CHAPTER 4

Triangle Congruence

4A Triangles and Congruence

4-1 Classifying Triangles

Lab Develop the Triangle Sum Theorem

4-2 Angle Relationships in Triangles

4-3 Congruent Triangles

CONCEPT CONNECTION

4B Proving Triangle Congruence

Lab Explore SSS and SAS Triangle Congruence

4-4 Triangle Congruence: SSS and SAS

Lab Predict Other Triangle Congruence Relationships

4-5 Triangle Congruence: ASA, AAS, and HL

4-6 Triangle Congruence: CPCTC

4-7 Introduction to Coordinate Proof

4-8 Isosceles and Equilateral Triangles

Ext Proving Constructions Valid

CONCEPT CONNECTION

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KEYWORD: MG7 ChProj

Congruent triangles can be seen in the structural design of the houses on Alamo Square.

Alamo Square
San Francisco, CA

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

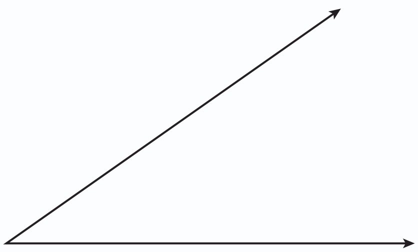
1. acute angle
2. congruent segments
3. obtuse angle
4. postulate
5. triangle

A. a statement that is accepted as true without proof
B. an angle that measures greater than 90° and less than 180°
C. a statement that you can prove
D. segments that have the same length
E. a three-sided polygon
F. an angle that measures greater than 0° and less than 90°

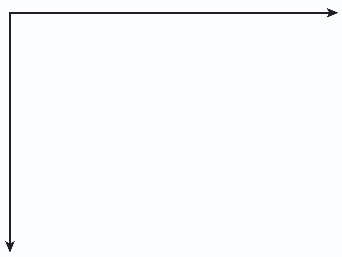
Measure Angles

Use a protractor to measure each angle.

6.



7.



Use a protractor to draw an angle with each of the following measures.

8. 20° **9.** 63° **10.** 105° **11.** 158°

Solve Equations with Fractions

Solve.

12. $\frac{9}{2}x + 7 = 25$

13. $3x - \frac{2}{3} = \frac{4}{3}$

14. $x - \frac{1}{5} = \frac{12}{5}$

15. $2y = 5y - \frac{21}{2}$

Connect Words and Algebra

Write an equation for each statement.

16. Tanya's age t is three times Martin's age m.
17. Twice the length of a segment x is 9 ft.
18. The sum of 53° and twice an angle measure y is 90°.
19. The price of a radio r is \$25 less than the price of a CD player p.
20. Half the amount of liquid j in a jar is 5 oz more than the amount of liquid b in a bowl.

CHAPTER 4

Unpacking the Standards

The information below "unpacks" the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

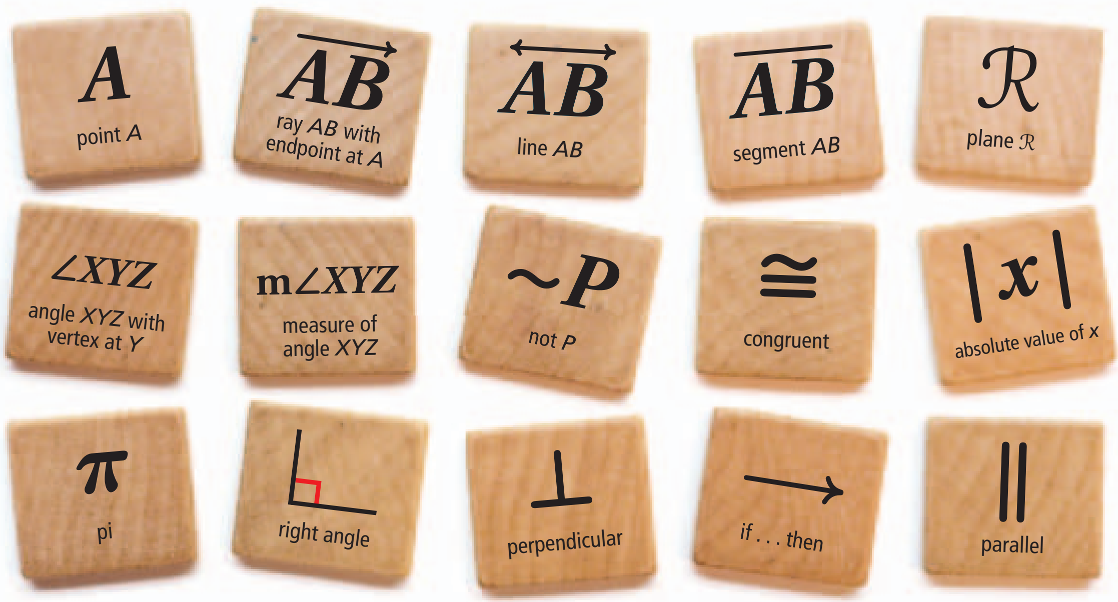
California Standard	Academic Vocabulary	Chapter Concept
4.0 Students prove basic theorems involving congruence and similarity. (Lessons 4-5, 4-8)	**involving** relating to	You learn how to use the ASA, AAS, and HL theorems to prove triangles congruent, construct triangles, and solve problems. You also learn theorems about isosceles and equilateral triangles.
5.0 Students prove that triangles are congruent or similar, **and they are able to use the concept of corresponding parts of congruent triangles.** (Lessons 4-3, 4-4, 4-5, 4-6, Extension)	**concept** idea **corresponding** matching	You learn how a triangle congruence statement indicates corresponding parts. You also learn how to use the SSS and SAS theorems to prove triangles congruent and to prove that constructions are valid.
12.0 Students find and use measures of sides and of interior and exterior angles of triangles and polygons **to classify figures and solve problems.** (Lessons 4-1, 4-2, 4-8)	**interior** inside **exterior** outside	You classify triangles by their angle measures and side lengths. You find the measures of interior and exterior angles of triangles and use theorems relating to these angles.
13.0 Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, **and exterior angles.** (Lesson 4-2)	**relationships** links	You learn how angles are related and apply the Triangle Sum Theorem, the Exterior Angle Theorem, and the Third Angles Theorem.
17.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles. (Lessons 4-7, 4-8)	**coordinate geometry** a form of geometry that uses a set of numbers to describe the exact position of a figure with reference to the *x*- and *y*-axes	You learn how to position figures in the coordinate plane for use in coordinate proofs.

Standards 1.0, 2.0, 3.0, and 16.0 are also covered in this chapter. To see these standards unpacked, go to Chapter 1, p. 4 and Chapter 2, p. 72.

Reading and Writing Math

Reading Strategy: Read Geometry Symbols

In Geometry we often use symbols to communicate information. When studying each lesson, read both the symbols and the words slowly and carefully. Reading aloud can sometimes help you translate symbols into words.



Throughout this course, you will use these symbols and combinations of these symbols to represent various geometric statements.

Symbol Combinations	Translated into Words
$\overleftrightarrow{ST} \parallel \overleftrightarrow{UV}$	Line *ST* is parallel to line *UV*.
$\overline{BC} \perp \overline{GH}$	Segment *BC* is perpendicular to segment *GH*.
$p \to q$	If *p*, then *q*.
$m\angle QRS = 45°$	The measure of angle *QRS* is 45 degrees.
$\angle CDE \cong \angle LMN$	Angle *CDE* is congruent to angle *LMN*.

Try This

Rewrite each statement using symbols.

1. the absolute value of 2 times pi
2. The measure of angle 2 is 125 degrees.
3. Segment *XY* is perpendicular to line *BC*.
4. If not *p*, then not *q*.

Translate the symbols into words.

5. $m\angle FGH = m\angle VWX$
6. $\overleftrightarrow{ZA} \parallel \overleftrightarrow{TU}$
7. $\sim p \to q$
8. $\overrightarrow{ST}$ bisects $\angle TSU$.

4-1 Classifying Triangles

Objectives
Classify triangles by their angle measures and side lengths.

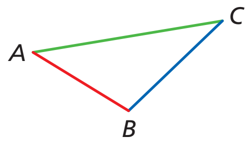
Use triangle classification to find angle measures and side lengths.

Vocabulary
acute triangle
equiangular triangle
right triangle
obtuse triangle
equilateral triangle
isosceles triangle
scalene triangle

Who uses this?
Manufacturers use properties of triangles to calculate the amount of material needed to make triangular objects. (See Example 4.)

A triangle is a steel percussion instrument in the shape of an *equilateral triangle*. Different-sized triangles produce different musical notes when struck with a metal rod.

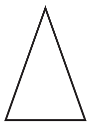
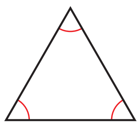
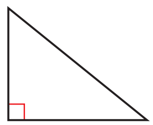
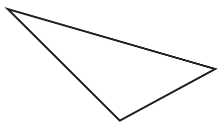
Recall that a *triangle* ($\triangle$) is a polygon with three sides. Triangles can be classified in two ways: by their angle measures or by their side lengths.



$\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are the *sides* of $\triangle ABC$.
A, B, and C are the triangle's *vertices*.

Know it! Note

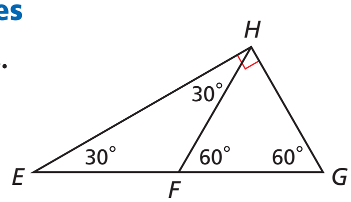
Triangle Classification — By Angle Measures

Acute Triangle	Equiangular Triangle	Right Triangle	Obtuse Triangle
			
Three acute angles	Three congruent acute angles	One right angle	One obtuse angle

EXAMPLE 1

Classifying Triangles by Angle Measures

Classify each triangle by its angle measures.



A $\triangle EHG$

$\angle EHG$ is a right angle. So $\triangle EHG$ is a right triangle.

B $\triangle EFH$

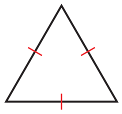
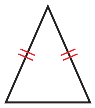
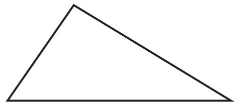
$\angle EFH$ and $\angle HFG$ form a linear pair, so they are supplementary. Therefore $m\angle EFH + m\angle HFG = 180°$. By substitution, $m\angle EFH + 60° = 180°$. So $m\angle EFH = 120°$. $\triangle EFH$ is an obtuse triangle by definition.

California Standards
12.0 Students find and use measures of sides and of interior and exterior **angles of triangles** and polygons **to classify figures and solve problems.**

CHECK IT OUT!
1. Use the diagram to classify $\triangle FHG$ by its angle measures.

Know it! Note

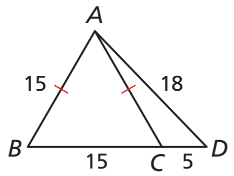
Triangle Classification — By Side Lengths

Equilateral Triangle	Isosceles Triangle	Scalene Triangle
		
Three congruent sides	At least two congruent sides	No congruent sides

Remember!
When you look at a figure, you cannot assume segments are congruent based on their appearance. They must be marked as congruent.

EXAMPLE 2 Classifying Triangles by Side Lengths

Classify each triangle by its side lengths.



A $\triangle ABC$

From the figure, $\overline{AB} \cong \overline{AC}$. So $AC = 15$, and $\triangle ABC$ is equilateral.

B $\triangle ABD$

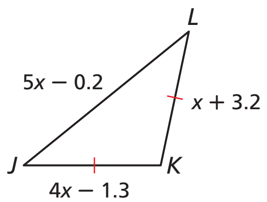
By the Segment Addition Postulate,
$BD = BC + CD = 15 + 5 = 20$.
Since no sides are congruent, $\triangle ABD$ is scalene.

CHECK IT OUT! **2.** Use the diagram to classify $\triangle ACD$ by its side lengths.

EXAMPLE 3 Using Triangle Classification

Algebra

Find the side lengths of the triangle.



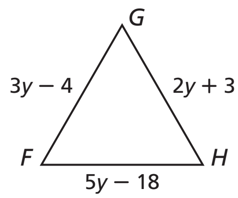
Step 1 Find the value of x.

$\overline{JK} \cong \overline{KL}$	*Given*
$JK = KL$	*Def. of ≅ segs.*
$(4x - 1.3) = (x + 3.2)$	*Substitute (4x – 13) for JK and (x + 3.2) for KL.*
$3x = 4.5$	*Add 1.3 and subtract x from both sides.*
$x = 1.5$	*Divide both sides by 3.*

Step 2 Substitute 1.5 into the expressions to find the side lengths.

$JK = 4x - 1.3$
$= 4(1.5) - 1.3 = 4.7$
$KL = x + 3.2$
$= (1.5) + 3.2 = 4.7$
$JL = 5x - 0.2$
$= 5(1.5) - 0.2 = 7.3$

CHECK IT OUT! **3.** Find the side lengths of equilateral $\triangle FGH$.



EXAMPLE 4 *Music Application*

A manufacturer produces musical triangles by bending pieces of steel into the shape of an equilateral triangle. The triangles are available in side lengths of 4 inches, 7 inches, and 10 inches. How many 4-inch triangles can the manufacturer produce from a 100 inch piece of steel?

The amount of steel needed to make one triangle is equal to the perimeter P of the equilateral triangle.

$$P = 3(4)$$
$$= 12 \text{ in.}$$

To find the number of triangles that can be made from 100 inches. of steel, divide 100 by the amount of steel needed for one triangle.

$$100 \div 12 = 8\frac{1}{3} \text{ triangles}$$

There is not enough steel to complete a ninth triangle. So the manufacturer can make 8 triangles from a 100 in. piece of steel.

CHECK IT OUT! **Each measure is the side length of an equilateral triangle. Determine how many triangles can be formed from a 100 in. piece of steel.**

4a. 7 in. **4b.** 10 in.

THINK AND DISCUSS

1. For $\triangle DEF$, name the three pairs of consecutive sides and the vertex formed by each.

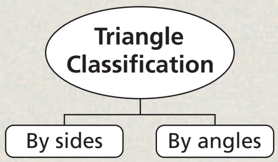
2. Sketch an example of an obtuse isosceles triangle, or explain why it is not possible to do so.

3. Is every acute triangle equiangular? Explain and support your answer with a sketch.

4. Use the Pythagorean Theorem to explain why you cannot draw an equilateral right triangle.

Know it! Note

5. GET ORGANIZED Copy and complete the graphic organizer. In each box, describe each type of triangle.



4-1 Exercises

California Standards
3.0, 12.0, 16.0, 7AF4.1, 7MG3.3, 1A2.0

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KEYWORD: MG7 4-1
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

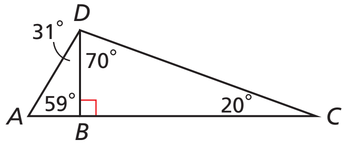
1. In △*JKL*, *JK*, *KL*, and *JL* are *equal*. How does this help you classify △*JKL* by its side lengths?

2. △*XYZ* is an *obtuse* triangle. What can you say about the types of angles in △*XYZ*?

SEE EXAMPLE 1
p. 216

Classify each triangle by its angle measures.

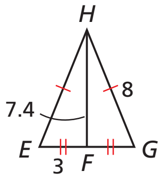
3. △*DBC* 4. △*ABD* 5. △*ADC*



SEE EXAMPLE 2
p. 217

Classify each triangle by its side lengths.

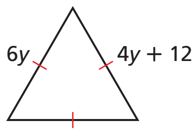
6. △*EGH* 7. △*EFH* 8. △*HFG*



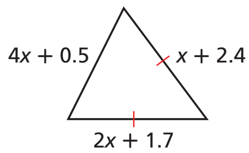
SEE EXAMPLE 3
p. 217

Multi-Step Find the side lengths of each triangle.

9.



10.



SEE EXAMPLE 4
p. 218

11. **Crafts** A jeweler creates triangular earrings by bending pieces of silver wire. Each earring is an isosceles triangle with the dimensions shown. How many earrings can be made from a piece of wire that is 50 cm long?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12–14	1
15–17	2
18–20	3
21–22	4

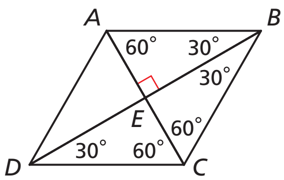
Extra Practice
Skills Practice p. S10
Application Practice p. S31

Classify each triangle by its angle measures.

12. △*BEA*

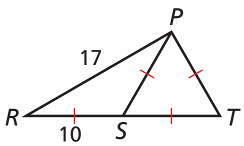
13. △*DBC*

14. △*ABC*



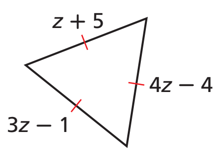
Classify each triangle by its side lengths.

15. △*PST* 16. △*RSP* 17. △*RPT*

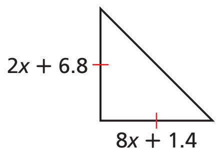


Multi-Step Find the side lengths of each triangle.

18.

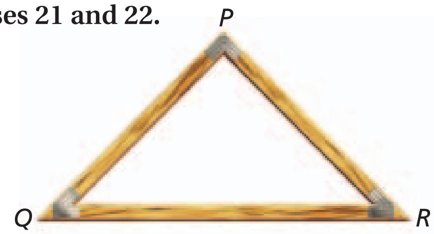


19.



20. Draw a triangle large enough to measure. Label the vertices *X*, *Y*, and *Z*.
 a. Name the three sides and three angles of the triangle.
 b. Use a ruler and protractor to classify the triangle by its side lengths and angle measures.

Carpentry **Use the following information for Exercises 21 and 22.** A manufacturer makes trusses, or triangular supports, for the roofs of houses. Each truss is the shape of an isosceles triangle in which $\overline{PQ} \cong \overline{PR}$. The length of the base $\overline{QR}$ is $\frac{4}{3}$ the length of each of the congruent sides.



21. The perimeter of each truss is 60 ft. Find each side length.

22. How many trusses can the manufacturer make from 150 feet of lumber?

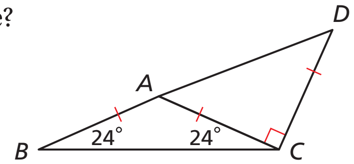
Draw an example of each type of triangle or explain why it is not possible.

23. isosceles right
24. equiangular obtuse
25. scalene right
26. equilateral acute
27. scalene equiangular
28. isosceles acute

29. An equilateral triangle has a perimeter of 105 in. What is the length of each side of the triangle?

Classify each triangle by its angles and sides.

30. $\triangle ABC$
31. $\triangle ACD$



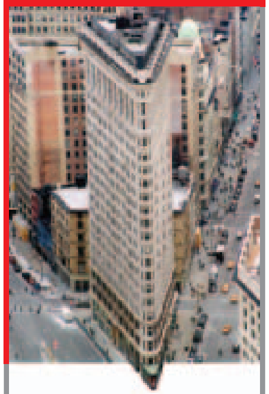
32. An isosceles triangle has a perimeter of 34 cm. The congruent sides measure $(4x - 1)$ cm. The length of the third side is x cm. What is the value of x?

33. **Architecture** The base of the Flatiron Building is a triangle bordered by three streets: Broadway, Fifth Avenue, and East Twenty-second Street. The Fifth Avenue side is 1 ft shorter than twice the East Twenty-second Street side. The East Twenty-second Street side is 8 ft shorter than half the Broadway side. The Broadway side is 190 ft.

a. Find the two unknown side lengths.
b. Classify the triangle by its side lengths.

LINK

Architecture



Daniel Burnham designed and built the 22-story Flatiron Building in New York City in 1902.

Source: www.greatbuildings.com

34. **Critical Thinking** Is every isosceles triangle equilateral? Is every equilateral triangle isosceles? Explain.

Tell whether each statement is sometimes, always, or never true. Support your answer with a sketch.

35. An acute triangle is a scalene triangle.

36. A scalene triangle is an obtuse triangle.

37. An equiangular triangle is an isosceles triangle.

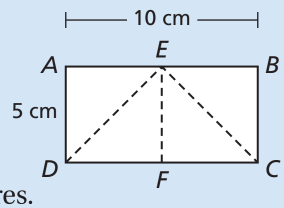
38. **Write About It** Write a formula for the side length s of an equilateral triangle, given the perimeter P. Explain how you derived the formula.

39. **Construction** Use the method for constructing congruent segments to construct an equilateral triangle.

Concept Connection

40. This problem will prepare you for the Concept Connection on page 238. Marc folded a rectangular sheet of paper, $ABCD$, in half along $\overline{EF}$. He folded the resulting square diagonally and then unfolded the paper to create the creases shown.

a. Use the Pythagorean Theorem to find DE and CE.
b. What is the $m\angle DEC$?
c. Classify $\triangle DEC$ by its side lengths and by its angle measures.



STANDARDIZED TEST PREP

41. What is the side length of an equilateral triangle with a perimeter of $36\frac{2}{3}$ inches?

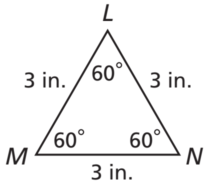
Ⓐ $36\frac{2}{3}$ inches
Ⓑ $18\frac{1}{3}$ inches
Ⓒ $12\frac{1}{3}$ inches
Ⓓ $12\frac{2}{9}$ inches

42. The vertices of $\triangle RST$ are $R(3, 2)$, $S(-2, 3)$, and $T(-2, 1)$. Which of these best describes $\triangle RST$?

Ⓕ Isosceles
Ⓖ Scalene
Ⓗ Equilateral
Ⓙ Right

43. Which of the following is NOT a correct classification of $\triangle LMN$?

Ⓐ Acute
Ⓑ Equiangular
Ⓒ Isosceles
Ⓓ Right

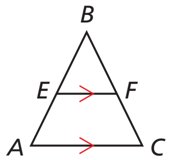


44. **Gridded Response** $\triangle ABC$ is isosceles, and $\overline{AB} \cong \overline{AC}$. $AB = \left(\frac{1}{2}x + \frac{1}{4}\right)$, and $BC = \left(\frac{5}{2} - x\right)$. What is the perimeter of $\triangle ABC$?

CHALLENGE AND EXTEND

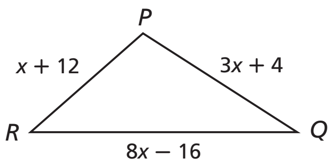
45. A triangle has vertices with coordinates $(0, 0)$, $(a, 0)$, and $(0, a)$, where $a \neq 0$. Classify the triangle in two different ways. Explain your answer.

46. Write a two-column proof.
Given: $\triangle ABC$ is equiangular.
$EF \parallel AC$
Prove: $\triangle EFB$ is equiangular.



47. Two sides of an equilateral triangle measure $(y + 10)$ units and $(y^2 - 2)$ units. If the perimeter of the triangle is 21 units, what is the value of y?

48. **Multi-Step** The average length of the sides of $\triangle PQR$ is 24. How much longer then the average is the longest side?



SPIRAL REVIEW

Name the parent function of each function. *(Previous course)*

49. $y = 5x^2 + 4$

50. $2y = 3x + 4$

51. $y = 2(x - 8)^2 + 6$

Determine if each biconditional is true. If false, give a counter example. *(Lesson 2-4)*

52. Two lines are parallel if and only if they do not intersect.

53. A triangle is equiangular if and only if it has three congruent angles.

54. A number is a multiple of 20 if and only if the number ends in a 0.

Determine whether each line is parallel to, is perpendicular to, or coincides with $y = 4x$. *(Lesson 3-6)*

55. $y = 4x + 2$

56. $4y = -x + 8$

57. $\frac{1}{2}y = 2x$

58. $-2y = \frac{1}{2}x$

4-2 Geometry LAB

Develop the Triangle Sum Theorem

In this lab, you will use patty paper to discover a relationship between the measures of the interior angles of a triangle.

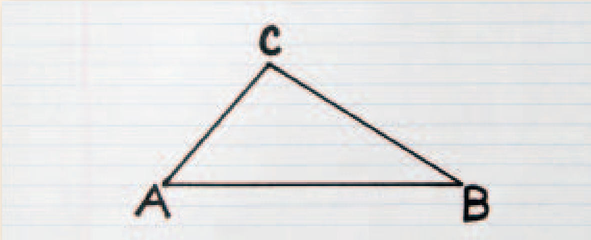
Use with Lesson 4-2

California Standards

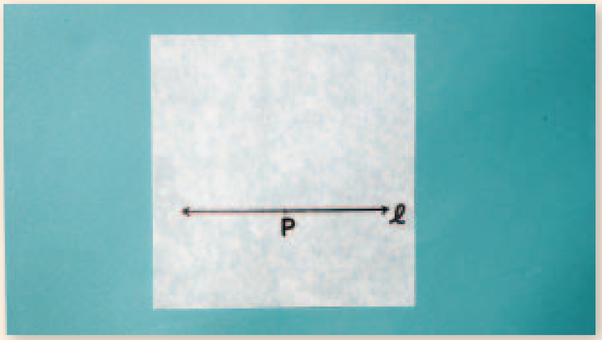
1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and **inductive** and deductive **reasoning.**

Activity

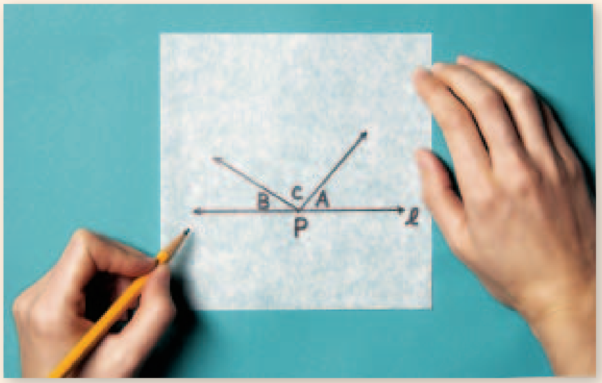
1. Draw and label $\triangle ABC$ on a sheet of notebook paper.



2. On patty paper draw a line ℓ and label a point P on the line.



3. Place the patty paper on top of the triangle you drew. Align the papers so that $\overline{AB}$ is on line ℓ and P and B coincide. Trace $\angle B$. Rotate the triangle and trace $\angle C$ adjacent to $\angle B$. Rotate the triangle again and trace $\angle A$ adjacent to $\angle C$. The diagram shows your final step.



Try This

1. What do you notice about the three angles of the triangle that you traced?
2. Repeat the activity two more times using two different triangles. Do you get the same results each time?
3. Write an equation describing the relationship among the measures of the angles of $\triangle ABC$.
4. Use inductive reasoning to write a conjecture about the sum of the measures of the angles of a triangle.

4-2 Angle Relationships in Triangles

Objectives

Find the measures of interior and exterior angles of triangles.

Apply theorems about the interior and exterior angles of triangles.

Vocabulary

auxiliary line
corollary
interior
exterior
interior angle
exterior angle
remote interior angle

Who uses this?

Surveyors use triangles to make measurements and create boundaries. (See Example 1.)

Triangulation is a method used in surveying. Land is divided into adjacent triangles. By measuring the sides and angles of one triangle and applying properties of triangles, surveyors can gather information about adjacent triangles.

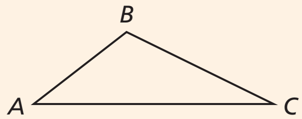
This engraving shows the county surveyor and commissioners laying out the town of Baltimore in 1730.

Know it! Note

Theorem 4-2-1 Triangle Sum Theorem

The sum of the angle measures of a triangle is 180°.

$m\angle A + m\angle B + m\angle C = 180°$

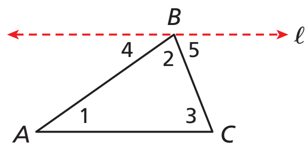


The proof of the Triangle Sum Theorem uses an *auxiliary line*. An **auxiliary line** is a line that is added to a figure to aid in a proof.

PROOF

Triangle Sum Theorem

Given: $\triangle ABC$
Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Proof:

Draw $\ell \parallel \overline{AC}$ through B. — Parallel Post.

$\angle 1 \cong \angle 4$ — Alt. Int. ∠s Thm.

$\angle 3 \cong \angle 5$ — Alt. Int. ∠s Thm.

$m\angle 1 = m\angle 4$ — Def. of ≅ ∠s

$m\angle 3 = m\angle 5$ — Def. of ≅ ∠s

$m\angle 4 + m\angle 2 + m\angle 5 = 180°$ — ∠ Add. Post. & def. of straight ∠

$m\angle 1 + m\angle 2 + m\angle 3 = 180°$ — Subst.

California Standards

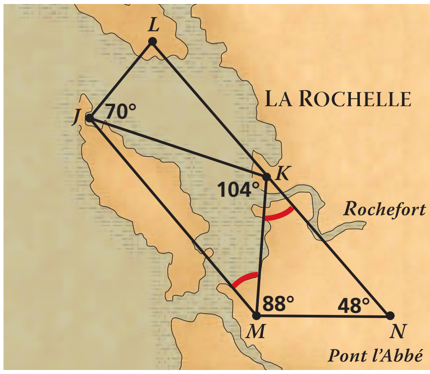
12.0 Students find and use measures of sides and of **interior and exterior angles of triangles** and polygons **to** classify figures and **solve problems.**

13.0 Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, **and exterior angles.**

Also covered: **2.0**

EXAMPLE 1 Surveying Application

The map of France commonly used in the 1600s was significantly revised as a result of a triangulation land survey. The diagram shows part of the survey map. Use the diagram to find the indicated angle measures.



Algebra

A $m\angle NKM$

$m\angle KMN + m\angle MNK + m\angle NKM = 180°$	*△ Sum Thm.*
$88 + 48 + m\angle NKM = 180$	*Substitute 88 for m∠KMN and 48 for m∠MNK.*
$136 + m\angle NKM = 180$	*Simplify.*
$m\angle NKM = 44°$	*Subtract 136 from both sides.*

B $m\angle JLK$

Step 1 Find $m\angle JKL$.

$m\angle NKM + m\angle MKJ + m\angle JKL = 180°$	*Lin. Pair Thm. & ∠ Add. Post.*
$44 + 104 + m\angle JKL = 180$	*Substitute 44 for m∠NKM and 104 for m∠MKJ.*
$148 + m\angle JKL = 180$	*Simplify.*
$m\angle JKL = 32°$	*Subtract 148 from both sides.*

Step 2 Use substitution and then solve for $m\angle JLK$.

$m\angle JLK + m\angle JKL + m\angle KJL = 180°$	*△ Sum Thm.*
$m\angle JLK + 32 + 70 = 180$	*Substitute 32 for m∠JKL and 70 for m∠KJL.*
$m\angle JLK + 102 = 180$	*Simplify.*
$m\angle JLK = 78°$	*Subtract 102 from both sides.*

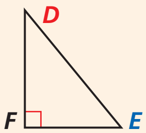
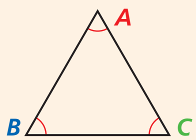
CHECK IT OUT!

1. Use the diagram to find $m\angle MJK$.

A **corollary** is a theorem whose proof follows directly from another theorem. Here are two corollaries to the Triangle Sum Theorem.

Know it! Note

Corollaries

COROLLARY		HYPOTHESIS	CONCLUSION
4-2-2	The acute angles of a right triangle are complementary.		∠D and ∠E are complementary. $m\angle D + m\angle E = 90°$
4-2-3	The measure of each angle of an equiangular triangle is 60°.		$m\angle A = m\angle B = m\angle C = 60°$

You will prove Corollaries 4-2-2 and 4-2-3 in Exercises 24 and 25.

EXAMPLE 2 Finding Angle Measures in Right Triangles

Algebra

One of the acute angles in a right triangle measures 22.9°. What is the measure of the other acute angle?

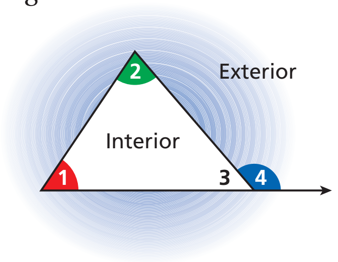
Let the acute angles be $\angle M$ and $\angle N$, with $m\angle M = 22.9°$.

$m\angle M + m\angle N = 90$	*Acute ∠s of rt. △ are comp.*
$22.9 + m\angle N = 90$	*Substitute 22.9 for m∠M.*
$m\angle N = 67.1°$	*Subtract 22.9 from both sides.*

CHECK IT OUT! **The measure of one of the acute angles in a right triangle is given. What is the measure of the other acute angle?**

2a. $63.7°$ **2b.** $x°$ **2c.** $48\frac{2}{5}°$

The **interior** is the set of all points inside the figure. The **exterior** is the set of all points outside the figure. An **interior angle** is formed by two sides of a triangle. An **exterior angle** is formed by one side of the triangle and the extension of an adjacent side. Each exterior angle has two *remote interior angles*. A **remote interior angle** is an interior angle that is not adjacent to the exterior angle.



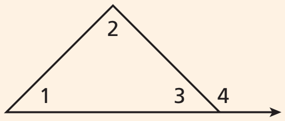
∠4 is an exterior angle. Its remote interior angles are ∠1 and ∠2.

Know it! Note

Theorem 4-2-4 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

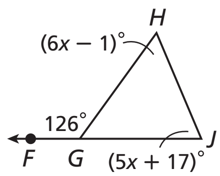
$m\angle 4 = m\angle 1 + m\angle 2$



You will prove Theorem 4-2-4 in Exercise 28.

EXAMPLE 3 Applying the Exterior Angle Theorem

Algebra

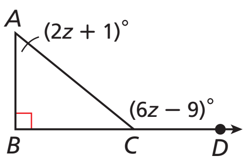


Find m∠J.

$m\angle J + m\angle H = m\angle FGH$	*Ext. ∠ Thm.*
$5x + 17 + 6x - 1 = 126$	*Substitute 5x + 17 for m∠J, 6x – 1 for m∠H, and 126 for m∠FGH.*
$11x + 16 = 126$	*Simplify.*
$11x = 110$	*Subtract 16 from both sides.*
$x = 10$	*Divide both sides by 11.*

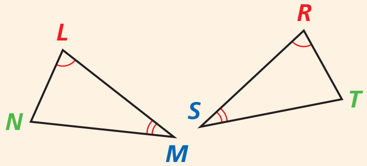
$m\angle J = 5x + 17 = 5(10) + 17 = 67°$

CHECK IT OUT! **3.** Find $m\angle ACD$.



Know it! Note

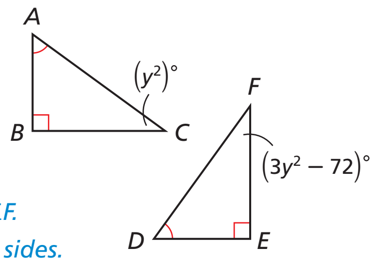
Theorem 4-2-5 Third Angles Theorem

THEOREM	HYPOTHESIS	CONCLUSION
If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.		$\angle N \cong \angle T$

You will prove Theorem 4-2-5 in Exercise 27.

EXAMPLE 4 Applying the Third Angles Theorem

Algebra



Find m∠C and m∠F.

$\angle C \cong \angle F$	*Third ∆s Thm.*
$m\angle C = m\angle F$	*Def. of ≅ ∆s.*
$y^2 = 3y^2 - 72$	*Substitute y^2 for m∠C and $3y^2 - 72$ for m∠F.*
$-2y^2 = -72$	*Subtract $3y^2$ from both sides.*
$y^2 = 36$	*Divide both sides by −2.*

So $m\angle C = 36°$.

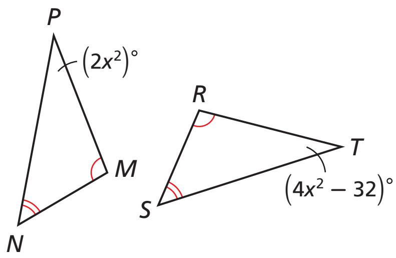
Since $m\angle F = m\angle C$, $m\angle F = 36°$.

Helpful Hint

You can use substitution to verify that $m\angle F = 36°$.
$m\angle F = (3 \cdot 36 - 72)$
$= 36°$.

CHECK IT OUT!

4. Find m∠P and m∠T.



THINK AND DISCUSS

1. Use the Triangle Sum Theorem to explain why the supplement of one of the angles of a triangle equals in measure the sum of the other two angles of the triangle. Support your answer with a sketch.

2. Sketch a triangle and draw all of its exterior angles. How many exterior angles are there at each vertex of the triangle? How many total exterior angles does the triangle have?

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write each theorem in words and then draw a diagram to represent it.

Theorem	Words	Diagram
Triangle Sum Theorem		
Exterior Angle Theorem		
Third Angles Theorem		

4-2 Exercises

California Standards
2.0, 12.0, 13.0, 7NS1.2, 7AF1.0, 7AF4.1, 1A2.0, 1A5.0

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Homework Help Online
KEYWORD: MG7 4-2
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. To remember the meaning of *remote interior angle*, think of a television remote control. What is another way to remember the term *remote*?

2. An *exterior angle* is drawn at vertex E of $\triangle DEF$. What are its *remote interior angles*?

3. What do you call segments, rays, or lines that are added to a given diagram?

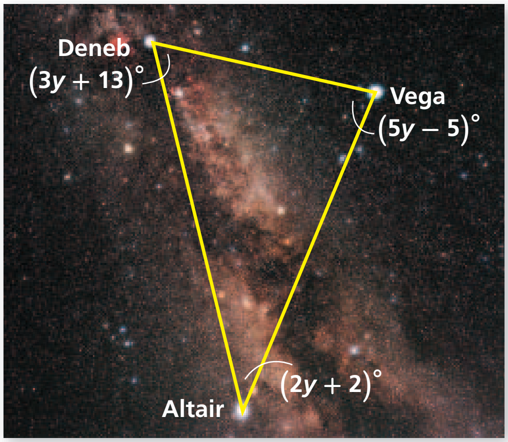
SEE EXAMPLE 1
p. 224

Astronomy **Use the following information for Exercises 4 and 5.**

An *asterism* is a group of stars that is easier to recognize than a constellation. One popular asterism is the Summer Triangle, which is composed of the stars Deneb, Altair, and Vega.

4. What is the value of y?

5. What is the measure of each angle in the Summer Triangle?



SEE EXAMPLE 2
p. 225

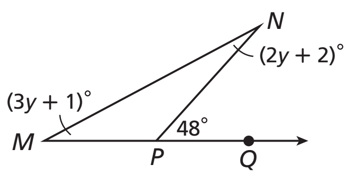
The measure of one of the acute angles in a right triangle is given. What is the measure of the other acute angle?

6. $20.8°$ **7.** $y°$ **8.** $24\frac{2}{3}°$

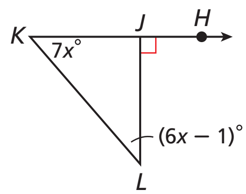
SEE EXAMPLE 3
p. 225

Find each angle measure.

9. $m\angle M$



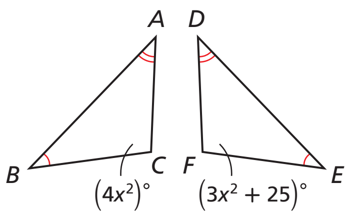
10. $m\angle L$



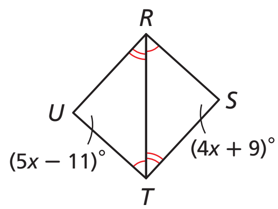
11. In $\triangle ABC$, $m\angle A = 65°$, and the measure of an exterior angle at C is $117°$. Find $m\angle B$ and the $m\angle BCA$.

SEE EXAMPLE 4
p. 226

12. $m\angle C$ and $m\angle F$



13. $m\angle S$ and $m\angle U$



14. For $\triangle ABC$ and $\triangle XYZ$, $m\angle A = m\angle X$ and $m\angle B = m\angle Y$. Find the measures of $\angle C$ and $\angle Z$ if $m\angle C = 4x + 7$ and $m\angle Z = 3(x + 5)$.

PRACTICE AND PROBLEM SOLVING

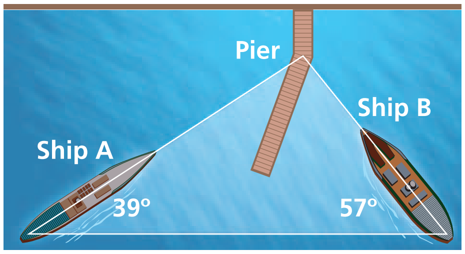
Independent Practice

For Exercises	See Example
15	1
16–18	2
19–20	3
21–22	4

Extra Practice

Skills Practice p. S10
Application Practice p. S31

15. Navigation A sailor on ship A measures the angle between ship B and the pier and finds that it is 39°. A sailor on ship B measures the angle between ship A and the pier and finds that it is 57°. What is the measure of the angle between ships A and B?

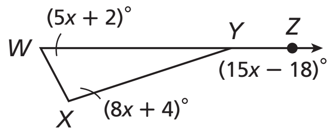


The measure of one of the acute angles in a right triangle is given. What is the measure of the other acute angle?

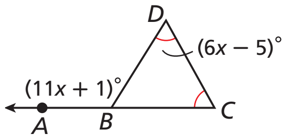
16. $76\frac{1}{4}°$ **17.** $2x°$ **18.** 56.8°

Find each angle measure.

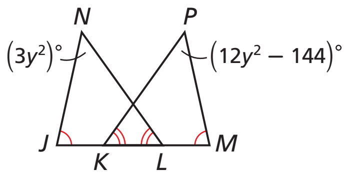
19. m∠XYZ



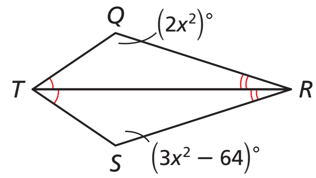
20. m∠C



21. m∠N and m∠P



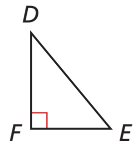
22. m∠Q and m∠S



23. Multi-Step The measures of the angles of a triangle are in the ratio 1 : 4 : 7. What are the measures of the angles? (*Hint:* Let x, $4x$, and $7x$ represent the angle measures.)

24. Complete the proof of Corollary 4-2-2.

Given: △DEF with right ∠F
Prove: ∠D and ∠E are complementary.



Proof:

Statements	Reasons
1. △DEF with rt. ∠F	1. a. ___?___
2. b. ___?___	2. Def. of rt. ∠
3. m∠D + m∠E + m∠F = 180°	3. c. ___?___
4. m∠D + m∠E + 90° = 180°	4. d. ___?___
5. e. ___?___	5. Subtr. Prop.
6. ∠D and ∠E are comp.	6. f. ___?___

25. Prove Corollary 4-2-3 using two different methods of proof.

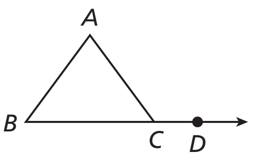
Given: △ABC is equiangular.
Prove: m∠A = m∠B = m∠C = 60°

26. Multi-Step The measure of one acute angle in a right triangle is $1\frac{1}{4}$ times the measure of the other acute angle. What is the measure of the larger acute angle?

27. Write a two-column proof of the Third Angles Theorem.

28. Prove the Exterior Angle Theorem.

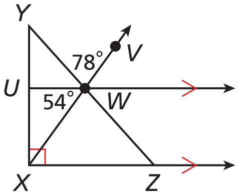
Given: $\triangle ABC$ with exterior angle $\angle ACD$
Prove: $m\angle ACD = m\angle A + m\angle B$
(*Hint:* $\angle BCA$ and $\angle DCA$ form a linear pair.)



Find each angle measure.

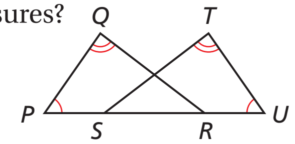
29. $\angle UXW$ **30.** $\angle UWY$

31. $\angle WZX$ **32.** $\angle XYZ$



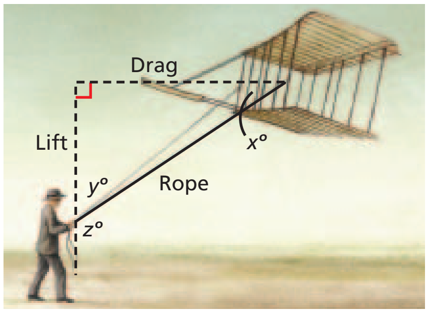
33. **Critical Thinking** What is the measure of any exterior angle of an equiangular triangle? What is the sum of the exterior angle measures?

34. Find $m\angle SRQ$, given that $\angle P \cong \angle U$, $\angle Q \cong \angle T$, and $m\angle RST = 37.5°$.



35. **Multi-Step** In a right triangle, one acute angle measure is 4 times the other acute angle measure. What is the measure of the smaller angle?

36. **Aviation** To study the forces of lift and drag, the Wright brothers built a glider, attached two ropes to it, and flew it like a kite. They modeled the two wind forces as the legs of a right triangle.



a. What part of a right triangle is formed by each rope?

b. Use the Triangle Sum Theorem to write an equation relating the angle measures in the right triangle.

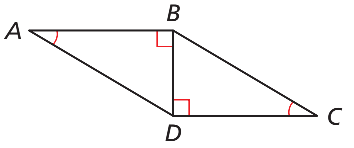
c. Simplify the equation from part **b**. What is the relationship between x and y?

d. Use the Exterior Angle Theorem to write an expression for z in terms of x.

e. If $x = 37°$, use your results from parts **c** and **d** to find y and z.

37. **Estimation** Draw a triangle and two exterior angles at each vertex. Estimate the measure of each angle. How are the exterior angles at each vertex related? Explain.

38. **Given:** $\overline{AB} \perp \overline{BD}$, $\overline{BD} \perp \overline{DC}$, $\angle A \cong \angle C$
Prove: $\overline{AD} \parallel \overline{CB}$



39. **Write About It** A triangle has angle measures of 115°, 40°, and 25°. Explain how to find the measures of the triangle's exterior angles. Support your answer with a sketch.

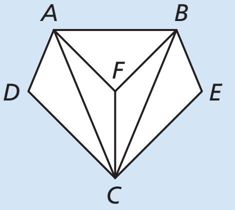
CONCEPT CONNECTION

40. This problem will prepare you for the Concept Connection on page 238.

One of the steps in making an origami crane involves folding a square sheet of paper into the shape shown.

a. $\angle DCE$ is a right angle. $\overline{FC}$ bisects $\angle DCE$, and $\overline{BC}$ bisects $\angle FCE$. Find $m\angle FCB$.

b. Use the Triangle Sum Theorem to find $m\angle CBE$.

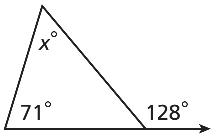


STANDARDIZED TEST PREP

41. What is the value of x?

Ⓐ 19　　Ⓒ 57

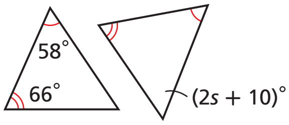
Ⓑ 52　　Ⓓ 71



42. Find the value of s.

Ⓕ 23　　Ⓗ 34

Ⓖ 28　　Ⓙ 56



43. $\angle A$ and $\angle B$ are the remote interior angles of $\angle BCD$ in $\triangle ABC$. Which of these equations must be true?

Ⓐ $m\angle A - 180° = m\angle B$

Ⓑ $m\angle A = 90° - m\angle B$

Ⓒ $m\angle BCD = m\angle BCA - m\angle A$

Ⓓ $m\angle B = m\angle BCD - m\angle A$

44. **Extended Response** The measures of the angles in a triangle are in the ratio 2:3:4. Describe how to use algebra to find the measures of these angles. Then find the measure of each angle and classify the triangle.

CHALLENGE AND EXTEND

45. An exterior angle of a triangle measures 117°. Its remote interior angles measure $(2y^2 + 7)°$ and $(61 - y^2)°$. Find the value of y.

46. Two parallel lines are intersected by a transversal. What type of triangle is formed by the intersection of the angle bisectors of two same-side interior angles? Explain. (*Hint:* Use geometry software or construct a diagram of the angle bisectors of two same-side interior angles.)

47. **Critical Thinking** Explain why an exterior angle of a triangle cannot be congruent to a remote interior angle.

48. **Probability** The measure of each angle in a triangle is a multiple of 30°. What is the probability that the triangle has at least two congruent angles?

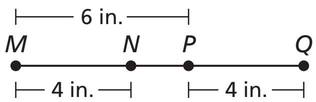
49. In $\triangle ABC$, $m\angle B$ is 5° less than $1\frac{1}{2}$ times $m\angle A$. $m\angle C$ is 5° less than $2\frac{1}{2}$ times $m\angle A$. What is $m\angle A$ in degrees?

SPIRAL REVIEW

Make a table to show the value of each function when x is −2, 0, 1, and 4. *(Previous course)*

50. $f(x) = 3x - 4$　　**51.** $f(x) = x^2 + 1$　　**52.** $f(x) = (x - 3)^2 + 5$

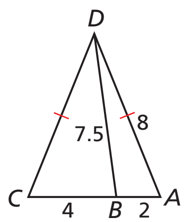
53. Find the length of $\overline{NQ}$. Name the theorem or postulate that justifies your answer. *(Lesson 1-2)*



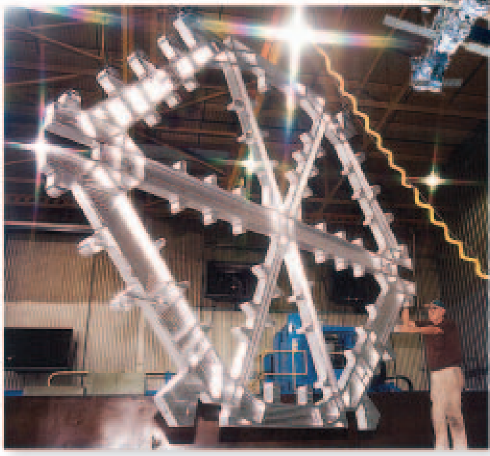
Classify each triangle by its side lengths. *(Lesson 4-1)*

54. $\triangle ACD$　　**55.** $\triangle BCD$　　**56.** $\triangle ABD$

57. **What if...?** If $CA = 8$, What is the effect on the classification of $\triangle ACD$?



4-3 Congruent Triangles



Objectives
Use properties of congruent triangles.

Prove triangles congruent by using the definition of congruence.

Vocabulary
corresponding angles
corresponding sides
congruent polygons

California Standards

5.0 Students prove that triangles are congruent or similar, **and they are able to use the concept of corresponding parts of congruent triangles.**
Also covered: **2.0**

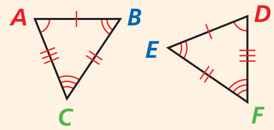
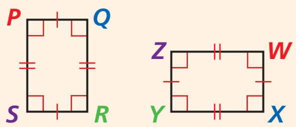
Helpful Hint

Two vertices that are the endpoints of a side are called consecutive vertices. For example, *P* and *Q* are consecutive vertices.

Who uses this?
Machinists used triangles to construct a model of the International Space Station's support structure.

Geometric figures are congruent if they are the same size and shape. **Corresponding angles** and **corresponding sides** are in the same position in polygons with an equal number of sides. Two polygons are **congruent polygons** if and only if their corresponding angles and sides are congruent. Thus triangles that are the same size and shape are congruent.

Properties of Congruent Polygons

DIAGRAM	CORRESPONDING ANGLES	CORRESPONDING SIDES
 $\triangle ABC \cong \triangle DEF$	$\angle A \cong \angle D$ $\angle B \cong \angle E$ $\angle C \cong \angle F$	$\overline{AB} \cong \overline{DE}$ $\overline{BC} \cong \overline{EF}$ $\overline{AC} \cong \overline{DF}$
 polygon $PQRS \cong$ polygon $WXYZ$	$\angle P \cong \angle W$ $\angle Q \cong \angle X$ $\angle R \cong \angle Y$ $\angle S \cong \angle Z$	$\overline{PQ} \cong \overline{WX}$ $\overline{QR} \cong \overline{XY}$ $\overline{RS} \cong \overline{YZ}$ $\overline{PS} \cong \overline{WZ}$

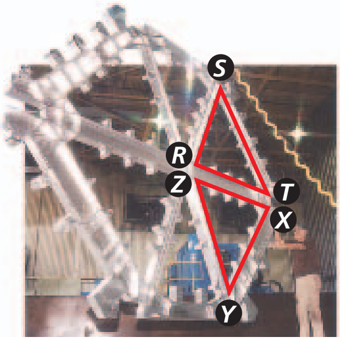
To name a polygon, write the vertices in consecutive order. For example, you can name polygon *PQRS* as *QRSP* or *SRQP*, but **not** as *PRQS*. In a congruence statement, the order of the vertices indicates the corresponding parts.

EXAMPLE 1 Naming Congruent Corresponding Parts

$\triangle RST$ and $\triangle XYZ$ represent the triangles of the space station's support structure. If $\triangle RST \cong \triangle XYZ$, identify all pairs of congruent corresponding parts.

Angles: $\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\angle T \cong \angle Z$

Sides: $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, $\overline{RT} \cong \overline{XZ}$

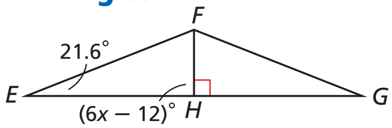


CHECK IT OUT! 1. If polygon *LMNP* ≅ polygon *EFGH*, identify all pairs of corresponding congruent parts.

EXAMPLE 2 Using Corresponding Parts of Congruent Triangles

Algebra

Given: $\triangle EFH \cong \triangle GFH$



A **Find the value of x.**

$\angle FHE$ and $\angle FHG$ are rt. $\angle$s.	*Def. of ⊥ lines*
$\angle FHE \cong \angle FHG$	*Rt. ∠ ≅ Thm.*
$m\angle FHE = m\angle FHG$	*Def. of ≅ ∠s*
$(6x - 12)^\circ = 90^\circ$	*Substitute values for m∠FHE and m∠FHG.*
$6x = 102$	*Add 12 to both sides.*
$x = 17$	*Divide both sides by 6.*

B **Find m∠GFH.**

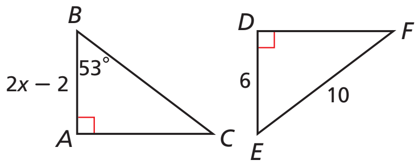
$m\angle EFH + m\angle FHE + m\angle E = 180^\circ$	*△ Sum Thm.*
$m\angle EFH + 90 + 21.6 = 180$	*Substitute values for m∠FHE and m∠E.*
$m\angle EFH + 111.6 = 180$	*Simplify.*
$m\angle EFH = 68.4$	*Subtract 111.6 from both sides.*
$\angle GFH \cong \angle EFH$	*Corr. ∠s of ≅ △s are ≅.*
$m\angle GFH = m\angle EFH$	*Def. of ≅ ∠s*
$m\angle GFH = 68.4^\circ$	*Trans. Prop. of =*

> **Helpful Hint**
>
> When you write a statement such as $\triangle ABC \cong \triangle DEF$, you are also stating which parts are congruent.

CHECK IT OUT! **Given:** $\triangle ABC \cong \triangle DEF$

2a. Find the value of x.

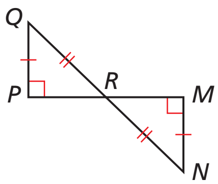
2b. Find m∠F.



EXAMPLE 3 Proving Triangles Congruent

Given: ∠P and ∠M are right angles.
R is the midpoint of $\overline{PM}$.
$\overline{PQ} \cong \overline{MN}$, $\overline{QR} \cong \overline{NR}$

Prove: $\triangle PQR \cong \triangle MNR$

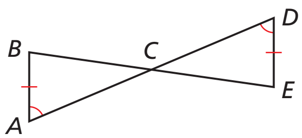


Proof:

Statements	Reasons
1. ∠P and ∠M are rt. ∠s	**1.** Given
2. ∠P ≅ ∠M	**2.** Rt. ∠ ≅ Thm.
3. ∠PRQ ≅ ∠MRN	**3.** Vert. ∠s Thm.
4. ∠Q ≅ ∠N	**4.** Third ∠s Thm.
5. R is the mdpt. of $\overline{PM}$.	**5.** Given
6. $\overline{PR} \cong \overline{MR}$	**6.** Def. of mdpt.
7. $\overline{PQ} \cong \overline{MN}$; $\overline{QR} \cong \overline{NR}$	**7.** Given
8. $\triangle PQR \cong \triangle MNR$	**8.** Def. of ≅ △s

CHECK IT OUT! **3. Given:** $\overline{AD}$ bisects $\overline{BE}$.
$\overline{BE}$ bisects $\overline{AD}$.
$\overline{AB} \cong \overline{DE}$, ∠A ≅ ∠D

Prove: $\triangle ABC \cong \triangle DEC$

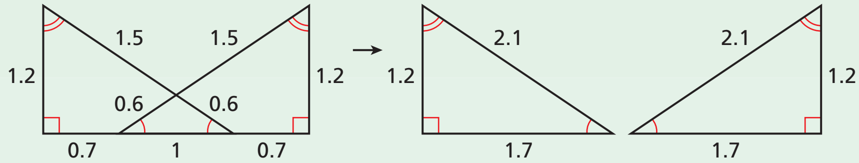


Student to Student

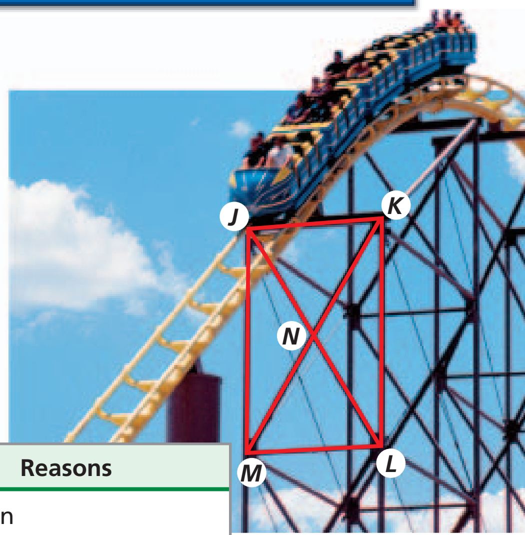
Overlapping Triangles

Cecelia Medina
Lamar High School

"With overlapping triangles, it helps me to redraw the triangles separately. That way I can mark what I know about one triangle without getting confused by the other one."



EXAMPLE 4 *Engineering Application*



The bars that give structural support to a roller coaster form triangles. Since the angle measures and the lengths of the corresponding sides are the same, the triangles are congruent.

Given: $\overline{JK} \perp \overline{KL}$, $\overline{ML} \perp \overline{KL}$, $\angle KLJ \cong \angle LKM$, $\overline{JK} \cong \overline{ML}$, $\overline{JL} \cong \overline{MK}$

Prove: $\triangle JKL \cong \triangle MLK$

Proof:

Statements	Reasons
1. $\overline{JK} \perp \overline{KL}$, $\overline{ML} \perp \overline{KL}$	**1.** Given
2. $\angle JKL$ and $\angle MLK$ are rt. ∠s.	**2.** Def. of ⊥ lines
3. $\angle JKL \cong \angle MLK$	**3.** Rt. ∠ ≅ Thm.
4. $\angle KLJ \cong \angle LKM$	**4.** Given
5. $\angle KJL \cong \angle LMK$	**5.** Third ∠s Thm.
6. $\overline{JK} \cong \overline{ML}$, $\overline{JL} \cong \overline{MK}$	**6.** Given
7. $\overline{KL} \cong \overline{LK}$	**7.** Reflex. Prop. of ≅
8. $\triangle JKL \cong \triangle MLK$	**8.** Def. of ≅ △s

CHECK IT OUT!

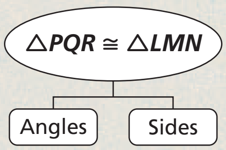
4. Use the diagram to prove the following.
Given: $\overline{MK}$ bisects $\overline{JL}$. $\overline{JL}$ bisects $\overline{MK}$. $\overline{JK} \cong \overline{ML}$, $\overline{JK} \parallel \overline{ML}$
Prove: $\triangle JKN \cong \triangle LMN$

THINK AND DISCUSS

1. A roof truss is a triangular structure that supports a roof. How can you be sure that two roof trusses are the same size and shape?

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, name the congruent corresponding parts.



4-3 Exercises

California Standards
2.0, 5.0, 12.0, 7AF1.0, 7AF4.1, 7MG3.4, 1A2.0, 1A5.0

go.hrw.com
Homework Help Online
KEYWORD: MG7 4-3
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. An everyday meaning of *corresponding* is "matching." How can this help you find the *corresponding* parts of two triangles?

2. If $\triangle ABC \cong \triangle RST$, what angle corresponds to $\angle S$?

SEE EXAMPLE 1
p. 231

Given: $\triangle RST \cong \triangle LMN$. Identify the congruent corresponding parts.

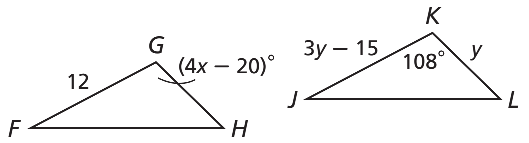
3. $\overline{RS} \cong$ ___?___ 4. $\overline{LN} \cong$ ___?___ 5. $\angle S \cong$ ___?___

6. $\overline{TS} \cong$ ___?___ 7. $\angle L \cong$ ___?___ 8. $\angle N \cong$ ___?___

SEE EXAMPLE 2
p. 232

Given: $\triangle FGH \cong \triangle JKL$. Find each value.

9. KL 10. x

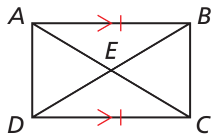


SEE EXAMPLE 3
p. 232

11. **Given:** E is the midpoint of $\overline{AC}$ and $\overline{BD}$. $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$

Prove: $\triangle ABE \cong \triangle CDE$

Proof:



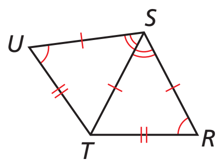
Statements	Reasons
1. $\overline{AB} \parallel \overline{CD}$	1. a. ___?___
2. $\angle ABE \cong \angle CDE$, $\angle BAE \cong \angle DCE$	2. b. ___?___
3. $\overline{AB} \cong \overline{CD}$	3. c. ___?___
4. E is the mdpt. of $\overline{AC}$ and $\overline{BD}$.	4. d. ___?___
5. e. ___?___	5. Def. of mdpt.
6. $\angle AEB \cong \angle CED$	6. f. ___?___
7. $\triangle ABE \cong \triangle CDE$	7. g. ___?___

SEE EXAMPLE 4
p. 233

12. **Engineering** The geodesic dome shown is a 14-story building that models Earth. Use the given information to prove that the triangles that make up the sphere are congruent.

Given: $\overline{SU} \cong \overline{ST} \cong \overline{SR}$, $\overline{TU} \cong \overline{TR}$, $\angle UST \cong \angle RST$, and $\angle U \cong \angle R$

Prove: $\triangle RTS \cong \triangle UTS$



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
13–16	1
17–18	2
19	3
20	4

Extra Practice

Skills Practice p. S10
Application Practice p. S31

Given: Polygon $CDEF \cong$ polygon $KLMN$. Identify the congruent corresponding parts.

13. $\overline{DE} \cong$ __?__

14. $\overline{KN} \cong$ __?__

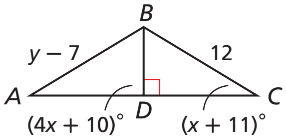
15. $\angle F \cong$ __?__

16. $\angle L \cong$ __?__

Given: $\triangle ABD \cong \triangle CBD$. Find each value.

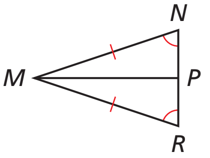
17. $m\angle C$

18. y



19. Given: $\overline{MP}$ bisects $\angle NMR$. P is the midpoint of $\overline{NR}$. $\overline{MN} \cong \overline{MR}$, $\angle N \cong \angle R$

Prove: $\triangle MNP \cong \triangle MRP$



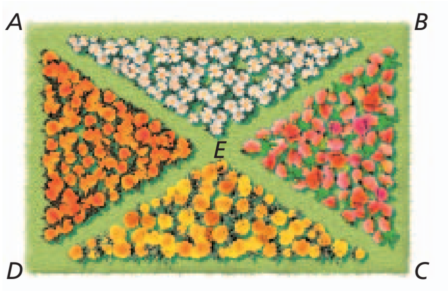
Proof:

Statements	Reasons
1. $\angle N \cong \angle R$	1. a. __?__
2. $\overline{MP}$ bisects $\angle NMR$.	2. b. __?__
3. c. __?__	3. Def. of $\angle$ bisector
4. d. __?__	4. Third $\triangle$s Thm.
5. P is the mdpt. of $\overline{NR}$.	5. e. __?__
6. f. __?__	6. Def. of mdpt.
7. $\overline{MN} \cong \overline{MR}$	7. g. __?__
8. $\overline{MP} \cong \overline{MP}$	8. h. __?__
9. $\triangle MNP \cong \triangle MRP$	9. Def. of $\cong$ $\triangle$s

20. Hobbies In a garden, triangular flower beds are separated by straight rows of grass as shown.

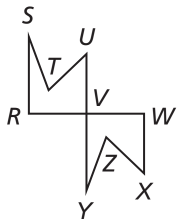
Given: $\angle ADC$ and $\angle BCD$ are right angles. $\overline{AC} \cong \overline{BD}$, $\overline{AD} \cong \overline{BC}$ $\angle DAC \cong \angle CBD$

Prove: $\triangle ADC \cong \triangle BCD$



21. For two triangles, the following corresponding parts are given: $\overline{GS} \cong \overline{KP}$, $\overline{GR} \cong \overline{KH}$, $\overline{SR} \cong \overline{PH}$, $\angle S \cong \angle P$, $\angle G \cong \angle K$, and $\angle R \cong \angle H$. Write three different congruence statements.

22. The two polygons in the diagram are congruent. Complete the following congruence statement for the polygons. polygon R __?__ $\cong$ polygon V __?__



Write and solve an equation for each of the following.

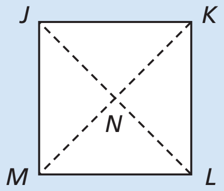
23. $\triangle ABC \cong \triangle DEF$. $AB = 2x - 10$, and $DE = x + 20$. Find the value of x and AB.

24. $\triangle JKL \cong \triangle MNP$. $m\angle L = (x^2 + 10)^\circ$, and $m\angle P = (2x^2 + 1)^\circ$. What is $m\angle L$?

25. Polygon $ABCD \cong$ polygon $PQRS$. $BC = 6x + 5$, and $QR = 5x + 7$. Find the value of x and BC.

CONCEPT CONNECTION

26. This problem will prepare you for the Concept Connection on page 238. Many origami models begin with a square piece of paper, *JKLM*, that is folded along both diagonals to make the creases shown. $\overline{JL}$ and $\overline{MK}$ are perpendicular bisectors of each other, and $\angle NML \cong \angle NKL$.



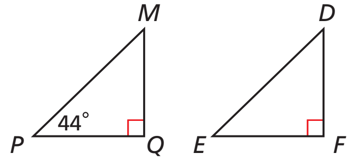
a. Explain how you know that $\overline{KL}$ and $\overline{ML}$ are congruent.

b. Prove $\triangle NML \cong \triangle NKL$.

27. Draw a diagram and then write a proof.
Given: $\overline{BD} \perp \overline{AC}$. *D* is the midpoint of $\overline{AC}$. $\overline{AB} \cong \overline{CB}$, and $\overline{BD}$ bisects $\angle ABC$.
Prove: $\triangle ABD \cong \triangle CBD$

28. **Critical Thinking** Draw two triangles that are not congruent but have an area of 4 cm^2 each.

29. **///ERROR ANALYSIS///** Given $\triangle MPQ \cong \triangle EDF$. Two solutions for finding $m\angle E$ are shown. Which is incorrect? Explain the error.



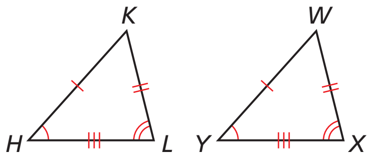
A

Since corr. parts of ≅ △s are ≅, $\angle E \cong \angle P$. So $m\angle E = m\angle P = 44°$.

B

Since the acute ∠s of a rt. △ are comp., $m\angle M = 46°$. $\angle E \cong \angle M$, so $m\angle E = 46°$.

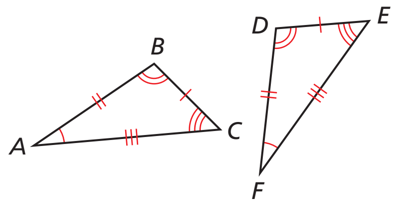
30. **Write About It** Given the diagram of the triangles, is there enough information to prove that $\triangle HKL$ is congruent to $\triangle YWX$? Explain.



STANDARDIZED TEST PREP

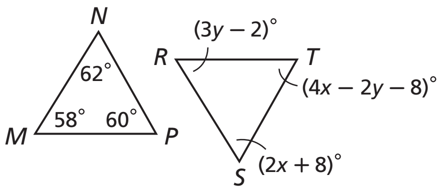
31. Which congruence statement correctly indicates that the two given triangles are congruent?

Ⓐ $\triangle ABC \cong \triangle EFD$ Ⓒ $\triangle ABC \cong \triangle DEF$
Ⓑ $\triangle ABC \cong \triangle FDE$ Ⓓ $\triangle ABC \cong \triangle FED$



32. $\triangle MNP \cong \triangle RST$. What are the values of *x* and *y*?

Ⓕ $x = 26, y = 21\frac{1}{3}$ Ⓗ $x = 25, y = 20\frac{2}{3}$
Ⓖ $x = 27, y = 20$ Ⓙ $x = 30\frac{1}{3}, y = 16\frac{2}{3}$

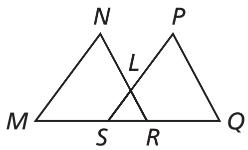


33. $\triangle ABC \cong \triangle XYZ$. $m\angle A = 47.1°$, and $m\angle C = 13.8°$. Find $m\angle Y$.

Ⓐ 13.8   Ⓒ 76.2
Ⓑ 42.9   Ⓓ 119.1

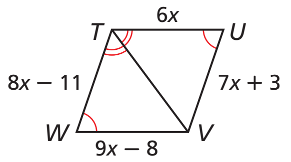
34. $\triangle MNR \cong \triangle SPQ$, $NL = 18$, $SP = 33$, $SR = 10$, $RQ = 24$, and $QP = 30$. What is the perimeter of $\triangle MNR$?

Ⓕ 79   Ⓗ 87
Ⓖ 85   Ⓙ 97



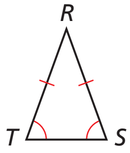
CHALLENGE AND EXTEND

35. **Multi-Step** Given that the perimeter of *TUVW* is 149 units, find the value of x. Is $\triangle TUV \cong \triangle TWV$? Explain.



36. **Multi-Step** Polygon $ABCD \cong$ polygon $EFGH$. $\angle A$ is a right angle. $m\angle E = (y^2 - 10)^\circ$, and $m\angle H = (2y^2 - 132)^\circ$. Find $m\angle D$.

37. **Given:** $\overline{RS} \cong \overline{RT}$, $\angle S \cong \angle T$
Prove: $\triangle RST \cong \triangle RTS$



SPIRAL REVIEW

Two number cubes are rolled. Find the probability of each outcome. *(Previous course)*

38. Both numbers rolled are even.

39. The sum of the numbers rolled is 5.

Classify each angle by its measure. *(Lesson 1-3)*

40. $m\angle DOC = 40°$

41. $m\angle BOA = 90°$

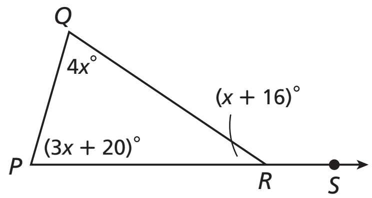
42. $m\angle COA = 140°$

Find each angle measure. *(Lesson 4-2)*

43. $\angle Q$

44. $\angle P$

45. $\angle QRS$



Career Path

go.hrw.com
Career Resources Online
KEYWORD: MG7 Career

Jordan Carter
Emergency Medical Services Program

Q: What math classes did you take in high school?

A: Algebra 1 and 2, Geometry, Precalculus

Q: What kind of degree or certification will you receive?

A: I will receive an associate's degree in applied science. Then I will take an exam to be certified as an EMT or paramedic.

Q: How do you use math in your hands-on training?

A: I calculate dosages based on body weight and age. I also calculate drug doses in milligrams per kilogram per hour or set up an IV drip to deliver medications at the correct rate.

Q: What are your future career plans?

A: When I am certified, I can work for a private ambulance service or with a fire department. I could also work in a hospital, transporting critically ill patients by ambulance or helicopter.

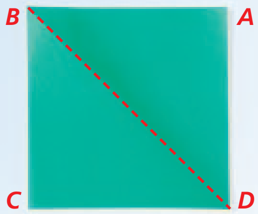
CONCEPT CONNECTION

Triangles and Congruence

Origami Origami is the Japanese art of paper folding. The Japanese word *origami* literally means "fold paper." This ancient art form relies on properties of geometry to produce fascinating and beautiful shapes.

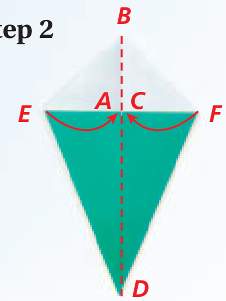
Each of the figures shows a step in making an origami swan from a square piece of paper. The final figure shows the creases of an origami swan that has been unfolded.

Step 1



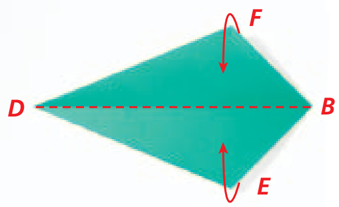
Fold the paper in half diagonally and crease it. Turn it over.

Step 2



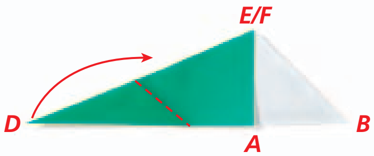
Fold corners *A* and *C* to the center line and crease. Turn it over.

Step 3



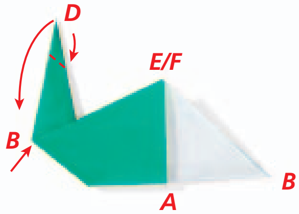
Fold in half along the center crease so that $\overline{DE}$ and $\overline{DF}$ are together.

Step 4



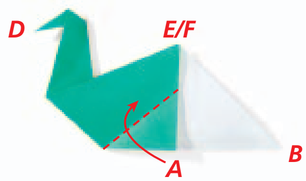
Fold the narrow point upward at a 90° angle and crease. Push in the fold so that the neck is inside the body.

Step 5

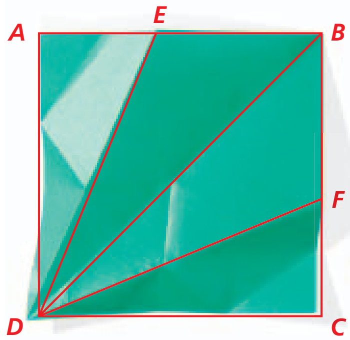


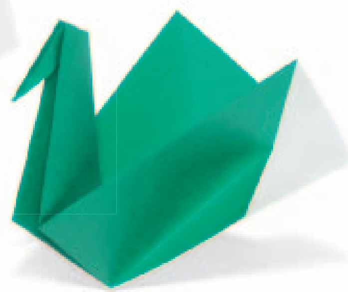
Fold the tip downward and crease. Push in the fold so that the head is inside the neck.

Step 6



Fold up the flap to form the wing.





1. Use the fact that *ABCD* is a square to classify $\triangle ABD$ by its side lengths and by its angle measures.
2. $\overline{DB}$ bisects $\angle ABC$ and $\angle ADC$. $\overline{DE}$ bisects $\angle ADB$. Find the measures of the angles in $\triangle EDB$. Explain how you found the measures.
3. Given that $\overline{DB}$ bisects $\angle ABC$ and $\angle EDF$, $\overline{BE} \cong \overline{BF}$, and $\overline{DE} \cong \overline{DF}$, prove that $\triangle EDB \cong \triangle FDB$.

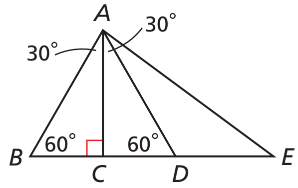
READY TO GO ON?

Quiz for Lessons 4-1 Through 4-3

4-1 Classifying Triangles

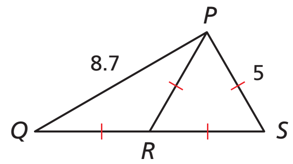
Classify each triangle by its angle measures.

1. $\triangle ACD$ 2. $\triangle ABD$ 3. $\triangle ADE$



Classify each triangle by its side lengths.

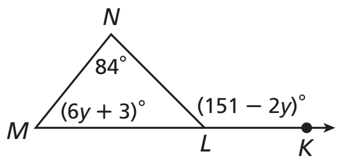
4. $\triangle PQR$ 5. $\triangle PRS$ 6. $\triangle PQS$



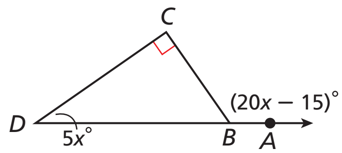
4-2 Angle Relationships in Triangles

Find each angle measure.

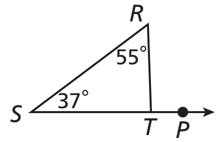
7. $m\angle M$



8. $m\angle ABC$



9. A carpenter built a triangular support structure for a roof. Two of the angles of the structure measure 37° and 55°. Find the measure of $\angle RTP$, the angle formed by the roof of the house and the roof of the patio.



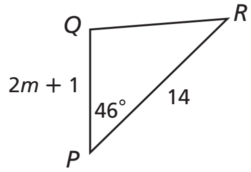
4-3 Congruent Triangles

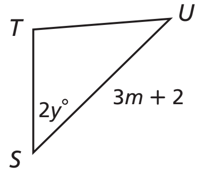
Given: $\triangle JKL \cong \triangle DEF$. Identify the congruent corresponding parts.

10. $\overline{KL} \cong$ __?__ 11. $\overline{DF} \cong$ __?__ 12. $\angle K \cong$ __?__ 13. $\angle F \cong$ __?__

Given: $\triangle PQR \cong \triangle STU$. Find each value.

14. PQ 15. y



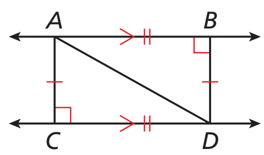


16. Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overline{AB} \cong \overline{CD}$, $\overline{AC} \cong \overline{BD}$, $\overline{AC} \perp \overline{CD}$, $\overline{DB} \perp \overline{AB}$

Prove: $\triangle ACD \cong \triangle DBA$

Proof:

Statements	Reasons
1. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	1. a. __?__
2. $\angle BAD \cong \angle CDA$	2. b. __?__
3. $\overline{AC} \perp \overline{CD}$, $\overline{DB} \perp \overline{AB}$	3. c. __?__
4. $\angle ACD$ and $\angle DBA$ are rt. ∆	4. d. __?__
5. e. __?__	5. Rt. ∠ ≅ Thm.
6. f. __?__	6. Third ∆ Thm.
7. $\overline{AB} \cong \overline{CD}$, $\overline{AC} \cong \overline{BD}$	7. g. __?__
8. h. __?__	8. Reflex Prop. of ≅
9. $\triangle ACD \cong \triangle DBA$	9. i. __?__



4-4 Geometry LAB

Use with Lesson 4-4

Explore SSS and SAS Triangle Congruence

In Lesson 4-3, you used the definition of congruent triangles to prove triangles congruent. To use the definition, you need to prove that all three pairs of corresponding sides and all three pairs of corresponding angles are congruent.

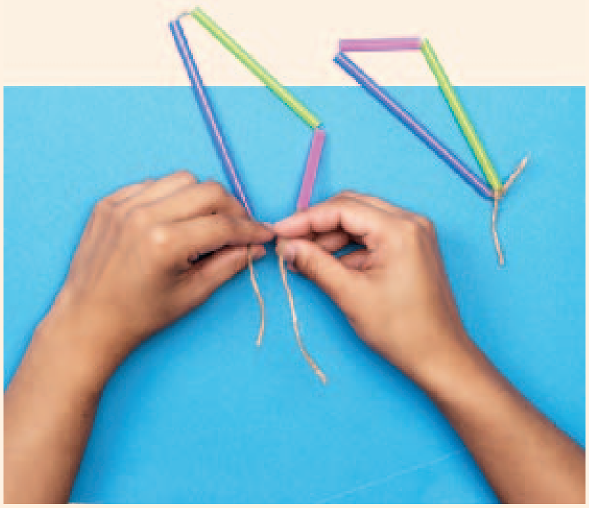
In this lab, you will discover some shortcuts for proving triangles congruent.

California Standards

1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and **inductive** and deductive **reasoning.**

Activity 1

1. Measure and cut six pieces from the straws: two that are 2 inches long, two that are 4 inches long, and two that are 5 inches long.
2. Cut two pieces of string that are each about 20 inches long.
3. Thread one piece of each size of straw onto a piece of string. Tie the ends of the string together so that the pieces of straw form a triangle.
4. Using the remaining pieces, try to make another triangle with the same side lengths that is *not* congruent to the first triangle.



Try This

1. Repeat Activity 1 using side lengths of your choice. Are your results the same?
2. Do you think it is possible to make two triangles that have the same side lengths but that are not congruent? Why or why not?
3. How does your answer to Problem 2 provide a shortcut for proving triangles congruent?
4. Complete the following conjecture based on your results. Two triangles are congruent if ______?______.

Activity 2

1. Measure and cut two pieces from the straws: one that is 4 inches long and one that is 5 inches long.
2. Use a protractor to help you bend a paper clip to form a 30° angle.
3. Place the pieces of straw on the sides of the 30° angle. The straws will form two sides of your triangle.
4. Without changing the angle formed by the paper clip, use a piece of straw to make a third side for your triangle, cutting it to fit as necessary. Use additional paper clips or string to hold the straws together in a triangle.

Try This

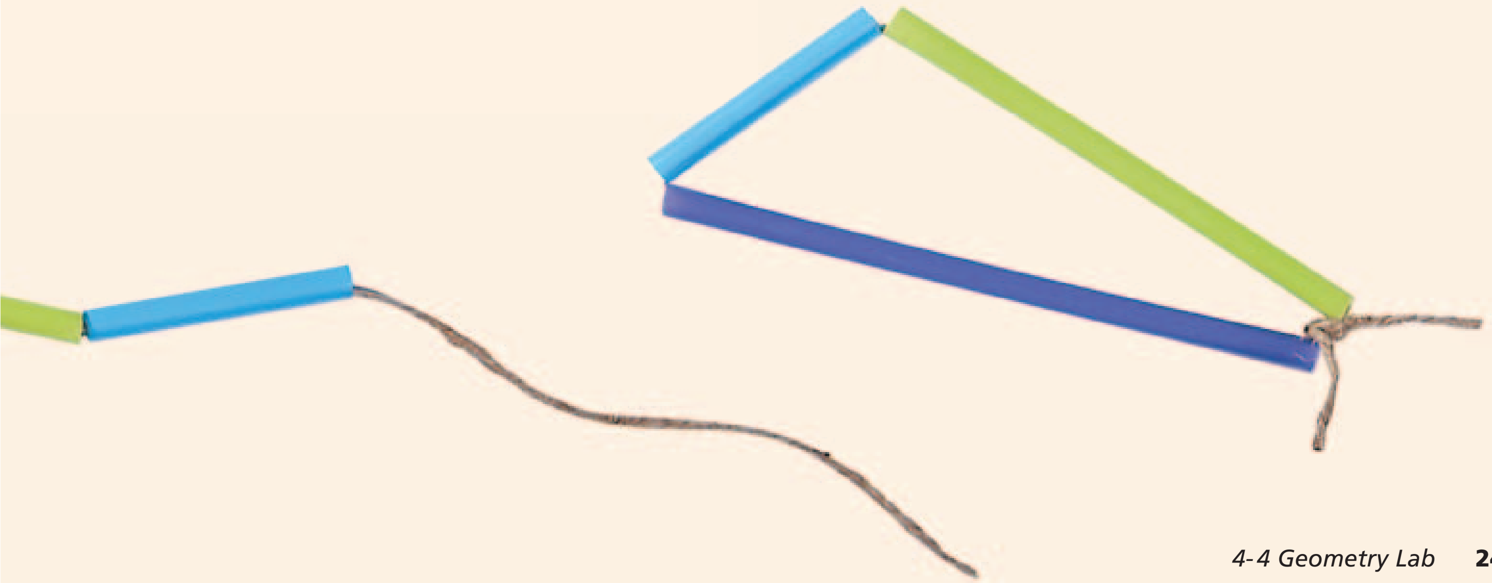
5. Repeat Activity 2 using side lengths and an angle measure of your choice. Are your results the same?

6. Suppose you know two side lengths of a triangle and the measure of the angle between these sides. Can the length of the third side be any measure? Explain.

7. How does your answer to Problem 6 provide a shortcut for proving triangles congruent?

8. Use the two given sides and the given angle from Activity 2 to form a triangle that is not congruent to the triangle you formed. (*Hint:* One of the given sides does not have to be adjacent to the given angle.)

9. Complete the following conjecture based on your results.
Two triangles are congruent if ____?____.



4-4 Triangle Congruence: SSS and SAS

Objectives

Apply SSS and SAS to construct triangles and to solve problems.

Prove triangles congruent by using SSS and SAS.

Vocabulary

triangle rigidity
included angle

California Standards

5.0 Students prove that triangles are congruent or similar, **and they are able to use the concept of corresponding parts of congruent triangles.**

Also covered: **2.0, 16.0**

Who uses this?

Engineers used the property of triangle rigidity to design the internal support for the Statue of Liberty and to build bridges, towers, and other structures. (See Example 2.)

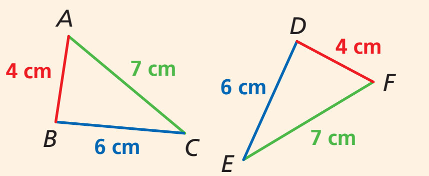
In Lesson 4-3, you proved triangles congruent by showing that all six pairs of corresponding parts were congruent.

The property of **triangle rigidity** gives you a shortcut for proving two triangles congruent. It states that if the side lengths of a triangle are given, the triangle can have only one shape.

For example, you only need to know that two triangles have three pairs of congruent corresponding sides. This can be expressed as the following postulate.

Know it! Note

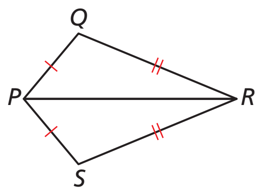
Postulate 4-4-1 Side-Side-Side (SSS) Congruence

POSTULATE	HYPOTHESIS	CONCLUSION
If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.		$\triangle ABC \cong \triangle FDE$

EXAMPLE 1 Using SSS to Prove Triangle Congruence

Use SSS to explain why $\triangle PQR \cong \triangle PSR$.

It is given that $\overline{PQ} \cong \overline{PS}$ and that $\overline{QR} \cong \overline{SR}$. By the Reflexive Property of Congruence, $\overline{PR} \cong \overline{PR}$. Therefore $\triangle PQR \cong \triangle PSR$ by SSS.

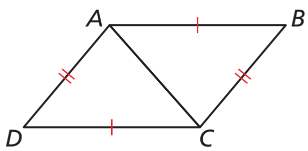


Remember!

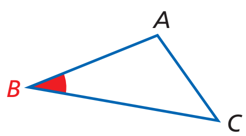
Adjacent triangles share a side, so you can apply the Reflexive Property to get a pair of congruent parts.

CHECK IT OUT!

1. Use SSS to explain why $\triangle ABC \cong \triangle CDA$.



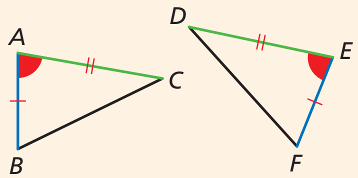
An **included angle** is an angle formed by two adjacent sides of a polygon. $\angle B$ is the included angle between sides $\overline{AB}$ and $\overline{BC}$.



It can also be shown that only two pairs of congruent corresponding sides are needed to prove the congruence of two triangles if the included angles are also congruent.

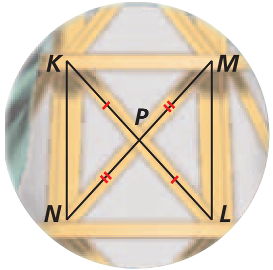
Know it! Note

Postulate 4-4-2 Side-Angle-Side (SAS) Congruence

POSTULATE	HYPOTHESIS	CONCLUSION
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.		$\triangle ABC \cong \triangle EFD$

EXAMPLE 2 *Engineering Application*

The figure shows part of the support structure of the Statue of Liberty. Use SAS to explain why $\triangle KPN \cong \triangle LPM$.



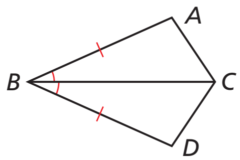
It is given that $\overline{KP} \cong \overline{LP}$ and that $\overline{NP} \cong \overline{MP}$. By the Vertical Angles Theorem, $\angle KPN \cong \angle LPM$. Therefore $\triangle KPN \cong \triangle LPM$ by SAS.

Caution!

The letters SAS are written in that order because the congruent angles must be between pairs of congruent corresponding sides.

CHECK IT OUT!

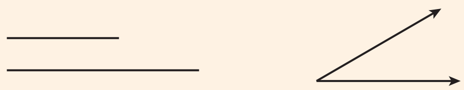
2. Use SAS to explain why $\triangle ABC \cong \triangle DBC$.

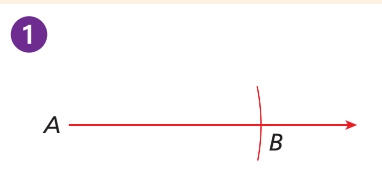


The SAS Postulate guarantees that if you are given the lengths of two sides and the measure of the included angle, you can construct one and only one triangle.

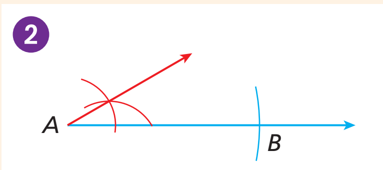
Construction Congruent Triangles Using SAS

Use a straightedge to draw two segments and one angle, or copy the given segments and angle.

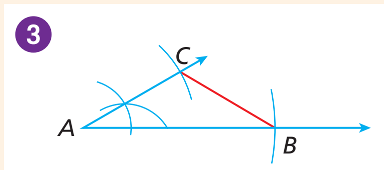




Construct $\overline{AB}$ congruent to one of the segments.



Construct $\angle A$ congruent to the given angle.



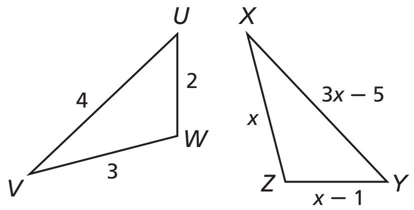
Construct $\overline{AC}$ congruent to the other segment. Draw $\overline{CB}$ to complete $\triangle ABC$.

EXAMPLE 3 Verifying Triangle Congruence

Algebra

Show that the triangles are congruent for the given value of the variable.

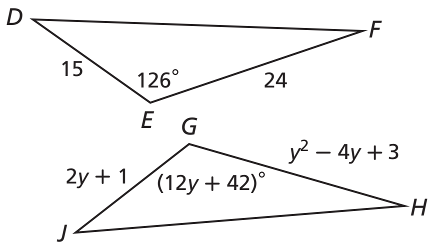
A $\triangle UVW \cong \triangle YXW, x = 3$



$ZY = x - 1$
$= 3 - 1 = 2$
$XZ = x = 3$
$XY = 3x - 5$
$= 3(3) - 5 = 4$

$\overline{UV} \cong \overline{YX}$. $\overline{VW} \cong \overline{XZ}$, and $\overline{UW} \cong \overline{YZ}$.
So $\triangle UVW \cong \triangle YXZ$ by SSS.

B $\triangle DEF \cong \triangle JGH, y = 7$

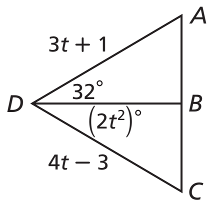


$JG = 2y + 1$
$= 2(7) + 1$
$= 15$
$GH = y^2 - 4y + 3$
$= (7)^2 - 4(7) + 3$
$= 24$
$m\angle G = 12y + 42$
$= 12(7) + 42$
$= 126°$

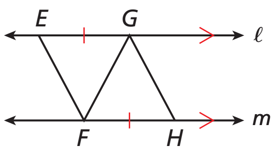
$\overline{DE} \cong \overline{JG}$. $\overline{EF} \cong \overline{GH}$, and $\angle E \cong \angle G$.
So $\triangle DEF \cong \triangle JGH$ by SAS.

CHECK IT OUT!

3. Show that $\triangle ADB \cong \triangle CDB$ when $t = 4$.



EXAMPLE 4 Proving Triangles Congruent

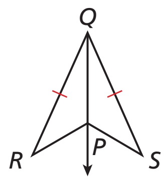


Given: $\ell \parallel m$, $\overline{EG} \cong \overline{HF}$
Prove: $\triangle EGF \cong \triangle HFG$
Proof:

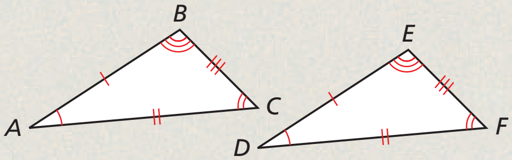
Statements	Reasons
1. $\overline{EG} \cong \overline{HF}$	**1.** Given
2. $\ell \parallel m$	**2.** Given
3. $\angle EGF \cong \angle HFG$	**3.** Alt. Int. ∠s Thm.
4. $\overline{FG} \cong \overline{GF}$	**4.** Reflex Prop. of ≅
5. $\triangle EGF \cong \triangle HFG$	**5.** SAS *Steps 1, 3, 4*

CHECK IT OUT!

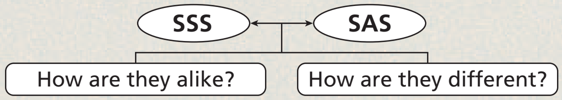
4. **Given:** $\overrightarrow{QP}$ bisects $\angle RQS$. $\overline{QR} \cong \overline{QS}$
Prove: $\triangle RQP \cong \triangle SQP$



THINK AND DISCUSS



1. Describe three ways you could prove that $\triangle ABC \cong \triangle DEF$.
2. Explain why the SSS and SAS Postulates are shortcuts for proving triangles congruent.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Use it to compare the SSS and SAS postulates.



4-4 Exercises

California Standards
2.0, 5.0, 16.0, 7AF4.1, 7MG3.3, 7MG3.4, 7MR1.0, 7MR1.1, 1A2.0, 1A4.0, 1A5.0

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Homework Help Online
KEYWORD: MG7 4-4
Parent Resources Online
KEYWORD: MG7 Parent

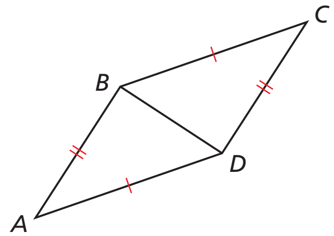
GUIDED PRACTICE

1. **Vocabulary** In $\triangle RST$ which angle is the included angle of sides $\overline{ST}$ and $\overline{TR}$?

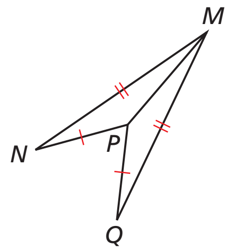
SEE EXAMPLE 1
p. 242

Use SSS to explain why the triangles in each pair are congruent.

2. $\triangle ABD \cong \triangle CDB$

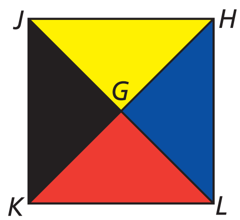


3. $\triangle MNP \cong \triangle MQP$



SEE EXAMPLE 2
p. 243

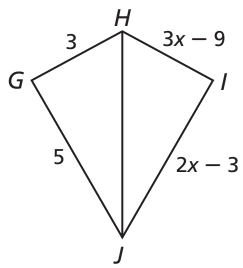
4. **Sailing** Signal flags are used to communicate messages when radio silence is required. The Zulu signal flag means, "I require a tug." $GJ = GH = GL = GK = 20$ in. Use SAS to explain why $\triangle JGK \cong \triangle LGH$.



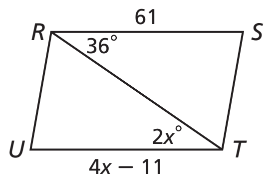
SEE EXAMPLE 3
p. 244

Show that the triangles are congruent for the given value of the variable.

5. $\triangle GHJ \cong \triangle IHJ$, $x = 4$

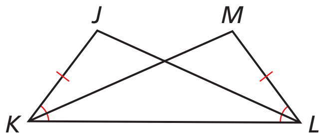


6. $\triangle RST \cong \triangle TUR$, $x = 18$



SEE EXAMPLE 4
p. 244

7. **Given:** $\overline{JK} \cong \overline{ML}$, $\angle JKL \cong \angle MLK$
Prove: $\triangle JKL \cong \triangle MLK$



Proof:

Statements	Reasons
1. $\overline{JK} \cong \overline{ML}$	1. a. __?__
2. b. __?__	2. Given
3. $\overline{KL} \cong \overline{LK}$	3. c. __?__
4. $\triangle JKL \cong \triangle MLK$	4. d. __?__

PRACTICE AND PROBLEM SOLVING

Independent Practice

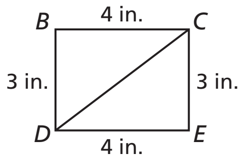
For Exercises	See Example
8–9	1
10	2
11–12	3
13	4

Extra Practice

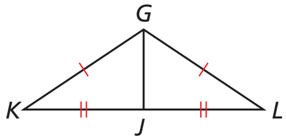
Skills Practice p. S11
Application Practice p. S31

Use SSS to explain why the triangles in each pair are congruent.

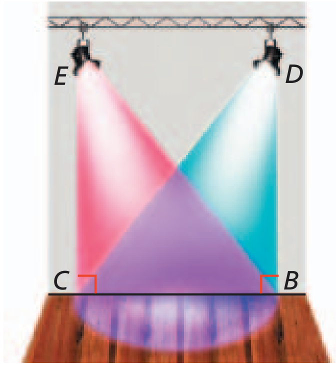
8. $\triangle BCD \cong \triangle EDC$



9. $\triangle GJK \cong \triangle GJL$

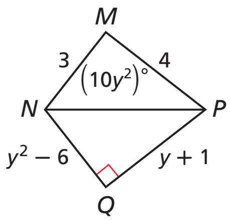


10. **Theater** The lights shining on a stage appear to form two congruent right triangles. Given $\overline{EC} \cong \overline{DB}$, use SAS to explain why $\triangle ECB \cong \triangle DBC$.

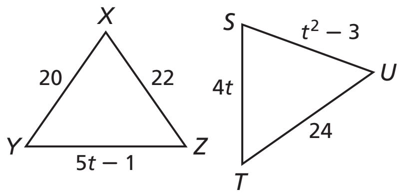


Show that the triangles are congruent for the given value of the variable.

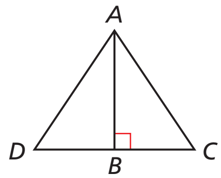
11. $\triangle MNP \cong \triangle QNP$, $y = 3$



12. $\triangle XYZ \cong \triangle STU$, $t = 5$



13. **Given:** B is the midpoint of $\overline{DC}$. $\overline{AB} \perp \overline{DC}$
Prove: $\triangle ABD \cong \triangle ABC$

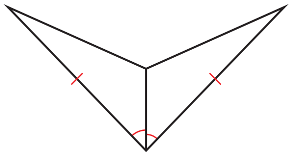


Proof:

Statements	Reasons
1. B is the mdpt. of $\overline{DC}$.	1. a. __?__
2. b. __?__	2. Def. of mdpt.
3. c. __?__	3. Given
4. $\angle ABD$ and $\angle ABC$ are rt. ⦜.	4. d. __?__
5. $\angle ABD \cong \angle ABC$	5. e. __?__
6. f. __?__	6. Reflex. Prop. of ≅
7. $\triangle ABD \cong \triangle ABC$	7. g. __?__

Which postulate, if any, can be used to prove the triangles congruent?

14.



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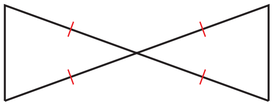
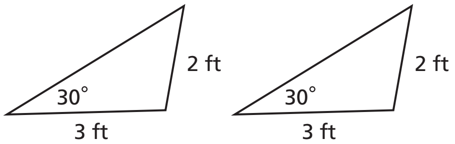
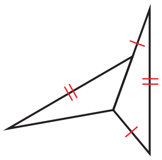


16.

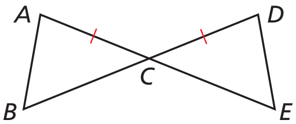


17.



18. Explain what additional information, if any, you would need to prove $\triangle ABC \cong \triangle DEC$ by each postulate.

a. SSS **b.** SAS

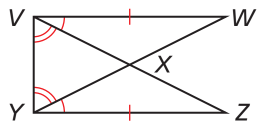


Multi-Step **Graph each triangle. Then use the Distance Formula and the SSS Postulate to determine whether the triangles are congruent.**

19. $\triangle QRS$ and $\triangle TUV$
$Q(-2, 0)$, $R(1, -2)$, $S(-3, -2)$
$T(5, 1)$, $U(3, -2)$, $V(3, 2)$

20. $\triangle ABC$ and $\triangle DEF$
$A(2, 3)$, $B(3, -1)$, $C(7, 2)$
$D(-3, 1)$, $E(1, 2)$, $F(-3, 5)$

21. Given: $\angle ZVY \cong \angle WYV$, $\angle ZVW \cong \angle WYZ$, $\overline{VW} \cong \overline{YZ}$
Prove: $\triangle ZVY \cong \triangle WYV$
Proof:



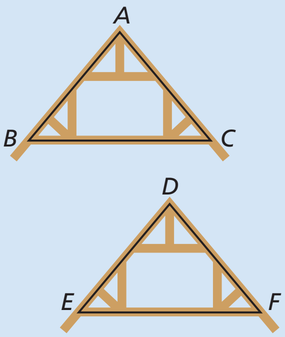
Statements	Reasons
1. $\angle ZVY \cong \angle WYV$, $\angle ZVW \cong WYZ$	1. a. __?__
2. $m\angle ZVY = m\angle WYV$, $m\angle ZVW = m\angle WYZ$	2. b. __?__
3. $m\angle ZVY + m\angle ZVW = m\angle WYV + m\angle WYZ$	3. Add. Prop. of =
4. c. __?__	4. ∠ Add. Post.
5. $\angle WVY \cong \angle ZYV$	5. d. __?__
6. $\overline{VW} \cong \overline{YZ}$	6. e. __?__
7. f. __?__	7. Reflex. Prop. of ≅
8. $\triangle ZVY \cong \triangle WYV$	8. g. __?__

Concept Connection

22. This problem will prepare you for the Concept Connection on page 280. The diagram shows two triangular trusses that were built for the roof of a doghouse.

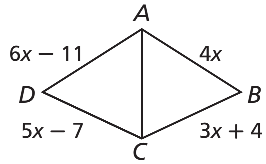
a. You can use a protractor to check that $\angle A$ and $\angle D$ are right angles. Explain how you could make just two additional measurements on each truss to ensure that the trusses are congruent.

b. You verify that the trusses are congruent and find that $AB = AC = 2.5$ ft. Find the length of $\overline{EF}$ to the nearest tenth. Explain.



23. **Critical Thinking** Draw two isosceles triangles that are not congruent but that have a perimeter of 15 cm each.

24. $\triangle ABC \cong \triangle ADC$ for what value of x? Explain why the SSS Postulate can be used to prove the two triangles congruent.

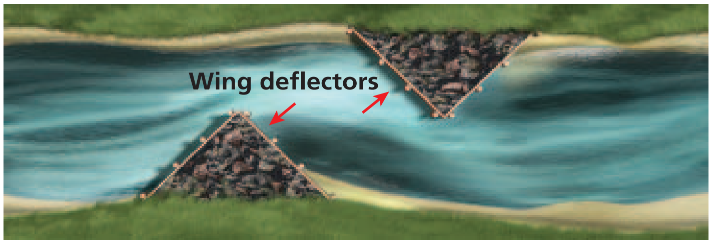


LINK

Ecology

Wing deflectors are designed to reduce the width-to-depth ratio of a stream. Reducing the width increases the velocity of the stream.

25. **Ecology** A *wing deflector* is a triangular structure made of logs that is filled with large rocks and placed in a stream to guide the current or prevent erosion. Wing deflectors are often used in pairs. Suppose an engineer wants to build two wing deflectors. The logs that form the sides of each wing deflector are perpendicular. How can the engineer make sure that the two wing deflectors are congruent?

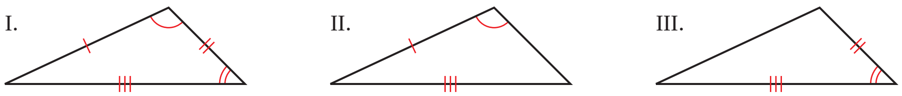


26. **Write About It** If you use the same two sides and included angle to repeat the construction of a triangle, are your two constructed triangles congruent? Explain.

27. **Construction** Use three segments (SSS) to construct a scalene triangle. Suppose you then use the same segments in a different order to construct a second triangle. Will the result be the same? Explain.

STANDARDIZED TEST PREP

28. Which of the three triangles below can be proven congruent by SSS or SAS?

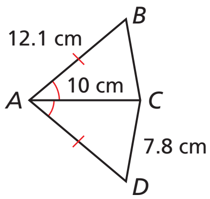


Ⓐ I and II Ⓑ II and III Ⓒ I and III Ⓓ I, II, and III

29. What is the perimeter of polygon *ABCD*?

Ⓕ 29.9 cm Ⓗ 49.8 cm

Ⓖ 39.8 cm Ⓙ 59.8 cm



30. Jacob wants to prove that $\triangle FGH \cong \triangle JKL$ using SAS. He knows that $\overline{FG} \cong \overline{JK}$ and $\overline{FH} \cong \overline{JL}$. What additional piece of information does he need?

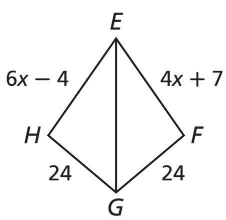
Ⓐ $\angle F \cong \angle J$ Ⓒ $\angle H \cong \angle L$

Ⓑ $\angle G \cong \angle K$ Ⓓ $\angle F \cong \angle G$

31. What must the value of x be in order to prove that $\triangle EFG \cong \triangle EHG$ by SSS?

Ⓕ 1.5 Ⓗ 4.67

Ⓖ 4.25 Ⓙ 5.5

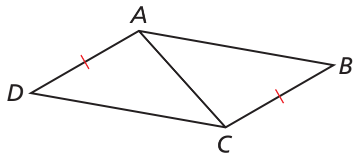


CHALLENGE AND EXTEND

32. Given:. $\angle ADC$ and $\angle BCD$ are supplementary. $\overline{AD} \cong \overline{CB}$

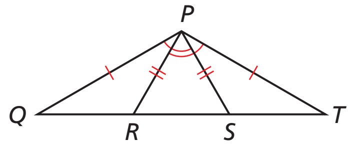
Prove: $\triangle ADB \cong \triangle CBD$

(*Hint:* Draw an auxiliary line.)

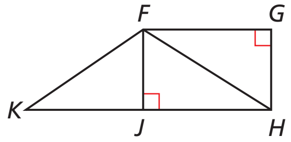


33. Given: $\angle QPS \cong \angle TPR$, $\overline{PQ} \cong \overline{PT}$, $\overline{PR} \cong \overline{PS}$

Prove: $\triangle PQR \cong \triangle PTS$



Algebra **Use the following information for Exercises 34 and 35.** Find the value of x. Then use SSS or SAS to write a paragraph proof showing that two of the triangles are congruent.



34. $m\angle FKJ = 2x°$
$m\angle KFJ = (3x + 10)°$
$KJ = 4x + 8$
$HJ = 6(x - 4)$

35. $\overline{FJ}$ bisects $\angle KFH$.
$m\angle KFJ = (2x + 6)°$
$m\angle HFJ = (3x - 21)°$
$FK = 8x - 45$
$FH = 6x + 9$

SPIRAL REVIEW

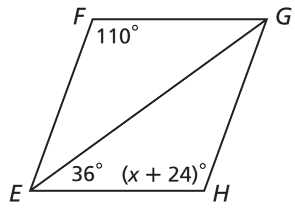
Solve and graph each inequality. *(Previous course)*

36. $\frac{x}{2} - 8 \leq 5$ **37.** $2a + 4 > 3a$ **38.** $-6m - 1 \leq -13$

Solve each equation. Write a justification for each step. *(Lesson 2-5)*

39. $4x - 7 = 21$ **40.** $\frac{a}{4} + 5 = -8$ **41.** $6r = 4r + 10$

Given: $\triangle EFG \cong \triangle GHE$. Find each value. *(Lesson 4-3)*



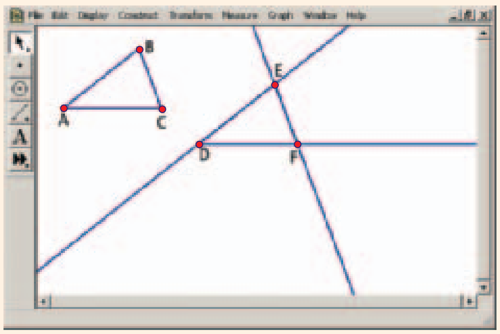
42. x

43. $m\angle FEG$

44. $m\angle FGH$

Using Technology

Use geometry software to complete the following.



1. Draw a triangle and label the vertices A, B, and C. Draw a point and label it D. Mark a vector from A to B and translate D by the marked vector. Label the image E. Draw $\overleftrightarrow{DE}$. Mark $\angle BAC$ and rotate $\overleftrightarrow{DE}$ about D by the marked angle. Mark $\angle ABC$ and rotate $\overleftrightarrow{DE}$ about E by the marked angle. Label the intersection F.
2. Drag A, B, and C to different locations. What do you notice about the two triangles?
3. Write a conjecture about $\triangle ABC$ and $\triangle DEF$.
4. Test your conjecture by measuring the sides and angles of $\triangle ABC$ and $\triangle DEF$.

4-5 Technology LAB

Use with Lesson 4-5

Predict Other Triangle Congruence Relationships

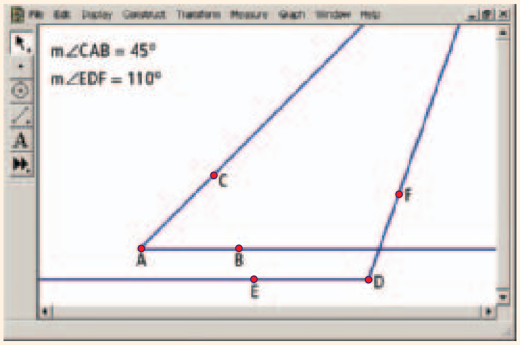
Geometry software can help you investigate whether certain combinations of triangle parts will make only one triangle. If a combination makes only one triangle, then this arrangement can be used to prove two triangles congruent.

California Standards

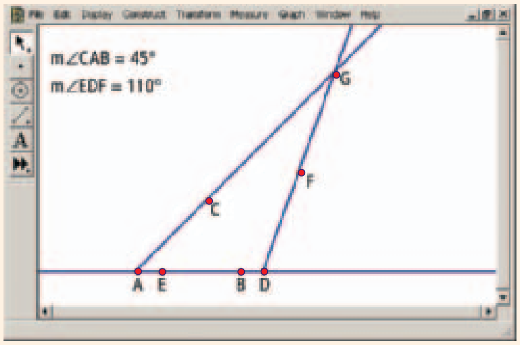
3.0 Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement.
Also covered: **1.0**

Activity 1

1. Construct $\angle CAB$ measuring 45° and $\angle EDF$ measuring 110°.



2. Move $\angle EDF$ so that $\overrightarrow{DE}$ overlays $\overrightarrow{BA}$. Where $\overrightarrow{DF}$ and $\overrightarrow{AC}$ intersect, label the point G. Measure $\angle DGA$.



3. Move $\angle CAB$ to the left and right without changing the measures of the angles. Observe what happens to the size of $\angle DGA$.

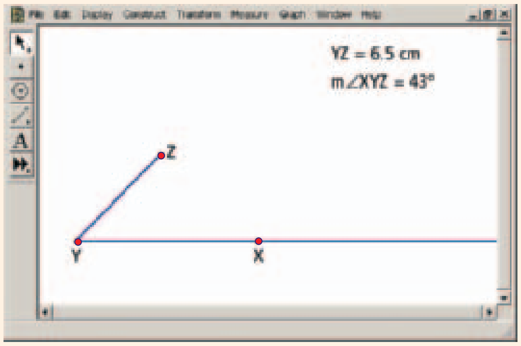
4. Measure the distance from A to D. Try to change the shape of the triangle without changing AD and the measures of $\angle A$ and $\angle D$.

Try This

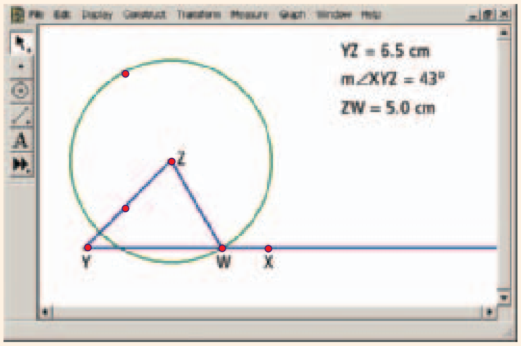
1. Repeat Activity 1 using angle measures of your choice. Are your results the same? Explain.
2. Do the results change if one of the given angles measures 90°?
3. What theorem proves that the measure of $\angle DGA$ in Step 2 will always be the same?
4. In Step 3 of the activity, the angle measures in $\triangle ADG$ stayed the same as the size of the triangle changed. Does Angle-Angle-Angle, like Side-Side-Side, make only one triangle? Explain.
5. Repeat Step 4 of the activity but measure the length of $\overline{AG}$ instead of $\overline{AD}$. Are your results the same? Does this lead to a new congruence postulate or theorem?
6. If you are given two angles of a triangle, what additional piece of information is needed so that only one triangle is made? Make a conjecture based on your findings in Step 5.

Activity 2

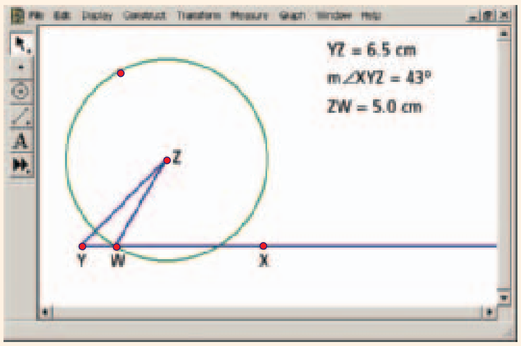
1. Construct $\overline{YZ}$ with a length of 6.5 cm.



2. Using $\overline{YZ}$ as a side, construct $\angle XYZ$ measuring 43°.



3. Draw a circle at Z with a radius of 5 cm. Construct $\overline{ZW}$, a radius of circle Z.



4. Move W around circle Z. Observe the possible shapes of $\triangle YZW$.

Try This

7. In Step 4 of the activity, how many different triangles were possible? Does Side-Side-Angle make only one triangle?

8. Repeat Activity 2 using an angle measure of 90° in Step 2 and a circle with a radius of 7 cm in Step 3. How many different triangles are possible in Step 4?

9. Repeat the activity again using a measure of 90° in Step 2 and a circle with a radius of 8.25 cm in Step 3. Classify the resulting triangle by its angle measures.

10. Based on your results, complete the following conjecture. In a Side-Side-Angle combination, if the corresponding nonincluded angles are __?__ , then only one triangle is possible.

4-5 Triangle Congruence: ASA, AAS, and HL

Objectives
Apply ASA, AAS, and HL to construct triangles and to solve problems.

Prove triangles congruent by using ASA, AAS, and HL.

Vocabulary
included side

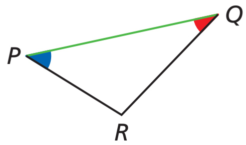
California Standards

5.0 Students prove that triangles are congruent or similar, **and they are able to use the concept of corresponding parts of congruent triangles.**
Also covered: **2.0, 4.0, 16.0**

Why use this?
Bearings are used to convey direction, helping people find their way to specific locations.

Participants in an *orienteering* race use a map and a compass to find their way to checkpoints along an unfamiliar course. Directions are given by *bearings*, which are based on compass headings. For example, to travel along the bearing S 43° E, you face south and then turn 43° to the east.

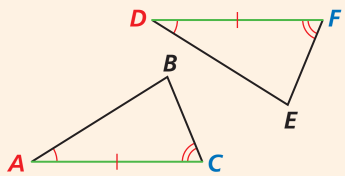
An **included side** is the common side of two consecutive angles in a polygon. The following postulate uses the idea of an *included side*.



$\overline{PQ}$ is the included side of $\angle P$ and $\angle Q$.

Know it! Note

Postulate 4-5-1 Angle-Side-Angle (ASA) Congruence

POSTULATE	HYPOTHESIS	CONCLUSION
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.		$\triangle ABC \cong \triangle DEF$

EXAMPLE 1 ***Problem-Solving Application***

PROBLEM SOLVING

Organizers of an orienteering race are planning a course with checkpoints *A*, *B*, and *C*. Does the table give enough information to determine the location of the checkpoints?

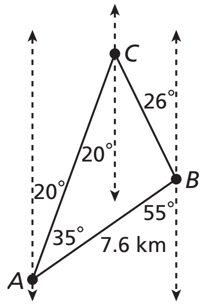
	Bearing	Distance
A* to *B	N 55° E	7.6 km
B* to *C	N 26° W	
C* to *A	S 20° W	

1 Understand the Problem

The **answer** is whether the information in the table can be used to find the position of checkpoints *A*, *B*, and *C*. List the **important information:** The bearing from *A* to *B* is N 55° E. From *B* to *C* is N 26° W, and from *C* to *A* is S 20° W. The distance from *A* to *B* is 7.6 km.

2 Make a Plan

Draw the course using vertical lines to show north-south directions. Then use these parallel lines and the alternate interior angles to help find angle measures of $\triangle ABC$.



3 Solve

$m\angle CAB = 55° - 20° = 35°$

$m\angle CBA = 180° - (26° + 55°) = 99°$

You know the measures of $\angle CAB$ and $\angle CBA$ and the length of the included side $\overline{AB}$. Therefore by ASA, a unique triangle ABC is determined.

4 Look Back

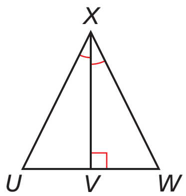
One and only one triangle can be made using the information in the table, so the table does give enough information to determine the location of all the checkpoints.

CHECK IT OUT! 1. **What if...?** If 7.6 km is the distance from B to C, is there enough information to determine the location of all the checkpoints? Explain.

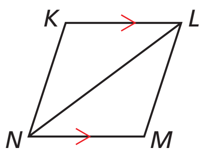
EXAMPLE 2 Applying ASA Congruence

Determine if you can use ASA to prove $\triangle UVX \cong \triangle WVX$. Explain.

$\angle UXV \cong \angle WXV$ as given. Since $\angle WVX$ is a right angle that forms a linear pair with $\angle UVX$, $\angle WVX \cong \angle UVX$. Also $\overline{VX} \cong \overline{VX}$ by the Reflexive Property. Therefore $\triangle UVX \cong \triangle WVX$ by ASA.

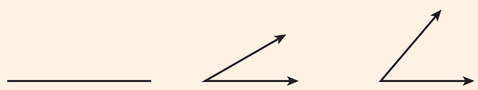


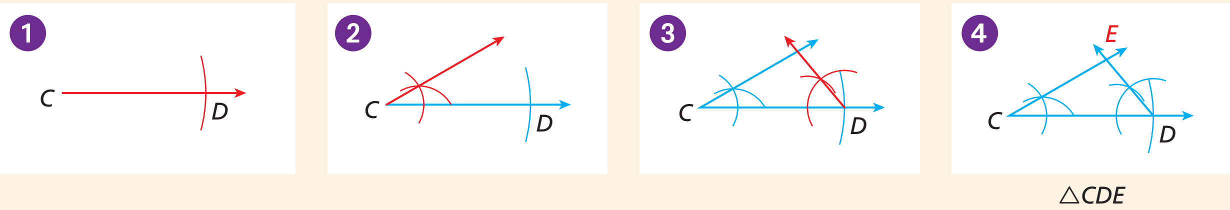
CHECK IT OUT! 2. Determine if you can use ASA to prove $\triangle NKL \cong \triangle LMN$. Explain.



Construction Congruent Triangles Using ASA

Use a straightedge to draw a segment and two angles, or copy the given segment and angles.



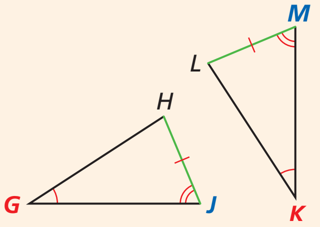


1. Construct $\overline{CD}$ congruent to the given segment.
2. Construct $\angle C$ congruent to one of the angles.
3. Construct $\angle D$ congruent to the other angle.
4. Label the intersection of the rays as E.

You can use the Third Angles Theorem to prove another congruence relationship based on ASA. This theorem is Angle-Angle-Side (AAS).

Know it! Note

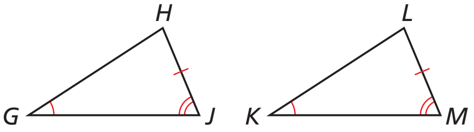
Theorem 4-5-2 **Angle-Angle-Side (AAS) Congruence**

THEOREM	HYPOTHESIS	CONCLUSION
If two angles and a nonincluded side of one triangle are congruent to the corresponding angles and nonincluded side of another triangle, then the triangles are congruent.		$\triangle GHJ \cong \triangle KLM$

PROOF

Angle-Angle-Side Congruence

Given: $\angle G \cong \angle K$, $\angle J \cong \angle M$, $\overline{HJ} \cong \overline{LM}$
Prove: $\triangle GHJ \cong \triangle KLM$



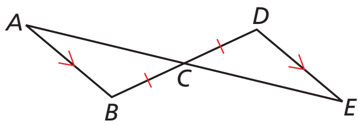
Proof:

Statements	Reasons
1. $\angle G \cong \angle K$, $\angle J \cong \angle M$	**1.** Given
2. $\angle H \cong \angle L$	**2.** Third ⦣ Thm.
3. $\overline{HJ} \cong \overline{LM}$	**3.** Given
4. $\triangle GHJ \cong \triangle KLM$	**4.** ASA *Steps 1, 3, and 2*

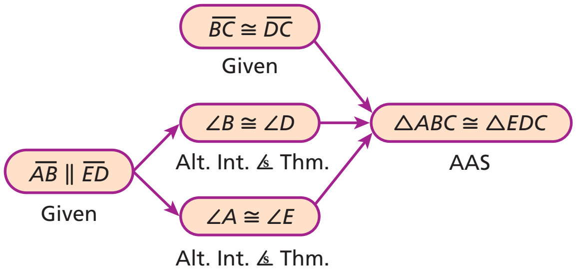
EXAMPLE 3

Using AAS to Prove Triangles Congruent

Use AAS to prove the triangles congruent.
Given: $\overline{AB} \parallel \overline{ED}$, $\overline{BC} \cong \overline{DC}$
Prove: $\triangle ABC \cong \triangle EDC$

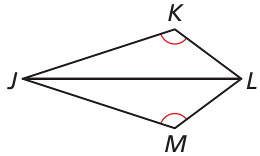


Proof:



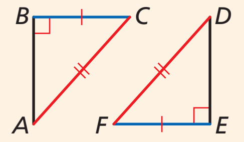
CHECK IT OUT!

3. Use AAS to prove the triangles congruent.
Given: $\overline{JL}$ bisects $\angle KLM$. $\angle K \cong \angle M$
Prove: $\triangle JKL \cong \triangle JML$



There are four theorems for right triangles that are not used for acute or obtuse triangles. They are Leg-Leg (LL), Hypotenuse-Angle (HA), Leg-Angle (LA), and Hypotenuse-Leg (HL). You will prove LL, HA, and LA in Exercises 21, 23, and 33.

Theorem 4-5-3 Hypotenuse-Leg (HL) Congruence

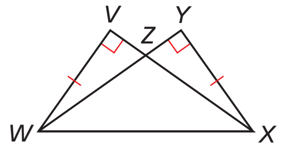
THEOREM	HYPOTHESIS	CONCLUSION
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.		$\triangle ABC \cong \triangle DEF$

You will prove the Hypotenuse-Leg Theorem in Lesson 4-8.

EXAMPLE 4 Applying HL Congruence

Determine if you can use the HL Congruence Theorem to prove the triangles congruent. If not, tell what else you need to know.

A **$\triangle VWX$ and $\triangle YXW$**



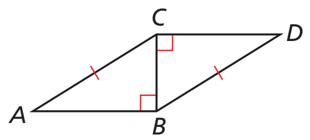
According to the diagram, $\triangle VWX$ and $\triangle YXW$ are right triangles that share hypotenuse $\overline{WX}$. $\overline{WX} \cong \overline{XW}$ by the Reflexive Property. It is given that $\overline{WV} \cong \overline{XY}$, therefore $\triangle VWX \cong \triangle YXW$ by HL.

B **$\triangle VWZ$ and $\triangle YXZ$**

This conclusion cannot be proved by HL. According to the diagram, $\triangle VWZ$ and $\triangle YXZ$ are right triangles, and $\overline{WV} \cong \overline{XY}$. You do not know that hypotenuse $\overline{WZ}$ is congruent to hypotenuse $\overline{XZ}$.

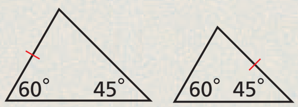
CHECK IT OUT!

4. Determine if you can use the HL Congruence Theorem to prove $\triangle ABC \cong \triangle DCB$. If not, tell what else you need to know.



THINK AND DISCUSS

1. Could you use AAS to prove that these two triangles are congruent? Explain.



2. The arrangement of the letters in ASA matches the arrangement of what parts of congruent triangles? Include a sketch to support your answer.

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each column, write a description of the method and then sketch two triangles, marking the appropriate congruent parts.

Proving Triangles Congruent						
	Def. of $\triangle \cong$	SSS	SAS	ASA	AAS	HL
Words						
Pictures						

4-5 Exercises

California Standards
2.0, 4.0, 5.0, 12.0, 16.0, AF1.0, 7AF4.1, 7MG3.4

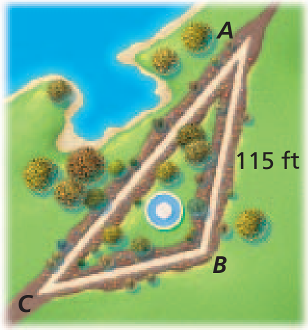
go.hrw.com
Homework Help Online
KEYWORD: MG7 4-5
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

1. **Vocabulary** A triangle contains ∠*ABC* and ∠*ACB* with $\overline{BC}$ "closed in" between them. How would this help you remember the definition of *included side*?

SEE EXAMPLE 1
p. 252

Surveying **Use the table for Exercises 2 and 3.** A landscape designer surveyed the boundaries of a triangular park. She made the following table for the dimensions of the land.



	A to *B*	*B* to *C*	*C* to *A*
Bearing	E	S 25° E	N 62° W
Distance	115 ft	?	?

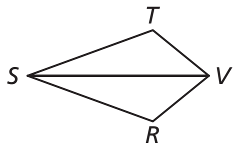
2. Draw the plot of land described by the table. Label the measures of the angles in the triangle.

3. Does the table have enough information to determine the locations of points *A*, *B*, and *C*? Explain.

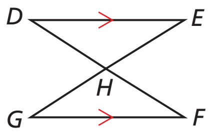
SEE EXAMPLE 2
p. 253

Determine if you can use ASA to prove the triangles congruent. Explain.

4. △*VRS* and △*VTS*, given that $\overline{VS}$ bisects ∠*RST* and ∠*RVT*



5. △*DEH* and △*FGH*

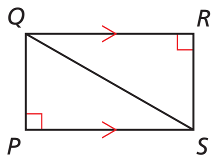


SEE EXAMPLE 3
p. 254

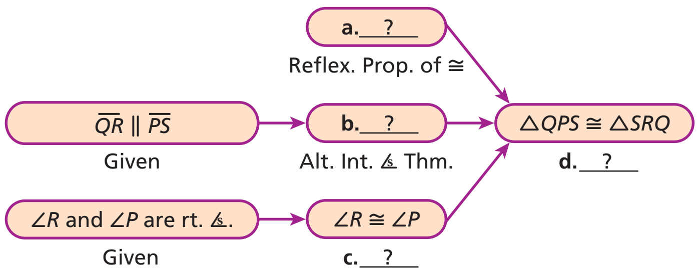
6. Use AAS to prove the triangles congruent.

Given: ∠*R* and ∠*P* are right angles.
$\overline{QR} \parallel \overline{SP}$

Prove: △*QPS* ≅ △*SRQ*



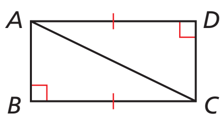
Proof:



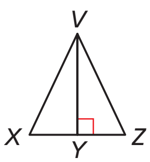
SEE EXAMPLE 4
p. 255

Determine if you can use the HL Congruence Theorem to prove the triangles congruent. If not, tell what else you need to know.

7. △*ABC* and △*CDA*



8. △*XYV* and △*ZYV*



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
9–10	1
11–12	2
13	3
14–15	4

Extra Practice

Skills Practice p. S11
Application Practice p. S31

Surveying **Use the table for Exercises 9 and 10.**
From two different observation towers a fire is sighted. The locations of the towers are given in the following table.

	X to *Y*	*X* to *F*	*Y* to *F*
Bearing	E	N 53° E	N 16° W
Distance	6 km	?	?

9. Draw the diagram formed by observation tower *X*, observation tower *Y*, and the fire *F*. Label the measures of the angles.

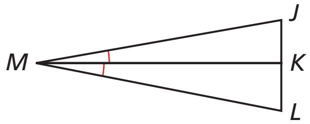
10. Is there enough information given in the table to pinpoint the location of the fire? Explain.

Math History

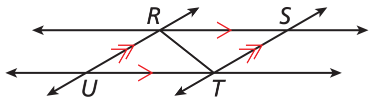
Euclid wrote the mathematical text *The Elements* around 2300 years ago. It may be the second most reprinted book in history.

Determine if you can use ASA to prove the triangles congruent. Explain.

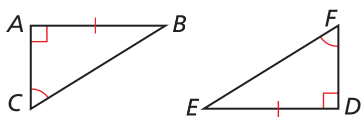
11. $\triangle MKJ$ and $\triangle MKL$



12. $\triangle RST$ and $\triangle TUR$



13. Given: $\overline{AB} \cong \overline{DE}$, $\angle C \cong \angle F$
Prove: $\triangle ABC \cong \triangle DEF$

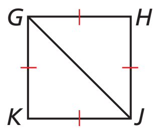


Proof:

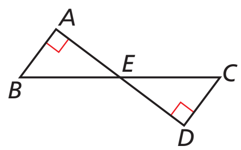
- $\angle A$ and $\angle D$ are rt. ∠s. — Given → **a.** ? — Rt. ∠ ≅ Thm. → $\triangle ABC \cong \triangle DEF$
- $\overline{AB} \cong \overline{DE}$ — **b.** ? → $\triangle ABC \cong \triangle DEF$
- **c.** ? — Given → $\triangle ABC \cong \triangle DEF$
- $\triangle ABC \cong \triangle DEF$ — **d.** ?

Determine if you can use the HL Congruence Theorem to prove the triangles congruent. If not, tell what else you need to know.

14. $\triangle GHJ$ and $\triangle JKG$

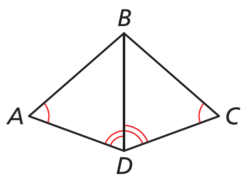


15. $\triangle ABE$ and $\triangle DCE$, given that *E* is the midpoint of $\overline{AD}$ and $\overline{BC}$

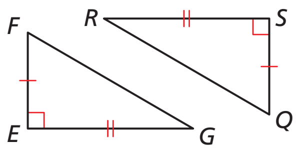


Multi-Step **For each pair of triangles write a triangle congruence statement. Identify the transformation that moves one triangle to the position of the other triangle.**

16.



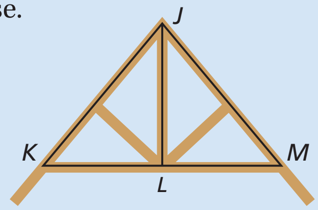
17.



18. **Critical Thinking** Side-Side-Angle (SSA) cannot be used to prove two triangles congruent. Draw a diagram that shows why this is true.

Concept Connection

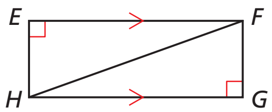
19. This problem will prepare you for the Concept Connection on page 280. A carpenter built a truss to support the roof of a doghouse.



a. The carpenter knows that $\overline{KJ} \cong \overline{MJ}$. Can the carpenter conclude that $\triangle KJL \cong \triangle MJL$? Why or why not?

b. Suppose the carpenter also knows that $\angle JLK$ is a right angle. Which theorem can be used to show that $\triangle KJL \cong \triangle MJL$?

20. **ERROR ANALYSIS** Two proofs that $\triangle EFH \cong \triangle GHF$ are given. Which is incorrect? Explain the error.

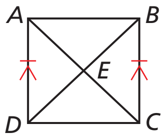


A It is given that $\overline{EF} \parallel \overline{GH}$. By the Alt. Int. ∠s Thm., $\angle EFH \cong \angle GHF$. $\angle E \cong \angle G$ by the Rt. ∠ ≅ Thm. By the Reflex. Prop. of ≅, $\overline{HF} \cong \overline{HF}$. So by AAS, $\triangle EFH \cong \triangle GHF$.

B $\overline{HF}$ is the hyp. of both rt. △s. $\overline{HF} \cong \overline{HF}$ by the Reflex. Prop. of ≅. Since the opp. sides of a rect. are ≅, $\overline{EF} \cong \overline{GH}$. So by HL, $\triangle EFH \cong \triangle FHG$.

21. Write a paragraph proof of the Leg-Leg (LL) Congruence Theorem. If the legs of one right triangle are congruent to the corresponding legs of another right triangle, the triangles are congruent.

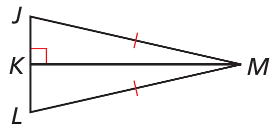
22. Use AAS to prove the triangles congruent.
Given: $\overline{AD} \parallel \overline{BC}$, $\overline{AD} \cong \overline{CB}$
Prove: $\triangle AED \cong \triangle CEB$



Proof:

Statements	Reasons
1. $\overline{AD} \parallel \overline{BC}$	1. a. ?
2. $\angle DAE \cong \angle BCE$	2. b. ?
3. c. ?	3. Vert. ∠s Thm.
4. d. ?	3. Given
5. e. ?	4. f. ?

23. Prove the Hypotenuse-Angle (HA) Theorem.
Given: $\overline{KM} \perp \overline{JL}$, $\overline{JM} \cong \overline{LM}$, $\angle JMK \cong \angle LMK$
Prove: $\triangle JKM \cong \triangle LKM$



24. **Write About It** The legs of both right $\triangle DEF$ and right $\triangle RST$ are 3 cm and 4 cm. They each have a hypotenuse 5 cm in length. Describe two different ways you could prove that $\triangle DEF \cong \triangle RST$.

25. **Construction** Use the method for constructing perpendicular lines to construct a right triangle.

Standardized Test Prep

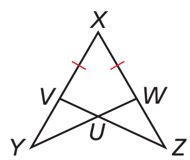
26. What additional congruence statement is necessary to prove $\triangle XWY \cong \triangle XVZ$ by ASA?

Ⓐ $\angle XVZ \cong \angle XWY$

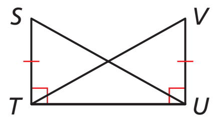
Ⓑ $\angle VUY \cong \angle WUZ$

Ⓒ $\overline{VZ} \cong \overline{WY}$

Ⓓ $\overline{XZ} \cong \overline{XY}$

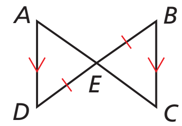


27. Which postulate or theorem justifies the congruence statement $\triangle STU \cong \triangle VUT$?



Ⓕ ASA Ⓗ HL
Ⓖ SSS Ⓙ SAS

28. Which of the following congruence statements is true?



Ⓐ $\angle A \cong \angle B$ Ⓒ $\triangle AED \cong \triangle CEB$
Ⓑ $\overline{CE} \cong \overline{DE}$ Ⓓ $\triangle AED \cong \triangle BEC$

29. In $\triangle RST$, $RT = 6y - 2$. In $\triangle UVW$, $UW = 2y + 7$. $\angle R \cong \angle U$, and $\angle S \cong \angle V$. What must be the value of y in order to prove that $\triangle RST \cong \triangle UVW$?

Ⓕ 1.25 Ⓖ 2.25 Ⓗ 9.0 Ⓙ 11.5

30. **Extended Response** Draw a triangle. Construct a second triangle that has the same angle measures but is not congruent. Compare the lengths of each pair of corresponding sides. Consider the relationship between the lengths of the sides and the measures of the angles. Explain why Angle-Angle-Angle (AAA) is not a congruence principle.

CHALLENGE AND EXTEND

31. **Sports** This bicycle frame includes $\triangle VSU$ and $\triangle VTU$, which lie in intersecting planes. From the given angle measures, can you conclude that $\triangle VSU \cong \triangle VTU$? Explain.

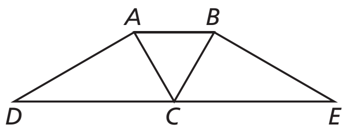
$m\angle VUS = (7y - 2)^\circ$ $m\angle VUT = \left(5\frac{1}{2}x - \frac{1}{2}\right)^\circ$

$m\angle USV = 5\frac{2}{3}y^\circ$ $m\angle UTV = (4x + 8)^\circ$

$m\angle SVU = (3y - 6)^\circ$ $m\angle TVU = 2x^\circ$

32. **Given:** $\triangle ABC$ is equilateral. C is the midpoint of $\overline{DE}$. $\angle DAC$ and $\angle EBC$ are congruent and supplementary.

Prove: $\triangle DAC \cong \triangle EBC$



33. Write a two-column proof of the Leg-Angle (LA) Congruence Theorem. If a leg and an acute angle of one right triangle are congruent to the corresponding parts of another right triangle, the triangles are congruent. (*Hint:* There are two cases to consider.)

34. If two triangles are congruent by ASA, what theorem could you use to prove that the triangles are also congruent by AAS? Explain.

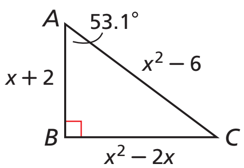
SPIRAL REVIEW

Identify the x- and y-intercepts. Use them to graph each line. *(Previous course)*

35. $y = 3x - 6$ 36. $y = -\frac{1}{2}x + 4$ 37. $y = -5x + 5$

38. Find AB and BC if $AC = 10$. *(Lesson 1-6)*

39. Find $m\angle C$. *(Lesson 4-2)*



4-6 Triangle Congruence: CPCTC

Objective
Use CPCTC to prove parts of triangles are congruent.

Vocabulary
CPCTC

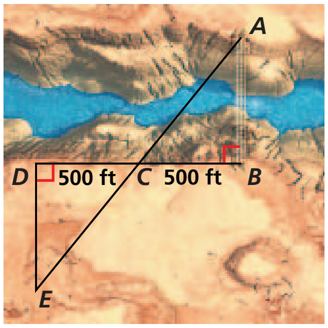
Why learn this?
You can use congruent triangles to estimate distances.

CPCTC is an abbreviation for the phrase "Corresponding Parts of Congruent Triangles are Congruent." It can be used as a justification in a proof after you have proven two triangles congruent.

Remember!
SSS, SAS, ASA, AAS, and HL use corresponding parts to prove triangles congruent. CPCTC uses congruent triangles to prove corresponding parts congruent.

EXAMPLE 1 ***Engineering Application***

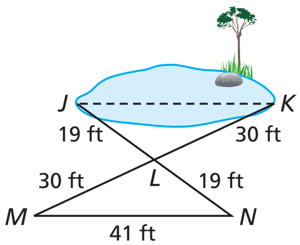
To design a bridge across a canyon, you need to find the distance from A to B. Locate points C, D, and E as shown in the figure. If $DE = 600$ ft, what is AB?



$\angle D \cong \angle B$, because they are both right angles.
$\overline{DC} \cong \overline{CB}$, because $DC = CB = 500$ ft.
$\angle DCE \cong \angle BCA$, because vertical angles are congruent. Therefore $\triangle DCE \cong \triangle BCA$ by ASA or LA. By CPCTC, $\overline{ED} \cong \overline{AB}$, so $AB = ED = 600$ ft.

CHECK IT OUT!

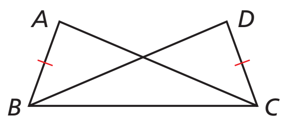
1. A landscape architect sets up the triangles shown in the figure to find the distance JK across a pond. What is JK?



EXAMPLE 2 **Proving Corresponding Parts Congruent**

California Standards
5.0 Students prove that triangles are congruent or similar, **and they are able to use the concept of corresponding parts of congruent triangles.**
Also covered: **2.0**

Given: $\overline{AB} \cong \overline{DC}$, $\angle ABC \cong \angle DCB$
Prove: $\angle A \cong \angle D$

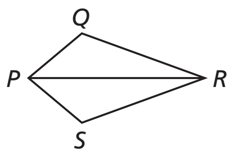


Proof:

- $\overline{AB} \cong \overline{DC}$ — Given
- $\angle ABC \cong \angle DCB$ — Given
- $\overline{BC} \cong \overline{CB}$ — Reflex. Prop. of $\cong$
- → $\triangle ABC \cong \triangle DCB$ — SAS
- → $\angle A \cong \angle D$ — CPCTC

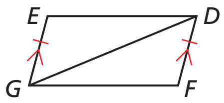
CHECK IT OUT!

2. **Given:** $\overline{PR}$ bisects $\angle QPS$ and $\angle QRS$.
Prove: $\overline{PQ} \cong \overline{PS}$



EXAMPLE 3 Using CPCTC in a Proof

Given: $\overline{EG} \parallel \overline{DF}$, $\overline{EG} \cong \overline{DF}$
Prove: $\overline{ED} \parallel \overline{GF}$
Proof:

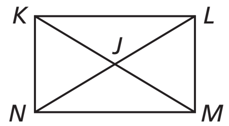


Statements	Reasons
1. $\overline{EG} \cong \overline{DF}$	1. Given
2. $\overline{EG} \parallel \overline{DF}$	2. Given
3. $\angle EGD \cong \angle FDG$	3. Alt. Int. ∠s Thm.
4. $\overline{GD} \cong \overline{DG}$	4. Reflex. Prop. of ≅
5. $\triangle EGD \cong \triangle FDG$	5. SAS *Steps 1, 3, and 4*
6. $\angle EDG \cong \angle FGD$	6. CPCTC
7. $\overline{ED} \parallel \overline{GF}$	7. Converse of Alt. Int. ∠s Thm.

Helpful Hint

Work backward when planning a proof. To show that $\overline{ED} \parallel \overline{GF}$, look for a pair of angles that are congruent. Then look for triangles that contain these angles.

CHECK IT OUT! **3. Given:** J is the midpoint of $\overline{KM}$ and $\overline{NL}$.
Prove: $\overline{KL} \parallel \overline{MN}$

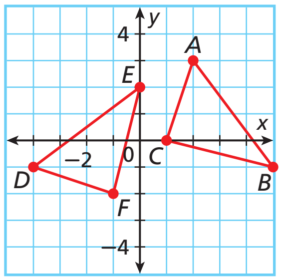


You can also use CPCTC when triangles are on a coordinate plane. You use the Distance Formula to find the lengths of the sides of each triangle. Then, after showing that the triangles are congruent, you can make conclusions about their corresponding parts.

EXAMPLE 4 Using CPCTC in the Coordinate Plane

Algebra

Given: $A(2, 3), B(5, -1), C(1, 0),$
$D(-4, -1), E(0, 2), F(-1, -2)$
Prove: $\angle ABC \cong \angle DEF$



Step 1 Plot the points on a coordinate plane.

Step 2 Use the Distance Formula to find the lengths of the sides of each triangle.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(5 - 2)^2 + (-1 - 3)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$DE = \sqrt{\left(0 - (-4)\right)^2 + \left(2 - (-1)\right)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$BC = \sqrt{(1 - 5)^2 + \left(0 - (-1)\right)^2} = \sqrt{16 + 1} = \sqrt{17}$$

$$EF = \sqrt{(-1 - 0)^2 + (-2 - 2)^2} = \sqrt{1 + 16} = \sqrt{17}$$

$$AC = \sqrt{(1 - 2)^2 + (0 - 3)^2} = \sqrt{1 + 9} = \sqrt{10}$$

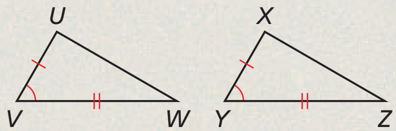
$$DF = \sqrt{\left(-1 - (-4)\right)^2 + \left(-2 - (-1)\right)^2} = \sqrt{9 + 1} = \sqrt{10}$$

So $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$. Therefore $\triangle ABC \cong \triangle DEF$ by SSS, and $\angle ABC \cong \angle DEF$ by CPCTC.

CHECK IT OUT! **4. Given:** $J(-1, -2), K(2, -1), L(-2, 0), R(2, 3), S(5, 2), T(1, 1)$
Prove: $\angle JKL \cong \angle RST$

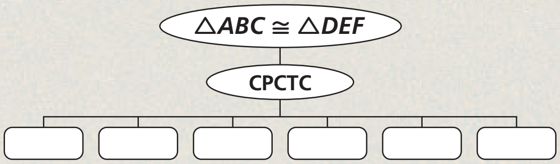
THINK AND DISCUSS

1. In the figure, $\overline{UV} \cong \overline{XY}$, $\overline{VW} \cong \overline{YZ}$, and $\angle V \cong \angle Y$. Explain why $\triangle UVW \cong \triangle XYZ$. By CPCTC, which additional parts are congruent?



Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. Write all conclusions you can make using CPCTC.



4-6 Exercises

California Standards
2.0, 5.0, 22.0, 6SDAP1.1, 7AF2.0, 7AF4.1, 7MG3.2, 7MG3.4, 7MR1.1, 1A2.0

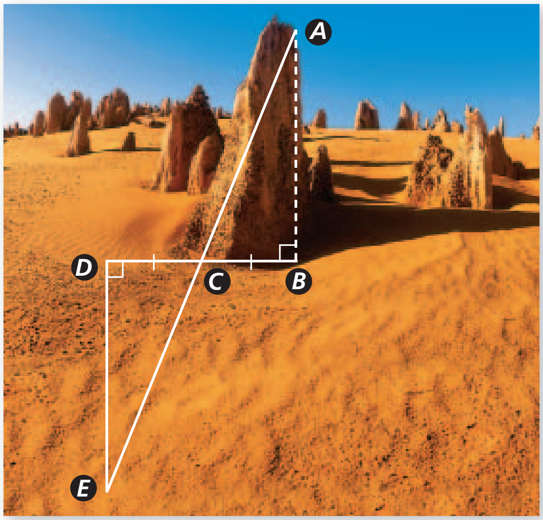
go.hrw.com
Homework Help Online
KEYWORD: MG7 4-6
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

1. **Vocabulary** You use CPCTC after proving triangles are congruent. Which parts of congruent triangles are referred to as corresponding parts?

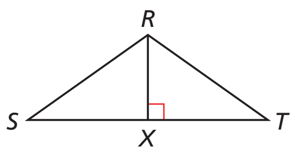
SEE EXAMPLE 1
p. 260

2. **Archaeology** An archaeologist wants to find the height AB of a rock formation. She places a marker at C and steps off the distance from C to B. Then she walks the same distance from C and places a marker at D. If $DE = 6.3$ m, what is AB?

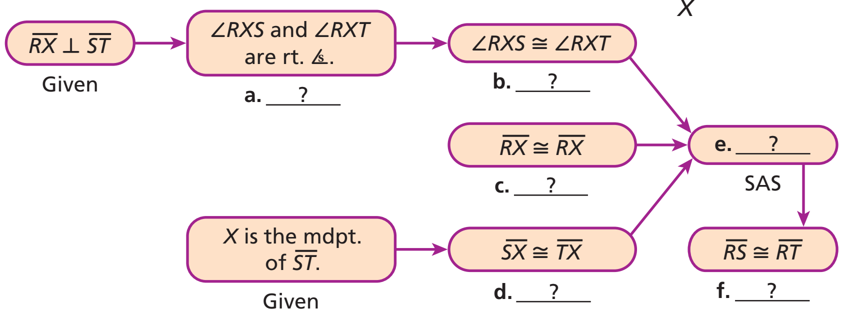


SEE EXAMPLE 2
p. 260

3. **Given:** X is the midpoint of $\overline{ST}$. $\overline{RX} \perp \overline{ST}$
Prove: $\overline{RS} \cong \overline{RT}$

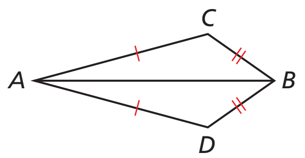


Proof:



SEE EXAMPLE 3
p. 261

4. **Given:** $\overline{AC} \cong \overline{AD}$, $\overline{CB} \cong \overline{DB}$
Prove: $\overline{AB}$ bisects $\angle CAD$.



Proof:

Statements	Reasons
1. $\overline{AC} \cong \overline{AD}$, $\overline{CB} \cong \overline{DB}$	1. a. ___?___
2. b. ___?___	2. Reflex. Prop. of $\cong$
3. $\triangle ACB \cong \triangle ADB$	3. c. ___?___
4. $\angle CAB \cong \angle DAB$	4. d. ___?___
5. $\overline{AB}$ bisects $\angle CAD$	5. e. ___?___

SEE EXAMPLE 4
p. 261

Multi-Step Use the given set of points to prove each congruence statement.

5. $E(-3, 3)$, $F(-1, 3)$, $G(-2, 0)$, $J(0, -1)$, $K(2, -1)$, $L(1, 2)$; $\angle EFG \cong \angle JKL$

6. $A(2, 3)$, $B(4, 1)$, $C(1, -1)$, $R(-1, 0)$, $S(-3, -2)$, $T(0, -4)$; $\angle ACB \cong \angle RTS$

PRACTICE AND PROBLEM SOLVING

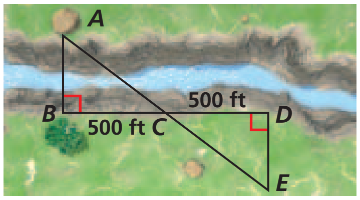
Independent Practice

For Exercises	See Example
7	1
8–9	2
10–11	3
12–13	4

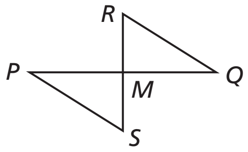
Extra Practice

Skills Practice p. S11
Application Practice p. S31

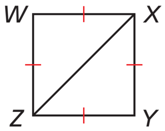
7. **Surveying** To find the distance AB across a river, a surveyor first locates point C. He measures the distance from C to B. Then he locates point D the same distance east of C. If $DE = 420$ ft, what is AB?



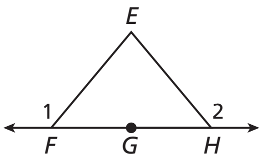
8. **Given:** M is the midpoint of $\overline{PQ}$ and $\overline{RS}$.
Prove: $\overline{QR} \cong \overline{PS}$



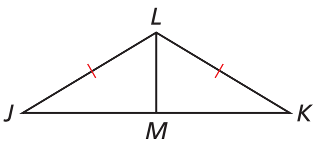
9. **Given:** $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$
Prove: $\angle W \cong \angle Y$



10. **Given:** G is the midpoint of $\overline{FH}$. $\overline{EF} \cong \overline{EH}$
Prove: $\angle 1 \cong \angle 2$



11. **Given:** $\overline{LM}$ bisects $\angle JLK$. $\overline{JL} \cong \overline{KL}$
Prove: M is the midpoint of $\overline{JK}$.



Multi-Step Use the given set of points to prove each congruence statement.

12. $R(0, 0)$, $S(2, 4)$, $T(-1, 3)$, $U(-1, 0)$, $V(-3, -4)$, $W(-4, -1)$; $\angle RST \cong \angle UVW$

13. $A(-1, 1)$, $B(2, 3)$, $C(2, -2)$, $D(2, -3)$, $E(-1, -5)$, $F(-1, 0)$; $\angle BAC \cong \angle EDF$

14. **Given:** $\triangle QRS$ is adjacent to $\triangle QTS$. $\overline{QS}$ bisects $\angle RQT$. $\angle R \cong \angle T$
Prove: $\overline{QS}$ bisects $\overline{RT}$.

15. **Given:** $\triangle ABE$ and $\triangle CDE$ with E the midpoint of $\overline{AC}$ and $\overline{BD}$
Prove: $\overline{AB} \parallel \overline{CD}$

CONCEPT CONNECTION

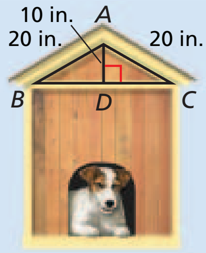
16. This problem will prepare you for the Concept Connection on page 280.

The front of a doghouse has the dimensions shown.

a. How can you prove that $\triangle ADB \cong \triangle ADC$?

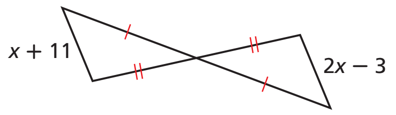
b. Prove that $\overline{BD} \cong \overline{CD}$.

c. What is the length of $\overline{BD}$ and $\overline{BC}$ to the nearest tenth?

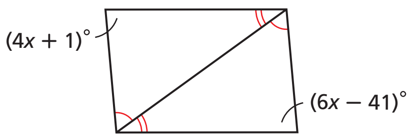


Multi-Step **Find the value of *x*.**

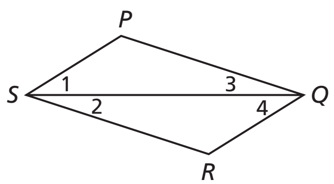
17.



18.



Use the diagram for Exercises 19–21.



19. Given: $PS = RQ$, $m\angle 1 = m\angle 4$

Prove: $m\angle 3 = m\angle 2$

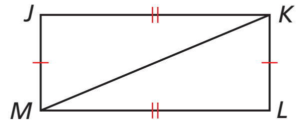
20. Given: $m\angle 1 = m\angle 2$, $m\angle 3 = m\angle 4$

Prove: $PS = RS$

21. Given: $PS = RQ$, $PQ = RS$

Prove: $\overline{PQ} \parallel \overline{RS}$

22. **Critical Thinking** Does the diagram contain enough information to allow you to conclude that $\overline{JK} \parallel \overline{ML}$? Explain.



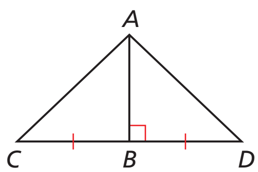
23. **Write About It** Draw a diagram and explain how a surveyor can set up triangles to find the distance across a lake. Label each part of your diagram. List which sides or angles must be congruent.

STANDARDIZED TEST PREP

24. Which of these will NOT be used as a reason in a proof of $\overline{AC} \cong \overline{AD}$?

Ⓐ SAS　Ⓒ ASA

Ⓑ CPCTC　Ⓓ Reflexive Property



25. Given the points $K(1, 2)$, $L(0, -4)$, $M(-2, -3)$, and $N(-1, 3)$, which of these is true?

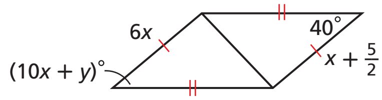
Ⓕ $\angle KNL \cong \angle MNL$　Ⓗ $\angle MLN \cong \angle KLN$

Ⓖ $\angle LNK \cong \angle NLM$　Ⓙ $\angle MNK \cong \angle NKL$

26. What is the value of *y*?

Ⓐ 10　Ⓒ 35

Ⓑ 20　Ⓓ 85



27. Which of these are NOT used to prove angles congruent?

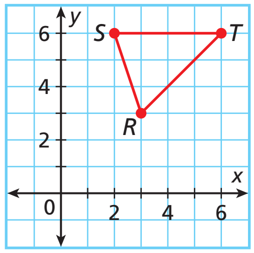
Ⓕ congruent triangles　Ⓗ parallel lines

Ⓖ noncorresponding parts　Ⓙ perpendicular lines

28. Which set of coordinates represents the vertices of a triangle congruent to $\triangle RST$? (*Hint:* Find the lengths of the sides of $\triangle RST$.)

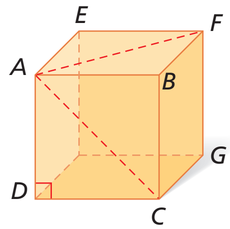
Ⓐ (3, 4), (3, 0), (0, 0)　　Ⓒ (3, 1), (3, 3), (4, 6)

Ⓑ (3, 3), (0, 4), (0, 0)　　Ⓓ (3, 0), (4, 4), (0, 6)

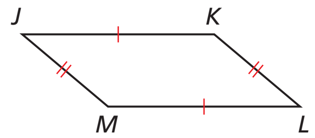


CHALLENGE AND EXTEND

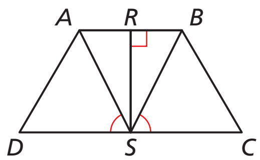
29. All of the edges of a cube are congruent. All of the angles on each face of a cube are right angles. Use CPCTC to explain why any two diagonals on the faces of a cube (for example, $\overline{AC}$ and $\overline{AF}$) must be congruent.



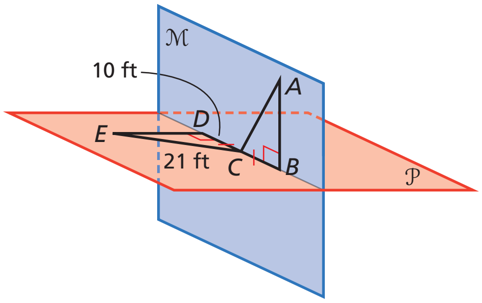
30. **Given:** $\overline{JK} \cong \overline{ML}$, $\overline{JM} \cong \overline{KL}$
Prove: $\angle J \cong \angle L$
(*Hint:* Draw an auxiliary line.)



31. **Given:** R is the midpoint of $\overline{AB}$.
S is the midpoint of $\overline{DC}$.
$\overline{RS} \perp \overline{AB}$, $\angle ASD \cong \angle BSC$
Prove: $\triangle ASD \cong \triangle BSC$



32. $\triangle ABC$ is in plane $\mathcal{M}$. $\triangle CDE$ is in plane $\mathcal{P}$. Both planes have C in common and $\angle A \cong \angle E$. What is the height AB to the nearest foot?



SPIRAL REVIEW

33. Lina's test scores in her history class are 90, 84, 93, 88, and 91. What is the minimum score Lina must make on her next test to have an average test score of 90? *(Previous course)*

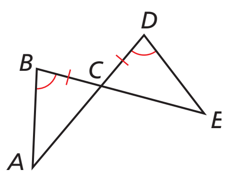
34. One long-distance phone plan costs \$3.95 per month plus \$0.08 per minute of use. A second long-distance plan costs \$0.10 per minute for the first 50 minutes used each month and then \$0.15 per minute after that. Which plan is cheaper if you use an average of 75 long-distance minutes per month? *(Previous course)*

A figure has vertices at (1, 3), (2, 2), (3, 2), and (4, 3). Identify the transformation of the figure that produces an image with each set of vertices. *(Lesson 1-7)*

35. (1, −3), (2, −2), (3, −2), (4, −3)

36. (−2, −1), (−1, −2), (0, −2), (1, −1)

37. Determine if you can use ASA to prove $\triangle ACB \cong \triangle ECD$. Explain. *(Lesson 4-5)*



Connecting Geometry to **Algebra**

See Skills Bank page S66

Quadratic Equations

A quadratic equation is an equation that can be written in the form $ax^2 + bx + c = 0$.

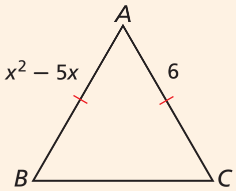
California Standards

12.0 Students find and **use measures of sides and of interior** and exterior **angles of triangles** and polygons **to** classify figures and **solve problems.**

Also covered: **Review of 1A14.0, 1A20.0**

Example

Given: $\triangle ABC$ is isosceles with $\overline{AB} \cong \overline{AC}$. Solve for x.



Step 1 Set $x^2 - 5x$ equal to 6 to get $x^2 - 5x = 6$.

Step 2 Rewrite the quadratic equation by subtracting 6 from each side to get $x^2 - 5x - 6 = 0$.

Step 3 Solve for x.

Method 1: Factoring	Method 2: Quadratic Formula
$x^2 - 5x - 6 = 0$	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$(x - 6)(x + 1) = 0$ *Factor.*	$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-6)}}{2(1)}$ *Substitute 1 for a, −5 for b, and −6 for c.*
$x - 6 = 0$ or $x + 1 = 0$ *Set each factor equal to 0.*	$x = \dfrac{5 \pm \sqrt{49}}{2}$ *Simplify.*
$x = 6$ or $x = -1$ *Solve.*	$x = \dfrac{5 \pm 7}{2}$ *Find the square root.*
	$x = \dfrac{12}{2}$ or $x = \dfrac{-2}{2}$ *Simplify.*
	$x = 6$ or $x = -1$

Step 4 Check each solution in the original equation.

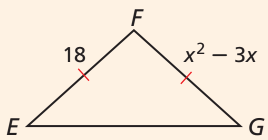
$x^2 - 5x$	$= 6$	
$(6)^2 - 5(6)$	6	
$36 - 30$	6	
6	6	✓

$x^2 - 5x$	$= 6$	
$(-1)^2 - 5(-1)$	6	
$1 + 5$	6	
6	6	✓

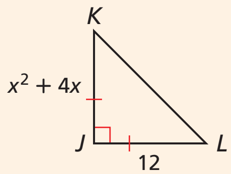
Try This

Solve for x in each isosceles triangle.

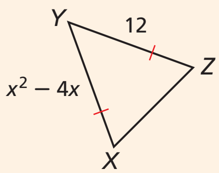
1. **Given:** $\overline{FE} \cong \overline{FG}$



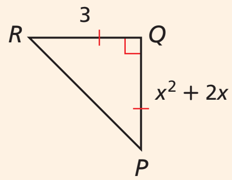
2. **Given:** $\overline{JK} \cong \overline{JL}$



3. **Given:** $\overline{YX} \cong \overline{YZ}$



4. **Given:** $\overline{QP} \cong \overline{QR}$



4-7 Introduction to Coordinate Proof

Objectives

Position figures in the coordinate plane for use in coordinate proofs.

Prove geometric concepts by using coordinate proof.

Vocabulary

coordinate proof

California Standards

17.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

Who uses this?

The Bushmen in South Africa use the Global Positioning System to transmit data about endangered animals to conservationists. (See Exercise 24.)

You have used coordinate geometry to find the midpoint of a line segment and to find the distance between two points. Coordinate geometry can also be used to prove conjectures.

A **coordinate proof** is a style of proof that uses coordinate geometry and algebra. The first step of a coordinate proof is to position the given figure in the plane. You can use any position, but some strategies can make the steps of the proof simpler.

Know it! Note

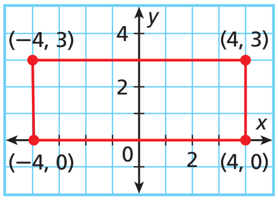
Strategies for Positioning Figures in the Coordinate Plane

- Use the origin as a vertex, keeping the figure in Quadrant I.
- Center the figure at the origin.
- Center a side of the figure at the origin.
- Use one or both axes as sides of the figure.

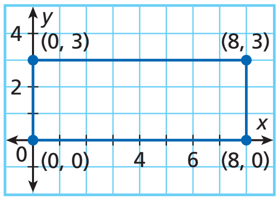
EXAMPLE 1 Positioning a Figure in the Coordinate Plane

Position a rectangle with a length of 8 units and a width of 3 units in the coordinate plane.

Method 1 You can center the longer side of the rectangle at the origin.



Method 2 You can use the origin as a vertex of the rectangle.



Depending on what you are using the figure to prove, one solution may be better than the other. For example, if you need to find the midpoint of the longer side, use the first solution.

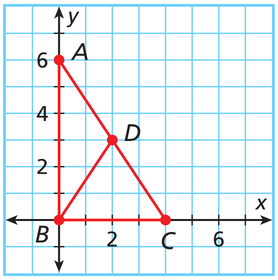
CHECK IT OUT!

1. Position a right triangle with leg lengths of 2 and 4 units in the coordinate plane. (*Hint:* Use the origin as the vertex of the right angle.)

Once the figure is placed in the coordinate plane, you can use slope, the coordinates of the vertices, the Distance Formula, or the Midpoint Formula to prove statements about the figure.

EXAMPLE 2 Writing a Proof Using Coordinate Geometry

Write a coordinate proof.



Given: Right $\triangle ABC$ has vertices $A(0, 6)$, $B(0, 0)$, and $C(4, 0)$. D is the midpoint of $\overline{AC}$.

Prove: The area of $\triangle DBC$ is one half the area of $\triangle ABC$.

Proof: $\triangle ABC$ is a right triangle with height AB and base BC.

area of $\triangle ABC = \frac{1}{2}bh$

$= \frac{1}{2}(4)(6) = 12$ square units

By the Midpoint Formula, the coordinates of $D = \left(\frac{0+4}{2}, \frac{6+0}{2}\right) = (2, 3)$. The y-coordinate of D is the height of $\triangle DBC$, and the base is 4 units.

area of $\triangle DBC = \frac{1}{2}bh$

$= \frac{1}{2}(4)(3) = 6$ square units

Since $6 = \frac{1}{2}(12)$, the area of $\triangle DBC$ is one half the area of $\triangle ABC$.

CHECK IT OUT! **2.** Use the information in Example 2 to write a coordinate proof showing that the area of $\triangle ADB$ is one half the area of $\triangle ABC$.

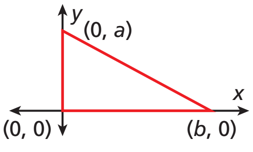
A coordinate proof can also be used to prove that a certain relationship is always true. You can prove that a statement is true for all right triangles without knowing the side lengths. To do this, assign variables as the coordinates of the vertices.

EXAMPLE 3 Assigning Coordinates to Vertices

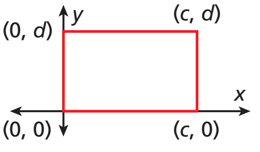
Caution!
Do not use both axes when positioning a figure unless you know the figure has a right angle.

Position each figure in the coordinate plane and give the coordinates of each vertex.

A **a right triangle with leg lengths *a* and *b***



B **a rectangle with length *c* and width *d***



CHECK IT OUT! **3.** Position a square with side length $4p$ in the coordinate plane and give the coordinates of each vertex.

If a coordinate proof requires calculations with fractions, choose coordinates that make the calculations simpler. For example, use multiples of 2 when you are to find coordinates of a midpoint. Once you have assigned the coordinates of the vertices, the procedure for the proof is the same, except that your calculations will involve variables.

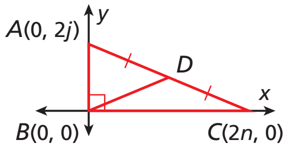
EXAMPLE 4 Writing a Coordinate Proof

Given: $\angle B$ is a right angle in $\triangle ABC$. D is the midpoint of $\overline{AC}$.
Prove: The area of $\triangle DBC$ is one half the area of $\triangle ABC$.

Step 1 Assign coordinates to each vertex.
The coordinates of A are $(0, 2j)$,
the coordinates of B are $(0, 0)$,
and the coordinates of C are $(2n, 0)$.

Since you will use the Midpoint Formula to find the coordinates of D, use multiples of 2 for the leg lengths.

Step 2 Position the figure in the coordinate plane.



Step 3 Write a coordinate proof.

Proof: $\triangle ABC$ is a right triangle with height $2j$ and base $2n$.

$$\text{area of } \triangle ABC = \frac{1}{2}bh$$
$$= \frac{1}{2}(2n)(2j)$$
$$= 2nj \text{ square units}$$

By the Midpoint Formula, the coordinates of $D = \left(\frac{0 + 2n}{2}, \frac{2j + 0}{2}\right) = (n, j)$.

The height of $\triangle DBC$ is j units, and the base is $2n$ units.

$$\text{area of } \triangle DBC = \frac{1}{2}bh$$
$$= \frac{1}{2}(2n)(j)$$
$$= nj \text{ square units}$$

Since $nj = \frac{1}{2}(2nj)$, the area of $\triangle DBC$ is one half the area of $\triangle ABC$.

Remember!

Because the x- and y-axes intersect at right angles, they can be used to form the sides of a right triangle.

CHECK IT OUT!

4. Use the information in Example 4 to write a coordinate proof showing that the area of $\triangle ADB$ is one half the area of $\triangle ABC$.

THINK AND DISCUSS

1. When writing a coordinate proof why are variables used instead of numbers as coordinates for the vertices of a figure?

2. How does the way you position a figure in the coordinate plane affect your calculations in a coordinate proof?

3. Explain why it might be useful to assign $2p$ as a coordinate instead of just p.

Know it! Note

4. GET ORGANIZED Copy and complete the graphic organizer. In each row, draw an example of each strategy that might be used when positioning a figure for a coordinate proof.

Positioning Strategy	Example
Use origin as a vertex.	
Center figure at origin.	
Center side of figure at origin.	
Use axes as sides of figure.	

4-7 Exercises

California Standards
5.0, 7.0, 17.0, 7AF1.0, 7AF2.0, 7MG2.1, 7MG3.2, 7MG3.4, 7MR2.3, 1A2.0, 1A5.0

go.hrw.com
Homework Help Online
KEYWORD: MG7 4-7
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

1. **Vocabulary** What is the relationship between *coordinate geometry*, *coordinate plane*, and *coordinate proof*?

SEE EXAMPLE 1 p. 267

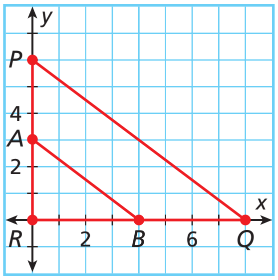
Position each figure in the coordinate plane.

2. a rectangle with a length of 4 units and width of 1 unit
3. a right triangle with leg lengths of 1 unit and 3 units

SEE EXAMPLE 2 p. 268

Write a proof using coordinate geometry.

4. **Given:** Right $\triangle PQR$ has coordinates $P(0, 6)$, $Q(8, 0)$, and $R(0, 0)$. A is the midpoint of $\overline{PR}$. B is the midpoint of $\overline{QR}$.
Prove: $AB = \frac{1}{2}PQ$



SEE EXAMPLE 3 p. 268

Position each figure in the coordinate plane and give the coordinates of each vertex.

5. a right triangle with leg lengths m and n
6. a rectangle with length a and width b

SEE EXAMPLE 4 p. 269

Multi-Step **Assign coordinates to each vertex and write a coordinate proof.**

7. **Given:** $\angle R$ is a right angle in $\triangle PQR$. A is the midpoint of $\overline{PR}$. B is the midpoint of $\overline{QR}$.
Prove: $AB = \frac{1}{2}PQ$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
8–9	1
10	2
11–12	3
13	4

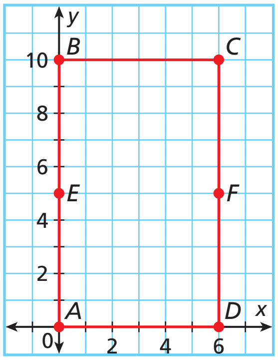
Extra Practice
Skills Practice p. S11
Application Practice p. S31

Position each figure in the coordinate plane.

8. a square with side lengths of 2 units
9. a right triangle with leg lengths of 1 unit and 5 units

Write a proof using coordinate geometry.

10. **Given:** Rectangle $ABCD$ has coordinates $A(0, 0)$, $B(0, 10)$, $C(6, 10)$, and $D(6, 0)$. E is the midpoint of $\overline{AB}$, and F is the midpoint of $\overline{CD}$.
Prove: $EF = BC$



Position each figure in the coordinate plane and give the coordinates of each vertex.

11. a square with side length $2m$
12. a rectangle with dimensions x and $3x$

Multi-Step **Assign coordinates to each vertex and write a coordinate proof.**

13. **Given:** E is the midpoint of $\overline{AB}$ in rectangle $ABCD$. F is the midpoint of $\overline{CD}$.
Prove: $EF = AD$

14. **Critical Thinking** Use variables to write the general form of the endpoints of a segment whose midpoint is $(0, 0)$.

15. Recreation A hiking trail begins at $E(0, 0)$. Bryan hikes from the start of the trail to a waterfall at $W(3, 3)$ and then makes a 90° turn to a campsite at $C(6, 0)$.

a. Draw Bryan's route in the coordinate plane.

b. If one grid unit represents 1 mile, what is the total distance Bryan hiked? Round to the nearest tenth.

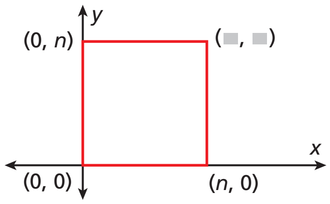
Find the perimeter and area of each figure.

16. a right triangle with leg lengths of a and $2a$ units

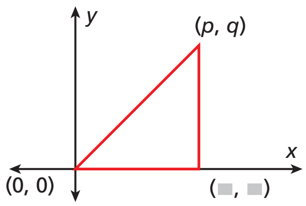
17. a rectangle with dimensions s and t units

Find the missing coordinates for each figure.

18.

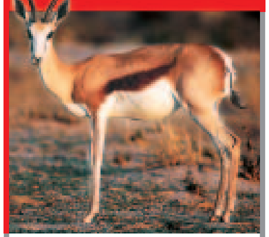


19.



LINK

Conservation



The origin of the springbok's name may come from its habit of *pronking*, or bouncing. When pronking, a springbok can leap up to 13 feet in the air. Springboks can run up to 53 miles per hour.

20. Conservation The Bushmen have sighted animals at the following coordinates: $(-25, 31.5)$, $(-23.2, 31.4)$, and $(-24, 31.1)$. Prove that the distance between two of these locations is approximately twice the distance between two other.

21. Navigation Two ships depart from a port at $P(20, 10)$. The first ship travels to a location at $A(-30, 50)$, and the second ship travels to a location at $B(70, -30)$. Each unit represents one nautical mile. Find the distance to the nearest nautical mile between the two ships. Verify that the port is at the midpoint between the two.

Write a coordinate proof.

22. Given: Rectangle $PQRS$ has coordinates $P(0, 2)$, $Q(3, 2)$, $R(3, 0)$, and $S(0, 0)$. $\overline{PR}$ and $\overline{QS}$ intersect at $T(1.5, 1)$.
Prove: The area of $\triangle RST$ is $\frac{1}{4}$ of the area of the rectangle.

23. Given: $A(x_1, y_1)$, $B(x_2, y_2)$, with midpoint $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Prove: $AM = \frac{1}{2}AB$

24. Plot the points on a coordinate plane and connect them to form $\triangle KLM$ and $\triangle MPK$. Write a coordinate proof.

Given: $K(-2, 1)$, $L(-2, 3)$, $M(1, 3)$, $P(1, 1)$
Prove: $\triangle KLM \cong \triangle MPK$

25. Write About It When you place two sides of a figure on the coordinate axes, what are you assuming about the figure?

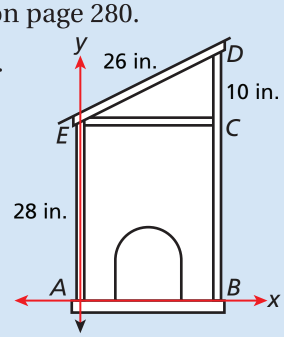
Concept Connection

26. This problem will prepare you for the Concept Connection on page 280.

Paul designed a doghouse to fit against the side of his house. His plan consisted of a right triangle on top of a rectangle.

a. Find BD and CE.

b. Before building the doghouse, Paul sketched his plan on a coordinate plane. He placed A at the origin and $\overline{AB}$ on the x-axis. Find the coordinates of B, C, D, and E, assuming that each unit of the coordinate plane represents one inch.



Standardized Test Prep

27. The coordinates of the vertices of a right triangle are (0, 0), (4, 0), and (0, 2). Which is a true statement?

(A) The vertex of the right angle is at (4, 2).

(B) The midpoints of the two legs are at (2, 0) and (0, 1).

(C) The hypotenuse of the triangle is $\sqrt{6}$ units.

(D) The shortest side of the triangle is positioned on the *x*-axis.

28. A rectangle has dimensions of $2g$ and $2f$ units. If one vertex is at the origin, which coordinates could NOT represent another vertex?

(F) $(2f, g)$ (G) $(2f, 0)$ (H) $(2g, 2f)$ (J) $(-2f, 2g)$

29. The coordinates of the vertices of a rectangle are (0, 0), $(a, 0)$, (a, b), and $(0, b)$. What is the perimeter of the rectangle?

(A) $a + b$ (B) ab (C) $\frac{1}{2}ab$ (D) $2a + 2b$

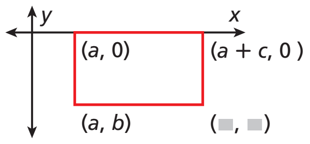
30. A coordinate grid is placed over a map. City A is located at $(-1, 2)$ and city C is located at (3, 5). If city C is at the midpoint between city A and city B, what are the coordinates of city B?

(F) (1, 3.5) (G) $(-5, -1)$ (H) (7, 8) (J) (2, 7)

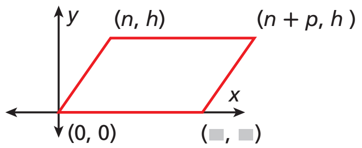
CHALLENGE AND EXTEND

Find the missing coordinates for each figure.

31.

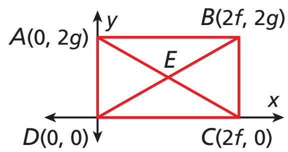


32.



33. The vertices of a right triangle are at $(-2s, 2s)$, $(0, 2s)$, and $(0, 0)$. What coordinates could be used so that a coordinate proof would be easier to complete?

34. Rectangle $ABCD$ has dimensions of $2f$ and $2g$ units. The equation of the line containing $\overline{BD}$ is $y = \frac{g}{f}x$, and the equation of the line containing $\overline{AC}$ is $y = -\frac{g}{f}x + 2g$. Use algebra to show that the coordinates of E are (f, g).



SPIRAL REVIEW

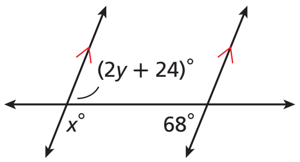
Use the quadratic formula to solve for *x*. Round to the nearest hundredth if necessary. *(Previous course)*

35. $0 = 8x^2 + 18x - 5$ **36.** $0 = x^2 + 3x - 5$ **37.** $0 = 3x^2 - x - 10$

Find each value. *(Lesson 3-2)*

38. x

39. y



40. Use $A(-4, 3)$, $B(-1, 3)$, $C(-3, 1)$, $D(0, -2)$, $E(3, -2)$, and $F(2, -4)$ to prove $\angle ABC \cong \angle EDF$. *(Lesson 4-6)*.

4-8 Isosceles and Equilateral Triangles

Objectives

Prove theorems about isosceles and equilateral triangles.

Apply properties of isosceles and equilateral triangles.

Vocabulary

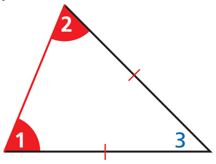
legs of an isosceles triangle
vertex angle
base
base angles

Who uses this?

Astronomers use geometric methods. (See Example 1.)

Recall that an isosceles triangle has at least two congruent sides. The congruent sides are called the **legs**. The **vertex angle** is the angle formed by the legs. The side opposite the vertex angle is called the **base**, and the **base angles** are the two angles that have the base as a side.

∠3 is the vertex angle.
∠1 and ∠2 are the base angles.



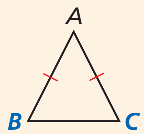
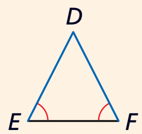
California Standards

12.0 Students find and use measures of sides and of interior and exterior **angles of triangles** and polygons **to** classify figures and **solve problems.**

Also covered: **2.0, 4.0, 17.0**

Know it! Note

Theorems Isosceles Triangle

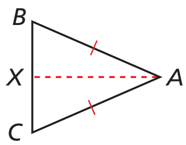
THEOREM		HYPOTHESIS	CONCLUSION
4-8-1	**Isosceles Triangle Theorem** If two sides of a triangle are congruent, then the angles opposite the sides are congruent.		∠B ≅ ∠C
4-8-2	**Converse of Isosceles Triangle Theorem** If two angles of a triangle are congruent, then the sides opposite those angles are congruent.		$\overline{DE} \cong \overline{DF}$

Theorem 4-8-1 is proven below. You will prove Theorem 4-8-2 in Exercise 35.

PROOF

Isosceles Triangle Theorem

Given: $\overline{AB} \cong \overline{AC}$
Prove: ∠B ≅ ∠C



Proof:

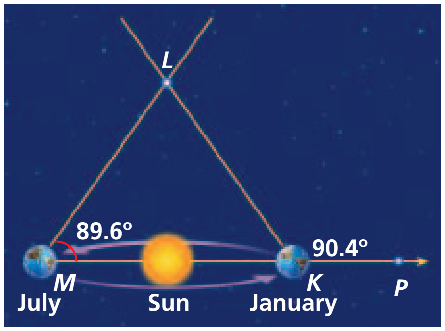
Statements	Reasons
1. Draw *X*, the mdpt. of $\overline{BC}$.	1. Every seg. has a unique mdpt.
2. Draw the auxiliary line $\overline{AX}$.	2. Through two pts. there is exactly one line.
3. $\overline{BX} \cong \overline{CX}$	3. Def. of mdpt.
4. $\overline{AB} \cong \overline{AC}$	4. Given
5. $\overline{AX} \cong \overline{AX}$	5. Reflex. Prop. of ≅
6. △ABX ≅ △ACX	6. SSS *Steps 3, 4, 5*
7. ∠B ≅ ∠C	7. CPCTC

Reading Math

The Isosceles Triangle Theorem is sometimes stated as "Base angles of an isosceles triangle are congruent."

EXAMPLE 1 Astronomy Application

The distance from Earth to nearby stars can be measured using the parallax method, which requires observing the positions of a star 6 months apart. If the distance *LM* to a star in July is 4.0×10^{13} km, explain why the distance *LK* to the star in January is the same. (Assume the distance from Earth to the Sun does not change.)



Not drawn to scale

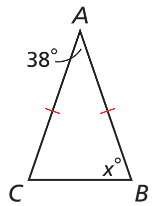
$m\angle LKM = 180 - 90.4$, so $m\angle LKM = 89.6°$. Since $\angle LKM \cong \angle M$, $\triangle LMK$ is isosceles by the Converse of the Isosceles Triangle Theorem. Thus $LK = LM = 4.0 \times 10^{13}$ km.

CHECK IT OUT! **1.** If the distance from Earth to a star in September is 4.2×10^{13} km, what is the distance from Earth to the star in March? Explain.

EXAMPLE 2 Finding the Measure of an Angle

Algebra

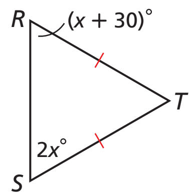
Find each angle measure.



A $m\angle C$

$m\angle C = m\angle B = x°$	*Isosc. △ Thm.*
$m\angle C + m\angle B + m\angle A = 180$	*△ Sum Thm.*
$x + x + 38 = 180$	*Substitute the given values.*
$2x = 142$	*Simplify and subtract 38 from both sides.*
$x = 71$	*Divide both sides by 2.*

Thus $m\angle C = 71°$.



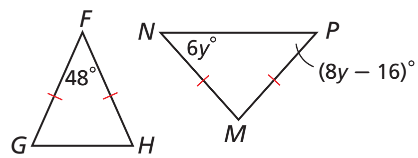
B $m\angle S$

$m\angle S = m\angle R$	*Isosc. △ Thm.*
$2x° = (x + 30)°$	*Substitute the given values.*
$x = 30$	*Subtract x from both sides.*

Thus $m\angle S = 2x° = 2(30) = 60°$.

CHECK IT OUT! **Find each angle measure.**

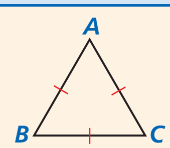
2a. $m\angle H$ **2b.** $m\angle N$



The following corollary and its converse show the connection between equilateral triangles and equiangular triangles.

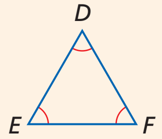
Know it! Note

Corollary 4-8-3 Equilateral Triangle

COROLLARY	HYPOTHESIS	CONCLUSION
If a triangle is equilateral, then it is equiangular. (equilateral △ → equiangular △)		$\angle A \cong \angle B \cong \angle C$

You will prove Corollary 4-8-3 in Exercise 36.

Corollary 4-8-4 Equiangular Triangle

COROLLARY	HYPOTHESIS	CONCLUSION
If a triangle is equiangular, then it is equilateral. (equiangular $\triangle \rightarrow$ equilateral $\triangle$)		$\overline{DE} \cong \overline{DF} \cong \overline{EF}$

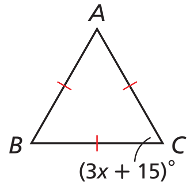
You will prove Corollary 4-8-4 in Exercise 37.

EXAMPLE 3 Using Properties of Equilateral Triangles

Algebra

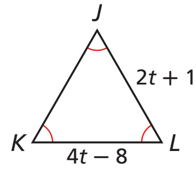
Find each value.

A x



$\triangle ABC$ is equiangular.	*Equilateral $\triangle \rightarrow$ equiangular $\triangle$*
$(3x + 15)^\circ = 60^\circ$	*The measure of each $\angle$ of an equiangular $\triangle$ is 60°.*
$3x = 45$	*Subtract 15 from both sides.*
$x = 15$	*Divide both sides by 3.*

B t



$\triangle JKL$ is equilateral.	*Equiangular $\triangle \rightarrow$ equilateral $\triangle$*
$4t - 8 = 2t + 1$	*Def. of equilateral $\triangle$*
$2t = 9$	*Subtract 2t and add 8 to both sides.*
$t = 4.5$	*Divide both sides by 2.*

CHECK IT OUT!

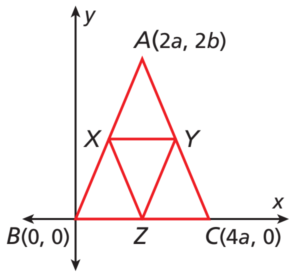
3. Use the diagram to find JL.

EXAMPLE 4 Using Coordinate Proof

Remember!

A coordinate proof may be easier if you place one side of the triangle along the x-axis and locate a vertex at the origin or on the y-axis.

Prove that the triangle whose vertices are the midpoints of the sides of an isosceles triangle is also isosceles.



Given: $\triangle ABC$ is isosceles. X is the mdpt. of $\overline{AB}$. Y is the mdpt. of $\overline{AC}$. Z is the mdpt. of $\overline{BC}$.

Prove: $\triangle XYZ$ is isosceles.

Proof:

Draw a diagram and place the coordinates of $\triangle ABC$ and $\triangle XYZ$ as shown.

By the Midpoint Formula, the coordinates of X are $\left(\frac{2a+0}{2}, \frac{2b+0}{2}\right) = (a, b)$, the coordinates of Y are $\left(\frac{2a+4a}{2}, \frac{2b+0}{2}\right) = (3a, b)$, and the coordinates of Z are $\left(\frac{4a+0}{2}, \frac{0+0}{2}\right) = (2a, 0)$.

By the Distance Formula, $XZ = \sqrt{(2a-a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$, and $YZ = \sqrt{(2a-3a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$.

Since $XZ = YZ$, $\overline{XZ} \cong \overline{YZ}$ by definition. So $\triangle XYZ$ is isosceles.

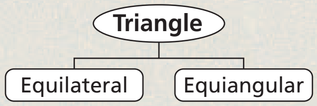
CHECK IT OUT!

4. **What if...?** The coordinates of $\triangle ABC$ are $A(0, 2b)$, $B(-2a, 0)$, and $C(2a, 0)$. Prove $\triangle XYZ$ is isosceles.

THINK AND DISCUSS

Know it! Note

1. Explain why each of the angles in an equilateral triangle measures 60°.
2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, draw and mark a diagram for each type of triangle.



4-8 Exercises

California Standards
2.0, 4.0, 17.0, 7AF4.1, 7MG3.4, 7MR1.2, 7MR2.3, 1A2.0

GUIDED PRACTICE

1. **Vocabulary** Draw isosceles $\triangle JKL$ with $\angle K$ as the vertex angle. Name the legs, base, and base angles of the triangle.

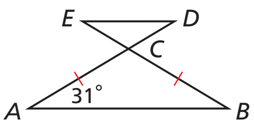
SEE EXAMPLE 1
p. 274

2. **Surveying** To find the distance QR across a river, a surveyor locates three points Q, R, and S. $QS = 41$ m, and $m\angle S = 35°$. The measure of exterior $\angle PQS = 70°$. Draw a diagram and explain how you can find QR.

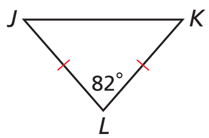
SEE EXAMPLE 2
p. 274

Find each angle measure.

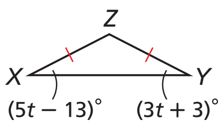
3. $m\angle ECD$



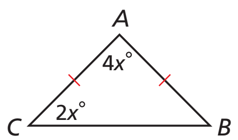
4. $m\angle K$



5. $m\angle X$



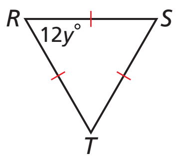
6. $m\angle A$



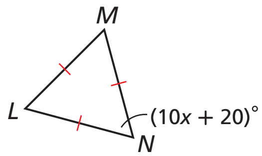
SEE EXAMPLE 3
p. 275

Find each value.

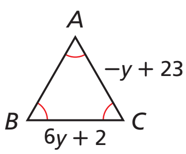
7. y



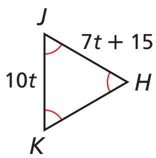
8. x



9. BC



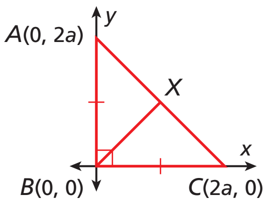
10. JK



SEE EXAMPLE 4
p. 275

11. **Given:** $\triangle ABC$ is right isosceles. X is the midpoint of $\overline{AC}$. $\overline{AB} \cong \overline{BC}$

Prove: $\triangle AXB$ is isosceles.



PRACTICE AND PROBLEM SOLVING

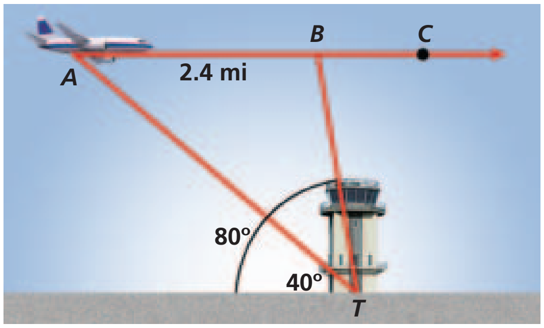
Independent Practice

For Exercises	See Example
12	1
13–16	2
17–20	3
21	4

Extra Practice

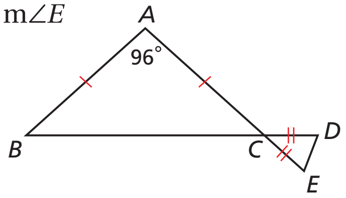
Skills Practice p. S11
Application Practice p. S31

12. Aviation A plane is flying parallel to the ground along $\overrightarrow{AC}$. When the plane is at A, an air-traffic controller in tower T measures the angle to the plane as 40°. After the plane has traveled 2.4 mi to B, the angle to the plane is 80°. How can you find BT?

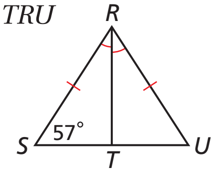


Find each angle measure.

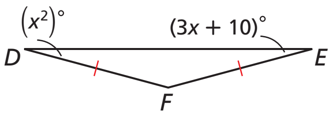
13. m∠E



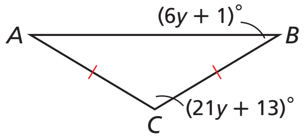
14. m∠TRU



15. m∠F

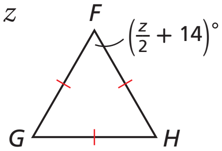


16. m∠A

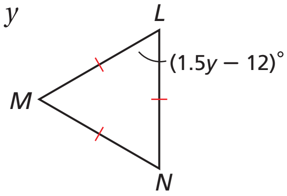


Find each value.

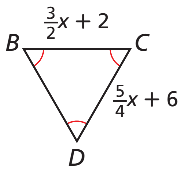
17. z



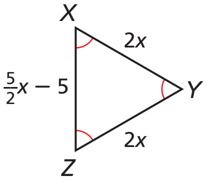
18. y



19. BC

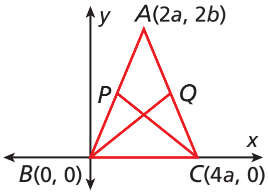


20. XZ



21. Given: $\triangle ABC$ is isosceles. P is the midpoint of $\overline{AB}$. Q is the midpoint of $\overline{AC}$. $\overline{AB} \cong \overline{AC}$

Prove: $\overline{PC} \cong \overline{QB}$



Tell whether each statement is sometimes, always, or never true. Support your answer with a sketch.

22. An equilateral triangle is an isosceles triangle.

23. The vertex angle of an isosceles triangle is congruent to the base angles.

24. An isosceles triangle is a right triangle.

25. An equilateral triangle and an obtuse triangle are congruent.

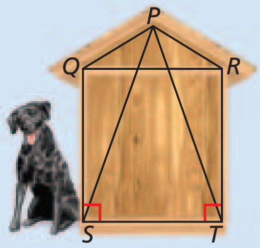
26. Critical Thinking Can a base angle of an isosceles triangle be an obtuse angle? Why or why not?

CONCEPT CONNECTION

27. This problem will prepare you for the Concept Connection page 280.
The diagram shows the inside view of the support structure of the back of a doghouse. $\overline{PQ} \cong \overline{PR}$, $\overline{PS} \cong \overline{PT}$, $m\angle PST = 71°$, and $m\angle QPS = m\angle RPT = 18°$.

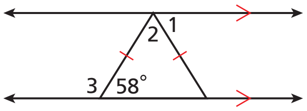
a. Find $m\angle SPT$.

b. Find $m\angle PQR$ and $m\angle PRQ$.

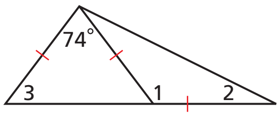


Multi-Step **Find the measure of each numbered angle.**

28.



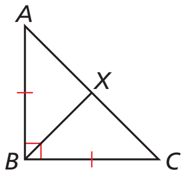
29.



30. Write a coordinate proof.

Given: $\angle B$ is a right angle in isosceles right $\triangle ABC$. X is the midpoint of $\overline{AC}$. $\overline{BA} \cong \overline{BC}$

Prove: $\triangle AXB \cong \triangle CXB$

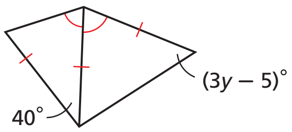


31. **Estimation** Draw the figure formed by $(-2, 1)$, $(5, 5)$, and $(-1, -7)$. Estimate the measure of each angle and make a conjecture about the classification of the figure. Then use a protractor to measure each angle. Was your conjecture correct? Why or why not?

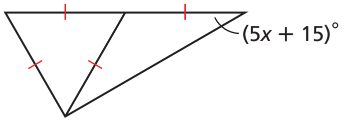
32. How many different isosceles triangles have a perimeter of 18 and sides whose lengths are natural numbers? Explain.

Multi-Step **Find the value of the variable in each diagram.**

33.



34.



LINK

Navigation

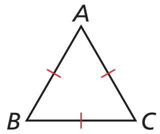
The taffrail log is dragged from the stern of a vessel to measure the speed or distance traveled during a voyage. The log consists of a rotator, recording device, and governor.

35. Prove the Converse of the Isosceles Triangle Theorem.

36. Complete the proof of Corollary 4-8-3.

Given: $\overline{AB} \cong \overline{AC} \cong \overline{BC}$

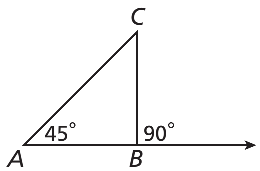
Prove: $\angle A \cong \angle B \cong \angle C$



Proof: Since $\overline{AB} \cong \overline{AC}$, **a.** __?__ by the Isosceles Triangle Theorem. Since $\overline{AC} \cong \overline{BC}$, $\angle A \cong \angle B$ by **b.** __?__ . Therefore $\angle A \cong \angle C$ by **c.** __?__ . By the Transitive Property of $\cong$, $\angle A \cong \angle B \cong \angle C$.

37. Prove Corollary 4-8-4.

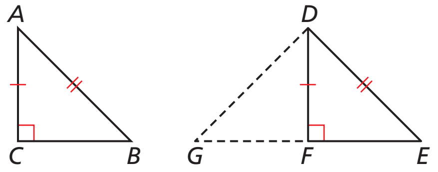
38. **Navigation** The captain of a ship traveling along $\overrightarrow{AB}$ sights an island C at an angle of 45°. The captain measures the distance the ship covers until it reaches B, where the angle to the island is 90°. Explain how to find the distance BC to the island.



39. **Given:** $\triangle ABC \cong \triangle CBA$
Prove: $\triangle ABC$ is isosceles.

40. **Write About It** Write the Isosceles Triangle Theorem and its converse as a biconditional.

41. Rewrite the paragraph proof of the Hypotenuse-Leg (HL) Congruence Theorem as a two-column proof.

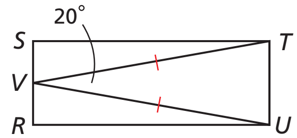


Given: $\triangle ABC$ and $\triangle DEF$ are right triangles.
$\angle C$ and $\angle F$ are right angles.
$\overline{AC} \cong \overline{DF}$, and $\overline{AB} \cong \overline{DE}$.
Prove: $\triangle ABC \cong \triangle DEF$

Proof: On $\triangle DEF$ draw $\overrightarrow{EF}$. Mark G so that $FG = CB$. Thus $\overline{FG} \cong \overline{CB}$. From the diagram, $\overline{AC} \cong \overline{DF}$ and $\angle C$ and $\angle F$ are right angles. $\overline{DF} \perp \overline{EG}$ by definition of perpendicular lines. Thus $\angle DFG$ is a right angle, and $\angle DFG \cong \angle C$. $\triangle ABC \cong \triangle DGF$ by SAS. $\overline{DG} \cong \overline{AB}$ by CPCTC. $\overline{AB} \cong \overline{DE}$ as given. $\overline{DG} \cong \overline{DE}$ by the Transitive Property. By the Isosceles Triangle Theorem $\angle G \cong \angle E$. $\angle DFG \cong \angle DFE$ since right angles are congruent. So $\triangle DGF \cong \triangle DEF$ by AAS. Therefore $\triangle ABC \cong \triangle DEF$ by the Transitive Property.

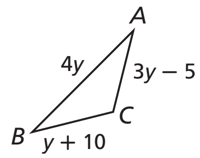
STANDARDIZED TEST PREP

42. Lorena is designing a window so that $\angle R$, $\angle S$, $\angle T$, and $\angle U$ are right angles, $\overline{VU} \cong \overline{VT}$, and $m\angle UVT = 20°$. What is $m\angle RUV$?



Ⓐ 10° Ⓒ 20°
Ⓑ 70° Ⓓ 80°

43. Which of these values of y makes $\triangle ABC$ isosceles?

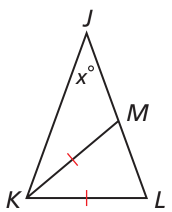


Ⓕ $1\frac{1}{4}$ Ⓗ $7\frac{1}{2}$
Ⓖ $2\frac{1}{2}$ Ⓙ $15\frac{1}{2}$

44. **Gridded Response** The vertex angle of an isosceles triangle measures $(6t - 9)°$, and one of the base angles measures $(4t)°$. Find t.

CHALLENGE AND EXTEND

45. In the figure, $\overline{JK} \cong \overline{JL}$, and $\overline{KM} \cong \overline{KL}$. Let $m\angle J = x°$. Prove $m\angle MKL$ must also be $x°$.



46. An equilateral $\triangle ABC$ is placed on a coordinate plane. Each side length measures $2a$. B is at the origin, and C is at $(2a, 0)$. Find the coordinates of A.

47. An isosceles triangle has coordinates $A(0, 0)$ and $B(a, b)$. What are all possible coordinates of the third vertex?

SPIRAL REVIEW

Find the solutions for each equation. *(Previous course)*

48. $x^2 + 5x + 4 = 0$ **49.** $x^2 - 4x + 3 = 0$ **50.** $x^2 - 2x + 1 = 0$

Find the slope of the line that passes through each pair of points. *(Lesson 3-5)*

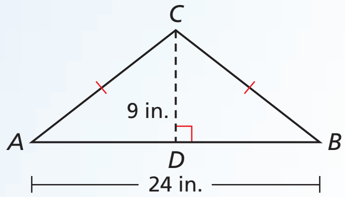
51. $(2, -1)$ and $(0, 5)$ **52.** $(-5, -10)$ and $(20, -10)$ **53.** $(4, 7)$ and $(10, 11)$

54. Position a square with a perimeter of $4s$ in the coordinate plane and give the coordinates of each vertex. *(Lesson 4-7)*

CONCEPT CONNECTION

Proving Triangles Congruent

Gone to the Dogs You are planning to build a doghouse for your dog. The pitched roof of the doghouse will be supported by four trusses. Each truss will be an isosceles triangle with the dimensions shown. To determine the materials you need to purchase and how you will construct the trusses, you must first plan carefully.



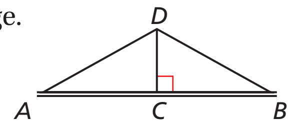
1. You want to be sure that all four trusses are exactly the same size and shape. Explain how you could measure three lengths on each truss to ensure this. Which postulate or theorem are you using?
2. Prove that the two triangular halves of the truss are congruent.
3. What can you say about $\overline{AD}$ and $\overline{DB}$? Why is this true? Use this to help you find the lengths of $\overline{AD}$, $\overline{DB}$, $\overline{AC}$, and $\overline{BC}$.
4. You want to make careful plans on a coordinate plane before you begin your construction of the trusses. Each unit of the coordinate plane represents 1 inch. How could you assign coordinates to vertices A, B, and C?
5. $m\angle ACB = 106°$. What is the measure of each of the acute angles in the truss? Explain how you found your answer.
6. You can buy the wood for the trusses at the building supply store for \$0.80 a foot. The store sells the wood in 6-foot lengths only. How much will you have to spend to get enough wood for the 4 trusses of the doghouse?

READY TO GO ON?

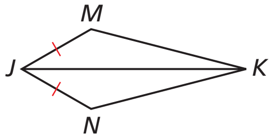
Quiz for Lessons 4-4 Through 4-8

4-4 Triangle Congruence: SSS and SAS

1. The figure shows one tower and the cables of a suspension bridge. Given that $\overline{AC} \cong \overline{BC}$, use SAS to explain why $\triangle ACD \cong \triangle BCD$.



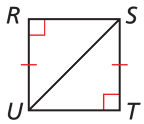
2. **Given:** $\overline{JK}$ bisects $\angle MJN$. $\overline{MJ} \cong \overline{NJ}$
   Prove: $\triangle MJK \cong \triangle NJK$



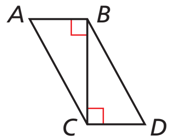
4-5 Triangle Congruence: ASA, AAS, and HL

Determine if you can use the HL Congruence Theorem to prove the triangles congruent. If not, tell what else you need to know.

3. $\triangle RSU$ and $\triangle TUS$



4. $\triangle ABC$ and $\triangle DCB$



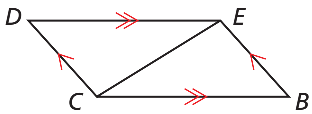
Observers in two lighthouses *K* and *L* spot a ship *S*.

5. Draw a diagram of the triangle formed by the lighthouses and the ship. Label each measure.
6. Is there enough data in the table to pinpoint the location of the ship? Why?

	K to *L*	*K* to *S*	*L* to *S*
Bearing	E	N 58° E	N 77° W
Distance	12 km	?	?

4-6 Triangle Congruence: CPCTC

7. **Given:** $\overline{CD} \parallel \overline{BE}$, $\overline{DE} \parallel \overline{CB}$
   Prove: $\angle D \cong \angle B$



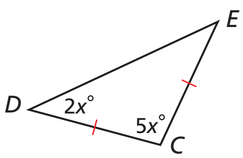
4-7 Introduction to Coordinate Proof

8. Position a square with side lengths of 9 units in the coordinate plane
9. Assign coordinates to each vertex and write a coordinate proof.
   Given: *ABCD* is a rectangle with *M* as the midpoint of $\overline{AB}$. *N* is the midpoint of $\overline{AD}$.
   Prove: The area of $\triangle AMN$ is $\frac{1}{8}$ the area of rectangle *ABCD*.

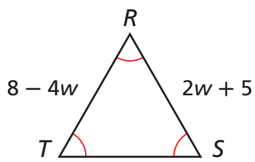
4-8 Isosceles and Equilateral Triangles

Find each value.

10. $m\angle C$



11. *ST*



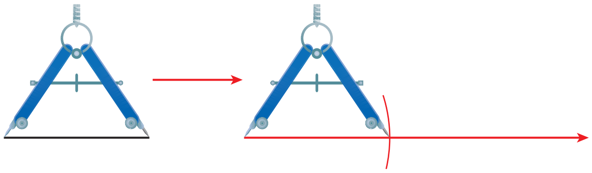
12. **Given:** Isosceles $\triangle JKL$ has coordinates $J(0, 0)$, $K(2a, 2b)$, and $L(4a, 0)$. *M* is the midpoint of $\overline{JK}$, and *N* is the midpoint of $\overline{KL}$.
   Prove: $\triangle KMN$ is isosceles.

EXTENSION Proving Constructions Valid

Objective
Use congruent triangles to prove constructions valid.

California Standards
2.0 Students write geometric proofs, including proofs by contradiction.
Also covered: **5.0**

When performing a compass and straight edge construction, the compass setting remains the same width until you change it. This fact allows you to construct a segment congruent to a given segment. You can assume that two distances constructed with the same compass setting are congruent.

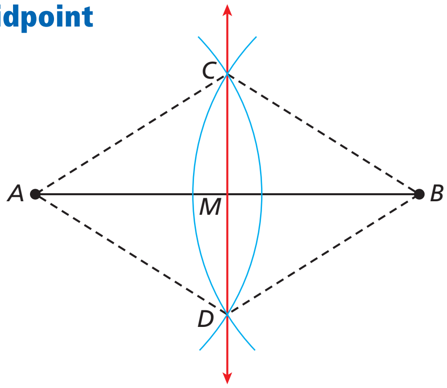


The steps in the construction of a figure can be justified by combining the assumptions of compass and straightedge constructions and the postulates and theorems that are used for proving triangles congruent.

You have learned that there exists exactly one midpoint on any line segment. The proof below justifies the construction of a midpoint.

EXAMPLE 1 **Proving the Construction of a Midpoint**

Given: diagram showing the steps in the construction
Prove: M is the midpoint of $\overline{AB}$.



Remember!
To construct a midpoint, see the construction of a perpendicular bisector on p. 172.

Proof:

Statements	Reasons
1. Draw $\overline{AC}$, $\overline{BC}$, $\overline{AD}$, and $\overline{BD}$.	1. Through any two pts. there is exactly one line.
2. $\overline{AC} \cong \overline{BC} \cong \overline{AD} \cong \overline{BD}$	2. Same compass setting used
3. $\overline{CD} \cong \overline{CD}$	3. Reflex. Prop. of $\cong$
4. $\triangle ACD \cong \triangle BCD$	4. SSS *Steps 2, 3*
5. $\angle ACD \cong \angle BCD$	5. CPCTC
6. $\overline{CM} \cong \overline{CM}$	6. Reflex. Prop. of $\cong$
7. $\triangle ACM \cong \triangle BCM$	7. SAS *Steps 2, 5, 6*
8. $\overline{AM} \cong \overline{BM}$	8. CPCTC
9. M is the midpt. of $\overline{AB}$.	9. Def. of mdpt.

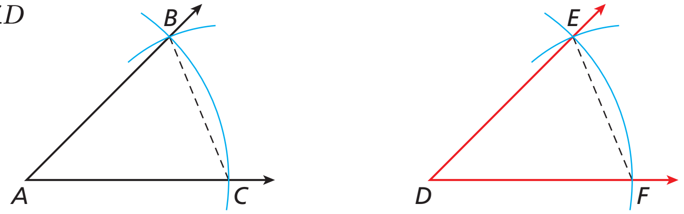
CHECK IT OUT!

1. **Given:** above diagram
   Prove: $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$.

EXAMPLE 2 Proving the Construction of an Angle

Given: diagram showing the steps in the construction

Prove: $\angle A \cong \angle D$



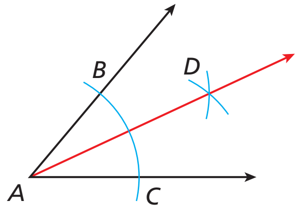
Proof: Since there is a straight line through any two points, you can draw $\overline{BC}$ and $\overline{EF}$. The same compass setting was used to construct $\overline{AC}$, $\overline{AB}$, $\overline{DF}$, and $\overline{DE}$, so $\overline{AC} \cong \overline{AB} \cong \overline{DF} \cong \overline{DE}$. The same compass setting was used to construct $\overline{BC}$ and $\overline{EF}$, so $\overline{BC} \cong \overline{EF}$. Therefore $\triangle BAC \cong \triangle EDF$ by SSS, and $\angle A \cong \angle D$ by CPCTC.

Remember!

To review the construction of an angle congruent to another angle, see page 22.

CHECK IT OUT!

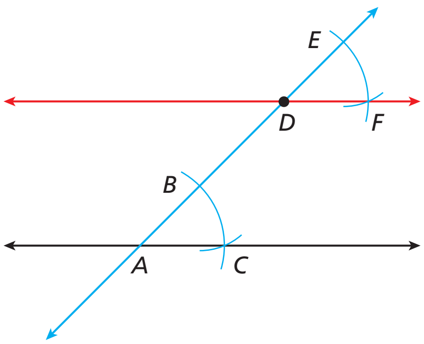
2. Prove the construction for bisecting an angle. (See page 23.)



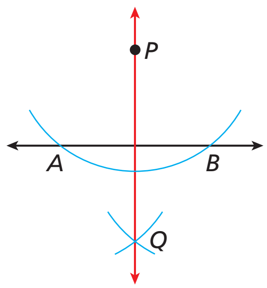
EXTENSION Exercises

Use each diagram to prove the construction valid.

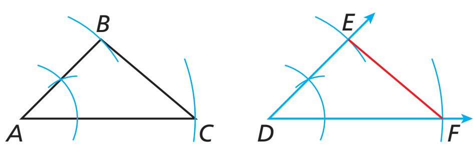
1. parallel lines
(See page 163 and page 170.)



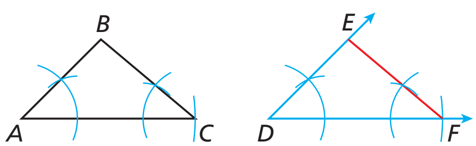
2. a perpendicular through a point not on the line (See page 179.)



3. constructing a triangle using SAS
(See page 243.)



4. constructing a triangle using ASA
(See page 253.)



CHAPTER 4

Study Guide: Review

Know it! Note — For a complete list of the postulates and theorems in this chapter, see p. S82.

Vocabulary

acute triangle 216
auxiliary line 223
base 273
base angle 273
congruent polygons 231
coordinate proof 267
corollary 224
corresponding angles 231
corresponding sides 231
CPCTC 260
equiangular triangle 216
equilateral triangle 217
exterior 225
exterior angle 225
included angle 242
included side 252
interior 225
interior angle 225
isosceles triangle 217
legs of an isosceles triangle . . 273
obtuse triangle 216
remote interior angle 225
right triangle 216
scalene triangle 217
triangle rigidity 242
vertex angle 273

Complete the sentences below with vocabulary words from the list above.

1. A(n) __?__ is a triangle with at least two congruent sides.

2. A name given to matching angles of congruent triangles is __?__ .

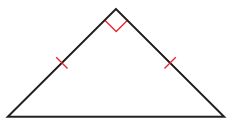
3. A(n) __?__ is the common side of two consecutive angles in a polygon.

4-1 Classifying Triangles *(pp. 216–221)*

12.0

EXAMPLE

- **Classify the triangle by its angle measures and side lengths.**

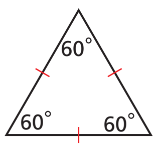


isosceles right triangle

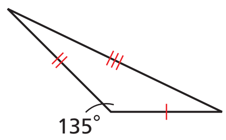
EXERCISES

Classify each triangle by its angle measures and side lengths.

4.



5.

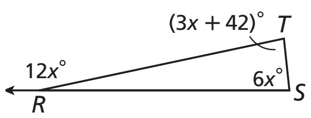


4-2 Angle Relationships in Triangles *(pp. 223–230)*

2.0, 12.0, 13.0

EXAMPLE

- **Find m∠S.**



$12x = 3x + 42 + 6x$

$12x = 9x + 42$

$3x = 42$

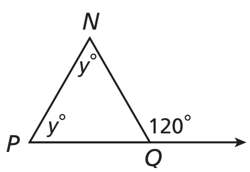
$x = 14$

$m\angle S = 6(14) = 84°$

EXERCISES

Find m∠N.

6.



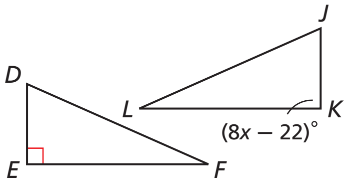
7. In $\triangle LMN$, $m\angle L = 8x°$, $m\angle M = (2x + 1)°$, and $m\angle N = (6x - 1)°$.

4-3 Congruent Triangles *(pp. 231–237)*

2.0, 5.0

EXAMPLE

- **Given: $\triangle DEF \cong \triangle JKL$. Identify all pairs of congruent corresponding parts. Then find the value of x.**



The congruent pairs follow: $\angle D \cong \angle J$, $\angle E \cong \angle K$, $\angle F \cong \angle L$, $\overline{DE} \cong \overline{JK}$, $\overline{EF} \cong \overline{KL}$, and $\overline{DF} \cong \overline{JL}$.

Since $m\angle E = m\angle K$, $90 = 8x - 22$. After 22 is added to both sides, $112 = 8x$. So $x = 14$.

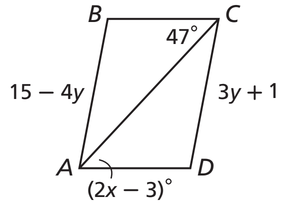
EXERCISES

Given: $\triangle PQR \cong \triangle XYZ$. Identify the congruent corresponding parts.

8. $\overline{PR} \cong$ __?__

9. $\angle Y \cong$ __?__

Given: $\triangle ABC \cong \triangle CDA$
Find each value.



10. x

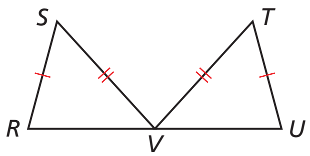
11. CD

4-4 Triangle Congruence: SSS and SAS *(pp. 242–249)*

2.0, 5.0, 16.0

EXAMPLES

- **Given: $\overline{RS} \cong \overline{UT}$, and $\overline{VS} \cong \overline{VT}$. V is the midpoint of $\overline{RU}$.**

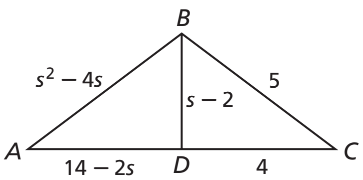


Prove: $\triangle RSV \cong \triangle UTV$

Proof:

Statements	Reasons
1. $\overline{RS} \cong \overline{UT}$	**1.** Given
2. $\overline{VS} \cong \overline{VT}$	**2.** Given
3. *V is the mdpt. of* $\overline{RU}$.	**3.** Given
4. $\overline{RV} \cong \overline{UV}$	**4.** Def. of mdpt.
5. $\triangle RSV \cong \triangle UTV$	**5.** SSS *Steps 1, 2, 4*

- **Show that $\triangle ADB \cong \triangle CDB$ when s = 5.**



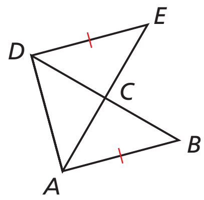
$AB = s^2 - 4s$
$= 5^2 - 4(5)$
$= 5$

$AD = 14 - 2s$
$= 14 - 2(5)$
$= 4$

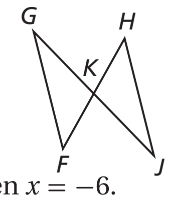
$\overline{BD} \cong \overline{BD}$ by the Reflexive Property. $\overline{AD} \cong \overline{CD}$ and $\overline{AB} \cong \overline{CB}$. So $\triangle ADB \cong \triangle CDB$ by SSS.

EXERCISES

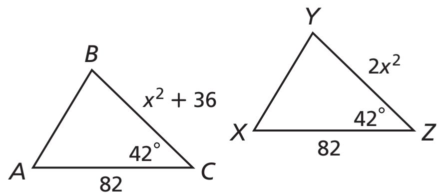
12. Given: $\overline{AB} \cong \overline{DE}$, $\overline{DB} \cong \overline{AE}$
Prove: $\triangle ADB \cong \triangle DAE$



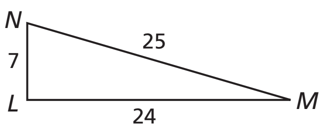
13. Given: $\overline{GJ}$ bisects $\overline{FH}$, and $\overline{FH}$ bisects $\overline{GJ}$.
Prove: $\triangle FGK \cong \triangle HJK$

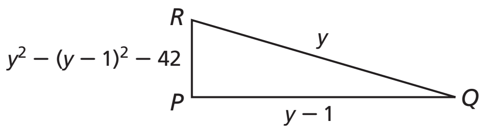


14. Show that $\triangle ABC \cong \triangle XYZ$ when $x = -6$.



15. Show that $\triangle LMN \cong \triangle PQR$ when $y = 25$.



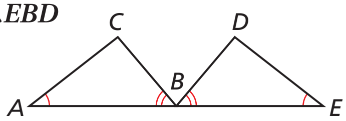


4-5 Triangle Congruence: ASA, AAS, and HL *(pp. 252–259)*

2.0, 4.0, 5.0, 16.0

EXAMPLES

■ **Given:** B is the midpoint of $\overline{AE}$.
$\angle A \cong \angle E$,
$\angle ABC \cong \angle EBD$
Prove: $\triangle ABC \cong \triangle EBD$

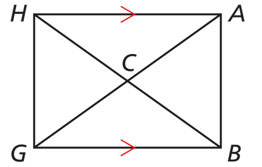


Proof:

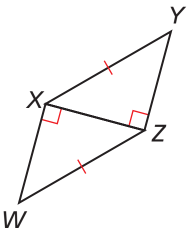
Statements	Reasons
1. $\angle A \cong \angle E$	**1.** Given
2. $\angle ABC \cong \angle EBD$	**2.** Given
3. B is the mdpt. of $\overline{AE}$.	**3.** Given
4. $\overline{AB} \cong \overline{EB}$	**4.** Def. of mdpt.
5. $\triangle ABC \cong \triangle EBD$	**5.** ASA *Steps 1, 4, 2*

EXERCISES

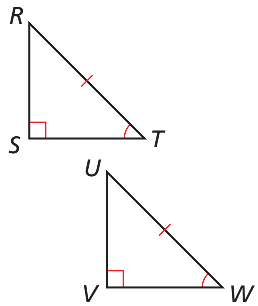
16. **Given:** C is the midpoint of $\overline{AG}$.
$\overline{HA} \parallel \overline{GB}$
Prove: $\triangle HAC \cong \triangle BGC$



17. **Given:** $\overline{WX} \perp \overline{XZ}$,
$\overline{YZ} \perp \overline{ZX}$,
$\overline{WZ} \cong \overline{YX}$
Prove: $\triangle WZX \cong \triangle YXZ$



18. **Given:** $\angle S$ and $\angle V$ are right angles.
$RT = UW$.
$m\angle T = m\angle W$
Prove: $\triangle RST \cong \triangle UVW$

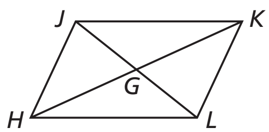


4-6 Triangle Congruence: CPCTC *(pp. 260–265)*

2.0, 5.0

EXAMPLES

■ **Given:** $\overline{JL}$ and $\overline{HK}$ bisect each other.
Prove: $\angle JHG \cong \angle LKG$

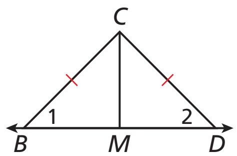


Proof:

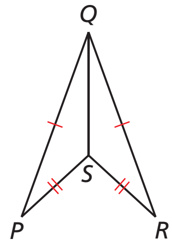
Statements	Reasons
1. $\overline{JL}$ and $\overline{HK}$ bisect each other.	**1.** Given
2. $\overline{JG} \cong \overline{LG}$, and $\overline{HG} \cong \overline{KG}$.	**2.** Def. of bisect
3. $\angle JGH \cong \angle LGK$	**3.** Vert. ∠s Thm.
4. $\triangle JHG \cong \triangle LKG$	**4.** SAS *Steps 2, 3*
5. $\angle JHG \cong \angle LKG$	**5.** CPCTC

EXERCISES

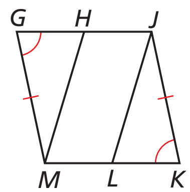
19. **Given:** M is the midpoint of $\overline{BD}$.
$\overline{BC} \cong \overline{DC}$
Prove: $\angle 1 \cong \angle 2$



20. **Given:** $\overline{PQ} \cong \overline{RQ}$,
$\overline{PS} \cong \overline{RS}$
Prove: $\overline{QS}$ bisects $\angle PQR$.



21. **Given:** H is the midpoint of $\overline{GJ}$.
L is the midpoint of $\overline{MK}$.
$\overline{GM} \cong \overline{KJ}$, $\overline{GJ} \cong \overline{KM}$,
$\angle G \cong \angle K$
Prove: $\angle GMH \cong \angle KJL$



4-7 Introduction to Coordinate Proof *(pp. 267–272)*

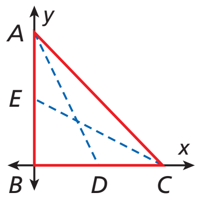
17.0

EXAMPLES

- **Given:** $\angle B$ **is a right angle in isosceles right** $\triangle ABC$. E **is the midpoint of** $\overline{AB}$. D **is the midpoint of** $\overline{CB}$. $\overline{AB} \cong \overline{CB}$

Prove: $\overline{CE} \cong \overline{AD}$

Proof: Use the coordinates $A(0, 2a)$, $B(0, 0)$, and $C(2a, 0)$. Draw $\overline{AD}$ and $\overline{CE}$.



By the Midpoint Formula,

$$E = \left(\frac{0+0}{2}, \frac{2a+0}{2}\right) = (0, a) \text{ and}$$

$$D = \left(\frac{0+2a}{2}, \frac{0+0}{2}\right) = (a, 0)$$

By the Distance Formula,

$$CE = \sqrt{(2a-0)^2 + (0-a)^2}$$
$$= \sqrt{4a^2 + a^2} = a\sqrt{5}$$
$$AD = \sqrt{(a-0)^2 + (0-2a)^2}$$
$$= \sqrt{a^2 + 4a^2} = a\sqrt{5}$$

Thus $\overline{CE} \cong \overline{AD}$ by the definition of congruence.

EXERCISES

Position each figure in the coordinate plane and give the coordinates of each vertex.

22. a right triangle with leg lengths r and s

23. a rectangle with length $2p$ and width p

24. a square with side length $8m$

For exercises 25 and 26 assign coordinates to each vertex and write a coordinate proof.

25. Given: In rectangle $ABCD$, E is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{BC}$, G is the midpoint of $\overline{CD}$, and H is the midpoint of $\overline{AD}$.
Prove: $\overline{EF} \cong \overline{GH}$

26. Given: $\triangle PQR$ has a right $\angle Q$. M is the midpoint of $\overline{PR}$.
Prove: $MP = MQ = MR$

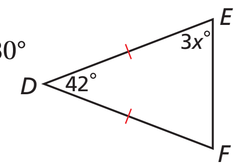
27. Show that a triangle with vertices at $(3, 5)$, $(3, 2)$, and $(2, 5)$ is a right triangle.

4-8 Isosceles and Equilateral Triangles *(pp. 273–279)*

2.0, 4.0, 12.0, 17.0

EXAMPLE

- **Find the value of** x.



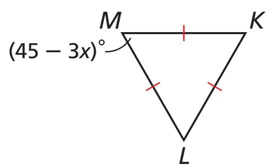
$m\angle D + m\angle E + m\angle F = 180°$ by the Triangle Sum Theorem. $m\angle E = m\angle F$ by the Isosceles Triangle Theorem.

$m\angle D + 2m\angle E = 180°$	*Substitution*
$42 + 2(3x) = 180$	*Substitute the given values.*
$6x = 138$	*Simplify.*
$x = 23$	*Divide both sides by 6.*

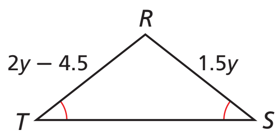
EXERCISES

Find each value.

28. x



29. RS

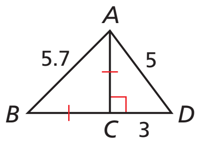


30. Given: $\triangle ACD$ is isosceles with $\angle D$ as the vertex angle. B is the midpoint of $\overline{AC}$. $AB = x + 5$, $BC = 2x - 3$, and $CD = 2x + 6$.
Find the perimeter of $\triangle ACD$.

CHAPTER 4

CHAPTER TEST

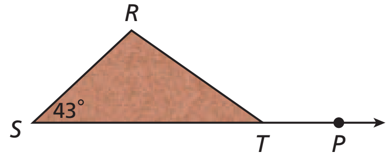
1. Classify $\triangle ACD$ by its angle measures.



Classify each triangle by its side lengths.

2. $\triangle ACD$ **3.** $\triangle ABC$ **4.** $\triangle ABD$

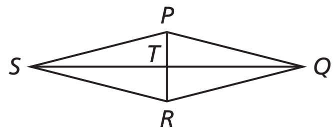
5. While surveying the triangular plot of land shown, a surveyor finds that $m\angle S = 43°$. The measure of $\angle RTP$ is twice that of $\angle RTS$. What is $m\angle R$?



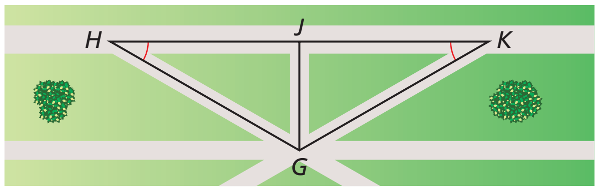
Given: $\triangle XYZ \cong \triangle JKL$
Identify the congruent corresponding parts.

6. $\overline{JL} \cong$ ___?___ **7.** $\angle Y \cong$ ___?___ **8.** $\angle L \cong$ ___?___ **9.** $\overline{YZ} \cong$ ___?___

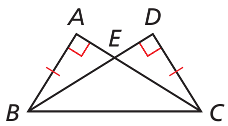
10. Given: T is the midpoint of $\overline{PR}$ and $\overline{SQ}$.
Prove: $\triangle PTS \cong \triangle RTQ$



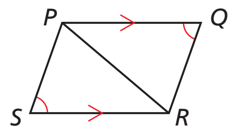
11. The figure represents a walkway with triangular supports. Given that $\overline{GJ}$ bisects $\angle HGK$ and $\angle H \cong \angle K$, use AAS to prove $\triangle HGJ \cong \triangle KGJ$



12. Given: $\overline{AB} \cong \overline{DC}$, $\overline{AB} \perp \overline{AC}$, $\overline{DC} \perp \overline{DB}$
Prove: $\triangle ABC \cong \triangle DCB$



13. Given: $\overline{PQ} \parallel \overline{SR}$, $\angle S \cong \angle Q$
Prove: $\overline{PS} \parallel \overline{QR}$

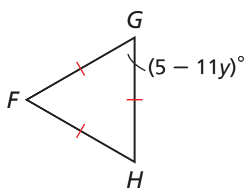


14. Position a right triangle with legs 3 m and 4 m long in the coordinate plane. Give the coordinates of each vertex.

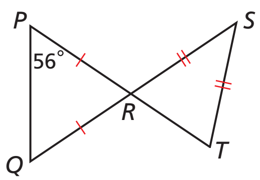
15. Assign coordinates to each vertex and write a coordinate proof.
Given: Square $ABCD$
Prove: $\overline{AC} \cong \overline{BD}$

Find each value.

16. y



17. $m\angle S$



18. Given: Isosceles $\triangle ABC$ has coordinates $A(2a, 0)$, $B(0, 2b)$, and $C(-2a, 0)$. D is the midpoint of $\overline{AC}$, and E is the midpoint of $\overline{AB}$.
Prove: $\triangle AED$ is isosceles.