CHAPTER 9

Extending Perimeter, Circumference, and Area

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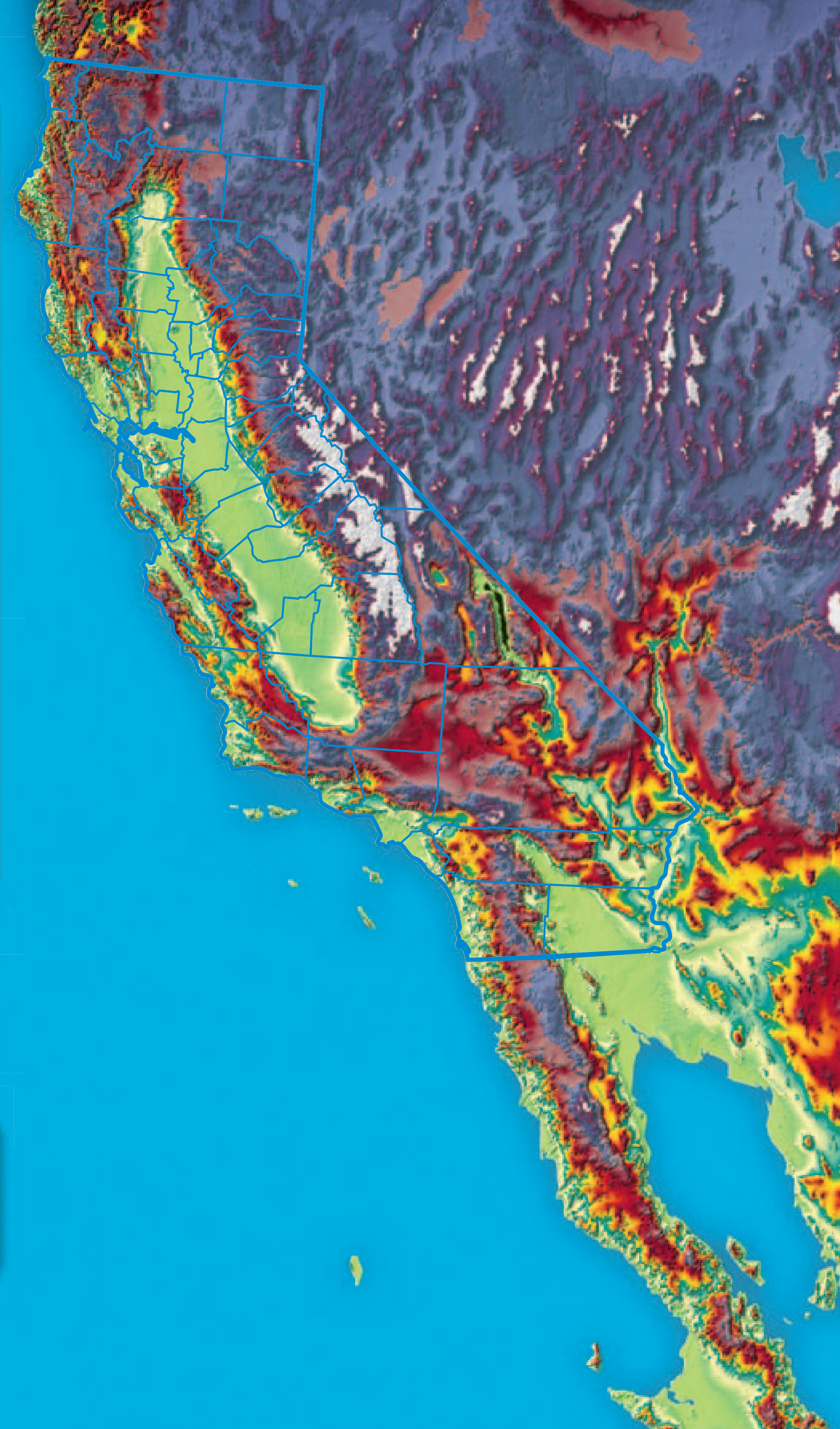
Chapter Project Online
KEYWORD: MG7 ChProj



You can calculate the perimeters and areas of California's 58 irregularly-shaped counties.

County Elevation Map
California

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. area
2. kite
3. perimeter
4. regular polygon

A. a polygon that is both equilateral and equiangular

B. a quadrilateral with exactly one pair of parallel sides

C. the number of nonoverlapping unit squares of a given size that exactly cover the interior of a figure

D. a quadrilateral with exactly two pairs of adjacent congruent sides

E. the distance around a closed plane figure

Convert Units

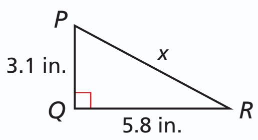
Use multiplication or division to change from one unit of measure to another.

5. 12 mi = ▢ yd
6. 7.3 km = ▢ m
7. 6 in. = ▢ ft
8. 15 m = ▢ mm

Length	
Metric	**Customary**
1 kilometer = 1000 meters 1 meter = 100 centimeters 1 centimeter = 10 millimeters	1 mile = 1760 yards 1 mile = 5280 feet 1 yard = 3 feet 1 foot = 12 inches

Pythagorean Theorem

Find x in each right triangle. Round to the nearest tenth, if necessary.

9.


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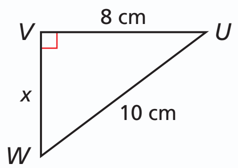
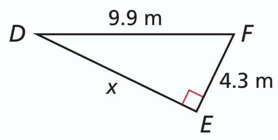


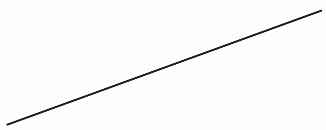
11.


Measure with Customary and Metric Units

Measure each segment to the nearest eighth of an inch and to the nearest half of a centimeter.

12.


13.

14.


Solve for a Variable

Solve each equation for the indicated variable.

15. $A = \frac{1}{2}bh$ for b

16. $P = 2b + 2h$ for h

17. $A = \frac{1}{2}(b_1 + b_2)h$ for b_1

18. $A = \frac{1}{2}d_1d_2$ for d_1

CHAPTER 9

Unpacking the Standards

The information below "unpacks" the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

California Standard	Academic Vocabulary	Chapter Concept
8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area **of common geometric figures.** (Lessons 9-1, 9-2, 9-3, 9-4, 9-5, 9-6) (Labs 9-2, 9-6)	**derive** develop a conclusion about something using a different method **solve** find the value of a variable that makes the left side of an equation equal to the right side of the equation ***Example:*** $2x = 6$ $2(3) = 6$ The value that makes $2x = 6$ true is **3**.	You will develop and apply formulas involving perimeter and area of triangles, circles, special quadrilaterals, and regular polygons.
10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids. (Lessons 9-1, 9-2, 9-3, 9-4, 9-5, 9-6) (Lab 9-3)	**compute** calculate or work out a problem **rhombi** plural of rhombus, a parallelogram with four sides of equal length	You will learn how to find the areas of composite figures and estimate the areas of irregular figures. Then use these skills to find geometric probabilities.
11.0 Students determine how changes in dimensions affect the perimeter, area, and volume **of common geometric figures** and solids. (Lesson 9-5)	**determine** find out **dimensions** sizes of objects	You will learn how to describe the effect on the perimeter and area of a figure if one dimension of the figure is changed.
12.0 Students find and use measures of sides and of interior and exterior angles **of triangles and polygons to classify figures and solve problems.** (Lesson 9-4)	**classify** assign polygons to groups according to their features	You will first draw a figure in a coordinate plane so you can classify it. Then you find the perimeter and area of the figure.

Standards 1.0 and 15.0 are also covered in this chapter. To see these standards unpacked, go to Chapter 1, p. 4 and Chapter 5, p. 298.

Reading and Writing Math

Study Strategy: Memorize Formulas

Throughout a geometry course, you will learn many formulas, theorems, postulates, and corollaries. You may be required to memorize some of these. In order not to become overwhelmed by the amount of information, it helps to use flash cards.

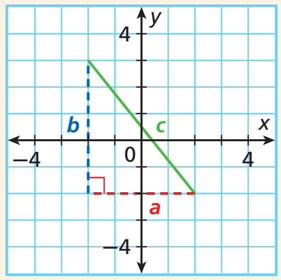
In a right triangle, the two sides that form the right angle are the **legs**. The side across from the right angle that stretches from one leg to the other is the **hypotenuse**. In the diagram, a and b are the lengths of the shorter sides, or legs, of the right triangle. The longest side is called the hypotenuse and has length c.

Know it! Note

Theorem 1-6-1 **Pythagorean Theorem**

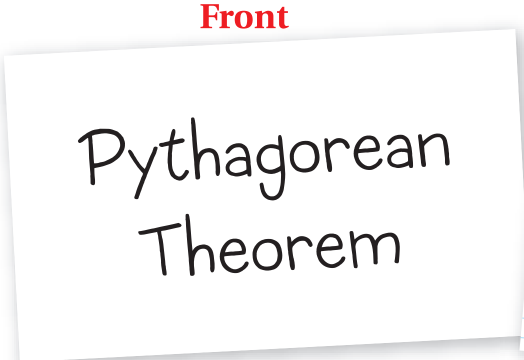
In a right triangle, the sum of the squares of the lengths of the *legs* is equal to the square of the length of the *hypotenuse*.

$$a^2 + b^2 = c^2$$



To create a flash card, write the name of the formula or theorem on the front of the card. Then clearly write the appropriate information on the back of the card. Be sure to include a labeled diagram.

Front



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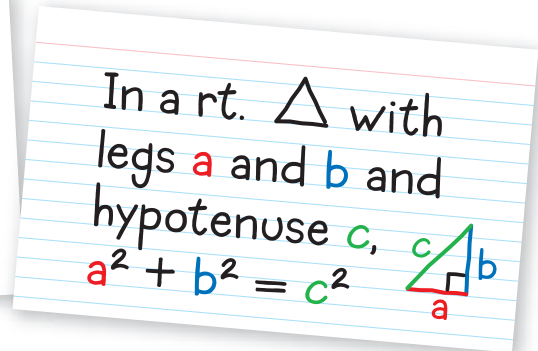


Try This

1. Choose a lesson from this book that you have already studied, and make flash cards of the formulas or theorems from the lesson.

2. Review your flash cards by looking at the front of each card and trying to recall the information on the back of the card.

Connecting Geometry to **Algebra**

See Skills Bank page S59

Literal Equations

A *literal equation* contains two or more variables. Formulas you have used to find perimeter, circumference, area, and side relationships of right triangles are examples of literal equations.

California Standards

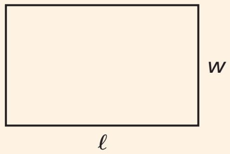
12.0 Students find and use measures of sides and of interior and exterior angles **of triangles and polygons to** classify figures and **solve problems.**

Also covered: **15.0, Extension of 1A5.0**

If you want to evaluate a formula for several different values of a given variable, it is helpful to solve for the variable first.

Example

Danielle plans to use 50 feet of fencing to build a dog run. Use the formula $P = 2\ell + 2w$ to find the length ℓ when the width w is 4, 5, 6, and 10 feet.



Solve the equation for ℓ.

First solve the formula for the variable.

$P = 2\ell + 2w$ — *Write the original equation.*

$P - 2w = 2\ell$ — *Subtract 2w from both sides.*

$\frac{P - 2w}{2} = \ell$ — *Divide both sides by 2.*

Use your result to find ℓ for each value of w.

$\ell = \frac{P - 2w}{2} = \frac{50 - 2(4)}{2} = 21$ ft — *Substitute 50 for P and 4 for w.*

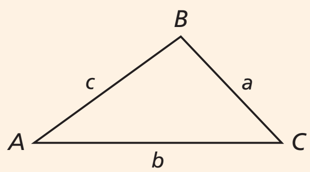
$\ell = \frac{P - 2w}{2} = \frac{50 - 2(5)}{2} = 20$ ft — *Substitute 50 for P and 5 for w.*

$\ell = \frac{P - 2w}{2} = \frac{50 - 2(6)}{2} = 19$ ft — *Substitute 50 for P and 6 for w.*

$\ell = \frac{P - 2w}{2} = \frac{50 - 2(10)}{2} = 15$ ft — *Substitute 50 for P and 10 for w.*

Try This

1. A rectangle has a perimeter of 24 cm. Use the formula $P = 2\ell + 2w$ to find the width when the length is 2, 3, 4, 6, and 8 cm.
2. A right triangle has a hypotenuse of length $c = 65$ ft. Use the Pythagorean Theorem to find the length of leg a when the length of leg b is 16, 25, 33, and 39 feet.
3. The perimeter of $\triangle ABC$ is 112 in. Write an expression for a in terms of b and c, and use it to complete the following table.



a	*b*	*c*
	48	35
	36	36
	14	50

9-1 Developing Formulas for Triangles and Quadrilaterals

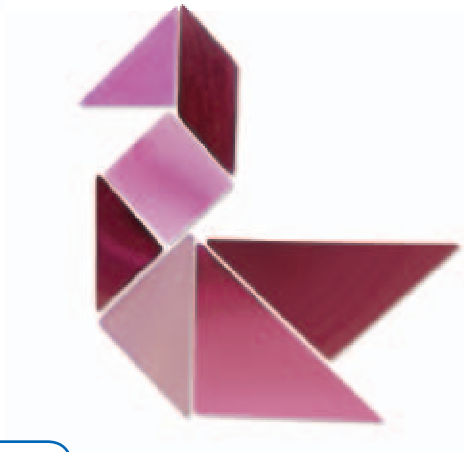
Objectives

Develop and apply the formulas for the areas of triangles and special quadrilaterals.

Solve problems involving perimeters and areas of triangles and special quadrilaterals.

Why learn this?

You can use formulas for area to help solve puzzles such as the tangram.



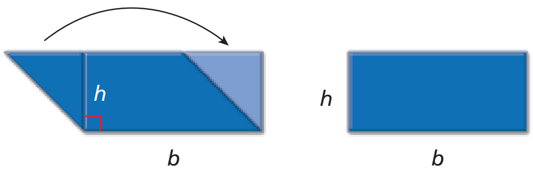
A tangram is an ancient Chinese puzzle made from a square. The pieces can be rearranged to form many different shapes. The area of a figure made with all the pieces is the sum of the areas of the pieces.

Know it! Note

Postulate 9-1-1 Area Addition Postulate

The area of a region is equal to the sum of the areas of its nonoverlapping parts.

Recall that a rectangle with base b and height h has an area of $A = bh$. You can use the Area Addition Postulate to see that a parallelogram has the same area as a rectangle with the same base and height.

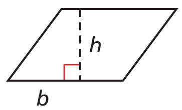


A triangle is cut off one side and translated to the other side.

Know it! Note

Area Parallelogram

The area of a parallelogram with base b and height h is $A = bh$.



Remember that rectangles and squares are also parallelograms. The area of a square with side s is $A = s^2$, and the perimeter is $P = 4s$.

EXAMPLE 1 Finding Measurements of Parallelograms

California Standards

8.0 Students know, derive, and solve problems involving the perimeter, circumference, **area,** volume, lateral area, and surface area **of common geometric figures.**
10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, **rhombi, parallelograms, and trapezoids.**

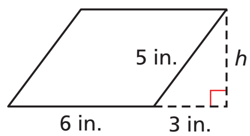
Find each measurement.

A **the area of the parallelogram**

Step 1 Use the Pythagorean Theorem to find the height h.

$3^2 + h^2 = 5^2$

$h = 4$



Step 2 Use h to find the area of the parallelogram.

$A = bh$ *Area of a parallelogram*

$A = 6(4)$ *Substitute 6 for b and 4 for h.*

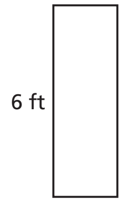
$A = 24 \text{ in}^2$ *Simplify.*

Find each measurement.

B **the height of a rectangle in which $b = 5$ cm and $A = (5x^2 - 5x)$ cm^2**

$A = bh$	*Area of a rectangle*
$5x^2 - 5x = 5h$	*Substitute $5x^2 - 5x$ for A and 5 for b.*
$5(x^2 - x) = 5h$	*Factor 5 out of the expression for A.*
$x^2 - x = h$	*Divide both sides by 5.*
$h = (x^2 - x)$ cm	*Sym. Prop. of =*

C **the perimeter of the rectangle, in which $A = 12x$ ft^2**



Step 1 Use the area and the height to find the base.

$A = bh$	*Area of a rectangle*
$12x = b(6)$	*Substitute 12x for A and 6 for h.*
$2x = b$	*Divide both sides by 6.*

Step 2 Use the base and the height to find the perimeter.

$P = 2b + 2h$	*Perimeter of a rectangle*
$P = 2(2x) + 2(6)$	*Substitute 2x for b and 6 for h.*
$P = (4x + 12)$ ft.	*Simplify.*

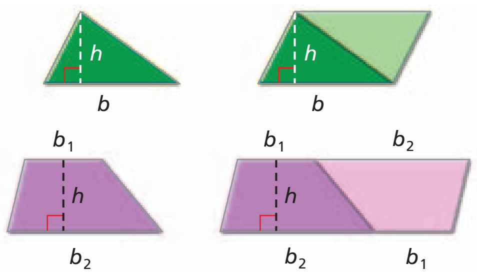
Remember!

The perimeter of a rectangle with base b and height h is $P = 2b + 2h$, or $P = 2(b + h)$.

CHECK IT OUT!

1. Find the base of a parallelogram in which $h = 56$ yd and $A = 28$ yd^2.

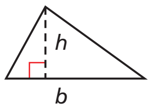
To understand the formula for the area of a triangle or trapezoid, notice that two congruent triangles or two congruent trapezoids fit together to form a parallelogram. Thus the area of a triangle or trapezoid is half the area of the related parallelogram.



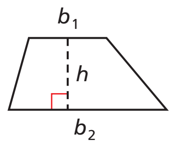
Know it! Note

Area — Triangles and Trapezoids

The area of a triangle with base b and height h is $A = \frac{1}{2}bh$.



The area of a trapezoid with bases b_1 and b_2 and height h is $A = \frac{1}{2}(b_1 + b_2)h$, or $A = \frac{(b_1 + b_2)h}{2}$.



EXAMPLE 2 Finding Measurements of Triangles and Trapezoids

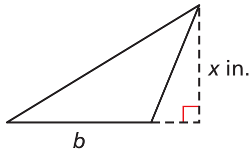
Find each measurement.

A **the area of a trapezoid in which $b_1 = 9$ cm, $b_2 = 12$ cm, and $h = 3$ cm**

$A = \frac{1}{2}(b_1 + b_2)h$	*Area of a trapezoid*
$A = \frac{1}{2}(9 + 12)3$	*Substitute 9 for b_1, 12 for b_2, and 3 for h.*
$A = 31.5$ cm^2	*Simplify.*

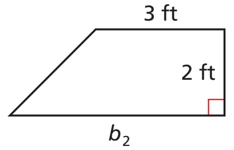
Find each measurement.

B **the base of the triangle, in which $A = x^2$ in^2**



$A = \frac{1}{2}bh$	*Area of a triangle*
$x^2 = \frac{1}{2}bx$	*Substitute x^2 for A and x for h.*
$x = \frac{1}{2}b$	*Divide both sides by x.*
$2x = b$	*Multiply both sides by 2.*
$b = 2x$ in.	*Sym. Prop. of =*

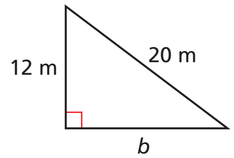
C **b_2 of the trapezoid, in which $A = 8$ ft^2**



$A = \frac{1}{2}(b_1 + b_2)h$	*Area of a trapezoid*
$8 = \frac{1}{2}(3 + b_2)(2)$	*Substitute 8 for A, 3 for b_1, and 2 for h.*
$8 = 3 + b_2$	*Multiply $\frac{1}{2}$ by 2.*
$5 = b_2$	*Subtract 3 from both sides.*
$b_2 = 5$ ft	*Sym. Prop. of =*

CHECK IT OUT!

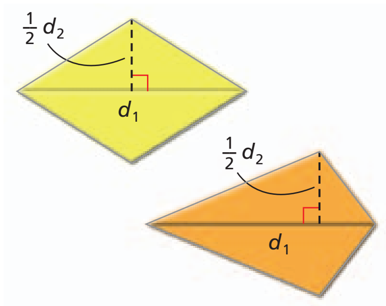
2. Find the area of the triangle.



A kite or a rhombus with diagonals d_1 and d_2 can be divided into two congruent triangles with a base of d_1 and a height of $\frac{1}{2}d_2$.

area of each triangle: $A = \frac{1}{2}d_1\left(\frac{1}{2}d_2\right) = \frac{1}{4}d_1d_2$

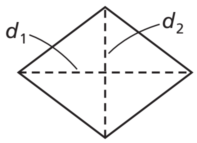
total area: $A = 2\left(\frac{1}{4}d_1d_2\right) = \frac{1}{2}d_1d_2$

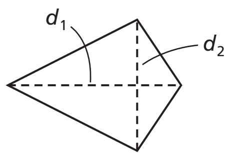


Know it! Note

Area — Rhombuses and Kites

The area of a rhombus or kite with diagonals d_1 and d_2 is $A = \frac{1}{2}d_1d_2$.





EXAMPLE 3 Finding Measurements of Rhombuses and Kites

Find each measurement.

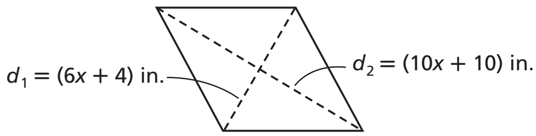
A **d_2 of a kite in which $d_1 = 16$ cm and $A = 48$ cm^2**

$A = \frac{1}{2}d_1d_2$	*Area of a kite*
$48 = \frac{1}{2}(16)d_2$	*Substitute 48 for A and 16 for d_1.*
$6 = d_2$	*Solve for d_2.*
$d_2 = 6$ cm	*Sym. Prop. of =*

Algebra

Find each measurement.

B **the area of the rhombus**



$A = \frac{1}{2}d_1 d_2$

$A = \frac{1}{2}(6x + 4)(10x + 10)$ — *Substitute $(6x + 4)$ for d_1 and $(10x + 10)$ for d_2.*

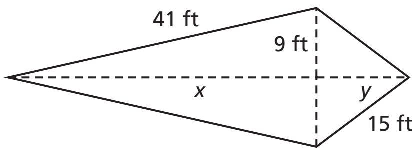
$A = \frac{1}{2}(60x^2 + 100x + 40)$ — *Multiply the binomials (FOIL).*

$A = (30x^2 + 50x + 20)$ in^2 — *Distrib. Prop.*

Remember!

The diagonals of a rhombus or kite are perpendicular, and the diagonals of a rhombus bisect each other.

C **the area of the kite**



Step 1 The diagonals d_1 and d_2 form four right triangles. Use the Pythagorean Theorem to find x and y.

$9^2 + x^2 = 41^2$	$9^2 + y^2 = 15^2$
$x^2 = 1600$	$y^2 = 144$
$x = 40$	$y = 12$

Step 2 Use d_1 and d_2 to find the area. d_1 is equal to $x + y$, which is 52. Half of d_2 is equal to 9, so d_2 is equal to 18.

$A = \frac{1}{2}d_1 d_2$ — *Area of a kite*

$A = \frac{1}{2}(52)(18)$ — *Substitute 52 for d_1 and 18 for d_2.*

$A = 468$ ft^2 — *Simplify.*

CHECK IT OUT! **3.** Find d_2 of a rhombus in which $d_1 = 3x$ m and $A = 12xy$ m^2.

EXAMPLE 4 *Games Application*

The pieces of a tangram are arranged in a square in which $s = 4$ cm. Use the grid to find the perimeter and area of the red square.

Perimeter:

Each side of the red square is the diagonal of a square of the grid. Each grid square has a side length of 1 cm, so the diagonal is $\sqrt{2}$ cm. The perimeter of the red square is $P = 4s = 4\sqrt{2}$ cm.

Area:

Method 1 The red square is also a rhombus. The diagonals d_1 and d_2 each measure 2 cm. So its area is

$A = \frac{1}{2}d_1 d_2 = \frac{1}{2}(2)(2) = 2$ cm^2.

Method 2 The side length of the red square is $\sqrt{2}$ cm, so the area is

$A = s^2 = \left(\sqrt{2}\right)^2 = 2$ cm.

CHECK IT OUT! **4.** In the tangram above, find the perimeter and area of the large green triangle.

THINK AND DISCUSS

1. Explain why the area of a triangle is half the area of a parallelogram with the same base and height.

2. Compare the formula for the area of a trapezoid with the formula for the area of a rectangle.

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. Name all the shapes whose area is given by each area formula and sketch an example of each shape.

Area Formula	Shape(s)	Example(s)
$A = bh$		
$A = \frac{1}{2}bh$		
$A = \frac{1}{2}(b_1 + b_2)h$		
$A = \frac{1}{2}d_1d_2$		

9-1 Exercises

California Standards
2.0, 8.0, 10.0, 14.0, 20.0, 7AF1.0, 7AF3.1, 7MG1.1, 7MG2.1, 7MR3.3

go.hrw.com
Homework Help Online
KEYWORD: MG7 9-1
Parent Resources Online
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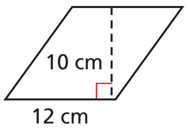
GUIDED PRACTICE

Find each measurement.

SEE EXAMPLE 1
p. 589

1. the area of the parallelogram

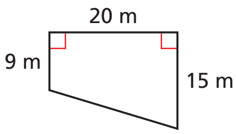


2. the height of the rectangle, in which $A = 10x^2$ ft^2

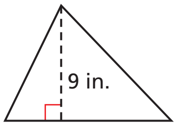
3. the perimeter of a square in which $A = 169$ cm^2

SEE EXAMPLE 2
p. 590

4. the area of the trapezoid



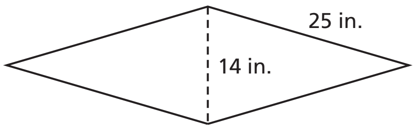
5. the base of the triangle, in which $A = 58.5$ in^2



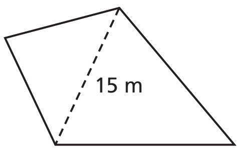
6. b_1 of a trapezoid in which $A = (48x + 68)$ in^2, $h = 8$ in., and $b_2 = (9x + 12)$ in.

SEE EXAMPLE 3
p. 591

7. the area of the rhombus



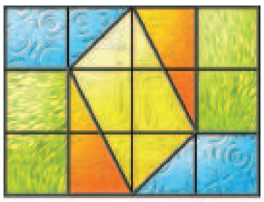
8. d_2 of the kite, in which $A = 187.5$ m^2



9. d_2 of a kite in which $A = 12x^2y^3$ cm^2, $d_1 = 3xy$ cm

SEE EXAMPLE 4
p. 592

10. **Art** The stained-glass window shown is a rectangle with a base of 4 ft and a height of 3 ft. Use the grid to find the area of each piece.



PRACTICE AND PROBLEM SOLVING

Independent Practice

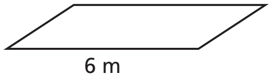
For Exercises	See Example
11–13	1
14–16	2
17–19	3
20–22	4

Extra Practice

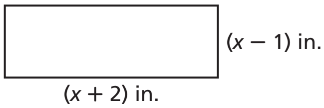
Skills Practice p. S20
Application Practice p. S36

Find each measurement.

11. the height of the parallelogram, in which $A = 7.5\text{ m}^2$

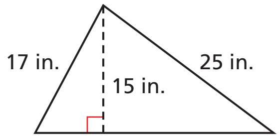


12. the perimeter of the rectangle

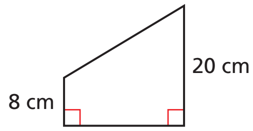


13. the area of a parallelogram in which $b = (3x + 5)$ ft and $h = (7x - 1)$ ft

14. the area of the triangle

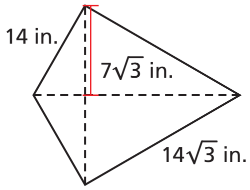


15. the height of the trapezoid, in which $A = 280\text{ cm}^2$

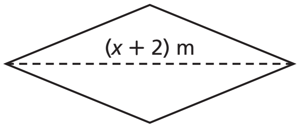


16. the area of a triangle in which $b = (x + 1)$ ft and $h = 8x$ ft

17. the area of the kite

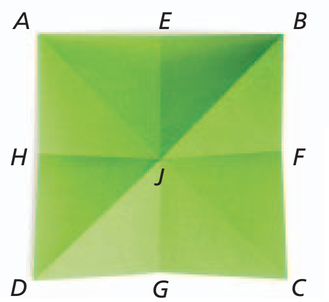


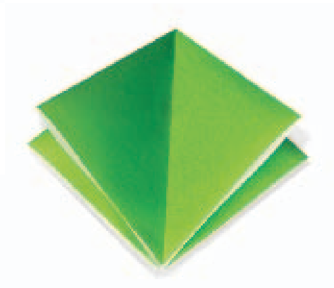
18. d_2 of the rhombus, in which $A = (3x^2 + 6x)\text{ m}^2$



19. the area of a kite in which $d_1 = (6x + 5)$ ft and $d_2 = (4x + 8)$ ft

Crafts **In origami, a *square base* is the starting point for the creation of many figures, such as a crane. In the pattern for the square base, *ABCD* is a square, and *E*, *F*, *G*, and *H* are the midpoints of the sides. If $AB = 6$ in., find the area of each shape.**





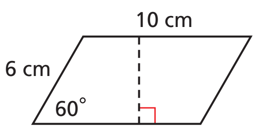
20. rectangle *ABFH*

21. $\triangle AEJ$

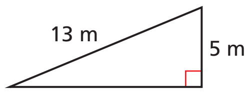
22. trapezoid *ABFJ*

Multi-Step **Find the area of each figure. Round to the nearest tenth, if necessary.**

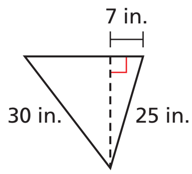
23.



24.

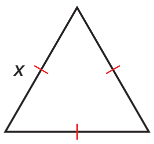


25.

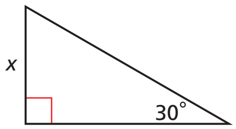


Write each area in terms of *x*.

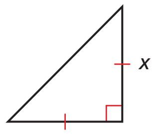
26. equilateral triangle



27. 30°-60°-90° triangle



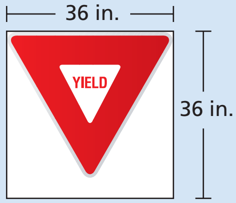
28. 45°-45°-90° triangle



CONCEPT CONNECTION

29. This problem will prepare you for the Concept Connection on page 614.

A sign manufacturer makes yield signs by cutting an equilateral triangle from a square piece of aluminum with the dimensions shown.



a. Find the height of the yield sign to the nearest tenth.

b. Find the area of the sign to the nearest tenth.

c. How much material is left after a sign is made?

Find the missing measurements for each rectangle.

	Base b	Height h	Area A	Perimeter P
30.	12	16		
31.	17		136	
32.		11		50
33.			216	66

34. The perimeter of a rectangle is 72 in. The base is 3 times the height. Find the area of the rectangle.

35. The area of a triangle is 50 cm^2. The base of the triangle is 4 times the height. Find the height of the triangle.

36. The perimeter of an isosceles trapezoid is 40 ft. The bases of the trapezoid are 11 ft and 19 ft. Find the area of the trapezoid.

Use the conversion table for Exercises 37–42.

Conversion Factors	
Metric	**Customary**
1 km = 1000 m	1 mi = 1760 yd
1 m = 100 cm	1 mi = 5280 ft
1 cm = 10 mm	1 yd = 3 ft
	1 ft = 12 in.

37. 1 yd^2 = ___?___ ft^2 **38.** 1 m^2 = ___?___ cm^2

39. 1 cm^2 = ___?___ mm^2 **40.** 1 mi^2 = ___?___ in^2

41. A triangle has a base of 3 yd and a height of 8 yd. Find the area in square feet.

42. A rhombus has diagonals 500 yd and 800 yd in length. Find the area in square miles.

LINK

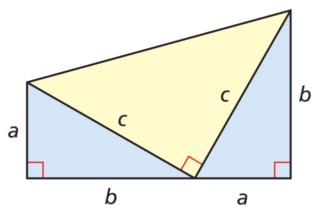
History

President James Garfield was a classics professor and a major general in the Union Army. He was assassinated in 1881.

Source: www.whitehouse.gov

The Granger Collection, New York

43. The following proof of the Pythagorean Theorem was discovered by President James Garfield in 1876 while he was a member of the House of Representatives.

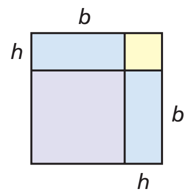


a. Write the area of the trapezoid in terms of a and b.

b. Write the areas of the three triangles in terms of a, b, and c.

c. Use the Area Addition Postulate to write an equation relating your results from parts **a** and **b.** Simplify the equation to prove the Pythagorean Theorem.

44. Use the diagram to prove the formula for the area of a rectangle, given the formula for the area of a square.



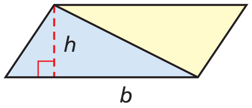
Given: Rectangle with base b and height h

Prove: The area of the rectangle is $A = bh$.

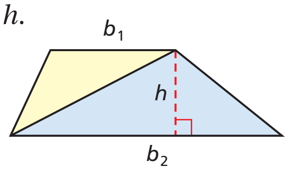
Plan: Use the formula for the area of a square to find the areas of the outer square and the two squares inside the figure. Write and solve an equation for the area of the rectangle.

Prove each area formula.

45. **Given:** Parallelogram with area $A = bh$
Prove: The area of the triangle is $A = \frac{1}{2}bh$.



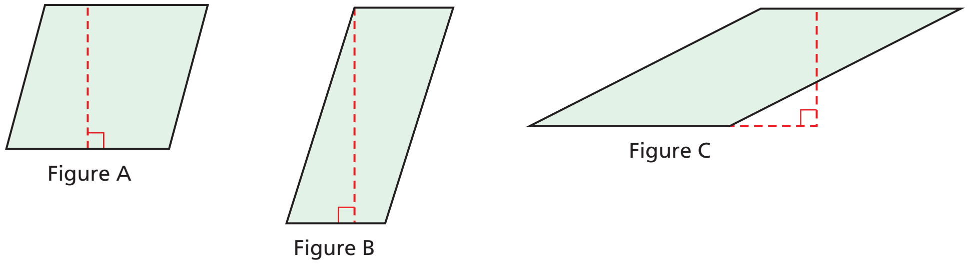
46. **Given:** Triangle with area $A = \frac{1}{2}bh$
Prove: The area of the trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$.



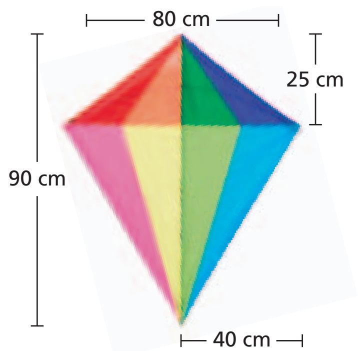
47. Measurement Choose an appropriate unit of measurement and measure the base and height of each parallelogram.

a. Find the area of each parallelogram. Give your answer with the correct precision.

b. Which has the greatest area?

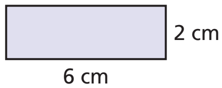


48. Hobbies Tina is making a kite according to the plans at right. The fabric weighs about 40 grams per square meter. The diagonal braces, or *spars*, weigh about 20 grams per meter. Estimate the weight of the kite.



49. Home Improvement Tom is buying tile for a 12 ft by 18 ft rectangular kitchen floor. He needs to buy 15% extra in case some of the tiles break. The tiles are squares with 4 in. sides that come in cases of 100. How many cases should he buy?

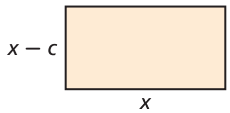
50. Critical Thinking If the maximum error in the given measurements of the rectangle is 0.1 cm, what is the greatest possible error in the area? Explain.



51. Write About It A square is also a parallelogram, a rectangle, and a rhombus. Prove that the area formula for each shape gives the same result as the formula for the area of a square.

STANDARDIZED TEST PREP

52. Which expression best represents the area of the rectangle?



Ⓐ $2x + 2(x - c)$
Ⓑ $x(x - c)$
Ⓒ $x^2 + (x - c)^2$
Ⓓ $2x(x - c)$

53. The length of a rectangle is 3 times the width. The perimeter is 48 inches. Which system of equations can be used to find the dimensions of the rectangle?

Ⓕ $\ell = w + 3$, $2(\ell + w) = 48$
Ⓖ $\ell = 3w$, $2\ell + 6w = 48$
Ⓗ $\ell = 3w$, $2(\ell + w) = 48$
Ⓙ $\ell = w + 3$, $2\ell + 6w = 48$

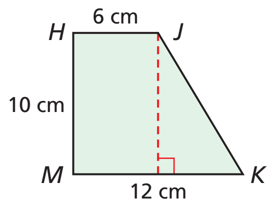
54. A 16- by 18-foot rectangular section of a wall will be covered by square tiles that measure 2 feet on each side. If the tiles are not cut, how many of them will be needed to cover the section of the wall?

Ⓐ 288 Ⓑ 144 Ⓒ 72 Ⓓ 17

55. The area of trapezoid *HJKM* is 90 square centimeters. Which is closest to the length of $\overline{JK}$?

Ⓕ 10 centimeters Ⓗ 11.7 centimeters

Ⓖ 10.5 centimeters Ⓙ 16 centimeters

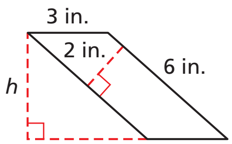


56. **Gridded Response** A driveway is shaped like a parallelogram with a base of 28 feet and a height of 17 feet. Covering the driveway with crushed stone will cost \$2.75 per square foot. How much will it cost to cover the driveway with crushed stone?

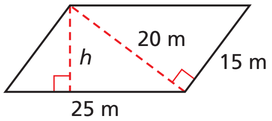
CHALLENGE AND EXTEND

Multi-Step **Find h in each parallelogram.**

57.



58.



59. **Algebra** A rectangle has a perimeter of $(26x + 16)$ cm and an area of $(42x^2 + 51x + 15)$ cm^2. Find the dimensions of the rectangle in terms of x.

60. Prove that the area of any quadrilateral with perpendicular diagonals is $\frac{1}{2}d_1d_2$.

61. **Gardening** A gardener has 24 feet of fencing to enclose a rectangular garden.

a. Let x and y represent the side lengths of the rectangle. Solve the perimeter formula $2x + 2y = 24$ for y, and substitute the expression into the area formula $A = xy$.

b. Graph the resulting function on a coordinate plane. What are the domain and range of the function?

c. What are the dimensions of the rectangle that will enclose the greatest area?

d. **Write About It** How would you find the dimensions of the rectangle with the least perimeter that would enclose a rectangular area of 100 square feet?

SPIRAL REVIEW

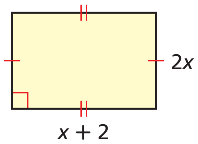
Determine the range of each function for the given domain. *(Previous course)*

62. $f(x) = x - 3$, domain: $-4 \le x \le 6$

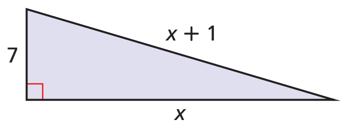
63. $f(x) = -x^2 + 2$, domain: $-2 \le x \le 2$

Find the perimeter and area of each figure. Express your answers in terms of x. *(Lesson 1-5)*

64.



65.



Write each vector in component form. *(Lesson 8-6)*

66. $\overrightarrow{LM}$ with $L(4, 3)$ and $M(5, 10)$

67. $\overrightarrow{ST}$ with $S(-2, -2)$ and $T(4, 6)$

9-2 Geometry LAB

Develop π

Use with Lesson 9-2

The ratio of the circumference of a circle to its diameter is defined as π. All circles are similar, so this ratio is the same for all circles:

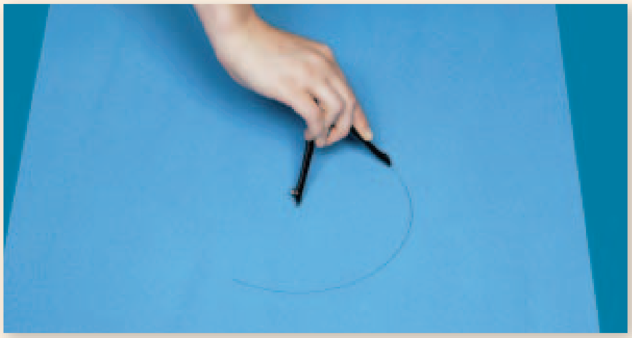
$$\pi = \frac{\text{circumference}}{\text{diameter}}.$$

California Standards

8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area **of common geometric figures.**

Activity 1

1. Use your compass to draw a large circle on a piece of cardboard and then cut it out.
2. Use a measuring tape to measure the circle's diameter and circumference as accurately as possible.
3. Use the results from your circle to estimate π. Compare your answers with the results of the rest of the class.



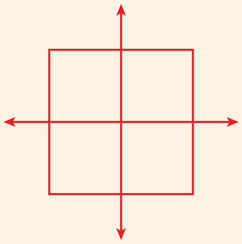
Try This

1. Do you think it is possible to draw a circle whose ratio of circumference to diameter is not π? Why or why not?
2. How does knowing the relationship between circumference, diameter, and π help you determine the formula for circumference?
3. Use a ribbon to make a π measuring tape. Mark off increments of π inches or π cm on your ribbon as accurately as possible. How could you use this π measuring tape to find the diameter of a circular object? Use your π measuring tape to measure 5 circular objects. Give the circumference and diameter of each object.

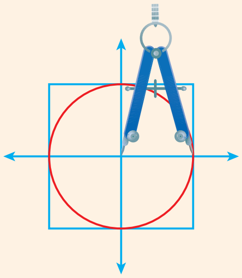
Archimedes used inscribed and circumscribed polygons to estimate the value of π. His "method of exhaustion" is considered to be an early version of calculus. In the figures below, the circumference of the circle is less than the perimeter of the larger polygon and greater than the perimeter of the smaller polygon. This fact is used to estimate π.

Activity 2

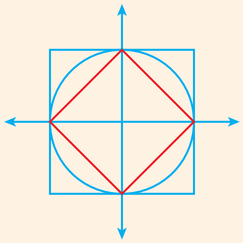
1. Construct a large square. Construct the perpendicular bisectors of two adjacent sides.



2. Use your compass to draw an inscribed circle as shown.



3. Connect the midpoints of the sides to form a square that is inscribed in the circle.



4. Let P_1 represent the perimeter of the smaller square, P_2 represent the perimeter of the larger square, and C represent the circumference of the circle. Measure the squares to find P_1 and P_2 and substitute the values into the inequality below.

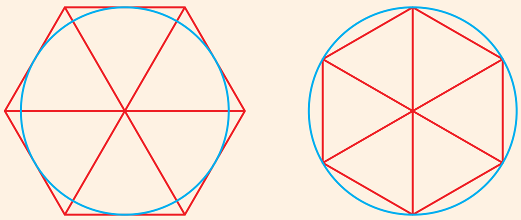
$$P_1 < C < P_2$$

5. Divide each expression in the inequality by the diameter of the circle. Why does this give you an inequality in terms of π? Complete the inequality below.

$$\underline{\quad ? \quad} < \pi < \underline{\quad ? \quad}$$

Try This

4. Use the perimeters of the inscribed and circumscribed regular hexagons to write an inequality for π. Assume the diameter of each circle is 2 units.



5. Compare the inequalities you found for π. What do you think would be true about your inequality if you used regular polygons with more sides? How could you use inscribed and circumscribed regular polygons to estimate π?

6. An alternate definition of π is the area of a circle with radius 1. How could you use this definition and the figures above to estimate the value of π?

9-2 Developing Formulas for Circles and Regular Polygons

Objectives

Develop and apply the formulas for the area and circumference of a circle.

Develop and apply the formula for the area of a regular polygon.

Vocabulary

circle
center of a circle
center of a regular polygon
apothem
central angle of a regular polygon

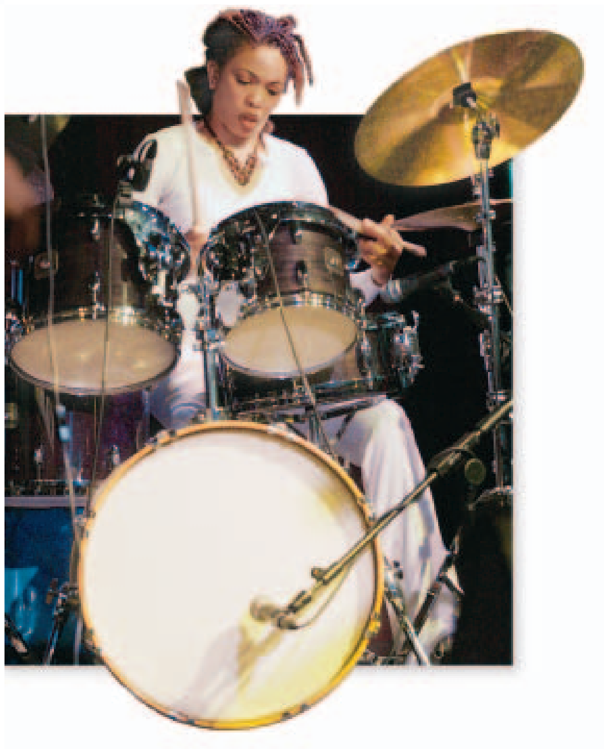
California Standards

8.0 Students know, derive, and solve problems involving the perimeter, **circumference, area,** volume, lateral area, and surface area **of common geometric figures.**

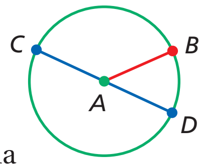
10.0 Students compute areas of polygons, including rectangles, scalene triangles, **equilateral triangles,** rhombi, parallelograms, and trapezoids.

Who uses this?

Drummers use drums of different sizes to produce different notes. The pitch is related to the area of the top of the drum. (See Example 2.)

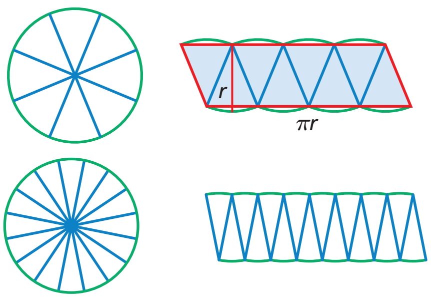


A **circle** is the locus of points in a plane that are a fixed distance from a point called the **center of the circle**. A circle is named by the symbol ⊙ and its center. ⊙A has radius $r = AB$ and diameter $d = CD$.



The irrational number π is defined as the ratio of the circumference C to the diameter d, or $\pi = \frac{C}{d}$. Solving for C gives the formula $C = \pi d$. Also $d = 2r$, so $C = 2\pi r$.

You can use the circumference of a circle to find its area. Divide the circle and rearrange the pieces to make a shape that resembles a parallelogram.



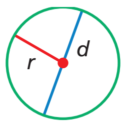
The base of the parallelogram is about half the circumference, or πr, and the height is close to the radius r. So $A \cong \pi r \cdot r = \pi r^2$.

The more pieces you divide the circle into, the more accurate the estimate will be.

Know it! Note

Circumference and Area | **Circle**

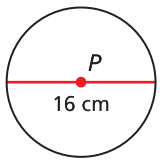
A circle with diameter d and radius r has circumference $C = \pi d$ or $C = 2\pi r$ and area $A = \pi r^2$.



EXAMPLE 1

Finding Measurements of Circles

Find each measurement.



A **the area of ⊙P in terms of π**

$A = \pi r^2$ — *Area of a circle*

$A = \pi(8)^2$ — *Divide the diameter by 2 to find the radius, 8.*

$A = 64\pi \text{ cm}^2$ — *Simplify.*

Find each measurement.

B **the radius of $\odot X$ in which $C = 24\pi$ in.**

$C = 2\pi r$	*Circumference of a circle*
$24\pi = 2\pi r$	*Substitute 24π for C.*
$r = 12$ *in.*	*Divide both sides by 2π.*

C **the circumference of $\odot S$ in which $A = 9x^2 \pi \text{ cm}^2$**

Algebra

Step 1 Use the given area to solve for r.

$A = \pi r^2$	*Area of a circle*
$9x^2\pi = \pi r^2$	*Substitute $9x^2\pi$ for A.*
$9x^2 = r^2$	*Divide both sides by π.*
$3x = r$	*Take the square root of both sides.*

Step 2 Use the value of r to find the circumference.

$C = 2\pi r$	
$C = 2\pi(3x)$	*Substitute 3x for r.*
$C = 6x\pi$ cm	*Simplify.*

CHECK IT OUT! **1.** Find the area of $\odot A$ in terms of π in which $C = (4x - 6)\pi$ m.

EXAMPLE 2 *Music Application*

A drum kit contains three drums with diameters of 10 in., 12 in., and 14 in. Find the area of the top of each drum. Round to the nearest tenth.

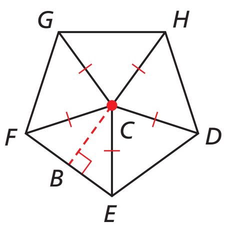
10 in. diameter	12 in. diameter	14 in. diameter
$A = \pi(5^2)$ $r = \frac{10}{2} = 5$	$A = \pi(6^2)$ $r = \frac{12}{2} = 6$	$A = \pi(7)^2$ $r = \frac{14}{2} = 7$
$\cong 78.5 \text{ in}^2$	$\cong 113.1 \text{ in}^2$	$\cong 153.9 \text{ in}^2$

Helpful Hint

The π key gives the best possible approximation for π on your calculator. Always wait until the last step to round.

CHECK IT OUT! **2.** Use the information above to find the circumference of each drum.

The **center of a regular polygon** is equidistant from the vertices. The **apothem** is the distance from the center to a side. A **central angle of a regular polygon** has its vertex at the center, and its sides pass through consecutive vertices. Each central angle measure of a regular n-gon is $\frac{360°}{n}$.



Regular pentagon *DEFGH* has center *C*, apothem *BC*, and central angle $\angle DCE$.

To find the area of a regular n-gon with side length s and apothem a, divide it into n congruent isosceles triangles.

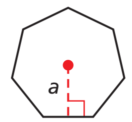
area of each triangle: $\frac{1}{2}as$

total area of the polygon: $A = n\left(\frac{1}{2}as\right)$, or $A = \frac{1}{2}aP$ *The perimeter is $P = ns$.*

Know it! Note

Area **Regular Polygon**

The area of a regular polygon with apothem a and perimeter P is $A = \frac{1}{2}aP$.

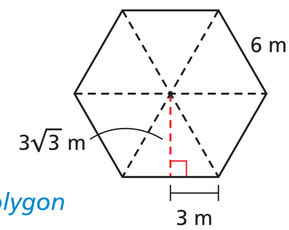


EXAMPLE 3 Finding the Area of a Regular Polygon

Find the area of each regular polygon. Round to the nearest tenth.

A **a regular hexagon with side length 6 m**

The perimeter is $6(6) = 36$ m. The hexagon can be divided into 6 equilateral triangles with side length 6 m. By the 30°-60°-90° Triangle Theorem, the apothem is $3\sqrt{3}$ m.



$A = \frac{1}{2}aP$ *Area of a regular polygon*

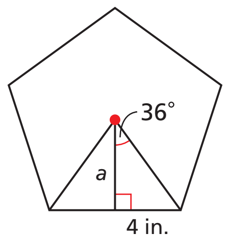
$A = \frac{1}{2}(3\sqrt{3})(36)$ *Substitute $3\sqrt{3}$ for a and 36 for P.*

$A = 54\sqrt{3} \cong 93.5 \text{ m}^2$ *Simplify.*

> **Remember!**
> The tangent of an angle in a right triangle is the ratio of the opposite leg length to the adjacent leg length. See page 525.

B **a regular pentagon with side length 8 in.**

Step 1 Draw the pentagon. Draw an isosceles triangle with its vertex at the center of the pentagon. The central angle is $\frac{360°}{5} = 72°$. Draw a segment that bisects the central angle and the side of the polygon to form a right triangle.



Step 2 Use the tangent ratio to find the apothem.

$\tan 36° = \frac{4}{a}$ *The tangent of an angle is $\frac{\text{opp. leg}}{\text{adj. leg}}$.*

$a = \frac{4}{\tan 36°}$ *Solve for a.*

Step 3 Use the apothem and the given side length to find the area.

$A = \frac{1}{2}aP$ *Area of a regular polygon*

$A = \frac{1}{2}\left(\frac{4}{\tan 36°}\right)(40)$ *The perimeter is $8(5) = 40$ in.*

$A \cong 110.1 \text{ in}^2$ *Simplify. Round to the nearest tenth.*

CHECK IT OUT! 3. Find the area of a regular octagon with a side length of 4 cm.

THINK AND DISCUSS

1. Describe the relationship between the circumference of a circle and π.
2. Explain how you would find the central angle of a regular polygon with n sides.
3. **GET ORGANIZED** Copy and complete the graphic organizer.

Know it! Note

Regular Polygons (Side Length = 1)					
Polygon	Number of Sides	Perimeter	Central Angle	Apothem	Area
Triangle					
Square					
Hexagon					

9-2 Exercises

California Standards
8.0, 10.0, 12.0, 19.0, 20.0, 7MG2.1

go.hrw.com
Homework Help Online
KEYWORD: MG7 9-2
Parent Resources Online
KEYWORD: MG7 Parent

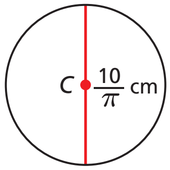
GUIDED PRACTICE

1. **Vocabulary** Describe how to find the *apothem* of a square with side length s.

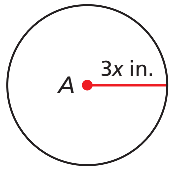
SEE EXAMPLE 1
p. 600

Find each measurement.

2. the circumference of $\odot C$



3. the area of $\odot A$ in terms of π



4. the circumference of $\odot P$ in which $A = 36\pi \text{ in}^2$

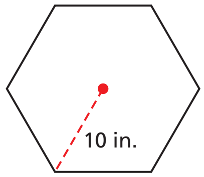
SEE EXAMPLE 2
p. 601

5. **Food** A pizza parlor offers pizzas with diameters of 8 in., 10 in., and 12 in. Find the area of each size pizza. Round to the nearest tenth.

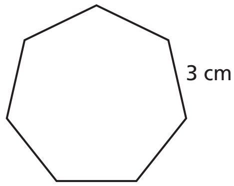
SEE EXAMPLE 3
p. 602

Find the area of each regular polygon. Round to the nearest tenth.

6.



7.



8. an equilateral triangle with an apothem of 2 ft

9. a regular dodecagon with a side length of 5 m

PRACTICE AND PROBLEM SOLVING

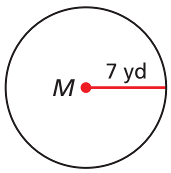
Independent Practice

For Exercises	See Example
10–12	1
13	2
14–17	3

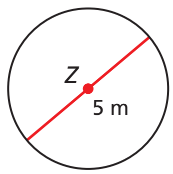
Extra Practice
Skills Practice p. S20
Application Practice p. S36

Find each measurement. Give your answers in terms of π.

10. the area of $\odot M$



11. the circumference of $\odot Z$

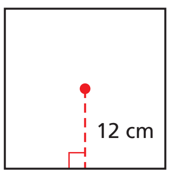


12. the diameter of $\odot G$ in which $C = 10$ ft.

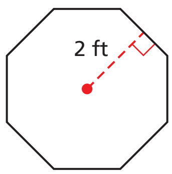
13. **Sports** A horse trainer uses circular pens that are 35 ft, 50 ft, and 66 ft in diameter. Find the area of each pen. Round to the nearest tenth.

Find the area of each regular polygon. Round to the nearest tenth, if necessary.

14.



15.



16. a regular nonagon with a perimeter of 144 in.

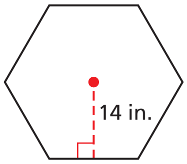
17. a regular pentagon with an apothem of 2 ft.

Find the central angle measure of each regular polygon. (*Hint:* To review polygon names, see page 382.)

18. equilateral triangle **19.** square **20.** pentagon **21.** hexagon

22. heptagon **23.** octagon **24.** nonagon **25.** decagon

Find the area of each regular polygon. Round to the nearest tenth.

26.


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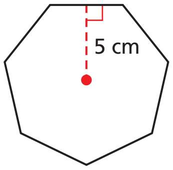

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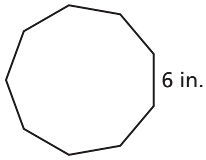

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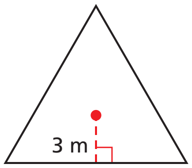

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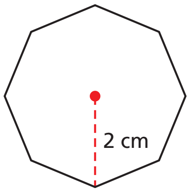
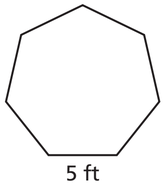
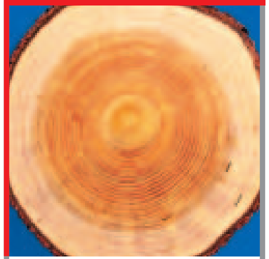


31.


LINK

Biology



Dendroclimatologists study tree rings for evidence of changes in weather patterns over time.

32. Biology You can estimate a tree's age in years by using the formula $a = \frac{r}{w}$, where r is the tree's radius without bark and w is the average thickness of the tree's rings. The circumference of a white oak tree is 100 in. The bark is 0.5 in. thick, and the average width of a ring is 0.2 in. Estimate the tree's age.

33. ERROR ANALYSIS A circle has a circumference of 2π in. Which calculation of the area is incorrect? Explain.

A

The circumference is 2π in., so the diameter is 2 in. The area is $A = \pi(2^2) = 4\pi$ in^2.

B

The circumference is 2π in., so the radius is 1 in. The area is $A = \pi(1^2) = 2\pi$ in^2.

Find the missing measurements for each circle. Give your answers in terms of π.

	Diameter d	Radius r	Area A	Circumference C
34.	6			
35.			100	
36.		17		
37.				36π

38. Multi-Step Janet is designing a garden around a gazebo that is a regular hexagon with side length 6 ft. The garden will be a circle that extends 10 feet from the vertices of the hexagon. What is the area of the garden? Round to the nearest square foot.

CONCEPT CONNECTION

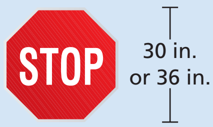
39. This problem will prepare you for the Concept Connection on page 614.

A stop sign is a regular octagon. The signs are available in two sizes: 30 in. or 36 in.

a. Find the area of a 30 in. sign. Round to the nearest tenth.

b. Find the area of a 36 in. sign. Round to the nearest tenth.

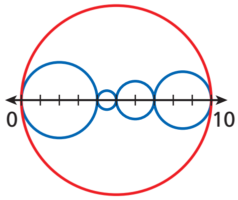
c. Find the percent increase in metal needed to make a 36 in. sign instead of a 30 in. sign.



40. Measurement A *trundle wheel* is used to measure distances by rolling it on the ground and counting its number of turns. If the circumference of a trundle wheel is 1 meter, what is its diameter?

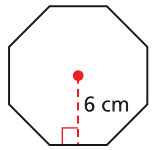
41. Critical Thinking Which do you think would seat more people, a 4 ft by 6 ft rectangular table or a circular table with a diameter of 6 ft? How many people would you sit at each table? Explain your reasoning.

42. Write About It The center of each circle in the figure lies on the number line. Describe the relationship between the circumference of the largest circle and the circumferences of the four smaller circles.



STANDARDIZED TEST PREP

43. Find the perimeter of the regular octagon to the nearest centimeter.



Ⓐ 5 Ⓑ 40 Ⓒ 20 Ⓓ 68

44. Which of the following ratios comparing a circle's circumference C to its diameter d gives the value of π?

Ⓕ $\frac{C}{d}$ Ⓖ $\frac{4C}{d^2}$ Ⓗ $\frac{d}{C}$ Ⓙ $\frac{d}{2C}$

45. Alisa has a circular tabletop with a 2-foot diameter. She wants to paint a pattern on the table top that includes a 2-foot-by-1-foot rectangle and 4 squares with sides 0.5 foot long. Which information makes this scenario impossible?

Ⓐ There will be no room left on the tabletop after the rectangle has been painted.

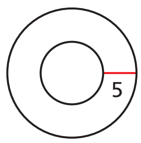
Ⓑ A 2-foot-long rectangle will not fit on the circular tabletop.

Ⓒ Squares cannot be painted on the circle.

Ⓓ There will not be enough room on the table to fit all the 0.5-foot squares.

CHALLENGE AND EXTEND

46. Two circles have the same center. The radius of the larger circle is 5 units longer than the radius of the smaller circle. Find the difference in the circumferences of the two circles.



47. Algebra Write the formula for the area of a circle in terms of its circumference.

48. Critical Thinking Show that the formula for the area of a regular n-gon approaches the formula for the area of a circle as n gets very large.

SPIRAL REVIEW

Write an equation for the linear function represented by the table. *(Previous course)*

49.

x	−2	0	5	10
y	−19	−13	2	17

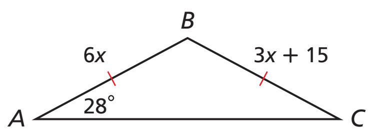
50.

x	−3	0	4	9
y	2	−1	−5	−10

Find each value. *(Lesson 4-8)*

51. m$\angle B$

52. AB



Find each measurement. *(Lesson 9-1)*

53. d_2 of a kite if $A = 14$ cm^2 and $d_1 = 20$ cm

54. the area of a trapezoid in which $b_1 = 3$ yd, $b_2 = 6$ yd, and $h = 4$ yd

9-3 Composite Figures

Objectives
Use the Area Addition Postulate to find the areas of composite figures.

Use composite figures to estimate the areas of irregular shapes.

Vocabulary
composite figure

Who uses this?
Landscape architects must compute areas of composite figures when designing gardens. (See Example 3.)

A **composite figure** is made up of simple shapes, such as triangles, rectangles, trapezoids, and circles. To find the area of a composite figure, find the areas of the simple shapes and then use the Area Addition Postulate.

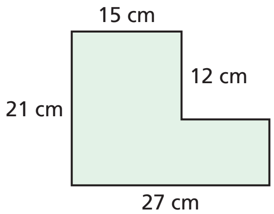
California Standards

8.0 Students know, derive, **and solve problems involving the** perimeter, circumference, **area,** volume, lateral area, and surface area **of common geometric figures.**
10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, **parallelograms, and trapezoids.**

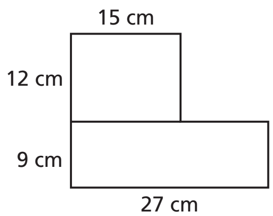
EXAMPLE 1 Finding the Areas of Composite Figures by Adding

Find the shaded area. Round to the nearest tenth, if necessary.

A



Divide the figure into rectangles.

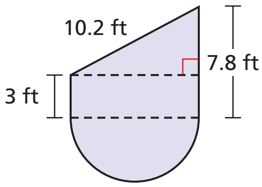


area of top rectangle:
$A = bh = 12(15) = 180 \text{ cm}^2$

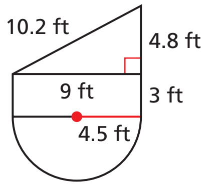
area of bottom rectangle:
$A = bh = 9(27) = 243 \text{ cm}^2$

shaded area:
$180 + 243 = 423 \text{ cm}^2$

B



Divide the figure into parts. The base of the triangle is $\sqrt{10.2^2 - 4.8^2} = 9$ ft.



area of triangle:
$A = \frac{1}{2}bh = \frac{1}{2}(9)(4.8) = 21.6 \text{ ft}^2$

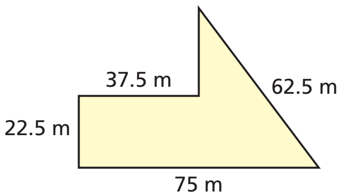
area of rectangle:
$A = bh = 9(3) = 27 \text{ ft}^2$

area of half circle:
$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(4.5^2) = 10.125\pi \text{ ft}^2$

shaded area:
$21.6 + 27 + 10.125\pi \approx 80.4 \text{ ft}^2$

CHECK IT OUT!

1. Find the shaded area. Round to the nearest tenth, if necessary.

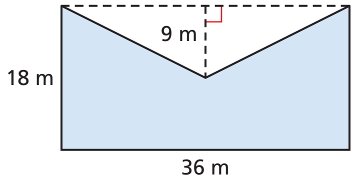


Sometimes you need to subtract to find the area of a composite figure.

EXAMPLE 2 Finding the Areas of Composite Figures by Subtracting

Find the shaded area. Round to the nearest tenth, if necessary.

A



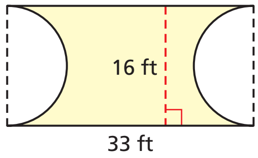
Subtract the area of the triangle from the area of the rectangle.

area of rectangle:
$A = bh = 18(36) = 648 \text{ m}^2$

area of triangle:
$A = \frac{1}{2}bh = \frac{1}{2}(36)(9) = 162 \text{ m}^2$

area of figure:
$A = 648 - 162 = 486 \text{ m}^2$

B



The two half circles have the same area as one circle. Subtract the area of the circle from the area of the rectangle.

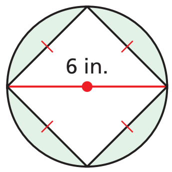
area of the rectangle:
$A = bh = 33(16) = 528 \text{ ft}^2$

area of circle:
$A = \pi r^2 = \pi(8^2) = 64\pi \text{ ft}^2$

area of figure:
$A = 528 - 64\pi \approx 326.9 \text{ ft}^2$

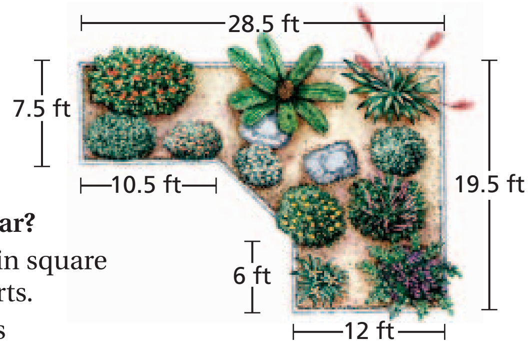
CHECK IT OUT!

2. Find the shaded area. Round to the nearest tenth, if necessary.



EXAMPLE 3 *Landscaping Application*

Katie is using the given plan to convert part of her lawn to a xeriscape garden. A newly planted xeriscape uses 17 gallons of water per square foot per year. How much water will the garden require in one year?



To find the area of the garden in square feet, divide the garden into parts.

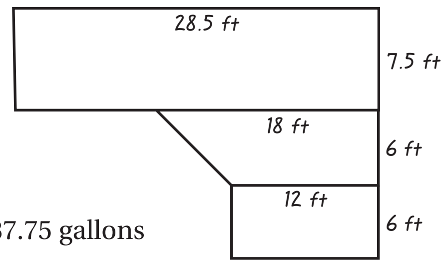
The area of the top rectangle is $28.5(7.5) = 213.75 \text{ ft}^2$.

The area of the center trapezoid is $\frac{1}{2}(12 + 18)(6) = 90 \text{ ft}^2$.

The area of the bottom rectangle is $12(6) = 72 \text{ ft}^2$.

The total area of the garden is $213.75 + 90 + 72 = 375.75 \text{ ft}^2$.

The garden will use $375.75(17) = 6387.75$ gallons of water per year.



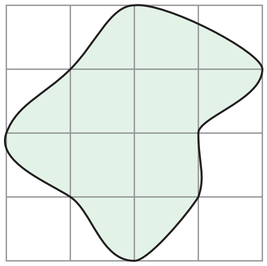
CHECK IT OUT!

3. The lawn that Katie is replacing requires 79 gallons of water per square foot per year. How much water will Katie save by planting the xeriscape garden?

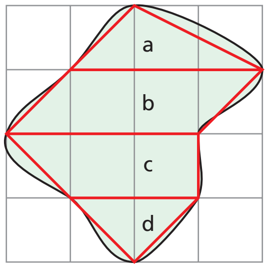
To estimate the area of an irregular shape, you can sometimes use a composite figure. First, draw a composite figure that resembles the irregular shape. Then divide the composite figure into simple shapes.

EXAMPLE 4 Estimating Areas of Irregular Shapes

Use a composite figure to estimate the shaded area. The grid has squares with side lengths of 1 cm.



Draw a composite figure that approximates the irregular shape. Find the area of each part of the composite figure.



area of triangle a:

$A = \frac{1}{2}bh = \frac{1}{2}(3)(1) = 1.5 \text{ cm}^2$

area of parallelogram b:

$A = bh = 3(1) = 3 \text{ cm}^2$

area of trapezoid c:

$A = \frac{1}{2}(3 + 2)(1) = 2.5 \text{ cm}^2$

area of triangle d:

$A = \frac{1}{2}(2)(1) = 1 \text{ cm}^2$

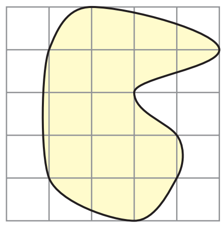
area of composite figure:

$1.5 + 3 + 2.5 + 1 = 8 \text{ cm}^2$

The shaded area is about 8 cm^2.

CHECK IT OUT!

4. Use a composite figure to estimate the shaded area. The grid has squares with side lengths of 1 ft.



THINK AND DISCUSS

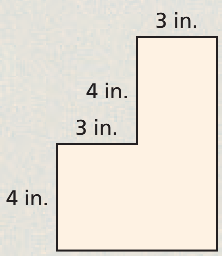
1. Describe a composite figure whose area you could find by using subtraction.

2. Explain how to find the area of an irregular shape by using a composite figure.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Use the given composite figure.

1. Divide the figure into parts. → **2.** Find the area of each part. → **3.** Add to find the total area.



9-3 Exercises

California Standards
7.0, 8.0, 10.0,
7NS1.7, 7MG2.1, 7MG2.2

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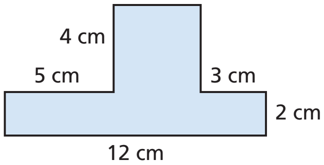
Homework Help Online
KEYWORD: MG7 9-3
Parent Resources Online
KEYWORD: MG7 Parent

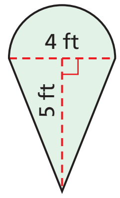
GUIDED PRACTICE

1. **Vocabulary** Draw a *composite figure* that is made up of two rectangles.

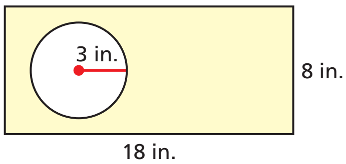
SEE EXAMPLE 1
p. 606

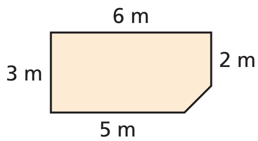
Multi-Step **Find the shaded area. Round to the nearest tenth, if necessary.**

2. 

3. 

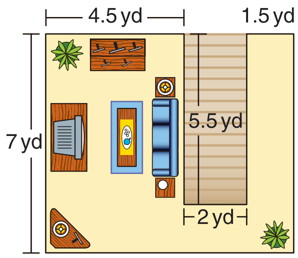
SEE EXAMPLE 2
p. 607

4. 

5. 

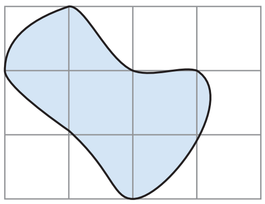
SEE EXAMPLE 3
p. 607

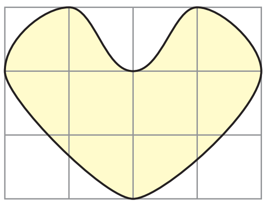
6. **Interior Decorating** Barbara is getting carpet installed in her living room and hallway. The cost of installation is \$6 per square yard. What is the total cost of installing the carpet?



SEE EXAMPLE 4
p. 608

Use a composite figure to estimate each shaded area. The grid has squares with side lengths of 1 in.

7. 

8. 

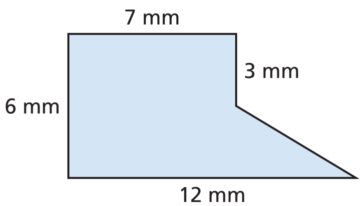
PRACTICE AND PROBLEM SOLVING

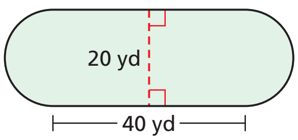
Independent Practice

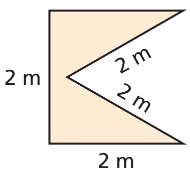
For Exercises	See Example
9–10	1
11–12	2
13	3
14–15	4

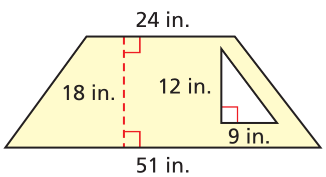
Extra Practice
Skills Practice p. S20
Application Practice p. S36

Multi-Step **Find the shaded area. Round to the nearest tenth, if necessary.**

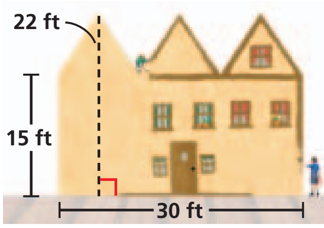
9. 

10. 

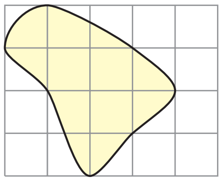
11. 

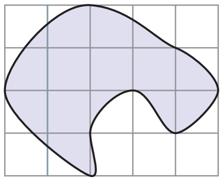
12. 

13. **Drama** Pat is painting a stage backdrop for a play. The paint he is using covers 90 square feet per quart. How many quarts of paint should Pat buy?

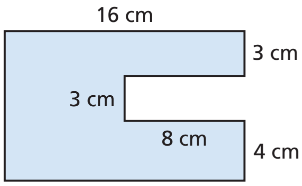


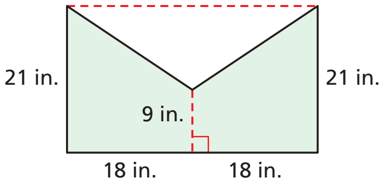
Use a composite figure to estimate each shaded area. The grid has squares with side lengths of 1 m.

14. 

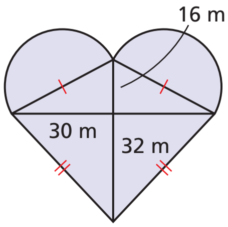
15. 

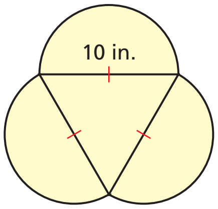
Find the area of each figure first by adding and then by subtracting. Compare your answers.

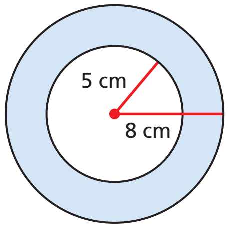
16. 

17. 

Find the area of each figure. Give your answers in terms of π.

18. 

19. 

20. 

21. **Geography** Use the grid on the map of Lake Superior to estimate the area of the surface of the lake. Each square on the grid has a side length of 100 miles.

22. **Critical Thinking** A trapezoid can be divided into a rectangle and two triangles. Show that the area formula for a trapezoid gives the same result as the sum of the areas of the rectangle and triangles.

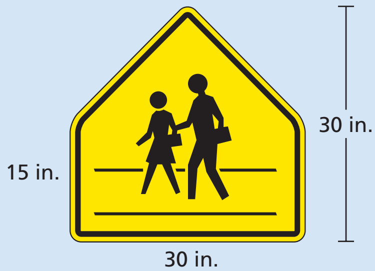
CONCEPT CONNECTION

23. This problem will prepare you for the Concept Connection on page 614. A school crossing sign has the dimensions shown.

  a. Find the area of the sign.

  b. A manufacturer has a rectangular sheet of metal measuring 45 in. by 105 in. Draw a figure that shows how 6 school crossing signs can be cut from this sheet of metal.

  c. How much metal will be left after the six signs are made?



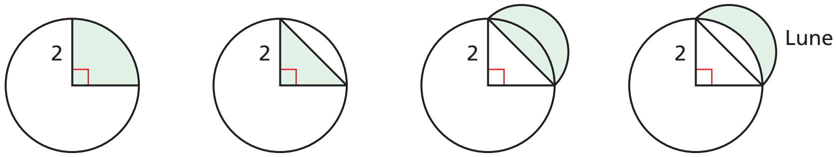
Multi-Step **Use a ruler and compass to draw each figure and then find the area.**

24. A rectangle with a base length of $b = 3$ cm and a height of $h = 4$ cm has a circle with a radius of $r = 1$ cm removed from the interior.

25. A square with a side length of $s = 4$ in. shares a side with a triangle with a height of $h = 5$ in. and a base length of $b = 4$ in. and shares another side with a half circle with $d = 4$ in.

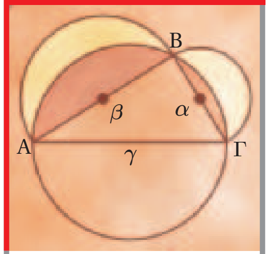
26. A circle with a radius of $r = 5$ cm has a right triangle with a base of $b = 8$ cm and a height of $h = 6$ cm removed from its interior.

27. **Multi-Step** A lune is a crescent-shaped figure bounded by two intersecting circles. Find the shaded area in each of the first three diagrams, and then use your results to find the area of the lune.



LINK

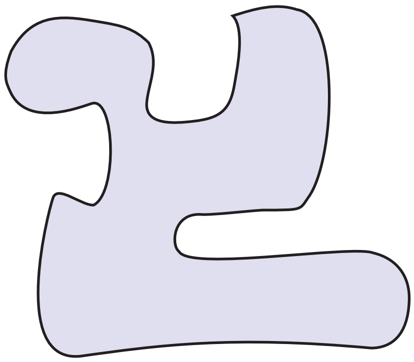
Math History



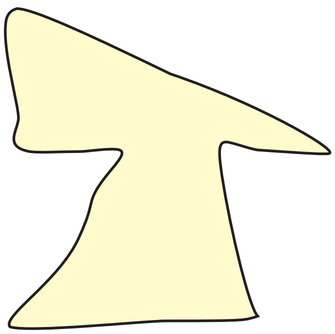
Hippocrates attempted to use lunes to solve a problem that has since been proven impossible: constructing a square with the same area as a given circle.

Estimation **Trace each irregular shape and draw a composite figure that approximates it. Measure the composite figure and use it to estimate the area of the irregular shape.**

28.



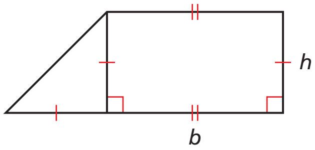
29.



30. **Write About It** Explain when you would use addition to find the area of a composite figure and when you would use subtraction.

STANDARDIZED TEST PREP

31. Which equation can be used to find the area of the composite figure?

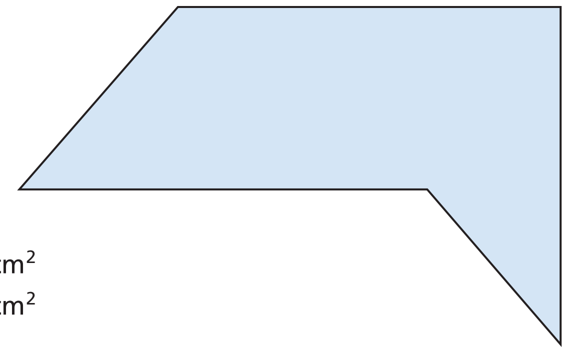


Ⓐ $A = bh + \frac{1}{2}(h)^2$ Ⓒ $A = h + 2b + h^2$

Ⓑ $A = bh + h^2$ Ⓓ $A = h + 2b + \frac{1}{2}h^2$

32. Use a ruler to measure the dimensions of the composite figure to the nearest tenth of a centimeter.

Which of the following best represents the area of the composite figure?



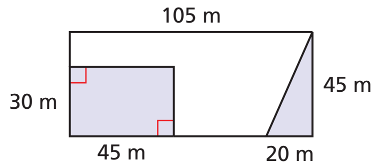
Ⓕ 4 cm^2 Ⓗ 22 cm^2

Ⓖ 19 cm^2 Ⓙ 42 cm^2

33. Find the area of the unshaded part of the rectangle.

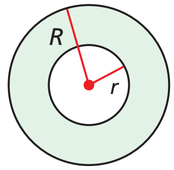
Ⓐ 1800 m^2 Ⓒ 2925 m^2

Ⓑ 2250 m^2 Ⓓ 4725 m^2



CHALLENGE AND EXTEND

34. An *annulus* is the region between two circles that have the same center. Write the formula for the area of the annulus in terms of the outer radius R and the inner radius r.



35. Draw two composite figures with the same area: one made up of two rectangles and the other made up of a rectangle and a triangle.

36. Draw a composite figure that has a total area of $10\pi \text{ cm}^2$ and is made up of a rectangle and a half circle. Label the dimensions of your figure.

SPIRAL REVIEW

Find each sale price. *(Previous course)*

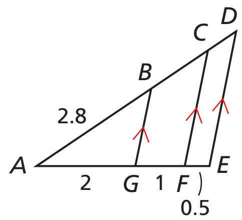
37. 20% off a regular price of \$19.95

38. 15% off a regular price of \$34.60

Find the length of each segment. *(Lesson 7-4)*

39. $\overline{BC}$

40. $\overline{CD}$



Find the area of each regular polygon. Round to the nearest tenth. *(Lesson 9-2)*

41. an equilateral triangle with a side length of 3 cm

42. a regular hexagon with an apothem of $4\sqrt{3}$ m

Career Path

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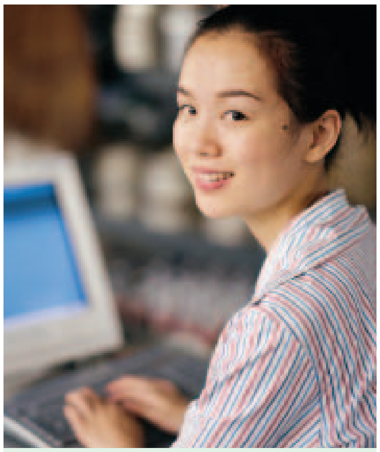


Anessa Liu
Technical writer

Q: What math classes did you take in high school?

A: In high school I took Algebra 1, Geometry, Algebra 2, and Trigonometry.

Q: What math classes did you take in college?

A: In college I took Precalculus, Calculus, and Statistics.

Q: What technical materials do you write?

A: I write training manuals for computer software packages.

Q: How do you use math?

A: Some manuals I write are for math programs, so I use a lot of formulas to describe patterns and measurements.

Q: What are your future plans?

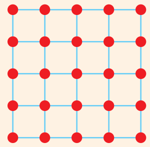
A: After I get a few more years experience writing manuals, I would like to train others who use these programs.

9-3 Geometry LAB

Use with Lesson 9-3

Develop Pick's Theorem for Area of Lattice Polygons

A *lattice polygon* is a polygon drawn on graph paper so that all its vertices are on intersections of grid lines, called *lattice points*. The lattice points of the grid at right are shown in red.



In this lab, you will discover a formula called Pick's Theorem, which is used to find the area of lattice polygons.

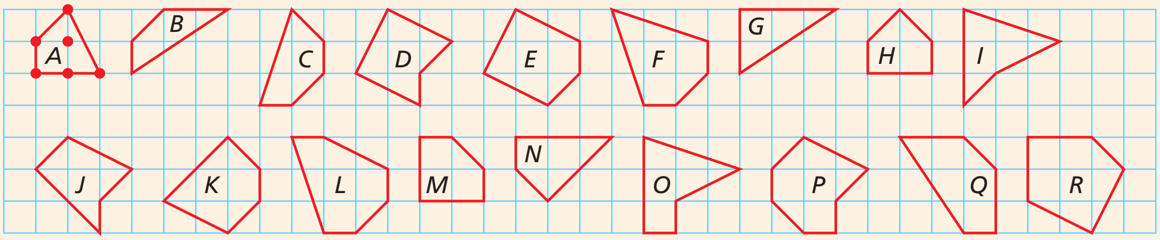
California Standards

1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and **inductive** and deductive **reasoning.**

10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids.

Activity

1. Find the area of each figure. Create a table like the one below with a row for each shape to record your answers. The first one is done for you.



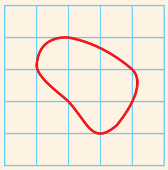
2. Count the number of lattice points on the boundary of each figure. Record your answers in the table.

3. Count the number of lattice points in the interior of each figure. Record your answers in the table.

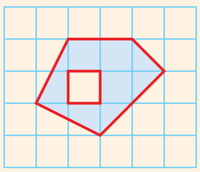
Figure	Area	Number of Lattice Points	
		On Boundary	In Interior
A	2.5	5	1
B			
C			
D			
E			
F			

Try This

1. **Make a Conjecture** What do you think is true about the relationship between the area of a figure and the number of lattice points on the boundary and in the interior of the figure? Write your conjecture as a formula in terms of the number of lattice points on the boundary B and the number of lattice points in the interior I.
2. Test your conjecture by drawing at least three different figures on graph paper and by finding their areas.
3. Estimate the area of the curved figure by using a lattice polygon.



4. Find the shaded area in the figure by subtracting. Test your formula on this figure. Does your formula work for figures with holes in them?

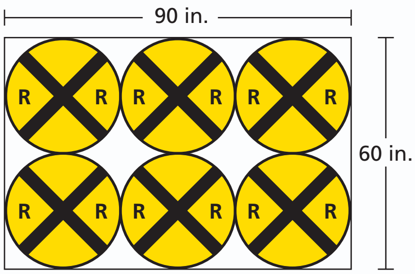


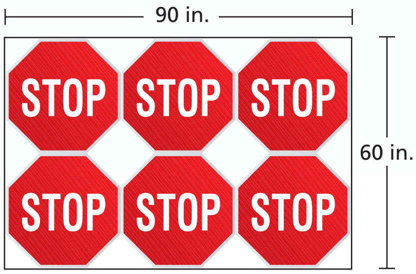
CONCEPT CONNECTION

Developing Geometric Formulas

Traffic Signs Traffic signs are usually made of reflective aluminum. A manufacturer of traffic signs begins with a rectangular sheet of aluminum that measures 60 in. by 90 in.

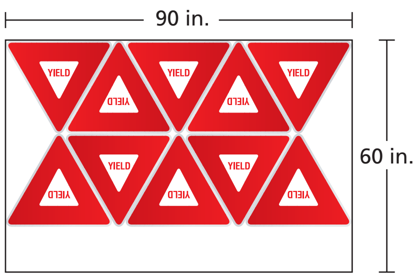
1. A railroad crossing sign is a circle with a diameter of 30 in. The manufacturer can make 6 of these signs from the sheet of aluminum by arranging the signs as shown. How much aluminum is left over once the signs have been made?





2. A stop sign is a regular octagon. The manufacturer can use the sheet of aluminum to make 6 stop signs as shown. How much aluminum is left over in this case?

3. A yield sign is an equilateral triangle with sides 30 in. long. By arranging the triangles as shown, the manufacturer can use the sheet of aluminum to make 10 yield signs. How much aluminum is left over when yield signs are made?



4. The making of which type of sign results in the least amount of waste?

READY TO GO ON?

CHAPTER 9

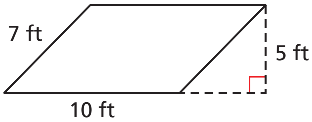
SECTION 9A

Quiz for Lessons 9-1 Through 9-3

9-1 Developing Formulas for Triangles and Quadrilaterals

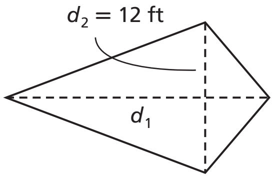
Find each measurement.

1. the area of the parallelogram

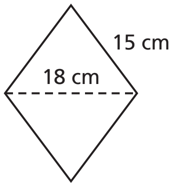


2. the base of the rectangle, in which $A = \left(24x^2 + 8x\right)\text{ m}^2$

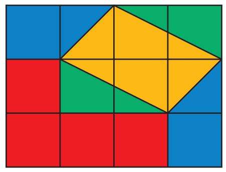
3. d_1 of the kite, in which $A = 126\text{ ft}^2$



4. the area of the rhombus



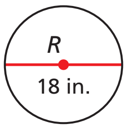
5. The tile mosaic shown is made up of 1 cm squares. Use the grid to find the perimeter and area of the green triangle, the blue trapezoid, and the yellow parallelogram.



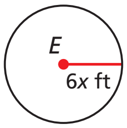
9-2 Developing Formulas for Circles and Regular Polygons

Find each measurement.

6. the circumference of $\odot R$ in terms of π



7. the area of $\odot E$ in terms of π



Find the area of each regular polygon. Round to the nearest tenth.

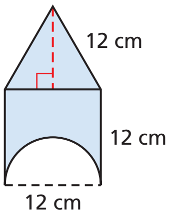
8. a regular hexagon with apothem 6 ft

9. a regular pentagon with side length 12 m

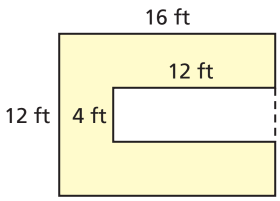
9-3 Composite Figures

Find the shaded area. Round to the nearest tenth, if necessary.

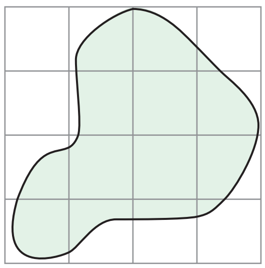
10.



11.



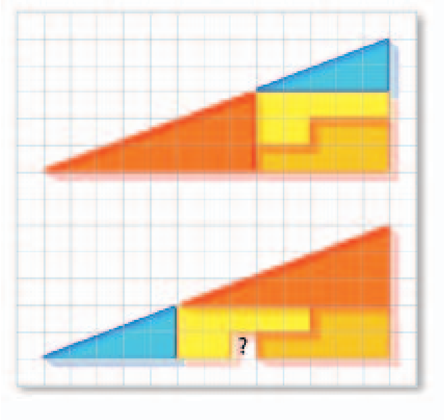
12. Shelby is planting grass in an irregularly shaped garden as shown. The grid has squares with side lengths of 1 yd. Estimate the area of the garden. Given that grass cost \$6.50 per square yard, find the cost of the grass.



9-4 Perimeter and Area in the Coordinate Plane

Objective
Find the perimeters and areas of figures in a coordinate plane.

Why learn this?
You can use figures in a coordinate plane to solve puzzles like the one at right. (See Example 4.)



In Lesson 9-3, you estimated the area of irregular shapes by drawing composite figures that approximated the irregular shapes and by using area formulas.

Another method of estimating area is to use a grid and count the squares on the grid.

California Standards

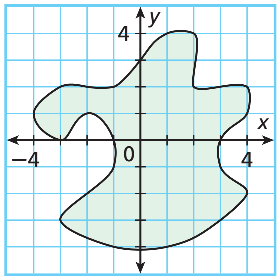
8.0 Students know, derive, **and solve problems involving the perimeter,** circumference, **area,** volume, lateral area, and surface area **of common geometric figures.**

10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids.

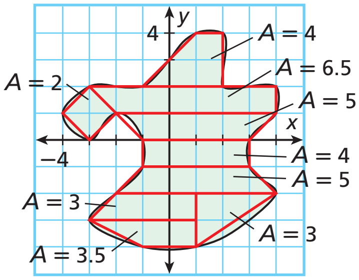
12.0 Students find and use measures of sides and of interior and exterior angles **of** triangles and **polygons to classify figures and solve problems.**

EXAMPLE 1 Estimating Areas of Irregular Shapes in the Coordinate Plane

Estimate the area of the irregular shape.

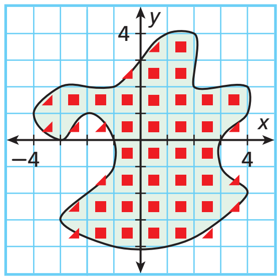


Method 1: Draw a composite figure that approximates the irregular shape and find the area of the composite figure.



The area is approximately $4 + 6.5 + 5 + 4 + 5 + 3.5 + 3 + 3 + 2 = 36$ units^2.

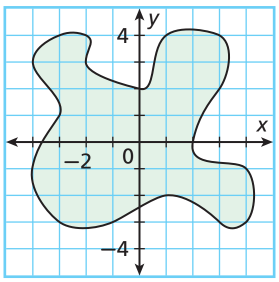
Method 2: Count the number of squares inside the figure, estimating half squares. Use a ■ for a whole square and a ◢ for a half square.



There are approximately 31 whole squares and 13 half squares, so the area is about $31 + \frac{1}{2}(13) = 37.5$ units^2.

CHECK IT OUT!

1. Estimate the area of the irregular shape.

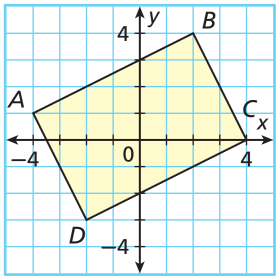


EXAMPLE 2 Finding Perimeter and Area in the Coordinate Plane

Algebra

Draw and classify the polygon with vertices $A(-4, 1)$, $B(2, 4)$, $C(4, 0)$, and $D(-2, -3)$. Find the perimeter and area of the polygon.

Step 1 Draw the polygon.



Step 2 $ABCD$ appears to be a rectangle. To verify this, use slopes to show that the sides are perpendicular.

slope of $\overline{AB}$: $\dfrac{4-1}{2-(-4)} = \dfrac{3}{6} = \dfrac{1}{2}$

slope of $\overline{BC}$: $\dfrac{0-4}{4-2} = \dfrac{-4}{2} = -2$

slope of $\overline{CD}$: $\dfrac{-3-0}{-2-4} = \dfrac{-3}{-6} = \dfrac{1}{2}$

slope of $\overline{DA}$: $\dfrac{1-(-3)}{-4-(-2)} = \dfrac{4}{-2} = -2$

The consecutive sides are perpendicular, so $ABCD$ is a rectangle.

Step 3 Let $\overline{CD}$ be the base and $\overline{BC}$ be the height of the rectangle. Use the Distance Formula to find each side length.

$b = CD = \sqrt{(-2-4)^2 + (-3-0)^2} = \sqrt{45} = 3\sqrt{5}$

$h = BC = \sqrt{(4-2)^2 + (0-4)^2} = \sqrt{20} = 2\sqrt{5}$

perimeter of $ABCD$: $P = 2b + 2h = 2(3\sqrt{5}) + 2(2\sqrt{5}) = 10\sqrt{5}$ units

area of $ABCD$: $A = bh = (3\sqrt{5})(2\sqrt{5}) = 30$ units^2.

Remember!

The distance from (x_1, y_1) to (x_2, y_2) in a coordinate plane is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, and the slope of the line containing the points is $m = \frac{y_2 - y_1}{x_2 - x_1}$. See pages 44 and 182.

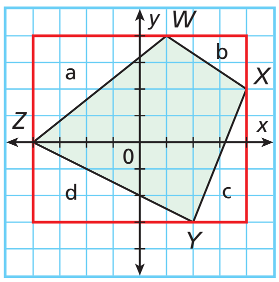
CHECK IT OUT!

2. Draw and classify the polygon with vertices $H(-3, 4)$, $J(2, 6)$, $K(2, 1)$, and $L(-3, -1)$. Find the perimeter and area of the polygon.

For a figure in a coordinate plane that does not have an area formula, it may be easier to enclose the figure in a rectangle and subtract the areas of the parts of the rectangle that are not included in the figure.

EXAMPLE 3 Finding Areas in the Coordinate Plane by Subtracting

Find the area of the polygon with vertices $W(1, 4)$, $X(4, 2)$, $Y(2, -3)$, and $Z(-4, 0)$.



Draw the polygon and enclose it in a rectangle.

area of the rectangle: $A = bh = 8(7) = 56$ units^2

area of the triangles:

a: $A = \frac{1}{2}bh = \frac{1}{2}(5)(4) = 10$ units^2

b: $A = \frac{1}{2}bh = \frac{1}{2}(3)(2) = 3$ units^2

c: $A = \frac{1}{2}bh = \frac{1}{2}(2)(5) = 5$ units^2

d: $A = \frac{1}{2}bh = \frac{1}{2}(6)(3) = 9$ units^2

The area of the polygon is $56 - 10 - 3 - 5 - 9 = 29$ units^2.

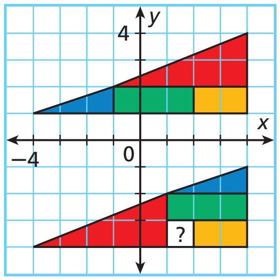
CHECK IT OUT!

3. Find the area of the polygon with vertices $K(-2, 4)$, $L(6, -2)$, $M(4, -4)$, and $N(-6, -2)$.

EXAMPLE 4 *Problem-Solving Application*

PROBLEM SOLVING

In the puzzle, the two figures are made up of the same pieces, but one figure appears to have a larger area. Use coordinates to show that the area does not change when the pieces are rearranged.



1 Understand the Problem

The parts of the puzzle appear to form two triangles with the same base and height that contain the same shapes, but one appears to have an area that is larger by one square unit.

2 Make a Plan

Find the areas of the shapes that make up each figure. If the corresponding areas are the same, then both figures have the same area by the Area Addition Postulate. To explain why the area appears to increase, consider the assumptions being made about the figure. Each figure is assumed to be a triangle with a base of 8 units and a height of 3 units. Both figures are divided into several smaller shapes.

3 Solve

Find the area of each shape.

Top figure	Bottom figure
red triangle:	red triangle:
$A = \frac{1}{2}bh = \frac{1}{2}(5)(2) = 5 \text{ units}^2$	$A = \frac{1}{2}bh = \frac{1}{2}(5)(2) = 5 \text{ units}^2$
blue triangle:	blue triangle:
$A = \frac{1}{2}bh = \frac{1}{2}(3)(1) = 1.5 \text{ units}^2$	$A = \frac{1}{2}bh = \frac{1}{2}(3)(1) = 1.5 \text{ units}^2$
green rectangle:	green rectangle:
$A = bh = (3)(1) = 3 \text{ units}^2$	$A = bh = (3)(1) = 3 \text{ units}^2$
yellow rectangle:	yellow rectangle:
$A = bh = (2)(1) = 2 \text{ units}^2$	$A = bh = (2)(1) = 2 \text{ units}^2$

The areas are the same. Both figures have an area of
$5 + 1.5 + 3 + 2 = 11.5 \text{ units}^2$.

If the figures were triangles, their areas would be $A = \frac{1}{2}(8)(3) = 12 \text{ units}^2$. By the Area Addition Postulate, the area is only 11.5 units^2, so the figures must not be triangles. Each figure is a quadrilateral whose shape is very close to a triangle.

4 Look Back

The slope of the hypotenuse of the red triangle is $\frac{2}{5}$. The slope of the hypotenuse of the blue triangle is $\frac{1}{3}$. Since the slopes are unequal, the hypotenuses do not form a straight line. This means the overall shapes are not triangles.

CHECK IT OUT!

4. Create a figure and divide it into pieces so that the area of the figure appears to increase when the pieces are rearranged.

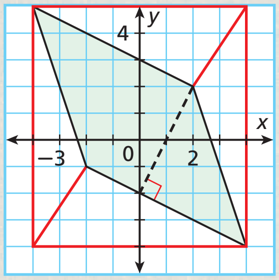
THINK AND DISCUSS

1. Describe two ways to estimate the area of an irregular shape in a coordinate plane.

2. Explain how you could use the Distance Formula to find the area of a special quadrilateral in a coordinate plane.

Know it! Note

3. GET ORGANIZED Copy the graph and the graphic organizer. Complete the graphic organizer by writing the steps used to find the area of the parallelogram.



Finding the Area

By using the formula | By subtracting

9-4 Exercises

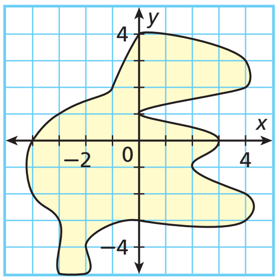
California Standards
5.0, 🔑 8.0, 🔑 10.0, 🔑 12.0, 7AF1.5, 🔑 7AF4.1, 7MG2.0, 7MG2.1, 7MG2.2, 7MG3.2, 7MR2.3, 1A8.0

GUIDED PRACTICE

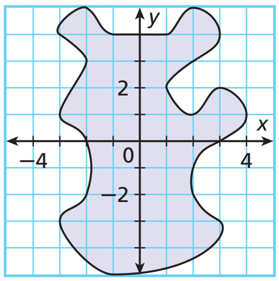
SEE EXAMPLE 1 p. 616

Estimate the area of each irregular shape.

1.



2.



SEE EXAMPLE 2 p. 617

Multi-Step Draw and classify the polygon with the given vertices. Find the perimeter and area of the polygon.

3. $V(-3, 0)$, $W(3, 0)$, $X(0, 3)$

4. $F(2, 8)$, $G(4, 4)$, $H(2, 0)$

5. $P(-2, 5)$, $Q(8, 5)$, $R(8, 1)$, $S(-2, 1)$

6. $A(-4, 2)$, $B(-2, 6)$, $C(6, 6)$, $D(8, 2)$

SEE EXAMPLE 3 p. 617

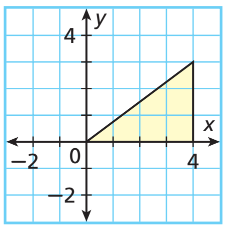
Find the area of each polygon with the given vertices.

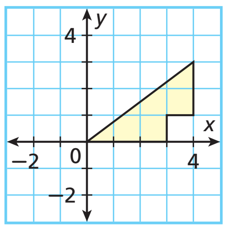
7. $S(3, 8)$, $T(8, 3)$, $U(2, 1)$

8. $L(3, 5)$, $M(6, 8)$, $N(9, 6)$, $P(5, 0)$

SEE EXAMPLE 4 p. 618

9. Find the area and perimeter of each polygon shown. Use your results to draw a polygon with a perimeter of 12 units and an area of 4 units^2 and a polygon with a perimeter of 12 units and an area of 3 units^2.





PRACTICE AND PROBLEM SOLVING

Independent Practice

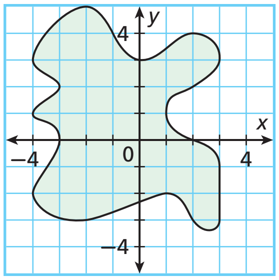
For Exercises	See Example
10–11	1
12–15	2
16–17	3
18	4

Extra Practice

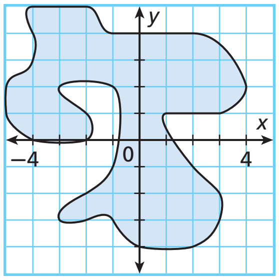
Skills Practice p. S21
Application Practice p. S36

Estimate the area of each irregular shape.

10.



11.



Multi-Step **Draw and classify the polygon with the given vertices. Find the perimeter and area of the polygon.**

12. $H(-3, -3)$, $J(-3, 3)$, $K(5, 3)$

13. $L(7, 5)$, $M(5, 0)$, $N(3, 5)$, $P(5, 10)$

14. $X(2, 1)$, $Y(5, 3)$, $Z(7, 1)$

15. $A(-3, 5)$, $B(2, 7)$, $C(2, 1)$, $D(-3, 3)$

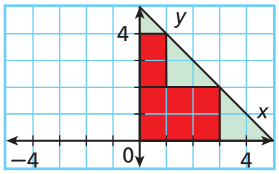
Find the area of each polygon with the given vertices.

16. $A(9, 9)$, $B(4, -4)$, $C(-4, 1)$

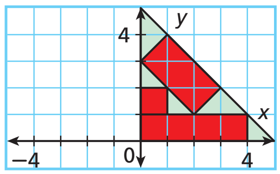
17. $T(-4, 4)$, $U(5, 3)$, $V(4, -5)$, $W(-5, 1)$

18. In which two figures do the rectangles cover the same area? Explain your reasoning.

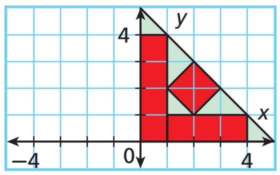
A



B



C

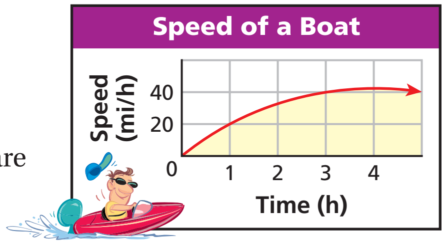


Algebra **Graph each set of lines to form a triangle. Find the area and perimeter.**

19. $y = 2$, $x = 5$, and $y = x$

20. $y = -5$, $x = 2$, and $y = -2x + 7$

21. **Transportation** The graph shows the speed of a boat versus time.



a. If the base of each square on the graph represents 1 hour and the height represents 20 miles per hour, what is the area of one square on the graph? Include units in your answer.

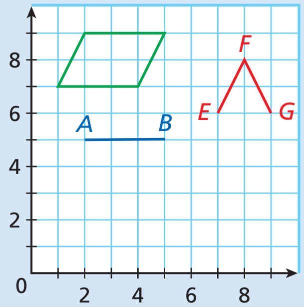
b. Estimate the shaded area in the graph.

c. **Critical Thinking** Use your results from part **a** to interpret the meaning of the area you found in part **b**. (*Hint:* Look at the units.)

22. **Write About It** Explain how to find the perimeter of the polygon with vertices $A(2, 3)$, $B(4, 0)$, $C(3, -2)$, $D(-1, -1)$, and $E(-2, 0)$.

CONCEPT CONNECTION

23. This problem will prepare you for the Concept Connection on page 638. A carnival game uses a 10-by-10 board with three targets. Each player throws a dart at the board and wins a prize if it hits a target.



a. One target is a parallelogram as shown. Find its area.

b. What should the coordinates be for points C and H so that the triangular target $\triangle ABC$ and the kite-shaped target $EFGH$ have the same area as the parallelogram?

STANDARDIZED TEST PREP

24. A circle with center $(0, 0)$ passes through the point $(3, 4)$. What is the area of the circle to the nearest tenth of a square unit?

Ⓐ 15.7 Ⓑ 25.0 Ⓒ 31.4 Ⓓ 78.5

25. $\triangle ABC$ with vertices $A(1, 1)$ and $B(3, 5)$ has an area of 10 units^2. Which is NOT a possible location of the third vertex?

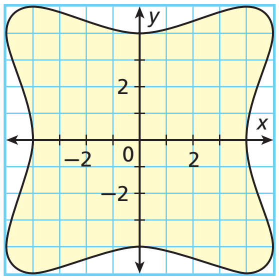
Ⓕ $C(-4, 1)$ Ⓖ $C(7, 3)$ Ⓗ $C(6, 1)$ Ⓙ $C(3, -3)$

26. **Extended Response** Mike estimated the area of the irregular figure to be 64 units^2.

a. Explain why his answer is not very accurate.

b. Explain how to use a composite figure to estimate the area.

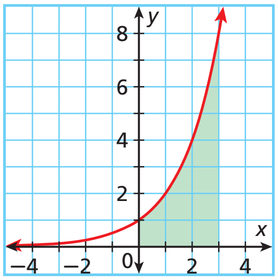
c. Explain how to estimate the area by averaging the areas of two squares.



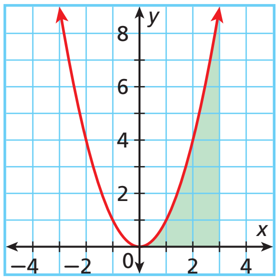
CHALLENGE AND EXTEND

Algebra **Estimate the shaded area under each curve.**

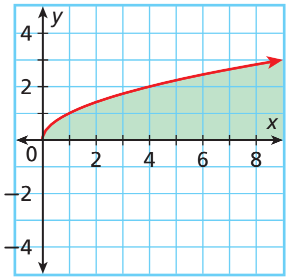
27. $y = 2^x$ for $0 \le x \le 3$



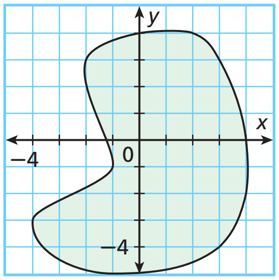
28. $y = x^2$ for $0 \le x \le 3$



29. $y = \sqrt{x}$ for $0 \le x \le 9$



30. **Estimation** Use a composite figure and the Distance Formula to estimate the perimeter of the irregular shape.



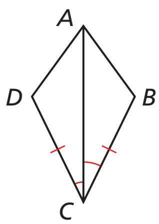
31. Graph a regular octagon on the coordinate plane with vertices on the x-and y-axes and on the lines $y = x$ and $y = -x$ so that the distance between opposite vertices is 2 units. Find the area and perimeter of the octagon.

SPIRAL REVIEW

Solve and graph each compound inequality. *(Previous course)*

32. $-4 < x + 3 < 7$ **33.** $0 < 2a + 4 < 10$ **34.** $12 \le -2m + 10 \le 20$

35. **Given:** $\overline{DC} \cong \overline{BC}$, $\angle DCA \cong \angle ACB$
Prove: $\angle DAC \cong \angle BAC$ *(Lesson 4-6)*



Find each measurement. *(Lesson 9-2)*

36. the area of $\odot C$ if the circumference is 16π cm

37. the diameter of $\odot H$ if the area is 121π ft^2

9-5 Effects of Changing Dimensions Proportionally

Objectives

Describe the effect on perimeter and area when one or more dimensions of a figure are changed.

Apply the relationship between perimeter and area in problem solving.

Why learn this?

You can analyze a graph to determine whether it is misleading or to explain why it is misleading. (See Example 4.)

In the graph, the height of each DVD is used to represent the number of DVDs shipped per year. However as the height of each DVD increases, the width also increases, which can create a misleading effect.

California Standards

8.0 Students know, derive, **and solve problems involving the perimeter, circumference, area,** volume, lateral area, and surface area **of common geometric figures.**

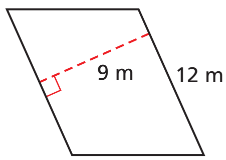
10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, **parallelograms,** and trapezoids.

11.0 Students determine how changes in dimensions affect the perimeter, area, and volume **of common geometric figures** and solids.

EXAMPLE 1 Effects of Changing One Dimension

Describe the effect of each change on the area of the given figure.

A The height of the parallelogram is doubled.



original dimensions:

$A = bh = 12(9)$

$= 108 \text{ cm}^2$

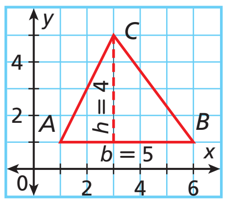
double the height:

$A = bh = 12(18)$

$= 216 \text{ cm}^2$

Notice that $216 = 2(108)$. If the height is doubled, the area is also doubled.

B The base length of the triangle with vertices $A(1, 1)$, $B(6, 1)$, and $C(3, 5)$ is multiplied by $\frac{1}{2}$.



Draw the triangle in a coordinate plane and find the base and height.

original dimensions:

$A = \frac{1}{2}bh = \frac{1}{2}(5)(4) = 10 \text{ units}^2$

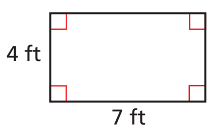
base multiplied by $\frac{1}{2}$:

$A = \frac{1}{2}bh = \frac{1}{2}(2.5)(4) = 5 \text{ units}^2$

Notice that $5 = \frac{1}{2}(10)$. If the base length is multiplied by $\frac{1}{2}$, the area is multiplied by $\frac{1}{2}$.

CHECK IT OUT!

1. The height of the rectangle is tripled. Describe the effect on the area.



EXAMPLE 2 Effects of Changing Dimensions Proportionally

Describe the effect of each change on the perimeter or circumference and the area of the given figure.

A **The base and height of a rectangle with base 8 m and height 3 m are both multiplied by 5.**

original dimensions:

$P = 2(8) + 2(3) = 22\text{ m}$ *$P = 2b + 2h$*

$A = 83 = 24\text{ m}^2$ *$A = bh$*

dimensions multiplied by 5:

$P = 2(40) + 2(15) = 110\text{ m}$ *$5(8) = 40;\ 5(3) = 15$*

$A = 40(15) = 600\text{ m}^2$

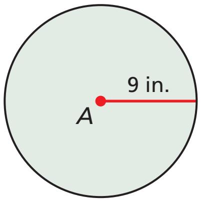
The perimeter is multiplied by 5. *$5(22) = 110$*

The area is multiplied by 5^2, or 25. *$25(24) = 600$*

Helpful Hint

If the radius of a circle or the side length of a square is changed, the size of the entire figure changes proportionally.

B **The radius of $\odot A$ is multiplied by $\frac{1}{3}$.**



original dimensions:

$C = 2\pi(9) = 18\pi\text{ in.}$ *$C = 2\pi r$*

$A = \pi(9)^2 = 81\pi\text{ in}^2$ *$A = \pi r^2$*

dimensions multiplied by $\frac{1}{3}$:

$C = 2\pi(3) = 6\pi\text{ in.}$ *$\frac{1}{3}(9) = 3$*

$A = \pi(3)^2 = 9\pi\text{ in}^2$

The perimeter is multiplied by $\frac{1}{3}$. *$\frac{1}{3}(18\pi) = 6\pi$*

The area is multiplied by $\left(\frac{1}{3}\right)^2$, or $\frac{1}{9}$. *$\frac{1}{9}(81\pi) = 9\pi$*

CHECK IT OUT!

2. The base and height of the triangle with vertices $P(2, 5)$, $Q(2, 1)$ and $R(7, 1)$ are tripled. Describe the effect on its area and perimeter.

Know it! Note

When all the dimensions of a figure are changed proportionally, the figure will be similar to the original figure.

Effects of Changing Dimensions Proportionally		
Change in Dimensions	**Perimeter or Circumference**	**Area**
All dimensions multiplied by a	Changes by a factor of a	Changes by a factor of a^2

EXAMPLE 3 Effects of Changing Area

A **A square has side length 5 cm. If the area is tripled, what happens to the side length?**

The area of the original square is $A = s^2 = 5^2 = 25\text{ cm}^2$.

If the area is tripled, the new area is 75 cm^2.

$s^2 = 75$ *Set the new area equal to s^2.*

$s = \sqrt{75} = 5\sqrt{3}$ *Take the square root of both sides and simplify.*

Notice that $5\sqrt{3} = \sqrt{3}(5)$. The side length is multiplied by $\sqrt{3}$.

B **A circle has a radius of 6 in. If the area is doubled, what happens to the circumference?**

The original area is $A = \pi r^2 = 36\pi \text{ in}^2$, and the circumference is $C = 2\pi r = 12\pi$ in. If the area is doubled, the new area is $72\pi \text{ in}^2$.

$\pi r^2 = 72\pi$ *Set the new area equal to πr^2.*

$r^2 = 72$ *Divide both sides by π.*

$r^2 = \sqrt{72} = 6\sqrt{2}$ *Take the square root of both sides and simplify.*

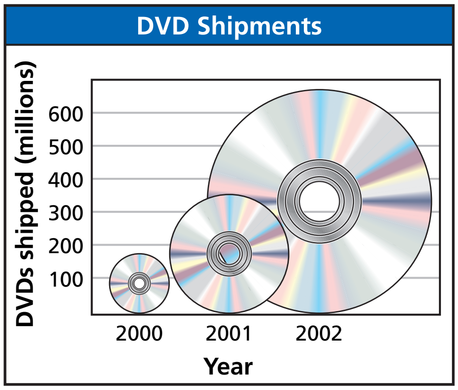
$C = 2\pi r = 2\pi\left(6\sqrt{2}\right) = 12\sqrt{2}\pi$ *Substitute $6\sqrt{2}$ for r and simplify.*

Notice that $12\sqrt{2}\pi = \sqrt{2}(12\pi)$. The circumference is multiplied by $\sqrt{2}$.

CHECK IT OUT! **3.** A square has a perimeter of 36 mm. If the area is multiplied by $\frac{1}{2}$, what happens to the side length?

EXAMPLE 4 Entertainment Application

The graph shows that DVD shipments totaled about 182 million in 2000, 364 million in 2001, and 685 million in 2002. The height of each DVD is used to represent the number of DVDs shipped. Explain why the graph is misleading.



The height of the DVD representing shipments in 2002 is about 3.8 times the height of the DVD representing shipments in 2002.

This means that the area of the DVD is multiplied by about 3.8^2, or 14.4, so the area of the larger DVD is about 14.4 times the area of the smaller DVD.

The graph gives the misleading impression that the number of shipments in 2002 was more than 14 times the number in 2000, but it was actually closer to 4 times the number shipped in 2000.

CHECK IT OUT! **4.** Use the information above to create a version of the graph that is not misleading.

THINK AND DISCUSS

1. Discuss how changing both dimensions of a rectangle affects the area and perimeter.

Know it! Note

2. **GET ORGANIZED** Copy and complete the graphic organizer.

If the radius of a circle is doubled …

The diameter __?__. | The circumference __?__. | The area __?__.

9-5 Exercises

California Standards
8.0, 10.0, 11.0, 12.0, 7AF1.1, 7MG2.3

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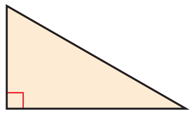
GUIDED PRACTICE

SEE EXAMPLE 1
p. 622

Describe the effect of each change on the area of the given figure.

1. The height of the triangle is doubled.



2. The height of a trapezoid with base lengths 12 cm and 18 cm and height 5 cm is multiplied by $\frac{1}{3}$.

SEE EXAMPLE 2
p. 623

Describe the effect of each change on the perimeter or circumference and the area of the given figure.

3. The base and height of a triangle with base 12 in. and height 6 in. are both tripled.
4. The base and height of the rectangle are both multiplied by $\frac{1}{2}$.

SEE EXAMPLE 3
p. 623

5. A square has an area of 36 m^2. If the area is doubled, what happens to the side length?
6. A circle has a diameter of 14 ft. If the area is tripled, what happens to the circumference?

SEE EXAMPLE 4
p. 624

7. **Business** A restaurant has a weekly ad in a local newspaper that is 2 inches wide and 4 inches high and costs $36.75 per week. The cost of each ad is based on its area. If the owner of the restaurant decides to double the width and height of the ad, how much will the new ad cost?

PRACTICE AND PROBLEM SOLVING

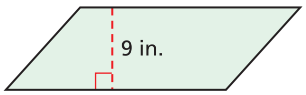
Independent Practice

For Exercises	See Example
8–9	1
10–11	2
12–13	3
14	4

Extra Practice
Skills Practice p. S21
Application Practice p. S36

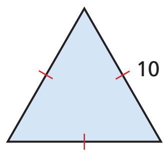
Describe the effect of each change on the area of the given figure.

8. The height of the triangle with vertices $(1, 5)$, $(2, 3)$, and $(-1, -6)$ is multiplied by 4.
9. The base of the parallelogram is multiplied by $\frac{2}{3}$.



Describe the effect of each change on the perimeter or circumference and the area of the given figure.

10. The base and height of the triangle are both doubled.



11. The radius of the circle with center $(0, 0)$ that passes through $(5, 0)$ is multiplied by $\frac{3}{5}$.
12. A circle has a circumference of 16π mm. If you multiply the area by $\frac{1}{3}$, what happens to the radius?
13. A square has vertices $(3, 2)$, $(8, 2,)$ $(8, 7)$, and $(3, 7)$. If you triple the area, what happens to the side length?
14. **Entertainment** Two televisions have rectangular screens with the same ratio of base to height. One has a 32 in. diagonal, and the other has a 36 in. diagonal.
    a. What is the ratio of the height of the larger screen to that of the smaller screen?
    b. What is the ratio of the area of the larger screen to that of the smaller screen?

Describe the effect of each change on the area of the given figure.

15. The diagonals of a rhombus are both multiplied by 8.

16. The circumference of a circle is multiplied by 2.4.

17. The base of a rectangle is multiplied by 4, and the height is multiplied by 7.

18. The apothem of a regular octagon is tripled.

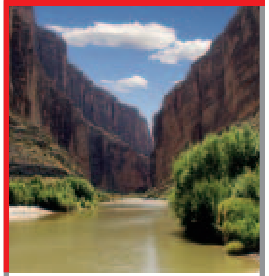
19. The diagonal of a square is divided by 4.

20. One diagonal of a kite is multiplied by $\frac{1}{7}$.

21. The perimeter of an equilateral triangle is doubled.

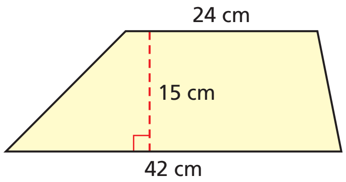
LINK

Geography



The altitude in Big Bend National Park ranges from approximately 1800 feet along the Rio Grande to 7800 feet in the Chisos Mountains.

22. Find the area of the trapezoid. Describe the effect of each change on the area.



a. The length of the top base is doubled.

b. The length of both bases is doubled.

c. The height is doubled.

d. Both bases and the height are doubled.

23. Geography A map has the scale 1 inch = 10 miles. On the map, the area of Big Bend National Park in Texas is about 12.5 square inches. Estimate the actual area of the park in acres. (*Hint:* 1 square mile = 640 acres)

24. Critical Thinking If you want to multiply the dimensions of a figure so that the area is 50% of the original area, what is your scale factor?

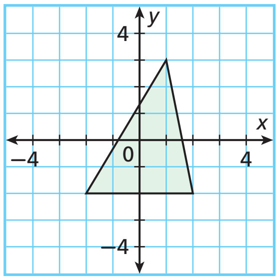
Multi-Step For each figure in the coordinate plane, describe the effect on the area that results from each change.

a. Only the x-coordinates of the vertices are multiplied by 3.

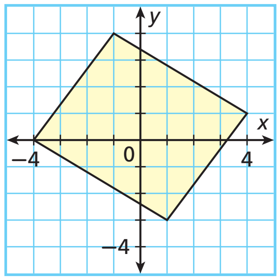
b. Only the y-coordinates of the vertices are multiplied by 3.

c. Both the x- and y-coordinates of the vertices are multiplied by 3.

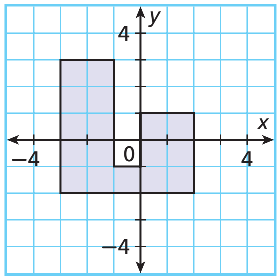
25.



26.



27.



28. Write About It How could you change the dimensions of a parallelogram to increase the area by a factor of 5 if the parallelogram does not have to be similar to the original parallelogram? if the parallelogram does have to be similar to the original parallelogram?

CONCEPT CONNECTION

29. This problem will prepare you for the Concept Connection on page 638.
To win a prize at a carnival, a player must toss a beanbag onto a circular disk with a diameter of 8 in.

a. The organizer of the game wants players to win twice as often, so he changes the disk so that it has twice the area. What is the diameter of the new disk?

b. Suppose the organizer wants players to win half as often. What should be the disk's diameter in this case?

STANDARDIZED TEST PREP

30. Which of the following describes the effect on the area of a square when the side length is doubled?

Ⓐ The area remains constant.
Ⓑ The area is reduced by a factor of $\frac{1}{2}$.
Ⓒ The area is doubled.
Ⓓ The area is increased by a factor of 4.

31. If the area of a circle is increased by a factor of 4, what is the change in the diameter of the circle?

Ⓕ The diameter is $\frac{1}{2}$ of the original diameter.
Ⓖ The diameter is 2 times the original diameter.
Ⓗ The diameter is 4 times the original diameter.
Ⓙ The diameter is 16 times the original diameter.

32. Tina and Kieu built rectangular play areas for their dogs. The play area for Tina's dog is 1.5 times as long and 1.5 times as wide as the play area for Kieu's dog. If the play area for Kieu's dog is 60 square feet, how big is the play area for Tina's dog?

Ⓐ 40 ft^2 Ⓑ 90 ft^2 Ⓒ 135 ft^2 Ⓓ 240 ft^2

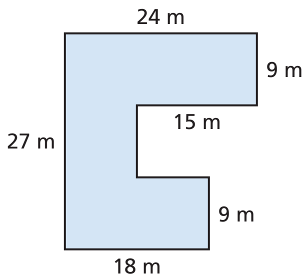
33. **Gridded Response** Suppose the dimensions of a triangle with a perimeter of 18 inches are doubled. Find the perimeter of the new triangle in inches.

CHALLENGE AND EXTEND

34. **Algebra** A square has a side length of $(2x + 5)$ cm . If the side length is multiplied by 5, what is the area of the new square?

35. **Algebra** A circle has a diameter of 6 in. If the circumference is multiplied by $(x + 3)$, what is the area of the new circle?

36. **Write About It** How could you change the dimensions of the composite figure to double the area if the resulting figure does not have to be similar to the original figure? if the resulting figure does have to be similar to the original figure?



SPIRAL REVIEW

Write an equation that can be used to determine the value of the variable in each situation. *(Previous course)*

37. Steve can make 2 tortillas per minute. He makes t tortillas in 36 minutes.

38. A car gets 25 mi/gal. At the beginning of a trip of m miles, the car's gas tank contains 13 gal of gas. At the end of the trip, the car has 8 gal of gasoline left.

Find the measure of each interior and each exterior angle of each regular polygon. Round to the nearest tenth, if necessary. *(Lesson 6-1)*

39. heptagon **40.** decagon **41.** 14-gon

Find the area of each polygon with the given vertices. *(Lesson 9-4)*

42. $L(-1, 1)$, $M(5, 2)$, and $N(1, -5)$ **43.** $A(-4, 2)$, $M(-2, 4)$, $C(4, 2)$ and $D(2, -4)$

Probability

Connecting Geometry to **Probability**

See Skills Bank page S77

An *experiment* is an activity in which results are observed. Each result of an experiment is called an *outcome*. The *sample space* is the set of all outcomes of an experiment. An *event* is any set of outcomes.

California Standards

Review of 6SDAP3.3 Represent probabilities as ratios, proportions, decimals **between 0 and 1,** and percentages between 0 and 100 and verify that the probabilities computed are reasonable; **know that if *P* is the probability of an event, 1 − *P* is the probability of an even not occurring.**

The probability of an event is a number from 0 to 1 that tells you how likely the event is to happen. The closer the probability is to 0, the less likely the event is to happen. The closer it is to 1, the more likely the event is to happen.

An experiment is *fair* if all outcomes are equally likely. The *theoretical probability* of an event is the ratio of the number of outcomes in the event to the number of outcomes in the sample space.

$$P(E) = \frac{\text{number of outcomes in event } E}{\text{number of possible outcomes}}$$

Example 1

A fair number cube has six faces, numbered 1 through 6. An experiment consists of rolling the number cube.

A What is the sample space of the experiment?

The sample space has 6 possible outcomes. The outcomes are 1, 2, 3, 4, 5, and 6.

B What is the probability of the event "rolling a 4"?

The event "rolling a 4" contains only 1 outcome. The probability is

$$P(E) = \frac{\text{number of outcomes in event } E}{\text{number of possible outcomes}} = \frac{1}{6}.$$

C What are the outcomes in the event "rolling an odd number"? What is the probability of rolling an odd number?

The event "rolling an odd number" contains 3 outcomes. The outcomes are 1, 3, and 5. The probability is

$$P(E) = \frac{\text{number of outcomes in event } E}{\text{number of possible outcomes}} = \frac{3}{6} = \frac{1}{2}.$$

If two events A and B have no outcomes in common, then the probability that A or B will happen is $P(A) + P(B)$.

The *complement of an event* is the set of outcomes that are *not* in the event. If the probability of an event is p, then the probability of the complement of the event is $1 - p$.

Example 2

The tiles shown below are placed in a bag. An experiment consists of drawing a tile at random from the bag.

A **What is the sample space of the experiment?**

The sample space has 9 possible outcomes. The outcomes are 1, 2, 3, 4, A, B, C, D, E, and F.

B **What is the probability of choosing a 3 or a vowel?**

The event "choosing a 3" contains only 1 outcome. The probability is

$$P(A) = \frac{\text{number of outcomes in event } A}{\text{number of possible outcomes}} = \frac{1}{9}.$$

The event "choosing a vowel" has 2 outcomes, A and E. The probability is

$$P(B) = \frac{\text{number of outcomes in event } B}{\text{number of possible outcomes}} = \frac{2}{9}.$$

The probability of choosing a 3 or a vowel is $\frac{1}{9} + \frac{2}{9} = \frac{3}{9} = \frac{1}{3}$.

C **What is the probability of not choosing a letter?**

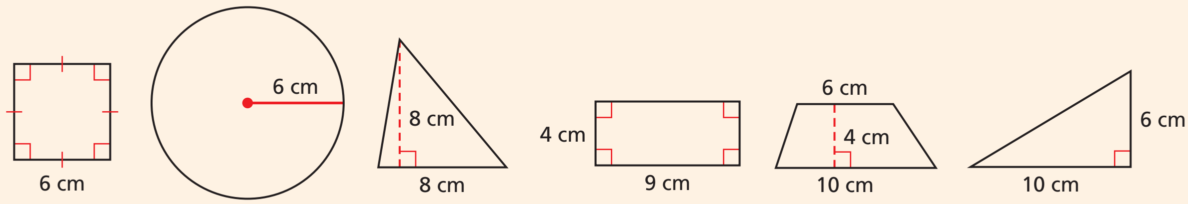
The event "choosing a letter" contains 5 outcomes, A, B, C, D, and E. The probability is

$$P(E) = \frac{\text{number of outcomes in event } E}{\text{number of possible outcomes}} = \frac{5}{9}.$$

The event of not choosing a letter is the complement of the event of choosing a letter. The probability of not choosing a letter is $1 - \frac{5}{9} = \frac{4}{9}$.

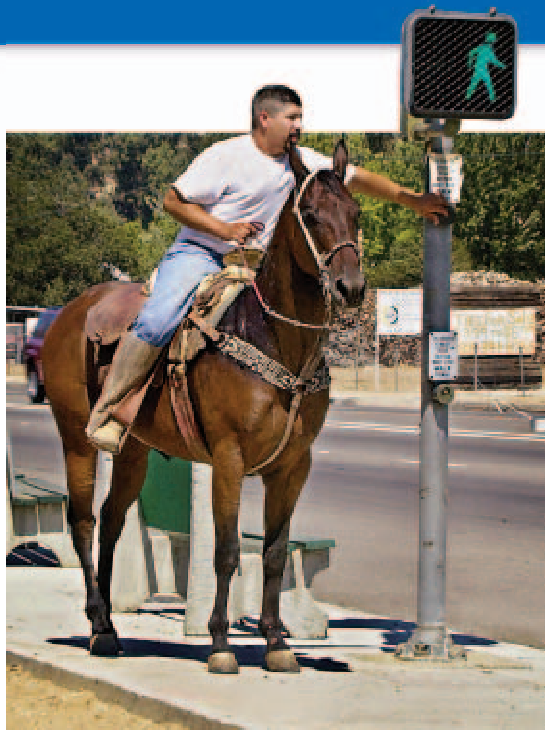
Try This

An experiment consists of randomly choosing one of the given shapes.



1. What is the probability of choosing a circle?
2. What is the probability of choosing a shape whose area is 36 cm^2?
3. What is the probability of choosing a quadrilateral or a triangle?
4. What is the probability of not choosing a triangle?

9-6 Geometric Probability



Objectives

Calculate geometric probabilities.

Use geometric probability to predict results in real-world situations.

Vocabulary

geometric probability

Why learn this?

You can use geometric probability to estimate how long you may have to wait to cross a street. (See Example 2.)

Remember that in probability, the set of all possible outcomes of an experiment is called the *sample space.* Any set of outcomes is called an *event.*

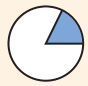
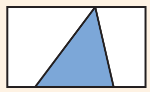
If every outcome in the sample space is equally likely, the *theoretical probability* of an event is

$$P = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}.$$

Geometric probability is used when an experiment has an infinite number of outcomes. In **geometric probability**, the probability of an event is based on a ratio of geometric measures such as length or area. The outcomes of an experiment may be points on a segment or in a plane figure. Three models for geometric probability are shown below.

Know it! Note

Geometric Probability

Model	Length	Angle Measure	Area
Example			
Sample space	All points on $\overline{AD}$	All points in the circle	All points in the rectangle
Event	All points on $\overline{BC}$	All points in the shaded region	All points in the triangle
Probability	$P = \frac{BC}{AD}$	$P = \frac{\text{measure of angle}}{360°}$	$P = \frac{\text{area of triangle}}{\text{area of rectangle}}$

EXAMPLE 1 Using Length to Find Geometric Probability

California Standards

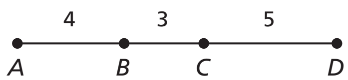
8.0 Students know, derive, **and solve problems involving the** perimeter, circumference, **area,** volume, lateral area, and surface area **of common geometric figures.**

10.0 Students compute areas of polygons, including rectangles, scalene triangles, **equilateral triangles,** rhombi, parallelograms, **and trapezoids.**

A point is chosen randomly on $\overline{AD}$. Find the probability of each event.

A **The point is on $\overline{AC}$.**

$P = \frac{AC}{AD} = \frac{7}{12}$



B **The point is not on $\overline{AB}$.**

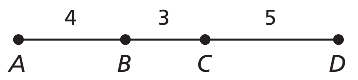
First find the probability that the point is on $\overline{AB}$.

$P(\overline{AB}) = \frac{AB}{AD} = \frac{4}{12} = \frac{1}{3}$

Subtract from 1 to find the probability that the point is not on $\overline{AB}$.

$P(\text{not on } \overline{AB}) = 1 - \frac{1}{3} = \frac{2}{3}$

A point is chosen randomly on $\overline{AD}$. Find the probability of each event.



C The point is on $\overline{AB}$ or $\overline{CD}$.

$$P(\overline{AB} \text{ or } \overline{CD}) = P(\overline{AB}) + P(\overline{CD}) = \frac{4}{12} + \frac{5}{12} = \frac{9}{12} = \frac{3}{4}$$

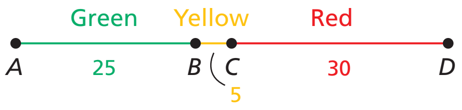
CHECK IT OUT! **1.** Use the figure above to find the probability that the point is on $\overline{BD}$.

EXAMPLE 2 *Transportation Application*

A stoplight has the following cycle: green for 25 seconds, yellow for 5 seconds, and red for 30 seconds.

A **What is the probability that the light will be yellow when you arrive?**

To find the probability, draw a segment to represent the number of seconds that each color light is on.



$P = \frac{5}{60} = \frac{1}{12} \approx 0.08$ *The light is yellow for 5 out of every 60 seconds.*

B **If you arrive at the light 50 times, predict about how many times you will have to stop and wait more than 10 seconds.**

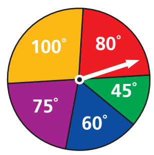
In the model, the event of stopping and waiting more than 10 seconds is represented by a segment that starts at C and ends 10 units from D. The probability of stopping and waiting more than 10 seconds is $P = \frac{20}{60} = \frac{1}{3}$.

If you arrive at the light 50 times, you will probably stop and wait more than 10 seconds about $\frac{1}{3}(50) \approx 17$ times.

CHECK IT OUT! **2.** Use the information above. What is the probability that the light will not be red when you arrive?

EXAMPLE 3 Using Angle Measures to Find Geometric Probability

Use the spinner to find the probability of each event.



A **the pointer landing on red**

$P = \frac{80}{360} = \frac{2}{9}$ *The angle measure in the red region is 80°.*

B **the pointer landing on purple or blue**

$P = \frac{75 + 60}{360} = \frac{135}{360} = \frac{3}{8}$ *The angle measure in the purple region is 75°. The angle measure in the blue region is 60°.*

C **the pointer not landing on yellow**

$$P = \frac{360 - 100}{360} = \frac{260}{360} = \frac{13}{18}$$

The angle measure in the yellow region is 100°. Substract this angle measure from 360°.

Helpful Hint

In Example 3C, you can also find the probability of the pointer landing on yellow, and subtract from 1.

CHECK IT OUT! **3.** Use the spinner above to find the probability of the pointer landing on red or yellow.

Student to Student

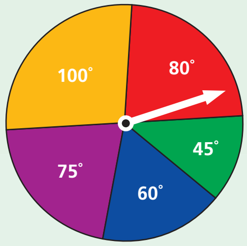
Geometric Probability

Jeremy Denton
Memorial High School

I like to write a probability as a percent to see if my answer is reasonable.

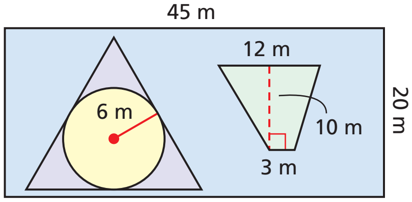
The probability of the pointer landing on red is $\frac{80°}{360°} = \frac{2}{9} \approx 22\%$.

The angle measure is close to 90°, which is 25% of the circle, so the answer is reasonable.



EXAMPLE 4 Using Area to Find Geometric Probability

Find the probability that a point chosen randomly inside the rectangle is in each given shape. Round to the nearest hundredth.



A **the equilateral triangle**

The area of the triangle is $A = \frac{1}{2}aP$

$= \frac{1}{2}(6)\left(36\sqrt{3}\right) \approx 187 \text{ m}^2.$

The area of the rectangle is $A = bh$

$= 45(20) = 900 \text{ m}^2.$

The probability is $P = \frac{187}{900} \approx 0.21.$

B **the trapezoid**

The area of the trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$

$= \frac{1}{2}(3 + 12)(10) = 75 \text{ m}^2.$

The area of the rectangle is $A = bh$

$= 45(20) = 900 \text{ m}^2.$

The probability is $P = \frac{75}{900} \approx 0.08.$

C **the circle**

The area of the circle is $A = \pi r^2$

$= \pi\left(6^2\right) = 36\pi \approx 113.1 \text{ m}^2.$

The area of the rectangle is $A = bh$

$= 45(20) = 900 \text{ m}^2.$

The probability is $P = \frac{113.1}{900} \approx 0.13.$

CHECK IT OUT!

4. Use the diagram above. Find the probability that a point chosen randomly inside the rectangle is not inside the triangle, circle, or trapezoid. Round to the nearest hundredth.

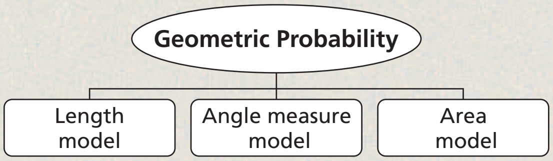
THINK AND DISCUSS

1. Explain why the ratio used in theoretical probability cannot be used to find geometric probability.

2. A spinner is one-half red and one-third blue, and the rest is yellow. How would you find the probability of the pointer landing on yellow?

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, give an example of the geometric probability model.



9-6 Exercises

California Standards
8.0, 10.0, 17.0, 7NS2.3, 6SDAP3.3, 7AF2.2, 1A10.0

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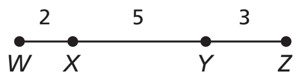
KEYWORD: MG7 9-6
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

1. **Vocabulary** Give an example of a model used to find *geometric probability.*

SEE EXAMPLE 1 p. 630

A point is chosen randomly on $\overline{WZ}$. Find the probability of each event.



2. The point is on $\overline{XZ}$.
3. The point is not on $\overline{XY}$.
4. The point is on $\overline{WX}$ or $\overline{YZ}$.
5. The point is on $\overline{WY}$.

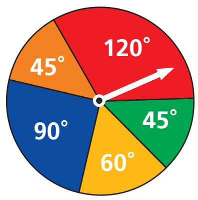
SEE EXAMPLE 2 p. 631

Transportation **A bus comes to a station once every 10 minutes and waits at the station for 1.5 minutes.**

6. Find the probability that the bus will be at the station when you arrive.
7. If you go to the station 20 times, predict about how many times you will have to wait less than 3 minutes.

SEE EXAMPLE 3 p. 631

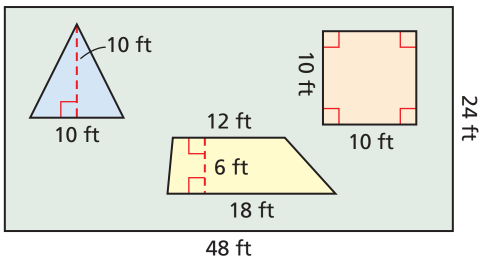
Use the spinner to find the probability of each event.



8. the pointer landing on green
9. the pointer landing on orange or blue
10. the pointer not landing on red
11. the pointer landing on yellow or blue

SEE EXAMPLE 4 p. 632

Multi-Step **Find the probability that a point chosen randomly inside the rectangle is in each shape. Round to the nearest hundredth.**



12. the triangle
13. the trapezoid
14. the square
15. the part of the rectangle that does not include the square, triangle, or trapezoid

PRACTICE AND PROBLEM SOLVING

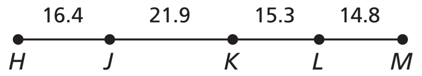
Independent Practice

For Exercises	See Example
16–19	1
20–22	2
23–26	3
27–30	4

Extra Practice

Skills Practice p. S21
Application Practice p. S36

A point is chosen randomly on $\overline{HM}$. Find the probability of each event. Round to the nearest hundredth.



16. The point is on $\overline{JK}$.

17. The point is not on $\overline{LM}$.

18. The point is on $\overline{HJ}$ or $\overline{KL}$.

19. The point is not on $\overline{JK}$ or $\overline{LM}$.

Communications **A radio station gives a weather report every 15 minutes. Each report lasts 45 seconds. Suppose you turn on the radio at a random time.**

20. Find the probability that the weather report will be on when you turn on the radio.

21. Find the probability that you will have to wait more than 5 minutes to hear the weather report.

22. If you turn on the radio at 50 random times, predict about how many times you will have to wait less than 1 minute before the start of the next weather report.

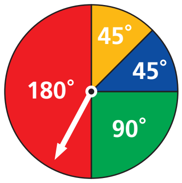
Use the spinner to find the probability of each event.

23. the pointer landing on red

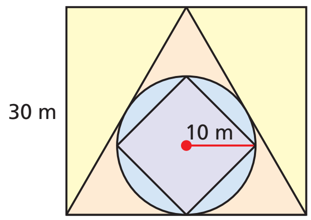
24. the pointer landing on yellow or blue

25. the pointer not landing on green

26. the pointer landing on red or green



Multi-Step **Find the probability that a point chosen randomly inside the rectangle is in each shape. Round to the nearest hundredth, if necessary.**



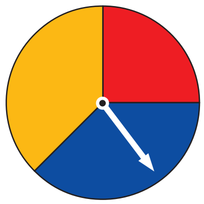
27. the equilateral triangle

28. the square

29. the part of the circle that does not include the square

30. the part of the rectangle that does not include the square, circle, or triangle

31. **ERROR ANALYSIS** In the spinner at right, the angle measure of the red region is 90°. The angle measure of the yellow region is 135°, and the angle measure of the blue region is 135°. Which value of the probability of the spinner landing on yellow is incorrect? Explain.



A

There are three outcomes, so the probability of the spinner landing on yellow is $\frac{1}{3}$.

B

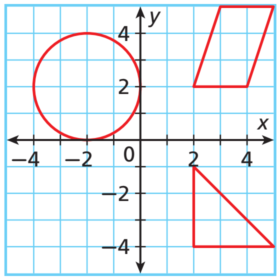
The angle measure of the yellow sector is 135°, so the probability of the spinner landing on yellow is $\frac{135}{360} = \frac{3}{8}$.

Algebra **A point is chosen randomly inside rectangle $ABCD$ with vertices $A(2, 8)$, $B(15, 8)$, $C(15, 1)$, and $D(2, 1)$. Find the probability of each event. Round to the nearest hundredth.**

32. The point lies in $\triangle KLM$ with vertices $K(4, 3)$, $L(5, 7)$, and $M(9, 5)$.

33. The point does not lie in $\odot P$ with center $P(2, 5)$ and radius 3. (*Hint:* draw the rectangle and circle.)

Algebra **A point is chosen at random in the coordinate plane such that $-5 \le x \le 5$ and $-5 \le y \le 5$. Find the probability of each event. Round to the nearest hundredth.**



34. The point is inside the parallelogram.

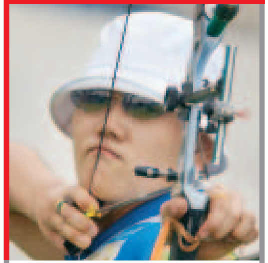
35. The point is inside the circle.

36. The point is inside the triangle or the circle.

37. The point is not inside the triangle, the parallelogram, or the circle.

LINK

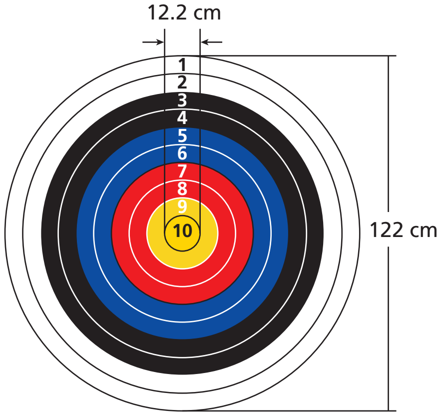
Sports



Olympic archers stand 70 m from their targets. From that distance, the target appears about the size of the head of a thumbtack held at arm's length.

Source: www.olympic.org

38. **Sports** The point value of each region of an Olympic archery target is shown in the diagram. The outer diameter of each ring is 12.2 cm greater than the inner diameter.



a. What is the probability of hitting the center?

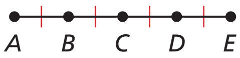
b. What is the probability of hitting a blue or black ring?

c. What is the probability of scoring higher than five points?

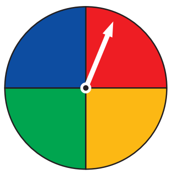
d. **Write About It** In an actual event, why might the probabilities be different from those you calculated in parts **a, b,** and **c**?

A point is chosen randomly in each figure. Describe an event with a probability of $\frac{1}{2}$.

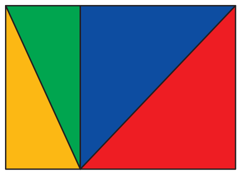
39.



40.



41.



42. If a fly lands randomly on the tangram, what is the probability that it will land on each of the following pieces?

a. the blue parallelogram

b. the medium purple triangle

c. the large yellow triangle

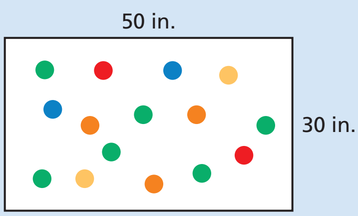
d. **Write About It** Do the probabilities change if you arrange the tangram pieces differently? Explain.

43. **Critical Thinking** If a rectangle is divided into 8 congruent regions and 4 of them are shaded, what is the probability that you will randomly pick a point in the shaded area? Does it matter which four regions are shaded? Explain.

Concept Connection

44. This problem will prepare you for the Concept Connection on page 638.

A carnival game board consists of balloons that are 3 inches in diameter and are attached to a rectangular board. A player who throws a dart at the board wins a prize if the dart pops a balloon.

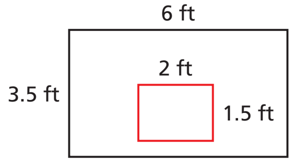


a. Find the probability of winning if there are 40 balloons on the board.

b. How many balloons must be on the board for the probability of winning to be at least 0.25?

STANDARDIZED TEST PREP

45. What is the probability that a ball thrown randomly at the backboard of the basketball goal will hit the inside rectangle?



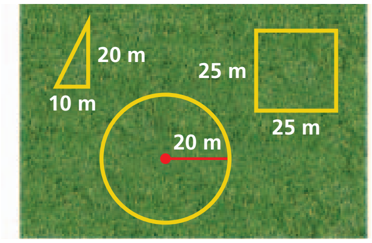
Ⓐ 0.14   Ⓒ 0.26

Ⓑ 0.21   Ⓓ 0.27

46. Point B is between A and C. If $AB = 18$ inches and $BC = 24$ inches, what is the probability that a point chosen at random is on $\overline{AB}$?

Ⓕ 0.18   Ⓖ 0.43   Ⓗ 0.57   Ⓙ 0.75

47. A skydiver jumps from an airplane and parachutes down to the 70-by-100-meter rectangular field shown. What is the probability that he will miss all three targets?



Ⓐ 0.014   Ⓒ 0.089

Ⓑ 0.180   Ⓓ 0.717

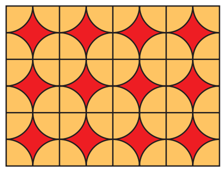
48. **Short Response** A spinner is divided into 12 congruent regions, colored red, blue, and green. Landing on red is twice as likely as landing on blue. Landing on blue and landing on green are equally likely.

a. What is the probability of landing on green? Show your work or explain in words how you got your answer.

b. How many regions of the spinner are colored green? Explain your reasoning.

CHALLENGE AND EXTEND

49. If you randomly choose a point on the grid, what is the probability that it will be in a red region?



50. You are designing a target that is a square inside an 18 ft by 24 ft rectangle. What size should the square be in order for the target to have a probability of $\frac{1}{3}$? to have a probability of $\frac{3}{4}$?

51. **Recreation** How would you design a spinner so that 1 point is earned for landing on yellow, 3 points for landing on blue and 6 points for landing on red? Explain.

SPIRAL REVIEW

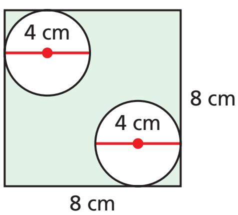
Simplify each expression. *(Previous course)*

52. $(3x^2y)(4x^3y^2)$ **53.** $(2m^5)^2$ **54.** $\dfrac{-8a^4b^6}{2ab^3}$

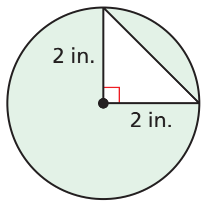
55. **Given:** $A(0, 4)$, $B(4, 6)$, $C(4, 2)$, $D(8, 8)$, and $E(8, 0)$
Prove: $\triangle ABC \sim \triangle ADE$ *(Lesson 7-6)*

Find the shaded area. Round to the nearest tenth, if necessary. *(Lesson 9-3)*

56.



57.



9-6 Geometry LAB

Use Geometric Probability to Estimate π

Use with Lesson 9-6

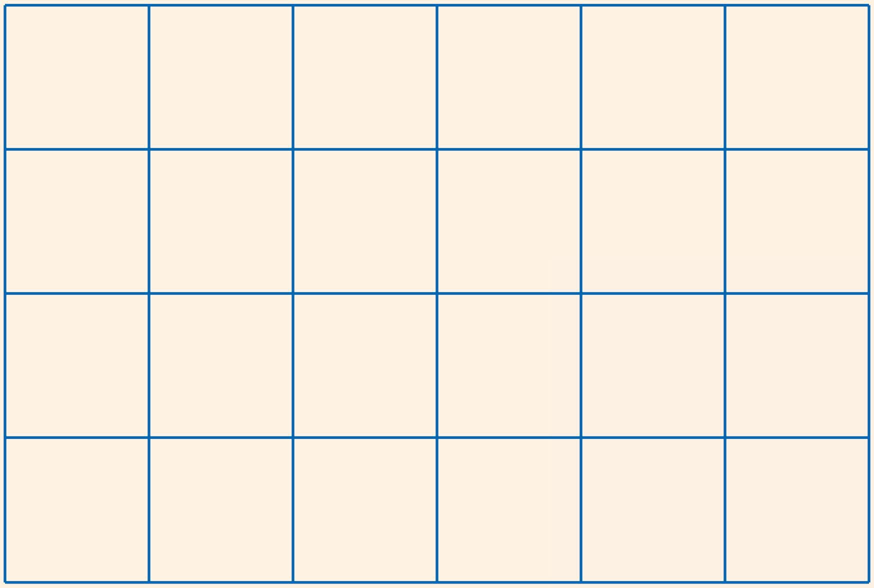
In this lab, you will use geometric probability to estimate π. The squares in the grid below are the same width as the diameter of a penny: 0.75 in., or 19.05 mm.

California Standards

8.0 Students know, derive, and solve problems involving the perimeter, circumference, **area,** volume, lateral area, and surface area **of common geometric figures.**

Activity

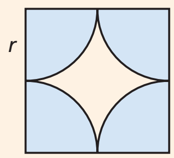
1. Toss a penny onto the grid 20 times. Let x represent the number of times the penny lands touching or covering an intersection of two grid lines.



2. Estimate π using the formula $\pi \approx 4 \cdot \frac{x}{20}$.

Try This

1. How close is your result to π? Average the results of the entire class to get a more accurate estimate.
2. In order for a penny to touch or cover an intersection, the center of the penny can land anywhere in the shaded area.



   a. Find the area of the shaded region. (*Hint:* Each corner part is one fourth of the circle. Put the four corner parts together to form a circle with radius r.)

   b. Find the area of the square.

   c. Write the expressions as a ratio and simplify to determine the probability of the center of the penny landing in the shaded area.
3. Explain why the formula in the activity can be used to estimate π.

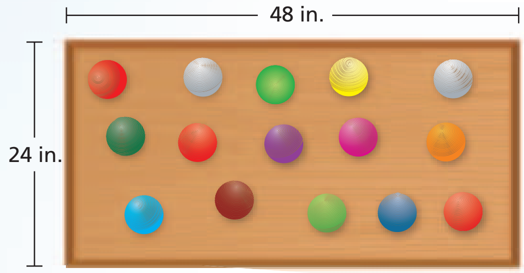
CHAPTER 9

CONCEPT CONNECTION

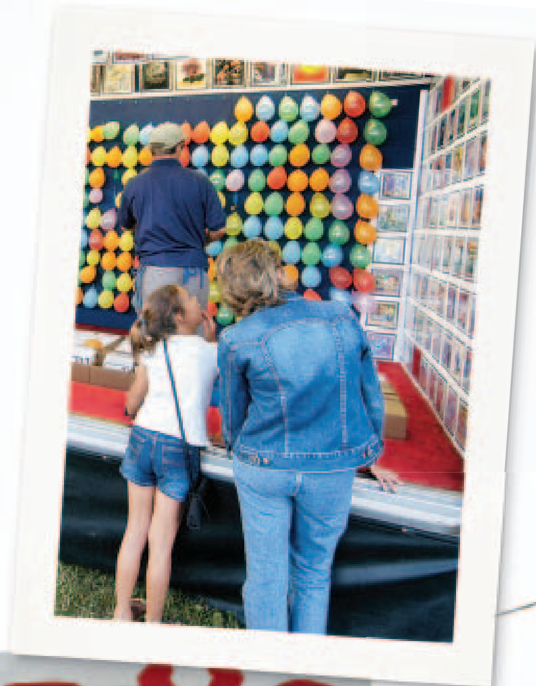
SECTION 9B

Applying Geometric Formulas

Step Right Up! A booster club organizes a carnival to raise money for sports uniforms. The carnival features several games that give visitors chances to win prizes.



1. The balloon game consists of 15 balloons attached to a vertical rectangular board with the dimensions shown. Each balloon has a diameter of 4 in. Each player throws a dart at the board and wins a prize if the dart pops a balloon. Assuming that all darts hit the board at random, what is the probability of winning a prize?
2. The organizers decide to make the game easier, so they double the diameter of the balloons. How does this affect the probability of winning?
3. The bean toss consists of a horizontal rectangular board that is divided into a grid. The board has coordinates $(0, 0)$, $(100, 0)$, $(100, 60)$, and $(0, 60)$. A quadrilateral on the board has coordinates $A(60,0)$, $B(100, 30)$, $C(40, 60)$, and $D(0, 40)$. Each player tosses a bean onto the board and wins a prize if the bean lands inside quadrilateral $ABCD$. Find the probability of winning a prize.
4. Of the three games described in Problems 1, 2, and 3, which one gives players the best chance of winning a prize?



READY TO GO ON?

Quiz for Lessons 9-4 Through 9-6

9-4 Perimeter and Area in the Coordinate Plane

Draw and classify the polygon with the given vertices. Find the perimeter and area of the polygon.

1. $A(-2, 2)$, $B(2, 4)$, $C(2, -4)$, $D(-2, -2)$

2. $E(-1, 5)$, $F(3, 5)$, $G(3, -3)$, $H(-1, -3)$

Find the area of each polygon with the given vertices.

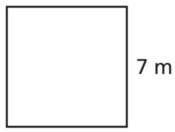
3. $J(-3, 3)$, $K(2, 2)$, $L(-1, -3)$, $M(-4, -1)$

4. $N(-3, 1)$, $P(3, 3)$, $Q(5, 1)$, $R(2, -4)$

9-5 Effects of Changing Dimensions Proportionally

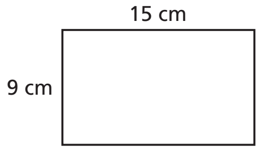
Describe the effect of each change on the perimeter and area of the given figure.

5. The side length of the square is tripled.



6. The diagonals of a rhombus in which $d_1 = 3$ ft and $d_2 = 9$ ft are both multiplied by $\frac{1}{3}$.

7. The base and height of the rectangle are both doubled.



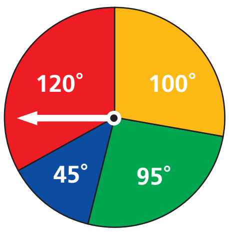
8. The base and height of a right triangle with base 15 in. and height 8 in. are multiplied by $\frac{1}{3}$.

9. A square has vertices $(-1, 2)$, $(3, 2)$, $(3, -2)$, and $(-1, -2)$. If you quadruple the area, what happens to the side length?

10. A restaurant sells pancakes in two sizes, silver dollar and regular. The silver-dollar pancakes have a 4-inch diameter and require $\frac{1}{8}$ cup of batter per pancake. The diameter of a regular pancake is 2.5 times the diameter of a silver-dollar pancake. About how much batter is required to make a regular pancake?

9-6 Geometric Probability

Use the spinner to find the probability of each event.



11. the pointer landing on red

12. the pointer landing on red or yellow

13. the pointer not landing on green

14. the pointer landing on yellow or blue

15. A radio station plays 12 commercials per hour. Each commercial is 1 minute long. If you turn on the radio at a random time, find the probability that a commercial will be playing.

CHAPTER 9 Study Guide: Review

Know it! Note: For a complete list of the postulates and theorems in this chapter, see p. S82.

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ is the length of a segment perpendicular to a side of a regular polygon.

2. The point that is equidistant from every point on a circle is the ___?___ .

3. ___?___ is based on a ratio of geometric measures.

9-1 Developing Formulas for Triangles and Quadrilaterals *(pp. 589–597)*

8.0, 10.0

EXAMPLES

Find each measurement.

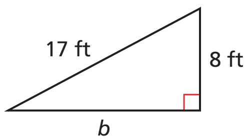
- the perimeter of a square in which $A = 36 \text{ in}^2$

  $A = s^2 = 36 \text{ in}^2$ *Use the Area Formula to find the side length.*

  $S = \sqrt{36} = 6$ in.

  $P = 4s = 4 \cdot 6 = 24$ in.

- the area of the triangle



  By the Pythagorean Theorem,

  $8^2 + b^2 = 17^2$

  $64 + b^2 = 289$

  $b^2 = 225$, so $b = 15$ ft.

  $A = \frac{1}{2}bh = \frac{1}{2}(15)(8) = 60 \text{ ft}^2$

- the diagonal d_2 of a rhombus in which $A = 6x^3y^3$ m and $d_1 = 4x^2y$ m

  $A = \frac{1}{2}d_1d_2$

  $6x^3y^3 = \frac{1}{2}(4x^2y)d_2$ *Substitute the given values.*

  $d_2 = 3xy^2$ *Solve for d_2.*

EXERCISES

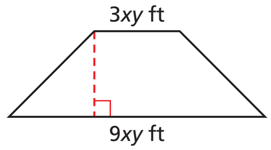
Find each measurement.

4. the area of a square in which $P = 36$ in.

5. the perimeter of a rectangle in which $b = 4$ cm and $A = 28 \text{ cm}^2$

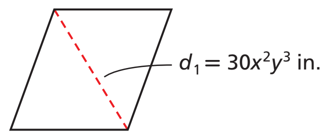
6. the height of a triangle in which $A = 6x^3y \text{ in}^2$ and $b = 4xy$ in.

7. the height of the trapezoid, in which $A = 48xy \text{ ft}^2$



8. the area of a rhombus in which $d_1 = 21$ yd and $d_2 = 24$ yd

9. the diagonal d_2 of the rhombus, in which $A = 630x^3y^7 \text{ in}^2$



10. the area of a kite in which $d_1 = 32$ m and $d_2 = 18$ m

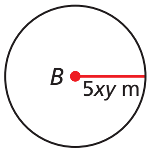
9-2 Developing Formulas for Circles and Regular Polygons *(pp. 600–605)*

8.0, 10.0

EXAMPLES

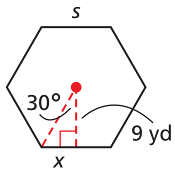
Find each measurement.

- the circumference and area of $\odot B$ in terms of π



$C = 2\pi r = 2\pi(5xy)$
$= 10xy\pi$ m
$A = \pi r^2 = \pi(5xy)^2 = 25x^2y^2\pi$ m^2

- the area, to the nearest tenth, of a regular hexagon with apothem 9 yd

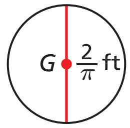


By the 30°-60°-90° Triangle Theorem, $x = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$.
So $s = 2x = 6\sqrt{3}$, and
$P = 6(6\sqrt{3}) = 36\sqrt{3}$.
$A = \frac{1}{2}aP = \frac{1}{2}(9)(36\sqrt{3}) = 162\sqrt{3} \approx 280.6$ yd^2

EXERCISES

Find each measurement. Round to the nearest tenth, if necessary.

11. the circumference of $\odot G$



12. the area of $\odot J$ in which $C = 14\pi$ yd

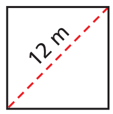
13. the diameter of $\odot K$ in which $A = 64x^2\pi$ m^2

14. the area of a regular pentagon with side length 10 ft

15. the area of an equilateral triangle with side length 4 in.

16. the area of a regular octagon with side length 8 cm

17. the area of the square

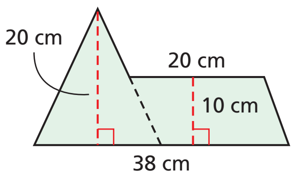


9-3 Composite Figures *(pp. 606–612)*

8.0, 10.0

EXAMPLE

- **Find the shaded area. Round to the nearest tenth, if necessary.**

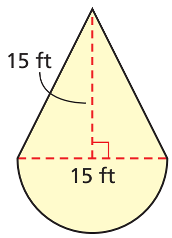


The area of the triangle is
$A = \frac{1}{2}(18)(20) = 180$ cm^2.
The area of the parallelogram is
$A = bh = 20(10) = 200$ cm^2.
The area of the figure is the sum of the two areas. $180 + 200 = 380$ cm^2

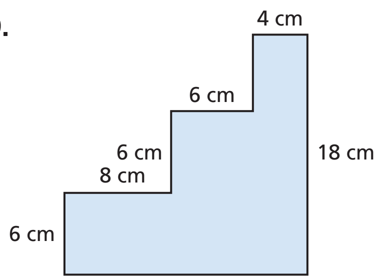
EXERCISES

Find the shaded area. Round to the nearest tenth, if necessary.

18.



19.



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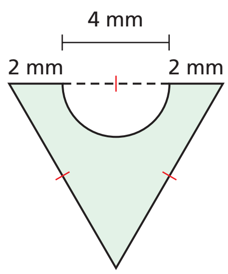
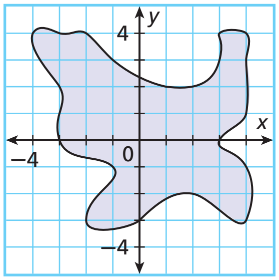


9-4 Perimeter and Area in the Coordinate Plane *(pp. 616–621)*

8.0, 10.0, 12.0

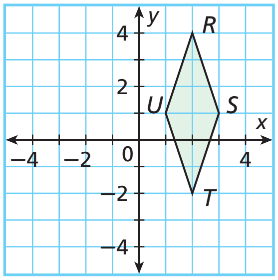
EXAMPLES

- **Estimate the area of the irregular shape.**



The shape has 28 approximately whole squares and 17 approximately half squares. The total area is approximately $28 + \frac{1}{2}(17) = 36.5$ units^2.

- **Draw and classify the polygons with vertices $R(2, 4)$, $S(3, 1)$, $T(2, -2)$, and $U(1, 1)$. Find the perimeter and area of the polygon.**



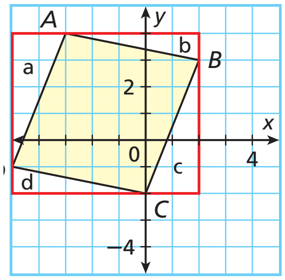
RSTU appears to be a rhombus.

Verify this by showing that the four sides are congruent. By the Distance Formula, $UR = RS = ST = TU = \sqrt{10}$ units.

The perimeter is $4\sqrt{10}$ units.

The area is $A = \frac{1}{2}d_1d_2 = \frac{1}{2}US \cdot RT = \frac{1}{2}(2 \cdot 6) = 6$ units^2.

- **Find the area of the polygon with vertices $A(-3, 4)$, $B(2, 3)$, $C(0, -2)$, and $D(-5, -1)$.**



area of rectangle:

$7(6) = 42$ units^2

area of triangles:

a: $A = \frac{1}{2}(2)(5) = 5$ units^2

b: $A = \frac{1}{2}(5)(1) = 2.5$ units^2

c: $A = \frac{1}{2}(2)(5) = 5$ units^2

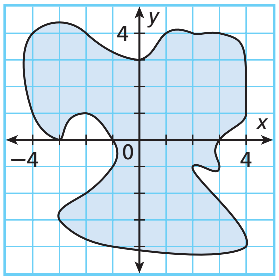
d: $A = \frac{1}{2}(5)(1) = 2.5$ units^2

area of polygon: $A = 42 - 5 - 2.5 - 5 - 2.5 = 27$ units^2

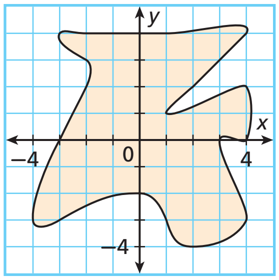
EXERCISES

Estimate the area of each irregular shape.

21.



22.



Draw and classify the polygon with the given vertices. Find the perimeter and area of the polygon.

23. $H(0, 3)$, $J(3, 0)$, $K(0, -3)$, $L(-3, 0)$

24. $M(-2, 5)$, $N(3, -2)$, $P(-2, -2)$

25. $A(-2, 3)$, $B(2, 3)$, $C(4, -1)$, $D(-4, -1)$

26. $E(-1, 3)$, $F(3, 3)$, $G(1, 0)$, $H(-3, 0)$

Find the area of the polygon with the given vertices.

27. $Q(1, 4)$, $R(4, 3)$, $S(2, -4)$, $T(-3, -2)$

28. $V(-2, 2)$, $W(4, 0)$, $X(2, -3)$, $Y(-3, 0)$

29. $A(1, 4)$, $B(2, 3)$, $C(0, -3)$, $D(-2, -1)$

30. $E(-1, 2)$, $F(2, 0)$, $G(1, -3)$, $H(-4, -1)$

9-5 Effects of Changing Dimensions Proportionally *(pp. 622–627)*

8.0, 10.0, 11.0

EXAMPLE

- **The base and height of a rectangle with base 10 cm and height 15 cm are both doubled. Describe the effect on the area and perimeter of the figure.**

  original: $P = 2b + 2h = 2(10) + 2(15) = 50$ cm

  $A = bh = 10(15) = 150 \text{ cm}^2$

  doubled: $P = 2b + 2h = 2(20) + 2(30)$
  $= 100$ cm

  $A = bh = 20(30) = 600 \text{ cm}^2$

  The perimeter increases by a factor of 2. The area increases by a factor of 4.

EXERCISES

Describe the effect of each change on the perimeter or circumference and area of the given figure.

31. The base and height of the triangle with vertices $X(-1, 3)$, $Y(-3, -2)$, and $Z(2, -2)$ are tripled.

32. The side length of the square with vertices $P(-1, 1)$, $Q(3, 1)$, $R(3, -3)$, and $S(-1, -3)$ is doubled.

33. The radius of $\odot A$ with radius 11 m is multiplied by $\frac{1}{2}$.

34. The base and height of a triangle with base 8 ft and height 20 ft are both multiplied by 4.

9-6 Geometric Probability *(pp. 630–636)*

8.0, 10.0

EXAMPLES

A point is chosen randomly on $\overline{WZ}$. Find the probability of each event.

- The point is on $\overline{XZ}$.

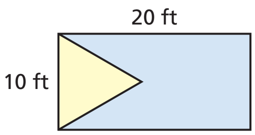
  $P(XZ) = \frac{XZ}{WZ} = \frac{15}{18} = \frac{5}{6}$

- The point is on $\overline{WX}$ or $\overline{YZ}$.

  $P(\overline{WX} \text{ or } \overline{YZ}) = P(\overline{WX}) + P(\overline{YZ}) = \frac{3}{18} + \frac{7}{18}$

  $= \frac{10}{18} = \frac{5}{9}$

- **Find the probability that a point chosen randomly inside the rectangle is inside the equilateral triangle.**



  area of rectangle

  $A = bh = 20(10) = 200 \text{ ft}^2$

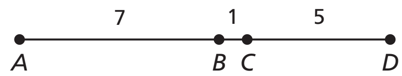
  area of triangle

  $A = \frac{1}{2}aP = \frac{1}{2}\left(\frac{5\sqrt{3}}{3}\right)(30) = 25\sqrt{3} \approx 43.3 \text{ ft}^2$

  $P = \frac{43.3}{200} \approx 0.22$

EXERCISES

A point is chosen randomly on $\overline{AD}$. Find the probability of each event.



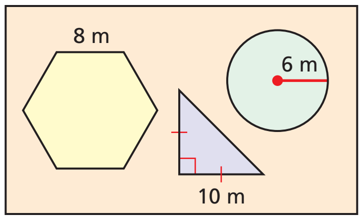
35. The point is on $\overline{AB}$.

36. The point is not on $\overline{CD}$.

37. The point is on $\overline{AB}$ or $\overline{CD}$.

38. The point is on $\overline{BC}$ or $\overline{CD}$.

Find the probability that a point chosen randomly inside the 40 m by 24 m rectangle is in each shape. Round to the nearest hundredth.



39. the regular hexagon

40. the triangle

41. the circle or the triangle

42. inside the rectangle but not inside the hexagon, triangle, or circle

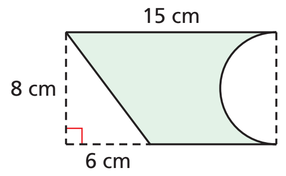
CHAPTER 9

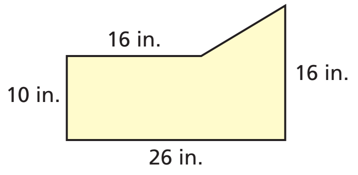
CHAPTER TEST

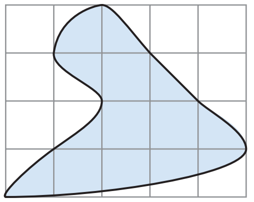
Find each measurement.

1. the height h of a triangle in which $A = 12x^2y$ ft^2 and $b = 3x$ ft
2. the base b_1 of a trapezoid in which $A = 161.5$ cm^2, $h = 17$ cm, and $b_2 = 13$ cm
3. the area A of a kite in which $d_1 = 25$ in. and $d_2 = 12$ in.
4. Find the circumference and area of $\odot A$ with diameter 12 in. Give your answers in terms of π.
5. Find the area of a regular hexagon with a side length of 14 m. Round to the nearest tenth.

Find the shaded area. Round to the nearest tenth, if necessary.

6. 

7. 



8. The diagram shows a plan for a pond. Use a composite figure to estimate the pond's area. The grid has squares with side lengths of 1 yd.
9. Draw and classify the polygon with vertices $A(1, 5)$, $B(2, 3)$, $C(-2, 1)$, and $D(-3, 3)$. Find the perimeter and area of the polygon.

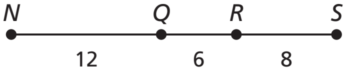
Find the area of each polygon with the given vertices.

10. $E(-3, 4)$, $F(1, 1)$, $G(0, -4)$, $H(-4, 1)$
11. $J(3, 4)$, $K(4, -1)$, $L(-2, -4)$, $M(-3, 3)$

Describe the effect of each change on the perimeter or circumference and area of the given figure.

12. The base and height of a triangle with base 10 cm and height 12 cm are multiplied by 3.
13. The radius of a circle with radius 12 m is multiplied by $\frac{1}{2}$.
14. A circular garden plot has a diameter of 21 ft. Janelle is planning a new circular plot with an area $\frac{1}{9}$ as large. How will the circumference of the new plot compare to the circumference of the old plot?

A point is chosen randomly on $\overline{NS}$. Find the probability of each event.



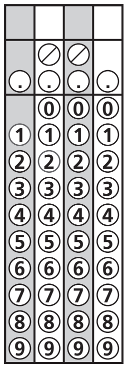
15. The point is on $\overline{NQ}$.
16. The point is not on $\overline{QR}$.
17. The point is on $\overline{NQ}$ or $\overline{RS}$.
18. A shuttle bus for a festival stops at the parking lot every 18 minutes and stays at the lot for 2 minutes. If you go to the festival at a random time, what is the probability that the shuttle bus will be at the parking lot when you arrive?

COLLEGE ENTRANCE EXAM PRACTICE

CHAPTER 9

FOCUS ON SAT STUDENT-PRODUCED RESPONSES

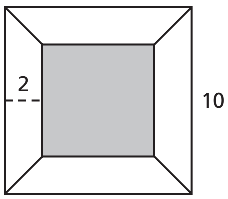
There are two types of questions in the mathematics sections of the SAT: multiple-choice questions, where you select the correct answer from five choices, and student-produced response questions, for which you enter the correct answer in a special grid.



HOT TIP! On the SAT, the student-produced response items do not have a penalty for incorrect answers. If you are uncertain of your answer and do not have time to rework the problem, you should still grid in the answer you have.

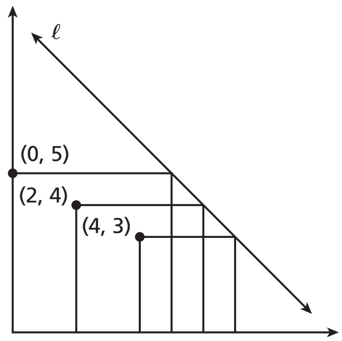
You may want to time yourself as you take this practice test. It should take you about 9 minutes to complete.

1. A triangle has two angles with a measure of 60° and one side with a length of 12. What is the perimeter of the triangle?

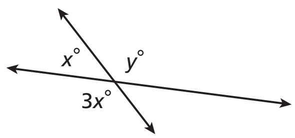


2. The figure above is composed of four congruent trapezoids arranged around a shaded square. What is the area of the shaded square?

3. If $\triangle PQR \sim \triangle STU$, $m\angle P = 22°$, $m\angle Q = 57°$, and $m\angle U = x°$, what is the value of x?



4. Three overlapping squares and the coordinates of a corner of each square are shown above. What is the y-intercept of line ℓ?



5. In the figure above, what is the value of y?

6. The three angles of a triangle have measures $12x°$, $3x°$, and $7y°$, where $7y > 60$. If x and y are integers, what is the value of x?

CHAPTER 9

STRATEGIES FOR SUCCESS

Any Question Type: Use a Formula Sheet

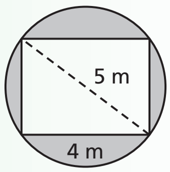
When you take a standardized mathematics test, you may be given a formula sheet or a mathematics chart that accompanies the test. Although many common formulas are given on these sheets, you still need to know when the formulas are applicable, and what the variables in the formulas represent.

EXAMPLE 1

Mathematics Chart

Perimeter	rectangle	$P = 2\ell + 2w$ or $P = 2(\ell + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	rectangle	$A = \ell w$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	circle	$A = \pi r^2$

Multiple Choice **In the figure, a rectangle is inscribed in a circle. Which best represents the shaded area to the nearest tenth of a square meter?**



Ⓐ 3.4 m^2 Ⓒ 12.6 m^2

Ⓑ 7.6 m^2 Ⓓ 17.2 m^2

Which formula(s) do I need?

area of a circle, area of a rectangle

What do I substitute for each variable in the formulas?

To use the formula for the area of a circle, I need to know the radius. The diameter of the circle is 5 m, so the radius is 2.5 m. I should substitute 2.5 for r and 3.14 for π.

To use the formula for the area of a rectangle, I need to know its base and height. The base b is 4 m. To find the height, I can use the Pythagorean Theorem.

$$4^2 + h^2 = 5^2$$
$$16 + h^2 = 25$$
$$h^2 = 9$$
$$h = 3$$

What are the areas of the shapes?

circle: $A = \pi r^2 = \pi(2.5)^2 = 6.25\pi\text{ m}^2$

rectangle: $A = bh = 4(3) = 12\text{ m}^2$

What do I do with the areas to find the answer?

shaded area = area of circle − area of rectangle

$= 6.25\pi - 12 \approx 7.6\text{ m}^2$

Choice B is the correct answer.

Read each test problem and answer the questions that follow. Use the formula sheet below, if applicable.

Perimeter	
rectangle	$P = 2\ell + 2w$ or $P = 2(\ell + w)$
Circumference	
circle	$C = 2\pi r$ or $C = \pi d$
Area	
rectangle	$A = \ell w$ or $A = bh$
triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
circle	$A = \pi r^2$
Pi	
π	$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$

Item A

The circumference of a circle is 48π meters. What is the radius in meters?

Ⓐ 6.9 meters

Ⓑ 24 meters

Ⓒ 12 meters

Ⓓ 36 meters

1. Which formula would you use to solve this problem?
2. After substituting the variables in the formula, what would you need to do to find the correct answer?

Item B

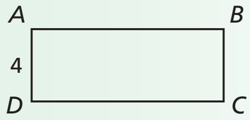
Gridded Response The area of a trapezoid is 171 square meters. The height is 9 meters, and one base length is 23 meters. What is the other base length of the trapezoid in meters?

3. What formula(s) would you use to solve this problem?
4. What would you substitute for each variable in the formula?

HOT TIP! Before you begin a test, quickly review the formulas included on your formula sheet.

Item C

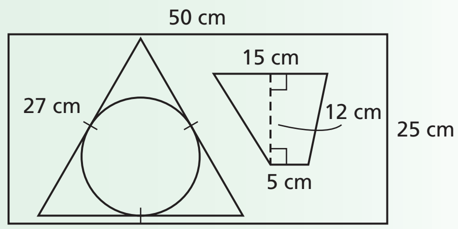
Gridded Response The area of the rectangle is 48 square miles. What is the perimeter in miles?



5. What formula(s) would you use to solve this problem?
6. What would you substitute for each variable in the formula?
7. After substituting the variables in the formula, what would you need to do to find the correct answer?

Item D

Short Response A point is chosen randomly inside the rectangle. Which is more likely: the point lies within the triangle, or the point does not lie inside the triangle or the trapezoid?



8. Which formulas would you use to solve this problem?
9. What would you substitute for each variable in the formula(s)?
10. After substituting the variables in the formula, what would you need to do to find the correct answer?

CHAPTER 9

STANDARDIZED TEST PREP

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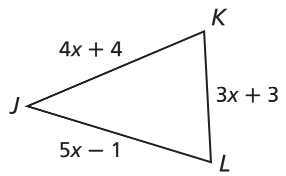
CUMULATIVE ASSESSMENT, CHAPTERS 1–9

Multiple Choice

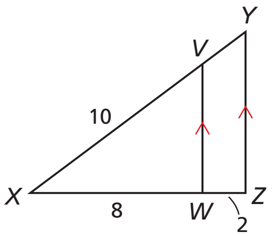
1. The floor of a tent is a regular hexagon. If the side length of the tent floor is 5 feet, what is the area of the floor? Round to the nearest tenth.
   - (A) 32.5 square feet
   - (B) 65.0 square feet
   - (C) 75.0 square feet
   - (D) 129.9 square feet

2. If J is on the perpendicular bisector of $\overline{KL}$, what is the length of $\overline{KL}$?



   - (F) 12
   - (G) 18
   - (H) 24
   - (J) 36

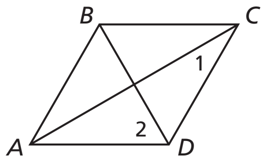
3. What is the length of $\overline{VY}$?



   - (A) 1.6
   - (B) 2
   - (C) 2.5
   - (D) 4

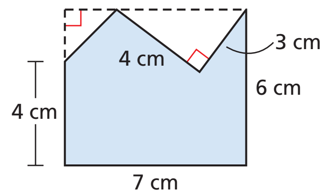
4. A sailor on a ship sights the light of a lighthouse at an angle of elevation of 15°. If the light in the lighthouse is 189 feet higher than the sailor's line of sight, what is the horizontal distance between the ship and the lighthouse? Round to the nearest foot.
   - (F) 49 feet
   - (G) 51 feet
   - (H) 705 feet
   - (J) 730 feet

5. If $ABCD$ is a rhombus in which $m\angle 1 = (x + 15)°$ and $m\angle 2 = (2x + 12)°$, what is the value of x?



   - (A) 3
   - (B) 9
   - (C) 18
   - (D) 21

6. What is the area of the shaded portion of the rectangle?



   - (F) 34 square centimeters
   - (G) 36 square centimeters
   - (H) 38 square centimeters
   - (J) 50 square centimeters

7. If $\triangle XYZ$ is isosceles and $m\angle Y > 100°$, which of the following must be true?
   - (A) $m\angle X < 40°$
   - (B) $m\angle X > 40°$
   - (C) $\overline{XZ} \cong \overline{YZ}$
   - (D) $\overline{XY} \cong \overline{XZ}$

8. The Eiffel Tower in Paris, France, is 300 meters tall. The first level of the tower has a height of 57 meters. A scale model of the Eiffel Tower in Shenzhen, China, is 108 meters tall. What is the height of the first level of the model? Round to the nearest tenth.
   - (F) 15.8 meters
   - (G) 20.5 meters
   - (H) 56.8 meters
   - (J) 61.6 meters

HOT TIP! There is often more than one way to find a missing side length or angle measure in a figure. For example, you might be able to find a side length of a right triangle by using either the Pythagorean Theorem or a trigonometric ratio. Check your answer by using a different method than the one you originally used.

9. The lengths of both bases of a trapezoid are tripled. What is the effect of the change on the area of the trapezoid?

Ⓐ The area remains the same.

Ⓑ The area is tripled.

Ⓒ The area increases by a factor of 6.

Ⓓ The area increases by a factor of 9.

10. If $\angle 1$ and $\angle 2$ form a linear pair, which of the following must also be true about these angles?

Ⓕ They are adjacent.

Ⓖ They are complementary.

Ⓗ They are congruent.

Ⓙ They are vertical.

11. In $\triangle ABC$, $AB = 8$, $BC = 17$, and $AC = 2x + 1$. Which of the following is a possible value of x?

Ⓐ 3 Ⓒ 9

Ⓑ 4 Ⓓ 12

12. Which line is parallel to the line with the equation $y = -3x + 4$?

Ⓕ $y - 3x = 8$

Ⓖ $4y - 12x = 1$

Ⓗ $3y - x = 3$

Ⓙ $2y + 6x = 5$

Gridded Response

13. What is the radius of a circle in inches if the ratio of its area to its circumference is 2.5 square inches : 1 inch?

14. $\triangle JLM \sim \triangle RST$. If $JL = 5$, $LM = 4$, $RS = 3x - 1$, and $ST = x + 2$, what is the value of x?

15. If the two diagonals of a kite measure 16 centimeters and 10 centimeters, what is the area of the kite in square centimeters?

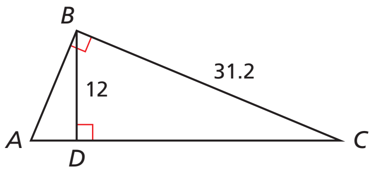
Short Response

16. Two gas stations on a straight highway are 8 miles apart. If a car runs out of gas at a random point between the two gas stations, what is the probability that the car will be at least 2 miles from either gas station? Draw a diagram or write and explanation to show how you determined your answer.

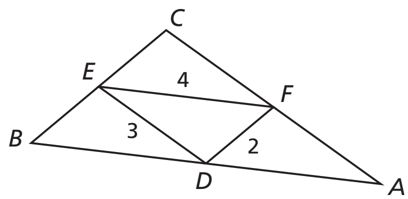
17. Use the figure below to find each measure. Show your work or explain in words how you found your answers. Round the angle measure to the nearest degree.

a. $m\angle A$

b. AC

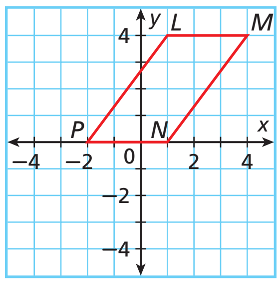


18. Given that $\overline{DE}$, $\overline{DF}$, and $\overline{EF}$ are midsegments of $\triangle ABC$, determine $m\angle C$ to the nearest degree. Show your work or explain in words how you determined your answer.



Extended Response

19. Quadrilateral $LMNP$ has vertices at $L(1, 4)$, $M(4, 4)$, $N(1, 0)$ and $P(-2, 0)$.



a. Write a coordinate proof showing that $LMNP$ is a parallelogram.

b. Draw a rectangle with the same area as figure $LMNP$. Explain how you know that the figures have the same area.

c. Does the rectangle you drew have the same perimeter as figure $LMNP$? Explain.