CHAPTER 5

Properties and Attributes of Triangles

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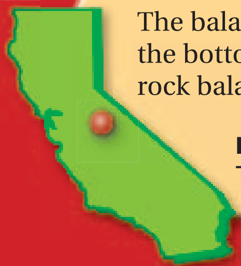
KEYWORD: MG7 ChProj



The balanced rock stack shows the bottom triangular shaped rock balancing on its vertex.

Balanced Rock Stack
Tuolumne Meadows, CA

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. angle bisector
2. conclusion
3. hypotenuse
4. leg of a right triangle
5. perpendicular bisector of a segment

A. the side opposite the right angle in a right triangle
B. a line that is perpendicular to a segment at its midpoint
C. the phrase following the word *then* in a conditional statement
D. one of the two sides that form the right angle in a right triangle
E. a line or ray that divides an angle into two congruent angles
F. the phrase following the word *if* in a conditional statement

Classify Triangles

Tell whether each triangle is acute, right, or obtuse.

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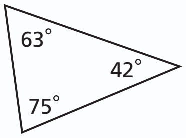

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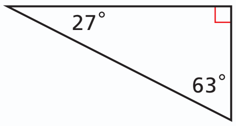

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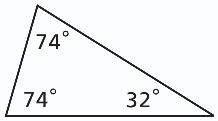

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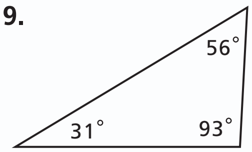


Squares and Square Roots

Simplify each expression.

10. 8^2
11. $(-12)^2$
12. $\sqrt{49}$
13. $-\sqrt{36}$

Simplify Radical Expressions

Simplify each expression.

14. $\sqrt{9 + 16}$
15. $\sqrt{100 - 36}$
16. $\sqrt{\frac{81}{25}}$
17. $\sqrt{2^2}$

Solve and Graph Inequalities

Solve each inequality. Graph the solutions on a number line.

18. $d + 5 < 1$
19. $-4 \leq w - 7$
20. $-3s \geq 6$
21. $-2 > \frac{m}{10}$

Logical Reasoning

Draw a conclusion from each set of true statements.

22. If two lines intersect, then they are not parallel.
Lines ℓ and m intersect at P.
23. If M is the midpoint of $\overline{AB}$, then $AM = MB$.
If $AM = MB$, then $AM = \frac{1}{2}AB$ and $MB = \frac{1}{2}AB$.

CHAPTER 5

Unpacking the Standards

The information below "unpacks" the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

California Standard	Academic Vocabulary	Chapter Concept
2.0 Students write geometric proofs, including proofs by contradiction. (Lessons 5-1, 5-2, 5-5, 5-6)	**contradiction** a statement that disagrees or conflicts with a known fact	You prove and use theorems about perpendicular bisectors, angle bisectors, and inequalities in triangles. You learn that a proof by contradiction is also called an *indirect proof* because you take a roundabout way to prove something.
6.0 Students know and are able to use the triangle inequality theorem. (Lessons 5-5, 5-7)	**able to use** have the skills you need	You use the Pythagorean Theorem and its converse to solve problems. You decide if three lengths can be the side lengths of triangles and then classify the triangles.
14.0 Students prove the Pythagorean theorem. (Lesson 5-7) (Lab 5-7)	**prove** explain why something is true	You prove the Pythagorean Theorem and learn which side lengths to substitute for *a*, *b*, and *c*. You also decide if *a*, *b*, and *c* make a Pythagorean triple.
15.0 Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles. (Lesson 5-7)	**determine** find	You use the Pythagorean Theorem to find the unknown measure of a side of a right triangle and to find the distance between two points.
20.0 Students know and are able to use angle and side relationships in problems with special right triangles, such as 30°, 60°, and 90° triangles and 45°, 45°, and 90° triangles. (Lesson 5-8)	**relationships** connections **special** particular	You find the side lengths of 45°-45°-90° and 30°-60°-90° triangles by using the special relationships between their measures.

Standards 1.0, 12.0, 16.0, and 17.0 are also covered in this chapter. To see these standards unpacked, go to Chapter 1, p. 4 and Chapter 4, p. 214

Reading and Writing Math

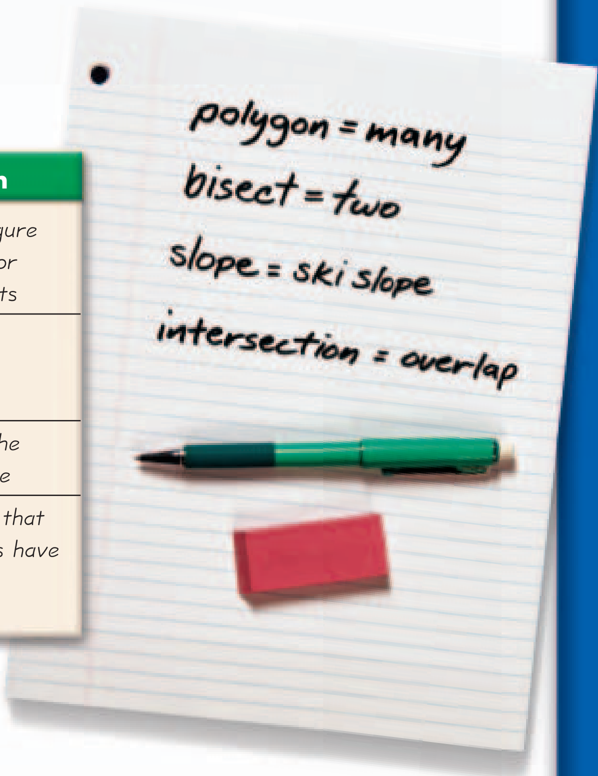
CHAPTER 5

Reading Strategy: Learn Math Vocabulary

Mathematics has a vocabulary all its own. To learn and remember new vocabulary words, use the following study strategies.

- Try to figure out the meaning of a new word based on its context.
- Use a dictionary to look up the root word or prefix.
- Relate the new word to familiar everyday words.

Once you know what a word means, write its definition in your own words.



Term	Study Notes	Definition
Polygon	*The prefix poly means "many" or "several."*	*A closed plane figure formed by three or more line segments*
Bisect	*The prefix bi means "two."*	*Cuts or divides something into two equal parts*
Slope	*Think of a ski slope.*	*The measure of the steepness of a line*
Intersection	*The root word intersect means "to overlap." Think of the intersection of two roads.*	*The set of points that two or more lines have in common*

Try This

Complete the table below.

	Term	Study Notes	Definition
1.	Trinomial		
2.	Equiangular triangle		
3.	Perimeter		
4.	Deductive reasoning		

Use the given prefix and its meanings to write a definition for each vocabulary word.

5. *circum* (about, around); circumference

6. *co* (with, together); coplanar

7. *trans* (across, beyond, through); translation

5-1 Perpendicular and Angle Bisectors

Objectives

Prove and apply theorems about perpendicular bisectors.

Prove and apply theorems about angle bisectors.

Vocabulary

equidistant
locus

California Standards

2.0 Students write geometric proofs, including proofs by contradiction.

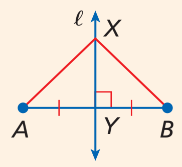
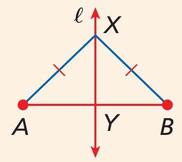
Who uses this?

The suspension and steering lines of a parachute keep the sky diver centered under the parachute. (See Example 3.)

When a point is the same distance from two or more objects, the point is said to be **equidistant** from the objects. Triangle congruence theorems can be used to prove theorems about equidistant points.

Know it! Note

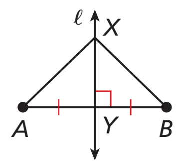
Theorems Distance and Perpendicular Bisectors

THEOREM		HYPOTHESIS	CONCLUSION
5-1-1	**Perpendicular Bisector Theorem** If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.	 $\overline{XY} \perp \overline{AB}$, $\overline{YA} \cong \overline{YB}$	$XA = XB$
5-1-2	**Converse of the Perpendicular Bisector Theorem** If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.	 $XA = XB$	$\overline{XY} \perp \overline{AB}$, $\overline{YA} \cong \overline{YB}$

You will prove Theorem 5-1-2 in Exercise 30.

PROOF

Perpendicular Bisector Theorem



Given: ℓ is the perpendicular bisector of $\overline{AB}$.
Prove: $XA = XB$

Proof:

Since ℓ is the perpendicular bisector of $\overline{AB}$, $\ell \perp \overline{AB}$ and Y is the midpoint of $\overline{AB}$. By the definition of perpendicular, $\angle AYX$ and $\angle BYX$ are right angles and $\angle AYX \cong \angle BYX$. By the definition of midpoint, $\overline{AY} \cong \overline{BY}$. By the Reflexive Property of Congruence, $\overline{XY} \cong \overline{XY}$. So $\triangle AYX \cong \triangle BYX$ by SAS, and $\overline{XA} \cong \overline{XB}$ by CPCTC. Therefore $XA = XB$ by the definition of congruent segments.

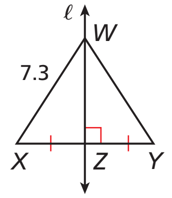
Reading Math

The word *locus* comes from the Latin word for location. The plural of *locus* is *loci*, which is pronounced LOW-sigh.

A **locus** is a set of points that satisfies a given condition. The perpendicular bisector of a segment can be defined as the locus of points in a plane that are equidistant from the endpoints of the segment.

EXAMPLE 1 Applying the Perpendicular Bisector Theorem and Its Converse

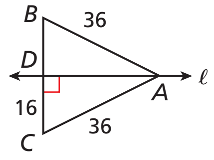
Find each measure.



A *YW*

$YW = XW$ — *⊥ Bisector Thm.*

$YW = 7.3$ — *Substitute 7.3 for XW.*



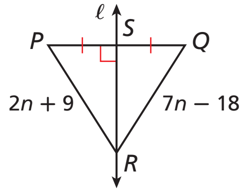
B *BC*

Since $AB = AC$ and $\ell \perp \overline{BC}$, ℓ is the perpendicular bisector of $\overline{BC}$ by the Converse of the Perpendicular Bisector Theorem.

$BC = 2CD$ — *Def. of seg. bisector*

$BC = 2(16) = 32$ — *Substitute 16 for CD.*

Algebra



C *PR*

$PR = RQ$ — *⊥ Bisector Thm.*

$2n + 9 = 7n - 18$ — *Substitute the given values.*

$9 = 5n - 18$ — *Subtract 2n from both sides.*

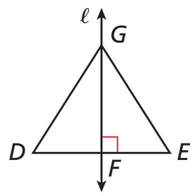
$27 = 5n$ — *Add 18 to both sides.*

$5.4 = n$ — *Divide both sides by 5.*

So $PR = 2(5.4) + 9 = 19.8$.

CHECK IT OUT!

Find each measure.



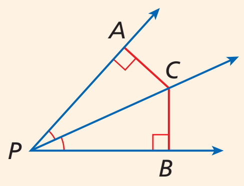
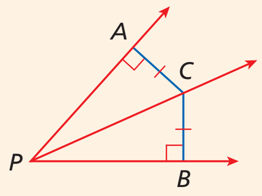
1a. Given that line ℓ is the perpendicular bisector of $\overline{DE}$ and $EG = 14.6$, find DG.

1b. Given that $DE = 20.8$, $DG = 36.4$, and $EG = 36.4$, find EF.

Remember that the distance between a point and a line is the length of the perpendicular segment from the point to the line.

Know it! Note

Theorems — Distance and Angle Bisectors

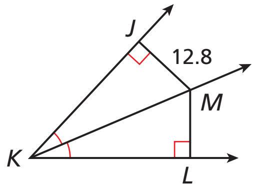
THEOREM	HYPOTHESIS	CONCLUSION
5-1-3 Angle Bisector Theorem If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.	 $\angle APC \cong \angle BPC$	$AC = BC$
5-1-4 Converse of the Angle Bisector Theorem If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.	 $AC = BC$	$\angle APC \cong \angle BPC$

You will prove these theorems in Exercises 31 and 40.

Based on these theorems, an angle bisector can be defined as the locus of all points in the interior of the angle that are equidistant from the sides of the angle.

EXAMPLE 2 Applying the Angle Bisector Theorems

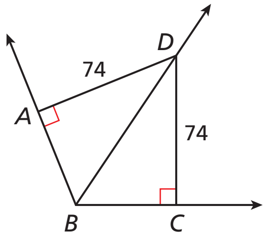
Find each measure.



A LM

$LM = JM$ — *∠ Bisector Thm.*

$LM = 12.8$ — *Substitute 12.8 for JM.*



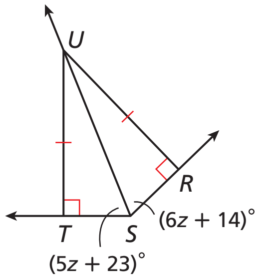
B **m∠*ABD*, given that m∠*ABC* = 112°**

Since $AD = DC$, $\overline{AD} \perp \overline{BA}$, and $\overline{DC} \perp \overline{BC}$, $\overrightarrow{BD}$ bisects $\angle ABC$ by the Converse of the Angle Bisector Theorem.

$m\angle ABD = \frac{1}{2} m\angle ABC$ — *Def. of ∠ bisector*

$m\angle ABD = \frac{1}{2}(112°) = 56°$ — *Substitute 112° for m∠ABC.*

Algebra



C **m∠*TSU***

Since $RU = UT$, $\overline{RU} \perp \overline{SR}$, and $\overline{UT} \perp \overline{ST}$, $\overrightarrow{SU}$ bisects $\angle RST$ by the Converse of the Angle Bisector Theorem.

$m\angle RSU = m\angle TSU$ — *Def. of ∠ bisector*

$6z + 14 = 5z + 23$ — *Substitute the given values.*

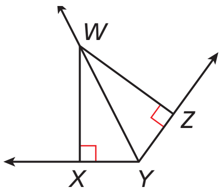
$z + 14 = 23$ — *Subtract 5z from both sides.*

$z = 9$ — *Subtract 14 from both sides.*

So $m\angle TSU = [5(9) + 23]° = 68°$.

CHECK IT OUT!

Find each measure.

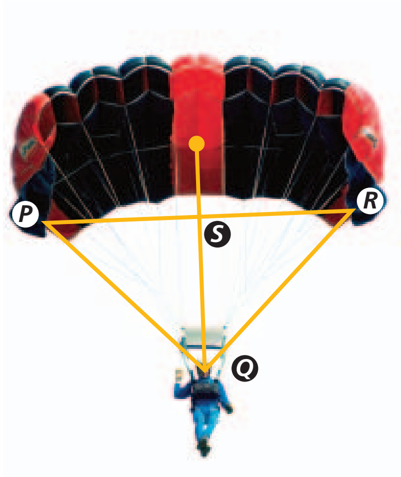


2a. Given that $\overrightarrow{YW}$ bisects $\angle XYZ$ and $WZ = 3.05$, find WX.

2b. Given that $m\angle WYZ = 63°$, $XW = 5.7$, and $ZW = 5.7$, find $m\angle XYZ$.

EXAMPLE 3 *Parachute Application*

Each pair of suspension lines on a parachute are the same length and are equally spaced from the center of the chute. How do these lines keep the sky diver centered under the parachute?



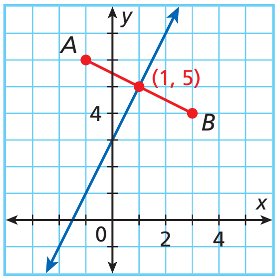
It is given that $\overline{PQ} \cong \overline{RQ}$. So Q is on the perpendicular bisector of $\overline{PR}$ by the Converse of the Perpendicular Bisector Theorem. Since S is the midpoint of $\overline{PR}$, $\overline{QS}$ is the perpendicular bisector of $\overline{PR}$. Therefore the sky diver remains centered under the chute.

CHECK IT OUT! 3. S is equidistant from each pair of suspension lines. What can you conclude about $\overrightarrow{QS}$?

EXAMPLE 4 Writing Equations of Bisectors in the Coordinate Plane

Algebra

Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints $A(-1, 6)$ and $B(3, 4)$.



Step 1 Graph $\overline{AB}$.

The perpendicular bisector of $\overline{AB}$ is perpendicular to $\overline{AB}$ at its midpoint.

Step 2 Find the midpoint of $\overline{AB}$.

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ *Midpoint formula*

mdpt. of $\overline{AB} = \left(\frac{-1+3}{2}, \frac{6+4}{2}\right) = (1, 5)$

Step 3 Find the slope of the perpendicular bisector.

slope $= \frac{y_2 - y_1}{x_2 - x_1}$ *Slope formula*

slope of $\overline{AB} = \frac{4-6}{3-(-1)} = \frac{-2}{4} = -\frac{1}{2}$

Since the slopes of perpendicular lines are opposite reciprocals, the slope of the perpendicular bisector is 2.

Step 4 Use point-slope form to write an equation.

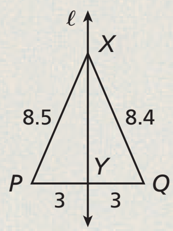
The perpendicular bisector of $\overline{AB}$ has slope 2 and passes through $(1, 5)$.

$y - y_1 = m(x - x_1)$ *Point-slope form*

$y - 5 = 2(x - 1)$ *Substitute 5 for y_1, 2 for m, and 1 for x_1.*

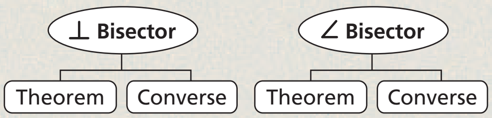
CHECK IT OUT! 4. Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints $P(5, 2)$ and $Q(1, -4)$.

THINK AND DISCUSS



1. Is line ℓ a bisector of $\overline{PQ}$? Is it a perpendicular bisector of $\overline{PQ}$? Explain.
2. Suppose that M is in the interior of $\angle JKL$ and $MJ = ML$. Can you conclude that $\overrightarrow{KM}$ is the bisector of $\angle JKL$? Explain.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write the theorem or its converse in your own words.

Know it! Note



5-1 Exercises

California Standards
2.0, 7AF1.0, 7AF4.1, 1A7.0

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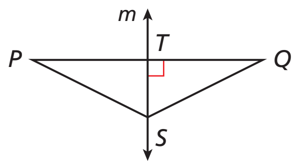
GUIDED PRACTICE

1. **Vocabulary** A __?__ is the *locus* of all points in a plane that are *equidistant* from the endpoints of a segment. (*perpendicular bisector* or *angle bisector*)

SEE EXAMPLE 1
p. 301

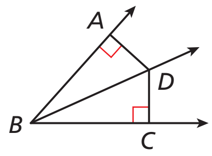
Use the diagram for Exercises 2–4.



2. Given that $PS = 53.4$, $QT = 47.7$, and $QS = 53.4$, find PQ.
3. Given that m is the perpendicular bisector of $\overline{PQ}$ and $SQ = 25.9$, find SP.
4. Given that m is the perpendicular bisector of $\overline{PQ}$, $PS = 4a$, and $QS = 2a + 26$, find QS.

SEE EXAMPLE 2
p. 302

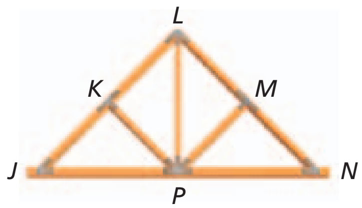
Use the diagram for Exercises 5–7.



5. Given that $\overrightarrow{BD}$ bisects $\angle ABC$ and $CD = 21.9$, find AD.
6. Given that $AD = 61$, $CD = 61$, and $m\angle ABC = 48°$, find $m\angle CBD$.
7. Given that $DA = DC$, $m\angle DBC = (10y + 3)°$, and $m\angle DBA = (8y + 10)°$, find $m\angle DBC$.

SEE EXAMPLE 3
p. 302

8. **Carpentry** For a king post truss to be constructed correctly, P must lie on the bisector of $\angle JLN$. How can braces $\overline{PK}$ and $\overline{PM}$ be used to ensure that P is in the proper location?



SEE EXAMPLE 4
p. 303

Write an equation in point-slope form for the perpendicular bisector of the segment with the given endpoints.

9. $M(-5, 4)$, $N(1, -2)$
10. $U(2, -6)$, $V(4, 0)$
11. $J(-7, 5)$, $K(1, -1)$

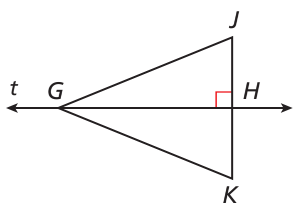
PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12–14	1
15–17	2
18	3
19–21	4

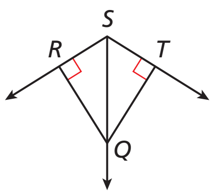
Extra Practice
Skills Practice p. S12
Application Practice p. S32

Use the diagram for Exercises 12–14.



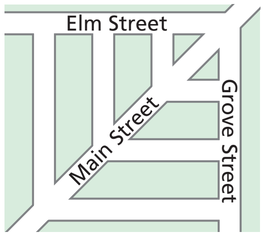
12. Given that line t is the perpendicular bisector of $\overline{JK}$ and $GK = 8.25$, find GJ.
13. Given that line t is the perpendicular bisector of $\overline{JK}$, $JG = x + 12$, and $KG = 3x - 17$, find KG.
14. Given that $GJ = 70.2$, $JH = 26.5$, and $GK = 70.2$, find JK.

Use the diagram for Exercises 15–17.



15. Given that $m\angle RSQ = m\angle TSQ$ and $TQ = 1.3$, find RQ.
16. Given that $m\angle RSQ = 58°$, $RQ = 49$, and $TQ = 49$, find $m\angle RST$.
17. Given that $RQ = TQ$, $m\angle QSR = (9a + 48)°$, and $m\angle QST = (6a + 50)°$, find $m\angle QST$.

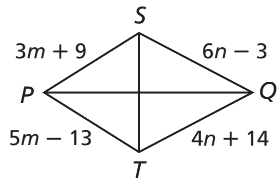
18. City Planning The planners for a new section of the city want every location on Main Street to be equidistant from Elm Street and Grove Street. How can the planners ensure that this is the case?



Write an equation in point-slope form for the perpendicular bisector of the segment with the given endpoints.

19. $E(-4, -7)$, $F(0, 1)$ **20.** $X(-7, 5)$, $Y(-1, -1)$ **21.** $M(-3, -1)$, $N(7, -5)$

22. $\overline{PQ}$ is the perpendicular bisector of $\overline{ST}$. Find the values of m and n.

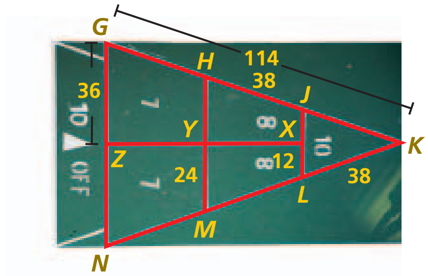


LINK

Shuffleboard

One of the first recorded shuffleboard games was played in England in 1532. In this game, Henry VIII supposedly lost £9 to Lord William.

Shuffleboard **Use the diagram of a shuffleboard and the following information to find each length in Exercises 23–28.**



$\overline{KZ}$ is the perpendicular bisector of $\overline{GN}$, $\overline{HM}$, and $\overline{JL}$.

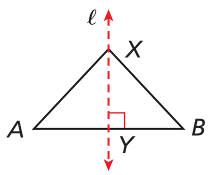
23. JK **24.** GN **25.** ML

26. HY **27.** JL **28.** NM

29. Multi-Step The endpoints of $\overline{AB}$ are $A(-2, 1)$ and $B(4, -3)$. Find the coordinates of a point C other than the midpoint of $\overline{AB}$ that is on the perpendicular bisector of $\overline{AB}$. How do you know it is on the perpendicular bisector?

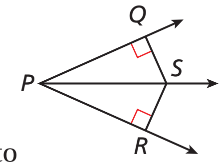
30. Write a paragraph proof of the Converse of the Perpendicular Bisector Theorem.

Given: $AX = BX$
Prove: X is on the perpendicular bisector of $\overline{AB}$.
Plan: Draw ℓ perpendicular to $\overline{AB}$ through X. Show that $\triangle AYX \cong \triangle BYX$ and thus $\overline{AY} \cong \overline{BY}$. By definition, ℓ is the perpendicular bisector of $\overline{AB}$.



31. Write a two-column proof of the Angle Bisector Theorem.

Given: $\overrightarrow{PS}$ bisects $\angle QPR$. $\overline{SQ} \perp \overrightarrow{PQ}$, $\overline{SR} \perp \overrightarrow{PR}$
Prove: $SQ = SR$
Plan: Use the definitions of angle bisector and perpendicular to identify two pairs of congruent angles. Show that $\triangle PQS \cong \triangle PRS$ and thus $\overline{SQ} \cong \overline{SR}$.



32. Critical Thinking In the Converse of the Angle Bisector Theorem, why is it important to say that the point must be in the interior of the angle?

Concept Connection

33. This problem will prepare you for the Concept Connection on page 328.
A music company has stores in Abby $(-3, -2)$ and Cardenas $(3, 6)$. Each unit in the coordinate plane represents 1 mile.

a. The company president wants to build a warehouse that is equidistant from the two stores. Write an equation that describes the possible locations.

b. A straight road connects Abby and Cardenas. The warehouse will be located exactly 4 miles from the road. How many locations are possible?

c. To the nearest tenth of a mile, how far will the warehouse be from each store?

34. **Write About It** How is the construction of the perpendicular bisector of a segment related to the Converse of the Perpendicular Bisector Theorem?

STANDARDIZED TEST PREP

35. If $\overleftrightarrow{JK}$ is perpendicular to $\overline{XY}$ at its midpoint M, which statement is true?

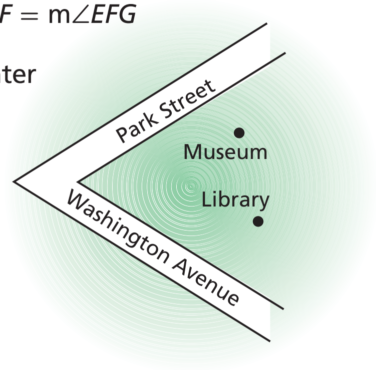
Ⓐ $JX = KY$ Ⓑ $JX = KX$ Ⓒ $JM = KM$ Ⓓ $JX = JY$

36. What information is needed to conclude that $\overrightarrow{EF}$ is the bisector of $\angle DEG$?

Ⓕ $m\angle DEF = m\angle DEG$ Ⓗ $m\angle GED = m\angle GEF$

Ⓖ $m\angle FEG = m\angle DEF$ Ⓙ $m\angle DEF = m\angle EFG$

37. **Short Response** The city wants to build a visitor center in the park so that it is equidistant from Park Street and Washington Avenue. They also want the visitor center to be equidistant from the museum and the library. Find the point V where the visitor center should be built. Explain your answer.

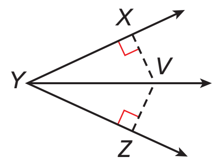


CHALLENGE AND EXTEND

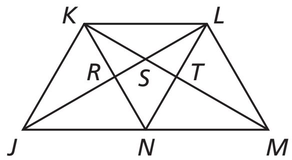
38. Consider the points $P(2, 0)$, $A(-4, 2)$, $B(0, -6)$, and $C(6, -3)$.
   a. Show that P is on the bisector of $\angle ABC$.
   b. Write an equation of the line that contains the bisector of $\angle ABC$.

39. Find the locus of points that are equidistant from the x-axis and y-axis.

40. Write a two-column proof of the Converse of the Angle Bisector Theorem.
   Given: $\overline{VX} \perp \overrightarrow{YX}$, $\overline{VZ} \perp \overrightarrow{YZ}$, $VX = VZ$
   Prove: $\overrightarrow{YV}$ bisects $\angle XYZ$.



41. Write a paragraph proof.
   Given: $\overline{KN}$ is the perpendicular bisector of $\overline{JL}$.
   $\overline{LN}$ is the perpendicular bisector of $\overline{KM}$.
   $\overline{JR} \cong \overline{MT}$
   Prove: $\angle JKM \cong \angle MLJ$



SPIRAL REVIEW

42. Lyn bought a sweater for \$16.95. The change c that she received can be described by $c = t - 16.95$, where t is the amount of money Lyn gave the cashier. What is the dependent variable? *(Previous course)*

For the points $R(-4, 2)$, $S(1, 4)$, $T(3, -1)$, and $V(-7, -5)$, determine whether the lines are parallel, perpendicular, or neither. *(Lesson 3-5)*

43. $\overleftrightarrow{RS}$ and $\overleftrightarrow{VT}$ 44. $\overleftrightarrow{RV}$ and $\overleftrightarrow{ST}$ 45. $\overleftrightarrow{RT}$ and $\overleftrightarrow{VR}$

Write the equation of each line in slope-intercept form. *(Lesson 3-6)*

46. the line through the points $(1, -1)$ and $(2, -9)$

47. the line with slope -0.5 through $(10, -15)$

48. the line with x-intercept -4 and y-intercept 5

5-2 Bisectors of Triangles

Objectives
Prove and apply properties of perpendicular bisectors of a triangle.

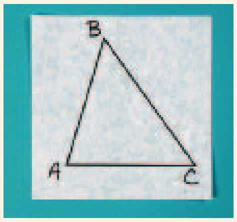
Prove and apply properties of angle bisectors of a triangle.

Vocabulary
concurrent
point of concurrency
circumcenter of a triangle
circumscribed
incenter of a triangle
inscribed

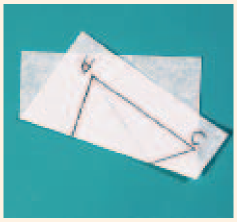
Who uses this?
An event planner can use perpendicular bisectors of triangles to find the best location for a fireworks display. (See Example 4.)

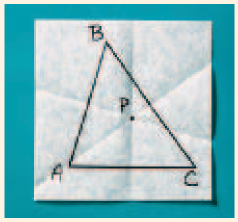
Since a triangle has three sides, it has three perpendicular bisectors. When you construct the perpendicular bisectors, you find that they have an interesting property.

Construction Circumcenter of a Triangle

1. 

Draw a large scalene acute triangle *ABC* on a piece of patty paper.

2. 
Fold the perpendicular bisector of each side.

3. 

Label the point where the three perpendicular bisectors intersect as *P*.

Helpful Hint
The perpendicular bisector of a side of a triangle does not always pass through the opposite vertex.

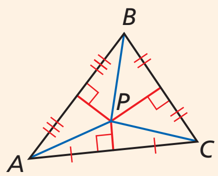
When three or more lines intersect at one point, the lines are said to be **concurrent**. The **point of concurrency** is the point where they intersect. In the construction, you saw that the three perpendicular bisectors of a triangle are concurrent. This point of concurrency is the **circumcenter of the triangle**.

Know it! Note

Theorem 5-2-1 **Circumcenter Theorem**

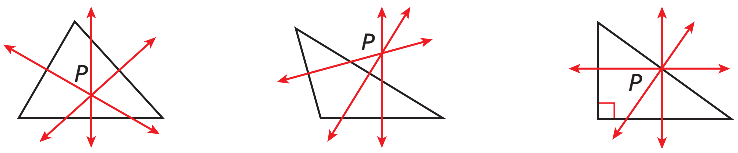
The circumcenter of a triangle is equidistant from the vertices of the triangle.

$$PA = PB = PC$$



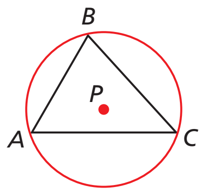
California Standards
2.0 Students write geometric proofs, including proofs by contradiction.
Also covered: **16.0**

The circumcenter can be inside the triangle, outside the triangle, or on the triangle.



Acute triangle Obtuse triangle Right triangle

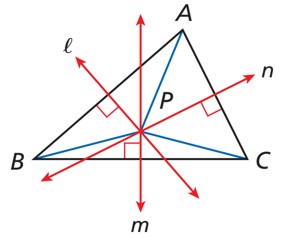
The circumcenter of $\triangle ABC$ is the center of its *circumscribed* circle. A circle that contains all the vertices of a polygon is **circumscribed** about the polygon.



PROOF

Circumcenter Theorem

Given: Lines ℓ, m, and n are the perpendicular bisectors of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively.
Prove: $PA = PB = PC$

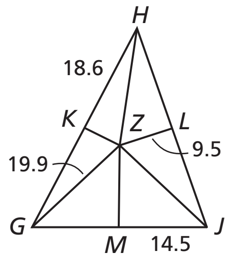


Proof:

P is the circumcenter of $\triangle ABC$. Since P lies on the perpendicular bisector of $\overline{AB}$, $PA = PB$ by the Perpendicular Bisector Theorem. Similarly, P also lies on the perpendicular bisector of $\overline{BC}$, so $PB = PC$. Therefore $PA = PB = PC$ by the Transitive Property of Equality.

EXAMPLE 1 Using Properties of Perpendicular Bisectors

$\overline{KZ}$, $\overline{LZ}$, and $\overline{MZ}$ are the perpendicular bisectors of $\triangle GHJ$. Find HZ.



Z is the circumcenter of $\triangle GHJ$. By the Circumcenter Theorem, Z is equidistant from the vertices of $\triangle GHJ$.

$HZ = GZ$	*Circumcenter Thm.*
$HZ = 19.9$	*Substitute 19.9 for GZ.*

CHECK IT OUT! **Use the diagram above. Find each length.**

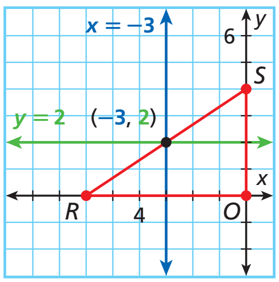
1a. GM **1b.** GK **1c.** JZ

EXAMPLE 2 Finding the Circumcenter of a Triangle

Algebra

Find the circumcenter of $\triangle RSO$ with vertices $R(-6, 0)$, $S(0, 4)$, and $O(0, 0)$.

Step 1 Graph the triangle.



Step 2 Find equations for two perpendicular bisectors.

Since two sides of the triangle lie along the axes, use the graph to find the perpendicular bisectors of these two sides. The perpendicular bisector of $\overline{RO}$ is $x = -3$, and the perpendicular bisector of $\overline{OS}$ is $y = 2$.

Step 3 Find the intersection of the two equations.

The lines $x = -3$ and $y = 2$ intersect at $(-3, 2)$, the circumcenter of $\triangle RSO$.

CHECK IT OUT! **2.** Find the circumcenter of $\triangle GOH$ with vertices $G(0, -9)$, $O(0, 0)$, and $H(8, 0)$.

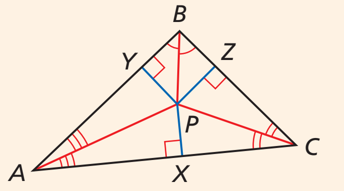
A triangle has three angles, so it has three angle bisectors. The angle bisectors of a triangle are also concurrent. This point of concurrency is the **incenter of the triangle**.

Know it! Note

Theorem 5-2-2 **Incenter Theorem**

The incenter of a triangle is equidistant from the sides of the triangle.

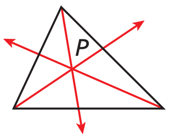
$PX = PY = PZ$



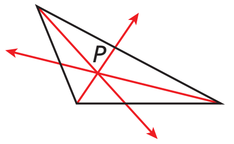
You will prove Theorem 5-2-2 in Exercise 35.

Remember!
The distance between a point and a line is the length of the perpendicular segment from the point to the line.

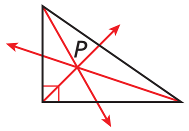
Unlike the circumcenter, the incenter is always inside the triangle.



Acute triangle

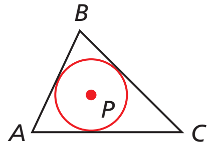


Obtuse triangle



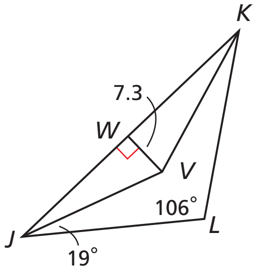
Right triangle

The incenter is the center of the triangle's *inscribed circle*. A circle **inscribed** in a polygon intersects each line that contains a side of the polygon at exactly one point.



EXAMPLE 3 Using Properties of Angle Bisectors

$\overline{JV}$ and $\overline{KV}$ are angle bisectors of $\triangle JKL$. Find each measure.



A **the distance from V to $\overline{KL}$**

V is the incenter of $\triangle JKL$. By the Incenter Theorem, V is equidistant from the sides of $\triangle JKL$.

The distance from V to $\overline{JK}$ is 7.3.
So the distance from V to $\overline{KL}$ is also 7.3.

B **$m\angle VKL$**

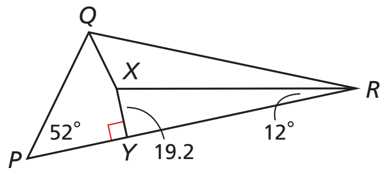
$m\angle KJL = 2m\angle VJL$	*$\overline{JV}$ is the bisector of $\angle KJL$.*
$m\angle KJL = 2(19°) = 38°$	*Substitute 19° for $m\angle VJL$.*
$m\angle KJL + m\angle JLK + m\angle JKL = 180°$	*△ Sum Thm.*
$38 + 106 + m\angle JKL = 180$	*Substitute the given values.*
$m\angle JKL = 36°$	*Subtract 144° from both sides.*
$m\angle VKL = \frac{1}{2}m\angle JKL$	*$\overline{KV}$ is the bisector of $\angle JKL$.*
$m\angle VKL = \frac{1}{2}(36°) = 18°$	*Substitute 36° for $m\angle JKL$.*

CHECK IT OUT!

$\overline{QX}$ and $\overline{RX}$ are angle bisectors of $\triangle PQR$. Find each measure.

3a. the distance from X to $\overline{PQ}$

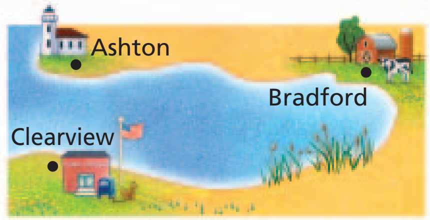
3b. $m\angle PQX$



EXAMPLE 4

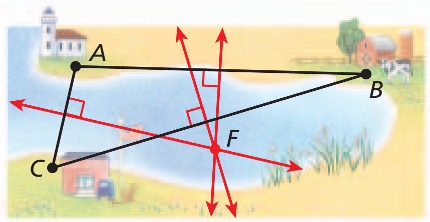
Community Application

For the next Fourth of July, the towns of Ashton, Bradford, and Clearview will launch a fireworks display from a boat in the lake. Draw a sketch to show where the boat should be positioned so that it is the same distance from all three towns. Justify your sketch.



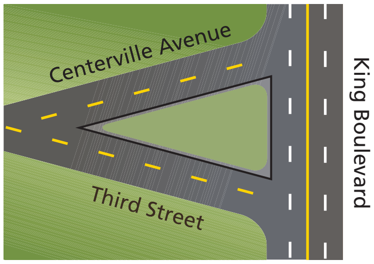
Let the three towns be vertices of a triangle. By the Circumcenter Theorem, the circumcenter of the triangle is equidistant from the vertices.

Trace the outline of the lake. Draw the triangle formed by the towns. To find the circumcenter, find the perpendicular bisectors of each side. The position of the boat is the circumcenter, F.



CHECK IT OUT!

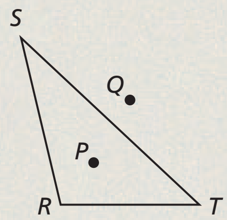
4. A city plans to build a firefighters' monument in the park between three streets. Draw a sketch to show where the city should place the monument so that it is the same distance from all three streets. Justify your sketch.



THINK AND DISCUSS

1. Sketch three lines that are concurrent.

2. P and Q are the circumcenter and incenter of $\triangle RST$, but not necessarily in that order. Which point is the circumcenter? Which point is the incenter? Explain how you can tell without constructing any of the bisectors.



Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Fill in the blanks to make each statement true.

	Circumcenter	Incenter
Definition	The point of concurrency of the __?__	The point of concurrency of the __?__
Distance	Equidistant from the __?__	Equidistant from the __?__
Location (Inside, Outside, or On)	Can be __?__ the triangle	__?__ the triangle

5-2 Exercises

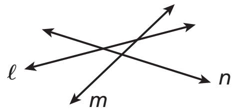
California Standards
2.0, 16.0, 7AF4.1, 7MG3.2, 7MR3.3, 1A2.0, 1A7.0

go.hrw.com
Homework Help Online
KEYWORD: MG7 5-2
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. Explain why lines ℓ, m, and n are NOT *concurrent*.

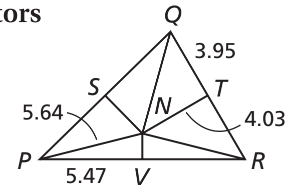


2. A circle that contains all the vertices of a polygon is ___?___ the polygon. (*circumscribed about* or *inscribed in*)

SEE EXAMPLE 1 p. 308

$\overline{SN}$, $\overline{TN}$, and $\overline{VN}$ are the perpendicular bisectors of $\triangle PQR$. Find each length.

3. NR
4. RV
5. TR
6. QN



SEE EXAMPLE 2 p. 308

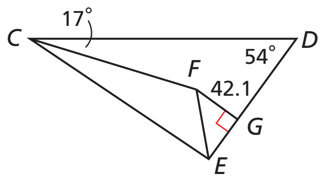
Multi-Step Find the circumcenter of a triangle with the given vertices.

7. $O(0, 0)$, $K(0, 12)$, $L(4, 0)$
8. $A(-7, 0)$, $O(0, 0)$, $B(0, -10)$

SEE EXAMPLE 3 p. 309

$\overline{CF}$ and $\overline{EF}$ are angle bisectors of $\triangle CDE$. Find each measure.

9. the distance from F to $\overline{CD}$
10. $m\angle FED$



SEE EXAMPLE 4 p. 310

11. **Design** The designer of the Newtown High School pennant wants the circle around the bear emblem to be as large as possible. Draw a sketch to show where the center of the circle should be located. Justify your sketch.

PRACTICE AND PROBLEM SOLVING

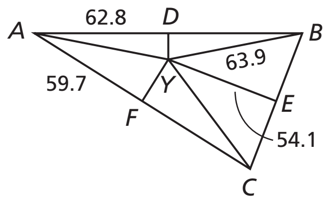
Independent Practice

For Exercises	See Example
12–15	1
16–17	2
18–19	3
20	4

Extra Practice
Skills Practice p. S12
Application Practice p. S32

$\overline{DY}$, $\overline{EY}$, and $\overline{FY}$ are the perpendicular bisectors of $\triangle ABC$. Find each length.

12. CF
13. YC
14. DB
15. AY

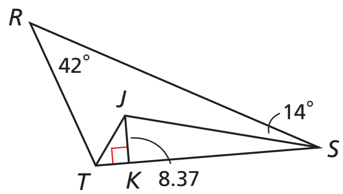


Multi-Step Find the circumcenter of a triangle with the given vertices.

16. $M(-5, 0)$, $N(0, 14)$, $O(0, 0)$
17. $O(0, 0)$, $V(0, 19)$, $W(-3, 0)$

$\overline{TJ}$ and $\overline{SJ}$ are angle bisectors of $\triangle RST$. Find each measure.

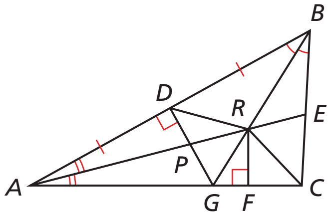
18. the distance from J to $\overline{RS}$
19. $m\angle RTJ$



20. **Business** A company repairs photocopiers in Harbury, Gaspar, and Knowlton. Draw a sketch to show where the company should locate its office so that it is the same distance from each city. Justify your sketch.

21. **Critical Thinking** If M is the incenter of $\triangle JKL$, explain why $\angle JML$ cannot be a right angle.

Tell whether each segment lies on a perpendicular bisector, an angle bisector, or neither. Justify your answer.



22. $\overline{AE}$	23. $\overline{DG}$	24. $\overline{BG}$
25. $\overline{CR}$	26. $\overline{FR}$	27. $\overline{DR}$

Tell whether each statement is sometimes, always, or never true. Support your answer with a sketch.

28. The angle bisectors of a triangle intersect at a point outside the triangle.

29. An angle bisector of a triangle bisects the opposite side.

30. A perpendicular bisector of a triangle passes through the opposite vertex.

31. The incenter of a right triangle is on the triangle.

32. The circumcenter of a scalene triangle is inside the triangle.

Algebra **Find the circumcenter of the triangle with the given vertices.**

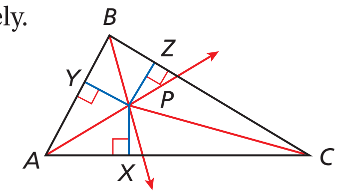
33. $O(0, 0)$, $A(4, 8)$, $B(8, 0)$

34. $O(0, 0)$, $Y(0, 12)$, $Z(6, 6)$

35. Complete this proof of the Incenter Theorem by filling in the blanks.

Given: $\overrightarrow{AP}$, $\overrightarrow{BP}$, and $\overrightarrow{CP}$ bisect $\angle A$, $\angle B$, and $\angle C$, respectively. $\overline{PX} \perp \overline{AC}$, $\overline{PY} \perp \overline{AB}$, $\overline{PZ} \perp \overline{BC}$

Prove: $PX = PY = PZ$



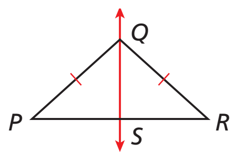
Proof: Let P be the incenter of $\triangle ABC$. Since P lies on the bisector of $\angle A$, $PX = PY$ by **a.** ___?___ . Similarly, P also lies on **b.** ___?___ , so $PY = PZ$. Therefore **c.** ___?___ by the Transitive Property of Equality.

36. Prove that the bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base.

Given: $\overleftrightarrow{QS}$ bisects $\angle PQR$. $\overline{PQ} \cong \overline{RQ}$

Prove: $\overleftrightarrow{QS}$ is the perpendicular bisector of $\overline{PR}$.

Plan: Show that $\triangle PQS \cong \triangle RQS$. Then use CPCTC to show that S is the midpoint of $\overline{PR}$ and that $\overleftrightarrow{QS} \perp \overline{PR}$.



37. This problem will prepare you for the Concept Connection on page 328.

A music company has stores at $A(0, 0)$, $B(8, 0)$, and $C(4, 3)$, where each unit of the coordinate plane represents one mile.

a. A new store will be built so that it is equidistant from the three existing stores. Find the coordinates of the new store's location.

b. Where will the new store be located in relation to $\triangle ABC$?

c. To the nearest tenth of a mile, how far will the new store be from each of the existing stores?

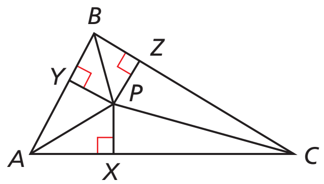
38. **Write About It** How are the inscribed circle and the circumscribed circle of a triangle alike? How are they different?

39. **Construction** Draw a large scalene acute triangle.
   a. Construct the angle bisectors to find the incenter. Inscribe a circle in the triangle.
   b. Construct the perpendicular bisectors to find the circumcenter. Circumscribe a circle around the triangle.

STANDARDIZED TEST PREP

40. P is the incenter of $\triangle ABC$. Which must be true?

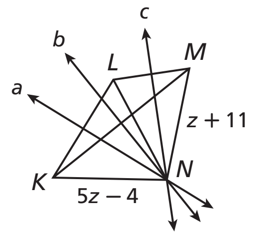
   Ⓐ $PA = PB$ Ⓒ $YA = YB$
   Ⓑ $PX = PY$ Ⓓ $AX = BZ$



41. Lines r, s, and t are concurrent. The equation of line r is $x = 5$, and the equation of line s is $y = -2$. Which could be the equation of line t?

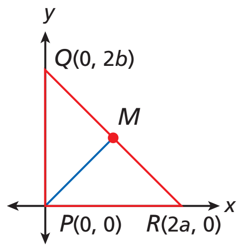
   Ⓕ $y = x - 7$ Ⓗ $y = x + 3$
   Ⓖ $y = x - 3$ Ⓙ $y = x + 7$

42. **Gridded Response** Lines a, b, and c are the perpendicular bisectors of $\triangle KLM$. Find LN.

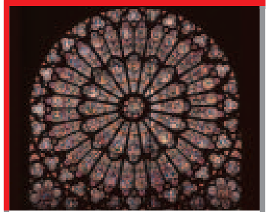


CHALLENGE AND EXTEND

43. Use the right triangle with the given coordinates.
   a. Prove that the midpoint of the hypotenuse of a right triangle is equidistant from all three vertices.
   b. Make a conjecture about the circumcenter of a right triangle.

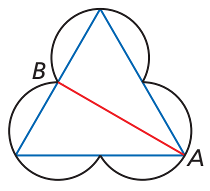


Design LINK



The trefoil shape, as seen in this stained glass window, has been used in design for centuries.

44. **Design** A *trefoil* is created by constructing a circle at each vertex of an equilateral triangle. The radius of each circle equals the distance from each vertex to the circumcenter of the triangle. If the distance from one vertex to the circumcenter is 14 cm, what is the distance AB across the trefoil?



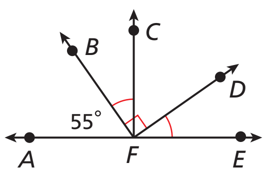
SPIRAL REVIEW

Solve each proportion. *(Previous course)*

45. $\frac{t}{26} = \frac{10}{65}$ 46. $\frac{2.5}{1.75} = \frac{6}{x}$ 47. $\frac{420}{y} = \frac{7}{2}$

Find each angle measure. *(Lesson 1-3)*

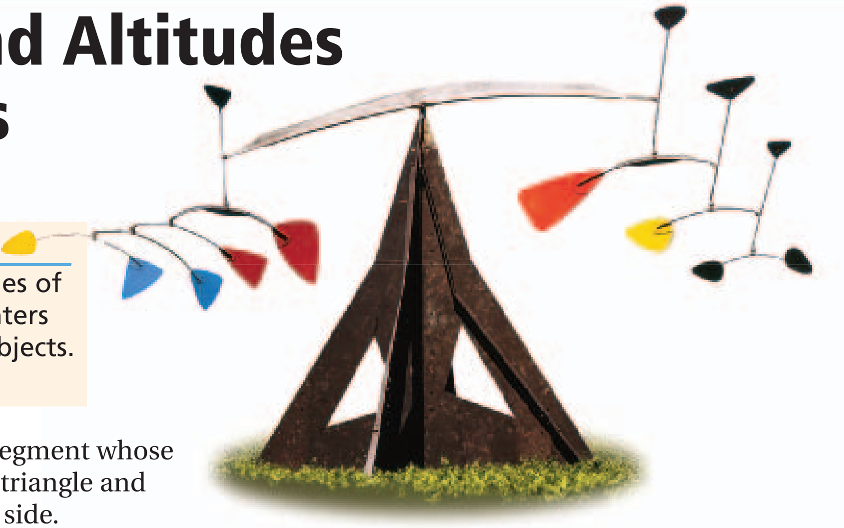
48. $m\angle BFE$ 49. $m\angle BFC$ 50. $m\angle CFE$



Determine whether each point is on the perpendicular bisector of the segment with endpoints $S(0, 8)$ and $T(4, 0)$. *(Lesson 5-1)*

51. $X(0, 3)$ 52. $Y(-4, 1)$ 53. $Z(-8, -2)$

5-3 Medians and Altitudes of Triangles



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Objectives
Apply properties of medians of a triangle.

Apply properties of altitudes of a triangle.

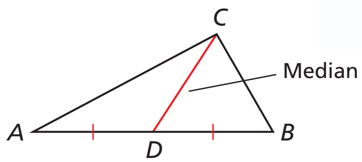
Vocabulary
median of a triangle
centroid of a triangle
altitude of a triangle
orthocenter of a triangle

California Standards

16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line.

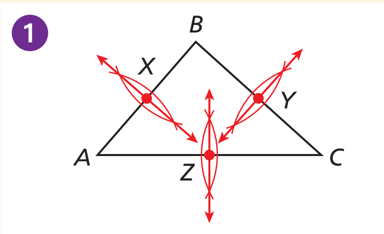
Who uses this?
Sculptors who create mobiles of moving objects can use centers of gravity to balance the objects. (See Example 2.)

A **median of a triangle** is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side.

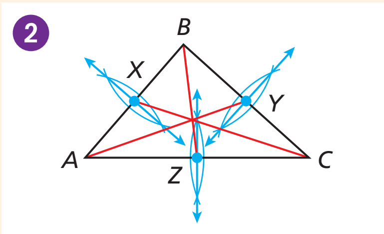


Every triangle has three medians, and the medians are concurrent, as shown in the construction below.

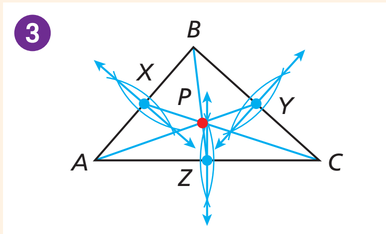
Construction Centroid of a Triangle



Draw $\triangle ABC$. Construct the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$. Label the midpoints of the sides ***X***, ***Y***, and ***Z***, respectively.



Draw $\overline{AY}$, $\overline{BZ}$, and $\overline{CX}$. These are the three medians of $\triangle ABC$.



Label the point where $\overline{AY}$, $\overline{BZ}$, and $\overline{CX}$ intersect as ***P***.

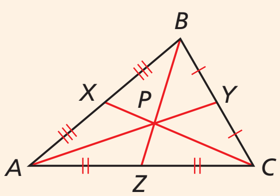
The point of concurrency of the medians of a triangle is the **centroid of the triangle**. The centroid is always inside the triangle. The centroid is also called the *center of gravity* because it is the point where a triangular region will balance.

Know it! Note

Theorem 5-3-1 **Centroid Theorem**

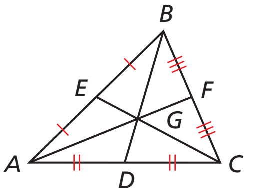
The centroid of a triangle is located $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.

$AP = \frac{2}{3}AY \qquad BP = \frac{2}{3}BZ \qquad CP = \frac{2}{3}CX$



EXAMPLE 1 Using the Centroid to Find Segment Lengths

In $\triangle ABC$, $AF = 9$, and $GE = 2.4$. Find each length.



A AG

$AG = \frac{2}{3}AF$ — *Centroid Thm.*

$AG = \frac{2}{3}(9)$ — *Substitute 9 for AF.*

$AG = 6$ — *Simplify.*

B CE

$CG = \frac{2}{3}CE$ — *Centroid Thm.*

$CG + GE = CE$ — *Seg. Add. Post.*

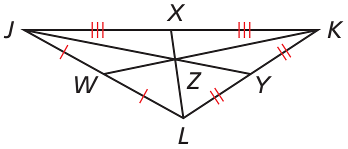
$\frac{2}{3}CE + GE = CE$ — *Substitute $\frac{2}{3}CE$ for CG.*

$GE = \frac{1}{3}CE$ — *Subtract $\frac{2}{3}CE$ from both sides.*

$2.4 = \frac{1}{3}CE$ — *Substitute 2.4 for GE.*

$7.2 = CE$ — *Multiply both sides by 3.*

CHECK IT OUT! **In $\triangle JKL$, $ZW = 7$, and $LX = 8.1$. Find each length.**



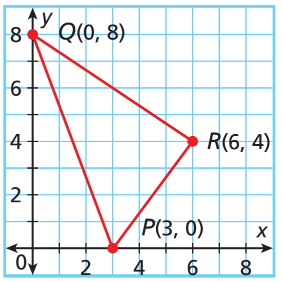
1a. KW

1b. LZ

EXAMPLE 2 *Problem-Solving Application*

PROBLEM SOLVING

The diagram shows the plan for a triangular piece of a mobile. Where should the sculptor attach the support so that the triangle is balanced?



1 Understand the Problem

The **answer** will be the coordinates of the centroid of $\triangle PQR$. The **important information** is the location of the vertices, $P(3, 0)$, $Q(0, 8)$, and $R(6, 4)$.

2 Make a Plan

The centroid of the triangle is the point of intersection of the three medians. So write the equations for two medians and find their point of intersection.

3 Solve

Let M be the midpoint of $\overline{QR}$ and N be the midpoint of $\overline{QP}$.

$$M = \left(\frac{0+6}{2}, \frac{8+4}{2}\right) = (3, 6) \qquad N = \left(\frac{0+3}{2}, \frac{8+0}{2}\right) = (1.5, 4)$$

$\overline{PM}$ is vertical. Its equation is $x = 3$. $\overline{RN}$ is horizontal. Its equation is $y = 4$. The coordinates of the centroid are $S(3, 4)$.

4 Look Back

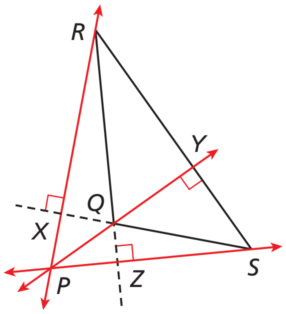
Let L be the midpoint of $\overline{PR}$. The equation for $\overline{QL}$ is $y = -\frac{4}{3}x + 8$, which intersects $x = 3$ at $S(3, 4)$.

CHECK IT OUT! **2.** Find the average of the x-coordinates and the average of the y-coordinates of the vertices of $\triangle PQR$. Make a conjecture about the centroid of a triangle.

Helpful Hint

The height of a triangle is the length of an altitude.

An **altitude of a triangle** is a perpendicular segment from a vertex to the line containing the opposite side. Every triangle has three altitudes. An altitude can be inside, outside, or on the triangle.

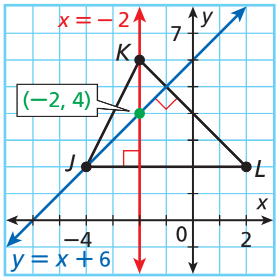


In $\triangle QRS$, altitude $\overline{QY}$ is inside the triangle, but $\overline{RX}$ and $\overline{SZ}$ are not. Notice that the lines containing the altitudes are concurrent at P. This point of concurrency is the **orthocenter of the triangle**.

EXAMPLE 3 Finding the Orthocenter

Algebra

Find the orthocenter of $\triangle JKL$ with vertices $J(-4, 2)$, $K(-2, 6)$, and $L(2, 2)$.



Step 1 Graph the triangle.

Step 2 Find an equation of the line containing the altitude from K to $\overline{JL}$.

Since $\overleftrightarrow{JL}$ is horizontal, the altitude is vertical. The line containing it must pass through $K(-2, 6)$, so the equation of the line is $x = -2$.

Step 3 Find an equation of the line containing the altitude from J to $\overline{KL}$.

$$\text{slope of } \overleftrightarrow{KL} = \frac{2 - 6}{2 - (-2)} = -1$$

The slope of a line perpendicular to $\overleftrightarrow{KL}$ is 1. This line must pass through $J(-4, 2)$.

$y - y_1 = m(x - x_1)$	*Point-slope form*
$y - 2 = 1[x - (-4)]$	*Substitute 2 for y_1, 1 for m, and −4 for x_1.*
$y - 2 = x + 4$	*Distribute 1.*
$y = x + 6$	*Add 2 to both sides.*

Step 4 Solve the system to find the coordinates of the orthocenter.

$$\begin{cases} x = -2 \\ y = x + 6 \end{cases}$$

$y = -2 + 6 = 4$ *Substitute −2 for x.*

The coordinates of the orthocenter are $(-2, 4)$.

CHECK IT OUT! **3.** Show that the altitude to $\overline{JK}$ passes through the orthocenter of $\triangle JKL$.

THINK AND DISCUSS

1. Draw a triangle in which a median and an altitude are the same segment. What type of triangle is it?
2. Draw a triangle in which an altitude is also a side of the triangle. What type of triangle is it?
3. The centroid of a triangle divides each median into two segments. What is the ratio of the two lengths of each median?
4. **GET ORGANIZED** Copy and complete the graphic organizer. Fill in the blanks to make each statement true.

	Centroid	Orthocenter
Definition	The point of concurrency of the _?_	The point of concurrency of the _?_
Location (Inside, Outside, or On)	_?_ the triangle	Can be _?_ the triangle

5-3 Exercises

California Standards
2.0, 3.0, 16.0, 7AF4.1, 7MG3.2, 7MG3.4, 7MR1.2, 1A7.0, 1A8.0, 1A9.0

go.hrw.com
Homework Help Online
KEYWORD: MG7 5-3
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

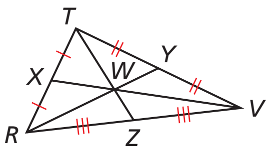
Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. The ___?___ of a triangle is located $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side. (*centroid* or *orthocenter*)
2. The ___?___ of a triangle is perpendicular to the line containing a side. (*altitude* or *median*)

SEE EXAMPLE 1
p. 315

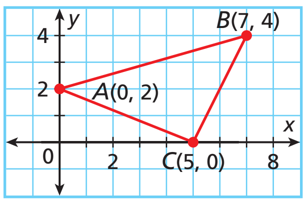
$VX = 204$, and $RW = 104$. Find each length.

3. VW
4. WX
5. RY
6. WY



SEE EXAMPLE 2
p. 315

7. **Design** The diagram shows a plan for a piece of a mobile. A chain will hang from the centroid of the triangle. At what coordinates should the artist attach the chain?



SEE EXAMPLE 3
p. 316

Multi-Step **Find the orthocenter of a triangle with the given vertices.**

8. $K(2, -2)$, $L(4, 6)$, $M(8, -2)$
9. $U(-4, -9)$, $V(-4, 6)$, $W(5, -3)$
10. $P(-5, 8)$, $Q(4, 5)$, $R(-2, 5)$
11. $C(-1, -3)$, $D(-1, 2)$, $E(9, 2)$

PRACTICE AND PROBLEM SOLVING

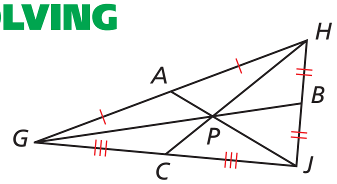
Independent Practice

For Exercises	See Example
12–15	1
16	2
17–20	3

Extra Practice

Skills Practice p. S12
Application Practice p. S32

$PA = 2.9$, and $HC = 10.8$. Find each length.



12. PC **13.** HP

14. JA **15.** JP

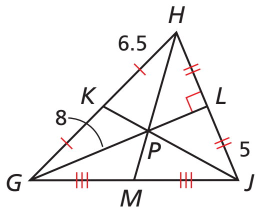
16. Design In the plan for a table, the triangular top has coordinates $(0, 10)$, $(4, 0)$, and $(8, 14)$. The tabletop will rest on a single support placed beneath it. Where should the support be attached so that the table is balanced?

Multi-Step **Find the orthocenter of a triangle with the given vertices.**

17. $X(-2, -2)$, $Y(6, 10)$, $Z(6, -6)$ **18.** $G(-2, 5)$, $H(6, 5)$, $J(4, -1)$

19. $R(-8, 9)$, $S(-2, 9)$, $T(-2, 1)$ **20.** $A(4, -3)$, $B(8, 5)$, $C(8, -8)$

Find each measure.



21. GL **22.** PL

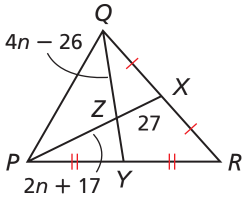
23. HL **24.** GJ

25. perimeter of $\triangle GHJ$ **26.** area of $\triangle GHJ$

Algebra **Find the centroid of a triangle with the given vertices.**

27. $A(0, -4)$, $B(14, 6)$, $C(16, -8)$ **28.** $X(8, -1)$, $Y(2, 7)$, $Z(5, -3)$

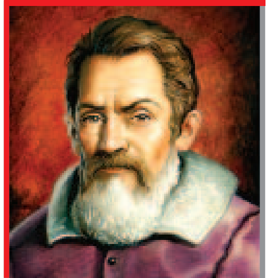
Find each length.



29. PZ **30.** PX

31. QZ **32.** YZ

Math History



In 1678, Giovanni Ceva published his famous theorem that states the conditions necessary for three *Cevians* (segments from a vertex of a triangle to the opposite side) to be concurrent. The medians and altitudes of a triangle meet these conditions.

33. Critical Thinking Draw an isosceles triangle and its line of symmetry. What are four other names for this segment?

Tell whether each statement is sometimes, always, or never true. Support your answer with a sketch.

34. A median of a triangle bisects one of the angles.

35. If one altitude of a triangle is in the triangle's exterior, then a second altitude is also in the triangle's exterior.

36. The centroid of a triangle lies in its exterior.

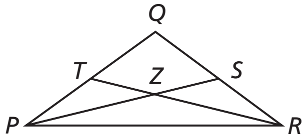
37. In an isosceles triangle, the altitude and median from the vertex angle are the same line as the bisector of the vertex angle.

38. Write a two-column proof.

Given: $\overline{PS}$ and $\overline{RT}$ are medians of $\triangle PQR$. $\overline{PS} \cong \overline{RT}$

Prove: $\triangle PQR$ is an isosceles triangle.

Plan: Show that $\triangle PTR \cong \triangle RSP$ and use CPCTC to conclude that $\angle QPR \cong \angle QRP$.



39. Write About It Draw a large triangle on a sheet of paper and cut it out. Find the centroid by paper folding. Try to balance the shape on the tip of your pencil at a point other than the centroid. Now try to balance the shape at its centroid. Explain why the centroid is also called the center of gravity.

CONCEPT CONNECTION

40. This problem will prepare you for the Concept Connection on page 328.

The towns of Davis, El Monte, and Fairview have the coordinates shown in the table, where each unit of the coordinate plane represents one mile. A music company has stores in each city and a distribution warehouse at the centroid of $\triangle DEF$.

City	Location
Davis	$D(0, 0)$
El Monte	$E(0, 8)$
Fairview	$F(8, 0)$

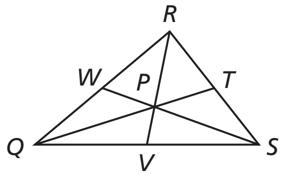
a. What are the coordinates of the warehouse?

b. Find the distance from the warehouse to the Davis store. Round your answer to the nearest tenth of a mile.

c. A straight road connects El Monte and Fairview. What is the distance from the warehouse to the road?

STANDARDIZED TEST PREP

41. $\overline{QT}$, $\overline{RV}$, and $\overline{SW}$ are medians of $\triangle QRS$. Which statement is NOT necessarily true?



Ⓐ $QP = \frac{2}{3}QT$ Ⓒ $RT = ST$

Ⓑ $RP = 2PV$ Ⓓ $QT = SW$

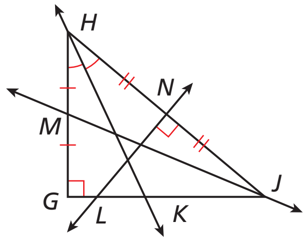
42. Suppose that the orthocenter of a triangle lies outside the triangle. Which points of concurrency are inside the triangle?

I. incenter **II.** circumcenter **III.** centroid

Ⓕ I and II only Ⓗ II and III only

Ⓖ I and III only Ⓙ I, II, and III

43. In the diagram, which of the following correctly describes $\overline{LN}$?



Ⓐ Altitude Ⓒ Median

Ⓑ Angle bisector Ⓓ Perpendicular bisector

CHALLENGE AND EXTEND

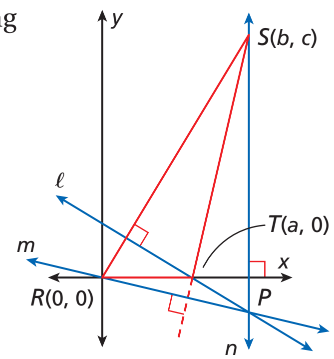
44. Draw an equilateral triangle.

a. Explain why the perpendicular bisector of any side contains the vertex opposite that side.

b. Explain why the perpendicular bisector through any vertex also contains the median, the altitude, and the angle bisector through that vertex.

c. Explain why the incenter, circumcenter, centroid, and orthocenter are the same point.

45. Use coordinates to show that the lines containing the altitudes of a triangle are concurrent.



a. Find the slopes of $\overline{RS}$, $\overline{ST}$, and $\overline{RT}$.

b. Find the slopes of lines ℓ, m, and n.

c. Write equations for lines ℓ, m, and n.

d. Solve a system of equations to find the point P where lines ℓ and m intersect.

e. Show that line n contains P.

f. What conclusion can you draw?

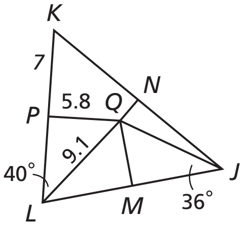
SPIRAL REVIEW

46. At a baseball game, a bag of peanuts costs \$0.75 more than a bag of popcorn. If a family purchases 5 bags of peanuts and 3 bags of popcorn for \$21.75, how much does one bag of peanuts cost? *(Previous course)*

Determine if each biconditional is true. If false, give a counterexample. *(Lesson 2-4)*

47. The area of a rectangle is 40 cm^2 if and only if the length of the rectangle is 4 cm and the width of the rectangle is 10 cm.

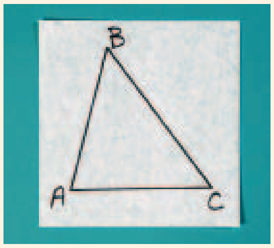
48. A nonzero number n is positive if and only if $-n$ is negative.



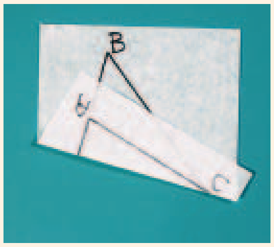
$\overline{NQ}$, $\overline{QP}$, and $\overline{QM}$ are perpendicular bisectors of $\triangle JKL$. Find each measure. *(Lesson 5-2)*

49. KL **50.** QJ **51.** $m\angle JQL$

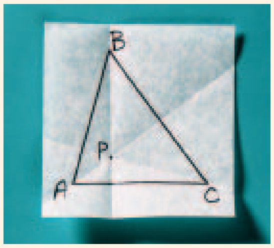
Construction Orientcenter of a Triangle

1 

Draw a large scalene acute triangle *ABC* on a piece of patty paper.

2 

Find the altitude of each side by folding the side so that it overlaps itself and so that the fold intersects the opposite vertex.

3 

Mark the point where the three lines containing the altitudes intersect and label it *P*. *P* is the orthocenter of $\triangle ABC$.

1. Repeat the construction for a scalene obtuse triangle and a scalene right triangle.

2. Make a conjecture about the location of the orthocenter in an acute, an obtuse, and a right triangle.

Career Path

go.hrw.com
Career Resources Online
KEYWORD: MG7 Career

Alex Peralta
Electrician

Q: What high school math classes did you take?

A: Algebra 1, Geometry, and Statistics.

Q: What type of training did you receive?

A: In high school, I took classes in electricity, electronics, and drafting. I began an apprenticeship program last year to prepare for the exam to get my license.

Q: How do you use math?

A: Determining the locations of outlets and circuits on blueprints requires good spatial sense. I also use ratios and proportions, calculate distances, work with formulas, and estimate job costs.

5-3 Technology LAB

Special Points in Triangles

Use with Lesson 5-3

In this lab you will use geometry software to explore properties of the four points of concurrency you have studied.

California Standards

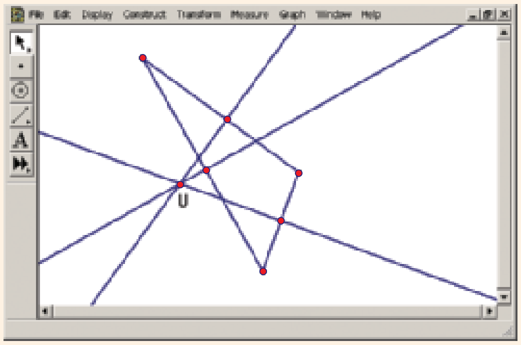
1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and **inductive** and deductive **reasoning.**

16.0 Students perform basic constructions with a straightedge and compass, **such as angle bisectors, perpendicular bisectors,** and the line parallel to a given line through a point off the line.

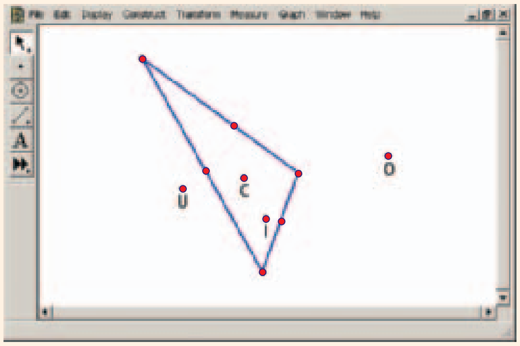
go.hrw.com
Lab Resources Online
KEYWORD: MG7 Lab5

Activity

1. Construct a triangle.
2. Construct the perpendicular bisector of each side of the triangle. Construct the point of intersection of these three lines. This is the circumcenter of the triangle. Label it *U* and hide the perpendicular bisectors.



3. In the same triangle, construct the bisector of each angle. Construct the point of intersection of these three lines. This is the incenter of the triangle. Label it *I* and hide the angle bisectors.
4. In the same triangle, construct the midpoint of each side. Then construct the three medians. Construct the point of intersection of these three lines. Label the centroid *C* and hide the medians.



5. In the same triangle, construct the altitude to each side. Construct the point of intersection of these three lines. Label the orthocenter *O* and hide the altitudes.
6. Move a vertex of the triangle and observe the positions of the four points of concurrency. In 1765, Swiss mathematician Leonhard Euler showed that three of these points are always collinear. The line containing them is called the *Euler line*.

Try This

1. Which three points of concurrency lie on the Euler line?
2. **Make a Conjecture** Which point on the Euler line is always between the other two? Measure the distances between the points. Make a conjecture about the relationship of the distances between these three points.
3. **Make a Conjecture** Move a vertex of the triangle until all four points of concurrency are collinear. In what type of triangle are all four points of concurrency on the Euler line?
4. **Make a Conjecture** Find a triangle in which all four points of concurrency coincide. What type of triangle has this special property?

5-4 The Triangle Midsegment Theorem

Objective

Prove and use properties of triangle midsegments.

Vocabulary

midsegment of a triangle

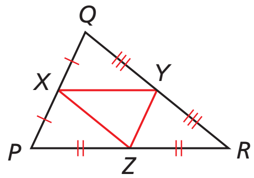
California Standards

17.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

Why learn this?

You can use triangle midsegments to make indirect measurements of distances, such as the distance across a volcano. (See Example 3.)

A **midsegment of a triangle** is a segment that joins the midpoints of two sides of the triangle. Every triangle has three midsegments, which form the *midsegment triangle*.

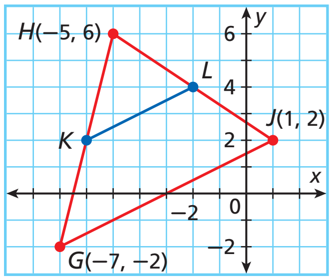


Midsegments: $\overline{XY}, \overline{YZ}, \overline{ZX}$

Midsegment triangle: $\triangle XYZ$

EXAMPLE 1 Examining Midsegments in the Coordinate Plane

In $\triangle GHJ$, show that midsegment $\overline{KL}$ is parallel to $\overline{GJ}$ and that $KL = \frac{1}{2}GJ$.



Step 1 Find the coordinates of K and L.

$$\text{mdpt. of } \overline{GH} = \left(\frac{-7+(-5)}{2}, \frac{-2+6}{2}\right)$$
$$= (-6, 2)$$

$$\text{mdpt. of } \overline{HJ} = \left(\frac{-5+1}{2}, \frac{6+2}{2}\right) = (-2, 4)$$

Step 2 Compare the slopes of $\overline{KL}$ and $\overline{GJ}$.

$$\text{slope of } \overline{KL} = \frac{4-2}{-2-(-6)} = \frac{1}{2} \qquad \text{slope of } \overline{GJ} = \frac{2-(-2)}{1-(-7)} = \frac{1}{2}$$

Since the slopes are the same, $\overline{KL} \parallel \overline{GJ}$.

Step 3 Compare the lengths of $\overline{KL}$ and $\overline{GJ}$.

$$KL = \sqrt{\left[-2-(-6)\right]^2 + (4-2)^2} = 2\sqrt{5}$$

$$GJ = \sqrt{\left[1-(-7)\right]^2 + \left[2-(-2)\right]^2} = 4\sqrt{5}$$

Since $2\sqrt{5} = \frac{1}{2}\left(4\sqrt{5}\right)$, $KL = \frac{1}{2}GJ$.

CHECK IT OUT!

1. The vertices of $\triangle RST$ are $R(-7, 0)$, $S(-3, 6)$, and $T(9, 2)$. M is the midpoint of $\overline{RT}$, and N is the midpoint of $\overline{ST}$. Show that $\overline{MN} \parallel \overline{RS}$ and $MN = \frac{1}{2}RS$.

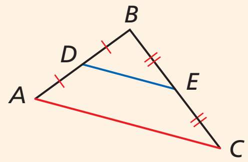
The relationship shown in Example 1 is true for the three midsegments of every triangle.

Know it! Note

Theorem 5-4-1 Triangle Midsegment Theorem

A midsegment of a triangle is parallel to a side of the triangle, and its length is half the length of that side.

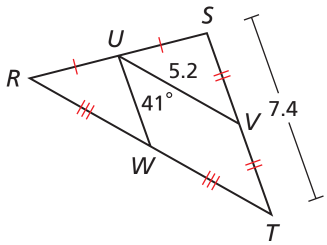
$\overline{DE} \parallel \overline{AC}, DE = \frac{1}{2}AC$



You will prove Theorem 5-4-1 in Exercise 38.

EXAMPLE 2 Using the Triangle Midsegment Theorem

Find each measure.



A UW

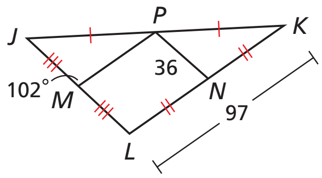
$UW = \frac{1}{2}ST$	*△ Midsegment Thm.*
$UW = \frac{1}{2}(7.4)$	*Substitute 7.4 for ST.*
$UW = 3.7$	*Simplify.*

B $m\angle SVU$

$\overline{UW} \parallel \overline{ST}$	*△ Midsegment Thm.*
$m\angle SVU = m\angle VUW$	*Alt. Int. ∠s Thm.*
$m\angle SVU = 41°$	*Substitute 41° for m∠VUW.*

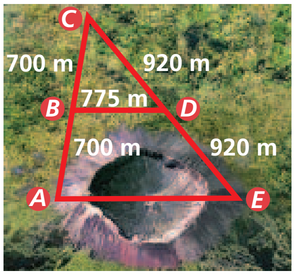
CHECK IT OUT! **Find each measure.**

2a. JL **2b.** PM **2c.** $m\angle MLK$



EXAMPLE 3 *Indirect Measurement Application*

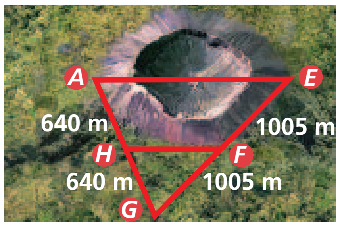
Anna wants to find the distance across the base of Capulin Volcano, an extinct volcano in New Mexico. She measures a triangle at one side of the volcano as shown in the diagram. What is AE?



$BD = \frac{1}{2}AE$	*△ Midsegment Thm.*
$775 = \frac{1}{2}AE$	*Substitute 775 for BD.*
$1550 = AE$	*Multiply both sides by 2.*

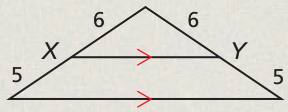
The distance AE across the base of the volcano is about 1550 meters.

CHECK IT OUT! **3. What if...?** Suppose Anna's result in Example 3 is correct. To check it, she measures a second triangle. How many meters will she measure between H and F?



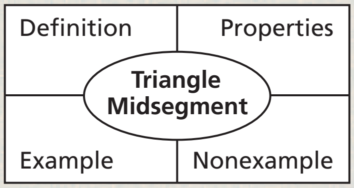
THINK AND DISCUSS

1. Explain why $\overline{XY}$ is NOT a midsegment of the triangle.



Know it! Note

2. **GET ORGANIZED** Copy and complete the graphic organizer. Write the definition of a triangle midsegment and list its properties. Then draw an example and a nonexample.



5-4 Exercises

California Standards
16.0, 17.0, 7AF1.0, 7AF2.0, 7AF4.1, 7MG2.1, 7MG3.2, 1A2.0, 1A8.0, 1A15.0

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Homework Help Online
KEYWORD: MG7 5-4
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

1. **Vocabulary** The *midsegment of a triangle* joins the ___?___ of two sides of the triangle. (*endpoints* or *midpoints*)

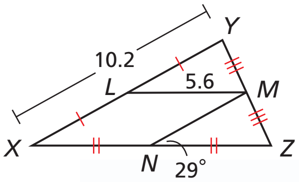
SEE EXAMPLE 1
p. 322

2. The vertices of $\triangle PQR$ are $P(-4, -1)$, $Q(2, 9)$, and $R(6, 3)$. S is the midpoint of $\overline{PQ}$, and T is the midpoint of $\overline{QR}$. Show that $\overline{ST} \parallel \overline{PR}$ and $ST = \frac{1}{2}PR$.

SEE EXAMPLE 2
p. 323

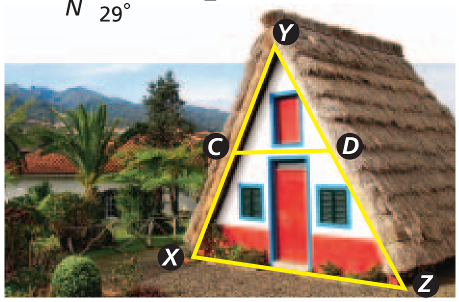
Find each measure.

3. NM
4. XZ
5. NZ
6. $m\angle LMN$
7. $m\angle YXZ$
8. $m\angle XLM$



SEE EXAMPLE 3
p. 323

9. **Architecture** In this A-frame house, the width of the first floor $\overline{XZ}$ is 30 feet. The second floor $\overline{CD}$ is slightly above and parallel to the midsegment of $\triangle XYZ$. Is the width of the second floor more or less than 5 yards? Explain.



PRACTICE AND PROBLEM SOLVING

Independent Practice

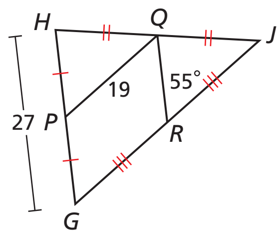
For Exercises	See Example
10	1
11–16	2
17	3

Extra Practice
Skills Practice p. S12
Application Practice p. S32

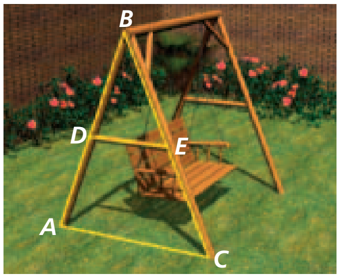
10. The vertices of $\triangle ABC$ are $A(-6, 11)$, $B(6, -3)$, and $C(-2, -5)$. D is the midpoint of $\overline{AC}$, and E is the midpoint of $\overline{AB}$. Show that $\overline{DE} \parallel \overline{CB}$ and $DE = \frac{1}{2}CB$.

Find each measure.

11. GJ
12. RQ
13. RJ
14. $m\angle PQR$
15. $m\angle HGJ$
16. $m\angle GPQ$



17. **Carpentry** In each support for the garden swing, the crossbar $\overline{DE}$ is attached at the midpoints of legs $\overline{BA}$ and $\overline{BC}$. The distance AC is $4\frac{1}{2}$ feet. The carpenter has a timber that is 30 inches long. Is this timber long enough to be used as one of the crossbars? Explain.

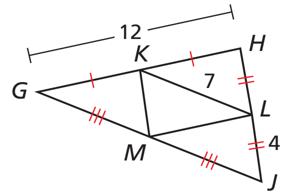


△*KLM* is the midsegment triangle of △*GHJ*.

18. What is the perimeter of $\triangle GHJ$?

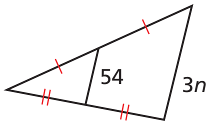
19. What is the perimeter of $\triangle KLM$?

20. What is the relationship between the perimeter of $\triangle GHJ$ and the perimeter of $\triangle KLM$?

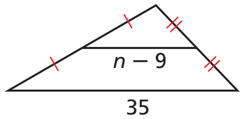


Algebra **Find the value of *n* in each triangle.**

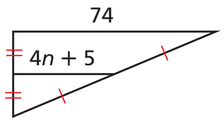
21.



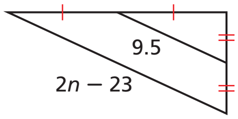
22.



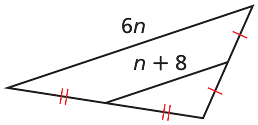
23.



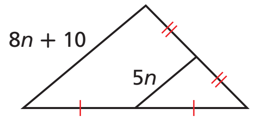
24.



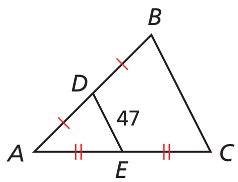
25.



26.



27. **ERROR ANALYSIS** Below are two solutions for finding BC. Which is incorrect? Explain the error.



A

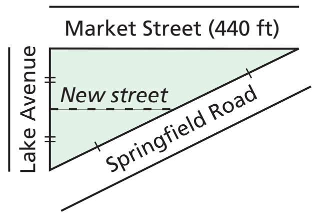
$DE = 0.5BC$
$47 = 0.5BC$
$94 = BC$

B

$BC = 0.5DE$
$BC = 0.5(47)$
$BC = 23.5$

28. **Critical Thinking** Draw scalene $\triangle DEF$. Label X as the midpoint of $\overline{DE}$, Y as the midpoint of $\overline{EF}$, and Z as the midpoint of $\overline{DF}$. Connect the three midpoints. List all of the congruent angles in your drawing.

29. **Estimation** The diagram shows the sketch for a new street. Parallel parking spaces will be painted on both sides of the street. Each parallel parking space is 23 feet long. About how many parking spaces can the city accommodate on both sides of the new street? Explain your answer.



$\overline{CG}$, $\overline{EH}$, and $\overline{FJ}$ are midsegments of △*ABD*, △*GCD*, and △*GHE*, respectively. Find each measure.

30. CG

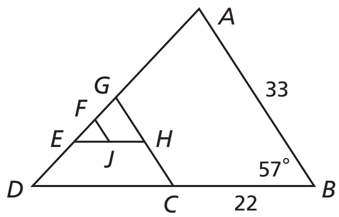
31. EH

32. FJ

33. $m\angle DCG$

34. $m\angle GHE$

35. $m\angle FJH$

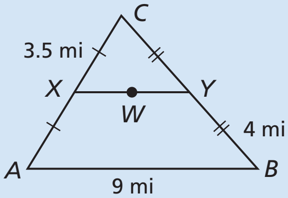


36. **Write About It** An isosceles triangle has two congruent sides. Does it also have two congruent midsegments? Explain.

CONCEPT CONNECTION

37. This problem will prepare you for the Concept Connection on page 328.

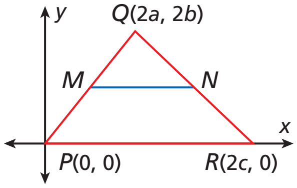
The figure shows the roads connecting towns *A*, *B*, and *C*. A music company has a store in each town and a distribution warehouse *W* at the midpoint of road $\overline{XY}$.



a. What is the distance from the warehouse to point *X*?

b. A truck starts at the warehouse, delivers instruments to the stores in towns *A*, *B*, and *C* (in this order) and then returns to the warehouse. What is the total length of the trip, assuming the driver takes the shortest possible route?

38. Use coordinates to prove the Triangle Midsegment Theorem.



a. *M* is the midpoint of $\overline{PQ}$. What are its coordinates?

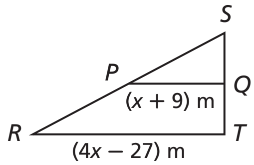
b. *N* is the midpoint of $\overline{QR}$. What are its coordinates?

c. Find the slopes of $\overline{PR}$ and $\overline{MN}$. What can you conclude?

d. Find *PR* and *MN*. What can you conclude?

STANDARDIZED TEST PREP

39. $\overline{PQ}$ is a midsegment of $\triangle RST$. What is the length of $\overline{RT}$?



Ⓐ 9 meters

Ⓑ 21 meters

Ⓒ 45 meters

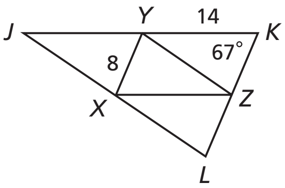
Ⓓ 63 meters

40. In $\triangle UVW$, *M* is the midpoint of $\overline{VU}$, and *N* is the midpoint of $\overline{VW}$. Which statement is true?

Ⓕ $VM = VN$ Ⓗ $VU = 2VM$

Ⓖ $MN = UV$ Ⓙ $VW = \frac{1}{2}VN$

41. $\triangle XYZ$ is the midsegment triangle of $\triangle JKL$, $XY = 8$, $YK = 14$, and $m\angle YKZ = 67°$. Which of the following measures CANNOT be determined?



Ⓐ *KL* Ⓒ $m\angle XZL$

Ⓑ *JY* Ⓓ $m\angle KZY$

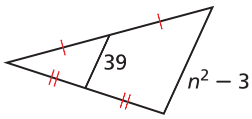
CHALLENGE AND EXTEND

42. **Multi-Step** The midpoints of the sides of a triangle are $A(-6, 3)$, $B(2, 1)$, and $C(0, -3)$. Find the coordinates of the vertices of the triangle.

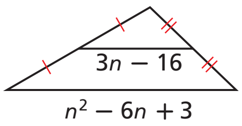
43. **Critical Thinking** Classify the midsegment triangle of an equilateral triangle by its side lengths and angle measures.

Algebra **Find the value of *n* in each triangle.**

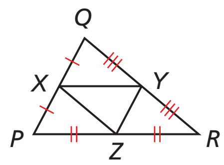
44.



45.



46. $\triangle XYZ$ is the midsegment triangle of $\triangle PQR$. Write a congruence statement involving all four of the smaller triangles. What is the relationship between the area of $\triangle XYZ$ and $\triangle PQR$?



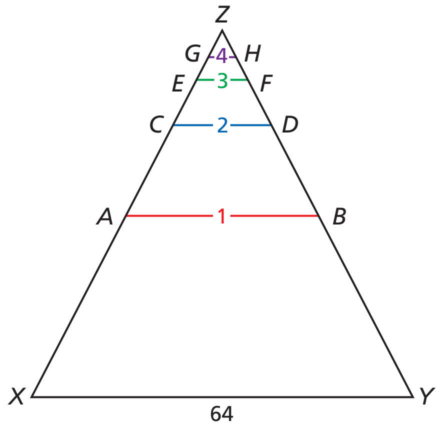
47. $\overline{AB}$ is a midsegment of $\triangle XYZ$. $\overline{CD}$ is a midsegment of $\triangle ABZ$. $\overline{EF}$ is a midsegment of $\triangle CDZ$, and $\overline{GH}$ is a midsegment of $\triangle EFZ$.

a. Copy and complete the table.

Number of Midsegment	1	2	3	4
Length of Midsegment				

b. If this pattern continues, what will be the length of midsegment 8?

c. Write an algebraic expression to represent the length of midsegment n. (*Hint*: Think of the midsegment lengths as powers of 2.)



SPIRAL REVIEW

Suppose a 2% acid solution is mixed with a 3% acid solution. Find the percent of acid in each mixture. *(Previous course)*

48. a mixture that contains an equal amount of 2% acid solution and 3% acid solution

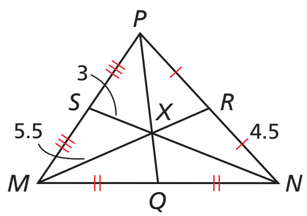
49. a mixture that contains 3 times more 2% acid solution than 3% acid solution

A figure has vertices $G(-3, -2)$, $H(0, 0)$, $J(4, 1)$, and $K(1, -2)$. Given the coordinates of the image of G under a translation, find the coordinates of the images of H, J, and K. *(Lesson 1-7)*

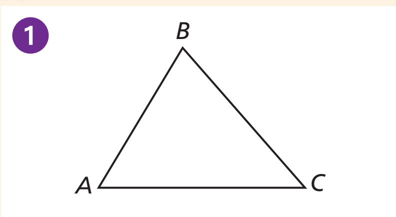
50. $(-3, 2)$ **51.** $(1, -4)$ **52.** $(3, 0)$

Find each length. *(Lesson 5-3)*

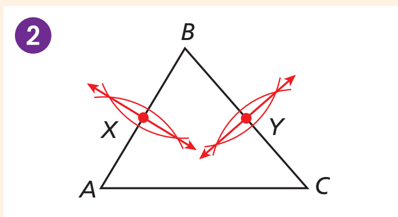
53. NX **54.** MR **55.** NP



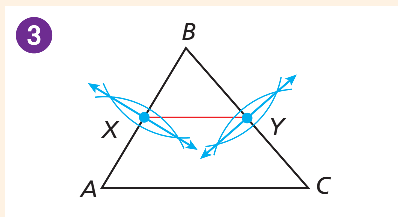
Construction Midsegment of a Triangle



Draw a large triangle. Label the vertices A, B, and C.



Construct the midpoints of $\overline{AB}$ and $\overline{BC}$. Label the midpoints X and Y, respectively.



Draw the midsegment $\overline{XY}$.

1. Using a ruler, measure $\overline{XY}$ and $\overline{AC}$. How are the two lengths related?

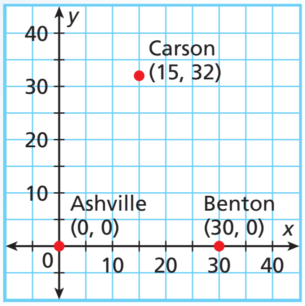
2. How can you use a protractor to verify that $\overline{XY}$ is parallel to $\overline{AC}$?

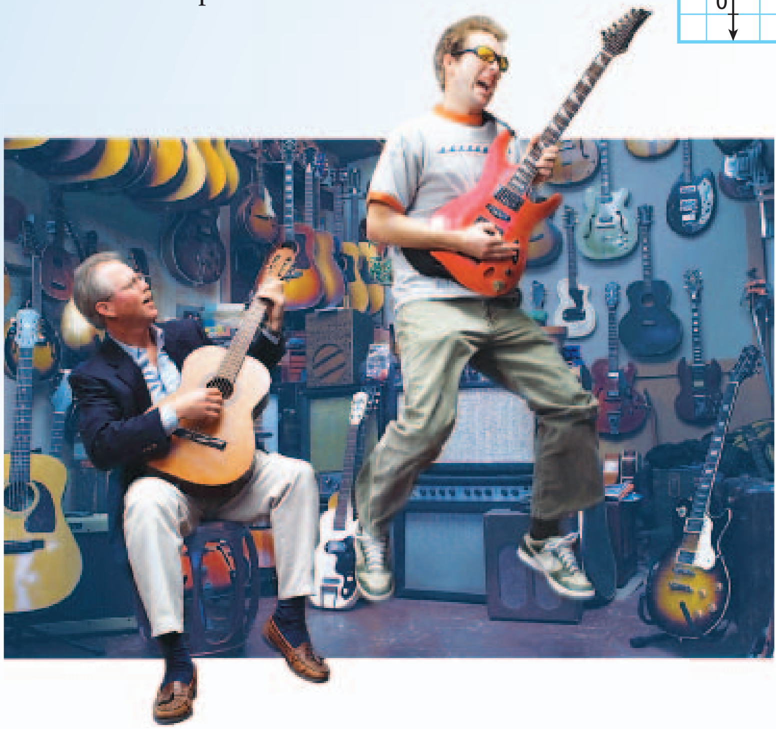
Concept Connection

Segments in Triangles

Location Contemplation

A chain of music stores has locations in Ashville, Benton, and Carson. The directors of the company are using a coordinate plane to decide on the location for a new distribution warehouse. Each unit on the plane represents one mile.





1. A plot of land is available at the centroid of the triangle formed by the three cities. What are the coordinates for this location?
2. If the directors build the warehouse at the centroid, about how far will it be from each of the cities?
3. Another plot of land is available at the orthocenter of the triangle. What are the coordinates for this location?
4. About how far would the warehouse be from each city if it were built at the orthocenter?
5. A third option is to build the warehouse at the circumcenter of the triangle. What are the coordinates for this location?
6. About how far would the warehouse be from each city if it were built at the circumcenter?
7. The directors decide that the warehouse should be equidistant from each city. Which location should they choose?

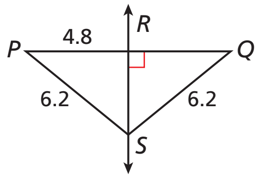
READY TO GO ON?

Quiz for Lessons 5-1 Through 5-4

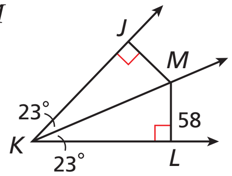
5-1 Perpendicular and Angle Bisectors

Find each measure.

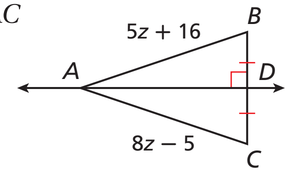
1. PQ



2. JM



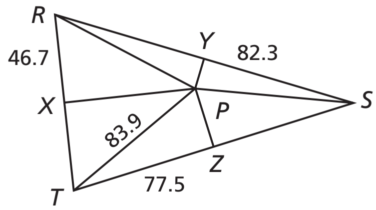
3. AC



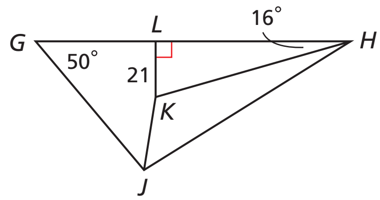
4. Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints $M(-1, -3)$ and $N(7, 1)$.

5-2 Bisectors of Triangles

5. $\overline{PX}$, $\overline{PY}$, and $\overline{PZ}$ are the perpendicular bisectors of $\triangle RST$. Find PS and XT.



6. $\overline{JK}$ and $\overline{HK}$ are angle bisectors of $\triangle GHJ$. Find $m\angle GJK$ and the distance from K to $\overline{HJ}$.



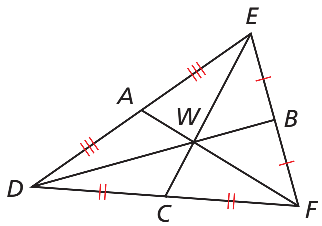
7. Find the circumcenter of $\triangle TVO$ with vertices $T(9, 0)$, $V(0, -4)$, and $O(0, 0)$.

5-3 Medians and Altitudes of Triangles

8. In $\triangle DEF$, $BD = 87$, and $WE = 38$. Find BW, CW, and CE.

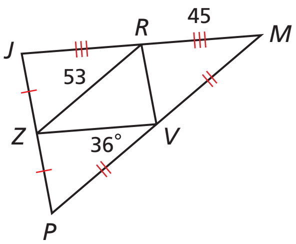
9. Paula cuts a triangle with vertices at coordinates $(0, 4)$, $(8, 0)$, and $(10, 8)$ from grid paper. At what coordinates should she place the tip of a pencil to balance the triangle?

10. Find the orthocenter of $\triangle PSV$ with vertices $P(2, 4)$, $S(8, 4)$, and $V(4, 0)$.

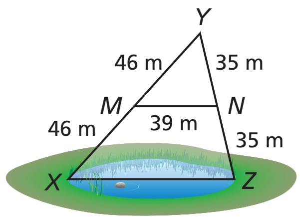


5-4 The Triangle Midsegment Theorem

11. Find ZV, PM, and $m\angle RZV$ in $\triangle JMP$.



12. What is the distance XZ across the pond?



Connecting Geometry to **Algebra**

See Skills Bank page S60

Solving Compound Inequalities

California Standards

Review of 1A5.0 Students solve multistep problems, including word problems, **involving** linear equations and **linear inequalities in one variable** and provide justification for each step.

To solve an inequality, you use the Properties of Inequality and inverse operations to undo the operations in the inequality one at a time.

Properties of Inequality

PROPERTY	ALGEBRA
Addition Property	If $a < b$, then $a + c < b + c$.
Subtraction Property	If $a < b$, then $a - c < b - c$.
Multiplication Property	If $a < b$ and $c > 0$, then $ac < bc$. If $a < b$ and $c < 0$, then $ac > bc$.
Division Property	If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$. If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$.
Transitive Property	If $a < b$ and $b < c$, then $a < c$.
Comparison Property	If $a + b = c$ and $b > 0$, then $a < c$.

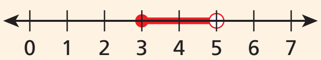
A compound inequality is formed when two simple inequalities are combined into one statement with the word *and* or *or*. To solve a compound inequality, solve each simple inequality and find the intersection or union of the solutions. The graph of a compound inequality may represent a line, a ray, two rays, or a segment.

Example

Solve the compound inequality $5 < 20 - 3a \leq 11$. What geometric figure does the graph represent?

$5 < 20 - 3a$	AND	$20 - 3a \leq 11$	*Rewrite the compound inequality as two simple inequalities.*
$-15 < -3a$	AND	$-3a \leq -9$	*Subtract 20 from both sides.*
$5 > a$	AND	$a \geq 3$	*Divide both sides by −3 and reverse the inequality symbols.*

$3 \leq a < 5$ *Combine the two solutions into a single statement.*



The graph represents a segment.

Try This

Solve. What geometric figure does each graph represent?

1. $-4 + x > 1$ OR $-8 + 2x < -6$
2. $2x - 3 \geq -5$ OR $x - 4 > -1$
3. $-6 < 7 - x \leq 12$
4. $22 < -2 - 2x \leq 54$
5. $3x \geq 0$ OR $x + 5 < 7$
6. $2x - 3 \leq 5$ OR $-2x + 3 \leq -9$

5-5 Geometry LAB

Use with Lesson 5-5

Explore Triangle Inequalities

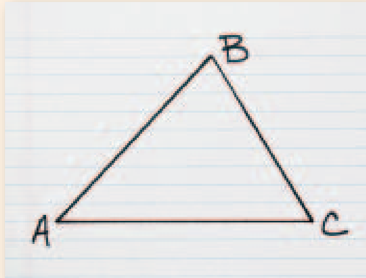
Many of the triangle relationships you have learned so far involve a statement of equality. For example, the circumcenter of a triangle is equidistant from the vertices of the triangle, and the incenter is equidistant from the sides of the triangle. Now you will investigate some triangle relationships that involve inequalities.

California Standards

Preparation for 6.0 Students know and are able to use the triangle inequality theorem.

Activity 1

1. Draw a large scalene triangle. Label the vertices *A*, *B*, and *C*.



2. Measure the sides and the angles. Copy the table below and record the measures in the first row.

	BC	*AC*	*AB*	m∠*A*	m∠*B*	m∠*C*
Triangle 1						
Triangle 2						
Triangle 3						
Triangle 4						

Try This

1. In the table, draw a circle around the longest side length, and draw a circle around the greatest angle measure of $\triangle ABC$. Draw a square around the shortest side length, and draw a square around the least angle measure.
2. **Make a Conjecture** Where is the longest side in relation to the largest angle? Where is the shortest side in relation to the smallest angle?
3. Draw three more scalene triangles and record the measures in the table. Does your conjecture hold?

Activity 2

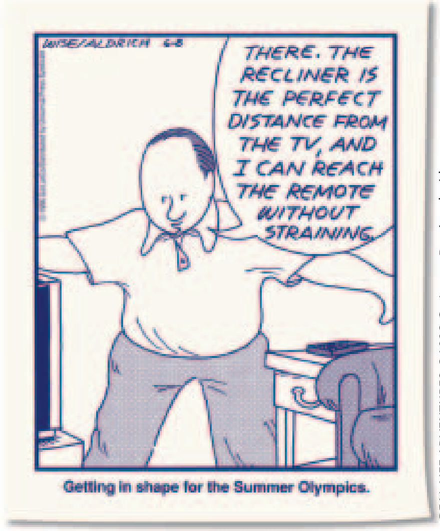
1. Cut three sets of chenille stems to the following lengths.
   3 inches, 4 inches, 6 inches
   3 inches, 4 inches, 7 inches
   3 inches, 4 inches, 8 inches

2. Try to make a triangle with each set of chenille stems.

Try This

4. Which sets of chenille stems make a triangle?
5. **Make a Conjecture** For each set of chenille stems, compare the sum of any two lengths with the third length. What is the relationship?
6. Select a different set of three lengths and test your conjecture. Does your conjecture hold?

5-5 Indirect Proof and Inequalities in One Triangle



Getting in shape for the Summer Olympics.

Objectives
Write indirect proofs.

Apply inequalities in one triangle.

Vocabulary
indirect proof

Why learn this?
You can use a triangle inequality to find a reasonable range of values for an unknown distance. (See Example 5.)

So far you have written proofs using *direct reasoning*. You began with a true hypothesis and built a logical argument to show that a conclusion was true. In an **indirect proof**, you begin by assuming that the conclusion is false. Then you show that this assumption leads to a contradiction. This type of proof is also called a *proof by contradiction*.

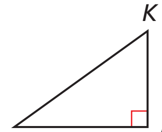
Helpful Hint
When writing an indirect proof, look for a contradiction of one of the following: the given information, a definition, a postulate, or a theorem.

Writing an Indirect Proof
1. Identify the conjecture to be proven.
2. Assume the opposite (the negation) of the conclusion is true.
3. Use direct reasoning to show that the assumption leads to a contradiction.
4. Conclude that since the assumption is false, the original conjecture must be true.

EXAMPLE 1 Writing an Indirect Proof

California Standards
2.0 Students write geometric proofs, including proofs by contradiction.
6.0 Students know and are able to use the triangle inequality theorem.

Write an indirect proof that a right triangle cannot have an obtuse angle.



Step 1 Identify the conjecture to be proven.
Given: $\triangle JKL$ is a right triangle.
Prove: $\triangle JKL$ does not have an obtuse angle.

Step 2 Assume the opposite of the conclusion.
Assume $\triangle JKL$ has an obtuse angle. Let $\angle K$ be obtuse.

Step 3 Use direct reasoning to lead to a contradiction.

$m\angle K + m\angle L = 90°$	*The acute ∠s of a rt. △ are comp.*
$m\angle K = 90° - m\angle L$	*Subtr. Prop. of =*
$m\angle K > 90°$	*Def. of obtuse ∠*
$90° - m\angle L > 90°$	*Substitute 90° − m∠L for m∠K.*
$m\angle L < 0°$	*Subtract 90° from both sides and solve for m∠L.*

However, by the Protractor Postulate, a triangle cannot have an angle with a measure less than 0°.

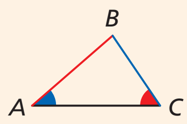
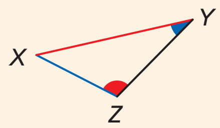
Step 4 Conclude that the original conjecture is true.
The assumption that $\triangle JKL$ has an obtuse angle is false.
Therefore $\triangle JKL$ does not have an obtuse angle.

CHECK IT OUT! **1.** Write an indirect proof that a triangle cannot have two right angles.

The positions of the longest and shortest sides of a triangle are related to the positions of the largest and smallest angles.

Know it! Note

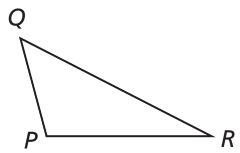
Theorems — Angle-Side Relationships in Triangles

THEOREM	HYPOTHESIS	CONCLUSION
5-5-1 If two sides of a triangle are not congruent, then the larger angle is opposite the longer side. (In △, larger ∠ is opp. longer side.)	 $AB > BC$	$m\angle C > m\angle A$
5-5-2 If two angles of a triangle are not congruent, then the longer side is opposite the larger angle. (In △, longer side is opp. larger ∠.)	 $m\angle Z > m\angle Y$	$XY > XZ$

You will prove Theorem 5-5-1 in Exercise 67.

PROOF

Theorem 5-5-2



Given: $m\angle P > m\angle R$
Prove: $QR > QP$

Indirect Proof:
Assume $QR \ngtr QP$. This means that either $QR < QP$ or $QR = QP$.

Case 1 If $QR < QP$, then $m\angle P < m\angle R$ because the larger angle is opposite the longer side. This contradicts the given information. So $QR \nless QP$.

Case 2 If $QR = QP$, then $m\angle P = m\angle R$ by the Isosceles Triangle Theorem. This also contradicts the given information, so $QR \neq QP$.

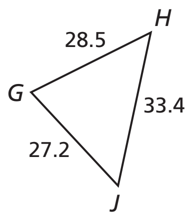
The assumption $QR \ngtr QP$ is false. Therefore $QR > QP$.

Caution!

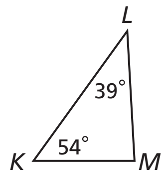
Consider all cases when you assume the opposite. If the conclusion is $QR > QP$, the negation includes $QR < QP$ and $QR = QP$.

EXAMPLE 2

Ordering Triangle Side Lengths and Angle Measures



A **Write the angles in order from smallest to largest.**
The shortest side is $\overline{GJ}$, so the smallest angle is $\angle H$.
The longest side is $\overline{HJ}$, so the largest angle is $\angle G$.
The angles from smallest to largest are $\angle H$, $\angle J$, and $\angle G$.



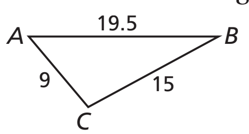
B **Write the sides in order from shortest to longest.**

$m\angle M = 180° - (39° + 54°) = 87°$ *△ Sum Thm.*

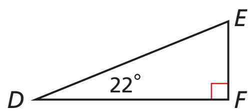
The smallest angle is $\angle L$, so the shortest side is $\overline{KM}$.
The largest angle is $\angle M$, so the longest side is $\overline{KL}$.
The sides from shortest to longest are $\overline{KM}$, $\overline{LM}$, and $\overline{KL}$.

CHECK IT OUT!

2a. Write the angles in order from smallest to largest.

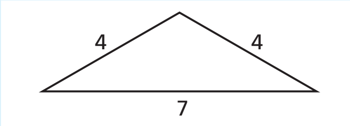


2b. Write the sides in order from shortest to longest.

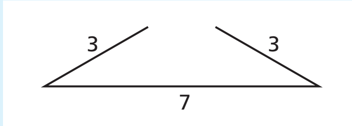


A triangle is formed by three segments, but not every set of three segments can form a triangle.

Segments with lengths of 7, 4, and 4 can form a triangle.



Segments with lengths of 7, 3, and 3 cannot form a triangle.



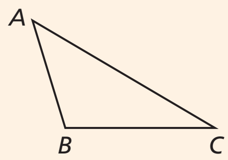
A certain relationship must exist among the lengths of three segments in order for them to form a triangle.

Know it! Note

Theorem 5-5-3 **Triangle Inequality Theorem**

The sum of any two side lengths of a triangle is greater than the third side length.

$AB + BC > AC$
$BC + AC > AB$
$AC + AB > BC$



You will prove Theorem 5-5-3 in Exercise 68.

EXAMPLE 3 Applying the Triangle Inequality Theorem

Tell whether a triangle can have sides with the given lengths. Explain.

A **3, 5, 7**

$3 + 5 \overset{?}{>} 7$ $\quad$ $3 + 7 \overset{?}{>} 5$ $\quad$ $5 + 7 \overset{?}{>} 3$

$8 > 7$ ✓ $\quad$ $10 > 5$ ✓ $\quad$ $12 > 3$ ✓

Yes—the sum of each pair of lengths is greater than the third length.

B **4, 6.5, 11**

$4 + 6.5 \overset{?}{>} 11$

$10.5 \not> 11$

No—by the Triangle Inequality Theorem, a triangle cannot have these side lengths.

C $\boldsymbol{n + 5, n^2, 2n}$**, when** $\boldsymbol{n = 3}$

Step 1 Evaluate each expression when $n = 3$.

$n + 5$	n^2	$2n$
$3 + 5$	3^2	$2(3)$
8	9	6

Step 2 Compare the lengths.

$8 + 9 \overset{?}{>} 6$ $\quad$ $8 + 6 \overset{?}{>} 9$ $\quad$ $9 + 6 \overset{?}{>} 8$

$17 > 6$ ✓ $\quad$ $14 > 9$ ✓ $\quad$ $15 > 8$ ✓

Yes—the sum of each pair of lengths is greater than the third length.

Helpful Hint

To show that three lengths cannot be the side lengths of a triangle, you only need to show that one of the three triangle inequalities is false.

CHECK IT OUT! **Tell whether a triangle can have sides with the given lengths. Explain.**

3a. 8, 13, 21 $\quad$ **3b.** 6.2, 7, 9 $\quad$ **3c.** $t - 2, 4t, t^2 + 1$, when $t = 4$

EXAMPLE 4 Finding Side Lengths

The lengths of two sides of a triangle are 6 centimeters and 11 centimeters. Find the range of possible lengths for the third side.

Let s represent the length of the third side. Then apply the Triangle Inequality Theorem.

$s + 6 > 11$	$s + 11 > 6$	$6 + 11 > s$
$s > 5$	$s > -5$	$17 > s$

Combine the inequalities. So $5 < s < 17$. The length of the third side is greater than 5 centimeters and less than 17 centimeters.

CHECK IT OUT!

4. The lengths of two sides of a triangle are 22 inches and 17 inches. Find the range of possible lengths for the third side.

EXAMPLE 5 *Travel Application*

The map shows the approximate distances from San Antonio to Mason and from San Antonio to Austin. What is the range of distances from Mason to Austin?

Let d be the distance from Mason to Austin.

$d + 111 > 78$	$d + 78 > 111$	$111 + 78 > d$	*△ Inequal. Thm.*
$d > -33$	$d > 33$	$189 > d$	*Subtr. Prop. of Inequal.*
	$33 < d < 189$		*Combine the inequalities.*

The distance from Mason to Austin is greater than 33 miles and less than 189 miles.

CHECK IT OUT!

5. The distance from San Marcos to Johnson City is 50 miles, and the distance from Seguin to San Marcos is 22 miles. What is the range of distances from Seguin to Johnson City?

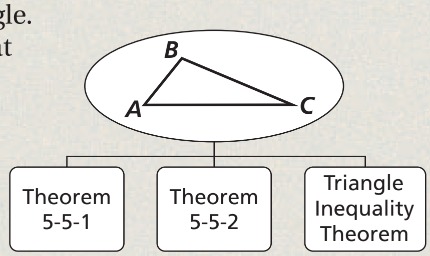
THINK AND DISCUSS

1. To write an indirect proof that an angle is obtuse, a student assumes that the angle is acute. Is this the correct assumption? Explain.

2. Give an example of three measures that can be the lengths of the sides of a triangle. Give an example of three lengths that cannot be the sides of a triangle.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, explain what you know about $\triangle ABC$ as a result of the theorem.



5-5 Exercises

California Standards
2.0, 6.0, 6SDAP3.0, 7AF1.0, 7AF4.0, 7MG3.4, 7MR1.1, 1A7.0

go.hrw.com
Homework Help Online
KEYWORD: MG7 5-5
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

1. **Vocabulary** Describe the process of an *indirect proof* in your own words.

SEE EXAMPLE 1 p. 332

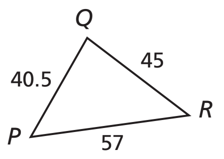
Write an indirect proof of each statement.

2. A scalene triangle cannot have two congruent angles.

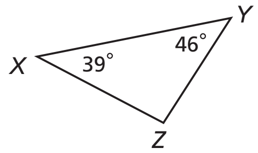
3. An isosceles triangle cannot have a base angle that is a right angle.

SEE EXAMPLE 2 p. 333

4. Write the angles in order from smallest to largest.



5. Write the sides in order from shortest to longest.



SEE EXAMPLE 3 p. 334

Tell whether a triangle can have sides with the given lengths. Explain.

6. 4, 7, 10
7. 2, 9, 12
8. $3\frac{1}{2}, 3\frac{1}{2}, 6$
9. 3, 1.1, 1.7
10. $3x, 2x - 1, x^2$, when $x = 5$
11. $7c + 6, 10c - 7, 3c^2$, when $c = 2$

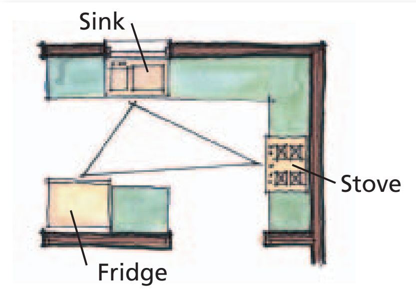
SEE EXAMPLE 4 p. 335

The lengths of two sides of a triangle are given. Find the range of possible lengths for the third side.

12. 8 mm, 12 mm
13. 16 ft, 16 ft
14. 11.4 cm, 12 cm

SEE EXAMPLE 5 p. 335

15. **Design** The refrigerator, stove, and sink in a kitchen are at the vertices of a path called the work triangle.
    a. If the angle at the sink is the largest, which side of the work triangle will be the longest?
    b. The designer wants the longest side of this triangle to be 9 feet long. Can the lengths of the other sides be 5 feet and 4 feet? Explain.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
16–17	1
18–19	2
20–25	3
26–31	4
32	5

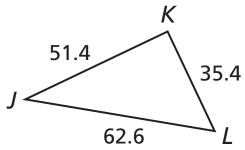
Extra Practice
Skills Practice p. S13
Application Practice p. S32

Write an indirect proof of each statement.

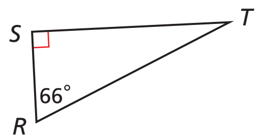
16. A scalene triangle cannot have two congruent midsegments.

17. Two supplementary angles cannot both be obtuse angles.

18. Write the angles in order from smallest to largest.



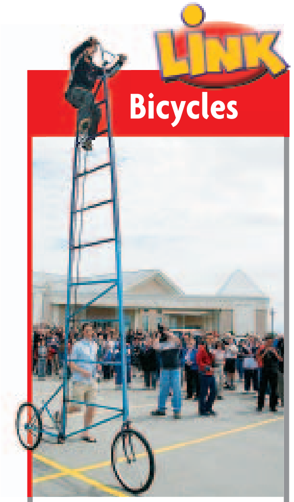
19. Write the sides in order from shortest to longest.



Tell whether a triangle can have sides with the given lengths. Explain.

20. 6, 10, 15
21. 14, 18, 32
22. 11.9, 5.8, 5.8
23. 103, 41.9, 62.5
24. $z + 8, 3z + 5, 4z - 11$, when $z = 6$
25. $m + 11, 8m, m^2 + 1$, when $m = 3$

LINK Bicycles



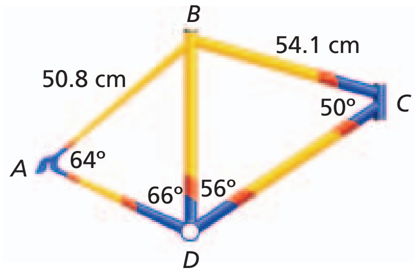
On June 26, 2004, Terry Goertzen of Winnipeg, Canada, attained the new Guinness world record for the tallest bicycle with his 5.5-meter-tall bike.

The lengths of two sides of a triangle are given. Find the range of possible lengths for the third side.

26. 4 yd, 19 yd **27.** 28 km, 23 km **28.** 9.2 cm, 3.8 cm

29. 3.07 m, 1.89 m **30.** $2\frac{1}{8}$ in., $3\frac{5}{8}$ in. **31.** $3\frac{5}{6}$ ft, $6\frac{1}{2}$ ft

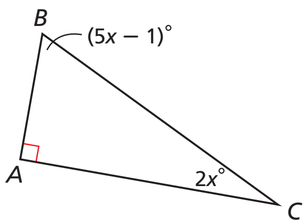
32. Bicycles The five steel tubes of this mountain bike frame form two triangles. List the five tubes in order from shortest to longest. Explain your answer.



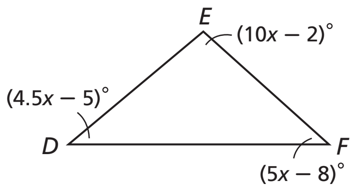
33. Critical Thinking The length of the base of an isosceles triangle is 15. What is the range of possible lengths for each leg? Explain.

List the sides of each triangle in order from shortest to longest.

34.



35.



In each set of statements, name the two that contradict each other.

36. $\triangle PQR$ is a right triangle.
$\triangle PQR$ is a scalene triangle.
$\triangle PQR$ is an acute triangle.

37. $\angle Y$ is supplementary to $\angle Z$.
$m\angle Y < 90°$
$\angle Y$ is an obtuse angle.

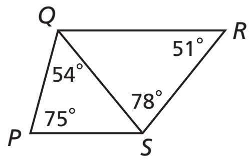
38. $\triangle JKL$ is isosceles with base $\overline{JL}$.
In $\triangle JKL$, $m\angle K > m\angle J$
In $\triangle JKL$, $JK > LK$

39. $\overline{AB} \perp \overline{BC}$
$\overline{AB} \cong \overline{CD}$
$\overline{AB} \parallel \overline{BC}$

40. Figure A is a polygon.
Figure A is a triangle.
Figure A is a quadrilateral.

41. x is even.
x is a multiple of 4.
x is prime.

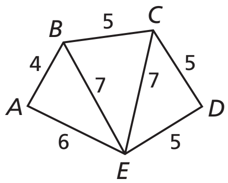
Compare. Write <, >, or =.



42. QS ▢ PS **43.** PQ ▢ QS

44. QS ▢ QR **45.** QS ▢ RS

46. PQ ▢ RS **47.** RS ▢ PS



48. $m\angle ABE$ ▢ $m\angle BEA$ **49.** $m\angle CBE$ ▢ $m\angle CEB$

50. $m\angle DCE$ ▢ $m\angle DEC$ **51.** $m\angle DCE$ ▢ $m\angle CDE$

52. $m\angle ABE$ ▢ $m\angle EAB$ **53.** $m\angle EBC$ ▢ $m\angle ECB$

List the angles of $\triangle JKL$ in order from smallest to largest.

54. $J(-3, -2)$, $K(3, 6)$, $L(8, -2)$ **55.** $J(-5, -10)$, $K(-5, 2)$, $L(7, -5)$

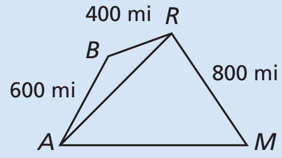
56. $J(-4, 1)$, $K(-3, 8)$, $L(3, 4)$ **57.** $J(-10, -4)$, $K(0, 3)$, $L(2, -8)$

58. Critical Thinking An attorney argues that her client did not commit a burglary because a witness saw her client in a different city at the time of the burglary. Explain how this situation is an example of indirect reasoning.

CONCEPT CONNECTION

59. This problem will prepare you for the Concept Connection on page 364.

The figure shows an airline's routes between four cities.



a. The airline's planes fly at an average speed of 500 mi/h. What is the range of time it might take to fly from Auburn (A) to Raymond (R)?

b. The airline offers one frequent-flier mile for every mile flown. Is it possible to earn 1800 miles by flying from Millford (M) to Auburn (A)? Explain.

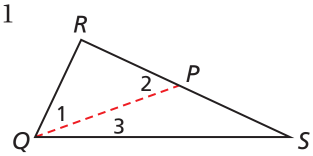
Multi-Step **Each set of expressions represents the lengths of the sides of a triangle. Find the range of possible values of *n*.**

60. $n, 6, 8$ **61.** $2n, 5, 7$ **62.** $n + 1, 3, 6$

63. $n + 1, n + 2, n + 3$ **64.** $n + 2, n + 3, 3n - 2$ **65.** $n, n + 2, 2n + 1$

66. Given that P is in the interior of $\triangle XYZ$, prove that $XY + XP + PZ > YZ$.

67. Complete the proof of Theorem 5-5-1 by filling in the blanks.



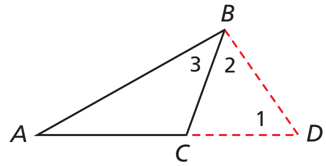
Given: $RS > RQ$

Prove: $m\angle RQS > m\angle S$

Proof:

Locate P on $\overline{RS}$ so that $RP = RQ$. So $\overline{RP} \cong \overline{RQ}$ by **a.** ___?___. Then $\angle 1 \cong \angle 2$ by **b.** ___?___, and $m\angle 1 = m\angle 2$ by **c.** ___?___. By the Angle Addition Postulate, $m\angle RQS =$ **d.** ___?___. So $m\angle RQS > m\angle 1$ by the Comparison Property of Inequality. Then $m\angle RQS > m\angle 2$ by **e.** ___?___. By the Exterior Angle Theorem, $m\angle 2 = m\angle 3 +$ **f.** ___?___. So $m\angle 2 > m\angle S$ by the Comparison Property of Inequality. Therefore $m\angle RQS > m\angle S$ by **g.** ___?___.

68. Complete the proof of the Triangle Inequality Theorem.



Given: $\triangle ABC$

Prove: $AB + BC > AC, AB + AC > BC, AC + BC > AB$

Proof:

One side of $\triangle ABC$ is as long as or longer than each of the other sides. Let this side be $\overline{AB}$. Then $AB + BC > AC$, and $AB + AC > BC$. Therefore what remains to be proved is $AC + BC > AB$.

Statements	Reasons
1. **a.** ___?___	**1.** Given
2. Locate D on $\overrightarrow{AC}$ so that $BC = DC$.	**2.** Ruler Post.
3. $AC + DC =$ **b.** ___?___	**3.** Seg. Add. Post.
4. $\angle 1 \cong \angle 2$	**4.** **c.** ___?___
5. $m\angle 1 = m\angle 2$	**5.** **d.** ___?___
6. $m\angle ABD = m\angle 2 +$ **e.** ___?___	**6.** $\angle$ Add. Post.
7. $m\angle ABD > m\angle 2$	**7.** Comparison Prop. of Inequal.
8. $m\angle ABD > m\angle 1$	**8.** **f.** ___?___
9. $AD > AB$	**9.** **g.** ___?___
10. $AC + DC > AB$	**10.** **h.** ___?___
11. **i.** ___?___	**11.** Subst.

69. **Write About It** Explain why the hypotenuse is always the longest side of a right triangle. Explain why the diagonal of a square is longer than each side.

STANDARDIZED TEST PREP

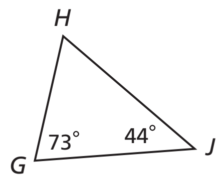
70. The lengths of two sides of a triangle are 3 feet and 5 feet. Which could be the length of the third side?

Ⓐ 3 feet Ⓑ 8 feet Ⓒ 15 feet Ⓓ 16 feet

71. Which statement about $\triangle GHJ$ is false?

Ⓕ $GH < GJ$ Ⓗ $GH + HJ < GJ$

Ⓖ $m\angle H > m\angle J$ Ⓙ $\triangle GHJ$ is a scalene triangle.



72. In $\triangle RST$, $m\angle S = 92°$. Which is the longest side of $\triangle RST$?

Ⓐ $\overline{RS}$ Ⓒ $\overline{RT}$

Ⓑ $\overline{ST}$ Ⓓ Cannot be determined

CHALLENGE AND EXTEND

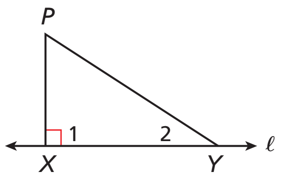
73. Probability A bag contains five sticks. The lengths of the sticks are 1 inch, 3 inches, 5 inches, 7 inches, and 9 inches. Suppose you pick three sticks from the bag at random. What is the probability you can form a triangle with the three sticks?

74. Complete this indirect argument that $\sqrt{2}$ is irrational. Assume that **a.** ___?___ . Then $\sqrt{2} = \frac{p}{q}$, where p and q are positive integers that have no common factors. Thus $2 =$ **b.** ___?___ , and $p^2 =$ **c.** ___?___ . This implies that p^2 is even, and thus p is even. Since p^2 is the square of an even number, p^2 is divisible by 4 because **d.** ___?___ . But then q^2 must be even because **e.** ___?___ , and so q is even. Then p and q have a common factor of 2, which contradicts the assumption that p and q have no common factors.

75. Prove that the perpendicular segment from a point to a line is the shortest segment from the point to the line.

Given: $\overline{PX} \perp \ell$. Y is any point on ℓ other than X.
Prove: $PY > PX$

Plan: Show that $\angle 2$ and $\angle P$ are complementary. Use the Comparison Property of Inequality to show that $90° > m\angle 2$. Then show that $m\angle 1 > m\angle 2$ and thus $PY > PX$.



SPIRAL REVIEW

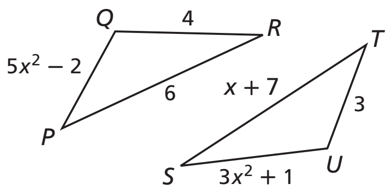
Write the equation of each line in standard form. *(Previous course)*

76. the line through points $(-3, 2)$ and $(-1, -2)$

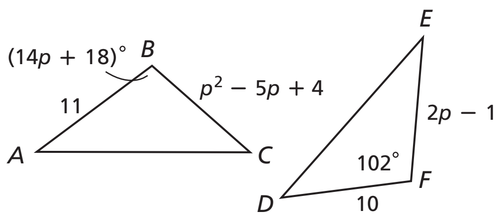
77. the line with slope 2 and x-intercept of -3

Show that the triangles are congruent for the given value of the variable. *(Lesson 4-4)*

78. $\triangle PQR \cong \triangle TUS$, when $x = -1$



79. $\triangle ABC \cong \triangle EFD$, when $p = 6$



Find the orthocenter of a triangle with the given vertices. *(Lesson 5-3)*

80. $R(0, 5)$, $S(4, 3)$, $T(0, 1)$

81. $M(0, 0)$, $N(3, 0)$, $P(0, 5)$

5-6 Inequalities in Two Triangles

Objective
Apply inequalities in two triangles.

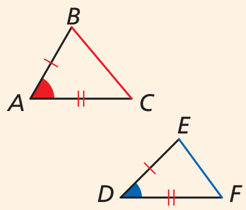
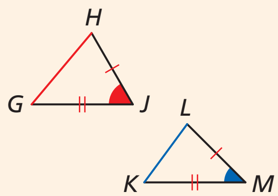
California Standards
2.0 Students write geometric proofs, including proofs by contradiction.

Who uses this?
Designers of this circular swing ride can use the angle of the swings to determine how high the chairs will be at full speed. (See Example 2.)

In this lesson, you will apply inequality relationships between two triangles.

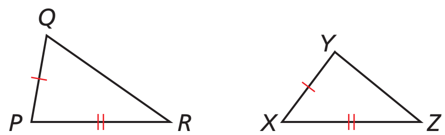
Know it! Note

Theorems: Inequalities in Two Triangles

THEOREM	HYPOTHESIS	CONCLUSION
5-6-1 **Hinge Theorem** If two sides of one triangle are congruent to two sides of another triangle and the included angles are not congruent, then the longer third side is across from the larger included angle.	 $m\angle A > m\angle D$	$BC > EF$
5-6-2 **Converse of the Hinge Theorem** If two sides of one triangle are congruent to two sides of another triangle and the third sides are not congruent, then the larger included angle is across from the longer third side.	 $GH > KL$	$m\angle J > m\angle M$

You will prove Theorem 5-6-1 in Exercise 35.

PROOF **Converse of the Hinge Theorem**



Given: $\overline{PQ} \cong \overline{XY}$, $\overline{PR} \cong \overline{XZ}$, $QR > YZ$
Prove: $m\angle P > m\angle X$

Indirect Proof:

Assume $m\angle P \not> m\angle X$. So either $m\angle P < m\angle X$, or $m\angle P = m\angle X$.

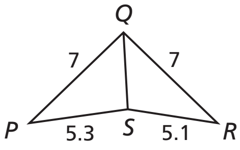
Case 1 If $m\angle P < m\angle X$, then $QR < YZ$ by the Hinge Theorem. This contradicts the given information that $QR > YZ$. So $m\angle P \not< m\angle X$.

Case 2 If $m\angle P = m\angle X$, then $\angle P \cong \angle X$. So $\triangle PQR \cong \triangle XYZ$ by SAS. Then $\overline{QR} \cong \overline{YZ}$ by CPCTC, and $QR = YZ$. This also contradicts the given information. So $m\angle P \neq m\angle X$.

The assumption $m\angle P \not> m\angle X$ is false. Therefore $m\angle P > m\angle X$.

EXAMPLE 1 Using the Hinge Theorem and Its Converse

A **Compare m∠*PQS* and m∠*RQS*.**

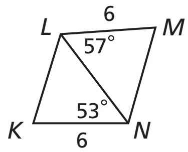


Compare the side lengths in △*PQS* and △*RQS*.

$PQ = RQ \qquad QS = QS \qquad PS > RS$

By the Converse of the Hinge Theorem, $m\angle PQS > m\angle RQS$.

B **Compare *KL* and *MN*.**



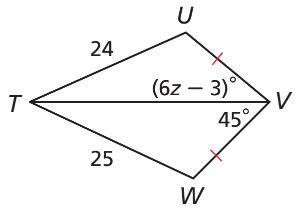
Compare the sides and angles in △*KLN* and △*MNL*.

$KN = ML \qquad LN = LN \qquad m\angle LNK < m\angle NLM$

By the Hinge Theorem, $KL < MN$.

Algebra

C **Find the range of values for *z*.**



Step 1 Compare the side lengths in △*TUV* and △*TWV*.

$TV = TV \qquad VU = VW \qquad TU < TW$

By the Converse of the Hinge Theorem, $m\angle UVT < m\angle WVT$.

$6z - 3 < 45$ *Substitute the given values.*

$z < 8$ *Add 3 to both sides and divide both sides by 6.*

Step 2 Since ∠*UVT* is in a triangle, $m\angle UVT > 0°$.

$6z - 3 > 0$ *Substitute the given value.*

$z > 0.5$ *Add 3 to both sides and divide both sides by 6.*

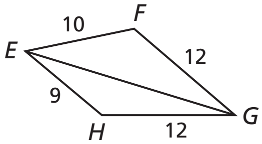
Step 3 Combine the inequalities.

The range of values for *z* is $0.5 < z < 8$.

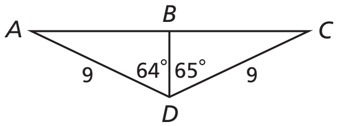
CHECK IT OUT!

Compare the given measures.

1a. m∠*EGH* and m∠*EGF*

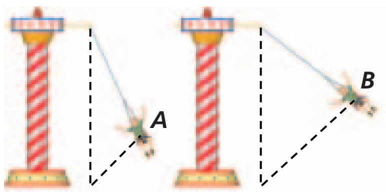


1b. *BC* and *AB*



EXAMPLE 2 *Entertainment Application*

The angle of the swings in a circular swing ride changes with the speed of the ride. The diagram shows the position of one swing at two different speeds. Which rider is farther from the base of the swing tower? Explain.



The height of the tower and the length of the cable holding the chair are the same in both triangles.

The angle formed by the swing in position *A* is smaller than the angle formed by the swing in position *B*. So rider *B* is farther from the base of the tower than rider *A* by the Hinge Theorem.

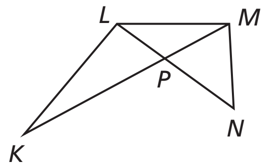
2. When the swing ride is at full speed, the chairs are farthest from the base of the swing tower. What can you conclude about the angles of the swings at full speed versus low speed? Explain.

EXAMPLE 3 Proving Triangle Relationships

Write a two-column proof.

Given: $\overline{KL} \cong \overline{NL}$

Prove: $KM > NM$

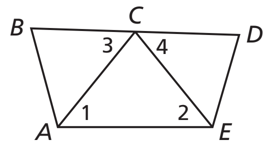


Proof:

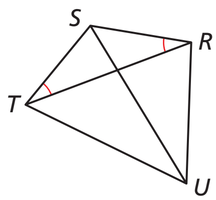
Statements	Reasons
1. $\overline{KL} \cong \overline{NL}$	**1.** Given
2. $\overline{LM} \cong \overline{LM}$	**2.** Reflex. Prop. of $\cong$
3. $m\angle KLM = m\angle NLM + m\angle KLN$	**3.** $\angle$ Add. Post.
4. $m\angle KLM > m\angle NLM$	**4.** Comparison Prop. of Inequal.
5. $KM > NM$	**5.** Hinge Thm.

Write a two-column proof.

3a. Given: C is the midpoint of $\overline{BD}$.
$m\angle 1 = m\angle 2$
$m\angle 3 > m\angle 4$
Prove: $AB > ED$



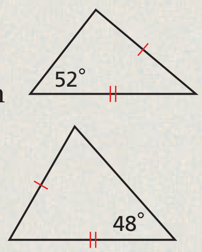
3b. Given: $\angle SRT \cong \angle STR$
$TU > RU$
Prove: $m\angle TSU > m\angle RSU$



THINK AND DISCUSS

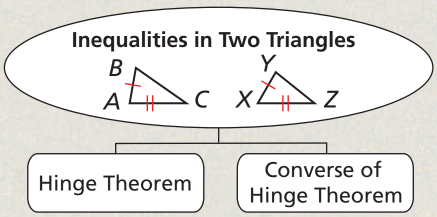
1. Describe a real-world object that shows the Hinge Theorem or its converse.

2. Can you make a conclusion about the triangles shown at right by applying the Hinge Theorem? Explain.



Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, use the given triangles to write a statement for the theorem.



5-6 Exercises

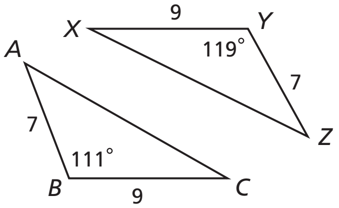
California Standards
2.0, 5.0, 7.0, 6SDAP1.1, 7AF1.0, 7AF4.1, 7MG3.4, 7MR1.1

GUIDED PRACTICE

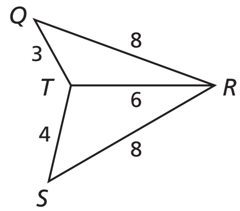
SEE EXAMPLE 1
p. 341

Compare the given measures.

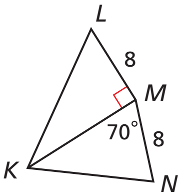
1. AC and XZ



2. m∠SRT and m∠QRT

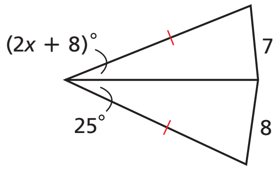


3. KL and KN



Find the range of values for x.

4.



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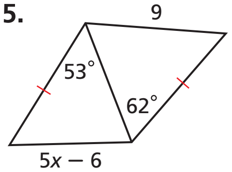

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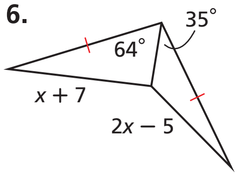
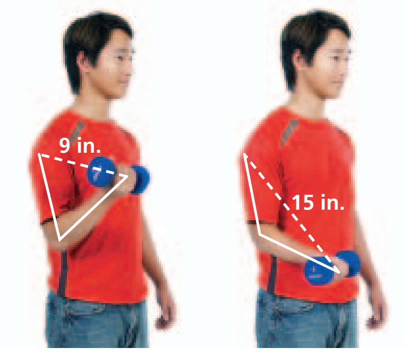


SEE EXAMPLE 2
p. 341

7. Health A therapist can take measurements to gauge the flexibility of a patient's elbow joint. In which position is the angle measure at the elbow joint greater? Explain.

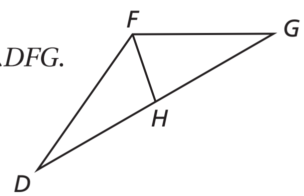


SEE EXAMPLE 3
p. 342

8. Write a two-column proof.

Given: $\overline{FH}$ is a median of $\triangle DFG$.
m∠DHF > m∠GHF

Prove: $DF > GF$



PRACTICE AND PROBLEM SOLVING

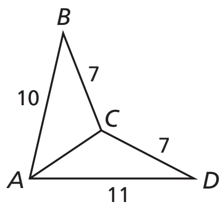
Independent Practice

For Exercises	See Example
9–14	1
15	2
16	3

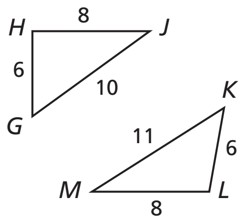
Extra Practice
Skills Practice p. S13
Application Practice p. S32

Compare the given measures.

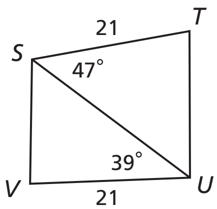
9. m∠DCA and m∠BCA



10. m∠GHJ and m∠KLM

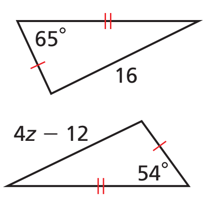


11. TU and SV

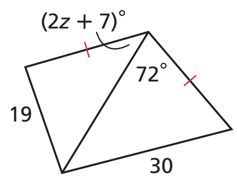


Find the range of values for z.

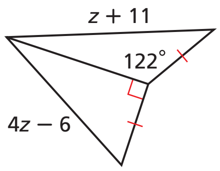
12.



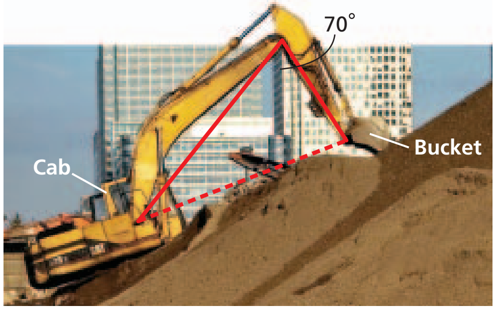
13.

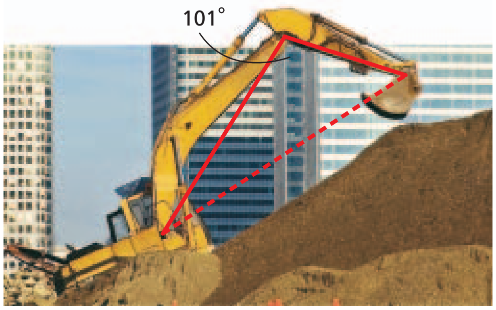


14.

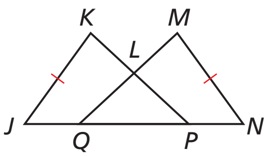


15. **Industry** The operator of a backhoe changes the distance between the cab and the bucket by changing the angle formed by the arms. In which position is the distance from the cab to the bucket greater? Explain.



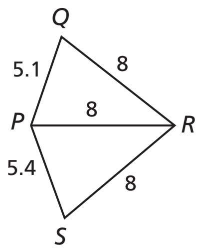


16. Write a two-column proof.
Given: $\overline{JK} \cong \overline{NM}$, $\overline{KP} \cong \overline{MQ}$, $JQ > NP$
Prove: $m\angle K > m\angle M$



17. **Critical Thinking** ABC is an isosceles triangle with base $\overline{BC}$. XYZ is an isosceles triangle with base $\overline{YZ}$. Given that $\overline{AB} \cong \overline{XY}$ and $m\angle A = m\angle X$, compare BC and YZ.

Compare. Write <, >, or =.



18. $m\angle QRP$ ▢ $m\angle SRP$
19. $m\angle QPR$ ▢ $m\angle QRP$
20. $m\angle PRS$ ▢ $m\angle RSP$
21. $m\angle RSP$ ▢ $m\angle RPS$
22. $m\angle QPR$ ▢ $m\angle RPS$
23. $m\angle PSR$ ▢ $m\angle PQR$

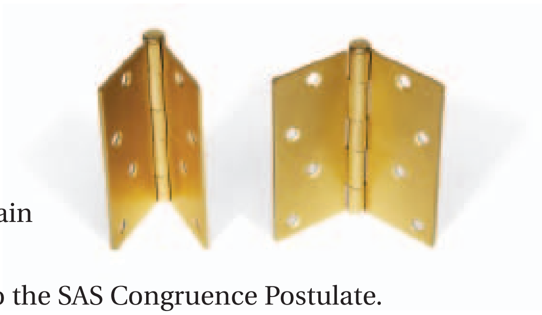
Make a conclusion based on the Hinge Theorem or its converse. (*Hint*: Draw a sketch.)

24. In $\triangle ABC$ and $\triangle DEF$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $m\angle B = 59°$, and $m\angle E = 47°$.

25. $\triangle RST$ is isosceles with base $\overline{RT}$. The endpoints of $\overline{SV}$ are vertex S and a point V on $\overline{RT}$. $RV = 4$, and $TV = 5$.

26. In $\triangle GHJ$ and $\triangle KLM$, $\overline{GH} \cong \overline{KL}$, and $\overline{GJ} \cong \overline{KM}$. $\angle G$ is a right angle, and $\angle K$ is an acute angle.

27. In $\triangle XYZ$, $\overline{XM}$ is the median to $\overline{YZ}$, and $YX > ZX$.

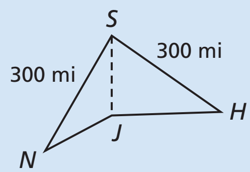


28. **Write About It** The picture shows a door hinge in two different positions. Use the picture to explain why Theorem 5-6-1 is called the Hinge Theorem.

29. **Write About It** Compare the Hinge Theorem to the SAS Congruence Postulate. How are they alike? How are they different?

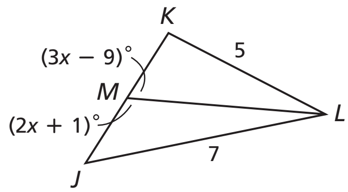
CONCEPT CONNECTION

30. This problem will prepare you for the Concept Connection on page 364.
The solid lines in the figure show an airline's routes between four cities.
  a. A traveler wants to fly from Jackson (J) to Shelby (S), but there is no direct flight between these cities. Given that $m\angle NSJ < m\angle HSJ$, should the traveler first fly to Newton Springs (N) or to Hollis (H) if he wants to minimize the number of miles flown? Why?
  b. The distance from Shelby (S) to Jackson (J) is 182 mi. What is the minimum number of miles the traveler will have to fly?



Standardized Test Prep

31. $\overline{ML}$ is a median of $\triangle JKL$. Which inequality best describes the range of values for x?



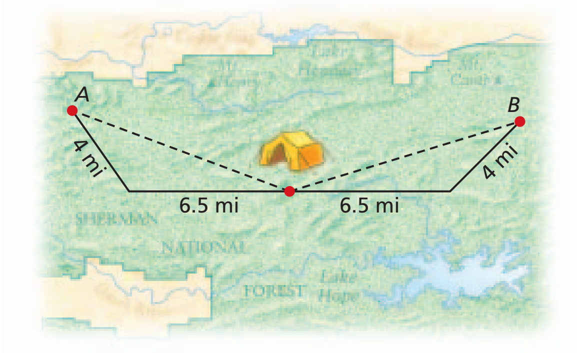
Ⓐ $x > 2$ Ⓒ $3 < x < 4\frac{2}{3}$

Ⓑ $x > 10$ Ⓓ $3 < x < 10$

32. $\overline{DC}$ is a median of $\triangle ABC$. Which of the following statements is true?

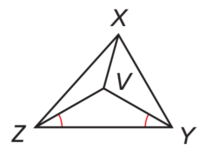
Ⓕ $BC < AC$ Ⓖ $BC > AC$ Ⓗ $AD = DB$ Ⓙ $DC = AB$

33. Short Response Two groups start hiking from the same camp. Group A hikes 6.5 miles due west and then hikes 4 miles in the direction N 35° W. Group B hikes 6.5 miles due east and then hikes 4 miles in the direction N 45° E. At this point, which group is closer to the camp? Explain.

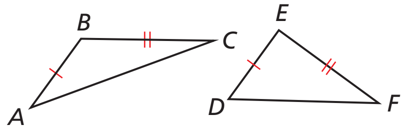


CHALLENGE AND EXTEND

34. Multi-Step In $\triangle XYZ$, $XZ = 5x + 15$, $XY = 8x - 6$, and $m\angle XVZ > m\angle XVY$. Find the range of values for x.



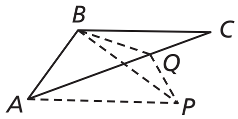
35. Use these steps to write a paragraph proof of the Hinge Theorem.



Given: $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $m\angle ABC > m\angle DEF$

Prove: $AC > DF$

a. Locate P outside $\triangle ABC$ so that $\angle ABP \cong \angle DEF$ and $\overline{BP} \cong \overline{EF}$. Show that $\triangle ABP \cong \triangle DEF$ and thus $\overline{AP} \cong \overline{DF}$.



b. Locate Q on $\overline{AC}$ so that $\overline{BQ}$ bisects $\angle PBC$. Draw $\overline{QP}$. Show that $\triangle BQP \cong \triangle BQC$ and thus $\overline{QP} \cong \overline{QC}$.

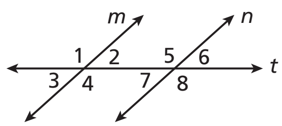
c. Justify the statements $AQ + QP > AP$, $AQ + QC = AC$, $AQ + QC > AP$, $AC > AP$, and $AC > DF$.

SPIRAL REVIEW

Find the range and mode, if any, of each set of data. *(Previous course)*

36. 2, 5, 1, 0.5, 0.75, 2 **37.** 95, 97, 89, 87, 85, 99 **38.** 5, 5, 7, 9, 4, 4, 8, 7

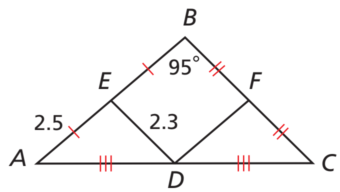
For the given information, show that $m \parallel n$. State any postulates or theorems used. *(Lesson 3-3)*



39. $m\angle 2 = (3x + 21)°$, $m\angle 6 = (7x + 1)°$, $x = 5$

40. $m\angle 4 = (2x + 34)°$, $m\angle 7 = (15x + 27)°$, $x = 7$

Find each measure. *(Lesson 5-4)*



41. DF **42.** BC **43.** $m\angle BFD$

Connecting Geometry to **Algebra**

See Skills Bank page S55

Simplest Radical Form

When a problem involves square roots, you may be asked to give the answer in simplest radical form. Recall that the radicand is the expression under the radical sign.

California Standards

Review of 1A2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, and **taking a root,** and raising to a fractional power. They understand and use the rules of exponents.

Simplest Form of a Square-Root Expression

An expression containing square roots is in simplest form when

- the radicand has no perfect square factors other than 1.
- the radicand has no fractions.
- there are no square roots in any denominator.

To simplify a radical expression, remember that the square root of a product is equal to the product of the square roots. Also, the square root of a quotient is equal to the quotient of the square roots.

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}, \text{ when } a \geq 0 \text{ and } b \geq 0$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, \text{ when } a \geq 0 \text{ and } b > 0$$

Examples

Write each expression in simplest radical form.

A $\sqrt{216}$

$\sqrt{216}$ — *216 has a perfect-square factor of 36, so the expression is not in simplest radical form.*

$\sqrt{(36)(6)}$ — *Factor the radicand.*

$\sqrt{36} \cdot \sqrt{6}$ — *Product Property of Square Roots*

$6\sqrt{6}$ — *Simplify.*

B $\frac{6}{\sqrt{2}}$

$\frac{6}{\sqrt{2}}$ — *There is a square root in the denominator, so the expression is not in simplest radical form.*

$\frac{6}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$ — *Multiply by a form of 1 to eliminate the square root in the denominator.*

$\frac{6\sqrt{2}}{2}$ — *Simplify.*

$3\sqrt{2}$ — *Divide.*

Try This

Write each expression in simplest radical form.

1. $\sqrt{720}$ **2.** $\sqrt{\frac{3}{16}}$ **3.** $\frac{10}{\sqrt{2}}$ **4.** $\sqrt{\frac{1}{3}}$ **5.** $\sqrt{45}$

5-7 Geometry LAB

Use with Lesson 5-7

Hands-on Proof of the Pythagorean Theorem

In Lesson 1-6, you used the Pythagorean Theorem to find the distance between two points in the coordinate plane. In this activity, you will build figures and compare their areas to justify the Pythagorean Theorem.

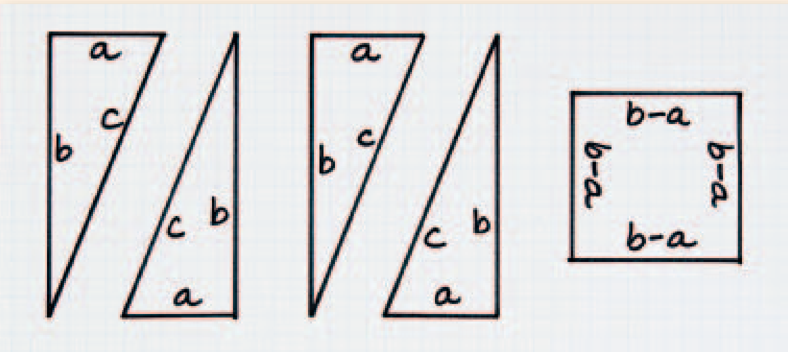
California Standards

14.0 Students prove the Pythagorean theorem.

Activity

1. Draw a large scalene right triangle on graph paper. Draw three copies of the triangle. On each triangle, label the shorter leg a, the longer leg b, and the hypotenuse c.

2. Draw a square with a side length of $b - a$. Label each side of the square.

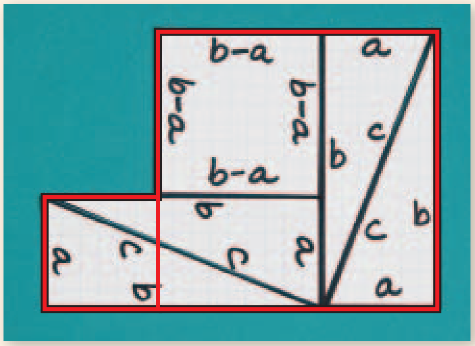


3. Cut out the five figures. Arrange them to make the composite figure shown at right.

4. You can think of this composite figure as being made of the two squares outlined in red. What are the side length and area of the small red square? of the large red square?

5. Use your results from Step 4 to write an algebraic expression for the area of the composite figure.

6. Now rearrange the five figures to make a single square with side length c. Write an algebraic expression for the area of this square.



Try This

1. Since the composite figure and the square with side length c are made of the same five shapes, their areas are equal. Write and simplify an equation to represent this relationship. What conclusion can you make?

2. Draw a scalene right triangle with different side lengths. Repeat the activity. Do you reach the same conclusion?

5-7 The Pythagorean Theorem

Objectives

Use the Pythagorean Theorem and its converse to solve problems.

Use Pythagorean inequalities to classify triangles.

Vocabulary

Pythagorean triple

California Standards

14.0 Students prove the Pythagorean theorem.
15.0 Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles.
Also covered: **6.0, 12.0**

Math Builders

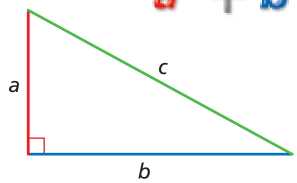
For more on the Pythagorean Theorem, see the Theorem Builder on page MB4.

Why learn this?

You can use the Pythagorean Theorem to determine whether a ladder is in a safe position. (See Example 2.)

The Pythagorean Theorem is probably the most famous mathematical relationship. As you learned in Lesson 1-6, it states that in a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse.

$$a^2 + b^2 = c^2$$



The Pythagorean Theorem is named for the Greek mathematician Pythagoras, who lived in the sixth century B.C.E. However, this relationship was known to earlier people, such as the Babylonians, Egyptians, and Chinese.

There are many different proofs of the Pythagorean Theorem. The one below uses area and algebra.

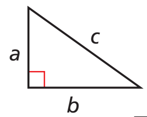
Remember!

The area A of a square with side length s is given by the formula $A = s^2$.

The area A of a triangle with base b and height h is given by the formula $A = \frac{1}{2}bh$.

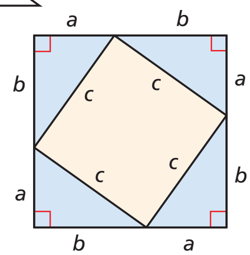
PROOF

Pythagorean Theorem



Given: A right triangle with leg lengths a and b and hypotenuse of length c

Prove: $a^2 + b^2 = c^2$



Proof: Arrange four copies of the triangle as shown. The sides of the triangles form two squares.

The area of the outer square is $(a + b)^2$. The area of the inner square is c^2. The area of each blue triangle is $\frac{1}{2}ab$.

area of outer square = area of 4 blue triangles + area of inner square

$$(a + b)^2 = 4\left(\frac{1}{2}ab\right) + c^2$$ *Substitute the areas.*

$$a^2 + 2ab + b^2 = 2ab + c^2$$ *Simplify.*

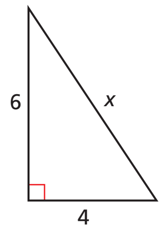
$$a^2 + b^2 = c^2$$ *Subtract 2ab from both sides.*

The Pythagorean Theorem gives you a way to find unknown side lengths when you know a triangle is a right triangle.

EXAMPLE 1 Using the Pythagorean Theorem

Find the value of x. Give your answer in simplest radical form.

A



$a^2 + b^2 = c^2$ — *Pythagorean Theorem*

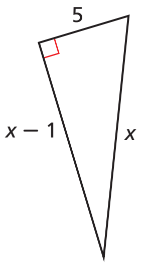
$6^2 + 4^2 = x^2$ — *Substitute 6 for a, 4 for b, and x for c.*

$52 = x^2$ — *Simplify.*

$\sqrt{52} = x$ — *Find the positive square root.*

$x = \sqrt{(4)(13)} = 2\sqrt{13}$ — *Simplify the radical.*

B



$a^2 + b^2 = c^2$ — *Pythagorean Theorem*

$5^2 + (x - 1)^2 = x^2$ — *Substitute 5 for a, x − 1 for b, and x for c.*

$25 + x^2 - 2x + 1 = x^2$ — *Multiply.*

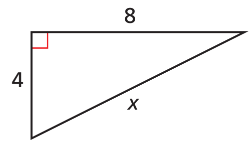
$-2x + 26 = 0$ — *Combine like terms.*

$26 = 2x$ — *Add 2x to both sides.*

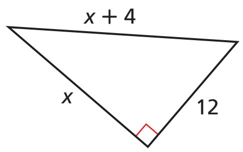
$x = 13$ — *Divide both sides by 2.*

CHECK IT OUT! **Find the value of x. Give your answer in simplest radical form.**

1a.

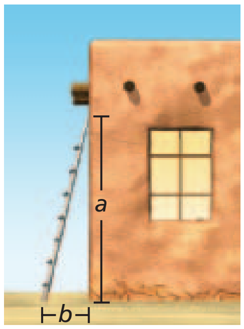


1b.



EXAMPLE 2 *Safety Application*

To prevent a ladder from shifting, safety experts recommend that the ratio of $a:b$ be 4:1. How far from the base of the wall should you place the foot of a 10-foot ladder? Round to the nearest inch.



Let x be the distance in feet from the foot of the ladder to the base of the wall. Then $4x$ is the distance in feet from the top of the ladder to the base of the wall.

$a^2 + b^2 = c^2$ — *Pythagorean Theorem*

$(4x)^2 + x^2 = 10^2$ — *Substitute.*

$17x^2 = 100$ — *Multiply and combine like terms.*

$x^2 = \frac{100}{17}$ — *Divide both sides by 17.*

$x = \sqrt{\frac{100}{17}} \approx 2$ ft 5 in. — *Find the positive square root and round it.*

CHECK IT OUT! **2. What if...?** According to the recommended ratio, how high will a 30-foot ladder reach when placed against a wall? Round to the nearest inch.

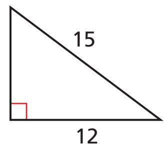
A set of three nonzero whole numbers a, b, and c such that $a^2 + b^2 = c^2$ is called a **Pythagorean triple**.

Common Pythagorean Triples			
3, 4, 5	5, 12, 13,	8, 15, 17	7, 24, 25

EXAMPLE 3 Identifying Pythagorean Triples

Find the missing side length. Tell if the side lengths form a Pythagorean triple. Explain.

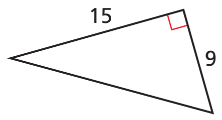
A



$a^2 + b^2 = c^2$	*Pythagorean Theorem*
$12^2 + b^2 = 15^2$	*Substitute 12 for a and 15 for c.*
$b^2 = 81$	*Multiply and subtract 144 from both sides.*
$b = 9$	*Find the positive square root.*

The side lengths are nonzero whole numbers that satisfy the equation $a^2 + b^2 = c^2$, so they form a Pythagorean triple.

B

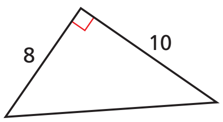


$a^2 + b^2 = c^2$	*Pythagorean Theorem*
$9^2 + 15^2 = c^2$	*Substitute 9 for a and 15 for b.*
$306 = c^2$	*Multiply and add.*
$c = \sqrt{306} = 3\sqrt{34}$	*Find the positive square root and simplify.*

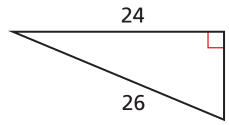
The side lengths do not form a Pythagorean triple because $3\sqrt{34}$ is not a whole number.

CHECK IT OUT! **Find the missing side length. Tell if the side lengths form a Pythagorean triple. Explain.**

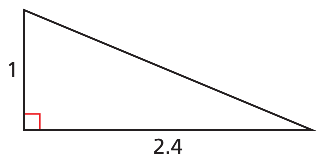
3a.



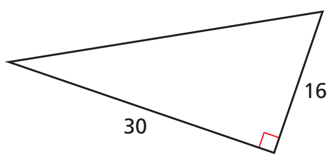
3b.



3c.



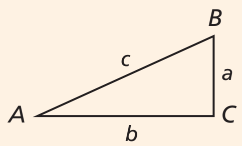
3d.



The converse of the Pythagorean Theorem gives you a way to tell if a triangle is a right triangle when you know the side lengths.

Know it! Note

Theorems 5-7-1 Converse of the Pythagorean Theorem

THEOREM	HYPOTHESIS	CONCLUSION
If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.	 $a^2 + b^2 = c^2$	$\triangle ABC$ is a right triangle.

You will prove Theorem 5-7-1 in Exercise 45.

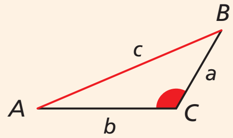
You can also use side lengths to classify a triangle as acute or obtuse.

Know it! Note

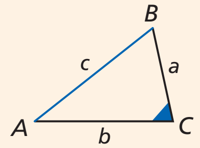
Theorems 5-7-2 **Pythagorean Inequalities Theorem**

In $\triangle ABC$, c is the length of the longest side.

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is an **obtuse** triangle.

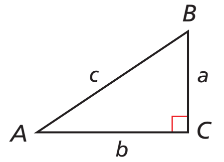


If $c^2 < a^2 + b^2$, then $\triangle ABC$ is an **acute** triangle.

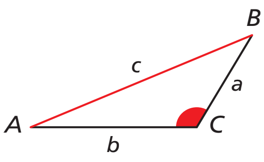


To understand why the Pythagorean inequalities are true, consider $\triangle ABC$.

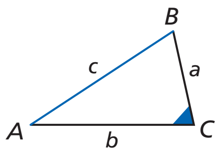
If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle by the Converse of the Pythagorean Theorem. So $m\angle C = 90°$.



If $c^2 > a^2 + b^2$, then c has increased. By the Converse of the Hinge Theorem, $m\angle C$ has also increased. So $\mathbf{m\angle C > 90°}$.



If $c^2 < a^2 + b^2$, then c has decreased. By the Converse of the Hinge Theorem, $m\angle C$ has also decreased. So $\mathbf{m\angle C < 90°}$.



EXAMPLE 4 Classifying Triangles

Tell if the measures can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.

A **8, 11, 13**

Step 1 Determine if the measures form a triangle.

By the Triangle Inequality Theorem, 8, 11, and 13 can be the side lengths of a triangle.

Step 2 Classify the triangle.

$c^2 \stackrel{?}{=} a^2 + b^2$	*Compare c^2 to $a^2 + b^2$.*
$13^2 \stackrel{?}{=} 8^2 + 11^2$	*Substitute the longest side length for c.*
$169 \stackrel{?}{=} 64 + 121$	*Multiply.*
$169 < 185$	*Add and compare.*

Since $c^2 < a^2 + b^2$, the triangle is **acute.**

B **5.8, 9.3, 15.6**

Step 1 Determine if the measures form a triangle.

Since $5.8 + 9.3 = 15.1$ and $15.1 \not> 15.6$, these cannot be the side lengths of a triangle.

Remember!

By the Triangle Inequality Theorem, the sum of any two side lengths of a triangle is greater than the third side length.

CHECK IT OUT!

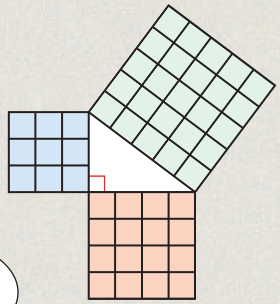
Tell if the measures can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.

4a. 7, 12, 16 **4b.** 11, 18, 34 **4c.** 3.8, 4.1, 5.2

THINK AND DISCUSS

1. How do you know which numbers to substitute for c, a, and b when using the Pythagorean Inequalities?

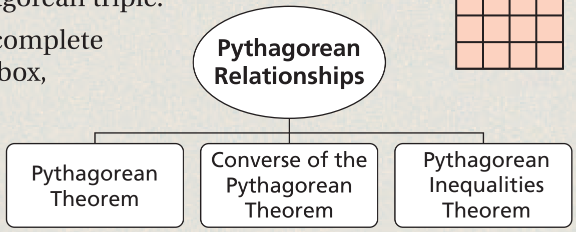
2. Explain how the figure at right demonstrates the Pythagorean Theorem.



3. List the conditions that a set of three numbers must satisfy in order to form a Pythagorean triple.

Know it! Note

4. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, summarize the Pythagorean relationship.



5-7 Exercises

California Standards
2.0, 5.0, 6.0, 8.0, 12.0, 15.0, 17.0, 7AF4.1, 7MG3.4, 7MR2.4, 1A2.0, 1A4.0, 1A10.0

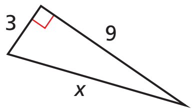
go.hrw.com
Homework Help Online
KEYWORD: MG7 5-7
Parent Resources Online
KEYWORD: MG7 Parent

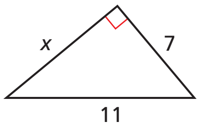
GUIDED PRACTICE

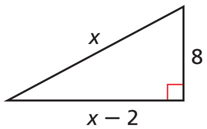
1. **Vocabulary** Do the numbers 2.7, 3.6, and 4.5 form a *Pythagorean triple*? Explain why or why not.

SEE EXAMPLE 1
p. 349

Find the value of x. Give your answer in simplest radical form.

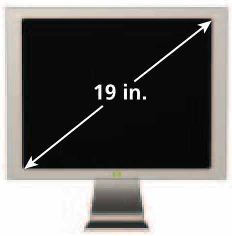
2. 

3. 

4. 

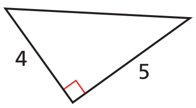
SEE EXAMPLE 2
p. 349

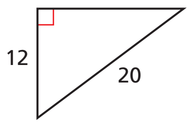
5. **Computers** The size of a computer monitor is usually given by the length of its diagonal. A monitor's aspect ratio is the ratio of its width to its height. This monitor has a diagonal length of 19 inches and an aspect ratio of 5 : 4. What are the width and height of the monitor? Round to the nearest tenth of an inch.

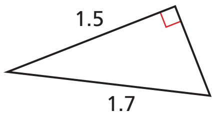


SEE EXAMPLE 3
p. 350

Find the missing side length. Tell if the side lengths form a Pythagorean triple. Explain.

6. 

7. 

8. 

SEE EXAMPLE 4
p. 351

Multi-Step **Tell if the measures can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.**

9. 7, 10, 12
10. 9, 11, 15
11. 9, 40, 41
12. $1\frac{1}{2}, 1\frac{3}{4}, 3\frac{1}{4}$
13. 5.9, 6, 8.4
14. $11, 13, 7\sqrt{6}$

PRACTICE AND PROBLEM SOLVING

Independent Practice

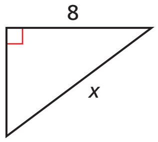
For Exercises	See Example
15–17	1
18	2
19–21	3
22–27	4

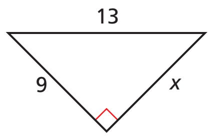
Extra Practice

Skills Practice p. S13

Application Practice p. S32

Find the value of x. Give your answer in simplest radical form.

15.


16.


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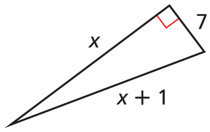
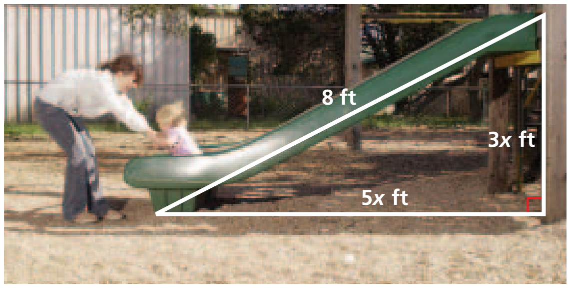
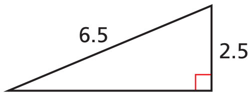
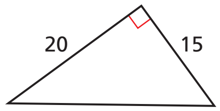


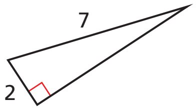
18. **Safety** The safety rules for a playground state that the height of the slide and the distance from the base of the ladder to the front of the slide must be in a ratio of 3:5. If a slide is about 8 feet long, what are the height of the slide and the distance from the base of the ladder to the front of the slide? Round to the nearest inch.



Find the missing side length. Tell if the side lengths form a Pythagorean triple. Explain.

19.


20.


21.


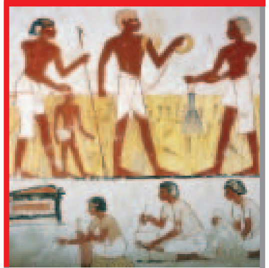
Multi-Step **Tell if the measures can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.**

22. 10, 12, 15 **23.** 8, 13, 23 **24.** 9, 14, 17

25. $1\frac{1}{2}, 2, 2\frac{1}{2}$ **26.** 0.7, 1.1, 1.7 **27.** $7, 12, 6\sqrt{5}$

LINK

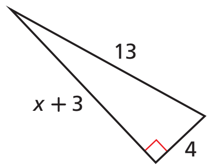
Surveying



Ancient Egyptian surveyors were referred to as *rope-stretchers*. The standard surveying rope was 100 royal cubits. A cubit is 52.4 cm long.

28. **Surveying** It is believed that surveyors in ancient Egypt laid out right angles using a rope divided into twelve sections by eleven equally spaced knots. How could the surveyors use this rope to make a right angle?

29. **ERROR ANALYSIS** Below are two solutions for finding x. Which is incorrect? Explain the error.



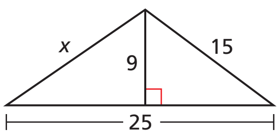
A

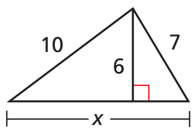
$a^2 + 4^2 = 13^2$
$a^2 = 169 - 16 = 153$
$a \approx 12.4$
$x + 3 \approx 12.4$
$x \approx 9.4$

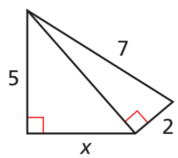
B

$(x + 3)^2 + 4^2 = 13^2$
$x^2 + 9 + 16 = 169$
$x^2 = 144$
$x = 12$

Find the value of x. Give your answer in simplest radical form.

30.


31.


32.


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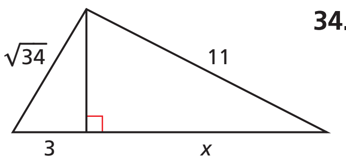

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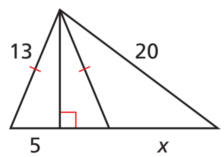
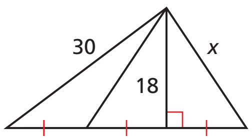
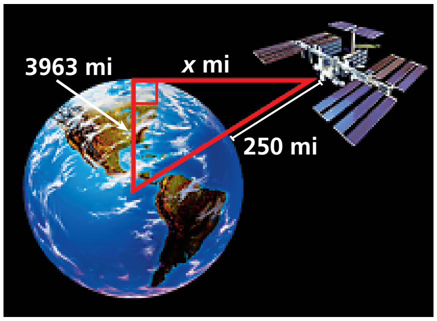


35.


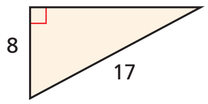
36. **Space Exploration** The International Space Station orbits at an altitude of about 250 miles above Earth's surface. The radius of Earth is approximately 3963 miles. How far can an astronaut in the space station see to the horizon? Round to the nearest mile.



Not drawn to scale

37. **Critical Thinking** In the proof of the Pythagorean Theorem on page 348, how do you know the outer figure is a square? How do you know the inner figure is a square?

Multi-Step **Find the perimeter and the area of each figure. Give your answer in simplest radical form.**

38.


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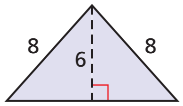

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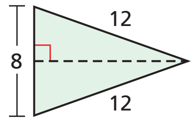
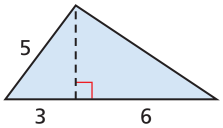
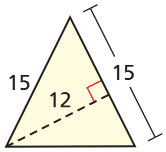
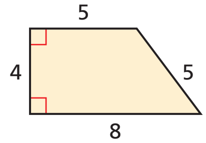


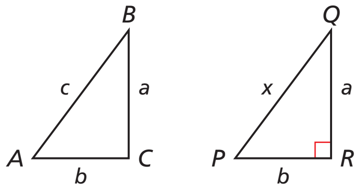
41.


42.


43.


44. **Write About It** When you apply both the Pythagorean Theorem and its converse, you use the equation $a^2 + b^2 = c^2$. Explain in your own words how the two theorems are different.

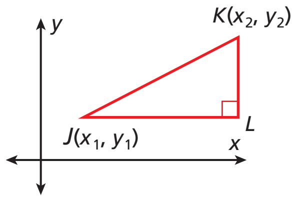
45. Use this plan to write a paragraph proof of the Converse of the Pythagorean Theorem.



Given: $\triangle ABC$ with $a^2 + b^2 = c^2$
Prove: $\triangle ABC$ is a right triangle.

Plan: Draw $\triangle PQR$ with $\angle R$ as the right angle, leg lengths of a and b, and a hypotenuse of length x. By the Pythagorean Theorem, $a^2 + b^2 = x^2$. Use substitution to compare x and c. Show that $\triangle ABC \cong \triangle PQR$ and thus $\angle C$ is a right angle.

46. Complete these steps to prove the Distance Formula.



Given: $J(x_1, y_1)$ and $K(x_2, y_2)$ with $x_1 \neq x_2$ and $y_1 \neq y_2$
Prove: $JK = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

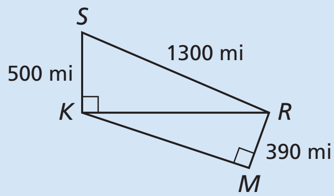
a. Locate L so that $\overline{JK}$ is the hypotenuse of right $\triangle JKL$. What are the coordinates of L?

b. Find JL and LK.

c. By the Pythagorean Theorem, $JK^2 = JL^2 + LK^2$. Find JK.

47. This problem will prepare you for the Concept Connection on page 364.

The figure shows an airline's routes between four cities.

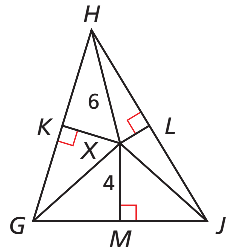


a. A traveler wants to go from Sanak (S) to Manitou (M). To minimize the total number of miles traveled, should she first fly to King City (K) or to Rice Lake (R)?

b. The airline decides to offer a direct flight from Sanak (S) to Manitou (M). Given that the length of this flight is more than 1360 mi, what can you say about $m\angle SRM$?

STANDARDIZED TEST PREP

48. **Gridded Response** $\overline{KX}$, $\overline{LX}$, and $\overline{MX}$ are the perpendicular bisectors of $\triangle GHJ$. Find GJ to the nearest tenth of a unit.



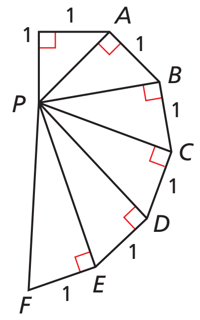
49. Which number forms a Pythagorean triple with 24 and 25?

(A) 1 (B) 7 (C) 26 (D) 49

50. The lengths of two sides of an obtuse triangle are 7 meters and 9 meters. Which could NOT be the length of the third side?

(F) 4 meters (G) 5 meters (H) 11 meters (J) 12 meters

51. **Extended Response** The figure shows the first six triangles in a pattern of triangles.



a. Find PA, PB, PC, PD, PE, and PF in simplest radical form.

b. If the pattern continues, what would be the length of the hypotenuse of the ninth triangle? Explain your answer.

c. Write a rule for finding the length of the hypotenuse of the nth triangle in the pattern. Explain your answer.

CHALLENGE AND EXTEND

52. **Algebra** Find all values of k so that $(-1, 2)$, $(-10, 5)$, and $(-4, k)$ are the vertices of a right triangle.

53. **Critical Thinking** Use a diagram of a right triangle to explain why $a + b > \sqrt{a^2 + b^2}$ for any positive numbers a and b.

54. In a right triangle, the leg lengths are a and b, and the length of the altitude to the hypotenuse is h. Write an expression for h in terms of a and b. (*Hint*: Think of the area of the triangle.)

55. **Critical Thinking** Suppose the numbers a, b, and c form a Pythagorean triple. Is each of the following also a Pythagorean triple? Explain.

a. $a + 1, b + 1, c + 1$

b. $2a, 2b, 2c$

c. a^2, b^2, c^2

d. $\sqrt{a}, \sqrt{b}, \sqrt{c}$

SPIRAL REVIEW

Solve each equation. *(Previous course)*

56. $(4 + x)12 - (4x + 1)6 = 0$

57. $\frac{2x - 5}{3} = x$

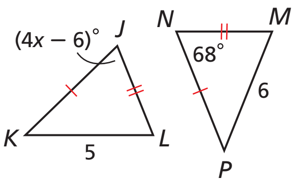
58. $4x + 3(x + 2) = -3(x + 3)$

Write a coordinate proof. *(Lesson 4-7)*

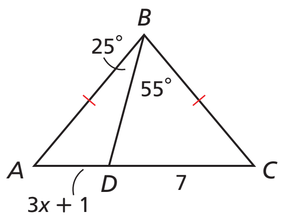
59. **Given:** $ABCD$ is a rectangle with $A(0, 0)$, $B(0, 2b)$, $C(2a, 2b)$, and $D(2a, 0)$. M is the midpoint of $\overline{AC}$.
Prove: $AM = MB$

Find the range of values for x. *(Lesson 5-6)*

60.



61.



5-8 Applying Special Right Triangles

Objectives

Justify and apply properties of 45°-45°-90° triangles.

Justify and apply properties of 30°-60°-90° triangles.

California Standards

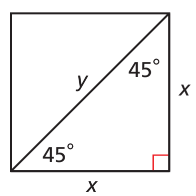
20.0 Students know and are able to use angle and side relationships in problems with special right triangles, such as 30°, 60°, and 90° triangles and 45°, 45°, and 90° triangles.

Who uses this?

You can use properties of special right triangles to calculate the correct size of a bandana for your dog. (See Example 2.)

A diagonal of a square divides it into two congruent isosceles right triangles. Since the base angles of an isosceles triangle are congruent, the measure of each acute angle is 45°. So another name for an isosceles right triangle is a 45°-45°-90° triangle.

A 45°-45°-90° triangle is one type of *special right triangle*. You can use the Pythagorean Theorem to find a relationship among the side lengths of a 45°-45°-90° triangle.



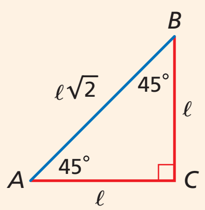
$a^2 + b^2 = c^2$	*Pythagorean Theorem*
$x^2 + x^2 = y^2$	*Substitute the given values.*
$2x^2 = y^2$	*Simplify.*
$\sqrt{2x^2} = \sqrt{y^2}$	*Find the square root of both sides.*
$x\sqrt{2} = y$	*Simplify.*

Know it! Note

Theorem 5-8-1 **45°-45°-90° Triangle Theorem**

In a 45°-45°-90° triangle, both legs are congruent, and the length of the hypotenuse is the length of a leg times $\sqrt{2}$.

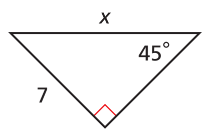
$AC = BC = \ell$ $\quad AB = \ell\sqrt{2}$



EXAMPLE 1 **Finding Side Lengths in a 45°-45°-90° Triangle**

Find the value of x. Give your answer in simplest radical form.

A

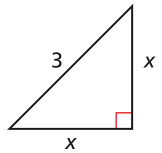


By the Triangle Sum Theorem, the measure of the third angle of the triangle is 45°. So it is a 45°-45°-90° triangle with a leg length of 7.

$x = 7\sqrt{2}$ *Hypotenuse = leg$\sqrt{2}$*

Find the value of x. Give your answer in simplest radical form.

B

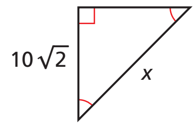


The triangle is an isosceles right triangle, which is a 45°-45°-90° triangle. The length of the hypotenuse is 3.

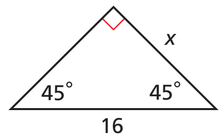
$3 = x\sqrt{2}$	*Hypotenuse = leg$\sqrt{2}$*
$\frac{3}{\sqrt{2}} = x$	*Divide both sides by $\sqrt{2}$.*
$\frac{3\sqrt{2}}{2} = x$	*Rationalize the denominator.*

CHECK IT OUT! **Find the value of x. Give your answer in simplest radical form.**

1a.

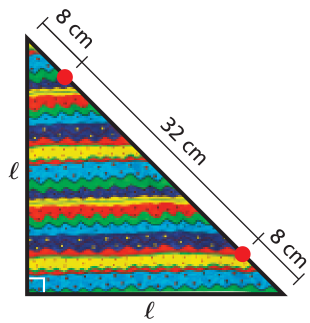


1b.



EXAMPLE 2 *Craft Application*

Tessa wants to make a bandana for her dog by folding a square of cloth into a 45°-45°-90° triangle. Her dog's neck has a circumference of about 32 cm. The folded bandana needs to be an extra 16 cm long so Tessa can tie it around her dog's neck. What should the side length of the square be? Round to the nearest centimeter.

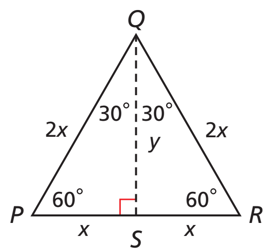


Tessa needs a 45°-45°-90° triangle with a hypotenuse of 48 cm.

$48 = \ell\sqrt{2}$	*Hypotenuse = leg$\sqrt{2}$*
$\ell = \frac{48}{\sqrt{2}} \approx 34$ cm	*Divide by $\sqrt{2}$ and round.*

CHECK IT OUT! **2. What if...?** Tessa's other dog is wearing a square bandana with a side length of 42 cm. What would you expect the circumference of the other dog's neck to be? Round to the nearest centimeter.

A 30°-60°-90° triangle is another special right triangle. You can use an equilateral triangle to find a relationship between its side lengths.



Draw an altitude in $\triangle PQR$. Since $\triangle PQS \cong \triangle RQS$, $\overline{PS} \cong \overline{RS}$. Label the side lengths in terms of x, and use the Pythagorean Theorem to find y.

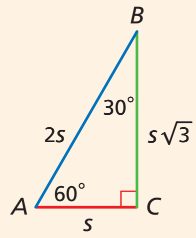
$a^2 + b^2 = c^2$	*Pythagorean Theorem*
$x^2 + y^2 = (2x)^2$	*Substitute x for a, y for b, and 2x for c.*
$y^2 = 3x^2$	*Multiply and combine like terms.*
$\sqrt{y^2} = \sqrt{3x^2}$	*Find the square root of both sides.*
$y = x\sqrt{3}$	*Simplify.*

Know it! Note

Theorem 5-8-2 **30°-60°-90° Triangle Theorem**

In a 30°-60°-90° triangle, the length of the hypotenuse is is 2 times the length of the shorter leg, and the length of the longer leg is the length of the shorter leg times $\sqrt{3}$.

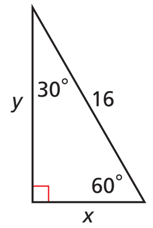
$AC = s$ $\quad\quad$ $AB = 2s$ $\quad\quad$ $BC = s\sqrt{3}$



EXAMPLE 3 Finding Side Lengths in a 30°-60°-90° Triangle

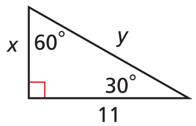
Find the values of x and y. Give your answers in simplest radical form.

A



$16 = 2x$	*Hypotenuse = 2(shorter leg)*
$8 = x$	*Divide both sides by 2.*
$y = x\sqrt{3}$	*Longer leg = (shorter leg)$\sqrt{3}$*
$y = 8\sqrt{3}$	*Substitute 8 for x.*

B



$11 = x\sqrt{3}$	*Longer leg = (shorter leg)$\sqrt{3}$*
$\frac{11}{\sqrt{3}} = x$	*Divide both sides by $\sqrt{3}$.*
$\frac{11\sqrt{3}}{3} = x$	*Rationalize the denominator.*
$y = 2x$	*Hypotenuse = 2(shorter leg)*
$y = 2\left(\frac{11\sqrt{3}}{3}\right)$	*Substitute $\frac{11\sqrt{3}}{3}$ for x.*
$y = \frac{22\sqrt{3}}{3}$	*Simplify.*

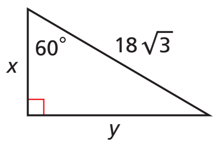
Remember!

If two angles of a triangle are not congruent, the shorter side lies opposite the smaller angle.

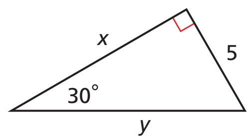
CHECK IT OUT!

Find the values of x and y. Give your answers in simplest radical form.

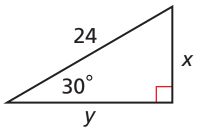
3a.



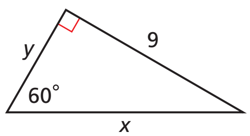
3b.



3c.

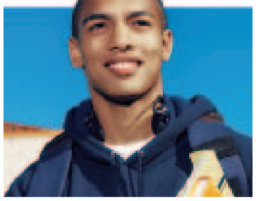


3d.



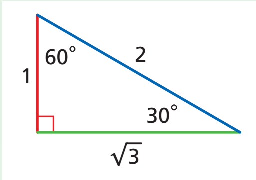
Student to Student

30°-60°-90° Triangles



Marcus Maiello
Johnson High School

To remember the side relationships in a 30°-60°-90° triangle, I draw a simple "1-2-$\sqrt{3}$" triangle like this.



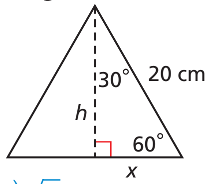
$2 = 2(1)$, so
***hypotenuse** $= 2($**shorter leg**$)$.*

$\sqrt{3} = \sqrt{3}(1)$, so
***longer leg** $= \sqrt{3}($**shorter leg**$)$.*

EXAMPLE 4 Using the 30°-60°-90° Triangle Theorem

The frame of the clock shown is an equilateral triangle. The length of one side of the frame is 20 cm. Will the clock fit on a shelf that is 18 cm below the shelf above it?

Step 1 Divide the equilateral triangle into two 30°-60°-90° triangles.
The height of the frame is the length of the longer leg.



Step 2 Find the length x of the shorter leg.

$20 = 2x$	*Hypotenuse = 2(shorter leg)*
$10 = x$	*Divide both sides by 2.*

Step 3 Find the length h of the longer leg.

$h = 10\sqrt{3} \approx 17.3$ cm   *Longer leg = (shorter leg)$\sqrt{3}$*

The frame is approximately 17.3 centimeters tall.
So the clock will fit on the shelf.

CHECK IT OUT!

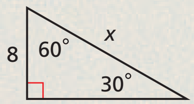
4. What if...? A manufacturer wants to make a larger clock with a height of 30 centimeters. What is the length of each side of the frame? Round to the nearest tenth.

THINK AND DISCUSS

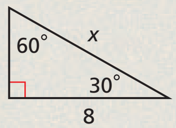
1. Explain why an isosceles right triangle is a 45°-45°-90° triangle.

2. Describe how finding x in triangle I is different from finding x in triangle II.

I.

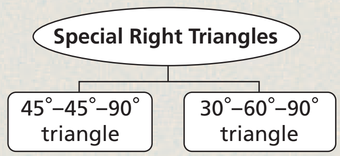


II.



Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, sketch the special right triangle and label its side lengths in terms of s.



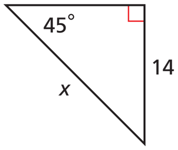
5-8 Exercises

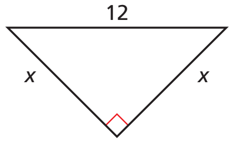
California Standards
2.0, 8.0, 12.0, 15.0, 20.0, 7AF2.0, 7MR2.0, 1A2.0

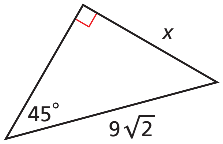
GUIDED PRACTICE

SEE EXAMPLE 1
p. 356

Find the value of x. Give your answer in simplest radical form.

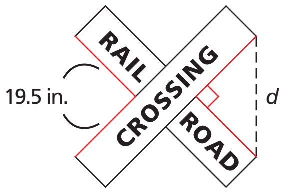
1. 

2. 

3. 

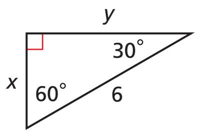
SEE EXAMPLE 2
p. 357

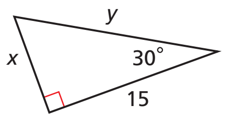
4. Transportation The two arms of the railroad sign are perpendicular bisectors of each other. In Pennsylvania, the lengths marked in red must be 19.5 inches. What is the distance labeled d? Round to the nearest tenth of an inch.

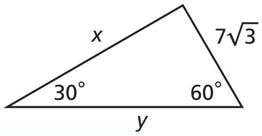


SEE EXAMPLE 3
p. 358

Find the values of x and y. Give your answers in simplest radical form.

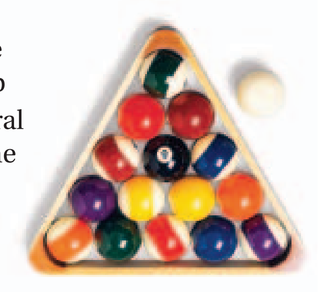
5. 

6. 

7. 

SEE EXAMPLE 4
p. 359

8. Entertainment Regulation billiard balls are $2\frac{1}{4}$ inches in diameter. The rack used to group 15 billiard balls is in the shape of an equilateral triangle. What is the approximate height of the triangle formed by the rack? Round to the nearest quarter of an inch.



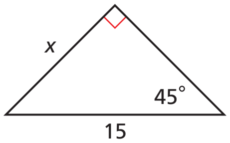
PRACTICE AND PROBLEM SOLVING

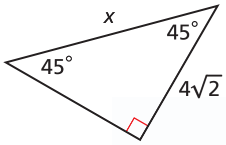
Independent Practice

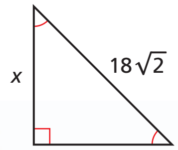
For Exercises	See Example
9–11	1
12	2
13–15	3
16	4

Extra Practice
Skills Practice p. S13
Application Practice p. S32

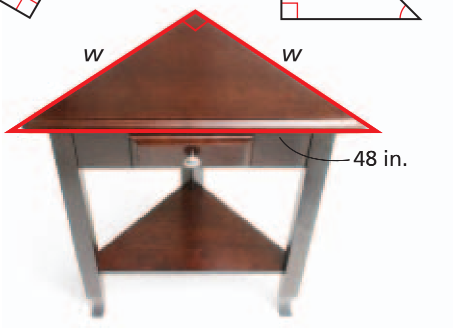
Find the value of x. Give your answer in simplest radical form.

9. 

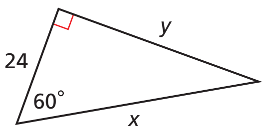
10. 

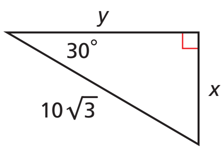
11. 

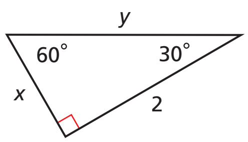
12. Design This tabletop is an isosceles right triangle. The length of the front edge of the table is 48 inches. What is the length w of each side edge? Round to the nearest tenth of an inch.



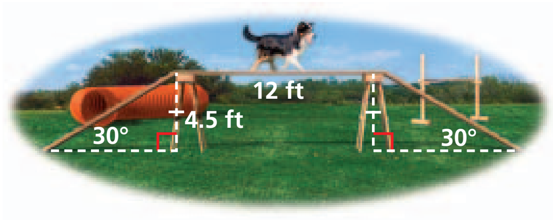
Find the value of x and y. Give your answers in simplest radical form.

13. 

14. 

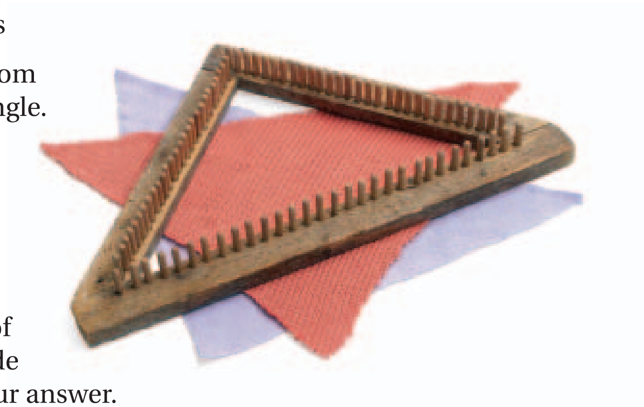
15. 

16. **Pets** A dog walk is used in dog agility competitions. In this dog walk, each ramp makes an angle of 30° with the ground.
   a. How long is one ramp?
   b. How long is the entire dog walk, including both ramps?



Multi-Step **Find the perimeter and area of each figure. Give your answers in simplest radical form.**

17. a 45°-45°-90° triangle with hypotenuse length 12 inches
18. a 30°-60°-90° triangle with hypotenuse length 28 centimeters
19. a square with diagonal length 18 meters
20. an equilateral triangle with side length 4 feet
21. an equilateral triangle with height 30 yards
22. **Estimation** The triangle loom is made from wood strips shaped into a 45°-45°-90° triangle. Pegs are placed every $\frac{1}{2}$ inch along the hypotenuse and every $\frac{1}{4}$ inch along each leg. Suppose you make a loom with an 18-inch hypotenuse. Approximately how many pegs will you need?



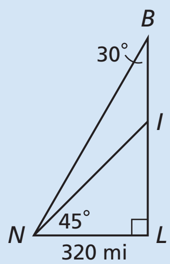
23. **Critical Thinking** The angle measures of a triangle are in the ratio 1 : 2 : 3. Are the side lengths also in the ratio 1 : 2 : 3? Explain your answer.

Find the coordinates of point *P* under the given conditions. Give your answers in simplest radical form.

24. $\triangle PQR$ is a 45°-45°-90° triangle with vertices $Q(4, 6)$ and $R(-6, -4)$, and $m\angle P = 90°$. P is in Quadrant II.
25. $\triangle PST$ is a 45°-45°-90° triangle with vertices $S(4, -3)$ and $T(-2, 3)$, and $m\angle S = 90°$. P is in Quadrant I.
26. $\triangle PWX$ is a 30°-60°-90° triangle with vertices $W(-1, -4)$ and $X(4, -4)$, and $m\angle W = 90°$. P is in Quadrant II.
27. $\triangle PYZ$ is a 30°-60°-90° triangle with vertices $Y(-7, 10)$ and $Z(5, 10)$, and $m\angle Z = 90°$. P is in Quadrant IV.
28. **Write About It** Why do you think 30°-60°-90° triangles and 45°-45°-90° triangles are called *special right triangles*?

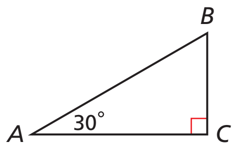
CONCEPT CONNECTION

29. This problem will prepare you for the Concept Connection on page 364.
   The figure shows an airline's routes among four cities. The airline offers one frequent-flier mile for each mile flown (rounded to the nearest mile). How many frequent-flier miles do you earn for each flight?
   a. Nelson (N) to Belton (B)
   b. Idria (I) to Nelson (N)
   c. Belton (B) to Idria (I)



STANDARDIZED TEST PREP

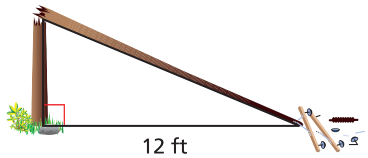
30. Which is a true statement?



Ⓐ $AB = BC\sqrt{2}$　　Ⓒ $AC = BC\sqrt{3}$

Ⓑ $AB = BC\sqrt{3}$　　Ⓓ $AC = AB\sqrt{2}$

31. An 18-foot pole is broken during a storm. The top of the pole touches the ground 12 feet from the base of the pole. How tall is the part of the pole left standing?



Ⓕ 5 feet　　Ⓗ 13 feet

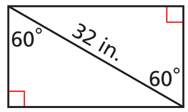
Ⓖ 6 feet　　Ⓙ 22 feet

32. The length of the hypotenuse of an isosceles right triangle is 24 inches. What is the length of one leg of the triangle, rounded to the nearest tenth of an inch?

Ⓐ 13.9 inches　　Ⓒ 33.9 inches

Ⓑ 17.0 inches　　Ⓓ 41.6 inches

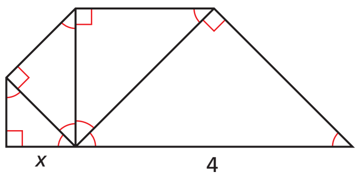
33. **Gridded Response** Find the area of the rectangle to the nearest tenth of a square inch.



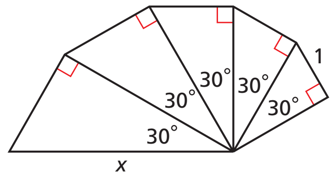
CHALLENGE AND EXTEND

Multi-Step **Find the value of x in each figure.**

34.



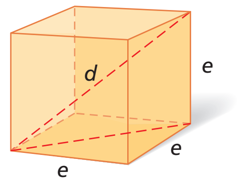
35.



36. Each edge of the cube has length e.

a. Find the diagonal length d when $e = 1$, $e = 2$, and $e = 3$. Give the answers in simplest radical form.

b. Write a formula for d for any positive value of e.



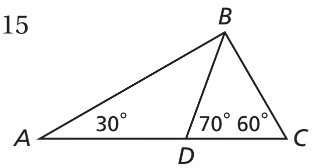
37. Write a paragraph proof to show that the altitude to the hypotenuse of a 30°-60°-90° triangle divides the hypotenuse into two segments, one of which is 3 times as long as the other.

SPIRAL REVIEW

Rewrite each function in the form $y = a(x - h)^2 - k$ and find the axis of symmetry. *(Previous course)*

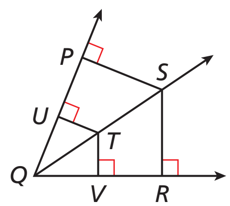
38. $y = x^2 + 4x$　　**39.** $y = x^2 - 10x - 2$　　**40.** $y = x^2 + 7x + 15$

Classify each triangle by its angle measures. *(Lesson 4-1)*



41. $\triangle ADB$　　**42.** $\triangle BDC$　　**43.** $\triangle ABC$

Use the diagram for Exercises 44–46. *(Lesson 5-1)*



44. Given that $PS = SR$ and $m\angle PSQ = 65°$, find $m\angle PQR$.

45. Given that $UT = TV$ and $m\angle PQS = 42°$, find $m\angle VTS$.

46. Given that $\angle PQS \cong \angle SQR$, $SR = 3TU$, and $PS = 7.5$, find TV.

5-8 Geometry LAB

Use with Lesson 5-8

Graph Irrational Numbers

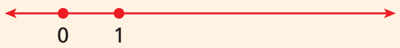
Numbers such as $\sqrt{2}$ and $\sqrt{3}$ are irrational. That is, they cannot be written as the ratio of two integers. In decimal form, they are infinite nonrepeating decimals. You can round the decimal form to estimate the location of these numbers on a number line, or you can use right triangles to construct their locations exactly.

California Standards

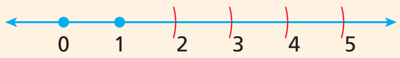
16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line.

Activity

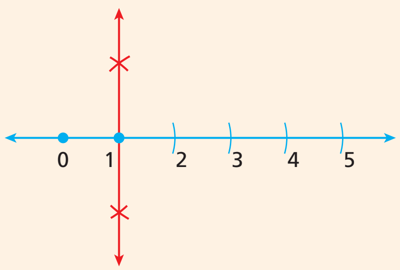
1. Draw a line. Mark two points near the left side of the line and label them 0 and 1. The distance from 0 to 1 is 1 unit.



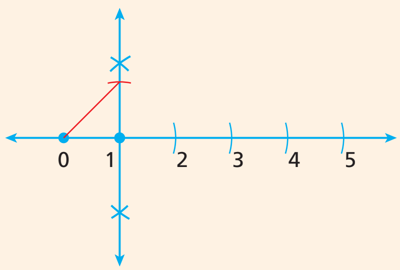
2. Set your compass to 1 unit and mark increments at 2, 3, 4, and 5 units to construct a number line.



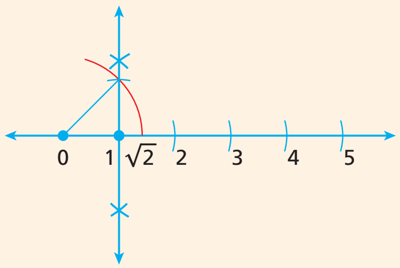
3. Construct a perpendicular to the line through 1.



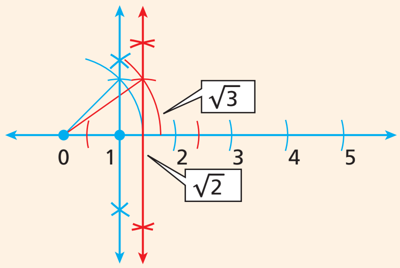
4. Using your compass, mark 1 unit up from the number line and then draw a right triangle. The legs both have length 1, so by the Pythagorean Theorem, the hypotenuse has a length of $\sqrt{2}$.



5. Set your compass to the length of the hypotenuse. Draw an arc centered at 0 that intersects the number line at $\sqrt{2}$.



6. Repeat Steps 3 through 5, starting at $\sqrt{2}$, to construct a segment of length $\sqrt{3}$.



Try This

1. Sketch the two right triangles from Step 6. Label the side lengths and use the Pythagorean Theorem to show why the construction is correct.
2. Construct $\sqrt{4}$ and verify that it is equal to 2.
3. Construct $\sqrt{5}$ through $\sqrt{9}$ and verify that $\sqrt{9}$ is equal to 3.
4. Set your compass to the length of the segment from 0 to $\sqrt{2}$. Mark off another segment of length $\sqrt{2}$ to show that $\sqrt{8}$ is equal to $2\sqrt{2}$.

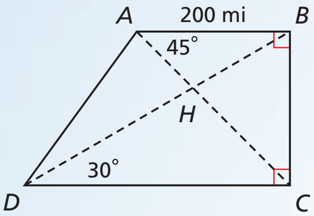
CHAPTER 5

CONCEPT CONNECTION

SECTION 5B

Relationships in Triangles

Fly Away! A commuter airline serves the four cities of Ashton, Brady, Colfax, and Dumas, located at points A, B, C, and D, respectively. The solid lines in the figure show the airline's existing routes. The airline is building an airport at H, which will serve as a hub. This will add four new routes to their schedule: $\overline{AH}$, $\overline{BH}$, $\overline{CH}$, and $\overline{DH}$.



1. The airline wants to locate the airport so that the combined distance to the cities $(AH + BH + CH + DH)$ is as small as possible. Give an indirect argument to explain why the airline should locate the airport at the intersection of the diagonals $\overline{AC}$ and $\overline{BD}$. (*Hint*: Assume that a different point X inside quadrilateral $ABCD$ results in a smaller combined distance. Then consider how $AX + CX$ compares to $AH + CH$.)
2. Currently, travelers who want to go from Ashton to Colfax must first fly to Brady. Once the airport is built, they will fly from Ashton to the new airport and then to Colfax. How many miles will this save compared to the distance of the current trip?
3. Currently, travelers who want to go from Brady to Dumas must first fly to Colfax. Once the airport is built, they will fly from Brady to the new airport and then to Dumas. How many miles will this save?
4. Once the airport is built, the airline plans to serve a meal only on its longest flight. On which route should they serve the meal? How do you know that this route is the longest?

READY TO GO ON?

CHAPTER 5

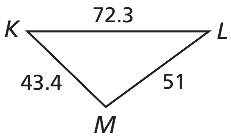
SECTION 5B

Quiz for Lessons 5-5 Through 5-8

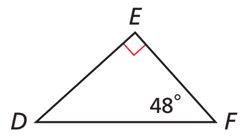
5-5 Indirect Proof and Inequalities in One Triangle

1. Write an indirect proof that the supplement of an acute angle cannot be an acute angle.

2. Write the angles of $\triangle KLM$ in order from smallest to largest.



3. Write the sides of $\triangle DEF$ in order from shortest to longest.



Tell whether a triangle can have sides with the given lengths. Explain.

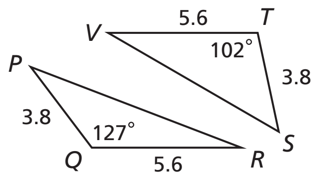
4. 8.3, 10.5, 18.8

5. $4s$, $s + 10$, s^2, when $s = 4$

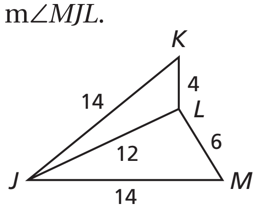
6. The distance from Kara's school to the theater is 9 km. The distance from her school to the zoo is 16 km. If the three locations form a triangle, what is the range of distances from the theater to the zoo?

5-6 Inequalities in Two Triangles

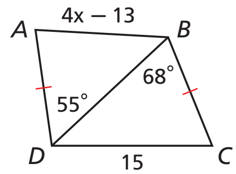
7. Compare PR and SV.



8. Compare $m\angle KJL$ and $m\angle MJL$.

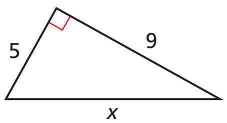


9. Find the range of values for x.

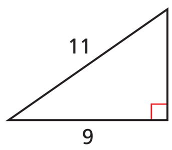


5-7 The Pythagorean Theorem

10. Find the value of x. Give the answer in simplest radical form.

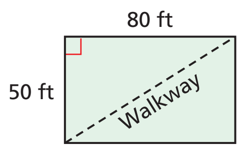


11. Find the missing side length. Tell if the side lengths form a Pythagorean triple. Explain.



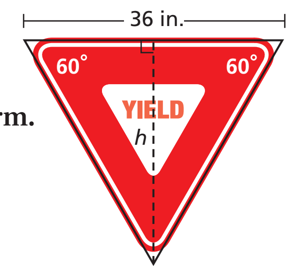
12. Tell if the measures 10, 12, and 16 can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.

13. A landscaper wants to place a stone walkway from one corner of the rectangular lawn to the opposite corner. What will be the length of the walkway? Round to the nearest inch.



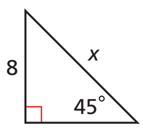
5-8 Applying Special Right Triangles

14. A yield sign is an equilateral triangle with a side length of 36 inches. What is the height h of the sign? Round to the nearest inch.

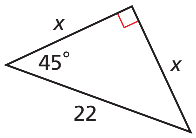


Find the values of the variables. Give your answers in simplest radical form.

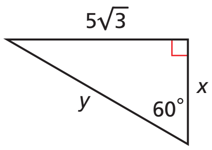
15.



16.



17.



CHAPTER 5

Study Guide: Review

Know it! Note For a complete list of the postulates and theorems in this chapter, see p. S82.

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. A point that is the same distance from two or more objects is __?__ from the objects.

2. A __?__ is a segment that joins the midpoints of two sides of the triangle.

3. The point of concurrency of the angle bisectors of a triangle is the __?__.

4. A __?__ is a set of points that satisfies a given condition.

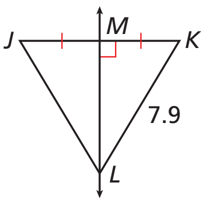
5-1 Perpendicular and Angle Bisectors *(pp. 300–306)*

2.0

EXAMPLES

Find each measure.

- JL



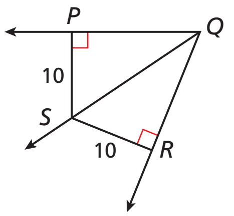
Because $\overline{JM} \cong \overline{MK}$ and $\overline{ML} \perp \overline{JK}$, $\overline{ML}$ is the perpendicular bisector of $\overline{JK}$.

$JL = KL$ *⊥ Bisector Thm.*

$JL = 7.9$ *Substitute 7.9 for KL.*

- $m\angle PQS$, given that $m\angle PQR = 68°$

Since $SP = SR$, $\overline{SP} \perp \overrightarrow{QP}$, and $\overline{SR} \perp \overrightarrow{QR}$, $\overrightarrow{QS}$ bisects $\angle PQR$ by the Converse of the Angle Bisector Theorem.



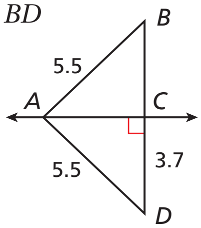
$m\angle PQS = \frac{1}{2} m\angle PQR$ *Def. of ∠ bisector*

$m\angle PQS = \frac{1}{2}(68°) = 34°$ *Substitute 68° for m∠PQR.*

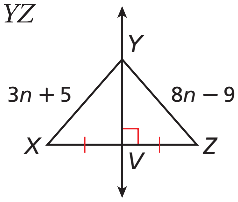
EXERCISES

Find each measure.

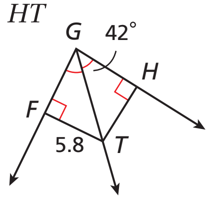
5. BD



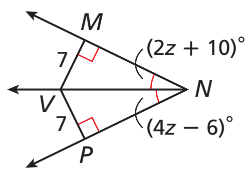
6. YZ



7. HT



8. $m\angle MNP$



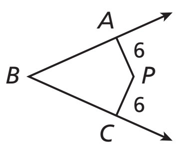
Write an equation in point-slope form for the perpendicular bisector of the segment with the given endpoints.

9. $A(-4, 5)$, $B(6, -5)$

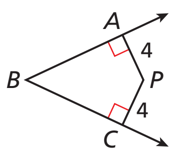
10. $X(3, 2)$, $Y(5, 10)$

Tell whether the given information allows you to conclude that P is on the bisector of $\angle ABC$.

11.



12.

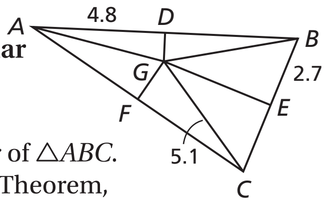


5-2 Bisectors of Triangles *(pp. 307–313)*

2.0, 16.0

EXAMPLES

- $\overline{DG}$, $\overline{EG}$, and $\overline{FG}$ are the perpendicular bisectors of $\triangle ABC$. Find AG.

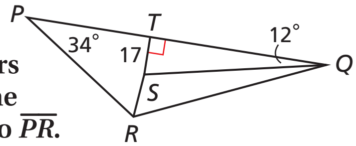


G is the circumcenter of $\triangle ABC$. By the Circumcenter Theorem, G is equidistant from the vertices of $\triangle ABC$.

$AG = CG$ *Circumcenter Thm.*

$AG = 5.1$ *Substitute 5.1 for CG.*

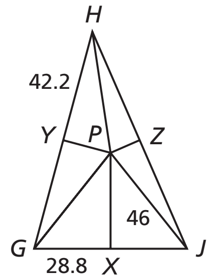
- $\overline{QS}$ and $\overline{RS}$ are angle bisectors of $\triangle PQR$. Find the distance from S to $\overline{PR}$.



S is the incenter of $\triangle PQR$. By the Incenter Theorem, S is equidistant from the sides of $\triangle PQR$. The distance from S to $\overline{PQ}$ is 17, so the distance from S to $\overline{PR}$ is also 17.

EXERCISES

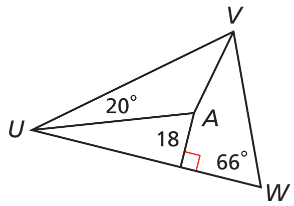
$\overline{PX}$, $\overline{PY}$, and $\overline{PZ}$ are the perpendicular bisectors of $\triangle GHJ$. Find each length.



13. GY **14.** GP

15. GJ **16.** PH

$\overline{UA}$ and $\overline{VA}$ are angle bisectors of $\triangle UVW$. Find each measure.



17. the distance from A to $\overline{UV}$

18. $m\angle WVA$

Find the circumcenter of a triangle with the given vertices.

19. $M(0, 6)$, $N(8, 0)$, $O(0, 0)$

20. $O(0, 0)$, $R(0, -7)$, $S(-12, 0)$

5-3 Medians and Altitudes of Triangles *(pp. 314–320)*

16.0

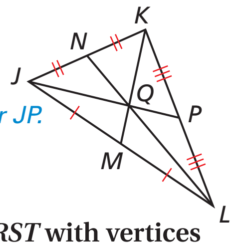
EXAMPLES

- **In $\triangle JKL$, $JP = 42$. Find JQ.**

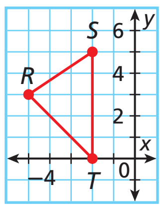
$JQ = \frac{2}{3}JP$ *Centroid Thm.*

$JQ = \frac{2}{3}(42)$ *Substitute 42 for JP.*

$JQ = 28$ *Multiply.*



- **Find the orthocenter of $\triangle RST$ with vertices $R(-5, 3)$, $S(-2, 5)$, and $T(-2, 0)$.**



Since $\overline{ST}$ is vertical, the equation of the line containing the altitude from R to $\overline{ST}$ is $y = 3$.

slope of $\overline{RT} = \dfrac{3 - 0}{-5 - (-2)} = -1$

The slope of the altitude to $\overline{RT}$ is 1. This line must pass through $S(-2, 5)$.

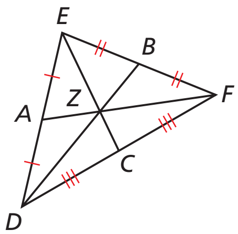
$y - y_1 = m(x - x_1)$ *Point-slope form*

$y - 5 = 1(x + 2)$ *Substitution*

Solve the system $\begin{cases} y = 3 \\ y = x + 7 \end{cases}$ to find that the coordinates of the orthocenter are $(-4, 3)$.

EXERCISES

In $\triangle DEF$, $DB = 24.6$, and $EZ = 11.6$. Find each length.



21. DZ **22.** ZB

23. ZC **24.** EC

Find the orthocenter of a triangle with the given vertices.

25. $J(-6, 7)$, $K(-6, 0)$, $L(-11, 0)$

26. $A(1, 2)$, $B(6, 2)$, $C(1, -8)$

27. $R(2, 3)$, $S(7, 8)$, $T(8, 3)$

28. $X(-3, 2)$, $Y(5, 2)$, $Z(3, -4)$

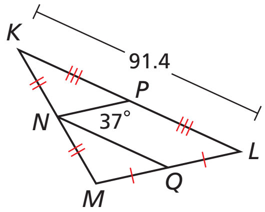
29. The coordinates of a triangular piece of a mobile are $(0, 4)$, $(3, 8)$, and $(6, 0)$. The piece will hang from a chain so that it is balanced. At what coordinates should the chain be attached?

5-4 The Triangle Midsegment Theorem *(pp. 322–327)*

17.0

EXAMPLES

Find each measure.



- *NQ*

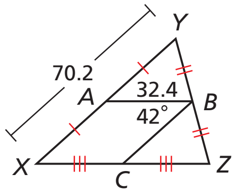
  By the △ Midsegment Thm., $NQ = \frac{1}{2}KL = 45.7$.

- m∠*NQM*

$\overline{NP} \parallel \overline{ML}$	*△ Midsegment Thm.*
m∠*NQM* = m∠*PNQ*	*Alt. Int. ∠ Thm.*
m∠*NQM* = **37°**	*Substitution*

EXERCISES

Find each measure.



30. *BC* **31.** *XZ*

32. *XC* **33.** m∠*BCZ*

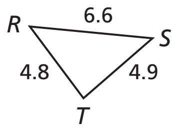
34. m∠*BAX* **35.** m∠*YXZ*

36. The vertices of △*GHJ* are *G*(−4, −7), *H*(2, 5), and *J*(10, −3). *V* is the midpoint of $\overline{GH}$, and *W* is the midpoint of $\overline{HJ}$. Show that $\overline{VW} \parallel \overline{GJ}$ and $VW = \frac{1}{2}GJ$.

5-5 Indirect Proof and Inequalities in One Triangle *(pp. 332–339)*

2.0, 6.0

EXAMPLES



- **Write the angles of △*RST* in order from smallest to largest.**

  The smallest angle is opposite the shortest side. In order, the angles are ∠*S*, ∠*R*, and ∠*T*.

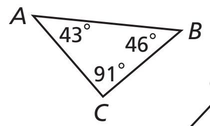
- **The lengths of two sides of a triangle are 15 inches and 12 inches. Find the range of possible lengths for the third side.**

  Let *s* be the length of the third side.

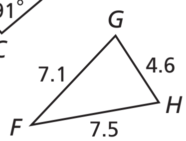
$s + 15 > 12$	$s + 12 > 15$	$15 + 12 > s$
$s > -3$	$s > 3$	$27 > s$

  By the Triangle Inequality Theorem, 3 in. < *s* < 27 in.

EXERCISES



37. Write the sides of △*ABC* in order from shortest to longest.



38. Write the angles of △*FGH* in order from smallest to largest.

39. The lengths of two sides of a triangle are 13.5 centimeters and 4.5 centimeters. Find the range of possible lengths for the third side.

Tell whether a triangle can have sides with the given lengths. Explain.

40. 6.2, 8.1, 14.2 **41.** *z*, *z*, 3*z*, when *z* = 5

42. Write an indirect proof that a triangle cannot have two obtuse angles.

5-6 Inequalities in Two Triangles *(pp. 340–345)*

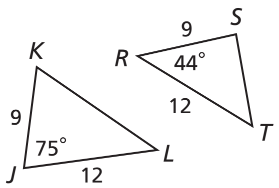
2.0

EXAMPLES

Compare the given measures.

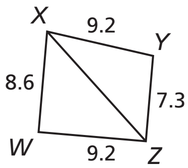
- *KL* and *ST*

  KJ = *RS*, *JL* = *RT*, and m∠*J* > m∠*R*. By the Hinge Theorem, *KL* > *ST*.



- m∠*ZXY* and m∠*XZW*

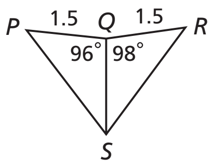
  XY = *WZ*, *XZ* = *XZ*, and *YZ* < *XW*. By the Converse of the Hinge Theorem, m∠*ZXY* < m∠*XZW*.



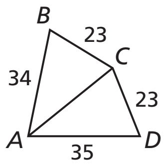
EXERCISES

Compare the given measures.

43. *PS* and *RS*

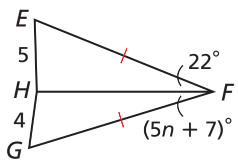


44. m∠*BCA* and m∠*DCA*

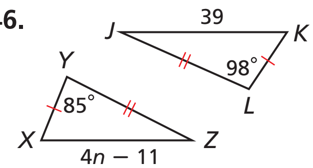


Find the range of values for *n*.

45.



46.

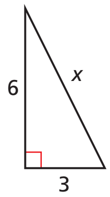


5-7 The Pythagorean Theorem *(pp. 348–355)*

6.0, 12.0, 14.0, 15.0

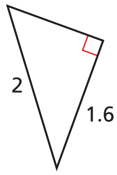
EXAMPLES

- **Find the value of x. Give your answer in simplest radical form.**



$a^2 + b^2 = c^2$	*Pyth. Thm.*
$6^2 + 3^2 = x^2$	*Substitution*
$45 = x^2$	*Simplify.*
$x = 3\sqrt{5}$	*Find the positive square root and simplify.*

- **Find the missing side length. Tell if the sides form a Pythagorean triple. Explain.**



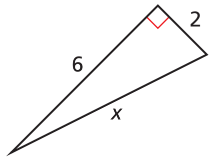
$a^2 + b^2 = c^2$	*Pyth. Thm.*
$a^2 + (1.6)^2 = 2^2$	*Substitution*
$a^2 = 1.44$	*Solve for a^2.*
$a = 1.2$	*Find the positive square root.*

The side lengths do not form a Pythagorean triple because 1.2 and 1.6 are not whole numbers.

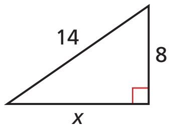
EXERCISES

Find the value of x. Give your answer in simplest radical form.

47.

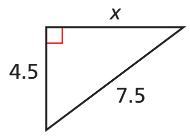


48.

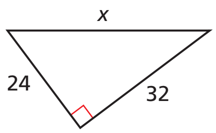


Find the missing side length. Tell if the sides form a Pythagorean triple. Explain.

49.



50.



Tell if the measures can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.

51. 9, 12, 16

52. 11, 14, 27

53. 1.5, 3.6, 3.9

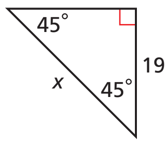
54. 2, 3.7, 4.1

5-8 Applying Special Right Triangles *(pp. 356–362)*

20.0

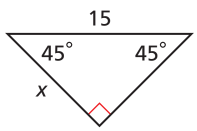
EXAMPLES

Find the values of the variables. Give your answers in simplest radical form.



This is a 45°-45°-90° triangle.

$x = 19\sqrt{2}$ *Hyp. = leg$\sqrt{2}$*

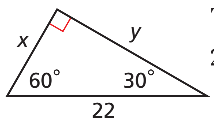


This is a 45°-45°-90° triangle.

$15 = x\sqrt{2}$ *Hyp. = leg$\sqrt{2}$*

$\frac{15}{\sqrt{2}} = x$ *Divide both sides by $\sqrt{2}$.*

$\frac{15\sqrt{2}}{2} = x$ *Rationalize the denominator.*



This is a 30°-60°-90° triangle.

$22 = 2x$ *Hyp. = 2(shorter leg)*

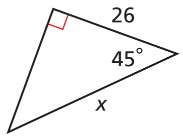
$11 = x$ *Divide both sides by 2.*

$y = 11\sqrt{3}$ *Longer leg = (shorter leg)$\sqrt{3}$*

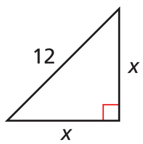
EXERCISES

Find the values of the variables. Give your answers in simplest radical form.

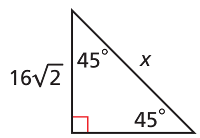
55.



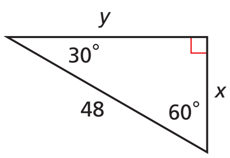
56.



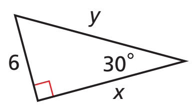
57.



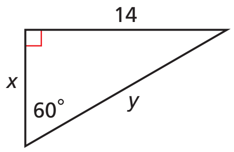
58.



59.

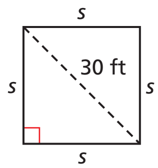


60.

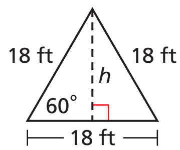


Find the value of each variable. Round to the nearest inch.

61.



62.

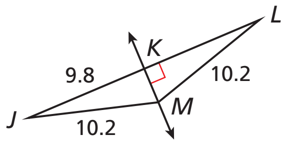


CHAPTER 5

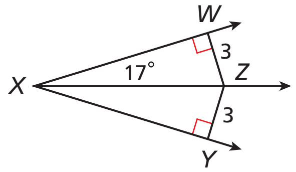
CHAPTER TEST

Find each measure.

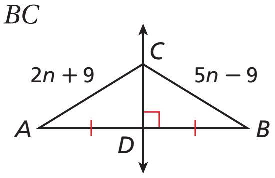
1. KL



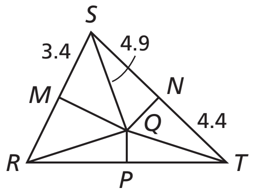
2. m$\angle WXY$



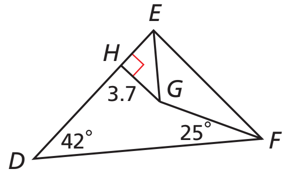
3. BC



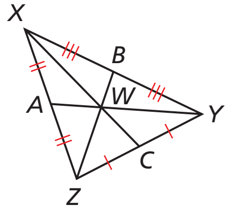
4. $\overline{MQ}$, $\overline{NQ}$, and $\overline{PQ}$ are the perpendicular bisectors of $\triangle RST$. Find RS and RQ.



5. $\overline{EG}$ and $\overline{FG}$ are angle bisectors of $\triangle DEF$. Find m$\angle GEF$ and the distance from G to $\overline{DF}$.

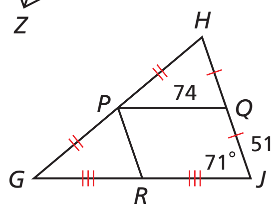


6. In $\triangle XYZ$, $XC = 261$, and $ZW = 118$. Find XW, BW, and BZ.



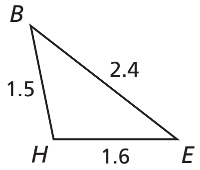
7. Find the orthocenter of $\triangle JKL$ with vertices $J(-5, 2)$, $K(-5, 10)$, and $L(1, 4)$.

8. In $\triangle GHJ$ at right, find PR, GJ, and m$\angle GRP$.

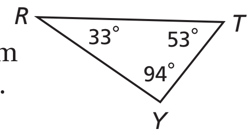


9. Write an indirect proof that two obtuse angles cannot form a linear pair.

10. Write the angles of $\triangle BEH$ in order from smallest to largest.

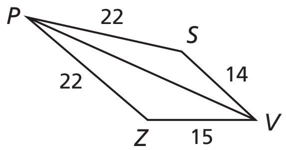


11. Write the sides of $\triangle RTY$ in order from shortest to longest.

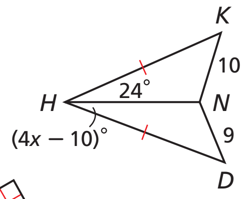


12. The distance from Arville to Branton is 114 miles. The distance from Branton to Camford is 247 miles. If the three towns form a triangle, what is the range of distances from Arville to Camford?

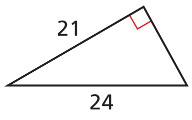
13. Compare m$\angle SPV$ and m$\angle ZPV$.



14. Find the range of values for x.



15. Find the missing side length in the triangle. Tell if the side lengths form a Pythagorean triple. Explain.

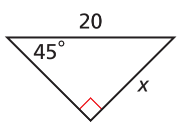


16. Tell if the measures 18, 20, and 27 can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.

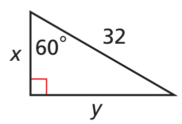
17. An IMAX screen is 62 feet tall and 82 feet wide. What is the length of the screen's diagonal? Round to the nearest inch.

Find the values of the variables. Give your answers in simplest radical form.

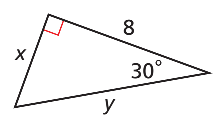
18.



19.



20.



College Entrance Exam Practice

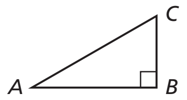
FOCUS ON SAT MATHEMATICS SUBJECT TESTS

Some questions on the SAT Mathematics Subject Tests require the use of a calculator. You can take the test without one, but it is not recommended. The calculator you use must meet certain criteria. For example, calculators that make noise or have typewriter-like keypads are not allowed.

HOT TIP! If you have both a scientific and a graphing calculator, bring the graphing calculator to the test. Make sure you spend time getting used to a new calculator before the day of the test.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. In $\triangle ABC$, $m\angle C = 2m\angle A$, and $CB = 3$ units. What is AB to the nearest hundredth unit?



(A) 1.73 units

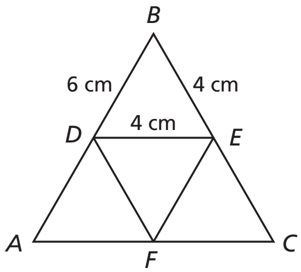
(B) 4.24 units

(C) 5.20 units

(D) 8.49 units

(E) 10.39 units

2. What is the perimeter of $\triangle ABC$ if D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$, and F is the midpoint of $\overline{AC}$?



Note: Figure not drawn to scale.

(A) 8 centimeters

(B) 14 centimeters

(C) 20 centimeters

(D) 28 centimeters

(E) 35 centimeters

3. The side lengths of a right triangle are 2, 5, and c, where $c > 5$. What is the value of c?

(A) $\sqrt{21}$

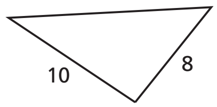
(B) $\sqrt{29}$

(C) 7

(D) 9

(E) $\sqrt{145}$

4. In the triangle below, which of the following CANNOT be the length of the unknown side?



(A) 2.2

(B) 6

(C) 12.8

(D) 17.2

(E) 18.1

5. Which of the following points is on the perpendicular bisector of the segment with endpoints $(3, 4)$ and $(9, 4)$?

(A) $(4, 2)$

(B) $(4, 5)$

(C) $(5, 4)$

(D) $(6, -1)$

(E) $(7, 4)$

CHAPTER 5

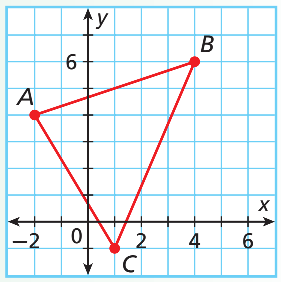
STRATEGIES FOR SUCCESS

Any Question Type: Check with a Different Method

It is important to check all of your answers on a test. An effective way to do this is to use a different method to answer the question a second time. If you get the same answer with two different methods, then your answer is probably correct.

EXAMPLE 1

Short Response **What are the coordinates of the centroid of $\triangle ABC$ with $A(-2, 4)$, $B(4, 6)$, and $C(1, -1)$? Show your work.**



Method 1: *The centroid of a triangle is the point of concurrency of the medians. Write the equations of two medians and find their point of intersection.*

Let D be the midpoint of $\overline{AB}$ and let E be the midpoint of $\overline{BC}$.

$$D = \left(\frac{-2+4}{2}, \frac{4+6}{2}\right) = (1, 5) \qquad E = \left(\frac{4+1}{2}, \frac{6+(-1)}{2}\right) = (2.5, 2.5)$$

The median from C to D contains $C(1, -1)$ and $D(1, 5)$. It is vertical, so its equation is $x = 1$.

The median from A to E contains $A(-2, 4)$ and $E(2.5, 2.5)$.

$$\text{slope of } \overline{AE} = \frac{4-2.5}{-2-2.5} = \frac{1.5}{-4.5} = -\frac{1}{3}$$

$y - y_1 = m(x - x_1)$ *Point-slope form*

$y - 4 = -\frac{1}{3}(x + 2)$ *Substitute 4 for y_1, $-\frac{1}{3}$ for m, and -2 for x_1.*

Solve the system $\begin{cases} x = 1 \\ y - 4 = -\frac{1}{3}(x + 2) \end{cases}$ *to find the point of intersection.*

$y - 4 = -\frac{1}{3}(1 + 2)$ *Substitute 1 for x.*

$y = 3$ *Simplify.*

The coordinates of the centroid are $(1, 3)$.

Method 2: *To check this answer, use a different method. By the Centroid Theorem, the centroid of a triangle is $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side. $\overline{CD}$ is vertical with a length of 6 units. $\frac{2}{3}(6) = 4$, and the coordinates of the point that is 4 units up from C is $(1, 3)$.*

This method confirms the first answer.

Problem Solving Strategies

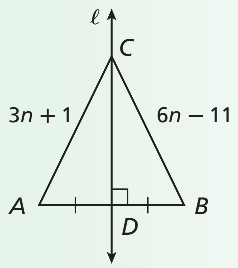
- Draw a Diagram
- Make a Model
- Guess and Test
- Work Backward
- Find a Pattern
- Make a Table
- Solve a Simpler Problem
- Use Logical Reasoning
- Use a Venn Diagram
- Make an Organized List

If you can't think of a different method to use to check your answer, circle the question and come back to it later.

Read each test item and answer the questions that follow.

Item A

Multiple Choice Given that ℓ is the perpendicular bisector of $\overline{AB}$, $AC = 3n + 1$, and $BC = 6n - 11$, what is the value of n?



Ⓐ -4 Ⓒ $\frac{4}{3}$

Ⓑ $\frac{3}{4}$ Ⓓ 4

1. How can you use the given answer choices to solve this problem?

2. Describe how to solve this problem differently.

Item B

Multiple Choice Which number forms a Pythagorean triple with 15 and 17?

Ⓕ 5 Ⓗ 8

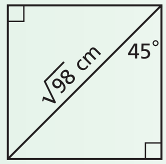
Ⓖ 7 Ⓙ 10

3. How can you use the given answer choices to find the answer?

4. Describe a different method you can use to check your answer.

Item C

Gridded Response Find the area of the square in square centimeters.



5. How can you use special right triangles to answer this question?

6. Explain how you can check your answer by using the Pythagorean Theorem.

Item D

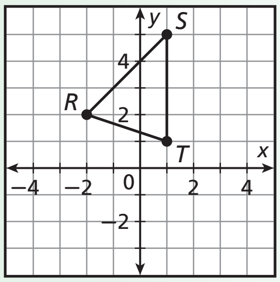
Short Response Do the ordered pairs $A(-8, 4)$, $B(0, -2)$, and $C(8, 4)$ form a right triangle? Explain your answer.

7. Explain how to use slope to determine if $\triangle ABC$ is a right triangle.

8. How can you use the Converse of the Pythagorean Theorem to check your answer?

Item E

Short Response Find the orthocenter of $\triangle RST$. Show your work.



9. Describe how you would solve this problem.

10. How can you use the third altitude of the triangle to confirm that your answer is correct?

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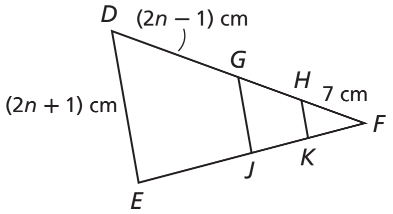
CUMULATIVE ASSESSMENT, CHAPTERS 1–5

Multiple Choice

1. $\overline{GJ}$ is a midsegment of $\triangle DEF$, and $\overline{HK}$ is a midsegment of $\triangle GFJ$. What is the length of $\overline{HK}$?



Ⓐ 2.25 centimeters
Ⓑ 4 centimeters
Ⓒ 7.5 centimeters
Ⓓ 9 centimeters

2. In $\triangle RST$, $SR < ST$, and $RT > ST$. If $m\angle R = (2x + 10)°$ and $m\angle T = (3x - 25)°$, which is a possible value of x?

Ⓕ 25
Ⓖ 30
Ⓗ 35
Ⓙ 40

3. The vertex angle of an isosceles triangle measures $(7a - 2)°$, and one of the base angles measures $(4a + 1)°$. Which term best describes this triangle?

Ⓐ Acute
Ⓑ Equiangular
Ⓒ Right
Ⓓ Obtuse

4. The lengths of two sides of an acute triangle are 8 inches and 10 inches. Which of the following could be the length of the third side?

Ⓕ 5 inches
Ⓖ 6 inches
Ⓗ 12 inches
Ⓙ 13 inches

5. For the coordinates $M(-1, 0)$, $N(-2, 2)$, $P(10, y)$, and $Q(4, 6)$, $\overline{MN} \parallel \overline{PQ}$. What is the value of y?

Ⓐ −18
Ⓑ −6
Ⓒ 6
Ⓓ 18

6. What is the area of an equilateral triangle that has a perimeter of 18 centimeters?

Ⓕ 9 square centimeters
Ⓖ $9\sqrt{3}$ square centimeters
Ⓗ 18 square centimeters
Ⓙ $18\sqrt{3}$ square centimeters

7. In $\triangle ABC$ and $\triangle DEF$, $\overline{AC} \cong \overline{DE}$, and $\angle A \cong \angle E$. Which of the following would allow you to conclude by SAS that these triangles are congruent?

Ⓐ $\overline{AB} \cong \overline{DF}$
Ⓑ $\overline{AC} \cong \overline{EF}$
Ⓒ $\overline{BA} \cong \overline{FE}$
Ⓓ $\overline{CB} \cong \overline{DF}$

8. For the segment below, $AB = \frac{1}{2}AC$, and $CD = 2BC$. Which expression is equal to the length of $\overline{AD}$?

Ⓕ $2AB + BC$
Ⓖ $2AC + AB$
Ⓗ $3AB$
Ⓙ $4BC$

9. In $\triangle DEF$, $m\angle D = 2(m\angle E + m\angle F)$. Which term best describes $\triangle DEF$?

Ⓐ Acute
Ⓑ Equiangular
Ⓒ Right
Ⓓ Obtuse

10. Which point of concurrency is always located inside the triangle?

Ⓕ The centroid of an obtuse triangle
Ⓖ The circumcenter of an obtuse triangle
Ⓗ The circumcenter of a right triangle
Ⓙ The orthocenter of a right triangle

HOT TIP! If a diagram is not provided, draw your own. Use the given information to label the diagram.

11. The length of one leg of a right triangle is 3 times the length of the other, and the length of the hypotenuse is 10. What is the length of the longest leg?

Ⓐ 3 Ⓒ $\sqrt{10}$

Ⓑ $3\sqrt{10}$ Ⓓ $12\sqrt{5}$

12. Which statement is true by the Transitive Property of Congruence?

Ⓕ If $\angle A \cong \angle T$, then $\angle T \cong \angle A$.

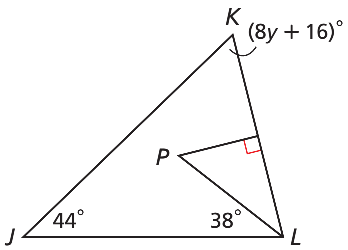
Ⓖ If $m\angle L = m\angle S$, then $\angle L \cong \angle S$.

Ⓗ $5QR + 10 = 5(QR + 2)$

Ⓙ If $\overline{BD} \cong \overline{DE}$ and $\overline{DE} \cong \overline{EF}$, then $\overline{BD} \cong \overline{EF}$.

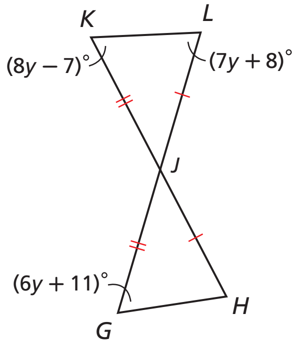
Gridded Response

13. P is the incenter of $\triangle JKL$. The distance from P to $\overline{KL}$ is $2y - 9$. What is the distance from P to $\overline{JK}$?



14. In a plane, $r \parallel s$, and $s \perp t$. How many right angles are formed by the lines r, s, and t?

15. What is the measure, in degrees, of $\angle H$?

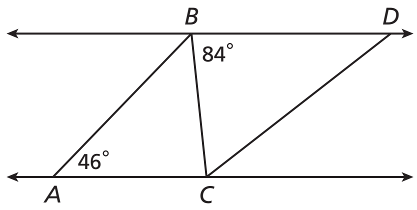


16. The point T is in the interior of $\angle XYZ$. If $m\angle XYZ = (25x + 10)^\circ$, $m\angle XYT = 90°$, and $m\angle TYZ = (9x)^\circ$, what is the value of x?

Short Response

17. In $\triangle RST$, S is on the perpendicular bisector of $\overline{RT}$, $m\angle S = (4n + 16)^\circ$, and $m\angle R = (3n - 18)^\circ$. Find $m\angle R$. Show your work and explain how you determined your answer.

18. Given that $\overline{BD} \parallel \overline{AC}$ and $\overline{AB} \cong \overline{BD}$, explain why $AC < DC$.

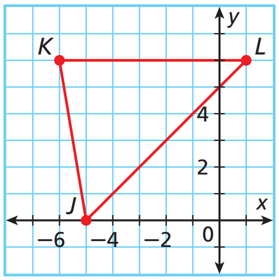


19. Write an indirect proof that an acute triangle cannot contain a pair of complementary angles.

Given: $\triangle XYZ$ is an acute triangle.

Prove: $\triangle XYZ$ does not contain a pair of complementary angles.

20. Find the coordinates of the orthocenter of $\triangle JKL$. Show your work and explain how you found your answer.



Extended Response

21. Consider the statement "If a triangle is equiangular, then it is acute."

a. Write the converse, inverse, and contrapositive of this conditional statement.

b. Write a biconditional statement from the conditional statement.

c. Determine the truth value of the biconditional statement. If it is false, give a counterexample.

d. Determine the truth value of each statement below. Give an example or counterexample to justify your reasoning.

"For any conditional, if the inverse and contrapositive are true, then the biconditional is true."

"For any conditional, if the inverse and converse are true, then the biconditional is true."