CHAPTER 8

Right Triangles and Trigonometry

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Chapter Project Online
KEYWORD: MG7 ChProj

You can use trigonometric ratios and angle measures to find the height of a tall object, such as a lighthouse.

Pigeon Point Lighthouse
Pescadero, CA

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. altitude
2. proportion
3. ratio
4. right triangle

A. a comparison of two numbers by division

B. a segment from a vertex to the midpoint of the opposite side of a triangle

C. an equation stating that two ratios are equal

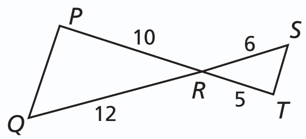
D. a perpendicular segment from the vertex of a triangle to a line containing the base

E. a triangle that contains a right angle

Identify Similar Figures

Determine if the two triangles are similar.

5.



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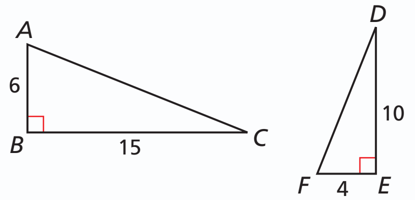

Special Right Triangles

Find the value of x. Give the answer in simplest radical form.

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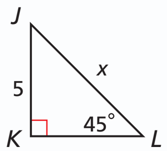

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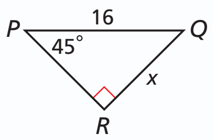

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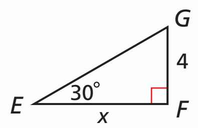

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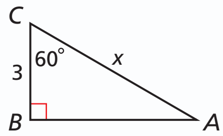


Solve Multi-Step Equations

Solve each equation.

11. $3(x - 1) = 12$
12. $-2(y + 5) = -1$
13. $6 = 8(x - 3)$
14. $2 = -1(z + 4)$

Solve Proportions

Solve each proportion.

15. $\frac{4}{y} = \frac{6}{18}$
16. $\frac{5}{8} = \frac{x}{32}$
17. $\frac{m}{9} = \frac{8}{12}$
18. $\frac{y}{4} = \frac{9}{y}$

Rounding and Estimation

Round each decimal to the indicated place value.

19. 13.118; hundredth
20. 37.91; tenth
21. 15.992; tenth
22. 173.05; whole number

CHAPTER 8

Unpacking the Standards

The information below "unpacks" the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

California Standard	Academic Vocabulary	Chapter Concept
4.0 Students prove basic theorems involving congruence and **similarity.** (Lesson 8-1)	**involving** relating to	You prove triangles are similar. Then you use the relationships in similar right triangles to solve problems. You also learn how to use the geometric mean to find the length of a segment.
15.0 Students use the Pythagorean theorem to determine distance and **find missing lengths of sides of right triangles.** (Lesson 8-3)	**Pythagorean** mathematics that relates to the ancient Greek mathematician, Pythagoras **lengths** distances along the side of a right triangle from end to end	You use the Pythagorean Theorem to find the lengths of the legs or hypotenuse of a right triangle.
18.0 Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, $\tan(x) = \sin(x)/\cos(x)$, $(\sin(x))^2 + (\cos(x))^2 = 1$. (Lessons 8-2, 8-3, Extension)	**trigonometric** mathematics that uses the proportional relationships between the sides and angles of right triangles ***Example:*** $\sin = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$ $\cos = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$ $\tan = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$	You learn how to define the sine, cosine, and tangent of an acute angle of a right triangle. You also learn how to define trigonometric ratios for angle measures greater than or equal to 90°.
19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side. (Lessons 8-2, 8-3, 8-4, 8-5, 8-6) (Lab 8-4)	**functions** quantities used to show the relationship between the parts of a triangle	You use trigonometric ratios to solve problems that involve angles of elevation and angles of depression. You learn the Law of Sines and the Law of Cosines and how to use these to solve triangles.

Reading and Writing Math

CHAPTER 8

Reading Strategy: Read to Understand

As you read a lesson, read with a purpose. Lessons are about one or two specific objectives. These objectives are at the top of the first page of every lesson. Reading with the objectives in mind can help you understand the lesson.

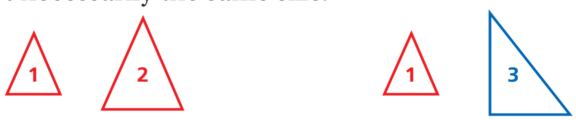
Objective

Identify similar polygons.

Identify the **objectives** of the lesson.

Lesson 7-2 Ratios in Similar Polygons

Figures that are **similar** ($\sim$) have the same shape but not necessarily the same size.



$\triangle 1$ is similar to $\triangle 2$ ($\triangle 1 \sim \triangle 2$).

$\triangle 1$ is not similar to $\triangle 3$ ($\triangle 1 \nsim \triangle 3$).

Read through the lesson to find where the objectives are explained.

Questions

- Can two polygons be both similar and congruent?
- In Example 1, the triangles are not oriented the same. How can you tell which angles are congruent and which sides are corresponding?

List any questions, problems, or trouble spots you may have.

New Vocabulary/Symbols

- Similarity is represented by the symbol $\sim$. Congruence is represented by the symbol $\cong$.
- Similar: same shape but not necessarily the same size

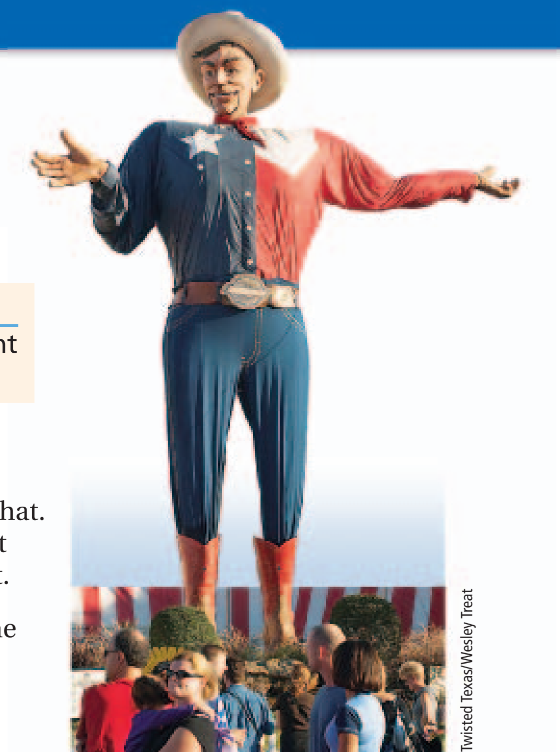
Write down any new vocabulary or symbols.

Try This

Use Lesson 8-1 to complete each of the following.

1. What are the objectives of the lesson?
2. Identify any new vocabulary, formulas, and symbols.
3. Identify any examples that you need clarified.
4. Make a list of questions you need answered during class.

8-1 Similarity in Right Triangles


Twisted Texas/Wesley Treat

Objectives

Use geometric mean to find segment lengths in right triangles.

Apply similarity relationships in right triangles to solve problems.

Vocabulary

geometric mean

Why learn this?

You can use similarity relationships in right triangles to find the height of Big Tex.

Big Tex debuted as the official symbol of the State Fair of Texas in 1952. This 6000-pound cowboy wears size 70 boots and a 75-gallon hat. In this lesson, you will learn how to use right triangle relationships to find Big Tex's height.

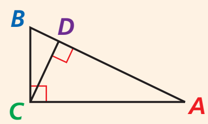
In a right triangle, an altitude drawn from the vertex of the right angle to the hypotenuse forms two right triangles.

Know it! Note

Theorem 8-1-1

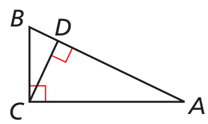
The altitude to the hypotenuse of a right triangle forms two triangles that are similar to each other and to the original triangle.

$\triangle ABC \sim \triangle ACD \sim \triangle CBD$



PROOF

Theorem 8-1-1



Given: $\triangle ABC$ is a right triangle with altitude $\overline{CD}$.
Prove: $\triangle ABC \sim \triangle ACD \sim \triangle CBD$

Proof: The right angles in $\triangle ABC$, $\triangle ACD$, and $\triangle CBD$ are all congruent. By the Reflexive Property of Congruence, $\angle A \cong \angle A$. Therefore $\triangle ABC \sim \triangle ACD$ by the AA Similarity Theorem. Similarly, $\angle B \cong \angle B$, so $\triangle ABC \sim \triangle CBD$. By the Transitive Property of Similarity, $\triangle ABC \sim \triangle ACD \sim \triangle CBD$.

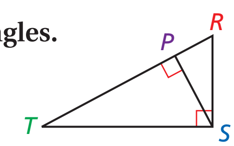
California Standards

4.0 Students prove basic theorems involving congruence and **similarity.**

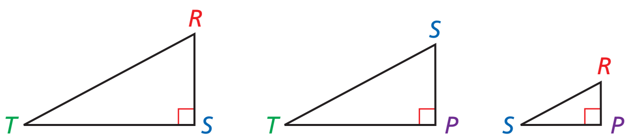
EXAMPLE 1

Identifying Similar Right Triangles

Write a similarity statement comparing the three triangles.



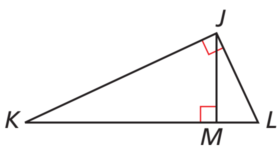
Sketch the three right triangles with the angles of the triangles in corresponding positions.



By Theorem 8-1-1, $\triangle RST \sim \triangle SPT \sim \triangle RPS$.

CHECK IT OUT!

1. Write a similarity statement comparing the three triangles.



Consider the proportion $\frac{a}{x} = \frac{x}{b}$. In this case, the means of the proportion are the same number, and that number is the *geometric mean* of the extremes. The **geometric mean** of two positive numbers is the positive square root of their product. So the geometric mean of a and b is the positive number x such that $x = \sqrt{ab}$, or $x^2 = ab$.

EXAMPLE 2 Finding Geometric Means

Find the geometric mean of each pair of numbers. If necessary, give the answer in simplest radical form.

A 4 and 9

Let x be the geometric mean.

$x^2 = (4)(9) = 36$ *Def. of geometric mean*

$x = 6$ *Find the positive square root.*

B 6 and 15

Let x be the geometric mean.

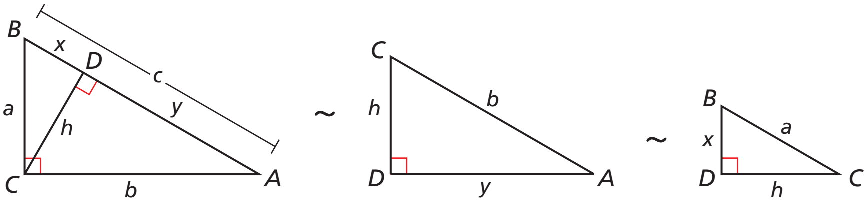
$x^2 = (6)(15) = 90$ *Def. of geometric mean*

$x = \sqrt{90} = 3\sqrt{10}$ *Find the positive square root.*

CHECK IT OUT! **Find the geometric mean of each pair of numbers. If necessary, give the answer in simplest radical form.**

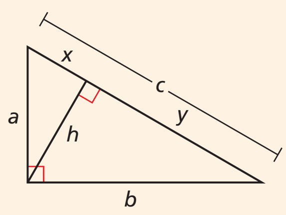
2a. 2 and 8 **2b.** 10 and 30 **2c.** 8 and 9

You can use Theorem 8-1-1 to write proportions comparing the side lengths of the triangles formed by the altitude to the hypotenuse of a right triangle. All the relationships in red involve geometric means.



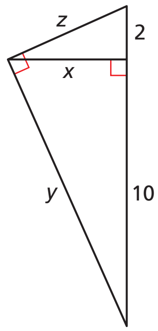
$\frac{b}{a} = \frac{y}{h} = \frac{h}{x}$ $\frac{c}{a} = \frac{b}{h} = \frac{a}{x}$ $\frac{c}{b} = \frac{b}{y} = \frac{a}{h}$

Know it! Note — Corollaries: Geometric Means

COROLLARY	EXAMPLE	DIAGRAM
8-1-2 The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the two segments of the hypotenuse.	$h^2 = xy$	
8-1-3 The length of a leg of a right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse adjacent to that leg.	$a^2 = xc$ $b^2 = yc$	

EXAMPLE 3 Finding Side Lengths in Right Triangles

Find x, y, and z.



> **Helpful Hint**
>
> Once you've found the unknown side lengths, you can use the Pythagorean Theorem to check your answers.

$x^2 = (2)(10) = 20$ — *x is the geometric mean of 2 and 10.*

$x = \sqrt{20} = 2\sqrt{5}$ — *Find the positive square root.*

$y^2 = (12)(10) = 120$ — *y is the geometric mean of 12 and 10.*

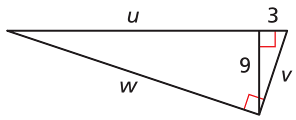
$y = \sqrt{120} = 2\sqrt{30}$ — *Find the positive square root.*

$z^2 = (12)(2) = 24$ — *z is the geometric mean of 12 and 2.*

$z = \sqrt{24} = 2\sqrt{6}$ — *Find the positive square root.*

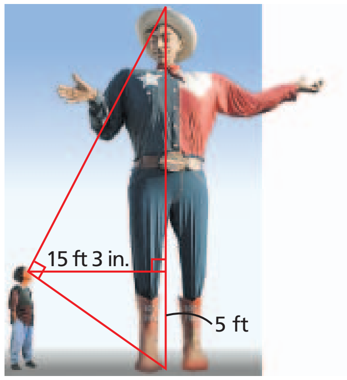
CHECK IT OUT!

3. Find u, v, and w.



EXAMPLE 4 *Measurement Application*

To estimate the height of Big Tex at the State Fair of Texas, Michael steps away from the statue until his line of sight to the top of the statue and his line of sight to the bottom of the statue form a 90° angle. His eyes are 5 ft above the ground, and he is standing 15 ft 3 in. from Big Tex. How tall is Big Tex to the nearest foot?



Not drawn to scale

Let x be the height of Big Tex above eye level.

15 ft 3 in. = 15.25 ft — *Convert 3 in. to 0.25 ft.*

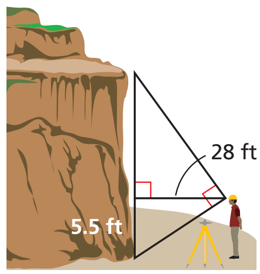
$(15.25)^2 = 5x$ — *15.25 is the geometric mean of 5 and x.*

$x = 46.5125 \approx 47$ — *Solve for x and round.*

Big Tex is about 47 + 5, or 52 ft tall.

CHECK IT OUT!

4. A surveyor positions himself so that his line of sight to the top of a cliff and his line of sight to the bottom form a right angle as shown. What is the height of the cliff to the nearest foot?

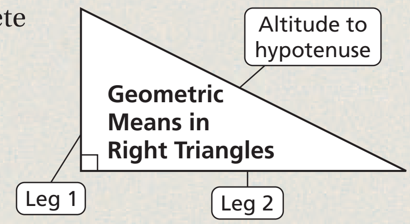


THINK AND DISCUSS

1. Explain how to find the geometric mean of 7 and 21.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. Label the right triangle and draw the altitude to the hypotenuse. In each box, write a proportion in which the given segment is a geometric mean.



8-1 Exercises

California Standards
4.0, 7.0, 12.0, 20.0, 7AF2.0, 1A2.0

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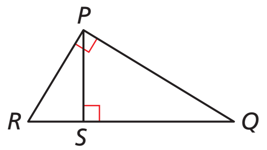
Homework Help Online
KEYWORD: MG7 8-1
Parent Resources Online
KEYWORD: MG7 Parent

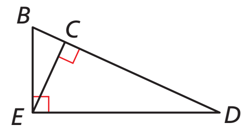
GUIDED PRACTICE

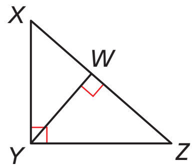
1. **Vocabulary** In the proportion $\frac{2}{8} = \frac{8}{32}$, which number is the *geometric mean* of the other two numbers?

SEE EXAMPLE 1 p. 518

Write a similarity statement comparing the three triangles in each diagram.

2. 

3. 

4. 

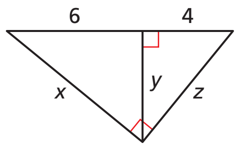
SEE EXAMPLE 2 p. 519

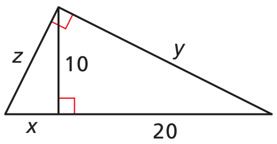
Find the geometric mean of each pair of numbers. If necessary, give the answer in simplest radical form.

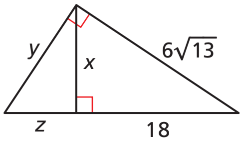
5. 2 and 50
6. 4 and 16
7. $\frac{1}{2}$ and 8
8. 9 and 12
9. 16 and 25
10. 7 and 11

SEE EXAMPLE 3 p. 520

Find *x*, *y*, and *z*.

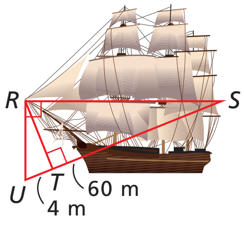
11. 

12. 

13. 

SEE EXAMPLE 4 p. 520

14. **Measurement** To estimate the length of the USS *Constitution* in Boston harbor, a student locates points *T* and *U* as shown. What is *RS* to the nearest tenth?



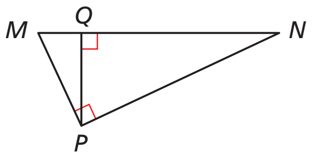
PRACTICE AND PROBLEM SOLVING

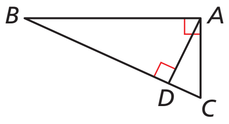
Independent Practice

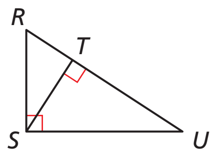
For Exercises	See Example
15–17	1
18–23	2
24–26	3
27	4

Extra Practice
Skills Practice p. S18
Application Practice p. S35

Write a similarity statement comparing the three triangles in each diagram.

15. 

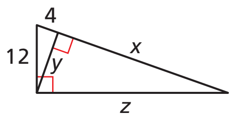
16. 

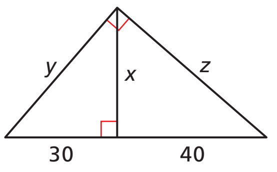
17. 

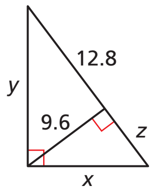
Find the geometric mean of each pair of numbers. If necessary, give the answer in simplest radical form.

18. 5 and 45
19. 3 and 15
20. 5 and 8
21. $\frac{1}{4}$ and 80
22. 1.5 and 12
23. $\frac{2}{3}$ and $\frac{27}{40}$

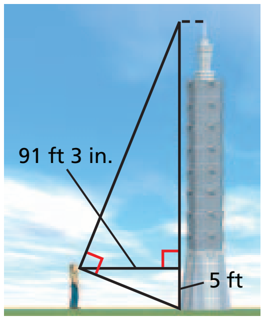
Find *x*, *y*, and *z*.

24. 

25. 

26. 

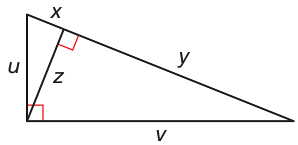
27. **Measurement** To estimate the height of the Taipei 101 tower, Andrew stands so that his lines of sight to the top and bottom of the tower form a 90° angle. What is the height of the tower to the nearest foot?



28. The geometric mean of two numbers is 8. One of the numbers is 2. Find the other number.

29. The geometric mean of two numbers is $2\sqrt{5}$. One of the numbers is 6. Find the other number.

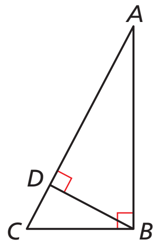
Use the diagram to complete each equation.



30. $\frac{x}{z} = \frac{z}{?}$ **31.** $\frac{?}{u} = \frac{u}{x}$ **32.** $\frac{x+y}{v} = \frac{v}{?}$

33. $\frac{y}{?} = \frac{z}{x}$ **34.** $(?)^2 = y(x+y)$ **35.** $u^2 = (x+y)(?)$

Give each answer in simplest radical form.



36. $AD = 12$, and $CD = 8$. Find BD.

37. $AC = 16$, and $CD = 5$. Find BC.

38. $AD = CD = \sqrt{2}$. Find BD.

39. $BC = \sqrt{5}$, and $AC = \sqrt{10}$. Find CD.

40. **Finance** An investment returns 3% one year and 10% the next year. The average rate of return is the geometric mean of the two annual rates. What is the average rate of return for this investment to the nearest tenth of a percent?

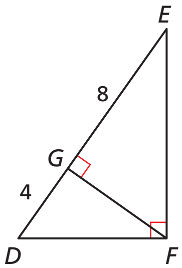
41. **ERROR ANALYSIS** Two students were asked to find EF. Which solution is incorrect? Explain the error.

A

$\frac{12}{EF} = \frac{EF}{8}$, so
$(EF)^2 = (12)(8) = 96.$
Thus $EF = \sqrt{96} =$
$\sqrt{(16)(6)} = 4\sqrt{6}.$

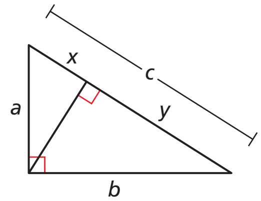
B

$\frac{8}{EF} = \frac{EF}{4}$, so
$(EF)^2 = (8)(4) = 32.$
Thus $EF = \sqrt{32} =$
$\sqrt{(16)(2)} = 4\sqrt{2}.$

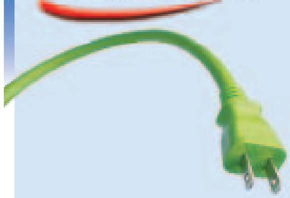


42. The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments that are 2 cm long and 5 cm long. Find the length of the altitude to the nearest tenth of a centimeter.

43. **Critical Thinking** Use the figure to show how Corollary 8-1-3 can be used to derive the Pythagorean Theorem. (*Hint:* Use the corollary to write expressions for a^2 and b^2. Then add the expressions.)

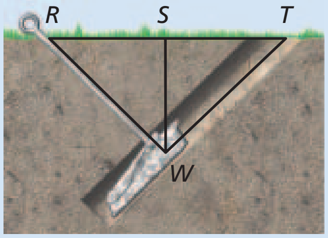


Concept Connection



44. This problem will prepare you for the Concept Connection on page 542.

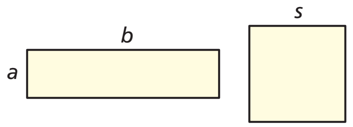
Before installing a utility pole, a crew must first dig a hole and install the anchor for the guy wire that supports the pole. In the diagram, $\overline{SW} \perp \overline{RT}$, $\overline{RW} \perp \overline{WT}$, $RS = 4$ ft, and $ST = 3$ ft.



a. Find the depth of the anchor $\overline{SW}$ to the nearest inch.

b. Find the length of the rod $\overline{RW}$ to the nearest inch.

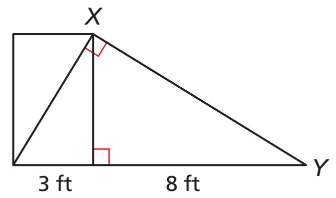
45. **Write About It** Suppose the rectangle and square have the same area. Explain why s must be the geometric mean of a and b.



46. **Write About It** Explain why the geometric mean of two perfect squares must be a whole number.

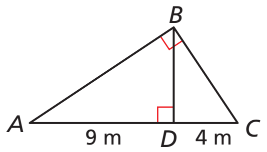
STANDARDIZED TEST PREP

47. Lee is building a skateboard ramp based on the plan shown. Which is closest to the length of the ramp from point X to point Y?



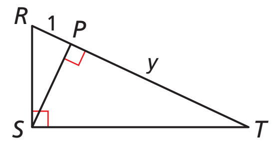
Ⓐ 4.9 feet
Ⓑ 5.7 feet
Ⓒ 8.5 feet
Ⓓ 9.4 feet

48. What is the area of $\triangle ABC$?



Ⓕ 18 square meters
Ⓖ 36 square meters
Ⓗ 39 square meters
Ⓙ 78 square meters

49. Which expression represents the length of $\overline{RS}$?

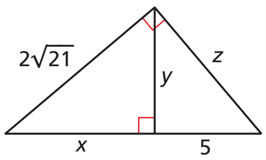


Ⓐ $\sqrt{y+1}$
Ⓑ $\sqrt{y}$
Ⓒ y^2
Ⓓ $y(y+1)$

CHALLENGE AND EXTEND

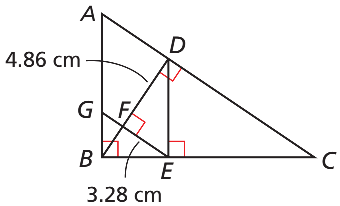
50. **Algebra** An 8-inch-long altitude of a right triangle divides the hypotenuse into two segments. One segment is 4 times as long as the other. What are the lengths of the segments of the hypotenuse?

51. Use similarity in right triangles to find x, y, and z.



52. Prove the following. If the altitude to the hypotenuse of a right triangle bisects the hypotenuse, then the triangle is a 45°-45°-90° right triangle.

53. **Multi-Step** Find AC and AB to the nearest hundredth.



SPIRAL REVIEW

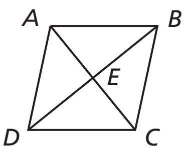
Find the x-intercept and y-intercept for each equation. *(Previous course)*

54. $3y + 4 = 6x$
55. $x + 4 = 2y$
56. $3y - 15 = 15x$

The leg lengths of a 30°-60°-90° triangle are given. Find the length of the hypotenuse. *(Lesson 5-8)*

57. 3 and $\sqrt{27}$
58. 7 and $7\sqrt{3}$
59. 2 and $2\sqrt{3}$

For rhombus $ABCD$, find each measure, given that $m\angle DEC = 30y°$, $m\angle EDC = (8y + 15)°$, $AB = 2x + 8$, and $BC = 4x$. *(Lesson 6-4)*



60. $m\angle EDC$
61. $m\angle EDA$
62. AB

8-2 Technology LAB

Explore Trigonometric Ratios

Use with Lesson 8-2

In a right triangle, the ratio of two side lengths is known as a *trigonometric ratio*.

California Standards

Preparation for 18.0 Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, $\tan(x) = \sin(x)/\cos(x)$, $(\sin(x))^2 + (\cos(x))^2 = 1$.

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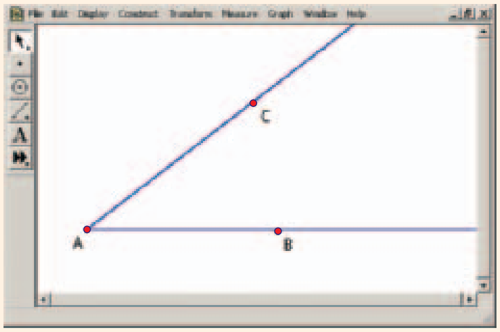
Lab Resources Online
KEYWORD: MG7 Lab8

Activity

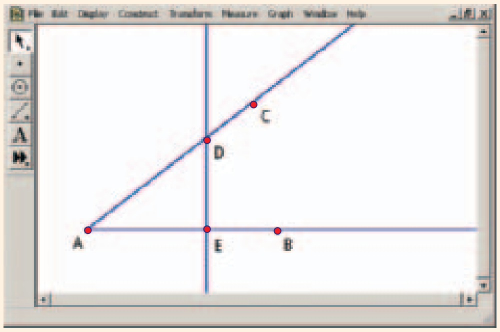
1. Construct three points and label them A, B, and C. Construct rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ with common endpoint A. Move C so that $\angle A$ is an acute angle.



2. Construct point D on $\overrightarrow{AC}$. Construct a line through D perpendicular to $\overrightarrow{AB}$. Label the intersection of the perpendicular line and $\overrightarrow{AB}$ as E.

3. Measure $\angle A$. Measure DE, AE, and AD, the side lengths of $\triangle AED$.

4. Calculate the ratios $\frac{DE}{AD}$, $\frac{AE}{AD}$, and $\frac{DE}{AE}$.



Try This

1. Drag D along $\overrightarrow{AC}$. What happens to the measure of $\angle A$ as D moves? What postulate or theorem guarantees that the different triangles formed are similar to each other?
2. As you move D, what happens to the values of the three ratios you calculated? Use the properties of similar triangles to explain this result.
3. Move C. What happens to the measure of $\angle A$? With a new value for $m\angle A$, note the values of the three ratios. What happens to the ratios if you drag D?
4. Move C until $\frac{DE}{AD} = \frac{AE}{AD}$. What is the value of $\frac{DE}{AE}$? What is the measure of $\angle A$? Use the properties of special right triangles to justify this result.

8-2 Trigonometric Ratios

Objectives

Find the sine, cosine, and tangent of an acute angle.

Use trigonometric ratios to find side lengths in right triangles and to solve real-world problems.

Vocabulary

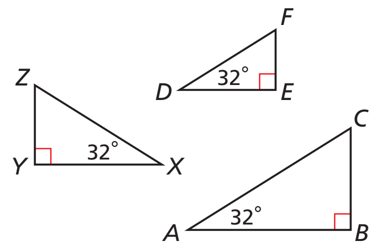
trigonometric ratio
sine
cosine
tangent

Who uses this?

Contractors use trigonometric ratios to build ramps that meet legal requirements.

According to the Americans with Disabilities Act (ADA), the maximum slope allowed for a wheelchair ramp is $\frac{1}{12}$, which is an angle of about 4.8°. Properties of right triangles help builders construct ramps that meet this requirement.

By the AA Similarity Postulate, a right triangle with a given acute angle is similar to every other right triangle with that same acute angle measure. So $\triangle ABC \sim \triangle DEF \sim \triangle XYZ$, and $\frac{BC}{AC} = \frac{EF}{DF} = \frac{YZ}{XZ}$. These are *trigonometric ratios*. A **trigonometric ratio** is a ratio of two sides of a right triangle.



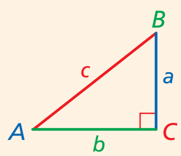
California Standards

18.0 Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, $\tan(x) = \sin(x)/\cos(x)$, $(\sin(x))^2 + (\cos(x))^2 = 1$.

19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.

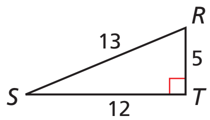
Know it! Note

Trigonometric Ratios

DEFINITION	SYMBOLS	DIAGRAM
The **sine** of an angle is the ratio of the length of the leg opposite the angle to the length of the hypotenuse.	$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{a}{c}$ $\sin B = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{b}{c}$	
The **cosine** of an angle is the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse.	$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{b}{c}$ $\cos B = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{a}{c}$	
The **tangent** of an angle is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle.	$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{a}{b}$ $\tan B = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{b}{a}$	

EXAMPLE 1 Finding Trigonometric Ratios

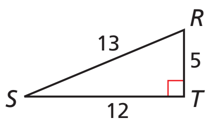
Write each trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



A $\sin R$

$\sin R = \frac{12}{13} \approx 0.92$ *The sine of an ∠ is $\frac{\text{opp. leg}}{\text{hyp.}}$.*

Write each trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



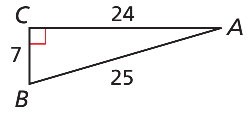
B $\cos R$

$\cos R = \frac{5}{13} \approx 0.38$ *The cosine of an ∠ is $\frac{\text{adj. leg}}{\text{hyp.}}$.*

C $\tan S$

$\tan S = \frac{5}{12} \approx 0.42$ *The tangent of an ∠ is $\frac{\text{opp. leg}}{\text{adj. leg}}$.*

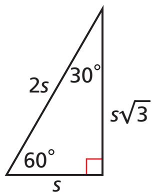
CHECK IT OUT! Write each trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



1a. $\cos A$ **1b.** $\tan B$ **1c.** $\sin B$

EXAMPLE 2 Finding Trigonometric Ratios in Special Right Triangles

Use a special right triangle to write sin 60° as a fraction.



Draw and label a 30°-60°-90° △.

$\sin 60° = \frac{s\sqrt{3}}{2s} = \frac{\sqrt{3}}{2}$ *The sine of an ∠ is $\frac{\text{opp. leg}}{\text{hyp.}}$.*

CHECK IT OUT! **2.** Use a special right triangle to write tan 45° as a fraction.

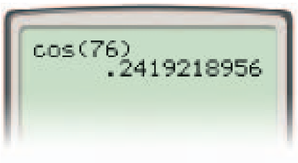
EXAMPLE 3 Calculating Trigonometric Ratios

Caution!

Be sure your calculator is in degree mode, not radian mode.

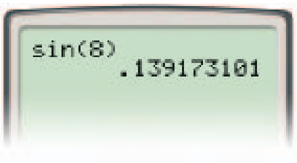
Use your calculator to find each trigonometric ratio. Round to the nearest hundredth.

A $\cos 76°$



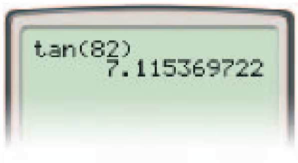
$\cos 76° \approx 0.24$

B $\sin 8°$



$\sin 8° \approx 0.14$

C $\tan 82°$



$\tan 82° \approx 7.12$

CHECK IT OUT! Use your calculator to find each trigonometric ratio. Round to the nearest hundredth.

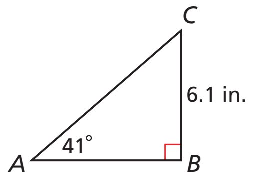
3a. $\tan 11°$ **3b.** $\sin 62°$ **3c.** $\cos 30°$

The hypotenuse is always the longest side of a right triangle. So the denominator of a sine or cosine ratio is always greater than the numerator. Therefore the sine and cosine of an acute angle are always positive numbers less than 1. Since the tangent of an acute angle is the ratio of the lengths of the legs, it can have any value greater than 0.

EXAMPLE 4 Using Trigonometric Ratios to Find Lengths

Find each length. Round to the nearest hundredth.

A *AB*



$\overline{AB}$ is adjacent to the given angle, $\angle A$. You are given *BC*, which is opposite $\angle A$. Since the adjacent and opposite legs are involved, use a tangent ratio.

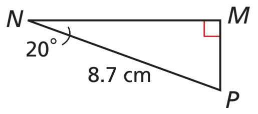
$$\tan A = \frac{\text{opp. leg}}{\text{adj. leg}} = \frac{BC}{AB}$$ *Write a trigonometric ratio.*

$$\tan 41° = \frac{6.1}{AB}$$ *Substitute the given values.*

$$AB = \frac{6.1}{\tan 41°}$$ *Multiply both sides by AB and divide by tan 41°.*

$$AB \approx 7.02 \text{ in.}$$ *Simplify the expression.*

B *MP*



$\overline{MP}$ is opposite the given angle, $\angle N$. You are given *NP*, which is the hypotenuse. Since the opposite side and hypotenuse are involved, use a sine ratio.

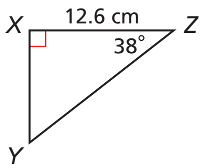
$$\sin N = \frac{\text{opp. leg}}{\text{hyp.}} = \frac{MP}{NP}$$ *Write a trigonometric ratio.*

$$\sin 20° = \frac{MP}{8.7}$$ *Substitute the given values.*

$$8.7(\sin 20°) = MP$$ *Multiply both sides by 8.7.*

$$MP \approx 2.98 \text{ cm}$$ *Simplify the expression.*

C *YZ*



YZ is the hypotenuse. You are given *XZ*, which is adjacent to the given angle, $\angle Z$. Since the adjacent side and hypotenuse are involved, use a cosine ratio.

$$\cos Z = \frac{\text{adj. leg}}{\text{hyp.}} = \frac{XZ}{YZ}$$ *Write a trigonometric ratio.*

$$\cos 38° = \frac{12.6}{YZ}$$ *Substitute the given values.*

$$YZ = \frac{12.6}{\cos 38°}$$ *Multiply both sides by YZ and divide by cos 38°.*

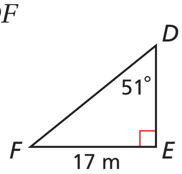
$$YZ \approx 15.99 \text{ cm}$$ *Simplify the expression.*

Caution!

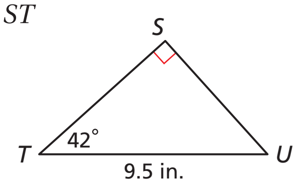
Do not round until the final step of your answer. Use the values of the trigonometric ratios provided by your calculator.

CHECK IT OUT! **Find each length. Round to the nearest hundredth.**

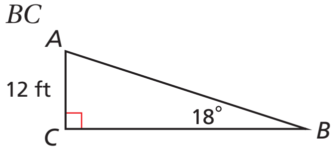
4a. *DF*



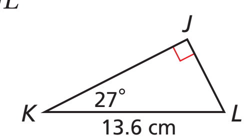
4b. *ST*



4c. *BC*



4d. *JL*

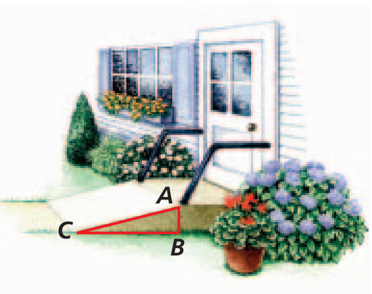


EXAMPLE 5

PROBLEM SOLVING

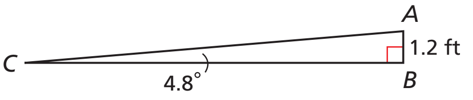
Problem Solving Application

A contractor is building a wheelchair ramp for a doorway that is 1.2 ft above the ground. To meet ADA guidelines, the ramp will make an angle of 4.8° with the ground. To the nearest hundredth of a foot, what is the horizontal distance covered by the ramp?



1 Understand the Problem

Make a sketch. The **answer** is BC.



2 Make a Plan

$\overline{BC}$ is the leg adjacent to $\angle C$. You are given AB, which is the leg opposite $\angle C$. Since the opposite and adjacent legs are involved, write an equation using the tangent ratio.

3 Solve

$$\tan C = \frac{AB}{BC}$$ *Write a trigonometric ratio.*

$$\tan 4.8° = \frac{1.2}{BC}$$ *Substitute the given values.*

$$BC = \frac{1.2}{\tan 4.8°}$$ *Multiply both sides by BC and divide by tan 4.8°.*

$$BC \approx 14.2904 \text{ ft}$$ *Simplify the expression.*

4 Look Back

The problem asks for BC rounded to the nearest hundredth, so round the length to 14.29. The ramp covers a horizontal distance of 14.29 ft.

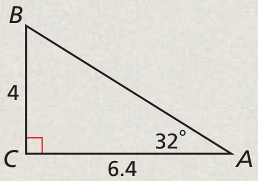
CHECK IT OUT!

5. Find AC, the length of the ramp in Example 5, to the nearest hundredth of a foot.

THINK AND DISCUSS

1. Tell how you could use a sine ratio to find AB.

2. Tell how you could use a cosine ratio to find AB.



Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each cell, write the meaning of each abbreviation and draw a diagram for each.

Abbreviation	Words	Diagram
$\sin = \frac{\text{opp. leg}}{\text{hyp.}}$		
$\cos = \frac{\text{adj. leg}}{\text{hyp.}}$		
$\tan = \frac{\text{opp. leg}}{\text{adj. leg}}$		

8-2 Exercises

California Standards
8.0, 18.0, 19.0, 20.0, 1A2.0

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KEYWORD: MG7 8-2
Parent Resources Online
KEYWORD: MG7 Parent

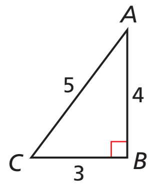
GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. In $\triangle JKL$, $\angle K$ is a right angle. Write the *sine* of $\angle J$ as a ratio of side lengths.
2. In $\triangle MNP$, $\angle M$ is a right angle. Write the *tangent* of $\angle N$ as a ratio of side lengths.

SEE EXAMPLE 1
p. 525

Write each trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



3. $\sin C$ 4. $\tan A$ 5. $\cos A$
6. $\cos C$ 7. $\tan C$ 8. $\sin A$

SEE EXAMPLE 2
p. 526

Use a special right triangle to write each trigonometric ratio as a fraction.

9. $\cos 60°$ 10. $\tan 30°$ 11. $\sin 45°$

SEE EXAMPLE 3
p. 526

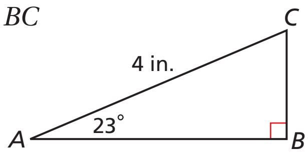
Use your calculator to find each trigonometric ratio. Round to the nearest hundredth.

12. $\tan 67°$ 13. $\sin 23°$ 14. $\sin 49°$
15. $\cos 88°$ 16. $\cos 12°$ 17. $\tan 9°$

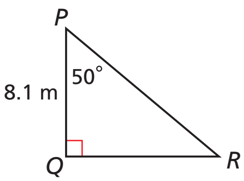
SEE EXAMPLE 4
p. 527

Find each length. Round to the nearest hundredth.

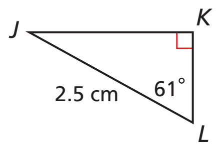
18. BC



19. QR



20. KL



SEE EXAMPLE 5
p. 528

21. **Architecture** A pediment has a pitch of 15°, as shown. If the width of the pediment, WZ, is 56 ft, what is XY to the nearest inch?

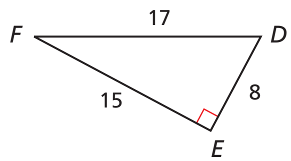
PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
22–27	1
28–30	2
31–36	3
37–42	4
43	5

Extra Practice
Skills Practice p. S18
Application Practice p. S35

Write each trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



22. $\cos D$ 23. $\tan D$ 24. $\tan F$
25. $\cos F$ 26. $\sin F$ 27. $\sin D$

Use a special right triangle to write each trigonometric ratio as a fraction.

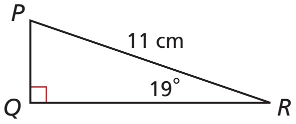
28. $\tan 60°$ 29. $\sin 30°$ 30. $\cos 45°$

Use your calculator to find each trigonometric ratio. Round to the nearest hundredth.

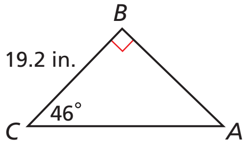
31. $\tan 51°$ 32. $\sin 80°$ 33. $\cos 77°$
34. $\tan 14°$ 35. $\sin 55°$ 36. $\cos 48°$

Find each length. Round to the nearest hundredth.

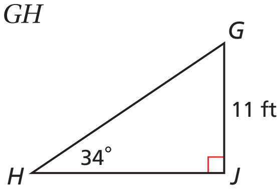
37. *PQ*



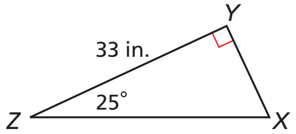
38. *AC*



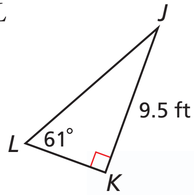
39. *GH*



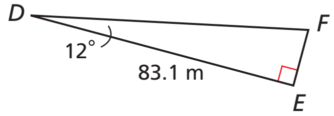
40. *XZ*



41. *KL*



42. *EF*



43. Sports A jump ramp for waterskiing makes an angle of 15° with the surface of the water. The ramp rises 1.58 m above the surface. What is the length of the ramp to the nearest hundredth of a meter?

Use special right triangles to complete each statement.

44. An angle that measures ___?___ has a tangent of 1.

45. For a 45° angle, the ___?___ and ___?___ ratios are equal.

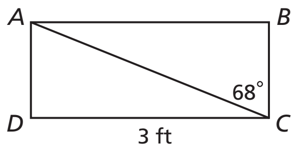
46. The sine of a ___?___ angle is 0.5.

47. The cosine of a 30° angle is equal to the sine of a ___?___ angle.

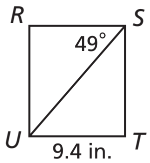
48. Safety According to the Occupational Safety and Health Administration (OSHA), a ladder that is placed against a wall should make a 75.5° angle with the ground for optimal safety. To the nearest tenth of a foot, what is the maximum height that a 10-ft ladder can safely reach?

Find the indicated length in each rectangle. Round to the nearest tenth.

49. *BC*



50. *SU*



51. Critical Thinking For what angle measures is the tangent ratio less than 1? greater than 1? Explain.

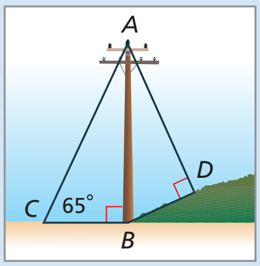
Concept Connection

52. This problem will prepare you for the Concept Connection on page 542.

A utility worker is installing a 25-foot pole $\overline{AB}$ at the foot of a hill. Two guy wires, $\overline{AC}$ and $\overline{AD}$, will help keep the pole vertical.

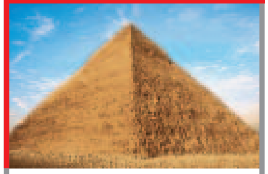
a. To the nearest inch, how long should $\overline{AC}$ be?

b. $\overline{AD}$ is perpendicular to the hill, which makes an angle of 28° with a horizontal line. To the nearest inch, how long should this guy wire be?



LINK

History



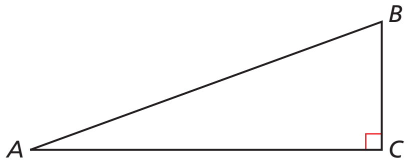
The Pyramid of Cheops consists of more than 2,000,000 blocks of stone with an average weight of 2.5 tons each.

53. Find the sine of the smaller acute angle in a triangle with side lengths of 3, 4, and 5 inches.

54. Find the tangent of the greater acute angle in a triangle with side lengths of 7, 24, and 25 centimeters.

55. **History** The Great Pyramid of Cheops in Giza, Egypt, was completed around 2566 B.C.E. Its original height was 482 ft. Each face of the pyramid forms a 52° angle with the ground. To the nearest foot, how long is the base of the pyramid?

56. **Measurement** Follow these steps to calculate trigonometric ratios.



a. Use a centimeter ruler to find AB, BC, and AC.

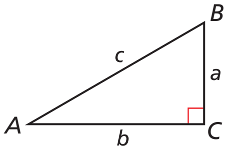
b. Use your measurements from part **a** to find the sine, cosine, and tangent of $\angle A$.

c. Use a protractor to find $m\angle A$.

d. Use a calculator to find the sine, cosine, and tangent of $\angle A$.

e. How do the values in part **d** compare to the ones you found in part **b**?

57. **Algebra** Recall from Algebra I that an *identity* is an equation that is true for all values of the variables.



a. Show that the identity $\tan A = \dfrac{\sin A}{\cos A}$ is true when $m\angle A = 30°$.

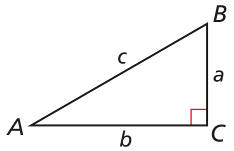
b. Write $\tan A$, $\sin A$, and $\cos A$ in terms of a, b, and c.

c. Use your results from part **b** to prove the identity $\tan A = \dfrac{\sin A}{\cos A}$.

Verify that $(\sin A)^2 + (\cos A)^2 = 1$ for each angle measure.

58. $m\angle A = 45°$ **59.** $m\angle A = 30°$ **60.** $m\angle A = 60°$

61. **Multi-Step** The equation $(\sin A)^2 + (\cos A)^2 = 1$ is known as a Pythagorean Identity.



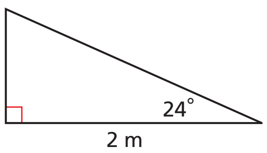
a. Write $\sin A$ and $\cos A$ in terms of a, b, and c.

b. Use your results from part **a** to prove the identity $(\sin A)^2 + (\cos A)^2 = 1$.

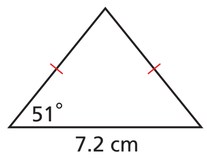
c. **Write About It** Why do you think this identity is called a Pythagorean identity?

Find the perimeter and area of each triangle. Round to the nearest hundredth.

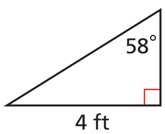
62.



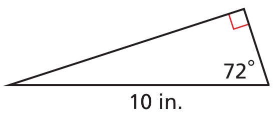
63.



64.



65.

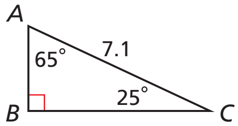


66. **Critical Thinking** Draw $\triangle ABC$ with $\angle C$ a right angle. Write $\sin A$ and $\cos B$ in terms of the side lengths of the triangle. What do you notice? How are $\angle A$ and $\angle B$ related? Make a conjecture based on your observations.

67. **Write About It** Explain how the tangent of an acute angle changes as the angle measure increases.

STANDARDIZED TEST PREP

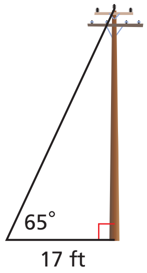
68. Which expression can be used to find AB?



Ⓐ $7.1(\sin 25°)$ Ⓒ $7.1(\sin 65°)$

Ⓑ $7.1(\cos 25°)$ Ⓓ $7.1(\tan 65°)$

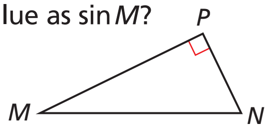
69. A steel cable supports an electrical tower as shown. The cable makes a 65° angle with the ground. The base of the cable is 17 ft from the tower. What is the height of the tower to the nearest foot?



Ⓕ 8 feet Ⓗ 36 feet

Ⓖ 15 feet Ⓙ 40 feet

70. Which of the following has the same value as $\sin M$?



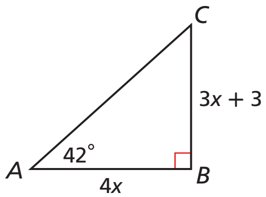
Ⓐ $\sin N$ Ⓒ $\cos N$

Ⓑ $\tan M$ Ⓓ $\cos M$

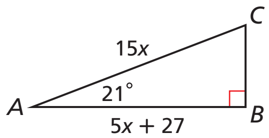
CHALLENGE AND EXTEND

Algebra **Find the value of x. Then find AB, BC, and AC. Round each to the nearest unit.**

71.

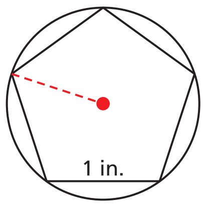


72.



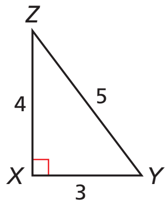
73. **Multi-Step** Prove the identity $(\tan A)^2 + 1 = \dfrac{1}{(\cos A)^2}$.

74. A regular pentagon with 1 in. sides is inscribed in a circle. Find the radius of the circle rounded to the nearest hundredth.



Each of the three trigonometric ratios has a reciprocal ratio, as defined below. These ratios are *cosecant* (csc), *secant* (sec), and *cotangent* (cot).

$$\csc A = \frac{1}{\sin A} \qquad \sec A = \frac{1}{\cos A} \qquad \cot A = \frac{1}{\tan A}$$



Find each trigonometric ratio to the nearest hundredth.

75. $\csc Y$ **76.** $\sec Z$ **77.** $\cot Y$

SPIRAL REVIEW

Find three ordered pairs that satisfy each function. *(Previous course)*

78. $f(x) = 3x - 6$ **79.** $f(x) = -0.5x + 10$ **80.** $f(x) = x^2 - 4x + 2$

Identify the property that justifies each statement. *(Lesson 2-5)*

81. $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{DE}$. So $\overline{AB} \cong \overline{DE}$.

82. $\overline{AB} \cong \overline{AB}$

83. If $\angle JKM \cong \angle MLK$, then $\angle MLK \cong \angle JKM$.

Find the geometric mean of each pair of numbers. *(Lesson 8-1)*

84. 3 and 27 **85.** 6 and 24 **86.** 8 and 32

Connecting Geometry to Algebra

Inverse Functions

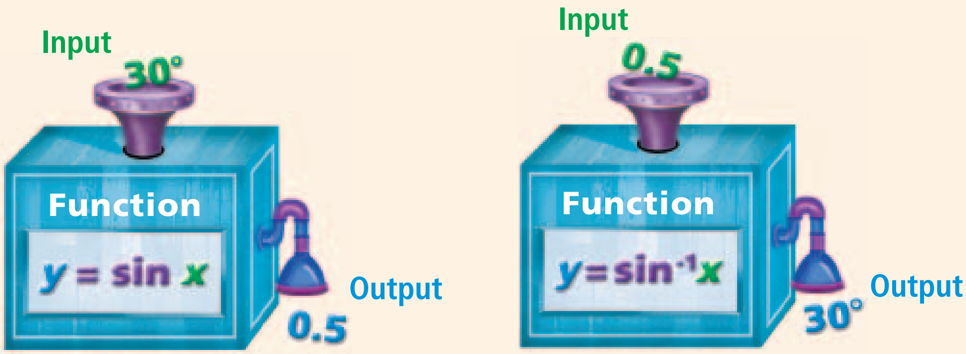
See Skills Bank page S62

California Standards

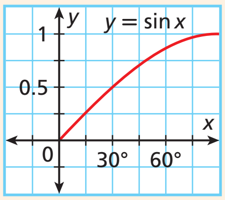
18.0 Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, $\tan(x) = \sin(x)/\cos(x)$, $(\sin(x))^2 + (\cos(x))^2 = 1$.

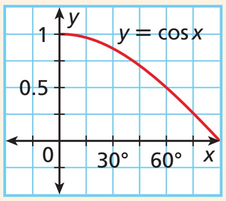
In Algebra, you learned that a function is a relation in which each element of the domain is paired with exactly one element of the range. If you switch the domain and range of a one-to-one function, you create an *inverse function*.

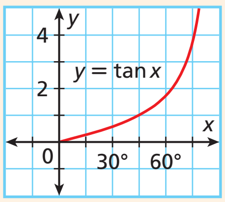
The function $y = \sin^{-1} x$ is the inverse of the function $y = \sin x$.



If you know the value of a trigonometric ratio, you can use the inverse trigonometric function to find the angle measure. You can do this either with a calculator or by looking at the graph of the function.







Example

Use the graphs above to find the value of x for $1 = \sin x$. Then write this expression using an inverse trigonometric function.

$1 = \sin x$ — *Look at the graph of $y = \sin x$. Find where the graph intersects the line $y = 1$ and read the corresponding x-coordinate.*

$x = 90°$

$90° = \sin^{-1}(1)$ — *Switch the x- and y-values.*

Try This

Use the graphs above to find the value of x for each of the following. Then write each expression using an inverse trigonometric function.

1. $0 = \sin x$

2. $\frac{1}{2} = \cos x$

3. $1 = \tan x$

4. $0 = \cos x$

5. $0 = \tan x$

6. $\frac{1}{2} = \sin x$

8-3 Solving Right Triangles

Objective
Use trigonometric ratios to find angle measures in right triangles and to solve real-world problems.

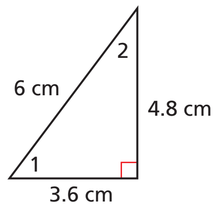
California Standards

19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.
Also covered: **15.0, 18.0**

Why learn this?
You can convert the percent grade of a road to an angle measure by solving a right triangle.

San Francisco, California, is famous for its steep streets. The steepness of a road is often expressed as a *percent grade.* Filbert Street, the steepest street in San Francisco, has a 31.5% grade. This means the road rises 31.5 ft over a horizontal distance of 100 ft, which is equivalent to a 17.5° angle. You can use trigonometric ratios to change a percent grade to an angle measure.

EXAMPLE 1 Identifying Angles from Trigonometric Ratios



Use the trigonometric ratio $\cos A = 0.6$ to determine which angle of the triangle is $\angle A$.

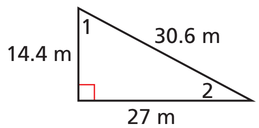
$\cos A = \dfrac{\text{adj. leg}}{\text{hyp.}}$ *Cosine is the ratio of the adjacent leg to the hypotenuse.*

$\cos\angle 1 = \dfrac{3.6}{6} = 0.6$ *The leg adjacent to $\angle 1$ is 3.6. The hypotenuse is 6.*

$\cos\angle 2 = \dfrac{4.8}{6} = 0.8$ *The leg adjacent to $\angle 2$ is 4.8. The hypotenuse is 6.*

Since $\cos A = \cos\angle 1$, $\angle 1$ is $\angle A$.

CHECK IT OUT! **Use the given trigonometric ratio to determine which angle of the triangle is $\angle A$.**



1a. $\sin A = \dfrac{8}{17}$ **1b.** $\tan A = 1.875$

In Lesson 8-2, you learned that $\sin 30° = 0.5$. Conversely, if you know that the sine of an acute angle is 0.5, you can conclude that the angle measures 30°. This is written as $\sin^{-1}(0.5) = 30°$.

Reading Math
The expression $\sin^{-1}x$ is read "the inverse sine of x." It does *not* mean $\frac{1}{\sin x}$. You can think of $\sin^{-1}x$ as "the angle whose sine is x."

If you know the sine, cosine, or tangent of an acute angle measure, you can use the inverse trigonometric functions to find the measure of the angle.

Inverse Trigonometric Functions
If $\sin A = x$, then $\sin^{-1}x = m\angle A$.
If $\cos A = x$, then $\cos^{-1}x = m\angle A$.
If $\tan A = x$, then $\tan^{-1}x = m\angle A$.

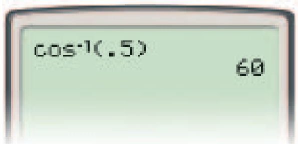
EXAMPLE 2 Calculating Angle Measures from Trigonometric Ratios

Helpful Hint

When using your calculator to find the value of an inverse trigonometric expression, you may need to press the [arc], [inv], or [2nd] key.

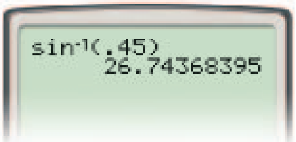
Use your calculator to find each angle measure to the nearest degree.

A $\cos^{-1}(0.5)$



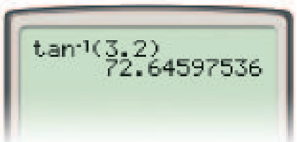
$\cos^{-1}(0.5) = 60°$

B $\sin^{-1}(0.45)$



$\sin^{-1}(0.45) \approx 27°$

C $\tan^{-1}(3.2)$



$\tan^{-1}(3.2) \approx 73°$

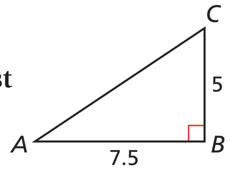
CHECK IT OUT! **Use your calculator to find each angle measure to the nearest degree.**

2a. $\tan^{-1}(0.75)$ **2b.** $\cos^{-1}(0.05)$ **2c.** $\sin^{-1}(0.67)$

Using given measures to find the unknown angle measures or side lengths of a triangle is known as *solving a triangle*. To solve a right triangle, you need to know two side lengths or one side length and an acute angle measure.

EXAMPLE 3 Solving Right Triangles

Find the unknown measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.



Method 1:

By the Pythagorean Theorem,

$AC^2 = AB^2 + BC^2.$

$= (7.5)^2 + 5^2 = 81.25$

So $AC = \sqrt{81.25} \approx 9.01.$

$m\angle A = \tan^{-1}\left(\frac{5}{7.5}\right) \approx 34°$

Since the acute angles of a right triangle are complementary, $m\angle C \approx 90° - 34° \approx 56°$.

Method 2:

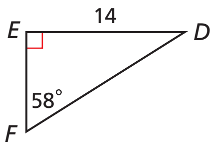
$m\angle A = \tan^{-1}\left(\frac{5}{7.5}\right) \approx 34°$

Since the acute angles of a right triangle are complementary, $m\angle C \approx 90° - 34° \approx 56°$.

$\sin A = \frac{5}{AC}$, so $AC = \frac{5}{\sin A}$.

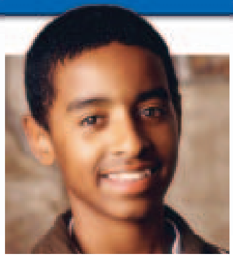
$$AC \approx \frac{5}{\sin\left[\tan^{-1}\left(\frac{5}{7.5}\right)\right]} \approx 9.01$$

CHECK IT OUT! **3.** Find the unknown measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.



Student to Student

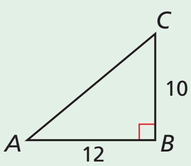
Solving Right Triangles



Kendell Waters
Marshall High School

Rounding can really make a difference! To find AC, I used the Pythagorean Theorem and got 15.62.

Then I did it a different way. I used $m\angle A = \tan^{-1}\left(\frac{10}{12}\right)$ to find $m\angle A = 39.8056°$, which I rounded to 40°. $\sin 40° = \frac{10}{AC}$, so $AC = \frac{10}{\sin 40°} \approx 15.56$.



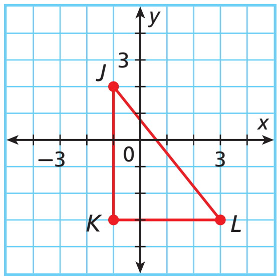
The difference in the two answers reminded me not to round values until the last step.

EXAMPLE 4 Solving a Right Triangle in the Coordinate Plane

The coordinates of the vertices of $\triangle JKL$ are $J(-1, 2)$, $K(-1, -3)$, and $L(3, -3)$. Find the side lengths to the nearest hundredth and the angle measures to the nearest degree.

Step 1 Find the side lengths.

Plot points J, K, and L.



$JK = 5 \qquad KL = 4$

By the Distance Formula,

$$JL = \sqrt{[3-(-1)]^2 + (-3-2)^2}.$$
$$= \sqrt{4^2 + (-5)^2}$$
$$= \sqrt{16 + 25} = \sqrt{41} \approx 6.40$$

Step 2 Find the angle measures.

$m\angle K = 90°$ — *$\overline{JK}$ and $\overline{KL}$ are $\perp$.*

$m\angle J = \tan^{-1}\left(\frac{4}{5}\right) \approx 39°$ — *$\overline{KL}$ is opp. $\angle J$, and $\overline{JK}$ is adj. to $\angle J$.*

$m\angle L \approx 90° - 39° \approx 51°$ — *The acute $\angle$s of a rt. $\triangle$ are comp.*

CHECK IT OUT!

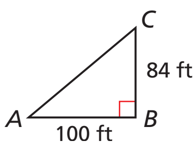
4. The coordinates of the vertices of $\triangle RST$ are $R(-3, 5)$, $S(4, 5)$, and $T(4, -2)$. Find the side lengths to the nearest hundredth and the angle measures to the nearest degree.

EXAMPLE 5 *Travel Application*

San Francisco's Lombard Street is known as one of "the crookedest streets in the world." The road's eight switchbacks were built in the 1920s to make the steep hill passable by cars. If the hill has a percent grade of 84%, what angle does the hill make with a horizontal line? Round to the nearest degree.

$84\% = \frac{84}{100}$ — *Change the percent grade to a fraction.*

An 84% grade means the hill rises 84 ft for every 100 ft of horizontal distance.



Draw a right triangle to represent the hill.

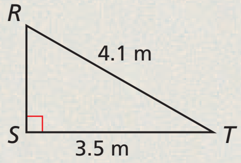
$\angle A$ is the angle the hill makes with a horizontal line.

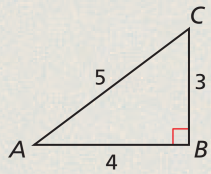
$m\angle A = \tan^{-1}\left(\frac{84}{100}\right) \approx 40°$

CHECK IT OUT!

5. Baldwin St. in Dunedin, New Zealand, is the steepest street in the world. It has a grade of 38%. To the nearest degree, what angle does Baldwin St. make with a horizontal line?

THINK AND DISCUSS





1. Describe the steps you would use to solve $\triangle RST$.

2. Given that $\cos Z = 0.35$, write an equivalent statement using an inverse trigonometric function.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write a trigonometric ratio for $\angle A$. Then write an equivalent statement using an inverse trigonometric function.

	Trigonometric Ratio	Inverse Trigonometric Function
Sine		
Cosine		
Tangent		

8-3 Exercises

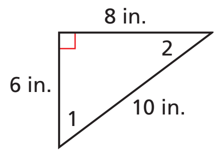
California Standards
5.0, 15.0, 18.0, 19.0, MG3.3, 1A2.0

go.hrw.com
Homework Help Online
KEYWORD: MG7 8-3
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 534

Use the given trigonometric ratio to determine which angle of the triangle is $\angle A$.



1. $\sin A = \frac{4}{5}$ **2.** $\tan A = 1\frac{1}{3}$ **3.** $\cos A = 0.6$

4. $\cos A = 0.8$ **5.** $\tan A = 0.75$ **6.** $\sin A = 0.6$

SEE EXAMPLE 2 p. 535

Use your calculator to find each angle measure to the nearest degree.

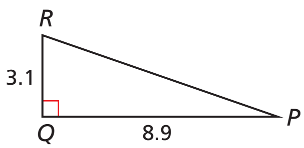
7. $\tan^{-1}(2.1)$ **8.** $\cos^{-1}\left(\frac{1}{3}\right)$ **9.** $\cos^{-1}\left(\frac{5}{6}\right)$

10. $\sin^{-1}(0.5)$ **11.** $\sin^{-1}(0.61)$ **12.** $\tan^{-1}(0.09)$

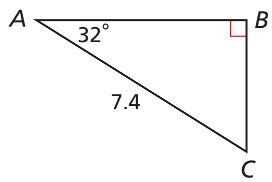
SEE EXAMPLE 3 p. 535

Multi-Step Find the unknown measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.

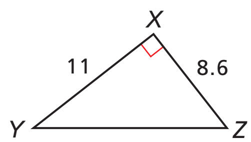
13.



14.



15.



SEE EXAMPLE 4 p. 536

Multi-Step For each triangle, find the side lengths to the nearest hundredth and the angle measures to the nearest degree.

16. $D(4, 1)$, $E(4, -2)$, $F(-2, -2)$ **17.** $R(3, 3)$, $S(-2, 3)$, $T(-2, -3)$

18. $X(4, -6)$, $Y(-3, 1)$, $Z(-3, -6)$ **19.** $A(-1, 1)$, $B(1, 1)$, $C(1, 5)$

SEE EXAMPLE 5
p. 536

20. **Cycling** A hill in the Tour de France bike race has a grade of 8%. To the nearest degree, what is the angle that this hill makes with a horizontal line?

PRACTICE AND PROBLEM SOLVING

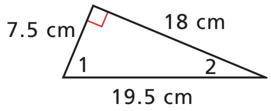
Independent Practice

For Exercises	See Example
21–26	1
27–32	2
33–35	3
36–37	4
38	5

Extra Practice

Skills Practice p. S18
Application Practice p. S35

Use the given trigonometric ratio to determine which angle of the triangle is $\angle A$.



21. $\tan A = \frac{5}{12}$ **22.** $\tan A = 2.4$ **23.** $\sin A = \frac{12}{13}$

24. $\sin A = \frac{5}{13}$ **25.** $\cos A = \frac{12}{13}$ **26.** $\cos A = \frac{5}{13}$

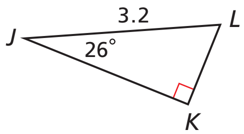
Use your calculator to find each angle measure to the nearest degree.

27. $\sin^{-1}(0.31)$ **28.** $\tan^{-1}(1)$ **29.** $\cos^{-1}(0.8)$

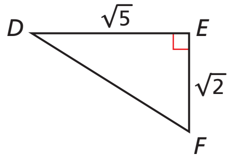
30. $\cos^{-1}(0.72)$ **31.** $\tan^{-1}(1.55)$ **32.** $\sin^{-1}\left(\frac{9}{17}\right)$

Multi-Step **Find the unknown measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.**

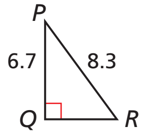
33.



34.



35.



Multi-Step **For each triangle, find the side lengths to the nearest hundredth and the angle measures to the nearest degree.**

36. $A(2, 0), B(2, -5), C(1, -5)$ **37.** $M(3, 2), N(3, -2), P(-1, -2)$

38. **Building** For maximum accessibility, a wheelchair ramp should have a slope between $\frac{1}{16}$ and $\frac{1}{20}$. What is the range of angle measures that a ramp should make with a horizontal line? Round to the nearest degree.

Complete each statement. If necessary, round angle measures to the nearest degree. Round other values to the nearest hundredth.

39. tan $\underline{\quad ? \quad} \approx 3.5$ **40.** sin $\underline{\quad ? \quad} \approx \frac{2}{3}$ **41.** $\underline{\quad ? \quad}$ $42° \approx 0.74$

42. $\cos^{-1}(\underline{\quad ? \quad}) \approx 12°$ **43.** $\sin^{-1}(\underline{\quad ? \quad}) \approx 69°$ **44.** $\underline{\quad ? \quad}$ $60° = \frac{1}{2}$

45. **Critical Thinking** Use trigonometric ratios to explain why the diagonal of a square forms a 45° angle with each of the sides.

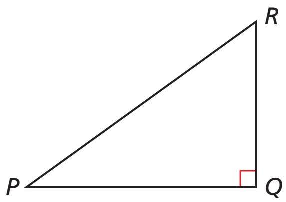
46. **Estimation** You can use trigonometry to find angle measures when a protractor is not available.

a. Estimate the measure of $\angle P$.

b. Use a centimeter ruler to find RQ and PQ.

c. Use your measurements from part **b** and an inverse trigonometric function to find $m\angle P$ to the nearest degree.

d. How does your result in part **c** compare to your estimate in part **a**?



CONCEPT CONNECTION

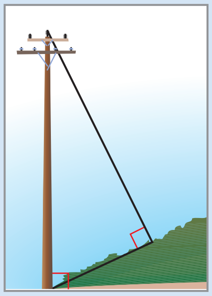
47. This problem will prepare you for the Concept Connection on page 542.

An electric company wants to install a vertical utility pole at the base of a hill that has an 8% grade.

a. To the nearest degree, what angle does the hill make with a horizontal line?

b. What is the measure of the angle between the pole and the hill? Round to the nearest degree.

c. A utility worker installs a 31-foot guy wire from the top of the pole to the hill. Given that the guy wire is perpendicular to the hill, find the height of the pole to the nearest inch.



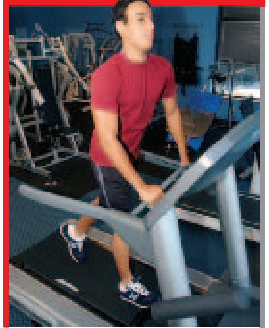
The side lengths of a right triangle are given below. Find the measures of the acute angles in the triangle. Round to the nearest degree.

48. 3, 4, 5 **49.** 5, 12, 13 **50.** 8, 15, 17

51. **What if...?** A right triangle has leg lengths of 28 and 45 inches. Suppose the length of the longer leg doubles. What happens to the measure of the acute angle opposite that leg?

LINK

Fitness



Running on a treadmill is slightly easier than running outdoors, since you don't have to overcome wind resistance. Set the treadmill to a 1% grade to match the intensity of an outdoor run.

52. **Fitness** As part of off-season training, the Houston Texans football team must sprint up a ramp with a 28% grade. To the nearest degree, what angle does this ramp make with a horizontal line?

53. The coordinates of the vertices of a triangle are $A(-1, 0)$, $B(6, 1)$, and $C(0, 3)$.

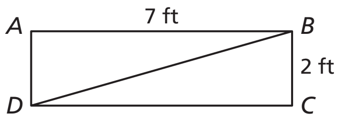
a. Use the Distance Formula to find AB, BC, and AC.

b. Use the Converse of the Pythagorean Theorem to show that $\triangle ABC$ is a right triangle. Identify the right angle.

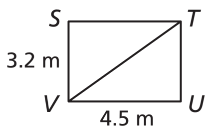
c. Find the measures of the acute angles of $\triangle ABC$. Round to the nearest degree.

Find the indicated measure in each rectangle. Round to the nearest degree.

54. $m\angle BDC$

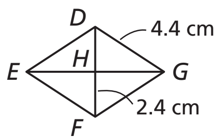


55. $m\angle STV$

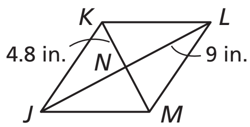


Find the indicated measure in each rhombus. Round to the nearest degree.

56. $m\angle DGF$

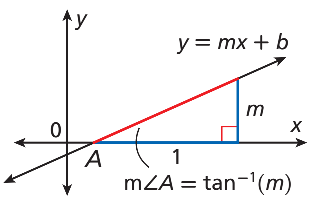


57. $m\angle LKN$



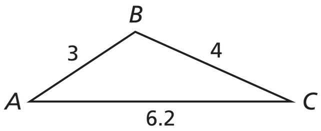
58. **Critical Thinking** Without using a calculator, compare the values of $\tan 60°$ and $\tan 70°$. Explain your reasoning.

The measure of an acute angle formed by a line with slope m and the x-axis can be found by using the expression $\tan^{-1}(m)$. Find the measure of the acute angle that each line makes with the x-axis. Round to the nearest degree.



59. $y = 3x + 5$ **60.** $y = \frac{2}{3}x + 1$ **61.** $5y = 4x + 3$

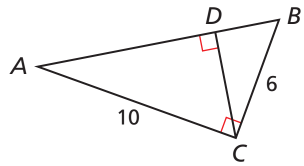
62. **ERROR ANALYSIS** A student was asked to find m$\angle C$. Explain the error in the student's solution.



> Since $\tan C = \frac{3}{4}$, m$\angle C = \tan^{-1}\left(\frac{3}{4}\right)$, and $\tan^{-1}(0.75) \approx 37^\circ$. So m$\angle C \approx 37^\circ$.

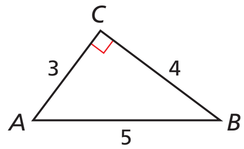
63. **Write About It** A student claims that you must know the three side lengths of a right triangle before you can use trigonometric ratios to find the measures of the acute angles. Do you agree? Why or why not?

64. $\overline{DC}$ is an altitude of right $\triangle ABC$. Use trigonometric ratios to find the missing lengths in the figure. Then use these lengths to verify the three relationships in the Geometric Mean Corollaries from Lesson 8-1.



STANDARDIZED TEST PREP

65. Which expression can be used to find m$\angle A$?



(A) $\tan^{-1}(0.75)$ (C) $\cos^{-1}(0.8)$

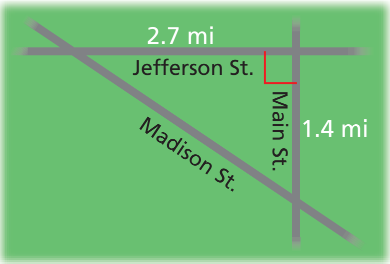
(B) $\sin^{-1}\left(\frac{3}{5}\right)$ (D) $\tan^{-1}\left(\frac{4}{3}\right)$

66. Which expression is NOT equivalent to $\cos 60^\circ$?

(F) $\frac{1}{2}$ (H) $\frac{\sin 60^\circ}{\tan 60^\circ}$

(G) $\sin 30^\circ$ (J) $\cos^{-1}\left(\frac{1}{2}\right)$

67. To the nearest degree, what is the measure of the acute angle formed by Jefferson St. and Madison St.?



(A) 27° (C) 59°

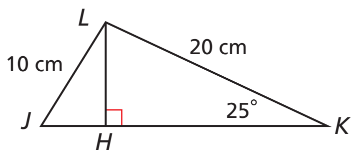
(B) 31° (D) 63°

68. **Gridded Response** A highway exit ramp has a slope of $\frac{3}{20}$. To the nearest degree, find the angle that the ramp makes with a horizontal line.

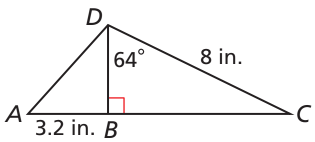
CHALLENGE AND EXTEND

Find each angle measure. Round to the nearest degree.

69. m$\angle J$



70. m$\angle A$



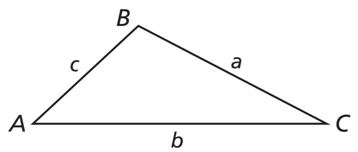
Simply each expression.

71. $\cos^{-1}(\cos 34^\circ)$ **72.** $\tan\left[\tan^{-1}(1.5)\right]$ **73.** $\sin\left(\sin^{-1} x\right)$

74. A ramp has a 6% grade. The ramp is 40 ft long. Find the vertical distance that the ramp rises. Round your answer to the nearest hundredth.

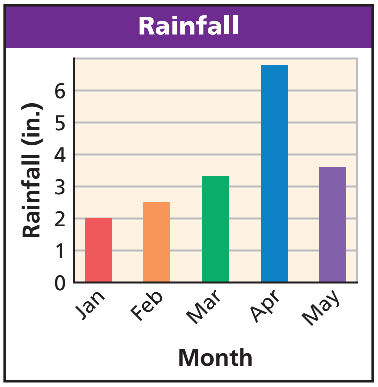
75. **Critical Thinking** Explain why the expression $\sin^{-1}(1.5)$ does not make sense.

76. If you are given the lengths of two sides of $\triangle ABC$ and the measure of the included angle, you can use the formula $\frac{1}{2}bc\sin A$ to find the area of the triangle. Derive this formula. (*Hint:* Draw an altitude from B to $\overline{AC}$. Use trigonometric ratios to find the length of this altitude.)



SPIRAL REVIEW

The graph shows the amount of rainfall in a city for the first five months of the year. Determine whether each statement is true or false. *(Previous course)*

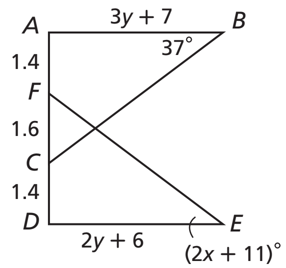


77. It rained more in April than it did in January, February, and March combined.

78. The average monthly rainfall for this five-month period was approximately 3.5 inches.

79. The rainfall amount increased at a constant rate each month over the five-month period.

Use the diagram to find each value, given that $\triangle ABC \cong \triangle DEF$. *(Lesson 4-3)*



80. x **81.** y **82.** DF

Use your calculator to find each trigonometric ratio. Round to the nearest hundredth. *(Lesson 8-2)*

83. $\sin 63°$ **84.** $\cos 27°$ **85.** $\tan 64°$

Using Technology

Use a spreadsheet to complete the following.

	A	B	C	D	E
1	a	b	c	m(angle A)	m(angle B)
2					

= SQRT(A2^2 + B2^2) = DEGREES(ATAN(A2/B2)) = DEGREES(ATAN(B2/A2))

1. In cells A2 and B2, enter values for the leg lengths of a right triangle.

2. In cell C2, write a formula to calculate c, the length of the hypotenuse.

3. Write a formula to calculate the measure of $\angle A$ in cell D2. Be sure to use the Degrees function so that the answer is given in degrees. Format the value to include no decimal places.

4. Write a formula to calculate the measure of $\angle B$ in cell E2. Again, be sure to use the Degrees function and format the value to include no decimal places.

5. Use your spreadsheet to check your answers for Exercises 48–50.

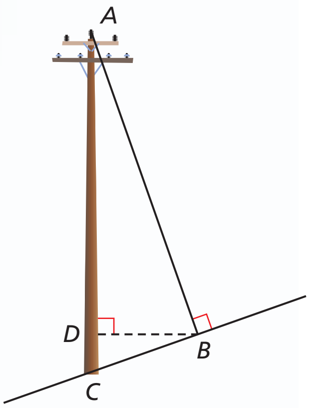
CHAPTER 8

SECTION 8A

CONCEPT CONNECTION

Trigonometric Ratios

It's Electrifying! Utility workers install and repair the utility poles and wires that carry electricity from generating stations to consumers. As shown in the figure, a crew of workers plans to install a vertical utility pole $\overline{AC}$ and a supporting guy wire $\overline{AB}$ that is perpendicular to the ground.



1. The utility pole is 30 ft tall. The crew finds that $DC = 6$ ft. What is the distance DB from the pole to the anchor point of the guy wire?
2. How long is the guy wire? Round to the nearest inch.
3. In the figure, $\angle ABD$ is called the *line angle*. In order to choose the correct weight of the cable for the guy wire, the crew needs to know the measure of the line angle. Find $m\angle ABD$ to the nearest degree.
4. To the nearest degree, what is the measure of the angle formed by the pole and the guy wire?
5. What is the percent grade of the hill on which the crew is working?

READY TO GO ON?

CHAPTER 8

SECTION 8A

Quiz for Lessons 8-1 Through 8-3

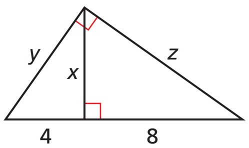
8-1 Similarity in Right Triangles

Find the geometric mean of each pair of numbers. If necessary, give the answer in simplest radical form.

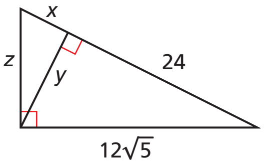
1. 5 and 12 **2.** 2.75 and 44 **3.** $\frac{5}{2}$ and $\frac{15}{8}$

Find *x*, *y*, and *z*.

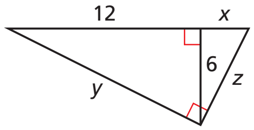
4.



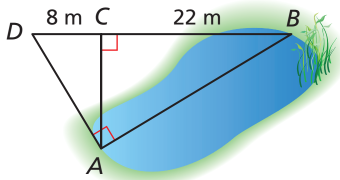
5.



6.



7. A land developer needs to know the distance across a pond on a piece of property. What is *AB* to the nearest tenth of a meter?



8-2 Trigonometric Ratios

Use a special right triangle to write each trigonometric ratio as a fraction.

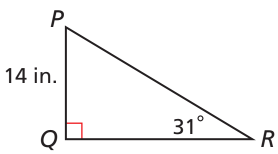
8. $\tan 45°$ **9.** $\sin 30°$ **10.** $\cos 30°$

Use your calculator to find each trigonometric ratio. Round to the nearest hundredth.

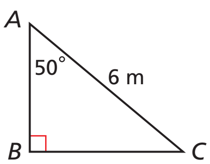
11. $\sin 16°$ **12.** $\cos 79°$ **13.** $\tan 27°$

Find each length. Round to the nearest hundredth.

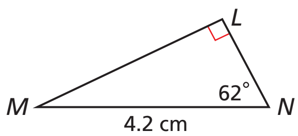
14. *QR*



15. *AB*



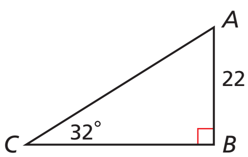
16. *LM*



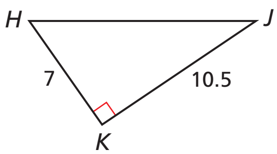
8-3 Solving Right Triangles

Find the unknown measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.

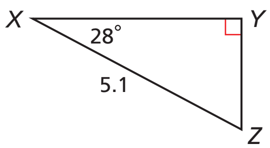
17.



18.



19.



20. The wheelchair ramp at the entrance of the Mission Bay Library has a slope of $\frac{1}{18}$. What angle does the ramp make with the sidewalk? Round to the nearest degree.

8-4 Angles of Elevation and Depression

Objective
Solve problems involving angles of elevation and angles of depression.

Vocabulary
angle of elevation
angle of depression

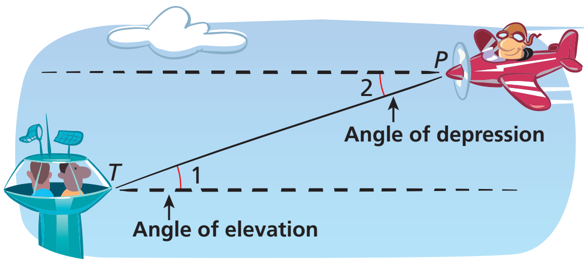
California Standards

19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.

Who uses this?
Pilots and air traffic controllers use angles of depression to calculate distances.

An **angle of elevation** is the angle formed by a horizontal line and a line of sight to a point *above* the line. In the diagram, ∠1 is the angle of elevation from the tower T to the plane P.

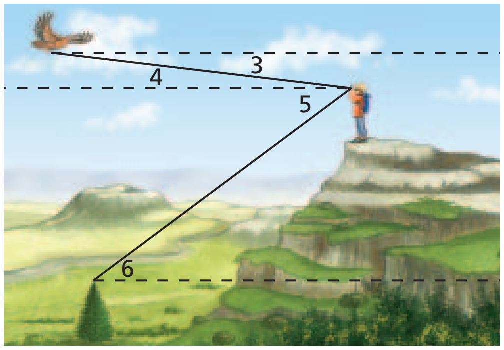
An **angle of depression** is the angle formed by a horizontal line and a line of sight to a point *below* the line. ∠2 is the angle of depression from the plane to the tower.



Since horizontal lines are parallel, ∠1 ≅ ∠2 by the Alternate Interior Angles Theorem. Therefore the angle of elevation from one point is congruent to the angle of depression from the other point.

EXAMPLE 1 Classifying Angles of Elevation and Depression

Classify each angle as an angle of elevation or angle of depression.



A ∠3

∠3 is formed by a horizontal line and a line of sight to a point below the line. It is an angle of depression.

B ∠4

∠4 is formed by a horizontal line and a line of sight to a point above the line. It is an angle of elevation.

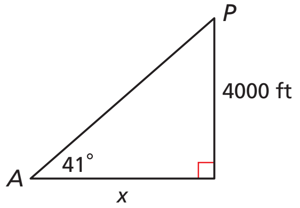
CHECK IT OUT! **Use the diagram above to classify each angle as an angle of elevation or angle of depression.**

1a. ∠5 **1b.** ∠6

EXAMPLE 2 Finding Distance by Using Angle of Elevation

An air traffic controller at an airport sights a plane at an angle of elevation of 41°. The pilot reports that the plane's altitude is 4000 ft. What is the horizontal distance between the plane and the airport? Round to the nearest foot.

Draw a sketch to represent the given information. Let A represent the airport and let P represent the plane. Let x be the horizontal distance between the plane and the airport.



$\tan 41° = \dfrac{4000}{x}$ *You are given the side opposite ∠A, and x is the side adjacent to ∠A. So write a tangent ratio.*

$x = \dfrac{4000}{\tan 41°}$ *Multiply both sides by x and divide both sides by tan 41°.*

$x \approx 4601$ ft *Simplify the expression.*

CHECK IT OUT!

2. **What if...?** Suppose the plane is at an altitude of 3500 ft and the angle of elevation from the airport to the plane is 29°. What is the horizontal distance between the plane and the airport? Round to the nearest foot.

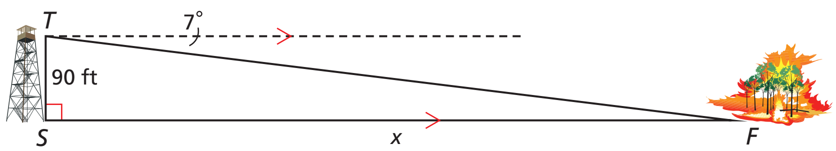
EXAMPLE 3 Finding Distance by Using Angle of Depression

A forest ranger in a 90-foot observation tower sees a fire. The angle of depression to the fire is 7°. What is the horizontal distance between the tower and the fire? Round to the nearest foot.

Caution!

The angle of depression may not be one of the angles in the triangle you are solving. It may be the complement of one of the angles in the triangle.

Draw a sketch to represent the given information. Let T represent the top of the tower and let F represent the fire. Let x be the horizontal distance between the tower and the fire.



By the Alternate Interior Angles Theorem, $m\angle F = 7°$.

$\tan 7° = \dfrac{90}{x}$ *Write a tangent ratio.*

$x = \dfrac{90}{\tan 7°}$ *Multiply both sides by x and divide both sides by tan 7°.*

$x \approx 733$ ft *Simplify the expression.*

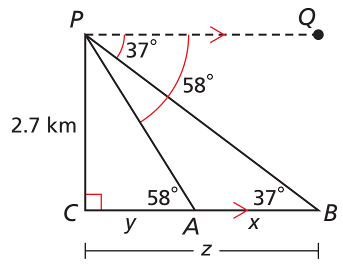
CHECK IT OUT!

3. **What if...?** Suppose the ranger sees another fire and the angle of depression to the fire is 3°. What is the horizontal distance to this fire? Round to the nearest foot.

EXAMPLE 4 Aviation Application

A pilot flying at an altitude of 2.7 km sights two control towers directly in front of her. The angle of depression to the base of one tower is 37°. The angle of depression to the base of the other tower is 58°. What is the distance between the two towers? Round to the nearest tenth of a kilometer.

Step 1 Draw a sketch. Let P represent the plane and let A and B represent the two towers. Let x be the distance between the towers.



Step 2 Find y.

By the Alternate Interior Angles Theorem, $m\angle CAP = 58°$.

In $\triangle APC$, $\tan 58° = \frac{2.7}{y}$.

So $y = \frac{2.7}{\tan 58°} \approx 1.6871$ km.

Step 3 Find z.

By the Alternate Interior Angles Theorem, $m\angle CBP = 37°$.

In $\triangle BPC$, $\tan 37° = \frac{2.7}{z}$.

So $z = \frac{2.7}{\tan 37°} \approx 3.5830$ km.

Step 4 Find x.

$x = z - y$

$x \approx 3.5830 - 1.6871 \approx 1.9$ km

So the two towers are about 1.9 km apart.

Helpful Hint

Always make a sketch to help you correctly place the given angle measure.

CHECK IT OUT!

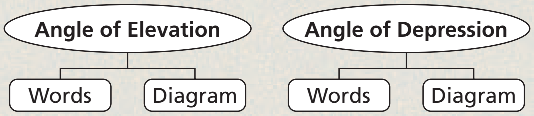
4. A pilot flying at an altitude of 12,000 ft sights two airports directly in front of him. The angle of depression to one airport is 78°, and the angle of depression to the second airport is 19°. What is the distance between the two airports? Round to the nearest foot.

THINK AND DISCUSS

1. Explain what happens to the angle of elevation from your eye to the top of a skyscraper as you walk toward the skyscraper.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer below. In each box, write a definition or make a sketch.



8-4 Exercises

California Standards 7.0, 12.0, 19.0, 7NS1.7

go.hrw.com
Homework Help Online
KEYWORD: MG7 8-4
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

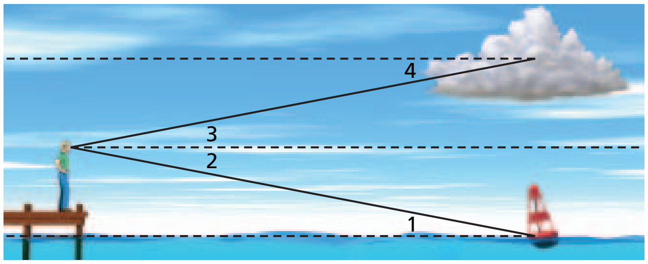
1. An angle of __?__ is measured from a horizontal line to a point above that line. (*elevation* or *depression*)

2. An angle of __?__ is measured from a horizontal line to a point below that line. (*elevation* or *depression*)

SEE EXAMPLE 1 p. 544

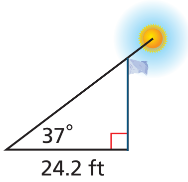
Classify each angle as an angle of elevation or angle of depression.

3. ∠1
4. ∠2
5. ∠3
6. ∠4



SEE EXAMPLE 2 p. 545

7. **Measurement** When the angle of elevation to the sun is 37°, a flagpole casts a shadow that is 24.2 ft long. What is the height of the flagpole to the nearest foot?

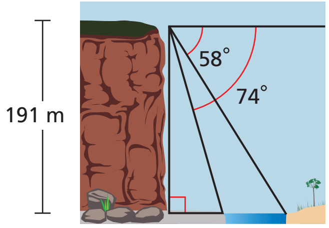


SEE EXAMPLE 3 p. 545

8. **Aviation** The pilot of a traffic helicopter sights an accident at an angle of depression of 18°. The helicopter's altitude is 1560 ft. What is the horizontal distance from the helicopter to the accident? Round to the nearest foot.

SEE EXAMPLE 4 p. 546

9. **Surveying** From the top of a canyon, the angle of depression to the far side of the river is 58°, and the angle of depression to the near side of the river is 74°. The depth of the canyon is 191 m. What is the width of the river at the bottom of the canyon? Round to the nearest tenth of a meter.



PRACTICE AND PROBLEM SOLVING

Independent Practice

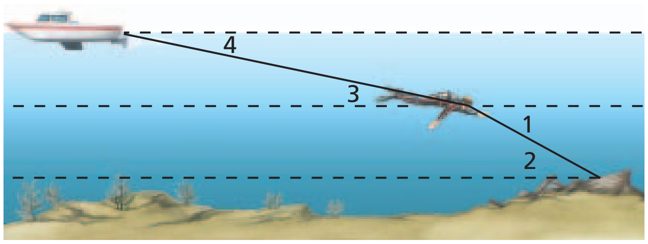
For Exercises	See Example
10–13	1
14	2
15	3
16	4

Extra Practice

Skills Practice p. S19
Application Practice p. S35

Classify each angle as an angle of elevation or angle of depression.

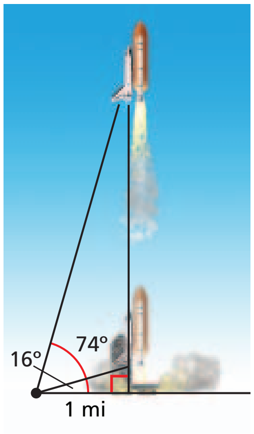
10. ∠1
11. ∠2
12. ∠3
13. ∠4



14. **Geology** To measure the height of a rock formation, a surveyor places her transit 100 m from its base and focuses the transit on the top of the formation. The angle of elevation is 67°. The transit is 1.5 m above the ground. What is the height of the rock formation? Round to the nearest meter.

15. **Forestry** A forest ranger in a 120 ft observation tower sees a fire. The angle of depression to the fire is 3.5°. What is the horizontal distance between the tower and the fire? Round to the nearest foot.

16. **Space Shuttle** Marion is observing the launch of a space shuttle from the command center. When she first sees the shuttle, the angle of elevation to it is 16°. Later, the angle of elevation is 74°. If the command center is 1 mi from the launch pad, how far did the shuttle travel while Marion was watching? Round to the nearest tenth of a mile.



LINK

Space Shuttle

During its launch, a space shuttle accelerates to more than 27,359 km/h in just over 8 minutes. So the shuttle travels 3219 km/h faster each minute.

Tell whether each statement is true or false. If false, explain why.

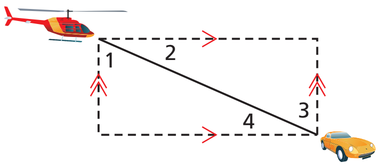
17. The angle of elevation from your eye to the top of a tree increases as you walk toward the tree.

18. If you stand at street level, the angle of elevation to a building's tenth-story window is greater than the angle of elevation to one of its ninth-story windows.

19. As you watch a plane fly above you, the angle of elevation to the plane gets closer to 0° as the plane approaches the point directly overhead.

20. An angle of depression can never be more than 90°.

Use the diagram for Exercises 21 and 22.

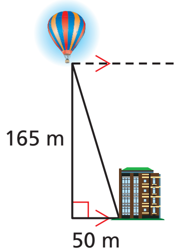


21. Which angles are not angles of elevation or angles of depression?

22. The angle of depression from the helicopter to the car is 30°. Find m∠1, m∠2, m∠3, and m∠4.

23. **Critical Thinking** Describe a situation in which the angle of depression to an object is decreasing.

24. An observer in a hot-air balloon sights a building that is 50 m from the balloon's launch point. The balloon has risen 165 m. What is the angle of depression from the balloon to the building? Round to the nearest degree.



25. **Multi-Step** A surveyor finds that the angle of elevation to the top of a 1000 ft tower is 67°.
    a. To the nearest foot, how far is the surveyor from the base of the tower?
    b. How far back would the surveyor have to move so that the angle of elevation to the top of the tower is 55°? Round to the nearest foot.

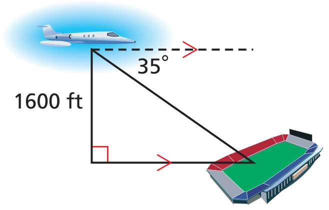
26. **Write About It** Two students are using shadows to calculate the height of a pole. One says that it will be easier if they wait until the angle of elevation to the sun is exactly 45°. Explain why the student made this suggestion.

CONCEPT CONNECTION

27. This problem will prepare you for the Concept Connection on page 568.
    The pilot of a rescue helicopter is flying over the ocean at an altitude of 1250 ft. The pilot sees a life raft at an angle of depression of 31°.
    a. What is the horizontal distance from the helicopter to the life raft, rounded to the nearest foot?
    b. The helicopter travels at 150 ft/s. To the nearest second, how long will it take until the helicopter is directly over the raft?

STANDARDIZED TEST PREP

28. Mai is flying a plane at an altitude of 1600 ft. She sights a stadium at an angle of depression of 35°. What is Mai's approximate horizontal distance from the stadium?



Ⓐ 676 feet Ⓒ 1450 feet

Ⓑ 1120 feet Ⓓ 2285 feet

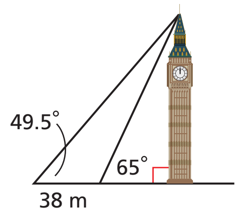
29. Jeff finds that an office building casts a shadow that is 93 ft long when the angle of elevation to the sun is 60°. What is the height of the building?

Ⓕ 54 feet Ⓖ 81 feet Ⓗ 107 feet Ⓙ 161 feet

30. **Short Response** Jim is rafting down a river that runs through a canyon. He sees a trail marker ahead at the top of the canyon and estimates the angle of elevation from the raft to the marker as 45°. Draw a sketch to represent the situation. Explain what happens to the angle of elevation as Jim moves closer to the marker.

CHALLENGE AND EXTEND

31. Susan and Jorge stand 38 m apart. From Susan's position, the angle of elevation to the top of Big Ben is 65°. From Jorge's position, the angle of elevation to the top of Big Ben is 49.5°. To the nearest meter, how tall is Big Ben?



32. A plane is flying at a constant altitude of 14,000 ft and a constant speed of 500 mi/h. The angle of depression from the plane to a lake is 6°. To the nearest minute, how much time will pass before the plane is directly over the lake?

33. A skyscraper stands between two school buildings. The two schools are 10 mi apart. From school A, the angle of elevation to the top of the skyscraper is 5°. From school B, the angle of elevation is 2°. What is the height of the skyscraper to the nearest foot?

34. Katie and Kim are attending a theater performance. Katie's seat is at floor level. She looks down at an angle of 18° to see the orchestra pit. Kim's seat is in the balcony directly above Katie. Kim looks down at an angle of 42° to see the pit. The horizontal distance from Katie's seat to the pit is 46 ft. What is the vertical distance between Katie's seat and Kim's seat? Round to the nearest inch.

SPIRAL REVIEW

35. Emma and her mother jog along a mile-long circular path in opposite directions. They begin at the same place and time. Emma jogs at a pace of 4 mi/h, and her mother runs at 6 mi/h. In how many minutes will they meet? *(Previous course)*

36. Greg bought a shirt that was discounted 30%. He used a coupon for an additional 15% discount. What was the original price of the shirt if Greg paid $17.85? *(Previous course)*

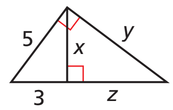
Tell which special parallelograms have each given property. *(Lesson 6-5)*

37. The diagonals are perpendicular. **38.** The diagonals are congruent.

39. The diagonals bisect each other. **40.** Opposite angles are congruent.

Find each length. *(Lesson 8-1)*

41. x **42.** y **43.** z



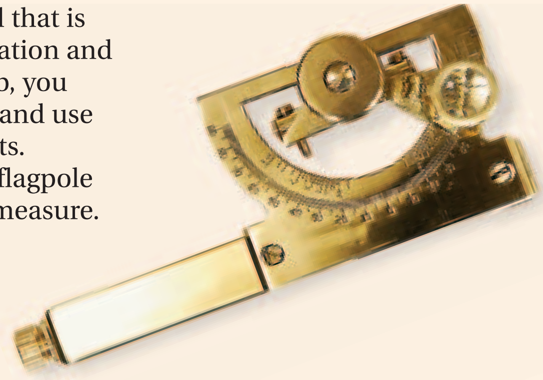
8-4

Geometry LAB

Use with Lesson 8-4

Indirect Measurement Using Trigonometry

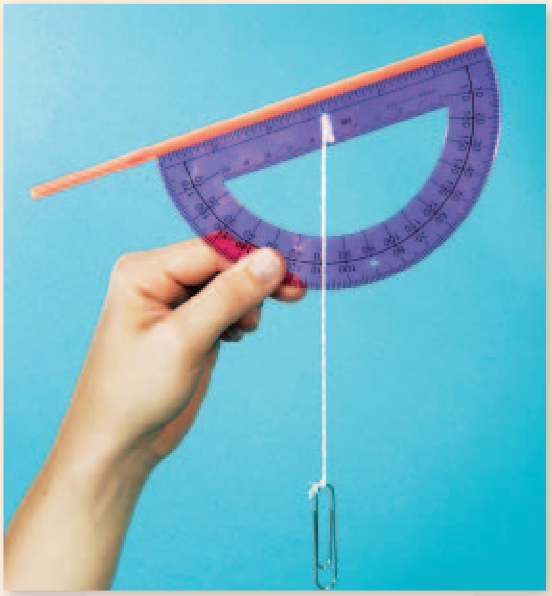
A *clinometer* is a surveying tool that is used to measure angles of elevation and angles of depression. In this lab, you will make a simple clinometer and use it to find indirect measurements. Choose a tall object, such as a flagpole or tree, whose height you will measure.



California Standards

19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.

Activity



1. Follow these instructions to make a clinometer.
   a. Tie a washer or paper clip to the end of a 6-inch string.
   b. Tape the string's other end to the midpoint of the straight edge of a protractor.
   c. Tape a straw along the straight edge of the protractor.

2. Stand back from the object you want to measure. Use a tape measure to measure and record the distance from your feet to the base of the object. Also measure the height of your eyes above the ground.

3. Hold the clinometer steady and look through the straw to sight the top of the object you are measuring. When the string stops moving, pinch it against the protractor and record the acute angle measure.

Try This

1. How is the angle reading from the clinometer related to the angle of elevation from your eye to the top of the object you are measuring?
2. Draw and label a diagram showing the object and the measurements you made. Then use trigonometric ratios to find the height of the object.
3. Repeat the activity, measuring the angle of elevation to the object from a different distance. How does your result compare to the previous one?
4. Describe possible measurement errors that can be made in the activity.
5. Explain why this method of indirect measurement is useful in real-world situations.

8-5 Law of Sines and Law of Cosines

Objective
Use the Law of Sines and the Law of Cosines to solve triangles.

California Standards

19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.

Who uses this?
Engineers can use the Law of Sines and the Law of Cosines to solve construction problems.

Since its completion in 1370, engineers have proposed many solutions for lessening the tilt of the Leaning Tower of Pisa. The tower does not form a right angle with the ground, so the engineers have to work with triangles that are not right triangles.

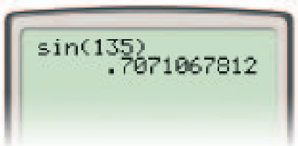
In this lesson, you will learn to solve *any* triangle. To do so, you will need to calculate trigonometric ratios for angle measures up to 180°. You can use a calculator to find these values.

EXAMPLE 1 Finding Trigonometric Ratios for Obtuse Angles

Helpful Hint
You will learn more about trigonometric ratios of angle measures greater than or equal to 90° in the Chapter Extension.

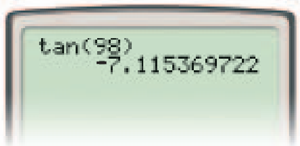
Use a calculator to find each trigonometric ratio. Round to the nearest hundredth.

A $\sin 135°$



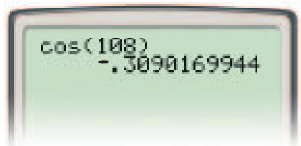
$\sin 135° \approx 0.71$

B $\tan 98°$



$\tan 98° \approx -7.12$

C $\cos 108°$



$\cos 108° \approx -0.31$

CHECK IT OUT! **Use a calculator to find each trigonometric ratio. Round to the nearest hundredth.**

1a. $\tan 175°$ **1b.** $\cos 92°$ **1c.** $\sin 160°$

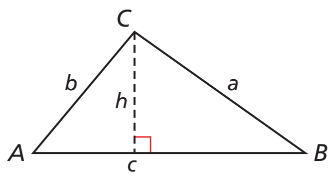
You can use the altitude of a triangle to find a relationship between the triangle's side lengths.

In $\triangle ABC$, let h represent the length of the altitude from C to $\overline{AB}$.

From the diagram, $\sin A = \frac{h}{b}$, and $\sin B = \frac{h}{a}$.

By solving for h, you find that $h = b \sin A$ and $h = a \sin B$. So $b \sin A = a \sin B$, and $\frac{\sin A}{a} = \frac{\sin B}{b}$.

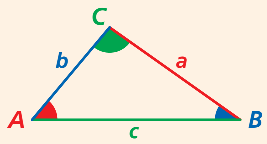
You can use another altitude to show that these ratios equal $\frac{\sin C}{c}$.



Theorem 8-5-1 The Law of Sines

For any $\triangle ABC$ with side lengths a, b, and c,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$



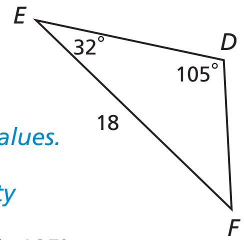
You can use the Law of Sines to solve a triangle if you are given

- two angle measures and any side length (ASA or AAS) or
- two side lengths and a non-included angle measure (SSA).

EXAMPLE 2 Using the Law of Sines

Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

A DF



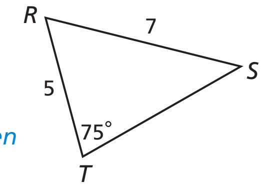
$\dfrac{\sin D}{EF} = \dfrac{\sin E}{DF}$ — *Law of Sines*

$\dfrac{\sin 105°}{18} = \dfrac{\sin 32°}{DF}$ — *Substitute the given values.*

$DF \sin 105° = 18 \sin 32°$ — *Cross Products Property*

$DF = \dfrac{18 \sin 32°}{\sin 105°} \approx 9.9$ — *Divide both sides by sin 105°.*

B $m\angle S$



$\dfrac{\sin T}{RS} = \dfrac{\sin S}{RT}$ — *Law of Sines*

$\dfrac{\sin 75°}{7} = \dfrac{\sin S}{5}$ — *Substitute the given values.*

$\sin S = \dfrac{5 \sin 75°}{7}$ — *Multiply both sides by 5.*

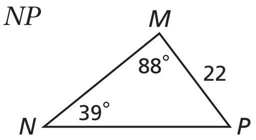
$m\angle S \approx \sin^{-1}\left(\dfrac{5 \sin 75°}{7}\right) \approx 44°$ — *Use the inverse sine function to find m∠S.*

Remember!

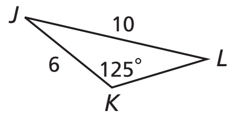
In a proportion with three parts, you can use any of the two parts together.

CHECK IT OUT! **Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.**

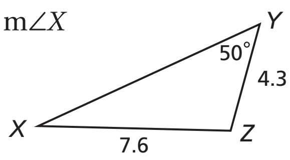
2a. NP



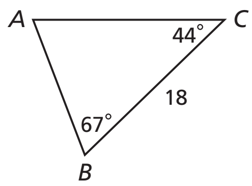
2b. $m\angle L$



2c. $m\angle X$



2d. AC



The Law of Sines cannot be used to solve every triangle. If you know two side lengths and the included angle measure or if you know all three side lengths, you cannot use the Law of Sines. Instead, you can apply the Law of Cosines.

Know it! Note

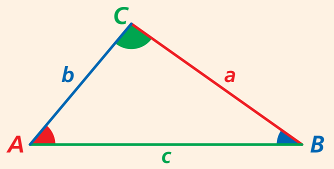
Theorem 8-5-2 The Law of Cosines

For any $\triangle ABC$ with side lengths a, b, and c:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



You will prove one case of the Law of Cosines in Exercise 57.

Helpful Hint

The angle referenced in the Law of Cosines is across the equal sign from its corresponding side.

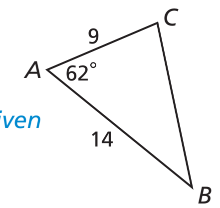
You can use the Law of Cosines to solve a triangle if you are given

- two side lengths and the included angle measure (SAS) or
- three side lengths (SSS).

EXAMPLE 3 Using the Law of Cosines

Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

A BC



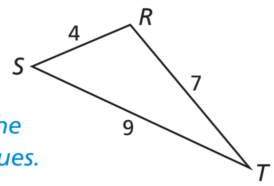
$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos A$ *Law of Cosines*

$= 14^2 + 9^2 - 2(14)(9)\cos 62°$ *Substitute the given values.*

$BC^2 \approx 158.6932$ *Simplify.*

$BC \approx 12.6$ *Find the square root of both sides.*

B $m\angle R$



$ST^2 = RS^2 + RT^2 - 2(RS)(RT)\cos R$ *Law of Cosines*

$9^2 = 4^2 + 7^2 - 2(4)(7)\cos R$ *Substitute the given values.*

$81 = 65 - 56\cos R$ *Simplify.*

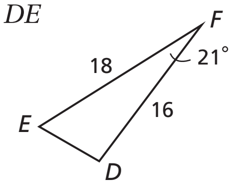
$16 = -56\cos R$ *Subtract 65 from both sides.*

$\cos R = -\dfrac{16}{56}$ *Solve for cos R.*

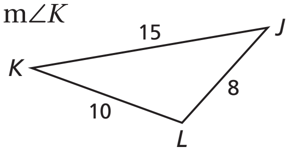
$m\angle R = \cos^{-1}\left(-\dfrac{16}{56}\right) \approx 107°$ *Use the inverse cosine function to find m∠R.*

CHECK IT OUT! **Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.**

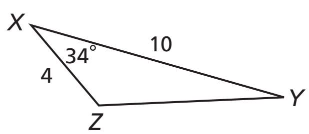
3a. DE



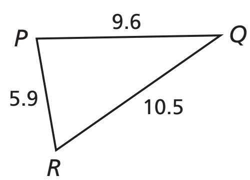
3b. $m\angle K$



3c. YZ

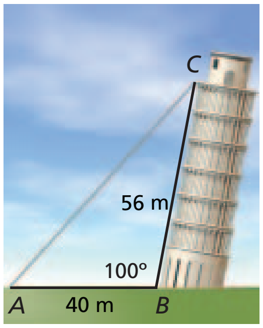


3d. $m\angle R$



EXAMPLE 4 Engineering Application

The Leaning Tower of Pisa is 56 m tall. In 1999, the tower made a 100° angle with the ground. To stabilize the tower, an engineer considered attaching a cable from the top of the tower to a point that is 40 m from the base. How long would the cable be, and what angle would it make with the ground? Round the length to the nearest tenth and the angle measure to the nearest degree.



Step 1 Find the length of the cable.

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos B \quad \textit{Law of Cosines}$$

$$= 40^2 + 56^2 - 2(40)(56)\cos 100° \quad \textit{Substitute the given values.}$$

$$AC^2 \approx 5513.9438 \quad \textit{Simplify.}$$

$$AC \approx 74.3 \text{ m} \quad \textit{Find the square root of both sides.}$$

Step 2 Find the measure of the angle the cable would make with the ground.

$$\frac{\sin A}{BC} = \frac{\sin B}{AC} \quad \textit{Law of Sines}$$

$$\frac{\sin A}{56} \approx \frac{\sin 100°}{74.2559} \quad \textit{Substitute the calculated value for AC.}$$

$$\sin A \approx \frac{56 \sin 100°}{74.2559} \quad \textit{Multiply both sides by 56.}$$

$$m\angle A \approx \sin^{-1}\left(\frac{56 \sin 100°}{74.2559}\right) \approx 48° \quad \textit{Use the inverse sine function to find } m\angle A.$$

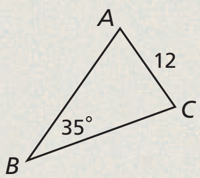
Helpful Hint

Do not round your answer until the final step of the computation. If a problem has multiple steps, store the calculated answers to each part in your calculator.

CHECK IT OUT!

4. **What if...?** Another engineer suggested using a cable attached from the top of the tower to a point 31 m from the base. How long would this cable be, and what angle would it make with the ground? Round the length to the nearest tenth and the angle measure to the nearest degree.

THINK AND DISCUSS



1. Tell what additional information, if any, is needed to find BC using the Law of Sines.

Know it! Note

2. **GET ORGANIZED** Copy and complete the graphic organizer. Tell which law you would use to solve each given triangle and then draw an example.

Given	Law	Example
Two angle measures and any side length		
Two side lengths and a nonincluded angle measure		
Two side lengths and the included angle measure		
Three side lengths		

8-5 Exercises

California Standards
2.0, 7.0, 8.0, 18.0, 19.0, 1A2.0

go.hrw.com
Homework Help Online
KEYWORD: MG7 8-5
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 551

Use a calculator to find each trigonometric ratio. Round to the nearest hundredth.

1. $\sin 100°$ **2.** $\cos 167°$ **3.** $\tan 92°$

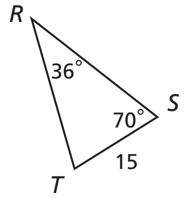
4. $\tan 141°$ **5.** $\cos 133°$ **6.** $\sin 150°$

7. $\sin 147°$ **8.** $\tan 164°$ **9.** $\cos 156°$

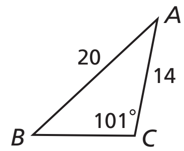
SEE EXAMPLE 2 p. 552

Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

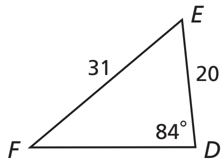
10. RT



11. $m\angle B$

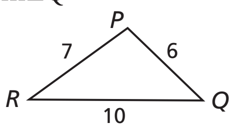


12. $m\angle F$

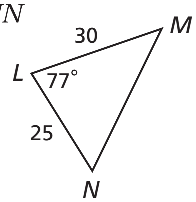


SEE EXAMPLE 3 p. 553

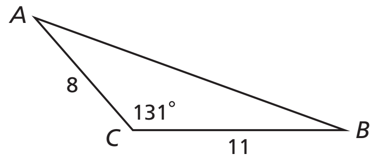
13. $m\angle Q$



14. MN



15. AB



SEE EXAMPLE 4 p. 554

16. Carpentry A carpenter makes a triangular frame by joining three pieces of wood that are 20 cm, 24 cm, and 30 cm long. What are the measures of the angles of the triangle? Round to the nearest degree.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–25	1
26–31	2
32–37	3
38	4

Extra Practice

Skills Practice p. S19
Application Practice p. S35

Use a calculator to find each trigonometric ratio. Round to the nearest hundredth.

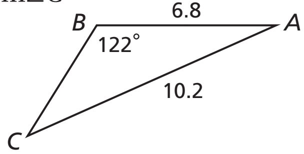
17. $\cos 95°$ **18.** $\tan 178°$ **19.** $\tan 118°$

20. $\sin 132°$ **21.** $\sin 98°$ **22.** $\cos 124°$

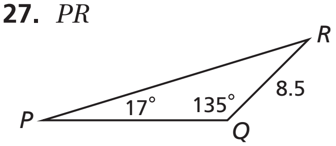
23. $\tan 139°$ **24.** $\cos 145°$ **25.** $\sin 128°$

Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

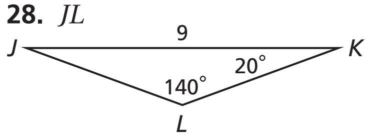
26. $m\angle C$



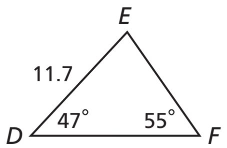
27. PR



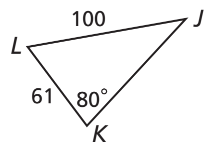
28. JL



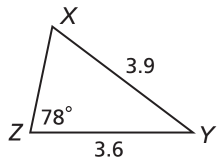
29. EF



30. $m\angle J$

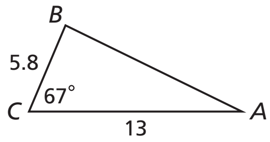


31. $m\angle X$

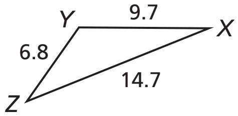


Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

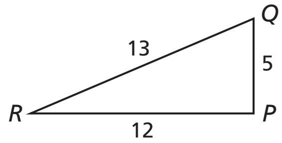
32. AB



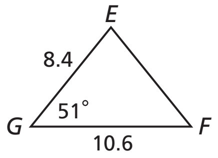
33. m$\angle Z$



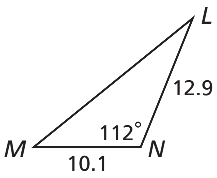
34. m$\angle R$



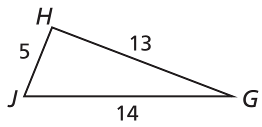
35. EF



36. LM



37. m$\angle G$

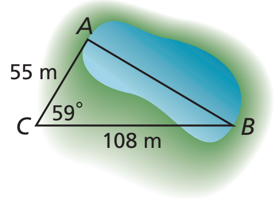


LINK

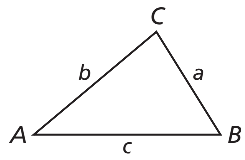
Surveying

Many modern surveys are done with GPS (Global Positioning System) technology. GPS uses orbiting satellites as reference points from which other locations are established.

38. Surveying To find the distance across a lake, a surveyor locates points A, B, and C as shown. What is AB to the nearest tenth of a meter, and what is m$\angle B$ to the nearest degree?



Use the figure for Exercises 39–42. Round lengths to the nearest tenth and angle measures to the nearest degree.



39. m$\angle A = 74°$, m$\angle B = 22°$, and $b = 3.2$ cm. Find a.

40. m$\angle C = 100°$, $a = 9.5$ in., and $b = 7.1$ in. Find c.

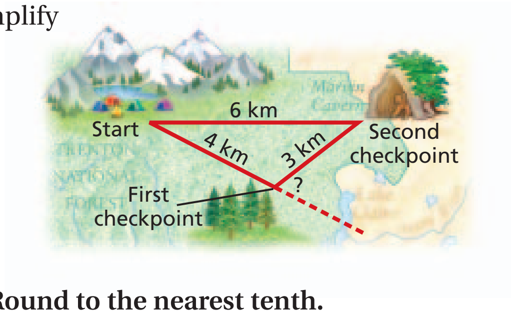
41. $a = 2.2$ m, $b = 3.1$ m, and $c = 4$ m. Find m$\angle B$.

42. $a = 10.3$ cm, $c = 8.4$ cm, and m$\angle A = 45°$. Find m$\angle C$.

43. Critical Thinking Suppose you are given the three angle measures of a triangle. Can you use the Law of Sines or the Law of Cosines to find the lengths of the sides? Why or why not?

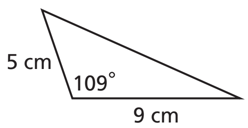
44. What if...? What does the Law of Cosines simplify to when the given angle is a right angle?

45. Orienteering The map of a beginning orienteering course is shown at right. To the nearest degree, at what angle should a team turn in order to go from the first checkpoint to the second checkpoint?

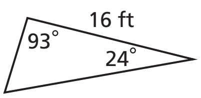


Multi-Step **Find the perimeter of each triangle. Round to the nearest tenth.**

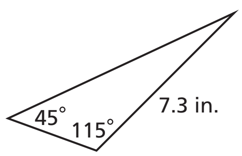
46.



47.

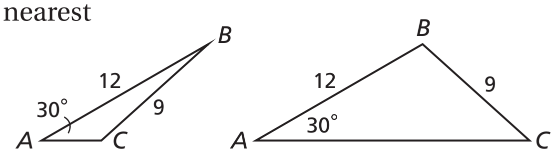


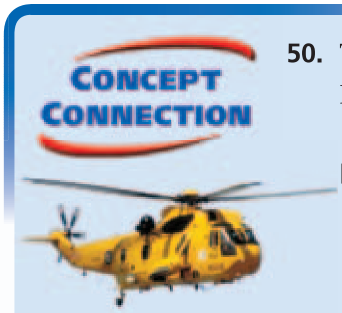
48.



49. The *ambiguous case* of the Law of Sines occurs when you are given an acute angle measure and when the side opposite this angle is shorter than the other given side. In this case, there are two possible triangles.

Find two possible values for m$\angle C$ to the nearest degree. (*Hint:* The inverse sine function on your calculator gives you only acute angle measures. Consider this angle *and* its supplement.)





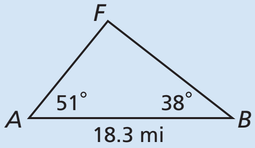
50. This problem will prepare you for the Concept Connection on page 568.

Rescue teams at two heliports, A and B, receive word of a fire at F.

a. What is $m\angle AFB$?

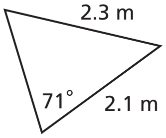
b. To the nearest mile, what are the distances from each heliport to the fire?

c. If a helicopter travels 150 mi/h, how much time is saved by sending a helicopter from A rather than B?

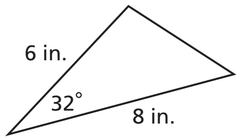


Identify whether you would use the Law of Sines or Law of Cosines as the first step when solving the given triangle.

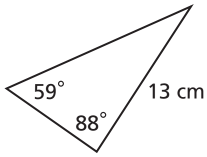
51.



52.



53.



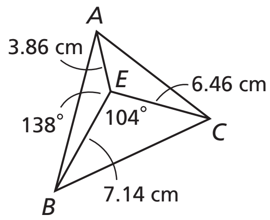
54. The coordinates of the vertices of $\triangle RST$ are $R(0, 3)$, $S(3, 1)$, and $T(-3, -1)$.

a. Find RS, ST, and RT.

b. Which angle of $\triangle RST$ is the largest? Why?

c. Find the measure of the largest angle in $\triangle RST$ to the nearest degree.

55. **Art** Jessika is creating a pattern for a piece of stained glass. Find BC, AB, and $m\angle ABC$. Round lengths to the nearest hundredth and angle measures to the nearest degree.



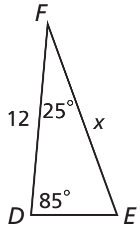
56. **ERROR ANALYSIS** Two students were asked to find x in $\triangle DEF$. Which solution is incorrect? Explain the error.

A

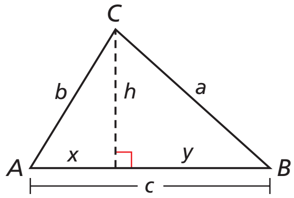
By the Law of Sines,
$\frac{\sin 85^\circ}{x} = \frac{\sin 25^\circ}{12}$. So
$12 \sin 85^\circ = x \sin 25^\circ$,
and $x = \frac{12 \sin 85^\circ}{\sin 25^\circ} \approx 28.3$.

B

By the Law of Sines,
$\frac{\sin 85^\circ}{x} = \frac{\sin 70^\circ}{12}$. So
$12 \sin 85^\circ = x \sin 70^\circ$,
and $x = \frac{12 \sin 85^\circ}{\sin 70^\circ} \approx 12.7$.

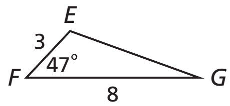


57. Complete the proof of the Law of Cosines for the case when $\triangle ABC$ is an acute triangle.
Given: $\triangle ABC$ is acute with side lengths a, b, and c.
Prove: $a^2 = b^2 + c^2 - 2bc\cos A$



Proof: Draw the altitude from C to $\overline{AB}$. Let h be the length of this altitude. It divides $\overline{AB}$ into segments of lengths x and y. By the Pythagorean Theorem, $a^2 =$ **a.** ___?___ , and **b.** ___?___ $= h^2 + x^2$. Substitute $y = c - x$ into the first equation to get **c.** ___?___ . Rearrange the terms to get $a^2 = (h^2 + x^2) + c^2 - 2cx$. Substitute the expression for b^2 to get **d.** ___?___ . From the diagram, $\cos A = \frac{x}{b}$. So $x =$ **e.** ___?___ . Therefore $a^2 = b^2 + c^2 - 2bc\cos A$ by **f.** ___?___ .

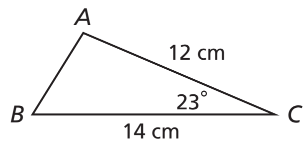
58. **Write About It** Can you use the Law of Sines to solve $\triangle EFG$? Explain why or why not.



STANDARDIZED TEST PREP

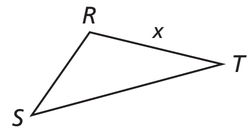
59. Which of these is closest to the length of $\overline{AB}$?

Ⓐ 5.5 centimeters Ⓒ 14.4 centimeters
Ⓑ 7.5 centimeters Ⓓ 22.2 centimeters



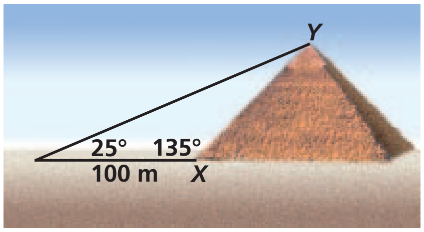
60. Which set of given information makes it possible to find x using the Law of Sines?

Ⓕ $m\angle T = 38°$, $RS = 8.1$, $ST = 15.3$
Ⓖ $RS = 4$, $m\angle S = 40°$, $ST = 9$
Ⓗ $m\angle R = 92°$, $m\angle S = 34°$, $ST = 7$
Ⓙ $m\angle R = 105°$, $m\angle S = 44°$, $m\angle T = 31°$



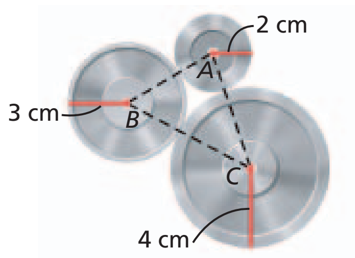
61. A surveyor finds that the face of a pyramid makes a 135° angle with the ground. From a point 100 m from the base of the pyramid, the angle of elevation to the top is 25°. How long is the face of the pyramid, $\overline{XY}$?

Ⓐ 48 meters Ⓒ 124 meters
Ⓑ 81 meters Ⓓ 207 meters



CHALLENGE AND EXTEND

62. Multi-Step Three circular disks are placed next to each other as shown. The disks have radii of 2 cm, 3 cm, and 4 cm. The centers of the disks form $\triangle ABC$. Find $m\angle ACB$ to the nearest degree.



63. Line ℓ passes through points $(-1, 1)$ and $(1, 3)$. Line m passes through points $(-1, 1)$ and $(3, 2)$. Find the measure of the acute angle formed by ℓ and m to the nearest degree.

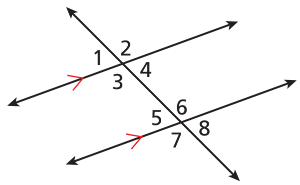
64. Navigation The port of Bonner is 5 mi due south of the port of Alston. A boat leaves the port of Alston at a bearing of N 32° E and travels at a constant speed of 6 mi/h. After 45 minutes, how far is the boat from the port of Bonner? Round to the nearest tenth of a mile.

SPIRAL REVIEW

Write a rule for the nth term in each sequence. *(Previous course)*

65. 3, 6, 9, 12, 15, … **66.** 3, 5, 7, 9, 11, … **67.** 4, 6, 8, 10, 12, …

State the theorem or postulate that justifies each statement. *(Lesson 3-2)*

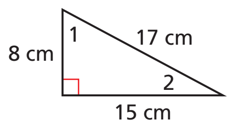


68. $\angle 1 \cong \angle 8$ **69.** $\angle 4 \cong \angle 5$

70. $m\angle 4 + m\angle 6 = 180°$ **71.** $\angle 2 \cong \angle 7$

Use the given trigonometric ratio to determine which angle of the triangle is $\angle A$. *(Lesson 8-3)*

72. $\cos A = \frac{15}{17}$ **73.** $\sin A = \frac{15}{17}$ **74.** $\tan A = 1.875$



8-6 Vectors

Objectives
Find the magnitude and direction of a vector.

Use vectors and vector addition to solve real-world problems.

Vocabulary
vector
component form
magnitude
direction
equal vectors
parallel vectors
resultant vector

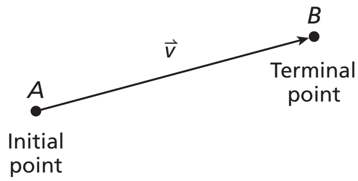
California Standards

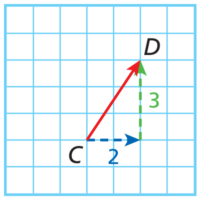
19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.

Who uses this?
By using vectors, a kayaker can take water currents into account when planning a course. (See Example 5.)

The speed and direction an object moves can be represented by a *vector*. A **vector** is a quantity that has both length and direction.

You can think of a vector as a directed line segment. The vector below may be named $\overrightarrow{AB}$ or $\vec{v}$.

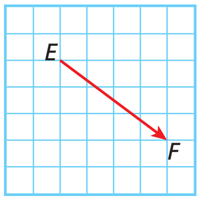




A vector can also be named using *component form*. The **component form** $\langle x, y\rangle$ of a vector lists the horizontal and vertical change from the initial point to the terminal point. The component form of $\overrightarrow{CD}$ is $\langle 2, 3\rangle$.

EXAMPLE 1 Writing Vectors in Component Form

Write each vector in component form.



A $\overrightarrow{EF}$

The horizontal change from E to F is 4 units.
The vertical change from E to F is -3 units.
So the component form of $\overrightarrow{EF}$ is $\langle 4, -3\rangle$.

B $\overrightarrow{PQ}$ **with $P(7, -5)$ and $Q(4, 3)$**

$\overrightarrow{PQ} = \langle x_2 - x_1, y_2 - y_1\rangle$ *Subtract the coordinates of the initial point from the coordinates of the terminal point.*

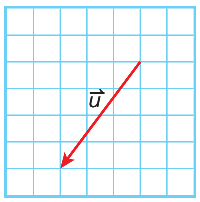
$\overrightarrow{PQ} = \langle 4 - 7, 3 - (-5)\rangle$ *Substitute the coordinates of the given points.*

$\overrightarrow{PQ} = \langle -3, 8\rangle$ *Simplify.*

CHECK IT OUT! **Write each vector in component form.**

1a. $\vec{u}$

1b. the vector with initial point $L(-1, 1)$ and terminal point $M(6, 2)$

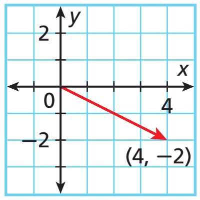


The **magnitude** of a vector is its length. The magnitude of a vector is written $|\overrightarrow{AB}|$ or $|\vec{v}|$.

When a vector is used to represent speed in a given direction, the magnitude of the vector equals the speed. For example, if a vector represents the course a kayaker paddles, the magnitude of the vector is the kayaker's speed.

EXAMPLE 2 Finding the Magnitude of a Vector

Draw the vector $\langle 4, -2\rangle$ on a coordinate plane. Find its magnitude to the nearest tenth.



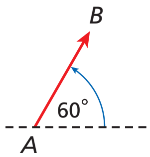
Step 1 Draw the vector on a coordinate plane.
Use the origin as the initial point. Then $(4, -2)$ is the terminal point.

Step 2 Find the magnitude.
Use the Distance Formula.

$$|\langle 4, -2\rangle| = \sqrt{(4-0)^2 + (-2-0)^2} = \sqrt{20} \approx 4.5$$

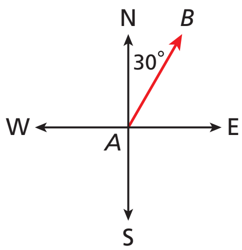
CHECK IT OUT! **2.** Draw the vector $\langle -3, 1\rangle$ on a coordinate plane. Find its magnitude to the nearest tenth.

The **direction** of a vector is the angle that it makes with a horizontal line. This angle is measured counterclockwise from the positive x-axis. The direction of $\overrightarrow{AB}$ is 60°.



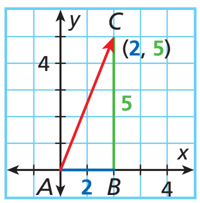
Remember!
See Lesson 4-5, page 252, to review bearings.

The direction of a vector can also be given as a bearing relative to the compass directions *north*, *south*, *east*, and *west*. $\overrightarrow{AB}$ has a bearing of N 30° E.



EXAMPLE 3 Finding the Direction of a Vector

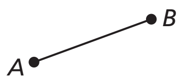
A wind velocity is given by the vector $\langle 2, 5\rangle$. Draw the vector on a coordinate plane. Find the direction of the vector to the nearest degree.



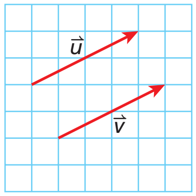
Step 1 Draw the vector on a coordinate plane.
Use the origin as the initial point.

Step 2 Find the direction.
Draw right triangle ABC as shown. $\angle A$ is the angle formed by the vector and the x-axis, and $\tan A = \frac{5}{2}$. So $m\angle A = \tan^{-1}\left(\frac{5}{2}\right) \approx 68°$.

CHECK IT OUT! **3.** The force exerted by a tugboat is given by the vector $\langle 7, 3\rangle$. Draw the vector on a coordinate plane. Find the direction of the vector to the nearest degree.

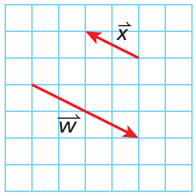
> **Caution!**
>
> 
>
>
> Note that $\overrightarrow{AB} \neq \overrightarrow{BA}$ since these vectors do not have the same direction.

Two vectors are **equal vectors** if they have the same magnitude and the same direction. For example, $\vec{u} = \vec{v}$. Equal vectors do not have to have the same initial point and terminal point.



$|\vec{u}| = |\vec{v}| = 2\sqrt{5}$

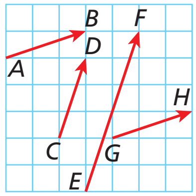
Two vectors are **parallel vectors** if they have the same direction or if they have opposite directions. They may have different magnitudes. For example, $\vec{w} \parallel \vec{x}$. Equal vectors are always parallel vectors.



$|\vec{w}| = 2\sqrt{5}$
$|\vec{x}| = \sqrt{5}$

EXAMPLE 4 Identifying Equal and Parallel Vectors

Identify each of the following.



A equal vectors

$\overrightarrow{AB} = \overrightarrow{GH}$ *Identify vectors with the same magnitude and direction.*

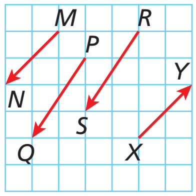
B parallel vectors

$\overrightarrow{AB} \parallel \overrightarrow{GH}$ and $\overrightarrow{CD} \parallel \overrightarrow{EF}$ *Identify vectors with the same or opposite directions.*

CHECK IT OUT! **Identify each of the following.**

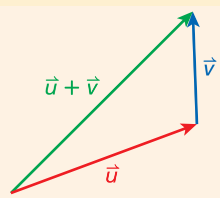
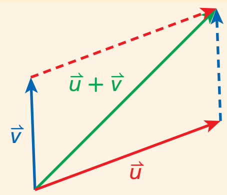
4a. equal vectors

4b. parallel vectors



The **resultant vector** is the vector that represents the sum of two given vectors. To add two vectors geometrically, you can use the head-to-tail method or the parallelogram method.

Know it! Note — Vector Addition

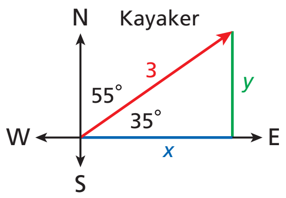
METHOD	EXAMPLE
Head-to-Tail Method	
Place the initial point (tail) of the **second vector** on the terminal point (head) of the **first vector**. The **resultant** is the vector that joins the initial point of the **first vector** to the terminal point of the **second vector**.	
Parallelogram Method	
Use the same initial point for both of the given vectors. Create a parallelogram by adding a copy of each vector at the terminal point (head) of the other vector. The **resultant vector** is a diagonal of the parallelogram formed.	

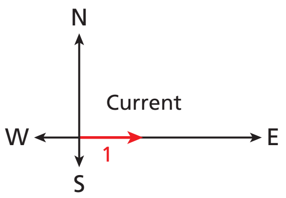
To add vectors numerically, add their components. If $\vec{u} = \langle x_1, y_1 \rangle$ and $\vec{v} = \langle x_2, y_2 \rangle$, then $\vec{u} + \vec{v} = \langle x_1 + x_2, y_1 + y_2 \rangle$.

EXAMPLE 5 Sports Application

A kayaker leaves shore at a bearing of N 55° E and paddles at a constant speed of 3 mi/h. There is a 1 mi/h current moving due east. What are the kayak's actual speed and direction? Round the speed to the nearest tenth and the direction to the nearest degree.

Step 1 Sketch vectors for the kayaker and the current.





Step 2 Write the vector for the kayaker in component form.

The kayaker's vector has a magnitude of 3 mi/h and makes an angle of 35° with the x-axis.

$$\cos 35° = \frac{x}{3}, \text{ so } x = 3\cos 35° \approx 2.5.$$

$$\sin 35° = \frac{y}{3}, \text{ so } y = 3\sin 35° \approx 1.7.$$

The kayaker's vector is $\langle 2.5, 1.7 \rangle$.

Step 3 Write the vector for the current in component form.

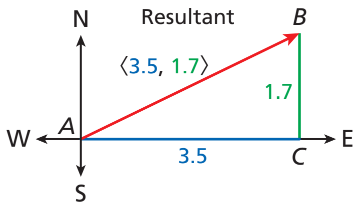
Since the current moves 1 mi/h in the direction of the x-axis, it has a horizontal component of 1 and a vertical component of 0. So its vector is $\langle 1, 0 \rangle$.

Step 4 Find and sketch the resultant vector $\overrightarrow{AB}$.

Add the components of the kayaker's vector and the current's vector.

$$\langle 2.5, 1.7 \rangle + \langle 1, 0 \rangle = \langle 3.5, 1.7 \rangle$$

The resultant vector in component form is $\langle 3.5, 1.7 \rangle$.



Step 5 Find the magnitude and direction of the resultant vector.

The magnitude of the resultant vector is the kayak's actual speed.

$$\left|\langle 3.5, 1.7 \rangle\right| = \sqrt{(3.5 - 0)^2 + (1.7 - 0)^2} \approx 3.9 \text{ mi/h}$$

The angle measure formed by the resultant vector gives the kayak's actual direction.

$$\tan A = \frac{1.7}{3.5}, \text{ so } A = \tan^{-1}\left(\frac{1.7}{3.5}\right) \approx 26°, \text{ or N } 64° \text{ E}.$$

Remember!

Component form gives the horizontal and vertical change from the initial point to the terminal point of the vector.

CHECK IT OUT!

5. What if...? Suppose the kayaker in Example 5 instead paddles at 4 mi/h at a bearing of N 20° E. What are the kayak's actual speed and direction? Round the speed to the nearest tenth and the direction to the nearest degree.

THINK AND DISCUSS

1. Explain why the segment with endpoints $(0, 0)$ and $(1, 4)$ is not a vector.

2. Assume you are given a vector in component form. Other than the Distance Formula, what theorem can you use to find the vector's magnitude?

3. Describe how to add two vectors numerically.

Know it! Note

4. GET ORGANIZED Copy and complete the graphic organizer.

Definition	Names
Vector	
Examples	Nonexamples

8-6 Exercises

California Standards
8.0, 19.0, 7NS2.5, 1A2.0, 1A9.0

GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. ___?___ vectors have the same magnitude and direction. (*equal*, *parallel*, or *resultant*)

2. ___?___ vectors have the same or opposite directions. (*equal*, *parallel*, or *resultant*)

3. The ___?___ of a vector indicates the vector's size. (*magnitude* or *direction*)

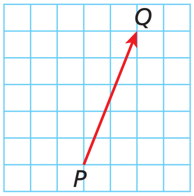
SEE EXAMPLE 1
p. 559

Write each vector in component form.

4. $\overrightarrow{AC}$ with $A(1, 2)$ and $C(6, 5)$

5. the vector with initial point $M(-4, 5)$ and terminal point $N(4, -3)$

6. $\overrightarrow{PQ}$



SEE EXAMPLE 2
p. 560

Draw each vector on a coordinate plane. Find its magnitude to the nearest tenth.

7. $\langle 1, 4\rangle$ **8.** $\langle -3, -2\rangle$ **9.** $\langle 5, -3\rangle$

SEE EXAMPLE 3
p. 560

Draw each vector on a coordinate plane. Find the direction of the vector to the nearest degree.

10. A river's current is given by the vector $\langle 4, 6\rangle$.

11. The velocity of a plane is given by the vector $\langle 5, 1\rangle$.

12. The path of a hiker is given by the vector $\langle 6, 3\rangle$.

SEE EXAMPLE 4
p. 561

Identify each of the following.

13. equal vectors in diagram 1

14. parallel vectors in diagram 1

15. equal vectors in diagram 2

16. parallel vectors in diagram 2

Diagram 1

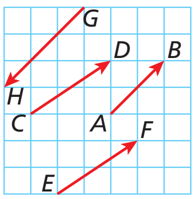
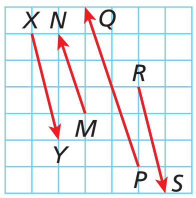
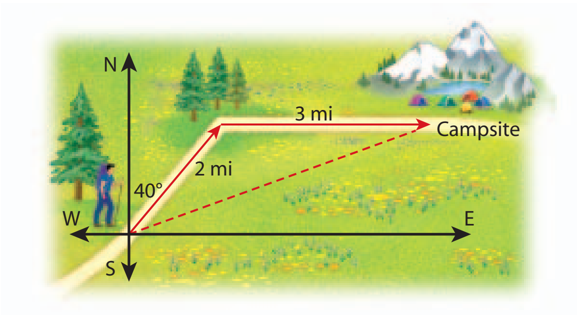


Diagram 2



SEE EXAMPLE 5
p. 562

17. **Recreation** To reach a campsite, a hiker first walks for 2 mi at a bearing of N 40° E. Then he walks 3 mi due east. What are the magnitude and direction of his hike from his starting point to the campsite? Round the distance to the nearest tenth of a mile and the direction to the nearest degree.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–20	1
21–23	2
24–26	3
27–30	4
31	5

Extra Practice

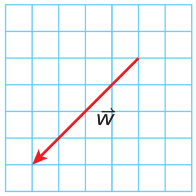
Skills Practice p. S19
Application Practice p. S35

Write each vector in component form.

18. $\overrightarrow{JK}$ with $J(-6, -7)$ and $K(3, -5)$

19. $\overrightarrow{EF}$ with $E(1.5, -3)$ and $F(-2, 2.5)$

20. $\vec{w}$



Draw each vector on a coordinate plane. Find its magnitude to the nearest tenth.

21. $\langle -2, 0\rangle$

22. $\langle 1.5, 1.5\rangle$

23. $\langle 2.5, -3.5\rangle$

Draw each vector on a coordinate plane. Find the direction of the vector to the nearest degree.

24. A boat's velocity is given by the vector $\langle 4, 1.5\rangle$.

25. The path of a submarine is given by the vector $\langle 3.5, 2.5\rangle$.

26. The path of a projectile is given by the vector $\langle 2, 5\rangle$.

Identify each of the following.

27. equal vectors in diagram 1

28. parallel vectors in diagram 1

29. equal vectors in diagram 2

30. parallel vectors in diagram 2

Diagram 1

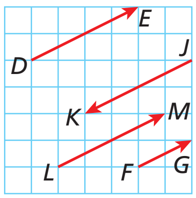
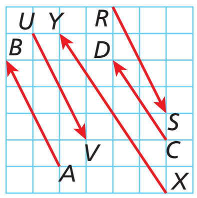
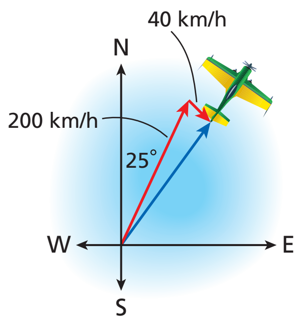


Diagram 2



31. **Aviation** The pilot of a single-engine airplane flies at a constant speed of 200 km/h at a bearing of N 25° E. There is a 40 km/h crosswind blowing southeast (S 45° E). What are the plane's actual speed and direction? Round the speed to the nearest tenth and the direction to the nearest degree.



Find each vector sum.

32. $\langle 1, 2\rangle + \langle 0, 6\rangle$

33. $\langle -3, 4\rangle + \langle 5, -2\rangle$

34. $\langle 0, 1\rangle + \langle 7, 0\rangle$

35. $\langle 8, 3\rangle + \langle -2, -1\rangle$

36. **Critical Thinking** Is vector addition commutative? That is, is $\vec{u} + \vec{v}$ equal to $\vec{v} + \vec{u}$? Use the head-to-tail method of vector addition to explain why or why not.

CONCEPT CONNECTION

37. This problem will prepare you for the Concept Connection on page 568.

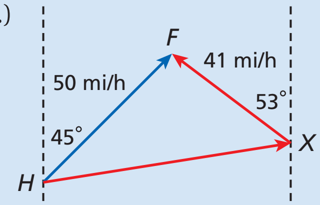
A helicopter at H must fly at 50 mi/h in the direction N 45° E to reach the site of a flood victim F. There is a 41 mi/h wind in the direction N 53° W. The pilot needs to the know the velocity vector $\overrightarrow{HX}$ he should use so that his resultant vector will be $\overrightarrow{HF}$.

a. What is $m\angle F$? (*Hint:* Consider a vertical line through F.)

b. Use the Law of Cosines to find the magnitude of $\overrightarrow{HX}$ to the nearest tenth.

c. Use the Law of Sines to find $m\angle FHX$ to the nearest degree.

d. What is the direction of $\overrightarrow{HX}$?



Write each vector in component form. Round values to the nearest tenth.

38. magnitude 15, direction 42°

39. magnitude 7.2, direction 9°

40. magnitude 12.1, direction N 57° E

41. magnitude 5.8, direction N 22° E

42. Physics A classroom has a window near the ceiling, and a long pole must be used to close it.

a. Carla holds the pole at a 45° angle to the floor and applies 10 lb of force to the upper edge of the window. Find the vertical component of the vector representing the force on the window. Round to the nearest tenth.

b. Taneka also applies 10 lb of force to close the window, but she holds the pole at a 75° angle to the floor. Find the vertical component of the force vector in this case. Round to the nearest tenth.

c. Who will have an easier time closing the window, Carla or Taneka? (*Hint:* Who applies more vertical force?)

43. Probability The numbers 1, 2, 3, and 4 are written on slips of paper and placed in a hat. Two different slips of paper are chosen at random to be the x- and y-components of a vector.

a. What is the probability that the vector will be equal to $\langle 1, 2\rangle$?

b. What is the probability that the vector will be parallel to $\langle 1, 2\rangle$?

44. Estimation Use the vector $\langle 4, 6\rangle$ to complete the following.

a. Draw the vector on a sheet of graph paper.

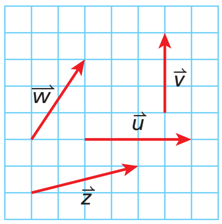
b. Estimate the vector's direction to the nearest degree.

c. Use a protractor to measure the angle the vector makes with a horizontal line.

d. Use the vector's components to calculate its direction.

e. How did your estimate in part **b** compare to your measurement in part **c** and your calculation in part **d**?

Multi-Step **Find the magnitude of each vector to the nearest tenth and the direction of each vector to the nearest degree.**



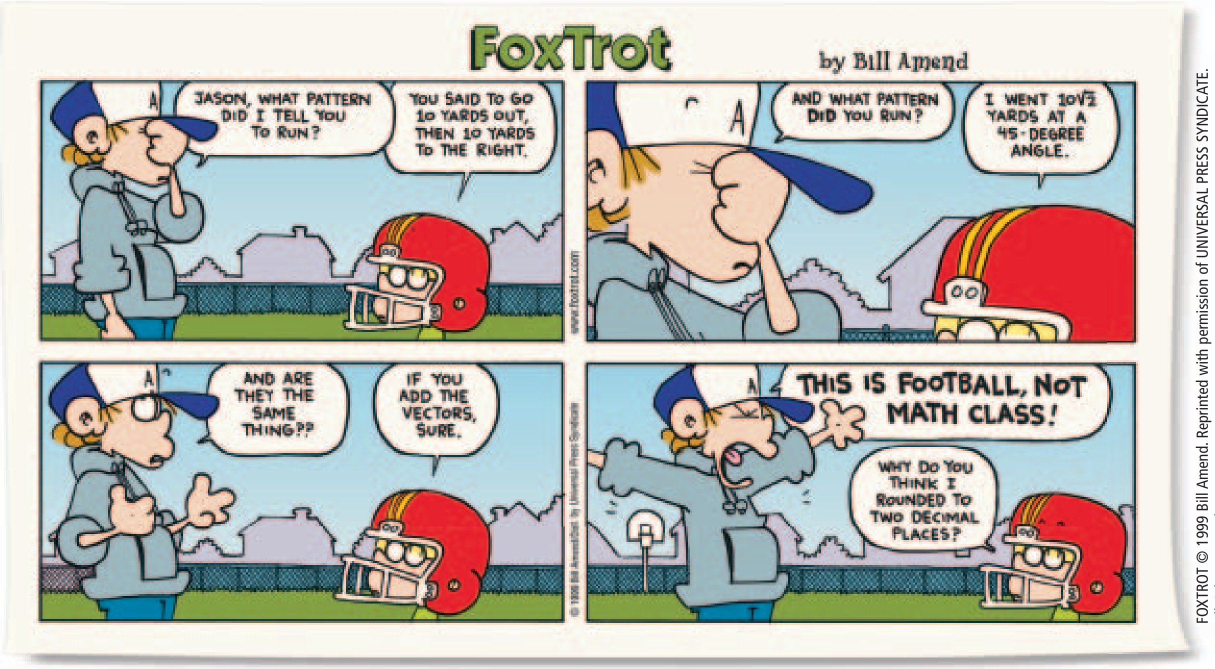
45. $\vec{u}$

46. $\vec{v}$

47. $\vec{w}$

48. $\vec{z}$

49. **Football** Write two vectors in component form to represent the pass pattern that Jason is told to run. Find the resultant vector and show that Jason's move is equivalent to the vector.



For each given vector, find another vector that has the same magnitude but a different direction. Then find a vector that has the same direction but a different magnitude.

50. $\langle -3, 6\rangle$ **51.** $\langle 12, 5\rangle$ **52.** $\langle 8, -11\rangle$

Multi-Step **Find the sum of each pair of vectors. Then find the magnitude and direction of the resultant vector. Round the magnitude to the nearest tenth and the direction to the nearest degree.**

53. $\vec{u} = \langle 1, 2\rangle$, $\vec{v} = \langle 2.5, -1\rangle$

54. $\vec{u} = \langle -2, 7\rangle$, $\vec{v} = \langle 4.8, -3.1\rangle$

55. $\vec{u} = \langle 6, 0\rangle$, $\vec{v} = \langle -2, 4\rangle$

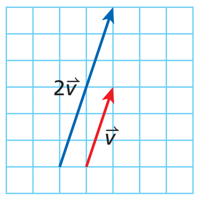
56. $\vec{u} = \langle -1.2, 8\rangle$, $\vec{v} = \langle 5.2, -2.1\rangle$

LINK

Math History

August Ferdinand Möbius is best known for experimenting with the Möbius strip, a three-dimensional figure that has only one side and one edge.

57. **Math History** In 1827, the mathematician August Ferdinand Möbius published a book in which he introduced directed line segments (what we now call vectors). He showed how to perform *scalar multiplication* of vectors. For example, consider a hiker who walks along a path given by the vector $\vec{v}$. The path of another hiker who walks twice as far in the same direction is given by the vector $2\vec{v}$.



a. Write the component form of the vectors $\vec{v}$ and $2\vec{v}$.

b. Find the magnitude of $\vec{v}$ and $2\vec{v}$. How do they compare?

c. Find the direction of $\vec{v}$ and $2\vec{v}$. How do they compare?

d. Given the component form of a vector, explain how to find the components of the vector $k\vec{v}$, where k is a constant.

e. Use scalar multiplication with $k = -1$ to write the *negation of a vector* $\vec{v}$ in component form.

58. **Critical Thinking** A vector $\vec{u}$ points due west with a magnitude of u units. Another vector $\vec{v}$ points due east with a magnitude of v units. Describe three possible directions and magnitudes for the resultant vector.

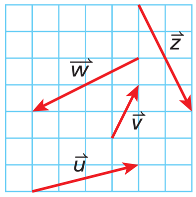
59. **Write About It** Compare a line segment, a ray, and a vector.

STANDARDIZED TEST PREP

60. Which vector is parallel to $\langle 2, 1\rangle$?

Ⓐ $\vec{u}$ Ⓒ $\vec{w}$

Ⓑ $\vec{v}$ Ⓓ $\vec{z}$



61. The vector $\langle 7, 9\rangle$ represents the velocity of a helicopter. What is the direction of this vector to the nearest degree?

Ⓕ 38° Ⓖ 52° Ⓗ 128° Ⓙ 142°

62. A canoe sets out on a course given by the vector $\langle 5, 11\rangle$. What is the length of the canoe's course to the nearest unit?

Ⓐ 6 Ⓑ 8 Ⓒ 12 Ⓓ 16

63. **Gridded Response** $\overrightarrow{AB}$ has an initial point of $(-3, 6)$ and a terminal point of $(-5, -2)$. Find the magnitude of $\overrightarrow{AB}$ to the nearest tenth.

CHALLENGE AND EXTEND

Recall that the angle of a vector's direction is measured counterclockwise from the positive *x*-axis. Find the direction of each vector to the nearest degree.

64. $\langle -2, 3\rangle$ **65.** $\langle -4, 0\rangle$ **66.** $\langle -5, -3\rangle$

67. **Navigation** The captain of a ship is planning to sail in an area where there is a 4 mi/h current moving due east. What speed and bearing should the captain maintain so that the ship's actual course (taking the current into account) is 10 mi/h at a bearing of N 70° E? Round the speed to the nearest tenth and the direction to the nearest degree.

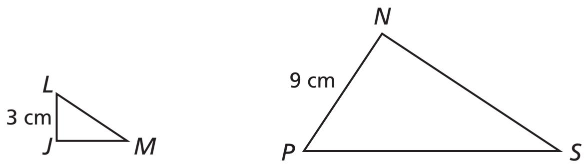
68. Aaron hikes from his home to a park by walking 3 km at a bearing of N 30° E, then 6 km due east, and then 4 km at a bearing of N 50° E. What are the magnitude and direction of the vector that represents the straight path from Aaron's home to the park? Round the magnitude to the nearest tenth and the direction to the nearest degree.

SPIRAL REVIEW

Solve each system of equations by graphing. *(Previous course)*

69. $\begin{cases} x - y = -5 \\ y = 3x + 1 \end{cases}$ **70.** $\begin{cases} x - 2y = 0 \\ 2y + x = 8 \end{cases}$ **71.** $\begin{cases} x + y = 5 \\ 3y + 15 = 2x \end{cases}$

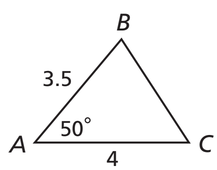
Given that $\triangle JLM \sim \triangle NPS$, the perimeter of $\triangle JLM$ is 12 cm, and the area of $\triangle JLM$ is 6 cm^2, find each measure. *(Lesson 7-5)*



72. the perimeter of $\triangle NPS$ **73.** the area of $\triangle NPS$

Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree. *(Lesson 8-5)*

74. BC **75.** $m\angle B$ **76.** $m\angle C$



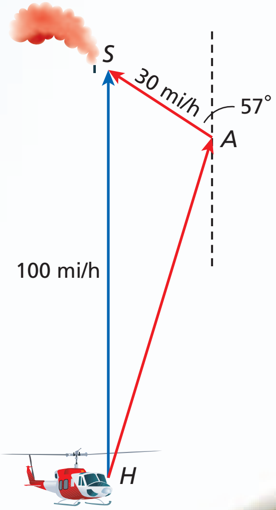
CHAPTER 8

SECTION 8B

Concept Connection

Applying Trigonometric Ratios

Help Is on the Way! Rescue helicopters were first used in the 1950s during the Korean War. The helicopters made it possible to airlift wounded soldiers to medical stations. Today, helicopters are used to rescue injured hikers, flood victims, and people who are stranded at sea.



1. The pilot of a helicopter is searching for an injured hiker. While flying at an altitude of 1500 ft, the pilot sees smoke at an angle of depression of 14°. Assuming that the smoke is a distress signal from the hiker, what is the helicopter's horizontal distance to the hiker? Round to the nearest foot.
2. The pilot plans to fly due north at 100 mi/h from the helicopter's current position H to the location of the smoke S. However there is a 30 mi/h wind in the direction N 57° W. The pilot needs to know the velocity vector $\overrightarrow{HA}$ that he should use so that his resultant vector will be $\overrightarrow{HS}$. Find $m\angle S$ and then use the Law of Cosines to find the magnitude of $\overrightarrow{HA}$ to the nearest mile per hour.
3. Use the Law of Sines to find the direction of $\overrightarrow{HA}$ to the nearest degree.

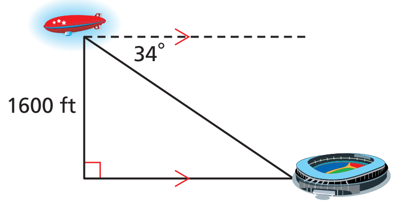
READY TO GO ON?

CHAPTER 8

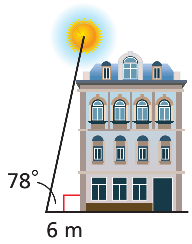
SECTION 8B

Quiz for Lessons 8-4 Through 8-6

8-4 Angles of Elevation and Depression



1. An observer in a blimp sights a football stadium at an angle of depression of 34°. The blimp's altitude is 1600 ft. What is the horizontal distance from the blimp to the stadium? Round to the nearest foot.

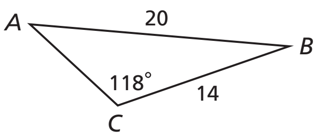


2. When the angle of elevation of the sun is 78°, a building casts a shadow that is 6 m long. What is the height of the building to the nearest tenth of a meter?

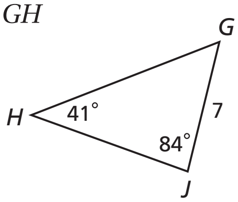
8-5 Law of Sines and Law of Cosines

Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

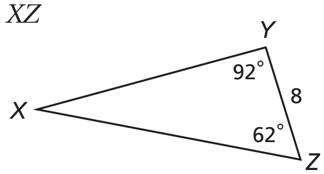
3. m∠A



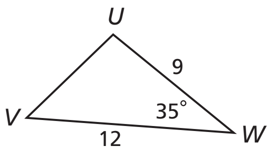
4. GH



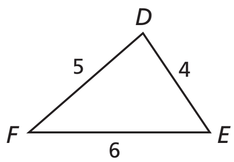
5. XZ



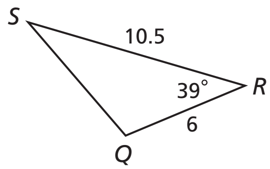
6. UV



7. m∠F



8. QS



8-6 Vectors

Draw each vector on a coordinate plane. Find its magnitude to the nearest tenth.

9. $\langle 3, 1\rangle$

10. $\langle -2, -4\rangle$

11. $\langle 0, 5\rangle$

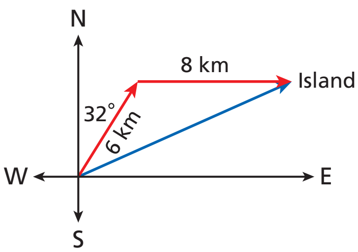
Draw each vector on a coordinate plane. Find the direction of the vector to the nearest degree.

12. A wind velocity is given by the vector $\langle 2, 1\rangle$.

13. The current of a river is given by the vector $\langle 5, 3\rangle$.

14. The force of a spring is given by the vector $\langle 4, 4\rangle$.

15. To reach an island, a ship leaves port and sails for 6 km at a bearing of N 32° E. It then sails due east for 8 km. What are the magnitude and direction of the voyage directly from the port to the island? Round the distance to the nearest tenth of a kilometer and the direction to the nearest degree.



EXTENSION Trigonometry and the Unit Circle

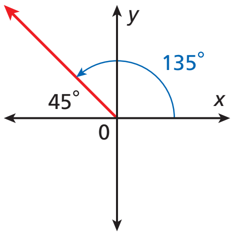
Objective
Define trigonometric ratios for angle measures greater than or equal to 90°.

Vocabulary
reference angle
unit circle

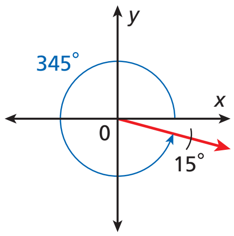
California Standards

18.0 Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, $\tan(x) = \sin(x)/\cos(x)$, $(\sin(x))^2 + (\cos(x))^2 = 1$.

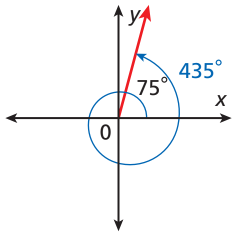
Rotations are used to extend the concept of trigonometric ratios to angle measures greater than or equal to 90°. Consider a ray with its endpoint at the origin, pointing in the direction of the positive x-axis. Rotate the ray counterclockwise around the origin. The acute angle formed by the ray and the nearest part of the positive or negative x-axis is called the **reference angle**. The rotated ray is called the *terminal side* of that angle.



Angle measure: **135°**
Reference angle: 45°



Angle measure: **345°**
Reference angle: 15°

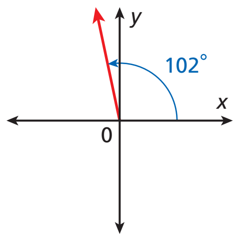


Angle measure: **435°**
Reference angle: 75°

EXAMPLE 1 **Finding Reference Angles**

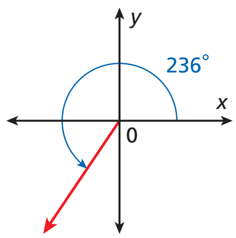
Sketch each angle on the coordinate plane. Find the measure of its reference angle.

A 102°



Reference angle:
180° − **102°** = 78°

B 236°



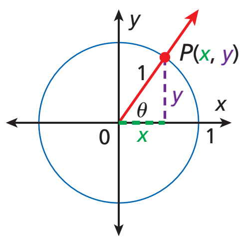
Reference angle:
236° − 180° = 56°

CHECK IT OUT! **Sketch each angle on the coordinate plane. Find the measure of its reference angle.**

1a. 309° **1b.** 410°

The **unit circle** is a circle with a radius of 1 unit, centered at the origin. It can be used to find the trigonometric ratios of an angle.

Consider the acute angle θ. Let $P(x, y)$ be the point where the terminal side of θ intersects the unit circle. Draw a vertical line from P to the x-axis. Since $\cos\theta = \frac{x}{1}$ and $\sin\theta = \frac{y}{1}$, the coordinates of P can be written as $(\cos\theta, \sin\theta)$. Thus if you know the coordinates of a point on the unit circle, you can find the trigonometric ratios for the associated angle.



Unit circle

Reading Math

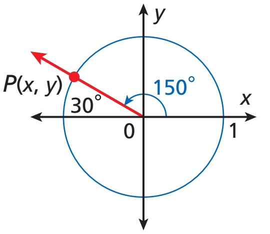
In trigonometry, the Greek letter *theta*, θ, is often used to represent angle measures.

EXAMPLE 2 Finding Trigonometric Ratios

Find each trigonometric ratio.

A **cos 150°**

Sketch the angle on the coordinate plane.
The reference angle is 30°.



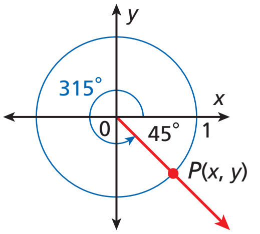
$$\cos 30° = \frac{\sqrt{3}}{2} \qquad \sin 30° = \frac{1}{2}$$

Let $P(x, y)$ be the point where the terminal side of the angle intersects the unit circle. Since P is in Quadrant II, its x-coordinate is negative, and its y-coordinate is positive. So the coordinates of P are $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

The cosine of 150° is the x-coordinate of P, so $\cos 150° = -\frac{\sqrt{3}}{2}$.

B **tan 315°**

Sketch the angle on the coordinate plane.
The reference angle is 45°.



$$\cos 45° = \frac{\sqrt{2}}{2} \qquad \sin 45° = \frac{\sqrt{2}}{2}$$

Since $P(x, y)$ is in Quadrant IV, its y-coordinate is negative. So the coordinates of P are $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

Remember that $\tan\theta = \frac{\sin\theta}{\cos\theta}$. So $\tan 315° = \frac{\sin 315°}{\cos 315°} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$.

Caution!

Be sure to use the correct sign when assigning coordinates to a point on the unit circle.

CHECK IT OUT! **Find each trigonometric ratio.**

2a. cos 240° **2b.** sin 135°

EXTENSION Exercises

Sketch each angle on the coordinate plane. Find the measure of its reference angle.

1. 125° **2.** 216° **3.** 359°

Find each trigonometric ratio.

4. cos 225° **5.** sin 120° **6.** cos 300°

7. tan 135° **8.** cos 420° **9.** tan 315°

10. sin 90° **11.** cos 180° **12.** sin 270°

13. **Critical Thinking** Given that $\cos\theta = 0.5$, what are the possible values for θ between 0° and 360°?

14. **Write About It** Explain how you can use the unit circle to find tan 180°.

15. **Challenge** If $\sin\theta \approx -0.891$, what are two values of θ between 0° and 360°?

CHAPTER 8 Study Guide: Review

Know it! Note: For a complete list of the postulates and theorems in this chapter, see p. S82.

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. The __?__ of a vector gives the horizontal and vertical change from the initial point to the terminal point.

2. Two vectors with the same magnitude and direction are called __?__ .

3. If a and b are positive numbers, then $\sqrt{ab}$ is the __?__ of a and b.

4. A(n) __?__ is the angle formed by a horizontal line and a line of sight to a point above the horizontal line.

5. The sine, cosine, and tangent are all examples of a(n) __?__ .

8-1 Similarity in Right Triangles *(pp. 518–523)*

4.0

EXAMPLES

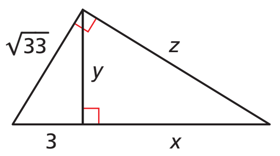
■ **Find the geometric mean of 5 and 30.**

Let x be the geometric mean.

$x^2 = (5)(30) = 150$ *Def. of geometric mean*

$x = \sqrt{150} = 5\sqrt{6}$ *Find the positive square root.*

■ **Find x, y, and z.**



$\left(\sqrt{33}\right)^2 = 3(3 + x)$ *$\sqrt{33}$ is the geometric mean of 3 and 3 + x.*

$33 = 9 + 3x$

$24 = 3x$

$x = 8$

$y^2 = (3)(8)$ *y is the geometric mean of 3 and 8.*

$y^2 = 24$

$y = \sqrt{24} = 2\sqrt{6}$

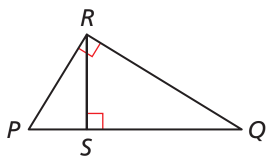
$z^2 = (8)(11)$ *z is the geometric mean of 8 and 11.*

$z^2 = 88$

$z = \sqrt{88} = 2\sqrt{22}$

EXERCISES

6. Write a similarity statement comparing the three triangles.



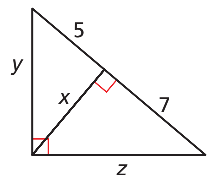
Find the geometric mean of each pair of numbers. If necessary, give the answer in simplest radical form.

7. $\frac{1}{4}$ and 100

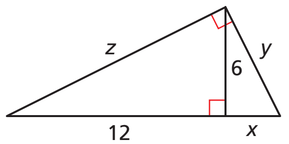
8. 3 and 17

Find x, y, and z.

9.



10.



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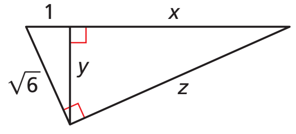


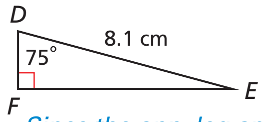
8-2 Trigonometric Ratios *(pp. 525–532)*

18.0, 19.0

EXAMPLES

Find each length. Round to the nearest hundredth.

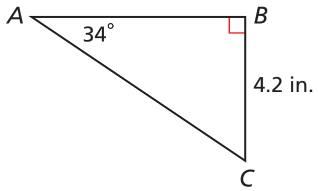
■ EF



$$\sin 75° = \frac{EF}{8.1}$$
$$EF = 8.1(\sin 75°)$$
$$EF \approx 7.82 \text{ cm}$$

Since the opp. leg and hyp. are involved, use a sine ratio.

■ AB



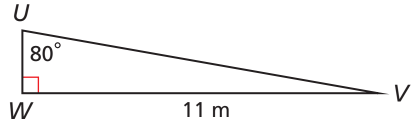
$$\tan 34° = \frac{4.2}{AB}$$
$$AB \tan 34° = 4.2$$
$$AB = \frac{4.2}{\tan 34°}$$
$$AB \approx 6.23 \text{ in.}$$

Since the opp. and adj. legs are involved, use a tangent ratio.

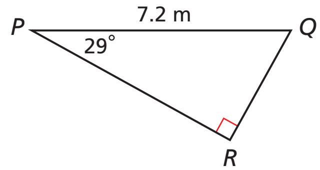
EXERCISES

Find each length. Round to the nearest hundredth.

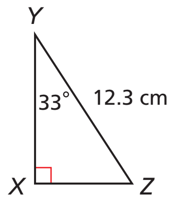
12. UV



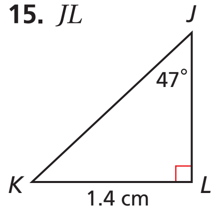
13. PR



14. XY



15. JL

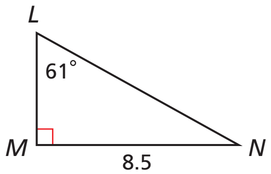


8-3 Solving Right Triangles *(pp. 534–541)*

15.0, 18.0, 19.0

EXAMPLE

■ **Find the unknown measures in $\triangle LMN$. Round lengths to the nearest hundredth and angle measures to the nearest degree.**



The acute angles of a right triangle are complementary. So $m\angle N = 90° - 61° = 29°$.

$\sin L = \frac{MN}{LN}$ — *Write a trig. ratio.*

$\sin 61° = \frac{8.5}{LN}$ — *Substitute the given values.*

$LN = \frac{8.5}{\sin 61°} \approx 9.72$ — *Solve for LN.*

$\tan L = \frac{MN}{LM}$ — *Write a trig. ratio.*

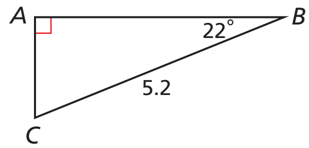
$\tan 61° = \frac{8.5}{LM}$ — *Substitute the given values.*

$LM = \frac{8.5}{\tan 61°} \approx 4.71$ — *Solve for LM.*

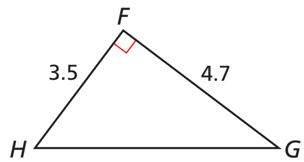
EXERCISES

Find the unknown measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.

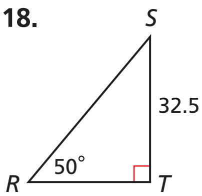
16.



17.



18.



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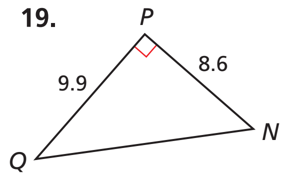


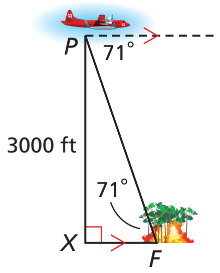
8-4 Angles of Elevation and Depression *(pp. 544–549)*

19.0

EXAMPLES

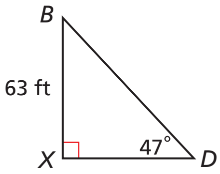
- **A pilot in a plane spots a forest fire on the ground at an angle of depression of 71°. The plane's altitude is 3000 ft. What is the horizontal distance from the plane to the fire? Round to the nearest foot.**

$$\tan 71° = \frac{3000}{XF}$$
$$XF = \frac{3000}{\tan 71°}$$
$$XF \approx 1033 \text{ ft}$$



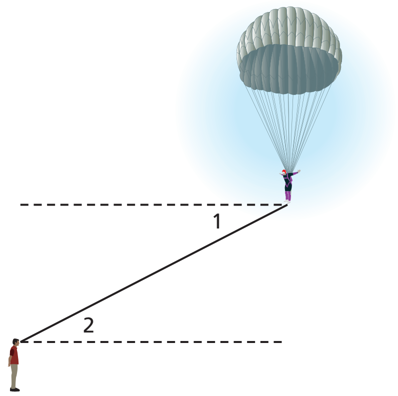
- **A diver is swimming at a depth of 63 ft below sea level. He sees a buoy floating at sea level at an angle of elevation of 47°. How far must the diver swim so that he is directly beneath the buoy? Round to the nearest foot.**

$$\tan 47° = \frac{63}{XD}$$
$$XD = \frac{63}{\tan 47°}$$
$$XD \approx 59 \text{ ft}$$



EXERCISES

Classify each angle as an angle of elevation or angle of depression.



20. $\angle 1$ **21.** $\angle 2$

22. When the angle of elevation to the sun is 82°, a monument casts a shadow that is 5.1 ft long. What is the height of the monument to the nearest foot?

23. A ranger in a lookout tower spots a fire in the distance. The angle of depression to the fire is 4°, and the lookout tower is 32 m tall. What is the horizontal distance to the fire? Round to the nearest meter.

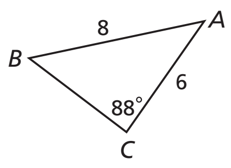
8-5 Law of Sines and Law of Cosines *(pp. 551–558)*

19.0

EXAMPLES

Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

- $m\angle B$



$\frac{\sin B}{AC} = \frac{\sin C}{AB}$ *Law of Sines*

$\frac{\sin B}{\mathbf{6}} = \frac{\sin \mathbf{88°}}{\mathbf{8}}$ *Substitute the given values.*

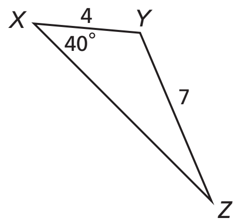
$\sin B = \frac{6 \sin 88°}{8}$ *Multiply both sides by 6.*

$$m\angle B = \sin^{-1}\left(\frac{6\sin 88°}{8}\right) \approx 49°$$

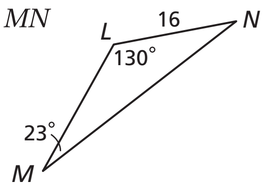
EXERCISES

Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

24. $m\angle Z$

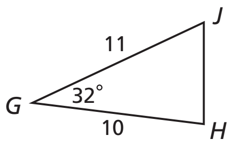


25. MN



Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

■ HJ



Use the Law of Cosines.

$HJ^2 = GH^2 + GJ^2 - 2(GH)(GJ)\cos G$

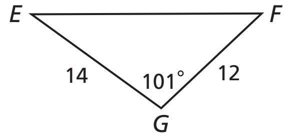
$= 10^2 + 11^2 - 2(10)(11)\cos 32°$

$HJ^2 \approx 34.4294$ *Simplify.*

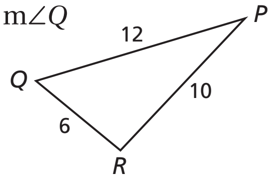
$HJ \approx 5.9$ *Find the square root.*

Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

26. EF



27. $m\angle Q$



8-6 Vectors *(pp. 559–567)*

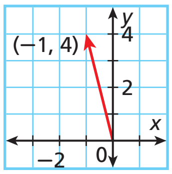
19.0

EXAMPLES

■ **Draw the vector $\langle -1, 4\rangle$ on a coordinate plane. Find its magnitude to the nearest tenth.**

$|\langle -1, 4\rangle| = \sqrt{(-1)^2 + (4)^2}$

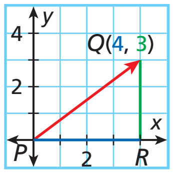
$= \sqrt{17} \approx 4.1$



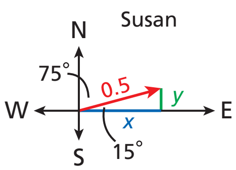
■ **The velocity of a jet is given by the vector $\langle 4, 3\rangle$. Draw the vector on a coordinate plane. Find the direction of the vector to the nearest degree.**

In $\triangle PQR$, $\tan P = \frac{3}{4}$, so

$m\angle P = \tan^{-1}\left(\frac{3}{4}\right) \approx 37°$.



■ **Susan swims across a river at a bearing of N 75° E at a speed of 0.5 mi/h. The river's current moves due east at 1 mi/h. Find Susan's actual speed to the nearest tenth and her direction to the nearest degree.**



$\cos 15° = \frac{x}{0.5}$, so $x \approx 0.48$.

$\sin 15° = \frac{y}{0.5}$, so $y \approx 0.13$.

Susan's vector is $\langle 0.48, 0.13\rangle$. The current is $\langle 1, 0\rangle$. Susan's actual speed is the magnitude of the resultant vector, $\langle 1.48, 0.13\rangle$.

$|\langle 1.48, 0.13\rangle| = \sqrt{(1.48)^2 + (0.13)^2} \approx 1.5$ mi/h

Her direction is $\tan^{-1}\left(\frac{0.13}{1.48}\right) \approx 5°$, or N 85° E.

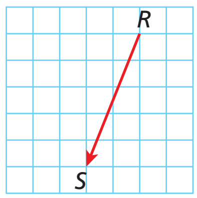
EXERCISES

Write each vector in component form.

28. $\overrightarrow{AB}$ with $A(5, 1)$ and $B(-2, 3)$

29. $\overrightarrow{MN}$ with $M(-2, 4)$ and $N(-1, -2)$

30. $\overrightarrow{RS}$



Draw each vector on a coordinate plane. Find its magnitude to the nearest tenth.

31. $\langle -5, -3\rangle$

32. $\langle -2, 0\rangle$

33. $\langle 4, -4\rangle$

Draw each vector on a coordinate plane. Find the direction of the vector to the nearest degree.

34. The velocity of a helicopter is given by the vector $\langle 4, 5\rangle$.

35. The force applied by a tugboat is given by the vector $\langle 7, 2\rangle$.

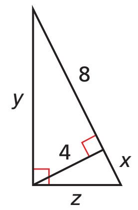
36. A plane flies at a constant speed of 600 mi/h at a bearing of N 55° E. There is a 50 mi/h crosswind blowing due east. What are the plane's actual speed and direction? Round the speed to the nearest tenth and the direction to the nearest degree.

CHAPTER 8

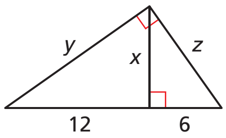
CHAPTER TEST

Find *x*, *y*, and *z*.

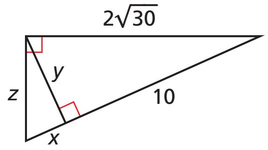
1.



2.



3.



Use a special right triangle to write each trigonometric ratio as a fraction.

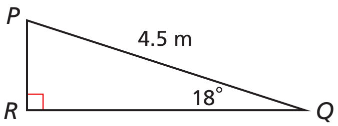
4. $\cos 60°$

5. $\sin 45°$

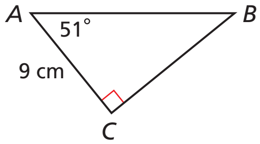
6. $\tan 60°$

Find each length. Round to the nearest hundredth.

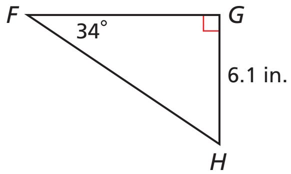
7. PR



8. AB



9. FG

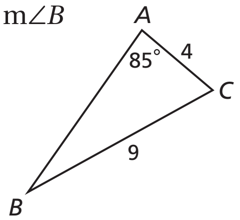


10. Nate built a skateboard ramp that covers a horizontal distance of 10 ft. The ramp rises a total of 3.5 ft. What angle does the ramp make with the ground? Round to the nearest degree.

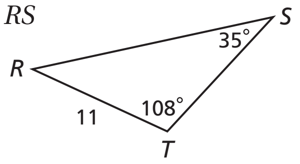
11. An observer at the top of a skyscraper sights a tour bus at an angle of depression of 61°. The skyscraper is 910 ft tall. What is the horizontal distance from the base of the skyscraper to the tour bus? Round to the nearest foot.

Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

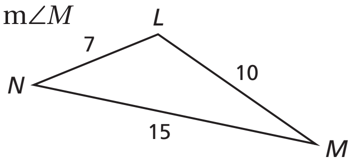
12. $m\angle B$



13. RS



14. $m\angle M$



Draw each vector on a coordinate plane. Find its magnitude to the nearest tenth.

15. $\langle 1, 3 \rangle$

16. $\langle -4, 1 \rangle$

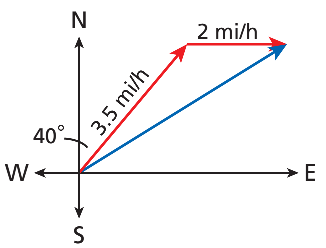
17. $\langle 2, -3 \rangle$

Draw each vector on a coordinate plane. Find the direction of the vector to the nearest degree.

18. The velocity of a plane is given by the vector $\langle 3, 5 \rangle$.

19. A wind velocity is given by the vector $\langle 4, 1 \rangle$.

20. Kate is rowing across a river. She sets out at a bearing of N 40° E and paddles at a constant rate of 3.5 mi/h. There is a 2 mi/h current moving due east. What are Kate's actual speed and direction? Round the speed to the nearest tenth and the direction to the nearest degree.



CHAPTER 8

College Entrance Exam Practice

FOCUS ON SAT MATHEMATICS SUBJECT TESTS

The SAT Mathematics Subject Tests each consist of 50 multiple-choice questions. You are not expected to have studied every topic on the SAT Mathematics Subject Tests, so some questions may be unfamiliar.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

HOT TIP! Though you can use a calculator on the SAT Mathematics Subject Tests, it may be faster to answer some questions without one. Remember to use test-taking strategies before you press buttons!

1. Let P be the acute angle formed by the line $-x + 4y = 12$ and the x-axis. What is the approximate measure of $\angle P$?

(A) $14°$

(B) $18°$

(C) $72°$

(D) $76°$

(E) $85°$

2. In right triangle DEF, $DE = 15$, $EF = 36$, and $DF = 39$. What is the cosine of $\angle F$?

(A) $\frac{5}{12}$

(B) $\frac{12}{5}$

(C) $\frac{5}{13}$

(D) $\frac{12}{13}$

(E) $\frac{13}{12}$

3. A triangle has angle measures of 19°, 61°, and 100°. What is the approximate length of the side opposite the 100° angle if the side opposite the 61° angle is 8 centimeters long?

(A) 2.5 centimeters

(B) 3 centimeters

(C) 9 centimeters

(D) 12 centimeters

(E) 13 centimeters

4. A swimmer jumps into a river and starts swimming directly across it at a constant velocity of 2 meters per second. The speed of the current is 7 meters per second. Given the current, what is the actual speed of the swimmer to the nearest tenth?

(A) 0.3 meters per second

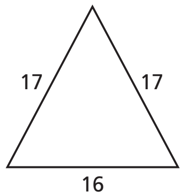
(B) 1.7 meters per second

(C) 5.0 meters per second

(D) 7.3 meters per second

(E) 9.0 meters per second

5. What is the approximate measure of the vertex angle of the isosceles triangle below?



(A) $28.1°$

(B) $56.1°$

(C) $62.0°$

(D) $112.2°$

(E) $123.9°$

CHAPTER 8

Strategies for Success

Any Question Type: Estimate

Once you find the answer to a test problem, take a few moments to check your answer by using estimation strategies. By doing so, you can verify that your final answer is reasonable.

EXAMPLE 1

Gridded Response **Find the geometric mean of 38 and 12 to the nearest hundredth.**

Let x be the geometric mean.

$x^2 = (38)(12) = 456$	*Def. of geometric mean*
$x \approx 21.35$	*Find the positive square root.*

Now use estimation to check that this answer is reasonable.

$x^2 \approx (40)(10) = 400$	*Round 38 to 40 and round 12 to 10.*
$x \approx 20$	*Find the positive square root.*

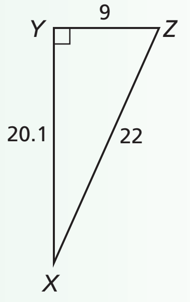
The estimate is close to the calculated answer, so 21.35 is a reasonable answer.

EXAMPLE 2

Multiple Choice **Which of the following is equal to sin X?**

Ⓐ 0.02　　Ⓒ 0.91

Ⓑ 0.41　　Ⓓ 2.44



Use a trigonometric ratio to find the answer.

$\sin X = \frac{YZ}{XZ}$	*The sine of an* $\angle$ *is* $\frac{\text{opp. leg}}{\text{hyp.}}$.
$\sin X = \frac{9}{22} \approx 0.41$	*Substitute the given values and simplify.*

Now use estimation to check that this answer is reasonable.

$\sin X \approx \frac{10}{20} \approx 0.5$	*Round 9 to 10 and round 22 to 20.*

The estimate is close to the calculated answer, so B is a reasonable answer.

HOT TIP! An extra minute spent checking your answers can result in a better test score.

Read each test item and answer the questions that follow.

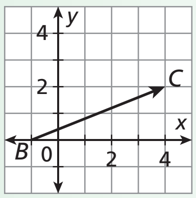
Item A

Gridded Response **A cell phone tower casts a shadow that is 121 ft long when the angle of elevation to the sun is 48°. How tall is the cell phone tower? Round to the nearest foot.**

1. A student estimated that the answer should be slightly greater than 121 by comparing $\tan 48°$ and $\tan 45°$. Explain why this estimation strategy works.
2. Describe how to use the inverse tangent function to estimate whether an answer of 134 ft makes sense.

Item B

Short Response $\overrightarrow{BC}$ **has an initial point of** $(-1, 0)$ **and a terminal point of** $(4, 2)$**.**



a. Write $\overrightarrow{BC}$ **in component form.**

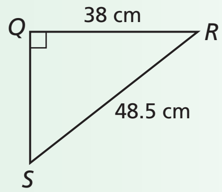
b. Find the magnitude of $\overrightarrow{BC}$**. Round to the nearest hundredth.**

c. Find the direction of $\overrightarrow{BC}$**. Round to the nearest degree.**

3. A student correctly found the magnitude of $\overrightarrow{BC}$ as $\sqrt{29}$. The student then calculated the value of this radical as 6.39. Explain how to use perfect squares to estimate the value of $\sqrt{29}$. Is 6.39 a reasonable answer?
4. A student calculated the measure of the angle the vector forms with a horizontal line as 68°. Use estimation to explain why this answer is not reasonable.

Item C

Multiple Choice **In** $\triangle QRS$**, what is the measure of** $\overline{SQ}$ **to the nearest tenth of a centimeter?**



(A) 9.3 centimeters

(B) 10.5 centimeters

(C) 30.1 centimeters

(D) 61.7 centimeters

5. A student calculated the answer as 30.1 cm. The student then used the diagram to estimate that SQ is more than half of RQ. So the student decided that his answer was reasonable. Is this estimation method a good way to check your answer? Why or why not?
6. Describe how to use estimation and the Pythagorean Theorem to check your answer to this problem.

Item D

Multiple Choice **The McCleods have a variable interest rate on their mortgage. The rate is 2.625% the first year and 4% the following year. The average interest rate is the geometric mean of these two rates. To the nearest hundredth of a percent, what is the average interest rate for their mortgage?**

(F) 1.38%

(G) 3.24%

(H) 3.89%

(J) 10.50%

7. Describe how to use estimation to show that choices **F** and **J** are unreasonable.
8. To find the answer, a student uses the equation $x^2 = (2.625)(4)$. Which compatible numbers should the student use to quickly check the answer?

CHAPTER 8

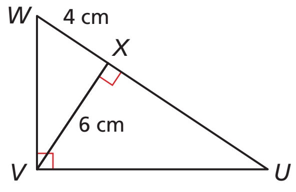
STANDARDIZED TEST PREP

go.hrw.com
Test Practice Online
KEYWORD: MG7 TestPrep

CUMULATIVE ASSESSMENT, CHAPTERS 1–8

Multiple Choice

1. What is the length of $\overline{UX}$ to the nearest centimeter?



(A) 3 centimeters
(B) 7 centimeters
(C) 9 centimeters
(D) 13 centimeters

2. $\triangle ABC$ is a right triangle. $m\angle A = 20°$, $m\angle B = 90°$, $AC = 8$, and $AB = 3$. Which expression can be used to find BC?

(F) $\dfrac{3}{\tan 70°}$
(G) $\dfrac{8}{\sin 20°}$
(H) $8 \tan 20°$
(J) $3 \cos 70°$

3. A slide at a park is 25 ft long, and the top of the slide is 10 ft above the ground. What is the approximate measure of the angle the slide makes with the ground?

(A) 21.8°
(B) 23.6°
(C) 66.4°
(D) 68.2°

4. Which of the following vectors is equal to the vector with an initial point at $(2, -1)$ and a terminal point at $(-2, 4)$?

(F) $\langle -4, -5 \rangle$
(G) $\langle -4, 5 \rangle$
(H) $\langle 5, -4 \rangle$
(J) $\langle 5, 4 \rangle$

5. Which statement is true by the Addition Property of Equality?

(A) If $3x + 6 = 9y$, then $x + 2 = 3y$.
(B) If $t = 1$ and $s = t + 5$, then $s = 6$.
(C) If $k + 1 = \ell + 2$, then $2k + 2 = 2\ell + 4$.
(D) If $a + 2 = 3b$, then $a + 5 = 3b + 3$.

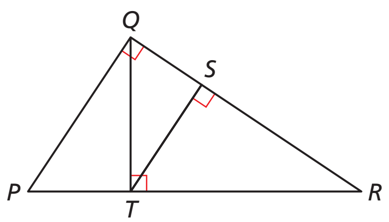
6. $\triangle ABC$ has vertices $A(-2, -2)$, $B(-3, 2)$, and $C(1, 3)$. Which translation produces an image with vertices at the coordinates $(-2, -2)$, $(2, -1)$, and $(-1, -6)$?

(F) $(x, y) \rightarrow (x + 1, y - 4)$
(G) $(x, y) \rightarrow (x + 2, y - 8)$
(H) $(x, y) \rightarrow (x - 3, y - 5)$
(J) $(x, y) \rightarrow (x - 4, y + 1)$

7. $\triangle ABC$ is a right triangle in which $m\angle A = 30°$ and $m\angle B = 60°$. Which of the following are possible lengths for the sides of this triangle?

(A) $AB = \sqrt{3}$, $AC = \sqrt{2}$, and $BC = 1$
(B) $AB = 4$, $AC = 2$, and $BC = 2\sqrt{3}$
(C) $AB = 6\sqrt{3}$, $AC = 27$, and $BC = 3\sqrt{3}$
(D) $AB = 8$, $AC = 4\sqrt{3}$, and $BC = 4$

8. Based on the figure below, which of the following similarity statements must be true?



(F) $\triangle PQR \sim \triangle TSR$
(G) $\triangle PQR \sim \triangle RTQ$
(H) $\triangle PQR \sim \triangle TSQ$
(J) $\triangle PQR \sim \triangle TQP$

9. $ABCD$ is a rhombus with vertices $A(1, 1)$ and $C(3, 4)$. Which of the following lines is parallel to diagonal $\overline{BD}$?

(A) $2x - 3y = 12$
(B) $2x + 3y = 12$
(C) $3x + 2y = 12$
(D) $3x - 4y = 12$

10. Which of the following is NOT equivalent to $\sin 60°$?

Ⓕ $\cos 30°$
Ⓗ $(\cos 60°)(\tan 60°)$
Ⓖ $\frac{\sqrt{3}}{2}$
Ⓙ $\frac{\tan 30°}{\sin 30°}$

11. *ABCDE* is a convex pentagon. $\angle A \cong \angle B \cong \angle C$, $\angle D \cong \angle E$, and $m\angle A = 2m\angle D$. What is the measure of $\angle C$?

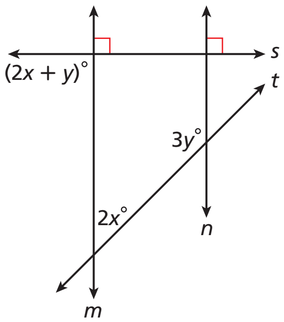
Ⓐ 67.5°
Ⓒ 154.2°
Ⓑ 135°
Ⓓ 225°

12. Which of the following sets of lengths can represent the side lengths of an obtuse triangle?

Ⓕ 4, 7.5, and 8.5
Ⓖ 7, 12, and 13
Ⓗ 9.5, 16.5, and 35
Ⓙ 36, 75, and 88

HOT TIP! **Be sure to correctly identify any pairs of parallel lines before using the Alternate Interior Angles Theorem or the Same-Side Interior Angles Theorem.**

13. What is the value of x?



Ⓐ 22.5
Ⓒ 90
Ⓑ 45
Ⓓ 135

Gridded Response

14. Find the next item in the pattern below.

1, 3, 7, 13, 21, ...

15. In $\triangle XYZ$, $\angle X$ and $\angle Z$ are remote interior angles of exterior $\angle XYT$. If $m\angle X = (x + 15)°$, $m\angle Z = (50 - 3x)°$, and $m\angle XYT = (4x - 25)°$, what is the value of x?

16. In $\triangle ABC$ and $\triangle DEF$, $\angle A \cong \angle F$. If $EF = 4.5$, $DF = 3$, and $AC = 1.5$, what length for $\overline{AB}$ would let you conclude that $\triangle ABC \sim \triangle FED$?

Short Response

17. A building casts a shadow that is 85 ft long when the angle of elevation to the sun is 34°.

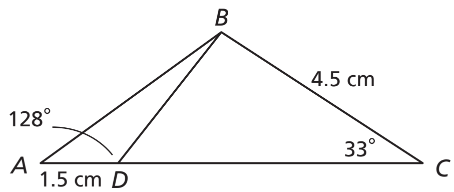
a. What is the height of the building? Round to the nearest inch and show your work.

b. What is the angle of elevation to the sun when the shadow is 42 ft 6 in. long? Round to the nearest tenth of a degree and show your work.

18. Use the figure to find each of the following. Round to the nearest tenth of a centimeter and show your work.

a. the length of $\overline{DC}$

b. the length of $\overline{AB}$



Extended Response

19. Tony and Paul are taking a vacation with their cousin, Greg. Tony and Paul live in the same house. Paul will go directly to the vacation spot, but Tony has to pick up Greg.

Tony travels 90 miles at a bearing of N 25° E to get to his cousin's house. He then travels due east for 50 miles to get to the vacation spot. Paul travels on one highway to get from his house to the vacation spot.

For each of the following, explain in words how you found your answer and round to the nearest tenth.

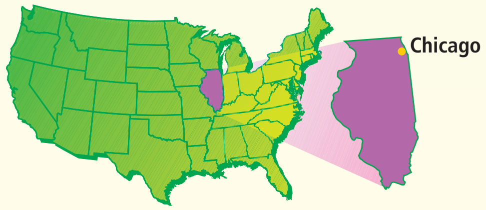
a. Write the vectors in component form for the route from Tony and Paul's house to their cousin's house and the route from their cousin's house to the vacation spot.

b. What are the direction and magnitude of Paul's direct route from his house to the vacation spot?

c. Tony and Paul leave the house at the same time and arrive at the vacation spot at the same time. If Tony traveled at an average speed of 50 mi/h, what was Paul's average speed?

Problem Solving on Location

ILLINOIS



The John Hancock Center

The 100-story John Hancock Center is one of the most distinctive features of the Chicago skyline. With its combination of stores, offices, and 49 floors of apartments, the John Hancock Center is the world's tallest multifunctional skyscraper.

Choose one or more strategies to solve each problem.

1. The building's observation deck is on the 94th floor, 1000 ft above street level. The deck is equipped with telescopes that offer close-up views of the surrounding city. Using one of the telescopes, a visitor spots a ship on Lake Michigan. The angle of depression to the ship is 10°. To the nearest foot, how far is the ship from the base of the building?

For 2–4, use the table.

2. At noon on May 15, the shadow of the John Hancock Center, including its antenna, is 818.2 ft long. Find the height of the building to the nearest foot.

3. How long is the shadow of the building at noon on October 15? Round to the nearest foot.

4. On which of the dates shown is the building's shadow the longest? What is the length of the shadow to the nearest foot?

Elevation of the Sun in Chicago, Illinois

Date	Angle of Elevation at Noon (°)
January 15	27
February 15	34
March 15	46
April 15	58
May 15	61
June 15	71
July 15	70
August 15	62
September 15	51
October 15	39
November 15	29
December 15	25

Ernest Hemingway's Birthplace

The Nobel Prize-winning author Ernest Hemingway (1899–1961) was born in Oak Park, Illinois. Visitors to Oak Park, a suburb of Chicago, can tour the home where Hemingway was born and spent much of his childhood. Thanks to a recent restoration, the house appears just as it did when Hemingway lived there.

Problem Solving Strategies

Draw a Diagram
Make a Model
Guess and Test
Work Backward
Find a Pattern
Make a Table
Solve a Simpler Problem
Use Logical Reasoning
Use a Venn Diagram
Make an Organized List

Choose one or more strategies to solve each problem.

1. The blueprint shown below was used during the restoration of the first floor of Hemingway's house. As part of the restoration project, a narrow border of wallpaper was placed along the edge of the ceiling around the perimeter of the dining room. Approximately how many feet of wallpaper were needed?

2. During the restoration, the floor of the parlor and living room was covered with red carpet. Estimate the number of square feet of carpet that were used.

3. Hemingway's childhood bedroom is located on the second floor of the house. The bedroom has a perimeter of 40 ft, and its length is 4 ft more than its width. Assuming the blueprint for the second floor uses the same scale as the blueprint below, what are the dimensions of the bedroom on the blueprint for the second floor?