimage.

EXAMPLE 1 Identifying Translations

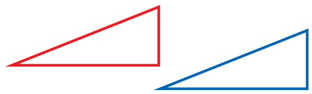
Tell whether each transformation appears to be a translation. Explain.

A



No; not all of the points have moved the same distance.

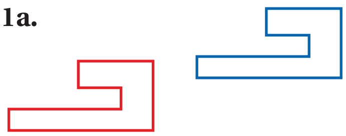
B



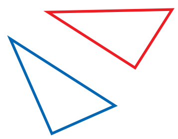
Yes; all of the points have moved the same distance in the same direction.

CHECK IT OUT! **Tell whether each transformation appears to be a translation.**

1a.

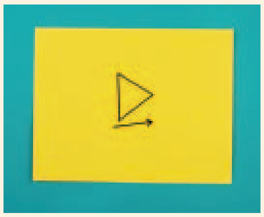


1b.



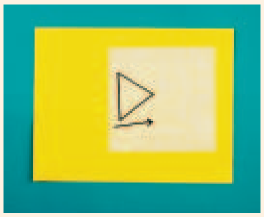
Construction Translate a Figure Using Patty Paper

1



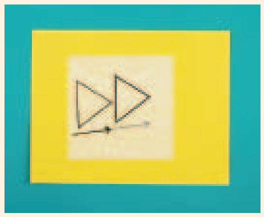
Draw a triangle and a translation vector on a sheet of paper.

2



Place a sheet of patty paper on top of the diagram. Trace the triangle and vector.

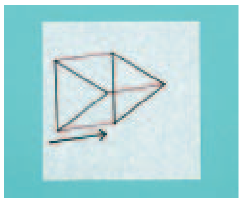
3



Slide the bottom paper in the direction of the vector until the head of the top vector aligns with the tail of the bottom vector. Trace the triangle.

Remember!
To review vectors, see Lesson 8-6, pages 559–567.

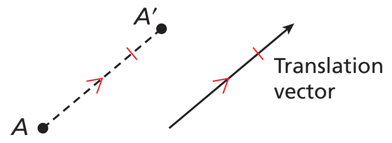
Draw a segment from each vertex of the preimage to the corresponding vertex of the image. Your construction should show that every segment connecting a point and its image is the same length as the translation vector. These segments are also parallel to the translation vector.



Translations

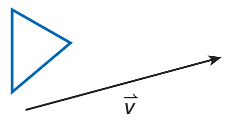
Know it! Note

A translation is a transformation along a vector such that each segment joining a point and its image has the same length as the vector and is parallel to the vector.

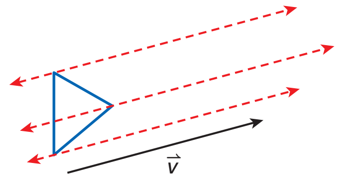


EXAMPLE 2 Drawing Translations

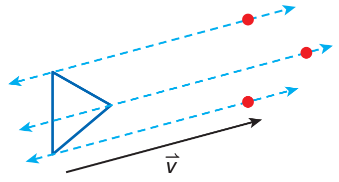
Copy the triangle and the translation vector. Draw the translation of the triangle along $\vec{v}$.



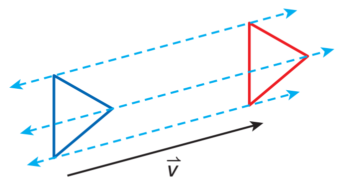
Step 1 Draw a line parallel to the vector through each vertex of the triangle.



Step 2 Measure the length of the vector. Then, from each vertex mark off this distance in the same direction as the vector, on each of the parallel lines.



Step 3 Connect the images of the vertices.

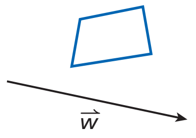


Math Builders

For more on translations, see the Transformation Builder on page MB2.

CHECK IT OUT!

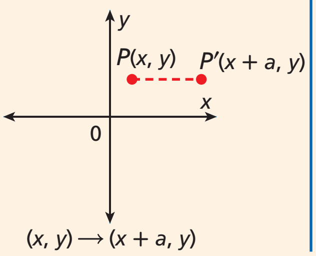
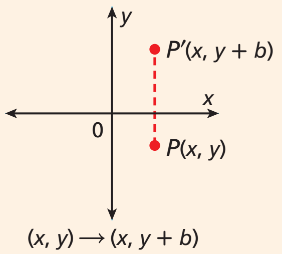
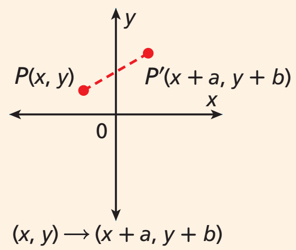
2. Copy the quadrilateral and the translation vector. Draw the translation of the quadrilateral along $\vec{w}$.



Recall that a vector in the coordinate plane can be written as $\langle a, b\rangle$, where a is the horizontal change and b is the vertical change from the initial point to the terminal point.

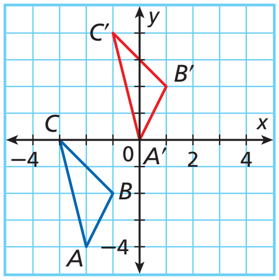
Translations in the Coordinate Plane

Know it! Note

HORIZONTAL TRANSLATION ALONG VECTOR $\langle a, 0\rangle$	VERTICAL TRANSLATION ALONG VECTOR $\langle 0, b\rangle$	GENERAL TRANSLATION ALONG VECTOR $\langle a, b\rangle$
		
$(x, y) \rightarrow (x + a, y)$	$(x, y) \rightarrow (x, y + b)$	$(x, y) \rightarrow (x + a, y + b)$

EXAMPLE 3 Drawing Translations in the Coordinate Plane

Translate the triangle with vertices $A(-2, -4)$, $B(-1, -2)$, and $C(-3, 0)$ along the vector $\langle 2, 4\rangle$.



The image of (x, y) is $(x + 2, y + 4)$.

$A(-2, -4) \rightarrow A'(-2 + 2, -4 + 4) = A'(0, 0)$

$B(-1, -2) \rightarrow B'(-1 + 2, -2 + 4) = B'(1, 2)$

$C(-3, 0) \rightarrow C'(-3 + 2, 0 + 4) = C'(-1, 4)$

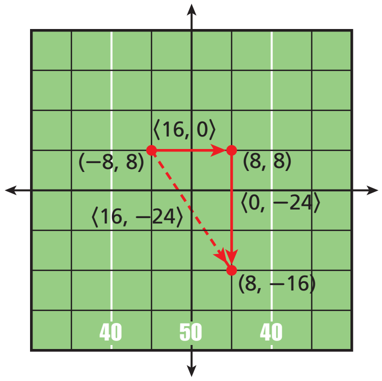
Graph the preimage and image.

CHECK IT OUT!

3. Translate the quadrilateral with vertices $R(2, 5)$, $S(0, 2)$, $T(1, -1)$, and $U(3, 1)$ along the vector $\langle -3, -3\rangle$.

EXAMPLE 4 *Entertainment Application*

In a marching drill, it takes 8 steps to march 5 yards. A drummer starts 8 steps to the left and 8 steps up from the center of the field. She marches 16 steps to the right to her second position. Then she marches 24 steps down the field to her final position. What is the drummer's final position? What single translation vector moves her from the starting position to her final position?



The drummer's starting coordinates are $(-8, 8)$.

Her second position is $(-8 + 16, 8) = (8, 8)$.

Her final position is $(8, 8 - 24) = (8, -16)$.

The vector that moves her directly from her starting position to her final position is $\langle 16, 0\rangle + \langle 0, -24\rangle = \langle 16, -24\rangle$.

CHECK IT OUT!

4. What if...? Suppose another drummer started at the center of the field and marched along the same vectors as above. What would this drummer's final position be?

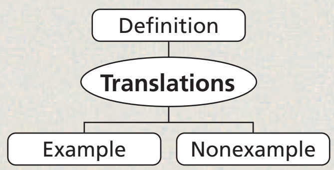
THINK AND DISCUSS

1. Point A' is a *translation* of point A along $\vec{v}$. What is the relationship of $\vec{v}$ to $\overline{AA'}$?

2. $\overline{AB}$ is translated to form $\overline{A'B'}$. Classify quadrilateral $AA'B'B$. Explain your reasoning.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer.



12-2 Exercises

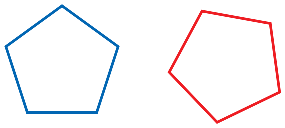
California Standards
2.0, 16.0, 22.0, 7MG3.2, 1A9.0

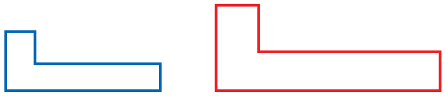
go.hrw.com
Homework Help Online
KEYWORD: MG7 12-2
Parent Resources Online
KEYWORD: MG7 Parent

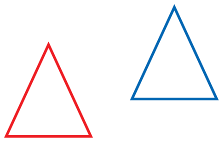
GUIDED PRACTICE

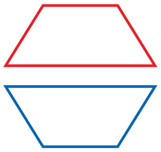
SEE EXAMPLE 1
p. 831

Tell whether each transformation appears to be a translation.

1.


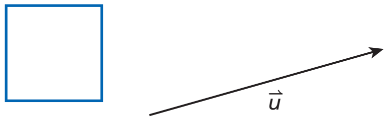
2.


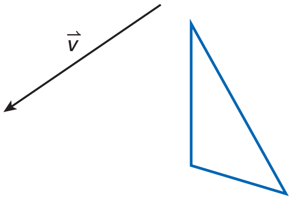
3.


4.


SEE EXAMPLE 2
p. 832

Multi-Step Copy each figure and the translation vector. Draw the translation of the figure along the given vector.

5.


6.


SEE EXAMPLE 3
p. 833

Translate the figure with the given vertices along the given vector.

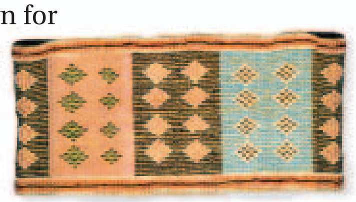
7. $A(-4, -4)$, $B(-2, -3)$, $C(-1, 3)$; $\langle 5, 0\rangle$

8. $R(-3, 1)$, $S(-2, 3)$, $T(2, 3)$, $U(3, 1)$; $\langle 0, -4\rangle$

9. $J(-2, 2)$, $K(-1, 2)$, $L(-1, -2)$, $M(-3, -1)$; $\langle 3, 2\rangle$

SEE EXAMPLE 4
p. 833

10. Art The Zulu people of southern Africa are known for their beadwork. To create a typical Zulu pattern, translate the polygon with vertices $(1, 5)$, $(2, 3)$, $(1, 1)$, and $(0, 3)$ along the vector $\langle 0, -4\rangle$. Translate the image along the same vector. Repeat to generate a pattern. What are the vertices of the fourth polygon in the pattern?



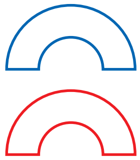
PRACTICE AND PROBLEM SOLVING

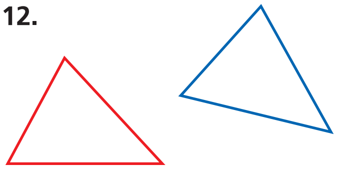
Independent Practice

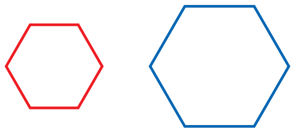
For Exercises	See Example
11–14	1
15–16	2
17–19	3
20	4

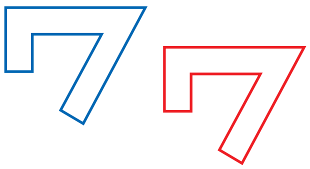
Extra Practice
Skills Practice p. S26
Application Practice p. S39

Tell whether each transformation appears to be a translation.

11.


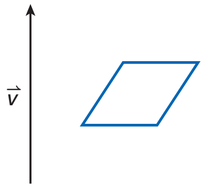
12.


13.


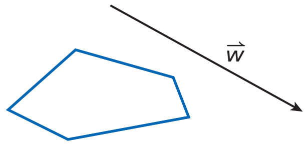
14.


Multi-Step Copy each figure and the translation vector. Draw the translation of the figure along the given vector.

15.



16.



LINK

Animation

Each frame of a computer-animated feature represents $\frac{1}{24}$ of a second of film.

Source: www.pixar.com

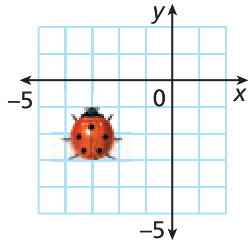
Translate the figure with the given vertices along the given vector.

17. $P(-1, 2)$, $Q(1, -1)$, $R(3, 1)$, $S(2, 3)$; $\langle -3, 0 \rangle$

18. $A(1, 3)$, $B(-1, 2)$, $C(2, 1)$, $D(4, 2)$; $\langle -3, -3 \rangle$

19. $D(0, 15)$, $E(-10, 5)$, $F(10, -5)$; $\langle 5, -20 \rangle$

20. **Animation** An animator draws the ladybug shown and then translates it along the vector $\langle 1, 1 \rangle$, followed by a translation of the new image along the vector $\langle 2, 2 \rangle$, followed by a translation of the second image along the vector $\langle 3, 3 \rangle$.

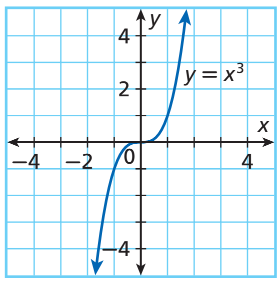


a. Sketch the ladybug's final position.

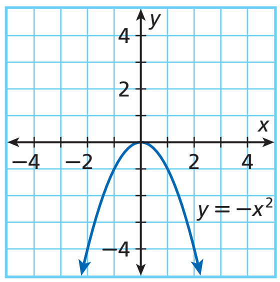
b. What single vector moves the ladybug from its starting position to its final position?

Draw the translation of the graph of each function along the given vector.

21. $\langle 3, 0 \rangle$



22. $\langle -1, -1 \rangle$



23. **Probability** The point $P(3, 2)$ is translated along one of the following four vectors chosen at random: $\langle -3, 0 \rangle$, $\langle -1, -4 \rangle$, $\langle 3, -2 \rangle$, and $\langle 2, 3 \rangle$. Find the probability of each of the following.

a. The image of P is in the fourth quadrant.

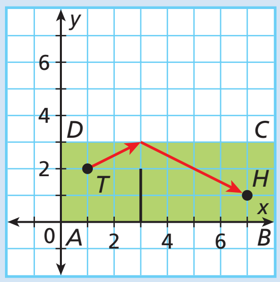
b. The image of P is on an axis.

c. The image of P is at the origin.

CONCEPT CONNECTION

24. This problem will prepare you for the Concept Connection on page 854.

The figure shows one hole of a miniature golf course and the path of a ball from the tee T to the hole H.



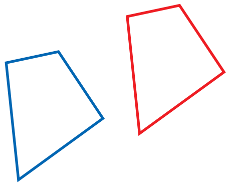
a. What translation vector represents the path of the ball from T to $\overline{DC}$?

b. What translation vector represents the path of the ball from $\overline{DC}$ to H?

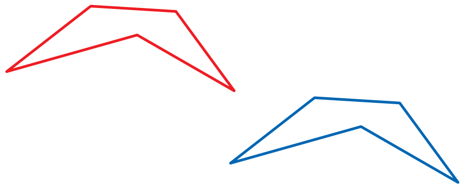
c. Show that the sum of these vectors is equal to the vector that represents the straight path from T to H.

Each figure shows a preimage (blue) and its image (red) under a translation. Copy the figure and draw the vector along which the polygon is translated.

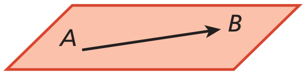
25.



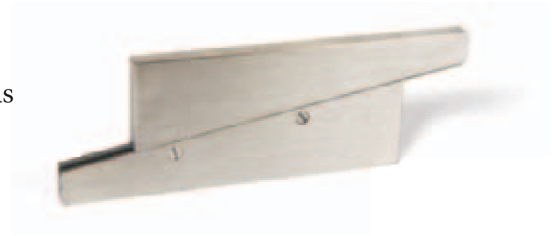
26.



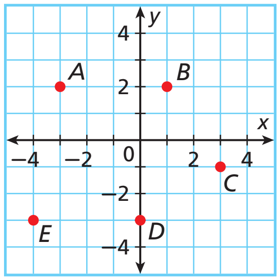
27. Critical Thinking The points of a plane are translated along the given vector $\overrightarrow{AB}$. Do any points remain fixed under this transformation? That is, are there any points for which the image coincides with the preimage? Explain.



28. Carpentry Carpenters use a tool called *adjustable parallels* to set up level work areas and to draw parallel lines. Describe how a carpenter could use this tool to translate a given point along a given vector. What additional tools, if any, would be needed?



Find the vector associated with each translation. Then use arrow notation to describe the mapping of the preimage to the image.



29. the translation that maps point A to point B

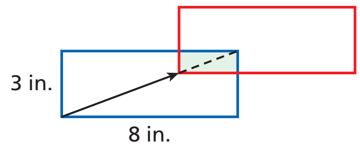
30. the translation that maps point B to point A

31. the translation that maps point C to point D

32. the translation that maps point E to point B

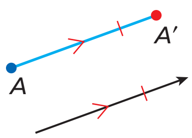
33. the translation that maps point C to the origin

34. Multi-Step The rectangle shown is translated two-thirds of the way along one of its diagonals. Find the area of the region where the rectangle and its image overlap.



35. Write About It Point P is translated along the vector $\langle a, b\rangle$. Explain how to find the distance between point P and its image.

Construction Use the construction of a line parallel to a given line through a given point (see page 163) and the construction of a segment congruent to a given segment (see page 14) to construct the translation of each figure along a vector.



36. a point **37.** a segment **38.** a triangle

STANDARDIZED TEST PREP

39. What is the image of $P(1, 3)$ when it is translated along the vector $\langle -3, 5\rangle$?

Ⓐ $(-2, 8)$ Ⓑ $(0, 6)$ Ⓒ $(1, 3)$ Ⓓ $(0, 4)$

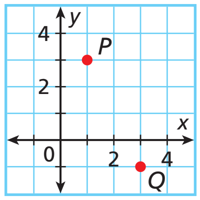
40. After a translation, the image of $A(-6, -2)$ is $B(-4, -4)$. What is the image of the point $(3, -1)$ after this translation?

Ⓕ $(-5, 1)$ Ⓖ $(5, -3)$ Ⓗ $(5, 1)$ Ⓙ $(-5, -3)$

41. Which vector translates point Q to point P?

Ⓐ $\langle -2, -4 \rangle$ Ⓒ $\langle -2, 4 \rangle$

Ⓑ $\langle 4, -2 \rangle$ Ⓓ $\langle 2, -4 \rangle$



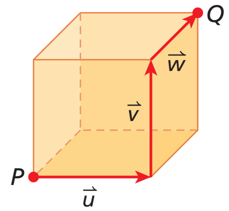
CHALLENGE AND EXTEND

42. The point $M(1, 2)$ is translated along a vector that is parallel to the line $y = 2x + 4$. The translation vector has magnitude $\sqrt{5}$. What are the possible images of point M?

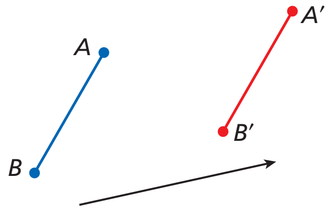
43. A cube has edges of length 2 cm. Point P is translated along $\vec{u}$, $\vec{v}$, and $\vec{w}$ as shown.

a. Describe a single translation vector that maps point P to point Q.

b. Find the magnitude of this vector to the nearest hundredth.



44. Prove that the translation image of a segment is congruent to the preimage.
Given: $\overline{A'B'}$ is the translation image of $\overline{AB}$.
Prove: $\overline{AB} \cong \overline{A'B'}$
(*Hint:* Draw auxiliary lines $\overline{AA'}$ and $\overline{BB'}$.
What can you conclude about $\overline{AA'}$ and $\overline{BB'}$?)



Once you have proved that the translation image of a segment is congruent to the preimage, how could you prove the following? Write a plan for each proof.

45. If $\overline{A'B'}$ is a translation of $\overline{AB}$, then $AB = A'B'$.

46. If $\angle A'B'C'$ is a translation of $\angle ABC$, then $m\angle ABC = m\angle A'B'C'$.

47. The translation $\triangle A'B'C'$ is congruent to the preimage $\triangle ABC$.

48. If point C is between points A and B, then the translation C' is between A' and B'.

49. If points A, B, and C are collinear, then the translations A', B', and C' are collinear.

SPIRAL REVIEW

Solve each system of equations and check your solution. *(Previous course)*

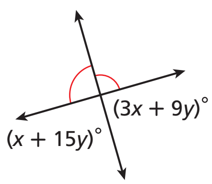
50. $\begin{cases} -5x - 2y = 17 \\ 6x - 2y = -5 \end{cases}$

51. $\begin{cases} 2x - 3y = -7 \\ 6x + 5y = 49 \end{cases}$

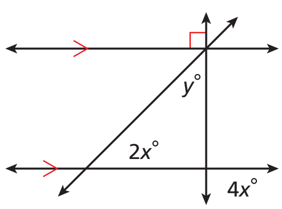
52. $\begin{cases} 4x + 4y = -1 \\ 12x - 8y = -8 \end{cases}$

Solve to find x and y in each diagram. *(Lesson 3-4)*

53.



54.



$\triangle MNP$ has vertices $M(-2, 0)$, $N(-3, 2)$, and $P(0, 4)$. Find the coordinates of the vertices of $\triangle M'N'P'$ after a reflection across the given line. *(Lesson 12-1)*

55. x-axis **56.** y-axis **57.** $y = x$

Connecting Geometry to **Algebra**

See Skills Bank page S63

Transformations of Functions

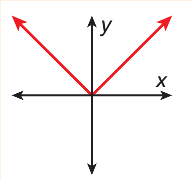
California Standards

Extension of 1A21.0 Students graph quadratic functions and know that their roots are the x-intercepts.

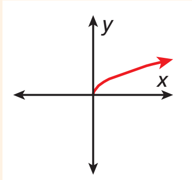
Also covered: **22.0**

Transformations can be used to graph complicated functions by using the graphs of simpler functions called *parent functions*. The following are examples of parent functions and their graphs.

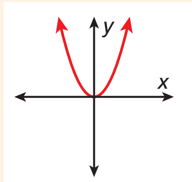
$y = |x|$



$y = \sqrt{x}$



$y = x^2$

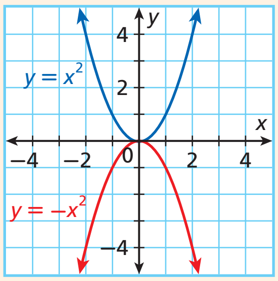


Transformation of Parent Function $y = f(x)$		
Reflection	**Vertical Translation**	**Horizontal Translation**
Across x-axis: $y = -f(x)$	$y = f(x) + k$	$y = f(x - h)$
Across y-axis: $y = f(-x)$	Up k units if $k > 0$	Right h units if $h > 0$
	Down k units if $k < 0$	Left h units if $h < 0$

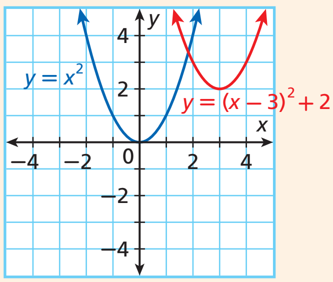
Example

For the parent function $y = x^2$, write a function rule for the given transformation and graph the preimage and image.

A a reflection across the x-axis
function rule: $y = -x^2$
graph:



B a translation up 2 units and right 3 units
function rule: $y = (x - 3)^2 + 2$
graph:

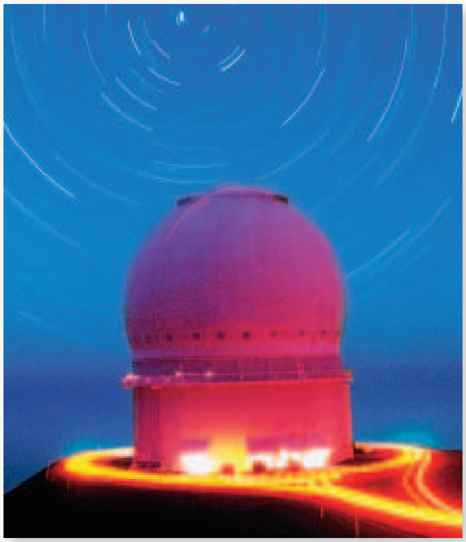


Try This

For each parent function, write a function rule for the given transformation and graph the preimage and image.

1. parent function: $y = x^2$
   transformation: a translation down 1 unit and right 4 units
2. parent function: $y = \sqrt{x}$
   transformation: a reflection across the x-axis
3. parent function: $y = |x|$
   transformation: a translation up 2 units and left 1 unit

12-3 Rotations



Objective
Identify and draw rotations.

Who uses this?
Astronomers can use properties of rotations to analyze photos of star trails. (See Exercise 35.)

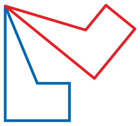
Remember that a rotation is a transformation that turns a figure around a fixed point, called the center of rotation. A rotation is an isometry, so the image of a rotated figure is congruent to the preimage.

California Standards
22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, **including rotations,** translations, and reflections.

EXAMPLE 1 Identifying Rotations

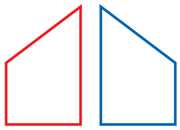
Tell whether each transformation appears to be a rotation. Explain.

A



Yes; the figure appears to be turned around a point.

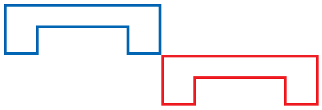
B



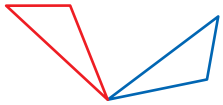
No; the figure appears to be flipped, not turned.

CHECK IT OUT! **Tell whether each transformation appears to be a rotation.**

1a.

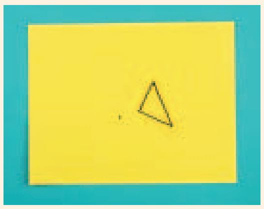


1b.



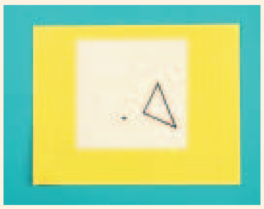
Construction Rotate a Figure Using Patty Paper

1



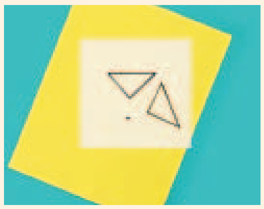
On a sheet of paper, draw a triangle and a point. The point will be the center of rotation.

2



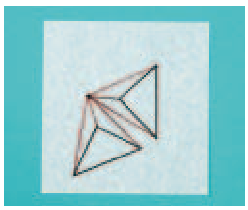
Place a sheet of patty paper on top of the diagram. Trace the triangle and the point.

3



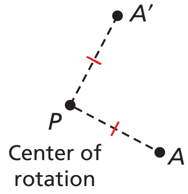
Hold your pencil down on the point and rotate the bottom paper counterclockwise. Trace the triangle.

Draw a segment from each vertex to the center of rotation. Your construction should show that a point's distance to the center of rotation is equal to its image's distance to the center of rotation. The angle formed by a point, the center of rotation, and the point's image is the angle by which the figure was rotated.



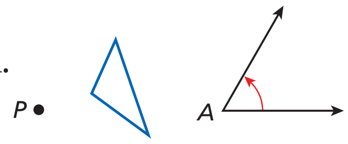
Know it! Note — Rotations

A rotation is a transformation about a point *P*, called the center of rotation, such that each point and its image are the same distance from *P*, and such that all angles with vertex *P* formed by a point and its image are congruent. In the figure, $\angle APA'$ is the angle of rotation.

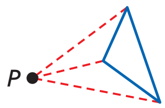


EXAMPLE 2 Drawing Rotations

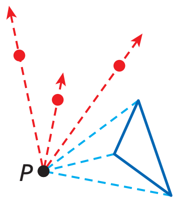
Copy the figure and the angle of rotation. Draw the rotation of the triangle about point *P* by m∠*A*.



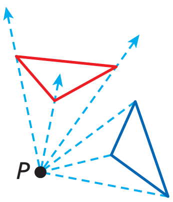
Step 1 Draw a segment from each vertex to point *P*.



Step 2 Construct an angle congruent to ∠*A* onto each segment. Measure the distance from each vertex to point *P* and mark off this distance on the corresponding ray to locate the image of each vertex.

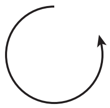


Step 3 Connect the images of the vertices.



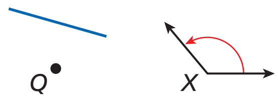
Helpful Hint

Unless otherwise stated, all rotations in this book are counterclockwise.

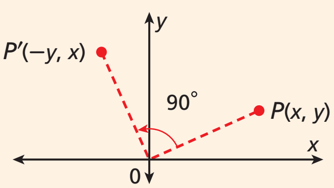
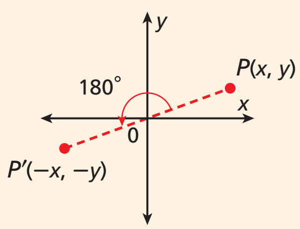


CHECK IT OUT!

2. Copy the figure and the angle of rotation. Draw the rotation of the segment about point *Q* by m∠*X*.



Know it! Note — Rotations in the Coordinate Plane

BY 90° ABOUT THE ORIGIN	BY 180° ABOUT THE ORIGIN
	
$(x, y) \rightarrow (-y, x)$	$(x, y) \rightarrow (-x, -y)$

Math Builders

For more on rotations, see the Transformation Builder on page MB2.

If the angle of a rotation in the coordinate plane is not a multiple of 90°, you can use sine and cosine ratios to find the coordinates of the image.

EXAMPLE 3 Drawing Rotations in the Coordinate Plane

Rotate $\triangle ABC$ with vertices $A(2, -1)$, $B(4, 1)$, and $C(3, 3)$ by 90° about the origin.

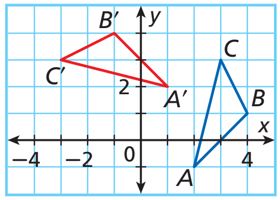
The rotation of (x, y) is $(-y, x)$.

$A(2, -1) \rightarrow A'(1, 2)$

$B(4, 1) \rightarrow B'(-1, 4)$

$C(3, 3) \rightarrow C'(-3, 3)$

Graph the preimage and image.



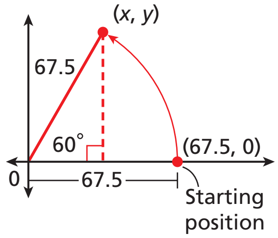
CHECK IT OUT! 3. Rotate $\triangle ABC$ by 180° about the origin.

EXAMPLE 4 *Engineering Application*

The London Eye observation wheel has a radius of 67.5 m and takes 30 minutes to make a complete rotation. A car starts at position $(67.5, 0)$. What are the coordinates of the car's location after 5 minutes?

Step 1 Find the angle of rotation. Five minutes is $\frac{5}{30} = \frac{1}{6}$ of a complete rotation, or $\frac{1}{6}(360°) = 60°$.

Step 2 Draw a right triangle to represent the car's location (x, y) after a rotation of 60° about the origin.



Step 3 Use the cosine ratio to find the x-coordinate.

$\cos 60° = \frac{x}{67.5}$ — *$\cos = \frac{\text{adj.}}{\text{hyp.}}$*

$x = 67.5 \cos 60° \approx 33.8$ — *Solve for x.*

Step 4 Use the sine ratio to find the y-coordinate.

$\sin 60° = \frac{y}{67.5}$ — *$\sin = \frac{\text{opp.}}{\text{hyp.}}$*

$y = 67.5 \sin 60° \approx 58.5$ — *Solve for y.*

The car's location after 5 minutes is approximately $(33.8, 58.5)$.

CHECK IT OUT! 4. Find the coordinates of the observation car after 6 minutes. Round to the nearest tenth.

Remember!
To review the sine and cosine ratios, see Lesson 8-2, pages 525–532.

THINK AND DISCUSS

1. Describe the image of a rotation of a figure by an angle of 360°.

2. Point A' is a rotation of point A about point P. What is the relationship of $\overline{AP}$ to $\overline{A'P}$?

3. **GET ORGANIZED** Copy and complete the graphic organizer.

Know it! Note

	Reflection	Translation	Rotation
Definition			
Example			

12-3 Exercises

California Standards
2.0, 7.0, 16.0, 22.0

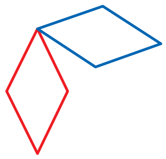
go.hrw.com
Homework Help Online
KEYWORD: MG7 12-3
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

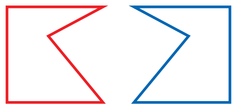
SEE EXAMPLE 1
p. 839

Tell whether each transformation appears to be a rotation.

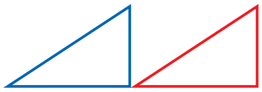
1.



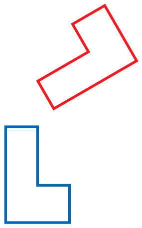
2.



3.



4.

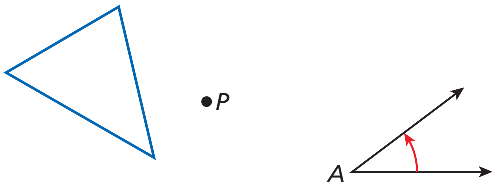


SEE EXAMPLE 2
p. 840

Copy each figure and the angle of rotation. Draw the rotation of the figure about point *P* by m∠*A*.

5.

6.



SEE EXAMPLE 3
p. 841

Rotate the figure with the given vertices about the origin using the given angle of rotation.

7. $A(1, 0)$, $B(3, 2)$, $C(5, 0)$; 90°

8. $J(2, 1)$, $K(4, 3)$, $L(2, 4)$, $M(-1, 2)$; 90°

9. $D(2, 3)$, $E(-1, 2)$, $F(2, 1)$; 180°

10. $P(-1, -1)$, $Q(-4, -2)$, $R(0, -2)$; 180°

SEE EXAMPLE 4
p. 841

11. Animation An artist uses a coordinate plane to plan the motion of an animated car. To simulate the car driving around a curve, the artist places the car at the point $(10, 0)$ and then rotates it about the origin by 30°. Give the car's final position, rounding the coordinates to the nearest tenth.

PRACTICE AND PROBLEM SOLVING

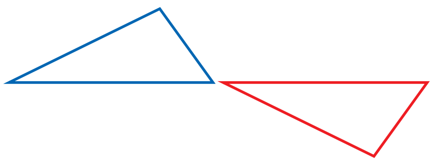
Independent Practice

For Exercises	See Example
12–15	1
16–17	2
18–21	3
22	4

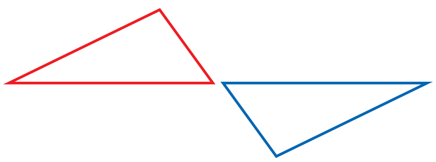
Extra Practice
Skills Practice p. S26
Application Practice p. S39

Tell whether each transformation appears to be a rotation.

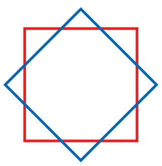
12.



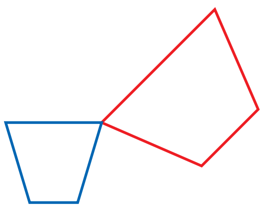
13.



14.

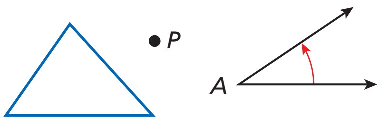


15.

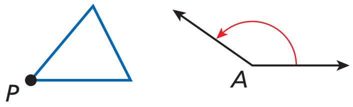


Copy each figure and the angle of rotation. Draw the rotation of the figure about point *P* by m∠*A*.

16.



17.



Rotate the figure with the given vertices about the origin using the given angle of rotation.

18. $E(-1, 2)$, $F(3, 1)$, $G(2, 3)$; 90°

19. $A(-1, 0)$, $B(-1, -3)$, $C(1, -3)$, $D(1, 0)$; 90°

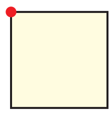
20. $P(0, -2)$, $Q(2, 0)$, $R(3, -3)$; 180°

21. $L(2, 0)$, $M(-1, -2)$, $N(2, -2)$; 180°

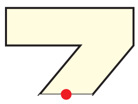
22. **Architecture** The CN Tower in Toronto, Canada, features a revolving restaurant that takes 72 minutes to complete a full rotation. A table that is 50 feet from the center of the restaurant starts at position $(50, 0)$. What are the coordinates of the table after 6 minutes? Round coordinates to the nearest tenth.

Copy each figure. Then draw the rotation of the figure about the red point using the given angle measure.

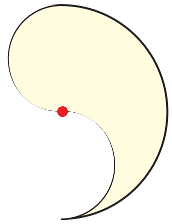
23. 90°



24. 180°



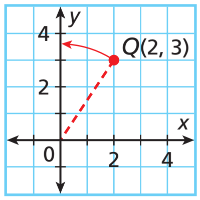
25. 180°



26. Point *Q* has coordinates $(2, 3)$. After a rotation about the origin, the image of point *Q* lies on the *y*-axis.

a. Find the angle of rotation to the nearest degree.

b. Find the coordinates of the image of point *Q*. Round to the nearest tenth.



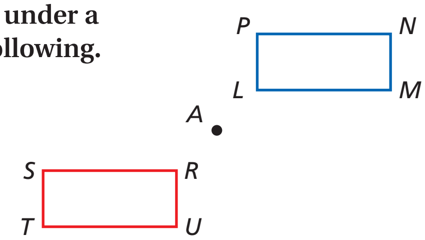
Rectangle *RSTU* is the image of rectangle *LMNP* under a 180° rotation about point *A*. Name each of the following.

27. the image of point *N*

28. the preimage of point *S*

29. the image of $\overline{MN}$

30. the preimage of $\overline{TU}$



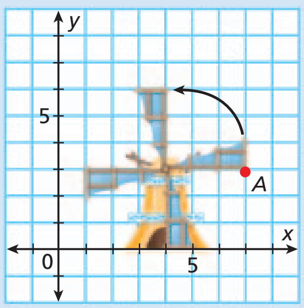
CONCEPT CONNECTION

31. This problem will prepare you for the Concept Connection on page 854.

A miniature golf course includes a hole with a windmill. Players must hit the ball through the opening at the base of the windmill while the blades rotate.

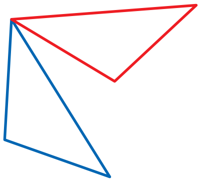
a. The blades take 20 seconds to make a complete rotation. Through what angle do the blades rotate in 4 seconds?

b. Find the coordinates of point *A* after 4 seconds. (*Hint:* $(4, 3)$ is the center of rotation.)

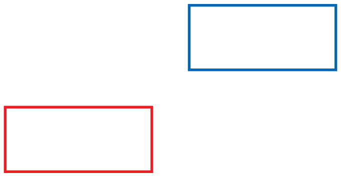


Each figure shows a preimage and its image under a rotation. Copy the figure and locate the center of rotation.

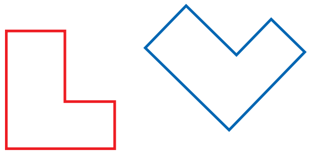
32.



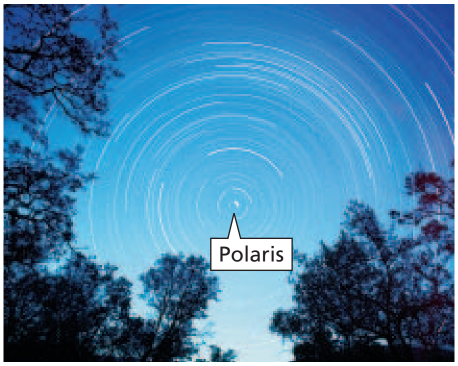
33.



34.



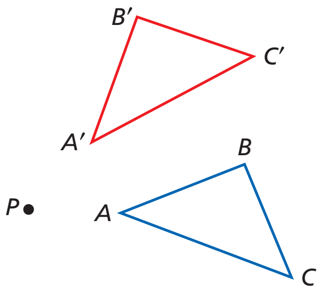
35. **Astronomy** The photograph was made by placing a camera on a tripod and keeping the camera's shutter open for a long time. Because of Earth's rotation, the stars appear to rotate around Polaris, also known as the North Star.



a. **Estimation** Estimate the angle of rotation of the stars in the photo.

b. **Estimation** Use your result from part **a** to estimate the length of time that the camera's shutter was open. (*Hint:* If the shutter was open for 24 hours, the stars would appear to make one complete rotation around Polaris.)

36. **Estimation** In the diagram, $\triangle ABC \rightarrow \triangle A'B'C'$ under a rotation about point P.



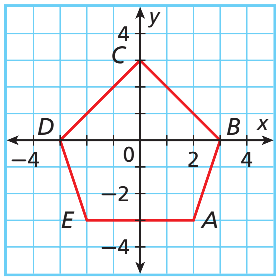
a. Estimate the angle of rotation.

b. Explain how you can draw two segments and can then use a protractor to measure the angle of rotation.

c. Copy the figure. Use the method from part **b** to find the angle of rotation. How does your result compare to your estimate?

37. **Critical Thinking** A student wrote the following in his math journal. "Under a rotation, every point moves around the center of rotation by the same angle measure. This means that every point moves the same distance." Do you agree? Explain.

Use the figure for Exercises 38–40.

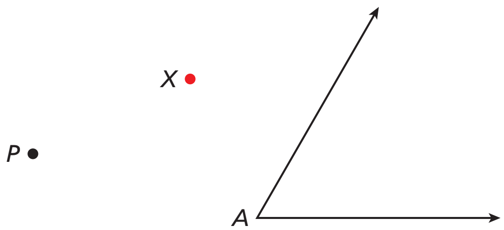


38. Sketch the image of pentagon $ABCDE$ under a rotation of 90° about the origin. Give the vertices of the image.

39. Sketch the image of pentagon $ABCDE$ under a rotation of 180° about the origin. Give the vertices of the image.

40. **Write About It** Is the image of $ABCDE$ under a rotation of 180° about the origin the same as its image under a reflection across the x-axis? Explain your reasoning.

41. **Construction** Copy the figure. Use the construction of an angle congruent to a given angle (see page 22) to construct the image of point X under a rotation about point P by $m\angle A$.



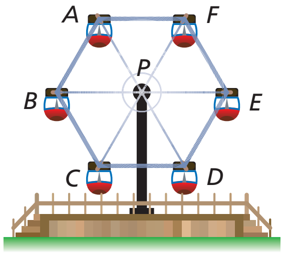
Standardized Test Prep

42. What is the image of the point $(-2, 5)$ when it is rotated about the origin by 90°?

Ⓐ $(-5, 2)$ Ⓑ $(5, -2)$ Ⓒ $(-5, -2)$ Ⓓ $(2, -5)$

43. The six cars of a Ferris wheel are located at the vertices of a regular hexagon. Which rotation about point P maps car A to car C?

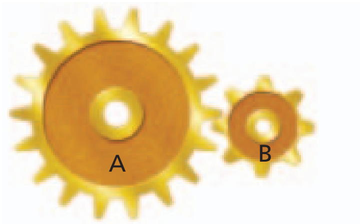
Ⓕ 60° Ⓖ 90° Ⓗ 120° Ⓙ 135°



44. **Gridded Response** Under a rotation about the origin, the point $(-3, 4)$ is mapped to the point $(3, -4)$. What is the measure of the angle of rotation?

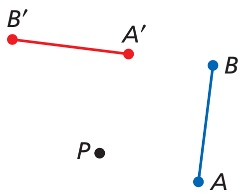
CHALLENGE AND EXTEND

45. **Engineering** Gears are used to change the speed and direction of rotating parts in pieces of machinery. In the diagram, suppose gear B makes one complete rotation in the counterclockwise direction. Give the angle of rotation and direction for the rotation of gear A. Explain how you got your answer.



46. **Given:** $\overline{A'B'}$ is the rotation image of $\overline{AB}$ about point P.
Prove: $\overline{AB} \cong \overline{A'B'}$

(*Hint:* Draw auxiliary lines $\overline{AP}$, $\overline{BP}$, $\overline{A'P}$, and $\overline{B'P}$ and show that $\triangle APB \cong \triangle A'PB'$.)



Once you have proved that the rotation image of a segment is congruent to the preimage, how could you prove the following? Write a plan for each proof.

47. If $\overline{A'B'}$ is a rotation of $\overline{AB}$, then $AB = A'B'$.

48. If $\angle A'B'C'$ is a rotation of $\angle ABC$, then $m\angle ABC = m\angle A'B'C'$.

49. The rotation $\triangle A'B'C'$ is congruent to the preimage $\triangle ABC$.

50. If point C is between points A and B, then the rotation C' is between A' and B'.

51. If points A, B, and C are collinear, then the rotations A', B', and C' are collinear.

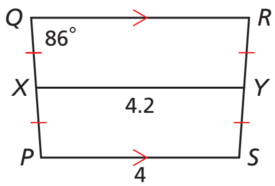
SPIRAL REVIEW

Find the value(s) of x when y is 3. *(Previous course)*

52. $y = x^2 - 4x + 7$ **53.** $y = 2x^2 - 5x - 9$ **54.** $y = x^2 - 2$

Find each measure. *(Lesson 6-6)*

55. $m\angle XYR$ **56.** QR



Given the points $A(1, 3)$, $B(5, 0)$, $C(-3, -2)$, and $D(5, -6)$, find the vector associated with each translation. *(Lesson 12-2)*

57. the translation that maps point A to point D

58. the translation that maps point D to point B

59. the translation that maps point C to the origin

12-3 Technology LAB

Explore Transformations with Matrices

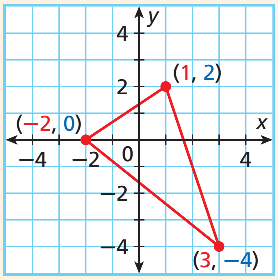
Use with Lesson 12-3

California Standards

22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, **including rotations, translations, and reflections.**

go.hrw.com
Lab Resources Online
KEYWORD: MG7 Lab12

The vertices of a polygon in the coordinate plane can be represented by a *point matrix* in which row 1 contains the x-values and row 2 contains the y-values. For example, the triangle with vertices $(1, 2)$, $(-2, 0)$, and $(3, -4)$ can be represented by $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & -4 \end{bmatrix}$.



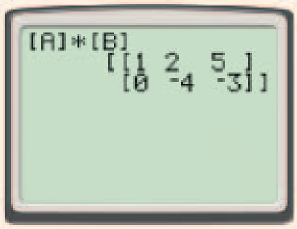
On the graphing calculator, enter a matrix using the **Matrix Edit** menu. Enter the number of rows and columns and then enter the values.

Matrix operations can be used to perform transformations.

Activity 1

1. Graph the triangle with vertices $(1, 0)$, $(2, 4)$, and $(5, 3)$ on graph paper. Enter the point matrix that represents the vertices into matrix **[B]** on your calculator.

2. Enter the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ into matrix **[A]** on your calculator. Multiply **[A]** * **[B]** and use the resulting matrix to graph the image of the triangle. Describe the transformation.



Try This

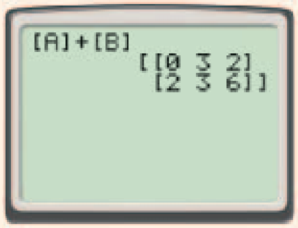
1. Enter the matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ into matrix **[A]**. Multiply **[A]** * **[B]** and use the resulting matrix to graph the image of the triangle. Describe the transformation.

2. Enter the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ into matrix **[A]**. Multiply **[A]** * **[B]** and use the resulting matrix to graph the image of the triangle. Describe the transformation.

Activity 2

1 Graph the triangle with vertices $(0, 0)$, $(3, 1)$, and $(2, 4)$ on graph paper. Enter the point matrix that represents the vertices into matrix **[B]** on your calculator.

2 Enter the matrix $\begin{bmatrix} 0 & 0 & 0 \\ 2 & 2 & 2 \end{bmatrix}$ into matrix **[A]**. Add **[A]** + **[B]** and use the resulting matrix to graph the image of the triangle. Describe the transformation.



Try This

3. Enter the matrix $\begin{bmatrix} -1 & -1 & -1 \\ 4 & 4 & 4 \end{bmatrix}$ into matrix **[A]**. Add **[A]** + **[B]** and use the resulting matrix to graph the image of the triangle. Describe the transformation.

4. **Make a Conjecture** How do you think you could use matrices to translate a triangle by the vector $\langle a, b \rangle$? Choose several values for a and b and test your conjecture.

Activity 3

1 Graph the triangle with vertices $(1, 1)$, $(4, 1)$, and $(1, 2)$ on graph paper. Enter the point matrix that represents the vertices into matrix **[B]** on your calculator.

2 Enter the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ into matrix **[A]**. Multiply **[A]** * **[B]** and use the resulting matrix to graph the image of the triangle. Describe the transformation.

Try This

5. Enter the values $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ into matrix **[A]**. Multiply **[A]** * **[B]** and use the resulting matrix to graph the image of the triangle. Describe the transformation.

6. Enter the values $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ into matrix **[A]**. Multiply **[A]** * **[B]** and use the resulting matrix to graph the image of the triangle. Describe the transformation.

12-4 Compositions of Transformations

Objectives
Apply theorems about isometries.

Identify and draw compositions of transformations, such as glide reflections.

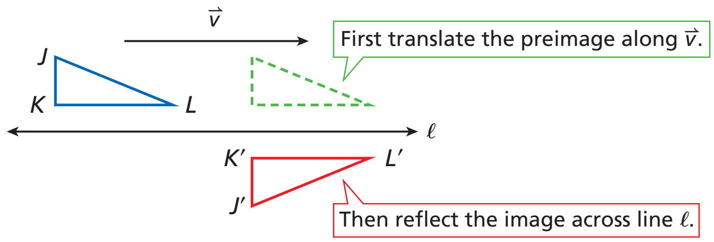
Vocabulary
composition of transformations
glide reflection

California Standards
🔑 **22.0 Students know the effect of rigid motions on figures in the coordinate plane** and space, **including rotations, translations, and reflections.**

Why learn this?
Compositions of transformations can be used to describe chess moves. (See Exercise 11.)

A **composition of transformations** is one transformation followed by another. For example, a **glide reflection** is the composition of a translation and a reflection across a line parallel to the translation vector.

The glide reflection that maps $\triangle JKL$ to $\triangle J'K'L'$ is the composition of a translation along $\vec{v}$ followed by a reflection across line ℓ.



The image after each transformation is congruent to the previous image. By the Transitive Property of Congruence, the final image is congruent to the preimage. This leads to the following theorem.

Know it! Note

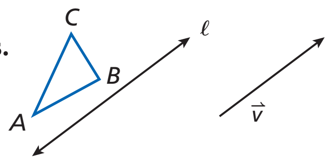
Theorem 12-4-1

A composition of two isometries is an isometry.

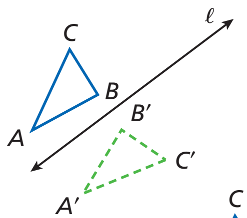
EXAMPLE 1 Drawing Compositions of Isometries

Draw the result of the composition of isometries.

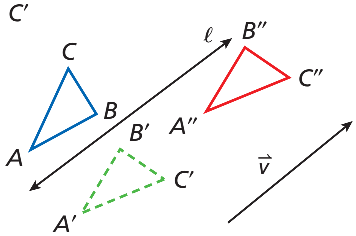
A **Reflect $\triangle ABC$ across line ℓ and then translate it along $\vec{v}$.**



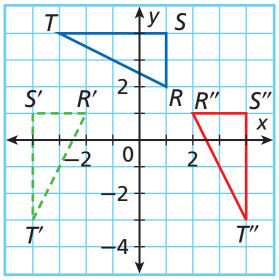
Step 1 Draw $\triangle A'B'C'$, the reflection image of $\triangle ABC$.



Step 2 Translate $\triangle A'B'C'$ along $\vec{v}$ to find the final image, $\triangle A''B''C''$.



B $\triangle RST$ **has vertices** $R(1, 2)$, $S(1, 4)$, **and** $T(-3, 4)$. **Rotate** $\triangle RST$ **90° about the origin and then reflect it across the** y**-axis.**



Step 1 The **rotation** image of (x, y) is $(-y, x)$.

$R(1, 2) \rightarrow R'(-2, 1)$, $S(1, 4) \rightarrow S'(-4, 1)$, and $T(-3, 4) \rightarrow T'(-4, -3)$.

Step 2 The **reflection** image of (x, y) is $(-x, y)$.

$R'(-2, 1) \rightarrow R''(2, 1)$, $S'(-4, 1) \rightarrow S''(4, 1)$, and $T'(-4, -3) \rightarrow T''(4, -3)$.

Step 3 Graph the preimage and images.

CHECK IT OUT!

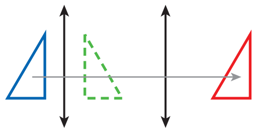
1. $\triangle JKL$ has vertices $J(1, -2)$, $K(4, -2)$, and $L(3, 0)$. Reflect $\triangle JKL$ across the x-axis and then rotate it 180° about the origin.

Know it! Note

Theorem 12-4-2

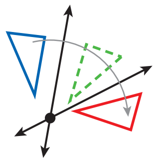
The composition of two reflections across two parallel lines is equivalent to a translation.

- The translation vector is perpendicular to the lines.
- The length of the translation vector is twice the distance between the lines.



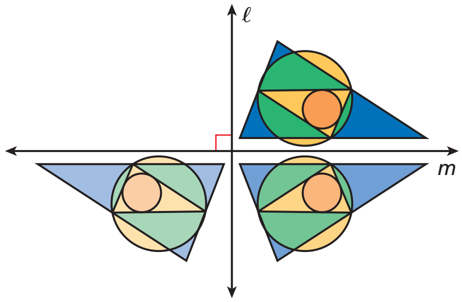
The composition of two reflections across two intersecting lines is equivalent to a rotation.

- The center of rotation is the intersection of the lines.
- The angle of rotation is twice the measure of the angle formed by the lines.



EXAMPLE 2 *Art Application*

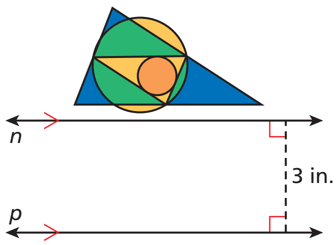
Tabitha is creating a design for an art project. She reflects a figure across line ℓ and then reflects the image across line m. Describe a single transformation that moves the figure from its starting position to its final position.



By Theorem 12-4-2, the composition of two reflections across intersecting lines is equivalent to a rotation about the point of intersection. Since the lines are perpendicular, they form a 90° angle. By Theorem 12-4-2, the angle of rotation is $2 \cdot 90° = 180°$.

CHECK IT OUT!

2. What if...? Suppose Tabitha reflects the figure across line n and then the image across line p. Describe a single transformation that is equivalent to the two reflections.



Know it! Note

Theorem 12-4-3

Any translation or rotation is equivalent to a composition of two reflections.

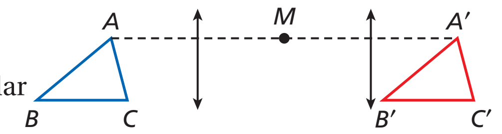
EXAMPLE 3 Describing Transformations in Terms of Reflections

Copy each figure and draw two lines of reflection that produce an equivalent transformation.

A **translation:** $\triangle ABC \rightarrow \triangle A'B'C'$

Step 1 Draw $\overline{AA'}$ and locate the midpoint M of $\overline{AA'}$.

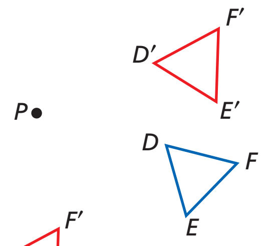
Step 2 Draw the perpendicular bisectors of $\overline{AM}$ and $\overline{A'M}$.



Remember!

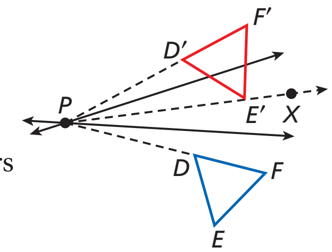
To draw the perpendicular bisector of a segment, use a ruler to locate the midpoint, and then use a right angle to draw a perpendicular line.

B **rotation with center P:** $\triangle DEF \rightarrow \triangle D'E'F'$



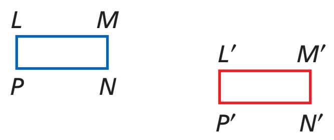
Step 1 Draw $\angle DPD'$. Draw the angle bisector $\overrightarrow{PX}$.

Step 2 Draw the bisectors of $\angle DPX$ and $\angle D'PX$.



CHECK IT OUT!

3. Copy the figure showing the translation that maps $LMNP \rightarrow L'M'N'P'$. Draw the lines of reflection that produce an equivalent transformation.



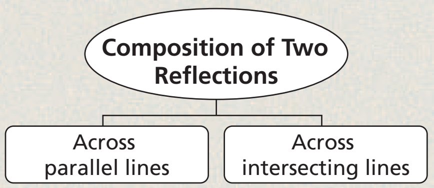
THINK AND DISCUSS

1. Which theorem explains why the image of a rectangle that is translated and then rotated is congruent to the preimage?

2. Point A' is a glide reflection of point A along $\vec{v}$ and across line ℓ. What is the relationship between $\vec{v}$ and ℓ? Explain the steps you would use to draw a glide reflection.

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe an equivalent transformation and sketch an example.



12-4 Exercises

California Standards
7.0, 21.0, 22.0, 7MG3.2

go.hrw.com
Homework Help Online
KEYWORD: MG7 12-4
Parent Resources Online
KEYWORD: MG7 Parent

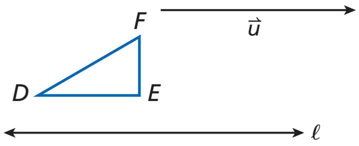
GUIDED PRACTICE

1. **Vocabulary** Explain the steps you would use to draw a *glide reflection*.

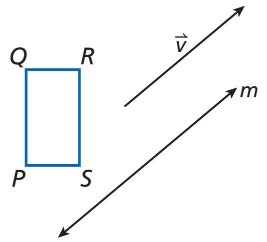
SEE EXAMPLE 1
p. 848

Draw the result of each composition of isometries.

2. Translate $\triangle DEF$ along $\vec{u}$ and then reflect it across line ℓ.



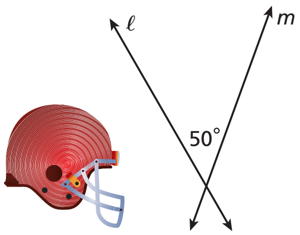
3. Reflect rectangle $PQRS$ across line m and then translate it along $\vec{v}$.



4. $\triangle ABC$ has vertices $A(1, -1)$, $B(4, -1)$, and $C(3, 2)$. Reflect $\triangle ABC$ across the y-axis and then translate it along the vector $\langle 0, -2\rangle$.

SEE EXAMPLE 2
p. 849

5. **Sports** To create the opening graphics for a televised football game, an animator reflects a picture of a football helmet across line ℓ. She then reflects its image across line m, which intersects line ℓ at a 50° angle. Describe a single transformation that moves the helmet from its starting position to its final position.

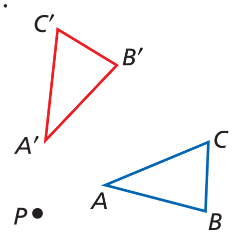


SEE EXAMPLE 3
p. 850

Copy each figure and draw two lines of reflection that produce an equivalent transformation.

6. translation: $\triangle EFG \to \triangle E'F'G'$

7. rotation with center P: $\triangle ABC \to \triangle A'B'C'$



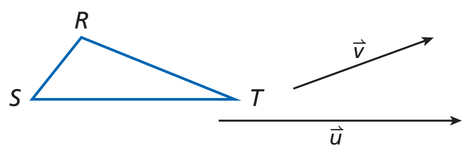
PRACTICE AND PROBLEM SOLVING

Independent Practice	
For Exercises	See Example
8–10	1
11	2
12–13	3

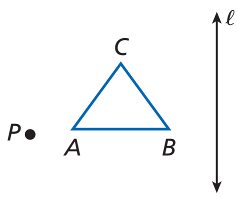
Extra Practice
Skills Practice p. S26
Application Practice p. S39

Draw the result of each composition of isometries.

8. Translate $\triangle RST$ along $\vec{u}$ and then translate it along $\vec{v}$.

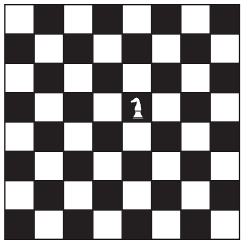


9. Rotate $\triangle ABC$ 90° about point P and then reflect it across line ℓ.



10. $\triangle GHJ$ has vertices $G(1, -1)$, $H(3, 1)$, and $J(3, -2)$. Reflect $\triangle GHJ$ across the line $y = x$ and then reflect it across the x-axis.

11. Games In chess, a knight moves in the shape of the letter L. The piece moves two spaces horizontally or vertically. Then it turns 90° in either direction and moves one more space.



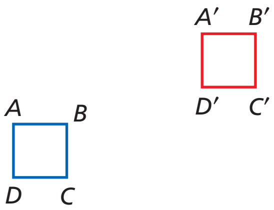
a. Describe a knight's move as a composition of transformations.

b. Copy the chessboard with the knight. Label all the positions the knight can reach in one move.

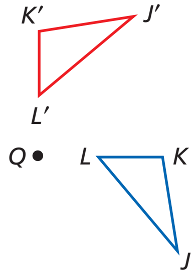
c. Label all the positions the knight can reach in two moves.

Copy each figure and draw two lines of reflection that produce an equivalent transformation.

12. translation: $ABCD \rightarrow A'B'C'D'$



13. rotation with center Q: $\triangle JKL \rightarrow \triangle J'K'L'$



14. ERROR ANALYSIS The segment with endpoints $A(4, 2)$ and $B(2, 1)$ is reflected across the y-axis. The image is reflected across the x-axis. What transformation is equivalent to the composition of these two reflections? Which solution is incorrect? Explain the error.

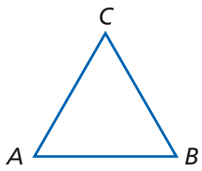
A

The image of $\overline{AB}$ reflected across the y-axis has endpoints $(-2, 1)$ and $(-4, 2)$. The image of $\overline{A'B'}$ reflected across the x-axis has endpoints $(-2, -1)$ and $(-4, -2)$. The reflections are equivalent to a translation along the vector $\langle -6, -3 \rangle$.

B

The angle between the x-axis and the y-axis is 90°. Therefore the composition of the two reflections is equivalent to a rotation about the origin by an angle measure of twice 90°, or 180°.

15. Equilateral $\triangle ABC$ is reflected across $\overline{AB}$. Then its image is translated along $\overrightarrow{BC}$. Copy $\triangle ABC$ and draw its final image.



Tell whether each statement is sometimes, always, or never true.

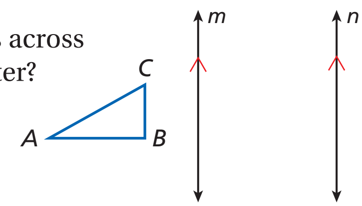
16. The composition of two reflections is equivalent to a rotation.

17. An isometry changes the size of a figure.

18. The composition of two isometries is an isometry.

19. A rotation is equivalent to a composition of two reflections.

20. Critical Thinking Given a composition of reflections across two parallel lines, does the order of the reflections matter? For example, does reflecting $\triangle ABC$ across m and then its image across n give the same result as reflecting $\triangle ABC$ across n and then its image across m? Explain.



21. Write About It Under a glide reflection, $\triangle RST \rightarrow \triangle R'S'T'$. The vertices of $\triangle RST$ are $R(-3, -2)$, $S(-1, -2)$, and $T(-1, 0)$. The vertices of $\triangle R'S'T'$ are $R'(2, 2)$, $S'(4, 2)$, and $T'(4, 0)$. Describe the reflection and translation that make up the glide reflection.

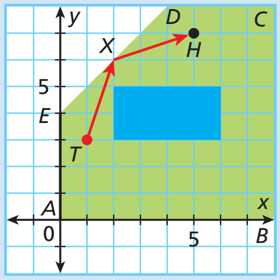
CONCEPT CONNECTION

22. This problem will prepare you for the Concept Connection on page 854.

The figure shows one hole of a miniature golf course where T is the tee and H is the hole.

a. Yuriko makes a hole in one as shown by the red arrows. Write the ball's path as a composition of translations.

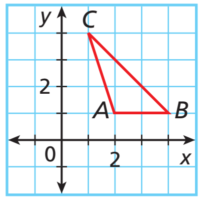
b. Find a different way to make a hole in one, and write the ball's path as a composition of translations.



STANDARDIZED TEST PREP

23. △*ABC* is reflected across the *y*-axis. Then its image is rotated 90° about the origin. What are the coordinates of the final image of point *A* under this composition of transformations?

Ⓐ (−1, −2) Ⓑ (−2, 1) Ⓒ (1, 2) Ⓓ (−2, −1)



24. Which composition of transformations maps △*ABC* into the fourth quadrant?

Ⓕ Reflect across the *x*-axis and then reflect across the *y*-axis.

Ⓖ Rotate about the origin by 180° and then reflect across the *y*-axis.

Ⓗ Translate along the vector ⟨−5, 0⟩ and then rotate about the origin by 90°.

Ⓙ Rotate about the origin by 90° and then translate along the vector ⟨1, −2⟩.

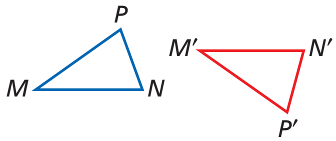
25. Which is equivalent to the composition of two translations?

Ⓐ Reflection Ⓑ Rotation Ⓒ Translation Ⓓ Glide reflection

CHALLENGE AND EXTEND

26. The point $A(3, 1)$ is rotated 90° about the point $P(-1, 2)$ and then reflected across the line $y = 5$. Find the coordinates of the image A'.

27. For any two congruent figures in a plane, one can be transformed to the other by a composition of no more than three reflections. Copy the figure. Show how to find a composition of three reflections that maps △*MNP* to △*M′N′P′*.



28. A figure in the coordinate plane is reflected across the line $y = x + 1$ and then across the line $y = x + 3$. Find a translation vector that is equivalent to the composition of the reflections. Write the vector in component form.

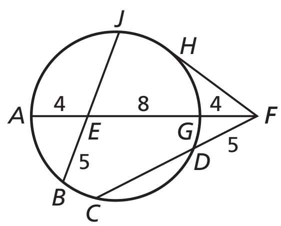
SPIRAL REVIEW

Determine whether the set of ordered pairs represents a function. *(Previous course)*

29. $\{(-6, -5), (-1, 0), (0, -5), (1, 0)\}$ **30.** $\{(-3, -1), (1, 2), (-3, 1), (5, 10)\}$

Find the length of each segment. *(Lesson 11-6)*

31. $\overline{EJ}$ **32.** $\overline{CD}$ **33.** $\overline{FH}$



Determine the coordinates of each point after a rotation about the origin by the given angle of rotation. *(Lesson 12-3)*

34. $F(2, 3)$; 90° **35.** $N(-1, -3)$; 180° **36.** $Q(-2, 0)$; 90°

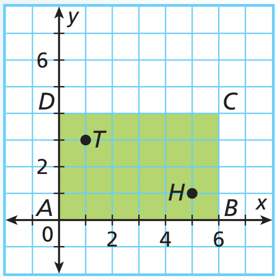
CHAPTER 12

CONCEPT CONNECTION

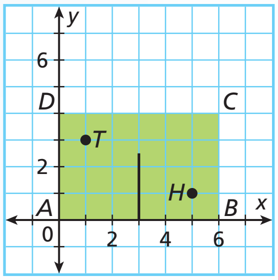
SECTION 12A

Congruence Transformations

A Hole in One The figure shows a plan for one hole of a miniature golf course. The tee is at point T and the hole is at point H. Each unit of the coordinate plane represents one meter.



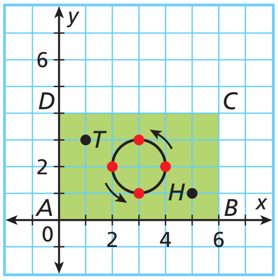
1. When a player hits the ball in a straight line from T to H, the path of the ball can be represented by a translation. What is the translation vector? How far does the ball travel? Round to the nearest tenth.



2. The designer of the golf course decides to make the hole more difficult by placing a barrier between the tee and the hole, as shown. To make a hole in one, a player must hit the ball so that it bounces off wall $\overline{DC}$. What point along the wall should a player aim for? Explain.

3. Write the path of the ball in Problem 2 as a composition of two translations. What is the total distance that the ball travels in this case? Round to the nearest tenth.

4. The designer decides to remove the barrier and put a revolving obstacle between the tee and the hole. The obstacle consists of a turntable with four equally spaced pillars, as shown. The designer wants the turntable to make one complete rotation in 16 seconds. What should be the coordinates of the pillar at $(4, 2)$ after 2 seconds?

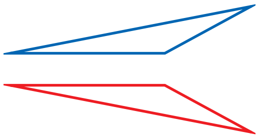


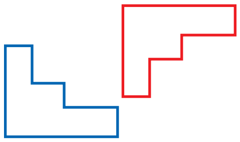
READY TO GO ON?

Quiz for Lessons 12-1 Through 12-4

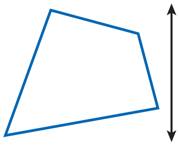
12-1 Reflections

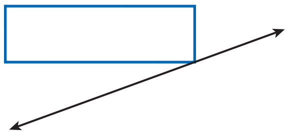
Tell whether each transformation appears to be a reflection.

1.


2.


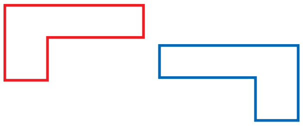
Copy each figure and the line of reflection. Draw the reflection of the figure across the line.

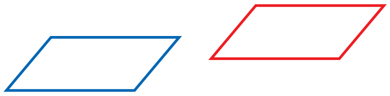
3.


4.


12-2 Translations

Tell whether each transformation appears to be a translation.

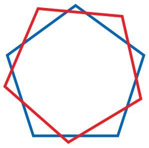
5.


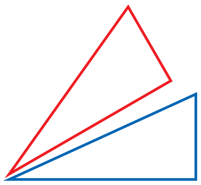
6.


7. A landscape architect represents a flower bed by a polygon with vertices $(1, 0)$, $(4, 0)$, $(4, 2)$, and $(1, 2)$. She decides to move the flower bed to a new location by translating it along the vector $\langle -4, -3 \rangle$. Draw the flower bed in its final position.

12-3 Rotations

Tell whether each transformation appears to be a rotation.

8.


9.


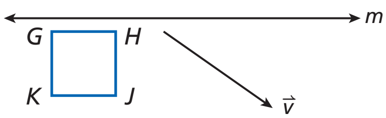
Rotate the figure with the given vertices about the origin using the given angle of rotation.

10. $A(1, 0)$, $B(4, 1)$, $C(3, 2)$; $180°$

11. $R(-2, 0)$, $S(-2, 4)$, $T(-3, 4)$, $U(-3, 0)$; $90°$

12-4 Compositions of Transformations

12. Draw the result of the following composition of transformations. Translate *GHJK* along $\vec{v}$ and then reflect it across line *m*.



13. $\triangle ABC$ with vertices $A(1, 0)$, $B(1, 3)$, and $C(2, 3)$ is reflected across the y-axis, and then its image is reflected across the x-axis. Describe a single transformation that moves the triangle from its starting position to its final position.

12-5 Symmetry

Objective
Identify and describe symmetry in geometric figures.

Vocabulary
symmetry
line symmetry
line of symmetry
rotational symmetry

California Standards
22.0 Students know the effect of rigid motions on figures in the coordinate plane and **space, including rotations,** translations, **and reflections.**

Who uses this?
Marine biologists use symmetry to classify diatoms.

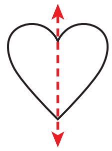
Diatoms are microscopic algae that are found in aquatic environments. Scientists use a system that was developed in the 1970s to classify diatoms based on their *symmetry*.

A figure has **symmetry** if there is a transformation of the figure such that the image coincides with the preimage.

Know it! Note

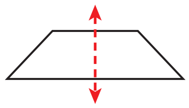
Line Symmetry

A figure has **line symmetry** (or reflection symmetry) if it can be reflected across a line so that the image coincides with the preimage. The **line of symmetry** (also called the axis of symmetry) divides the figure into two congruent halves.

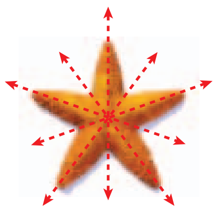


EXAMPLE 1 **Identifying Line Symmetry**

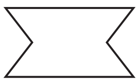
Tell whether each figure has line symmetry. If so, copy the shape and draw all lines of symmetry.

A 
yes; one line of symmetry


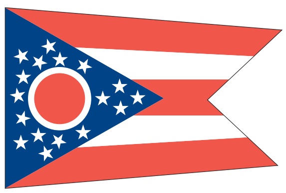
B 
no line symmetry

C 
yes; five lines of symmetry


CHECK IT OUT! **Tell whether each figure has line symmetry. If so, copy the shape and draw all lines of symmetry.**

1a. 

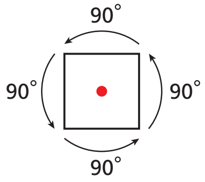
1b. B

1c. 

Rotational Symmetry

A figure has **rotational symmetry** (or *radial symmetry*) if it can be rotated about a point by an angle greater than 0° and less than 360° so that the image coincides with the preimage.

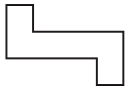
The *angle of rotational symmetry* is the smallest angle through which a figure can be rotated to coincide with itself. The number of times the figure coincides with itself as it rotates through 360° is called the *order* of the rotational symmetry.



Angle of rotational symmetry: 90°
Order: 4

EXAMPLE 2 Identifying Rotational Symmetry

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

A 

yes; 180°; order: 2

B 

no rotational symmetry

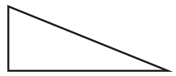
C 

yes; 60°; order: 6

CHECK IT OUT! **Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.**

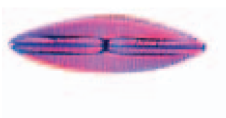
2a. 

2b. 

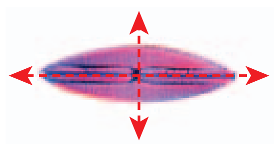
2c. 

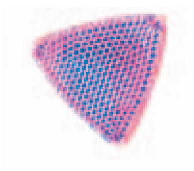
EXAMPLE 3 *Biology Application*

Describe the symmetry of each diatom. Copy the shape and draw any lines of symmetry. If there is rotational symmetry, give the angle and order.

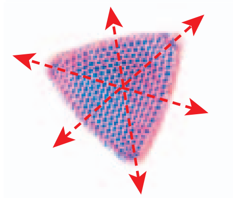
A 

line symmetry and rotational symmetry; angle of rotational symmetry: 180°; order: 2

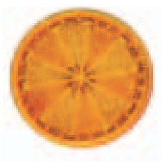


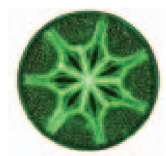
B 

line symmetry and rotational symmetry; angle of rotational symmetry: 120°; order: 3

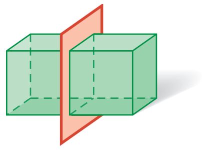


CHECK IT OUT! **Describe the symmetry of each diatom. Copy the shape and draw any lines of symmetry. If there is rotational symmetry, give the angle and order.**

3a. 

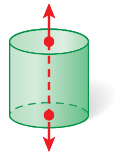
3b. 

A three-dimensional figure has *plane symmetry* if a plane can divide the figure into two congruent reflected halves.



Plane symmetry

A three-dimensional figure has *symmetry about an axis* if there is a line about which the figure can be rotated (by an angle greater than 0° and less than 360°) so that the image coincides with the preimage.

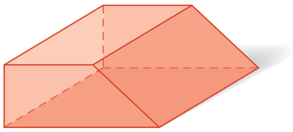


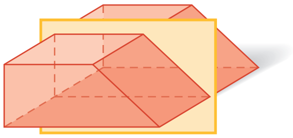
Symmetry about an axis

EXAMPLE 4 Identifying Symmetry in Three Dimensions

Tell whether each figure has plane symmetry, symmetry about an axis, or neither.

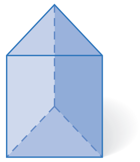
A **trapezoidal prism**

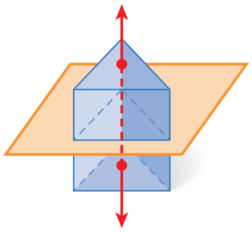




plane symmetry

B **equilateral triangular prism**

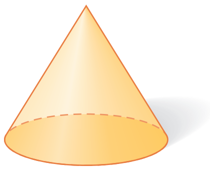




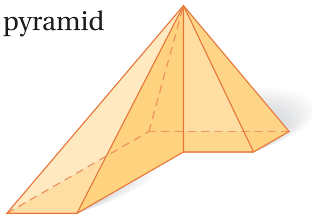
plane symmetry and symmetry about an axis

CHECK IT OUT! **Tell whether each figure has plane symmetry, symmetry about an axis, or no symmetry.**

4a. cone



4b. pyramid

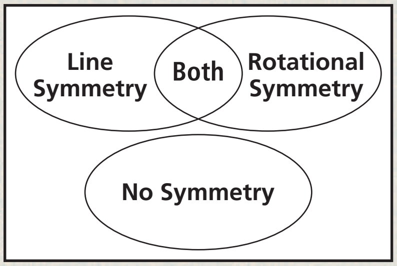


THINK AND DISCUSS

1. Explain how you could use scissors and paper to cut out a shape that has line symmetry.

2. Describe how you can find the angle of rotational symmetry for a regular polygon with *n* sides.

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each region, draw a figure with the given type of symmetry.



Know it! Note

12-5 Exercises

California Standards
8.0, 22.0, 7AF3.1

go.hrw.com
Homework Help Online
KEYWORD: MG7 12-5
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. Describe the *line of symmetry* of an isosceles triangle.

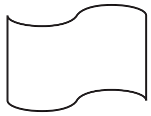
2. The capital letter T has __?__ . (*line symmetry* or *rotational symmetry*)

SEE EXAMPLE 1
p. 856

Tell whether each figure has line symmetry. If so, copy the shape and draw all lines of symmetry.

3. 

4. 

5. 

SEE EXAMPLE 2
p. 857

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

6. 

7. 

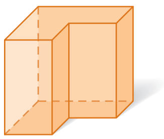
8. 

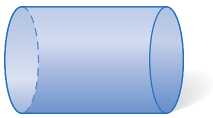
SEE EXAMPLE 3
p. 857

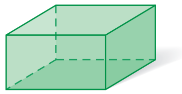
9. **Architecture** The Pentagon in Alexandria, Virginia, is the world's largest office building. Copy the shape of the building and draw all lines of symmetry. Give the angle and order of rotational symmetry.

SEE EXAMPLE 4
p. 858

Tell whether each figure has plane symmetry, symmetry about an axis, or neither.

10. prism 

11. cylinder 

12. rectangular prism 

PRACTICE AND PROBLEM SOLVING

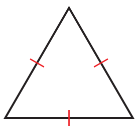
Independent Practice

For Exercises	See Example
13–15	1
16–18	2
19	3
20–22	4

Extra Practice
Skills Practice p. S27
Application Practice p. S39

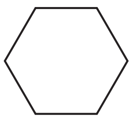
Tell whether each figure has line symmetry. If so, copy the shape and draw all lines of symmetry.

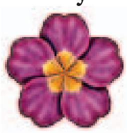
13. 

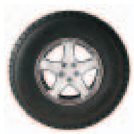
14. 

15. 

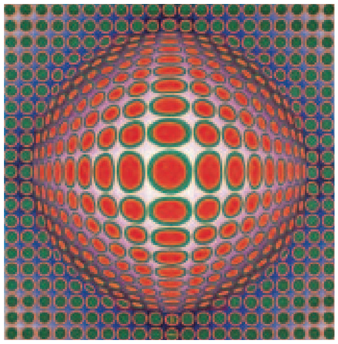
Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

16. 

17. 

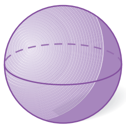
18. 

19. **Art** *Op art* is a style of art that uses optical effects to create an impression of movement in a painting or sculpture. The painting at right, *Vega-Tek*, by Victor Vasarely, is an example of op art. Sketch the shape in the painting and draw any lines of symmetry. If there is rotational symmetry, give the angle and order.

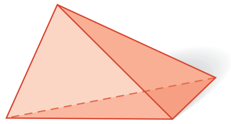


Tell whether each figure has plane symmetry, symmetry about an axis, or neither.

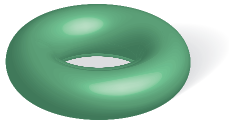
20. sphere



21. triangular pyramid



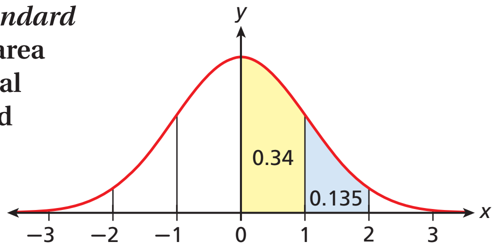
22. torus



Draw a triangle with the following number of lines of symmetry. Then classify the triangle.

23. exactly one line of symmetry
24. three lines of symmetry
25. no lines of symmetry

Data Analysis **The graph shown, called the *standard normal curve*, is used in statistical analysis. The area under the curve is 1 square unit. There is a vertical line of symmetry at $x = 0$. The areas of the shaded regions are indicated on the graph.**

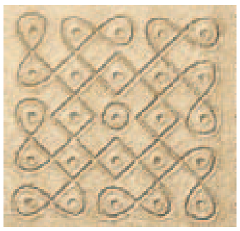


26. Find the area under the curve for $x > 0$.
27. Find the area under the curve for $x > 3$.
28. If a point under the curve is selected at random, what is the probability that the x-value of the point will be between -1 and 1?

Tell whether the figure with the given vertices has line symmetry and/or rotational symmetry. Give the angle and order if there is rotational symmetry. Draw the figure and any lines of symmetry.

29. $A(-2, 2)$, $B(2, 2)$, $C(1, -2)$, $D(-1, -2)$
30. $R(-3, 3)$, $S(3, 3)$, $T(3, -3)$, $U(-3, -3)$
31. $J(4, 4)$, $K(-2, 2)$, $L(2, -2)$
32. $A(3, 1)$, $B(0, 2)$, $C(-3, 1)$, $D(-3, -1)$, $E(0, -2)$, $F(3, -1)$

33. **Art** The Chokwe people of Angola are known for their traditional sand designs. These complex drawings are traced out to illustrate stories that are told at evening gatherings. Classify the symmetry of the Chokwe design shown.



Algebra **Graph each function. Tell whether the graph has line symmetry and/or rotational symmetry. If there is rotational symmetry, give the angle and order. Write the equations of any lines of symmetry.**

34. $y = x^2$
35. $y = (x - 2)^2$
36. $y = x^3$

CONCEPT CONNECTION

37. This problem will prepare you for the Concept Connection on page 880.

This woodcut, entitled *Circle Limit III*, was made by Dutch artist M. C. Escher.

a. Does the woodcut have line symmetry? If so, describe the lines of symmetry. If not, explain why not.

b. Does the woodcut have rotational symmetry? If so, give the angle and order of the symmetry. If not, explain why not.

c. Does your answer to part **b** change if color is not taken into account? Explain.

Classify the quadrilateral that meets the given conditions. First make a conjecture and then verify your conjecture by drawing a figure.

38. two lines of symmetry perpendicular to the sides and order-2 rotational symmetry

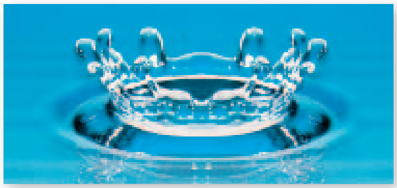
39. no line symmetry and order-2 rotational symmetry

40. two lines of symmetry through opposite vertices and order-2 rotational symmetry

41. four lines of symmetry and order-4 rotational symmetry

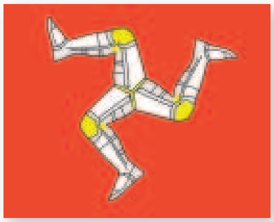
42. one line of symmetry through a pair of opposite vertices and no rotational symmetry

43. **Physics** High-speed photography makes it possible to analyze the physics behind a water splash. When a drop lands in a bowl of liquid, the splash forms a crown of evenly spaced points. What is the angle of rotational symmetry for a crown with 24 points?



44. **Critical Thinking** What can you conclude about a rectangle that has four lines of symmetry? Explain.

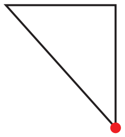
45. **Geography** The Isle of Man is an island in the Irish Sea. The island's symbol is a *triskelion* that consists of three running legs radiating from the center. Describe the symmetry of the triskelion.



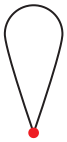
46. **Critical Thinking** Draw several examples of figures that have two perpendicular lines of symmetry. What other type of symmetry do these figures have? Make a conjecture based on your observation.

Each figure shows part of a shape with a center of rotation and a given rotational symmetry. Copy and complete each figure.

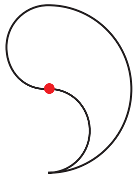
47. order 4



48. order 6



49. order 2



50. **Write About It** Explain the connection between the angle of rotational symmetry and the order of the rotational symmetry. That is, if you know one of these, explain how you can find the other.

STANDARDIZED TEST PREP

51. What is the order of rotational symmetry for the hexagon shown?

Ⓐ 2 Ⓑ 3 Ⓒ 4 Ⓓ 6

52. Which of these figures has exactly four lines of symmetry?

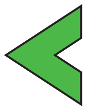
Ⓕ Regular octagon Ⓗ Isosceles triangle

Ⓖ Equilateral triangle Ⓙ Square

53. Consider the graphs of the following equations. Which graph has the y-axis as a line of symmetry?

Ⓐ $y = (x - 3)^2$ Ⓑ $y = x^3$ Ⓒ $y = x^2 - 3$ Ⓓ $y = |x + 3|$

54. Donnell designed a garden plot that has rotational symmetry, but not line symmetry. Which of these could be the shape of the plot?

Ⓕ  Ⓖ  Ⓗ  Ⓙ 

CHALLENGE AND EXTEND

55. A regular polygon has an angle of rotational symmetry of 5°. How many sides does the polygon have?

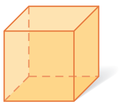
56. How many lines of symmetry does a regular n-gon have if n is even? if n is odd? Explain your reasoning.

Find the equation of the line of symmetry for the graph of each function.

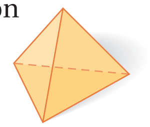
57. $y = (x + 4)^2$ **58.** $y = |x - 2|$ **59.** $y = 3x^2 + 5$

Give the number of axes of symmetry for each regular polyhedron. Describe all axes of symmetry.

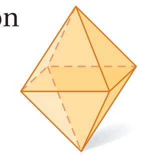
60. cube



61. tetrahedron



62. octahedron



SPIRAL REVIEW

63. Shari worked 16 hours last week and earned $197.12. The amount she earns in one week is directly proportional to the number of hours she works in that week. If Shari works 20 hours one week, how much does she earn? *(Previous course)*

Find the slant height of each figure. *(Lesson 10-5)*

64. a right cone with radius 5 in. and surface area 61π in^2

65. a square pyramid with lateral area 45 cm^2 and surface area 65.25 cm^2

66. a regular triangular pyramid with base perimeter $24\sqrt{3}$ m and surface area $120\sqrt{3}$ m^2

Determine the coordinates of the final image of the point $P(-1, 4)$ under each composition of isometries. *(Lesson 12-4)*

67. Reflect point P across the line $y = x$ and then translate it along the vector $\langle 2, -4 \rangle$.

68. Rotate point P by 90° about the origin and then reflect it across the y-axis.

69. Translate point P along the vector $\langle 1, 0 \rangle$ and then rotate it 180° about the origin.

12-6 Tessellations

Objectives

Use transformations to draw tessellations.

Identify regular and semiregular tessellations and figures that will tessellate.

Vocabulary

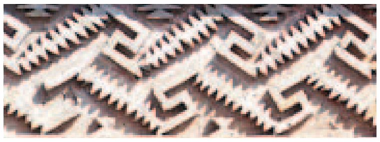
translation symmetry
frieze pattern
glide reflection symmetry
tessellation
regular tessellation
semiregular tessellation

Who uses this?

Repeating patterns play an important role in traditional Native American art.

A pattern has **translation symmetry** if it can be translated along a vector so that the image coincides with the preimage. A **frieze pattern** is a pattern that has translation symmetry along a line.

Both of the frieze patterns shown below have translation symmetry. The pattern on the right also has *glide reflection symmetry*. A pattern with **glide reflection symmetry** coincides with its image after a glide reflection.



EXAMPLE 1 Art Application

Helpful Hint

When you are given a frieze pattern, you may assume that the pattern continues forever in both directions.

Identify the symmetry in each frieze pattern.

A

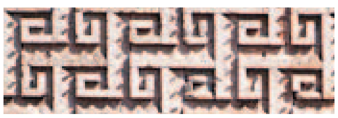
translation symmetry and glide reflection symmetry

B

translation symmetry

CHECK IT OUT! **Identify the symmetry in each frieze pattern.**

1a.



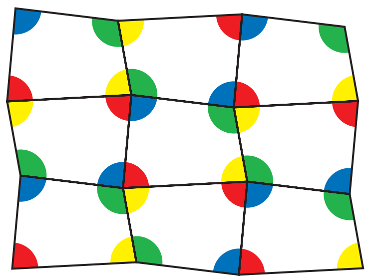
1b.

California Standards

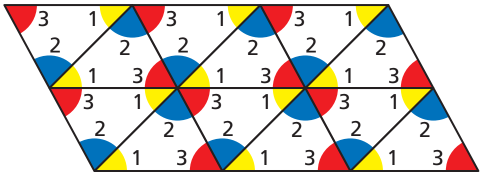
22.0 Students know the effect of rigid motions on figures in the coordinate plane and **space, including rotations, translations, and reflections.**

A **tessellation**, or *tiling*, is a repeating pattern that completely covers a plane with no gaps or overlaps. The measures of the angles that meet at each vertex must add up to 360°.

In the tessellation shown, each angle of the quadrilateral occurs once at each vertex. Because the angle measures of any quadrilateral add to 360°, any quadrilateral can be used to tessellate the plane. Four copies of the quadrilateral meet at each vertex.



The angle measures of any triangle add up to 180°. This means that any triangle can be used to tessellate a plane. Six copies of the triangle meet at each vertex, as shown.



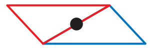
$m\angle 1 + m\angle 2 + m\angle 3 = 180°$
$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 1 + m\angle 2 + m\angle 3 = 360°$

EXAMPLE 2 Using Transformations to Create Tessellations

Copy the given figure and use it to create a tessellation.

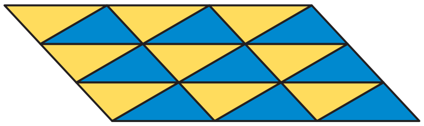
A 

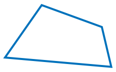
Step 1 Rotate the triangle 180° about the midpoint of one side.



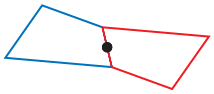
Step 2 Translate the resulting pair of triangles to make a row of triangles.

Step 3 Translate the row of triangles to make a tessellation.

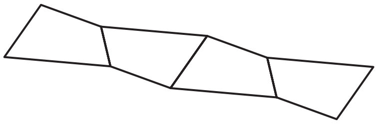


B 

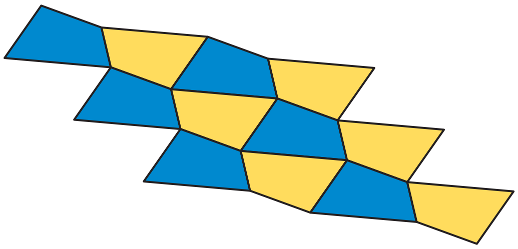
Step 1 Rotate the quadrilateral 180° about the midpoint of one side.



Step 2 Translate the resulting pair of quadrilaterals to make a row of quadrilaterals.

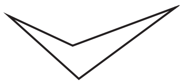


Step 3 Translate the row of quadrilaterals to make a tessellation.

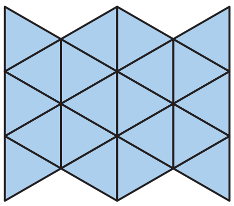


CHECK IT OUT!

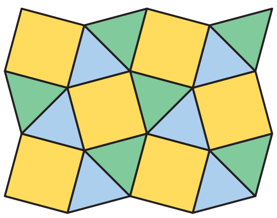
2. Copy the given figure and use it to create a tessellation.



A **regular tessellation** is formed by congruent regular polygons. A **semiregular tessellation** is formed by two or more different regular polygons, with the same number of each polygon occurring in the same order at every vertex.



Regular tessellation

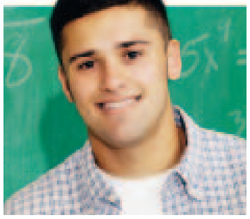


Semiregular tessellation

Every vertex has two squares and three triangles in this order: square, triangle, square, triangle, triangle.

Student to Student

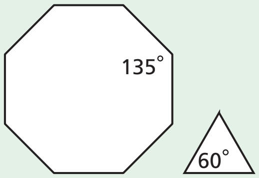
Tessellations



Ryan Gray
Sunset High School

When I need to decide if given figures can be used to tessellate a plane, I look at angle measures. To form a regular tessellation, the angle measures of a regular polygon must be a divisor of 360°. To form a semiregular tessellation, the angle measures around a vertex must add up to 360°.

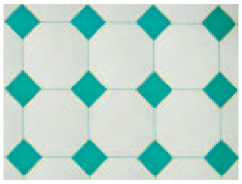
For example, regular octagons and equilateral triangles cannot be used to make a semiregular tessellation because no combination of 135° and 60° adds up to exactly 360°.



EXAMPLE 3 Classifying Tessellations

Classify each tessellation as regular, semiregular, or neither.

A

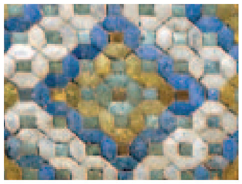


Two regular octagons and one square meet at each vertex. The tessellation is semiregular.

B

Only squares are used. The tessellation is regular.

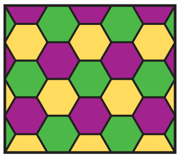
C



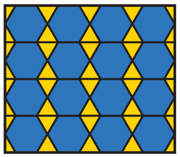
Irregular hexagons are used in the tessellation. It is neither regular nor semiregular.

CHECK IT OUT! **Classify each tessellation as regular, semiregular, or neither.**

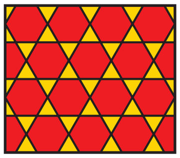
3a.



3b.



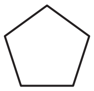
3c.



EXAMPLE 4 Determining Whether Polygons Will Tessellate

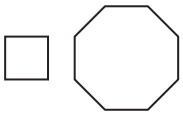
Determine whether the given regular polygon(s) can be used to form a tessellation. If so, draw the tessellation.

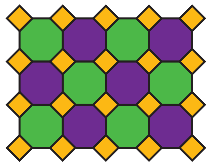
A



No; each angle of the pentagon measures 108°, and 108 is not a divisor of 360.

B

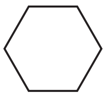




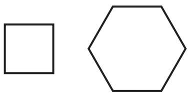
Yes; two octagons and one square meet at each vertex.
$135° + 135° + 90° = 360°$

CHECK IT OUT! **Determine whether the given regular polygon(s) can be used to form a tessellation. If so, draw the tessellation.**

4a.

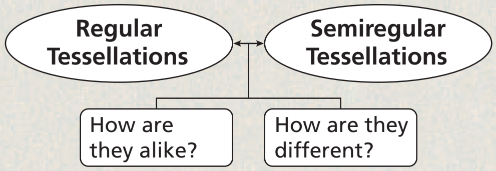


4b.



THINK AND DISCUSS

1. Explain how you can identify a frieze pattern that has glide reflection symmetry.
2. Is it possible to tessellate a plane using circles? Why or why not?
3. **GET ORGANIZED** Copy and complete the graphic organizer.



12-6 Exercises

California Standards
7.0, 22.0, 7AF4.2

go.hrw.com
Homework Help Online
KEYWORD: MG7 12-6
Parent Resources Online
KEYWORD: MG7 Parent

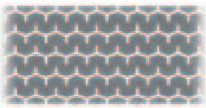
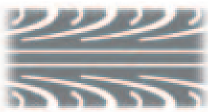
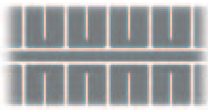
GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. Sketch a pattern that has *glide reflection symmetry*.
2. Describe a real-world example of a *regular tessellation*.

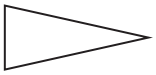
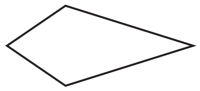
SEE EXAMPLE 1 p. 863

Transportation The tread of a tire is the part that makes contact with the ground. Various tread patterns help improve traction and increase durability. Identify the symmetry in each tread pattern.

3. 
4. 
5. 

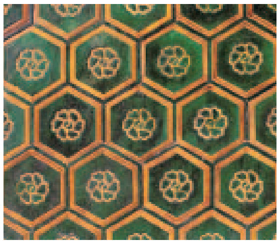
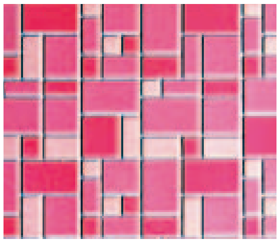
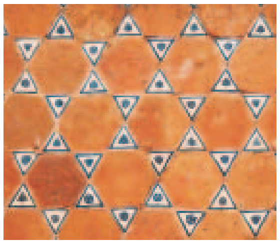
SEE EXAMPLE 2 p. 864

Copy the given figure and use it to create a tessellation.

6. 
7. 
8. 

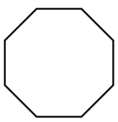
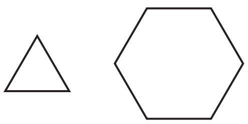
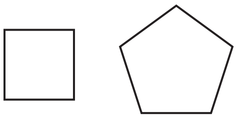
SEE EXAMPLE 3 p. 865

Classify each tessellation as regular, semiregular, or neither.

9. 
10. 
11. 

SEE EXAMPLE 4 p. 865

Determine whether the given regular polygon(s) can be used to form a tessellation. If so, draw the tessellation.

12. 
13. 
14. 

PRACTICE AND PROBLEM SOLVING

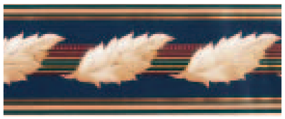
Independent Practice

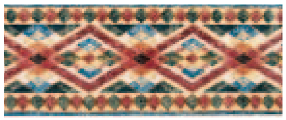
For Exercises	See Example
15–17	1
18–20	2
21–23	3
24–26	4

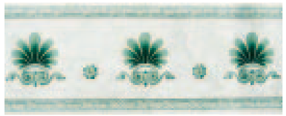
Extra Practice

Skills Practice p. S27
Application Practice p. S39

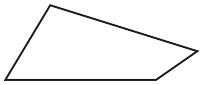
Interior Decorating **Identify the symmetry in each wallpaper border.**

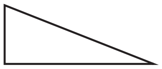
15. 

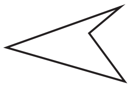
16. 

17. 

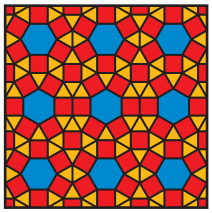
Copy the given figure and use it to create a tessellation.

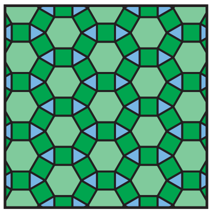
18. 

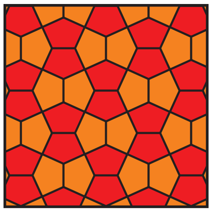
19. 

20. 

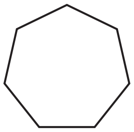
Classify each tessellation as regular, semiregular, or neither.

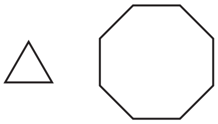
21. 

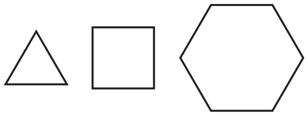
22. 

23. 

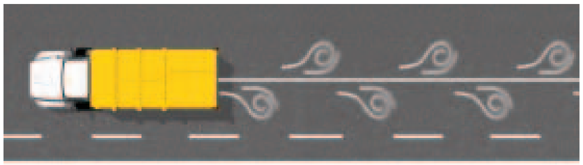
Determine whether the given regular polygon(s) can be used to form a tessellation. If so, draw the tessellation.

24. 

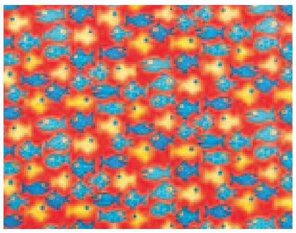
25. 

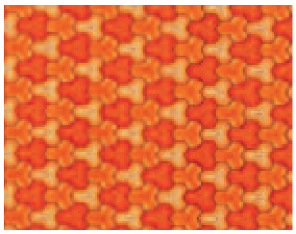
26. 

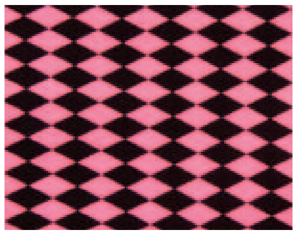
27. **Physics** A truck moving down a road creates whirling pockets of air called a *vortex train*. Use the figure to classify the symmetry of a vortex train.



Identify all of the types of symmetry (translation, glide reflection, and/or rotation) in each tessellation.

28. 

29. 

30. 

Tell whether each statement is sometimes, always, or never true.

31. A triangle can be used to tessellate a plane.

32. A frieze pattern has glide reflection symmetry.

33. The angles at a vertex of a tessellation add up to 360°.

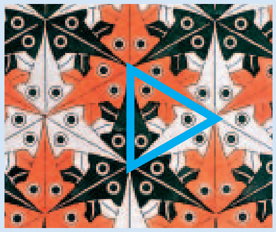
34. It is possible to use a regular pentagon to make a regular tessellation.

35. A semiregular tessellation includes scalene triangles.

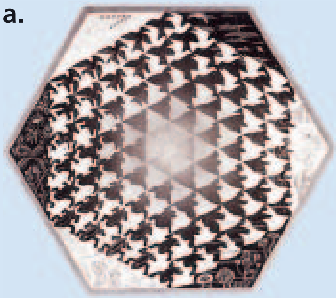
CONCEPT CONNECTION

36. This problem will prepare you for the Concept Connection on page 880.

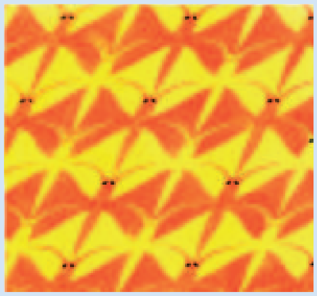
Many of the patterns in M. C. Escher's works are based on simple tessellations. For example, the pattern at right is based on a tessellation of equilateral triangles. Identify the figure upon which each pattern is based.



a.



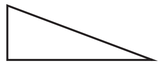
b.



Use the given figure to draw a frieze pattern with the given symmetry.

37. translation symmetry

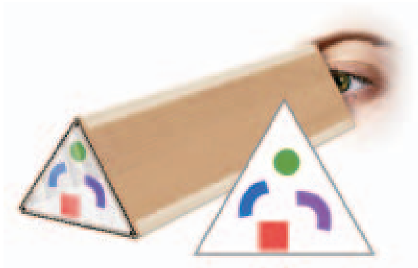
38. glide reflection symmetry



39. translation symmetry

40. glide reflection symmetry

41. Optics A kaleidoscope is formed by three mirrors joined to form the lateral surface of a triangular prism. Copy the triangular faces and reflect it over each side. Repeat to form a tessellation. Describe the symmetry of the tessellation.

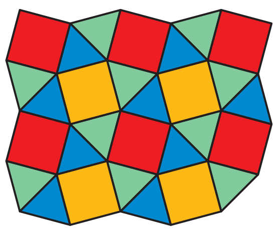


42. Critical Thinking The pattern on a soccer ball is a tessellation of a sphere using regular hexagons and regular pentagons. Can these two shapes be used to tessellate a plane? Explain your reasoning.

43. Chemistry A *polymer* is a substance made of repeating chemical units or molecules. The *repeat unit* is the smallest structure that can be repeated to create the chain. Draw the repeat unit for polypropylene, the polymer shown below.

$$-CH_2-\underset{\substack{|\\CH_3}}{CH}-CH_2-\underset{\substack{|\\CH_3}}{CH}-CH_2-\underset{\substack{|\\CH_3}}{CH}-CH_2-\underset{\substack{|\\CH_3}}{CH}-$$

44. The *dual* of a tessellation is formed by connecting the centers of adjacent polygons with segments. Copy or trace the semiregular tessellation shown and draw its dual. What type of polygon makes up the dual tessellation?



45. Write About It You can make a regular tessellation from an equilateral triangle, a square, or a regular hexagon. Explain why these are the only three regular tessellations that are possible.

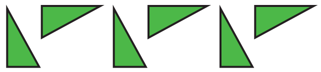
Standardized Test Prep

46. Which frieze pattern has glide reflection symmetry?

Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

47. Which shape CANNOT be used to make a regular tessellation?

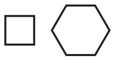
Ⓕ Equilateral triangle

Ⓖ Square

Ⓗ Regular pentagon

Ⓙ Regular hexagon

48. Which pair of regular polygons can be used to make a semiregular tessellation?

Ⓐ 

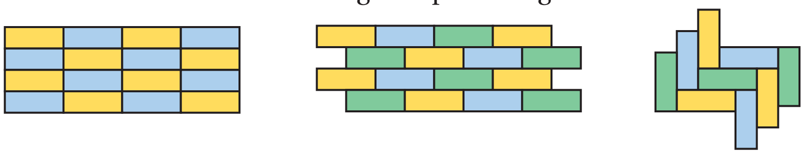
Ⓑ 

Ⓒ 

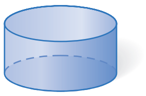
Ⓓ 

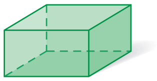
CHALLENGE AND EXTEND

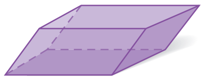
49. Some shapes can be used to tessellate a plane in more than one way. Three tessellations that use the same rectangle are shown. Draw a parallelogram and draw at least three tessellations using that parallelogram.



Determine whether each figure can be used to tessellate three-dimensional space.

50. 

51. 

52. 

SPIRAL REVIEW

53. A book is on sale for 15% off the regular price of $8.00. If Harold pays with a $10 bill and receives $2.69 in change, what is the sales tax rate on the book? *(Previous course)*

54. Louis lives 5 miles from school and jogs at a rate of 6 mph. Andrea lives 3.9 miles from school and jogs at a rate of 6.5 mph. Andrea leaves her house at 7:00 A.M. When should Louis leave his house to arrive at school at the same time as Andrea? *(Previous course)*

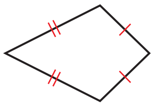
Write the equation of each circle. *(Lesson 11-7)*

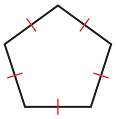
55. $\odot P$ with center $(-2, 3)$ and radius $\sqrt{5}$

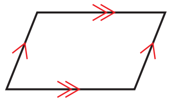
56. $\odot Q$ that passes through $(3, 4)$ and has center $(0, 0)$

57. $\odot T$ that passes through $(1, -1)$ and has center $(5, -3)$

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry. *(Lesson 12-5)*

58. 

59. 

60. 

12-6 Geometry LAB

Use with Lesson 12-6

Use Transformations to Extend Tessellations

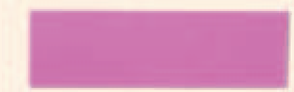
In Lesson 12-6, you saw that you can use any triangle or quadrilateral to tessellate a plane. In this lab, you will learn how to use transformations to turn these basic patterns into more-complex tessellations.

California Standards

22.0 Students know the effect of rigid motions on figures in the coordinate plane and **space, including rotations, translations, and reflections.**

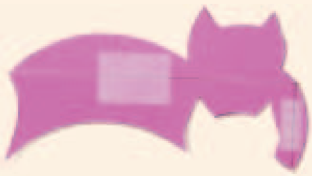
Activity 1

1. Cut a rectangle out of heavy paper.

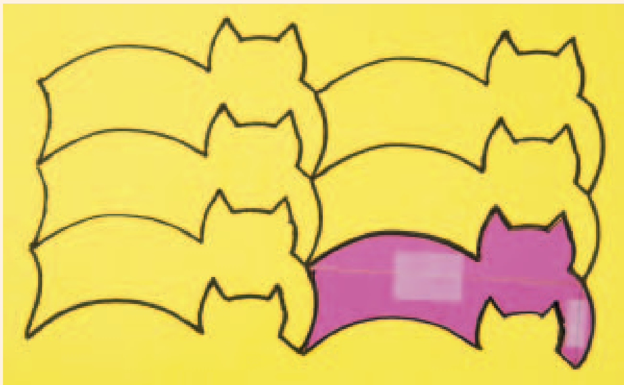


2. Cut a piece from one side of the rectangle and translate it to the opposite side. Tape it into place.

3. Repeat the process with the other pair of sides.

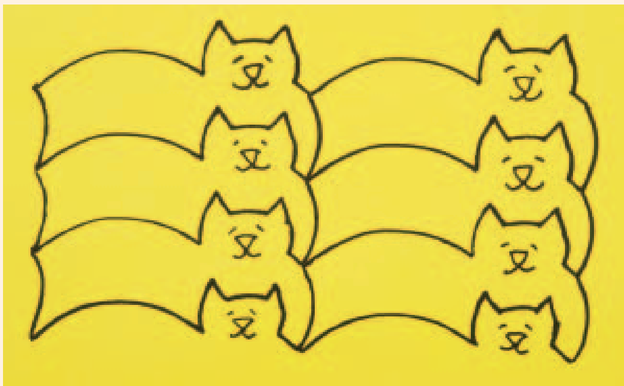


4. The resulting shape will tessellate the plane. Trace around the shape to create a tessellation.



Try This

1. Repeat Activity 1, starting with a parallelogram.
2. Repeat Activity 1, starting with a hexagon whose opposite sides are congruent and parallel.
3. Add details to one of your tessellations to make it look like a pattern of people, animals, flowers, or other objects.



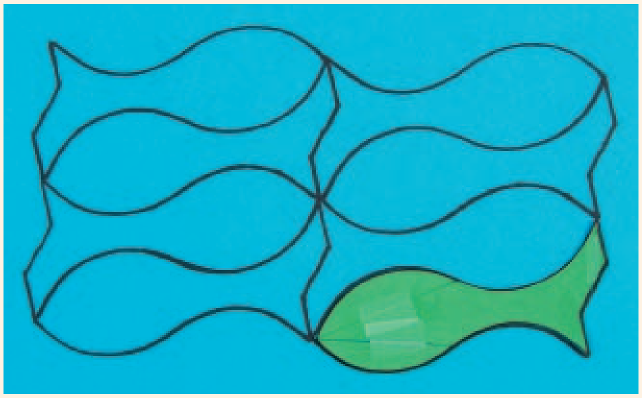
Activity 2

1. Cut a triangle out of heavy paper.

2. Find the midpoint of one side. Cut a piece from one half of this side of the triangle and rotate the piece 180°. Tape it to the other half of this side.

3. Repeat the process with the other two sides.

4. The resulting shape will tessellate the plane. Trace around the shape to create a tessellation.

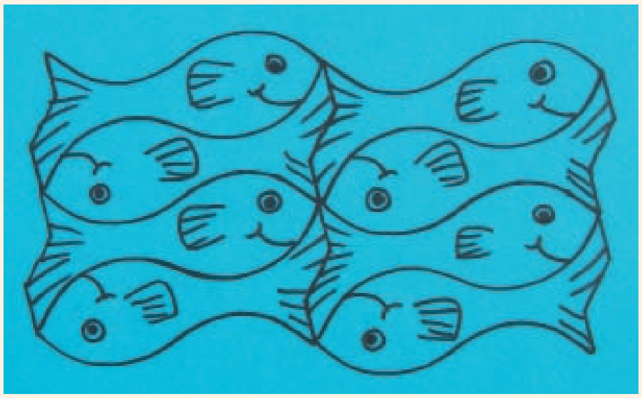


Try This

4. Repeat Activity 2, starting with a quadrilateral.

5. How is this tessellation different from the ones you created in Activity 1?

6. Add details to one of your tessellations to make it look like a pattern of people, animals, flowers, or other objects.



12-7 Dilations

Objective
Identify and draw dilations.

Vocabulary
center of dilation
enlargement
reduction

Who uses this?
Artists use dilations to turn sketches into large-scale paintings. (See Example 3.)

Recall that a dilation is a transformation that changes the size of a figure but not the shape. The image and the preimage of a figure under a dilation are similar.

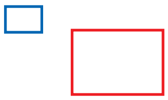
California Standards

8.0 Students know, derive, **and solve problems involving the perimeter,** circumference, **area,** volume, lateral area, and surface area **of common geometric figures.**
11.0 Students determine how changes in dimensions affect the perimeter, area, and volume **of common geometric figures** and solids.
Also covered: **16.0**

EXAMPLE 1 Identifying Dilations

Tell whether each transformation appears to be a dilation. Explain.

A



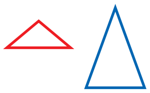
Yes; the figures are similar, and the image is not turned or flipped.

B

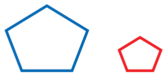
No; the figures are not similar.

CHECK IT OUT! **Tell whether each transformation appears to be a dilation.**

1a.

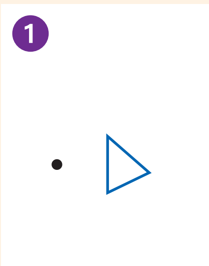


1b.



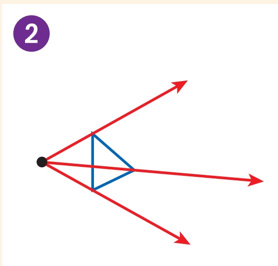
Construction Dilate a Figure by a Scale Factor of 2

1



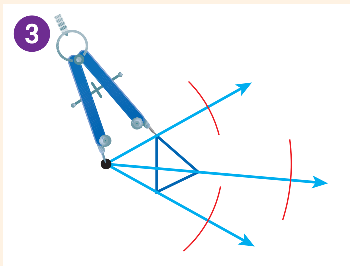
Draw a triangle and a point outside the triangle. The point is the *center of dilation*.

2



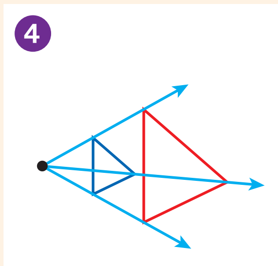
Use a straightedge to draw a line through the center of dilation and each vertex of the triangle.

3



Set the compass to the distance from the center of dilation to a vertex. Mark this distance along the line for each vertex as shown.

4



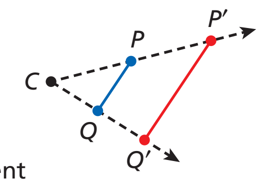
Connect the vertices of the image.

In the construction, the lines connecting points of the image with the corresponding points of the preimage all intersect at the center of dilation. Also, the distance from the center to each point of the image is twice the distance to the corresponding point of the preimage.

Dilations

A dilation, or *similarity transformation*, is a transformation in which the lines connecting every point P with its image P' all intersect at a point C, called the **center of dilation**. $\frac{CP'}{CP}$ is the same for every point P.

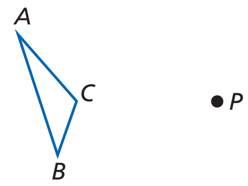
The scale factor k of a dilation is the ratio of a linear measurement of the image to a corresponding measurement of the preimage. In the figure, $k = \frac{P'Q'}{PQ}$.



A dilation enlarges or reduces all dimensions proportionally. A dilation with a scale factor greater than 1 is an **enlargement**, or *expansion*. A dilation with a scale factor greater than 0 but less than 1 is a **reduction**, or *contraction*.

EXAMPLE 2 Drawing Dilations

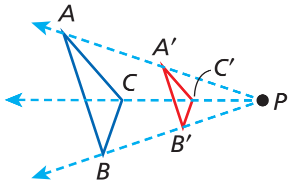
Copy the triangle and the center of dilation P. Draw the image of $\triangle ABC$ under a dilation with a scale factor of $\frac{1}{2}$.



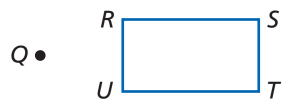
Step 1 Draw a line through P and each vertex.

Step 2 On each line, mark half the distance from P to the vertex.

Step 3 Connect the vertices of the image.



CHECK IT OUT! **2.** Copy the figure and the center of dilation. Draw the dilation of $RSTU$ using center Q and a scale factor of 3.



EXAMPLE 3 *Art Application*

An artist is creating a large painting from a photograph by dividing the photograph into squares and dilating each square by a scale factor of 4. If the photograph is 20 cm by 25 cm, what is the perimeter of the painting?

The scale factor of the dilation is 4, so a 1 cm by 1 cm square on the photograph represents a 4 cm by 4 cm square on the painting.

Find the dimensions of the painting.

$b = 4(25) = 100$ cm
$h = 4(20) = 80$ cm

Multiply each dimension by the scale factor, 4.

Find the perimeter of the painting.

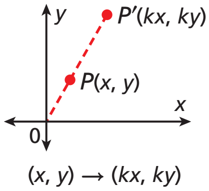
$P = 2(100 + 80) = 360$ cm   *$P = 2(b + h)$*

CHECK IT OUT! **3. What if...?** In Example 3, suppose the photograph is a square with sides of length 10 in. Find the area of the painting.

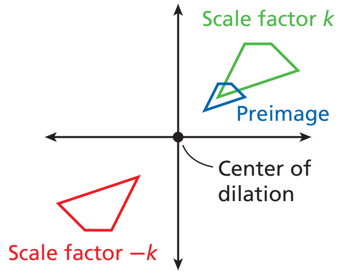
Know it! Note

Dilations in the Coordinate Plane

If $P(x, y)$ is the preimage of a point under a dilation centered at the origin with scale factor k, then the image of the point is $P'(kx, ky)$.



If the scale factor of a dilation is negative, the preimage is rotated by 180°. For $k > 0$, a dilation with a scale factor of $-k$ is equivalent to the composition of a dilation with a scale factor of k that is rotated 180° about the center of dilation.



EXAMPLE 4 Drawing Dilations in the Coordinate Plane

Draw the image of a triangle with vertices $A(-1, 1)$, $B(-2, -1)$, and $C(-1, -2)$ under a dilation with a scale factor of -2 centered at the origin.

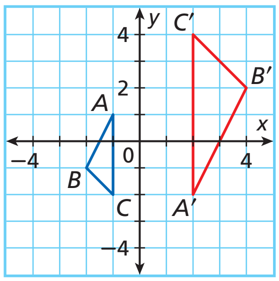
The dilation of (x, y) is $(-2x, -2y)$.

$A(-1, 1) \rightarrow A'(-2(-1), -2(1)) = A'(2, -2)$

$B(-2, -1) \rightarrow B'(-2(-2), -2(-1)) = B'(4, 2)$

$C(-1, -2) \rightarrow C'(-2(-1), -2(-2)) = C'(2, 4)$

Graph the preimage and image.



CHECK IT OUT!

4. Draw the image of a parallelogram with vertices $R(0, 0)$, $S(4, 0)$, $T(2, -2)$, and $U(-2, -2)$ under a dilation centered at the origin with a scale factor of $-\frac{1}{2}$.

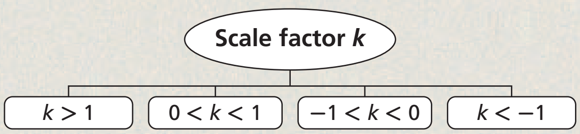
THINK AND DISCUSS

1. Given a triangle and its image under a dilation, explain how you could use a ruler to find the scale factor of the dilation.

2. A figure is dilated by a scale factor of k, and then the image is rotated 180° about the center of dilation. What single transformation would produce the same image?

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, describe the dilation with the given scale factor.

Know it! Note



12-7 Exercises

California Standards
8.0, 10.0, 11.0, 16.0, 22.0, 7AF4.2, 7MG1.2, 7MG2.0

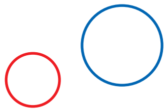
go.hrw.com
Homework Help Online
KEYWORD: MG7 12-7
Parent Resources Online
KEYWORD: MG7 Parent

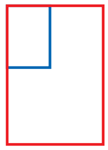
GUIDED PRACTICE

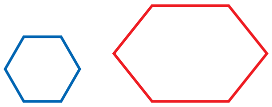
1. **Vocabulary** What are the *center of dilation* and scale factor for the transformation $(x, y) \rightarrow (3x, 3y)$?

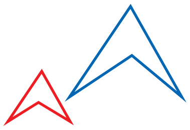
SEE EXAMPLE 1
p. 872

Tell whether each transformation appears to be a dilation.

2. 

3. 

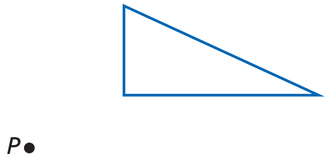
4. 

5. 

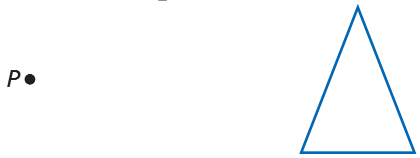
SEE EXAMPLE 2
p. 873

Copy each triangle and center of dilation *P*. Draw the image of the triangle under a dilation with the given scale factor.

6. Scale factor: 2



7. Scale factor: $\frac{1}{2}$



SEE EXAMPLE 3
p. 873

8. **Architecture** A blueprint shows a reduction of a room using a scale factor of $\frac{1}{50}$. In the blueprint, the room's length is 8 in., and its width is 6 in. Find the perimeter of the room.

SEE EXAMPLE 4
p. 874

Draw the image of the figure with the given vertices under a dilation with the given scale factor centered at the origin.

9. $A(1, 0)$, $B(2, 2)$, $C(4, 0)$; scale factor: 2
10. $J(-2, 2)$, $K(4, 2)$, $L(4, -2)$, $M(-2, -2)$; scale factor: $\frac{1}{2}$
11. $D(-3, 3)$, $E(3, 6)$, $F(3, 0)$; scale factor: $-\frac{1}{3}$
12. $P(-2, 0)$, $Q(-1, 0)$, $R(0, -1)$, $S(-3, -1)$; scale factor: -2

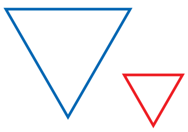
PRACTICE AND PROBLEM SOLVING

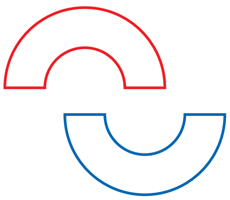
Independent Practice

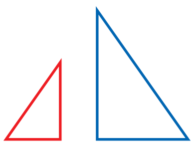
For Exercises	See Example
13–16	1
17–18	2
19	3
20–23	4

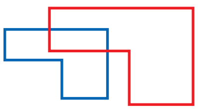
Extra Practice
Skills Practice p. S27
Application Practice p. S39

Tell whether each transformation appears to be a dilation.

13. 

14. 

15. 

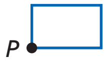
16. 

Art LINK

Mosaic is an ancient art form that is over 4000 years old and is still popular today. Creators of early mosaics used pebbles and other objects, but mosaic titles, or *tesserae,* have been in use since at least 200 B.C.E.

Copy each rectangle and the center of dilation *P*. Draw the image of the rectangle under a dilation with the given scale factor.

17. scale factor: 3



18. scale factor: $\frac{1}{2}$

19. **Art** Jeff is making a mosaic by gluing 1 cm square tiles onto a photograph. He starts with a 6 cm by 8 cm rectangular photo and enlarges it by a scale factor of 1.5. How many tiles will Jeff need in order to cover the enlarged photo?

Draw the image of the figure with the given vertices under a dilation with the given scale factor centered at the origin.

20. $M(0, 3)$, $N(6, 0)$, $P(0, -3)$; scale factor: $-\frac{1}{3}$

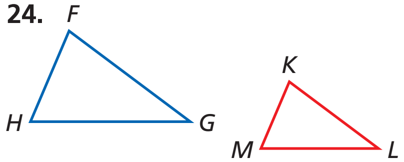
21. $A(-1, 3)$, $B(1, 1)$, $C(-4, 1)$; scale factor: -1

22. $R(1, 0)$, $S(2, 0)$, $T(2, -2)$, $U(-1, -2)$; scale factor: -2

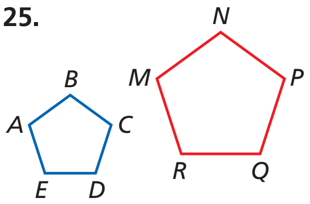
23. $D(4, 0)$, $E(2, -4)$, $F(-2, -4)$, $G(-4, 0)$, $H(-2, 4)$, $J(2, 4)$; scale factor: $-\frac{1}{2}$

Each figure shows the preimage (blue) and image (red) under a dilation. Write a similarity statement based on the figure.

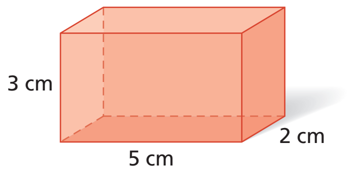
24.



25.

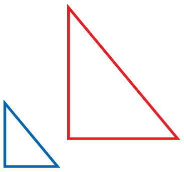


26. The rectangular prism shown is enlarged by a dilation with scale factor 4. Find the surface area and volume of the image.

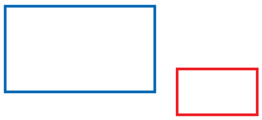


Copy each figure and locate the center of dilation.

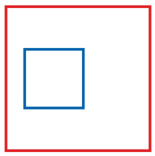
27.



28.



29.



CONCEPT CONNECTION

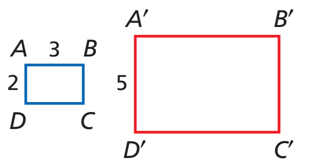
30. This problem will prepare you for the Concept Connection on page 880.

This lithograph, *Drawing Hands*, was made by M. C. Escher in 1948.

a. In the original drawing, the rectangular piece of paper from which the hands emerge measures 27.6 cm by 19.9 cm. On a poster of the drawing, the paper is 82.8 cm long. What is the scale factor of the dilation that was used to make the poster?

b. What is the area of the paper on the poster?

31. **ERROR ANALYSIS** Rectangle $A'B'C'D'$ is the image of rectangle $ABCD$ under a dilation. Which calculation of the area of rectangle $A'B'C'D'$ is incorrect? Explain the error.



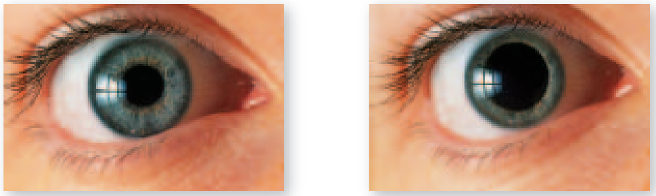
A

The scale factor of the dilation is $\frac{5}{2}$, or 2.5, so the length of $\overline{A'B'}$ must be $2.5 \times 3 = 7.5$. Then the area of rectangle $A'B'C'D'$ is $5 \times 7.5 = 37.5$.

B

The area of rectangle $ABCD$ is $2 \times 3 = 6$, and the scale factor of the dilation is $\frac{5}{2}$, or 2.5. Therefore the area of rectangle $A'B'C'D'$ is $2.5 \times 6 = 15$.

32. **Optometry** The pupil is the circular opening that allows light into the eye.

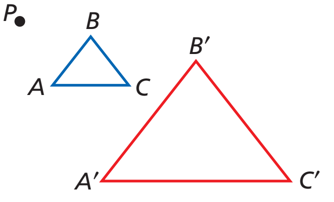


a. An optometrist dilates a patient's pupil from 6 mm to 8 mm. What is the scale factor for this dilation?

b. To the nearest tenth, find the area of the pupil before and after the dilation.

c. As a percentage, how much more light is admitted to the eye after the dilation?

33. **Estimation** In the diagram, $\triangle ABC \rightarrow \triangle A'B'C'$ under a dilation with center P.



a. Estimate the scale factor of the dilation.

b. Explain how you can use a ruler to make measurements and calculate the scale factor.

c. Use the method from part **b** to calculate the scale factor. How does your result compare to your estimate?

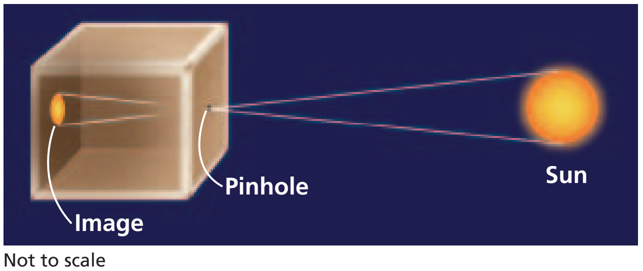
34. $\triangle ABC$ has vertices $A(-1, 1)$, $B(2, 1)$, and $C(2, 2)$.

a. Draw the image of $\triangle ABC$ under a dilation centered at the origin with scale factor 2 followed by a reflection across the x-axis.

b. Draw the image of $\triangle ABC$ under a reflection across the x-axis followed by a dilation centered at the origin with scale factor 2.

c. Compare the results of parts **a** and **b**. Does the order of the transformations matter?

35. **Astronomy** The image of the sun projected through the hole of a pinhole camera (the center of dilation) has a diameter of $\frac{1}{4}$ in. The diameter of the sun is 870,000 mi. What is the scale factor of the dilation?



Not to scale

Multi-Step $\triangle ABC$ **with vertices** $A(-2, 2)$, $B(1, 3)$, **and** $C(1, -1)$ **is transformed by a dilation centered at the origin. For each given image point, find the scale factor of the dilation and the coordinates of the remaining image points. Graph the preimage and image on a coordinate plane.**

36. $A'(-4, 4)$ **37.** $C'(-2, 2)$ **38.** $B'(-1, -3)$

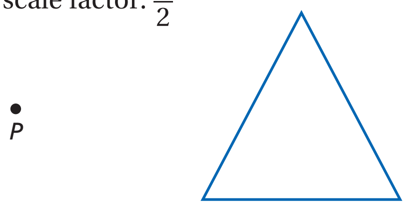
39. **Critical Thinking** For what values of the scale factor is the image of a dilation congruent to the preimage? Explain.

40. **Write About It** When is a dilation equivalent to a rotation by 180°? Why?

41. **Write About It** Is the composition of a dilation with scale factor m followed by a dilation with scale factor n equivalent to a single dilation with scale factor mn? Explain your reasoning.

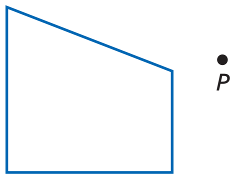
Construction **Copy each figure. Then use a compass and straightedge to construct the dilation of the figure with the given scale factor and point** P **as the center of dilation.**

42. scale factor: $\frac{1}{2}$

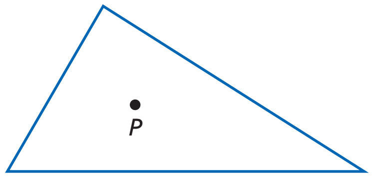


43. scale factor: 2

44. scale factor: -1

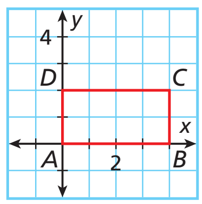


45. scale factor: -2



Standardized Test Prep

46. Rectangle $ABCD$ is transformed by a dilation centered at the origin. Which scale factor produces an image that has a vertex at $(0, -2)$?



Ⓐ $-\frac{1}{2}$ Ⓒ -2

Ⓑ -1 Ⓓ -4

47. Rectangle $ABCD$ is enlarged under a dilation centered at the origin with scale factor 2.5. What is the perimeter of the image?

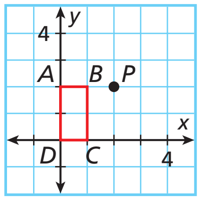
Ⓕ 15 Ⓖ 24 Ⓗ 30 Ⓙ 50

48. **Gridded Response** What is the scale factor of a dilation centered at the origin that maps the point $(-2, 3)$ to the point $(-8.4, 12.6)$?

49. **Short Response** The rules for a photo contest state that entries must have an area no greater than 100 cm^2. Amber has a 6 cm by 8 cm digital photo, and she uses software to enlarge it by a scale factor of 1.5. Does the enlargement meet the requirements of the contest? Show the steps you used to decide your answer.

CHALLENGE AND EXTEND

50. Rectangle *ABCD* has vertices $A(0, 2)$, $B(1, 2)$, $C(1, 0)$, and $D(0, 0)$.



- **a.** Draw the image of *ABCD* under a dilation centered at point *P* with scale factor 2.
- **b.** Describe the dilation in part **a** as a composition of a dilation centered at the origin followed by a translation.
- **c.** Explain how a dilation with scale factor *k* and center of dilation (a, b) can be written as a composition of a dilation centered at the origin and a translation.

51. The equation of line ℓ is $y = -x + 2$. Find the equation of the image of line ℓ after a dilation centered at the origin with scale factor 3.

SPIRAL REVIEW

52. Jerry has a part-time job waiting tables. He kept records comparing the number of customers served to his total amount of tips for the day. If this trend continues, how many customers would he need to serve in order to make $68.00 in tips for the day? *(Previous course)*

Customers per Day	15	20	25	30
Tips per Day ($)	20	28	36	44

Find the perimeter and area of each polygon with the given vertices. *(Lesson 9-4)*

53. $J(-3, -2)$, $K(0, 2)$, $L(7, 2)$, and $M(4, -2)$

54. $D(-3, 0)$, $E(1, 2)$, and $F(-1, -4)$

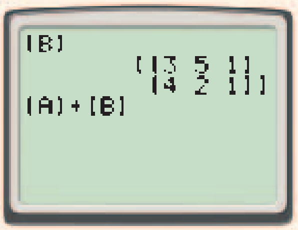
Determine whether the polygons can be used to tessellate a plane. *(Lesson 12-6)*

55. a right triangle and a square

56. a regular nonagon and an equilateral triangle

Using Technology

Use a graphing calculator to complete the following.



1. $\triangle ABC$ with vertices $A(3, 4)$, $B(5, 2)$, and $C(1, 1)$ can be represented by the point matrix $\begin{bmatrix} 3 & 5 & 1 \\ 4 & 2 & 1 \end{bmatrix}$. Enter these values into matrix **[B]** on your calculator. (See page 846.)

2. The matrix $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ can be used to perform a dilation with scale factor 2. Enter these values into matrix **[A]** on your calculator and find **[A] * [B].** Graph the triangle represented by the resulting point matrix.

3. Make a conjecture about the matrix that could be used to perform a dilation with scale factor $-\frac{1}{2}$. Enter the values into matrix **[A]** on your calculator.

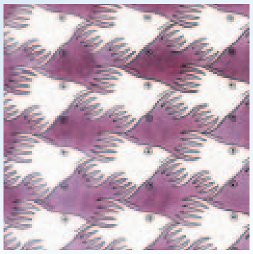
4. Test your conjecture by finding **[A] * [B]** and graphing the triangle represented by the resulting point matrix.

CONCEPT CONNECTION

Patterns

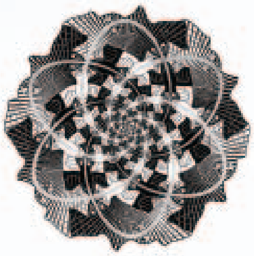
Tessellation Fascination A museum is planning an exhibition of works by the Dutch artist M. C. Escher (1898–1972). The exhibit will include the five drawings shown here.

1. Tell whether each drawing has parallel lines of symmetry, intersecting lines of symmetry, or no lines of symmetry.
2. Tell whether each drawing has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.
3. Tell whether each drawing is a tessellation. If so, identify the basic figure upon which the tessellation is based.

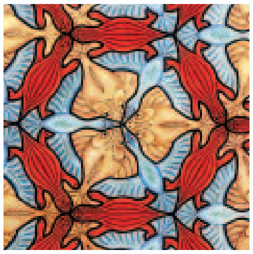


Drawing A

Drawing B



Drawing C



Drawing D

4. The entrance to the exhibit will include a large mural based on drawing E. In the original drawing, the cover of the book measures 13.2 cm by 11.1 cm. In the mural, the book cover will have an area of 21,098.88 cm^2. What is the scale factor of the dilation that will be used to make the mural?

Drawing E

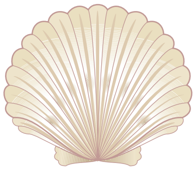
READY TO GO ON?

Quiz for Lessons 12-5 Through 12-7

12-5 Symmetry

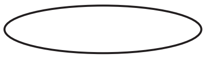
Explain whether each figure has line symmetry. If so, copy the figure and draw all lines of symmetry.

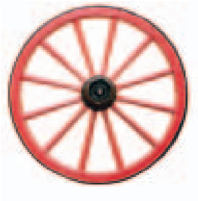
1. 

2. 

3. 

Explain whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

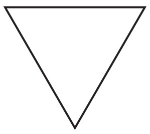
4. 

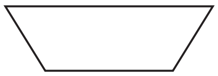
5. 

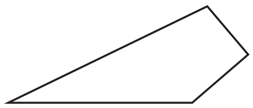
6. 

12-6 Tessellations

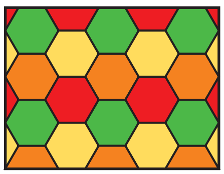
Copy the given figure and use it to create a tessellation.

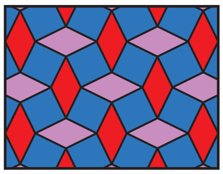
7. 

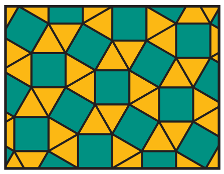
8. 

9. 

Classify each tessellation as regular, semiregular, or neither.

10. 

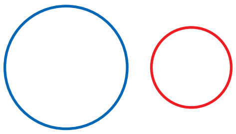
11. 

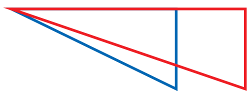
12. 

13. Determine whether it is possible to tessellate a plane with regular octagons. If so, draw the tessellation. If not, explain why.

12-7 Dilations

Tell whether each transformation appears to be a dilation.

14. 

15. 

16. 

Draw the image of the figure with the given vertices under a dilation with the given scale factor centered at the origin.

17. $A(0, 2)$, $B(-1, 0)$, $C(0, -1)$, $D(1, 0)$; scale factor: 2

18. $P(-4, -2)$, $Q(0, -2)$, $R(0, 0)$, $S(-4, 0)$; scale factor: $-\frac{1}{2}$

EXTENSION

Using Patterns to Generate Fractals

Objective
Describe iterative patterns that generate fractals.

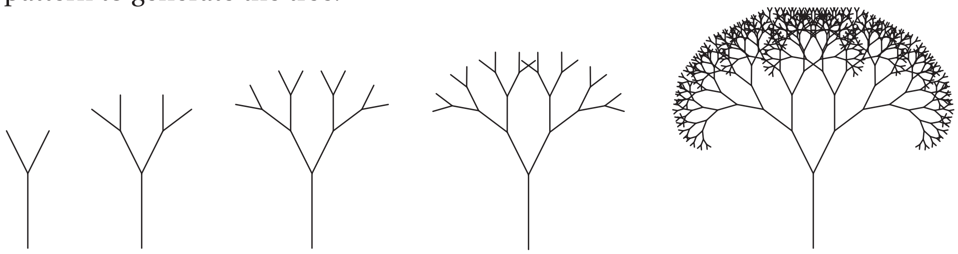
Vocabulary
self-similar
iteration
fractal

California Standards
1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and **inductive** and deductive **reasoning.**

Look closely at one of the large spirals in the Romanesco broccoli. You will notice that it is composed of many smaller spirals, each of which has the same shape as the large one. This is an example of *self-similarity*.

A figure is **self-similar** if it can be divided into parts that are similar to the entire figure. You can draw self-similar figures by **iteration**, the repeated application of a rule.

To create a self-similar tree, start with the figure shown in stage 0. Replace each of its branches with the original figure to get the figure in stage 1. Again replace the branches with the original figure to get the figure in stage 2. Continue the pattern to generate the tree.

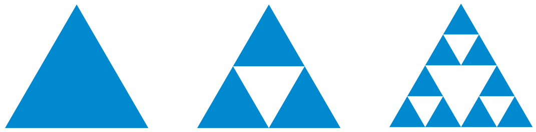


Stage 0 Stage 1 Stage 2 Stage 3 Stage 8

A figure that is generated by iteration is called a **fractal**.

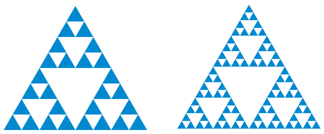
EXAMPLE 1 Creating Fractals

Continue the pattern to draw stages 3 and 4 of this fractal, which is called the Sierpinski triangle.



Stage 0 Stage 1 Stage 2

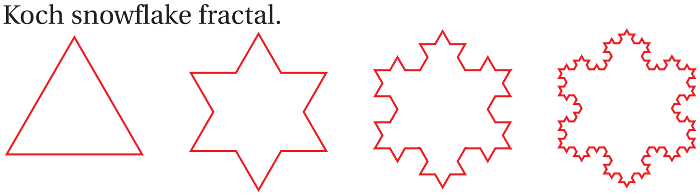
To go from one stage to the next, remove an equilateral triangle from each remaining black triangle.



Stage 3 Stage 4

CHECK IT OUT!

1. Explain how to go from one stage to the next to create the Koch snowflake fractal.



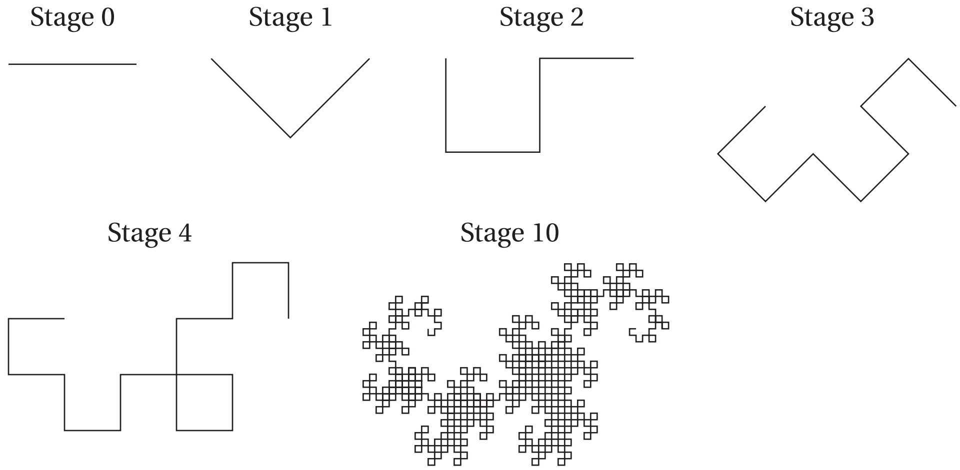
Stage 0 Stage 1 Stage 2 Stage 3

EXTENSION

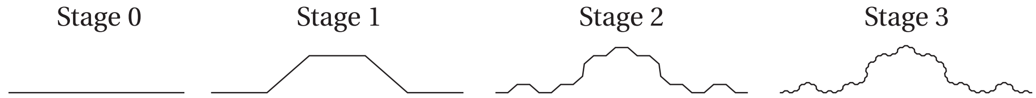
Exercises

Explain how to go from one stage to the next to generate each fractal.

1.



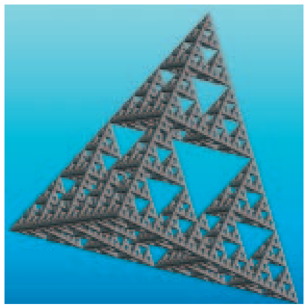
2.



3. The three-dimensional figure in the photo is called a Sierpinski tetrahedron.

a. Describe stage 0 for this fractal.

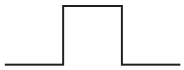
b. Explain how to go from one stage to the next to generate the Sierpinski tetrahedron.



4. A fractal is generated according to the following rules.

Stage 0 is a segment.
To go from one stage to the next, replace each segment with the figure at right.

Draw Stage 2 of this fractal.

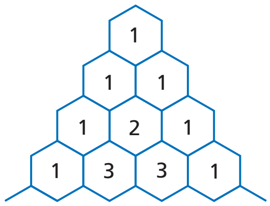


5. The first four rows of Pascal's triangle are shown in the hexagonal tessellation at right. The beginning and end of each row is a 1. To find each remaining number, add the two numbers to the left and right from the row above.

a. Continue the pattern to write the first eight rows of Pascal's triangle.

b. Shade all the hexagons that contain an odd number.

c. What fractal does the resulting pattern resemble?



6. **Write About It** Explain why the fern leaf at right is an example of self-similarity.

CHAPTER 12

Study Guide: Review

For a complete list of the postulates and theorems in this chapter, see p. S82.

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ is a pattern formed by congruent regular polygons.

2. A pattern that has translation symmetry along a line is called a(n) ___?___ .

3. A transformation that does not change the size or shape of a figure is a(n) ___?___ .

4. One transformation followed by another is called a(n) ___?___ .

12-1 Reflections *(pp. 824–830)*

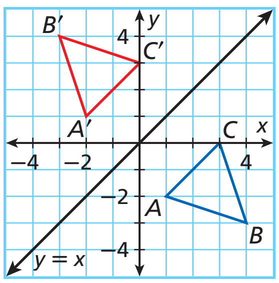
22.0

EXAMPLE

- **Reflect the figure with the given vertices across the given line.**

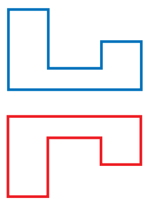
$A(1, -2), B(4, -3), C(3, 0); y = x$

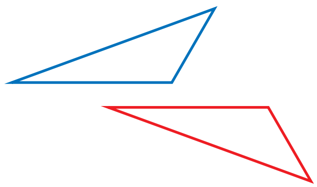
To reflect across the line $y = x$, interchange the x- and y-coordinates of each point. The images of the vertices are $A'(-2, 1)$, $B'(-3, 4)$, and $C'(0, 3)$.

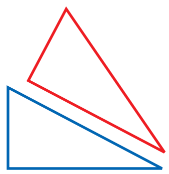


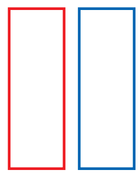
EXERCISES

Tell whether each transformation appears to be a reflection.

5.


6.


7.


8.


Reflect the figure with the given vertices across the given line.

9. $E(-3, 2), F(0, 2), G(-2, 5)$; x-axis

10. $J(2, -1), K(4, -2), L(4, -3), M(2, -3)$; y-axis

11. $P(2, -2), Q(4, -2), R(3, -4)$; $y = x$

12. $A(2, 2), B(-2, 2), C(-1, 4)$; $y = x$

12-2 Translations *(pp. 831–837)*

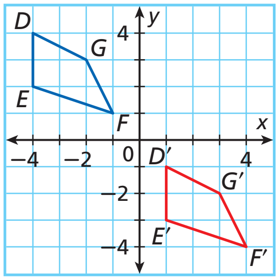
22.0

EXAMPLE

- **Translate the figure with the given vertices along the given vector.**

  $D(-4, 4), E(-4, 2), F(-1, 1), G(-2, 3); \langle 5, -5\rangle$

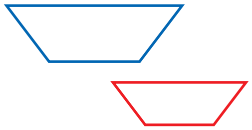
  To translate along $\langle 5, -5\rangle$, add 5 to the x-coordinate of each point and add -5 to the y-coordinate of each point. The vertices of the image are $D'(1, -1)$, $E'(1, -3)$, $F'(4, -4)$, and $G'(3, -2)$.



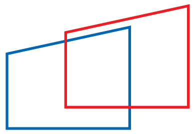
EXERCISES

Tell whether each transformation appears to be a translation.

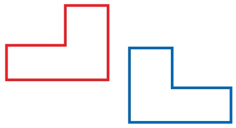
13.



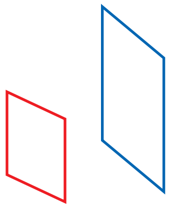
14.



15.



16.



Translate the figure with the given vertices along the given vector.

17. $R(1, -1), S(1, -3), T(4, -3), U(4, -1); \langle -5, 2\rangle$

18. $A(-4, -1), B(-3, 2), C(-1, -2); \langle 6, 0\rangle$

19. $M(1, 4), N(4, 4), P(3, 1); \langle -3, -3\rangle$

20. $D(3, 1), E(2, -2), F(3, -4), G(4, -2); \langle -6, 2\rangle$

12-3 Rotations *(pp. 839–845)*

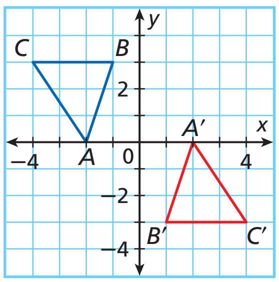
22.0

EXAMPLE

- **Rotate the figure with the given vertices about the origin using the given angle of rotation.**

  $A(-2, 0), B(-1, 3), C(-4, 3); 180°$

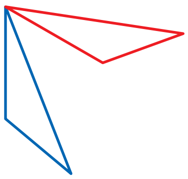
  To rotate by 180°, find the opposite of the x- and y-coordinate of each point. The vertices of the image are $A'(2, 0)$, $B'(1, -3)$, and $C'(4, -3)$.



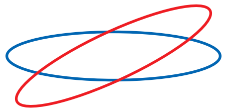
EXERCISES

Tell whether each transformation appears to be a rotation.

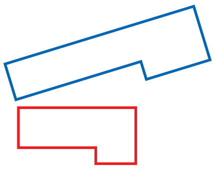
21.



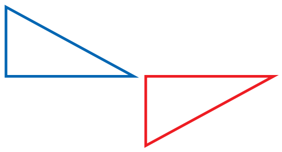
22.



23.



24.



Rotate the figure with the given vertices about the origin using the given angle of rotation.

25. $A(1, 3), B(4, 1), C(4, 4); 90°$

26. $A(1, 3), B(4, 1), C(4, 4); 180°$

27. $M(2, 2), N(5, 2), P(3, -2), Q(0, -2); 90°$

28. $G(-2, 1), H(-3, -2), J(-1, -4); 180°$

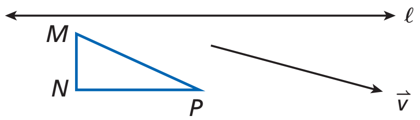
12-4 Compositions of Transformations *(pp. 848–853)*

22.0

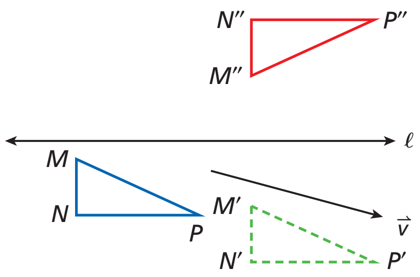
EXAMPLE

■ **Draw the result of the composition of isometries.**

Translate $\triangle MNP$ along $\vec{v}$ and then reflect it across line ℓ.



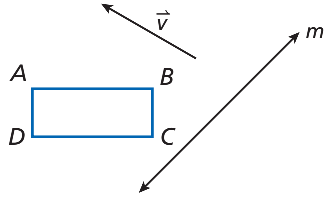
First draw $\triangle M'N'P'$, the translation image of $\triangle MNP$. Then reflect $\triangle M'N'P'$ across line ℓ to find the final image, $\triangle M''N''P''$.



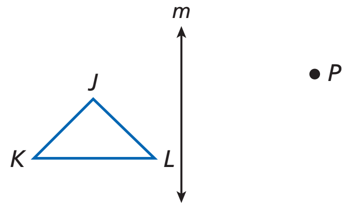
EXERCISES

Draw the result of the composition of isometries.

29. Translate $ABCD$ along $\vec{v}$ and then reflect it across line m.



30. Reflect $\triangle JKL$ across line m and then rotate it 90° about point P.

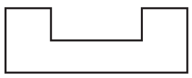


12-5 Symmetry *(pp. 856–862)*

22.0

EXAMPLES

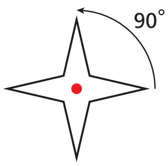
Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

■ 

no rotational symmetry

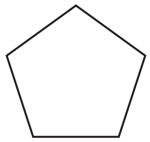
■ 

The figure coincides with itself when it is rotated by 90°. Therefore the angle of rotational symmetry is 90°. The order of symmetry is 4.



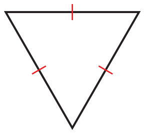
EXERCISES

Tell whether each figure has line symmetry. If so, copy the figure and draw all lines of symmetry.

31. 

32. 

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of symmetry.

33. 

34. 

35. 

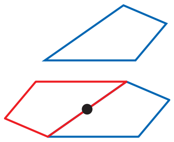
36. 

12-6 Tessellations *(pp. 863–869)*

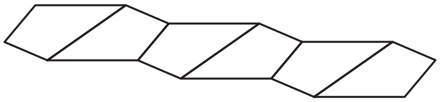
22.0

EXAMPLES

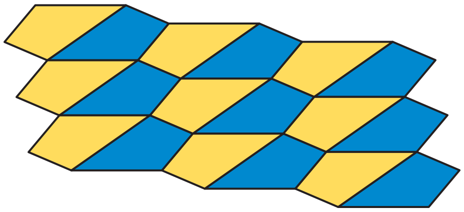
- **Copy the given figure and use it to create a tessellation. Rotate the quadrilateral 180° about the midpoint of one side.**



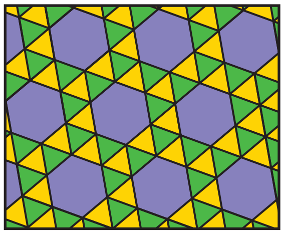
Translate the resulting pair of quadrilaterals to make a row.



Translate the row to make a tessellation.



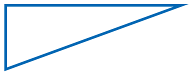
- **Classify the tessellation as regular, semiregular, or neither.**

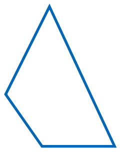


The tessellation is made of two different regular polygons, and each vertex has the same polygons in the same order. Therefore the tessellation is semiregular.

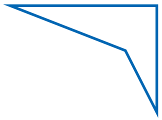
EXERCISES

Copy the given figure and use it to create a tessellation.

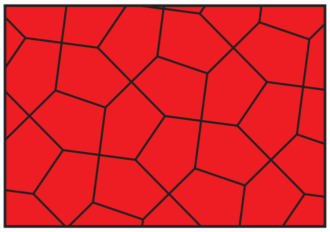
37. 

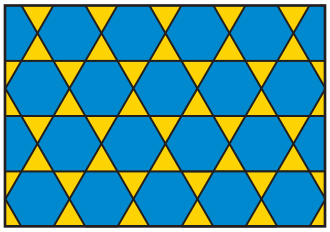
38. 

39. 

40. 

Classify each tessellation as regular, semiregular, or neither.

41. 

42. 

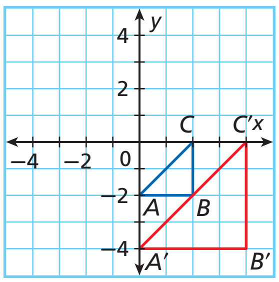
12-7 Dilations *(pp. 872–879)*

8.0, 11.0, 16.0

EXAMPLE

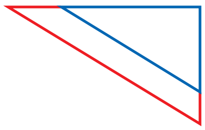
- **Draw the image of the figure with the given vertices under a dilation centered at the origin using the given scale factor.**
$A(0, -2)$, $B(2, -2)$, $C(2, 0)$; scale factor: 2

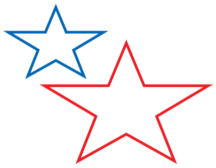
Multiply the x- and y-coordinates of each point by 2. The vertices of the image are $A'(0, -4)$, $B'(4, -4)$, and $C'(4, 0)$.



EXERCISES

Tell whether each transformation appears to be a dilation.

43. 

44. 

Draw the image of the figure with the given vertices under a dilation centered at the origin using the given scale factor.

45. $R(0, 0)$, $S(4, 4)$, $T(4, -4)$; scale factor: $-\frac{1}{2}$

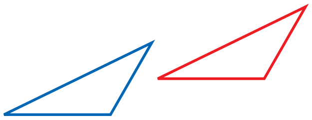
46. $D(0, 2)$, $E(-2, 2)$, $F(-2, 0)$; scale factor: -2

CHAPTER 12

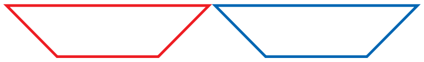
Chapter Test

Tell whether each transformation appears to be a reflection.

1.

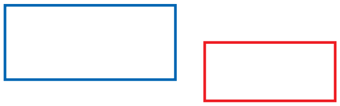


2.

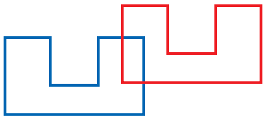


Tell whether each transformation appears to be a translation.

3.



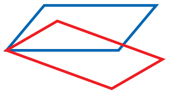
4.



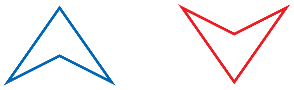
5. An interior designer is using a coordinate grid to place furniture in a room. The position of a sofa is represented by a rectangle with vertices $(1, 3)$, $(2, 2)$, $(5, 5)$, and $(4, 6)$. He decides to move the sofa by translating it along the vector $\langle -1, -1 \rangle$. Draw the sofa in its final position.

Tell whether each transformation appears to be a rotation.

6.



7.



8. Rotate rectangle $DEFG$ with vertices $D(1, -1)$, $E(4, -1)$, $F(4, -3)$, and $G(1, -3)$ about the origin by 180°.

9. Rectangle $ABCD$ with vertices $A(3, -1)$, $B(3, -2)$, $C(1, -2)$, and $D(1, -1)$ is reflected across the y-axis, and then its image is reflected across the x-axis. Describe a single transformation that moves the rectangle from its starting position to its final position.

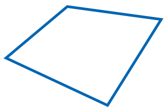
10. Tell whether the "no entry" sign has line symmetry. If so, copy the sign and draw all lines of symmetry.

11. Tell whether the "no entry" sign has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

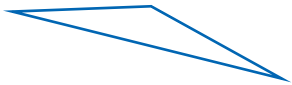
Copy the given figure and use it to create a tessellation.

12.

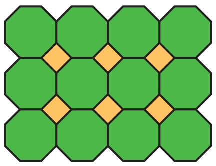
13.



14.

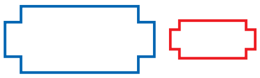


15. Classify the tessellation shown as regular, semiregular, or neither.

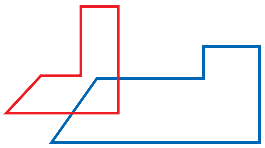


Tell whether each transformation appears to be a dilation.

16.



17.



18. Draw the image of $\triangle ABC$ with vertices $A(2, -1)$, $B(1, -4)$, and $C(4, -4)$ under a dilation centered at the origin with scale factor $-\frac{1}{2}$.

College Entrance Exam Practice

FOCUS ON ACT

No question on the ACT Mathematics Test requires the use of a calculator, but you may bring certain types of calculators to the test. Check www.actstudent.org for a descriptive list of calculators that are prohibited or allowed with slight modifications.

You may want to time yourself as you take this practice test. It should take you about 5 minutes to complete.

HOT TIP! If you are not sure how to solve a problem, looking through the answer choices may provide you with a clue to the solution method. It may take longer to work backward from the answers provided, so make sure you are monitoring your time.

1. Which of the following functions has a graph that is symmetric with respect to the *y*-axis?

(A) $f(x) = x^4 - 2$

(B) $f(x) = (x + 2)^4$

(C) $f(x) = 2x - 4$

(D) $f(x) = x^2 + 4x$

(E) $f(x) = (x - 4)^2$

2. What is the image of the point $(-4, 5)$ after the translation that maps the point $(1, -3)$ to the point $(-1, -7)$?

(F) $(4, 1)$

(G) $(-6, 1)$

(H) $(-8, 3)$

(J) $(-2, 9)$

(K) $(0, 7)$

3. When the point $(-2, -5)$ is reflected across the *x*-axis, what is the resulting image?

(A) $(-5, -2)$

(B) $(2, 5)$

(C) $(2, -5)$

(D) $(-2, 5)$

(E) $(5, 2)$

4. After a composition of transformations, the line segment from $A(1, 4)$ to $B(4, 2)$ maps to the line segment from $C(-1, -2)$ to $D(-4, -4)$. Which of the following describes the composition that is applied to $\overline{AB}$ to obtain $\overline{CD}$?

(F) Translate 5 units to the left and then reflect across the *y*-axis.

(G) Reflect across the *y*-axis and then reflect across the *x*-axis.

(H) Reflect across the *y*-axis and then translate 6 units down.

(J) Reflect across the *x*-axis and then reflect across the *y*-axis.

(K) Translate 6 units down and then reflect across the *x*-axis.

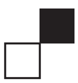
5. What is the image of the following figure after rotating it counterclockwise by 270°?

(A) 

(B) 

(C) 

(D) 

(E) 

CHAPTER 12

STRATEGIES FOR SUCCESS

Any Question Type: Highlight Main Ideas

Before answering a test item, identify the important information given in the problem and make sure you clearly identify the question being asked. Outlining the question or breaking a problem into parts can help you to understand the main idea.

A common error in answering multi-step questions is to complete only the first step. In multiple-choice questions, partial answers are often used as the incorrect answer choices. If you start by outlining all steps needed to solve the problem, you are less likely to choose one of these incorrect answers.

EXAMPLE 1

Gridded Response

A blueprint shows a rectangular building's layout reduced using a scale factor of $\frac{1}{30}$. On the blueprint, the building's width is 15 in. and its length is 6 in. Find the area of the actual building in square feet.

What are you asked to find?

the area of the actual building in square feet

List the given information you need to solve the problem.

The scale factor is $\frac{1}{30}$.

On the blueprint, the width is 15 in. and the length is 6 in.

EXAMPLE 2

Short Response

An animator uses a coordinate plane to show the motion of a flying bird. The bird begins at the point $(12, 0)$ and is then rotated about the origin by 15° every 0.005 second. Give the bird's position after 0.015 second. Round the coordinates to the nearest tenth. Explain the steps you used to get your answer.

What are you asked to find?

the coordinates of the bird's position after 0.015 seconds, to the nearest tenth

What information are you given?

the initial position of the bird and the angle of rotation for every 0.005 second

Sometimes important information is given in a diagram.

Read each test item and answer the questions that follow.

Item A

Multiple Choice Jonas is using a coordinate plane to plan an archaeological dig. He outlines a rectangle with vertices at $(5, 2)$, $(5, 9)$, $(10, 9)$, and $(10, 2)$. Then he outlines a second rectangle by reflecting the first area across the x-axis and then across the y-axis. Which is a vertex of the second outlined rectangle?

Ⓐ $(-5, 2)$
Ⓑ $(-5, -9)$
Ⓒ $(-2, -10)$
Ⓓ $(10, -9)$

1. Identify the sentence that gives the information regarding the coordinates of the initial rectangle.
2. What are you being asked to do?
3. How many transformations does Jonas perform before he sketches the second rectangle? Which sentence leads you to this answer?
4. A student incorrectly marked choice A as her response. What part of the test item did she fail to complete?

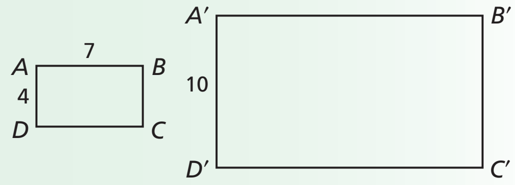
Item B

Short Response A picture frame can hold a picture that is no greater than 320 in^2. Gabby has a digital photo with dimensions 3.5 in. by 5 in., and she uses software to enlarge it by a scale factor of 5. Does the enlargement fit the frame? Show the steps you used to decide your answer.

5. Make a list stating the information given and what you are being asked to do.
6. Are there any intermediate steps you have to make to obtain a solution for the problem? If so, describe the steps.

Item C

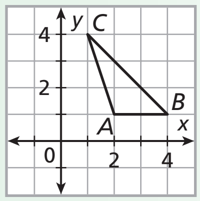
Short Response Rectangle $A'B'C'D'$ is the image of rectangle $ABCD$ under a dilation. Identify the scale factor and determine the area of rectangle $A'B'C'D'$.



7. How many parts are there to this item? Make a list of what needs to be included in your response.
8. Where in the test item can you find the important information (data) needed to solve the problem? Make a list of this information.

Item D

Multiple Choice $\triangle ABC$ is reflected across the x-axis. Then its image is rotated 180° about the origin. What are the coordinates of the image of point B after the reflection?



Ⓐ $(-4, -1)$
Ⓑ $(-1, 4)$
Ⓒ $(1, -4)$
Ⓓ $(4, -1)$

9. Identify the transformations described in the problem statement.
10. What are you being asked to do?
11. Identify any part of the problem statement that you will not use to answer the question.
12. There are only two pieces of information given in this test item that are important to answering this question. What are they?

CHAPTER 12

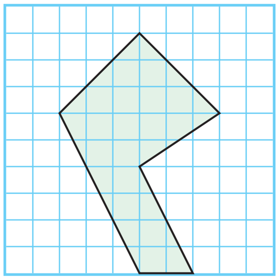
STANDARDIZED TEST PREP

go.hrw.com
Test Practice Online
KEYWORD: MG7 TestPrep

CUMULATIVE ASSESSMENT, CHAPTERS 1–12

Multiple Choice

1. Which of the following best represents the area of the shaded figure if each square in the grid has a side length of 1 centimeter?



(A) 17 square centimeters
(B) 21 square centimeters
(C) 25 square centimeters
(D) 29 square centimeters

2. Which of the following expressions represents the number of edges of a polyhedron with n vertices and n faces?

(F) $n - 2$ (H) $2(n - 1)$
(G) $2n - 1$ (J) $2(n + 1)$

3. The image of point A under a 90° rotation about the origin is $A'(10, -4)$. What are the coordinates of point A?

(A) $(-10, -4)$ (C) $(-4, -10)$
(B) $(-10, 4)$ (D) $(4, 10)$

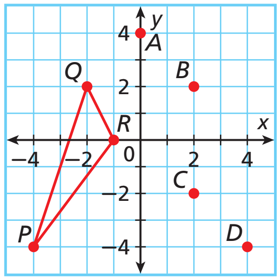
4. A cylinder has a volume of 24 cubic centimeters. The height of a cone with the same radius is two times the height of the cylinder. What is the volume of the cone?

(F) 8 cubic centimeters
(G) 12 cubic centimeters
(H) 16 cubic centimeters
(J) 48 cubic centimeters

5. Marty conjectures that the sum of any two prime numbers is even. Which of the following is a counterexample that shows Marty's conjecture is false?

(A) $2 + 2 = 4$ (C) $2 + 9 = 11$
(B) $2 + 7 = 9$ (D) $3 + 5 = 8$

Use the graph for Items 6–8.



6. What are the coordinates of the image of point C under the same translation that maps point D to point B?

(F) $(4, 4)$ (H) $(0, 8)$
(G) $(0, 4)$ (J) $(4, -8)$

7. $\triangle PQR$ is the image of a triangle under a dilation centered at the origin with scale factor $-\frac{1}{2}$. Which point is a vertex of the preimage of $\triangle PQR$ under this dilation?

(A) A (C) C
(B) B (D) D

8. What is the measure of $\angle PRQ$? Round to the nearest degree.

(F) 63° (H) 117°
(G) 127° (J) 45°

9. Which mapping represents a rotation of 270° about the origin?

(A) $(x, y) \rightarrow (-x, -y)$
(B) $(x, y) \rightarrow (x, -y)$
(C) $(x, y) \rightarrow (-y, -x)$
(D) $(x, y) \rightarrow (y, -x)$

HOT TIP!

When problems involve geometric figures in the coordinate plane, it may be useful to describe properties of the figures algebraically. For example, you can use slope to verify that sides of a figure are parallel or perpendicular, or you can use the Distance Formula to find side lengths of the figure.

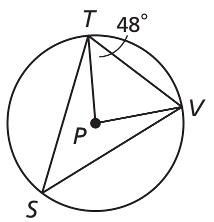
10. What are the coordinates of the center of the circle $(x + 1)^2 + (y + 4)^2 = 4$?

Ⓕ $(-1, -4)$ Ⓗ $(1, 2)$
Ⓖ $(-1, -2)$ Ⓙ $(1, 4)$

11. Which regular polygon can be used with an equilateral triangle to tessellate a plane?

Ⓐ Heptagon
Ⓑ Octagon
Ⓒ Nonagon
Ⓓ Dodecagon

12. What is the measure of $\angle TSV$ in $\odot P$?



Ⓕ 24° Ⓗ 45°
Ⓖ 42° Ⓙ 48°

13. Given the points $B(-1, 2)$, $C(-7, y)$, $D(1, -3)$, and $E(-3, -2)$, what is the value of y if $\overline{BD} \parallel \overline{CE}$?

Ⓐ -12 Ⓒ 3.5
Ⓑ -8 Ⓓ 8

Gridded Response

14. $\triangle ABC$ is a right triangle such that $m\angle B = 90°$. If $AC = 12$ and $BC = 9$, what is the perimeter of $\triangle ABC$? Round to the nearest tenth.

15. A blueprint for an office space uses a scale of 3 inches: 20 feet. What is the area in square inches of the office space on the blueprint if the actual office space has area 1300 square feet?

16. How many lines of symmetry does a regular hexagon have?

17. What is the x-coordinate of the image of the point $A(12, -7)$ if A is reflected across the x-axis?

Short Response

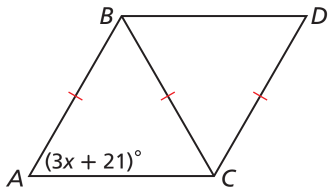
18. $A(-4, -2)$, $B(-2, -3)$, and $C(-3, -5)$ are three of the vertices of rhombus $ABCD$. Show that $ABCD$ is a square. Justify your answer.

19. $ABCD$ is a square with vertices $A(3, -1)$, $B(3, -3)$, $C(1, -3)$, and $D(1, -1)$. $\odot P$ is a circle with equation $(x - 2)^2 + (y - 2)^2 = 4$.

a. What is the center and radius of $\odot P$?

b. Describe a reflection and dilation of $ABCD$ so that $\odot P$ is inscribed in the image of $ABCD$. Justify your answer.

20. Determine the value of x if $\triangle ABC \cong \triangle BDC$. Justify your answer.



21. $\triangle ABC$ is reflected across line m.

a. What observations can be made about $\triangle ABC$ and its reflected image $\triangle A'B'C'$ regarding the following properties: collinearity, betweenness, angle measure, triangle congruence, and orientation?

b. Explain.

22. Given the coordinates of points A, B, and C, explain how you could demonstrate that the three points are collinear.

23. Proving that the diagonals of rectangle $KLMN$ are equal using a coordinate proof involves placement of the rectangle and selection of coordinates.

a. Is it possible to always position rectangle $KLMN$ so that one vertex coincides with the origin?

b. Why is it convenient to place rectangle $KLMN$ so that one vertex is at the origin?

Extended Response

24. $\overline{AB}$ has endpoints $A(0, 3)$ and $B(2, 5)$.

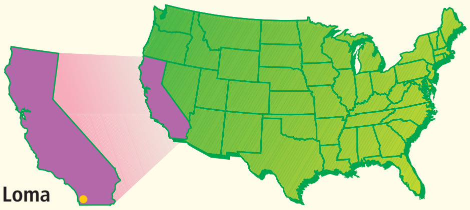
a. Draw $\overline{AB}$ and its image, $\overline{A'B'}$, under the translation $\langle 0, -8 \rangle$.

b. Find the equations of two lines such that the composition of the two reflections across the lines will also map $\overline{AB}$ to $\overline{A'B'}$. Show your work or explain in words how you found your answer.

c. Show that any glide reflection is equivalent to a composition of three reflections.

Problem Solving on Location

CALIFORNIA



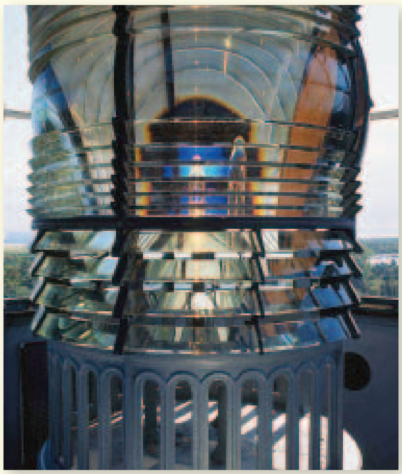
New Point Loma Lighthouse

New Point Loma Lighthouse at the southern tip of Point Loma in San Diego has been guiding ships off the California coast for 116 years. The tower is located 88 feet above sea level and contains a fog signal and a radio beacon.

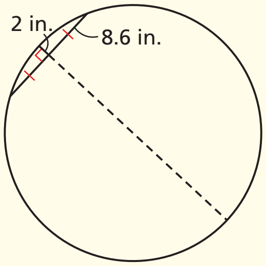
Choose one or more strategies to solve each problem.

1. The beam from the lighthouse is visible for up to 15 miles at sea. To the nearest square mile, what is the area of water covered by the beam if the beam rotates through an angle of 60°?

2. Given that Earth's radius is approximately 4000 miles, find the distance from the top of the tower to the horizon.



3. Most lighthouses use *Fresnel lenses*, named after their inventor, Augustine Fresnel. The chart shows the sizes, or orders, of the circular lenses. Use the diagram of the lens to determine the order of the Fresnel lens at the New Point Loma Lighthouse.



Fresnel Lenses

Order	Diameter
First	6 ft 1 in.
Second	4 ft 7 in.
Third	3 ft 3 in.
Fourth	1 ft 8 in.
Fifth	1 ft 3 in.
Sixth	1 ft 0 in.

Moveable Bridges

California has numerous moveable bridges. A moveable bridge has a section that can be lifted, tilted, or swung out of the way so that ships can pass.

1. The I Street Bridge in Sacramento is a swing bridge. Part of the roadbed can pivot horizontally to let ships pass. What transformation describes the motion of the bridge? The pivoting section moves through an angle of 90°. How far does a point 10 ft from the pivot travel as the bridge opens?

A *lift bridge* contains a section that can be translated vertically. For 2–4, use the table.

2. Suppose it takes 2 min to completely lift the roadbed of the Tower Bridge. At what speed in feet per minute does the lifting mechanism translate the roadbed?

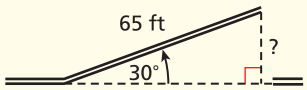
3. To the nearest second, how long would it take the Tower Bridge's lifting mechanism to translate the roadbed 10 ft?

Lift Bridges

Name	Vertical Clearance in Lowered Position	Vertical Clearance in Raised Position
Tower Bridge	54 ft	100 ft
Schuyler F. Heim Bridge	38 ft	163 ft

4. Suppose the Schuyler F. Heim Bridge can be raised at the same speed as the Tower Bridge. To the nearest second, how long would it take to completely lift the roadbed of the Schuyler F. Heim Bridge?

5. The Islais Creek Bridge in San Francisco is a *bascule bridge*. Weights are used to raise part of its deck at an angle. The moveable section of the bridge is 65 feet long. Find the height of the deck above the roadway after the deck has been rotated by an angle of 30°.



Problem Solving Strategies

- Draw a Diagram
- Make a Model
- Guess and Test
- Work Backward
- Find a Pattern
- Make a Table
- Solve a Simpler Problem
- Use Logical Reasoning
- Use a Venn Diagram
- Make an Organized List