

Geometric Reasoning

2A Inductive and Deductive Reasoning

- 2-1 Using Inductive Reasoning to Make Conjectures
- 2-2 Conditional Statements
- 2-3 Using Deductive Reasoning to Verify Conjectures
- Lab Solve Logic Puzzles
- 2-4 Biconditional Statements and Definitions

CONCEPT CONNECTION

2B Mathematical Proof

- 2-5 Algebraic Proof
- 2-6 Geometric Proof
- Lab Design Plans for Proofs
- 2-7 Flowchart and Paragraph Proofs
- Ext Introduction to Symbolic Logic

CONCEPT CONNECTION



Chapter Project Online

KEYWORD: MG7 ChProj

Planning and logical thinking were necessary to make the corn maze resemble the Bear Flag of California.

Woodland Corn Maze
Woodland, CA

ARE YOU READY?

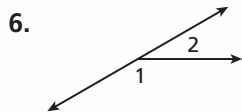
Vocabulary

Match each term on the left with a definition on the right.

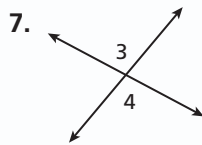
- | | |
|-------------|---|
| 1. angle | A. a straight path that has no thickness and extends forever |
| 2. line | B. a figure formed by two rays with a common endpoint |
| 3. midpoint | C. a flat surface that has no thickness and extends forever |
| 4. plane | D. a part of a line between two points |
| 5. segment | E. names a location and has no size |
| | F. a point that divides a segment into two congruent segments |

Angle Relationships

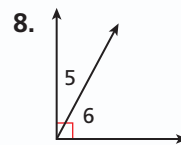
Select the best description for each labeled angle pair.



linear pair or
vertical angles



adjacent angles or
vertical angles



supplementary angles or
complementary angles

Classify Real Numbers

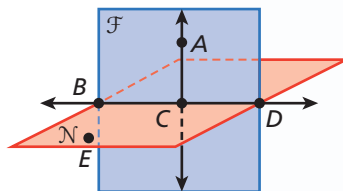
Tell if each number is a natural number, a whole number, an integer, or a rational number. Give all the names that apply.

- | | | |
|-----------|-------------------|----------|
| 9. 6 | 10. -0.8 | 11. -3 |
| 12. 5.2 | 13. $\frac{3}{8}$ | 14. 0 |

Points, Lines, and Planes

Name each of the following.

15. a point
16. a line
17. a ray
18. a segment
19. a plane






Solve One-Step Equations

Solve.

- | | | |
|-----------------|-----------------------|-------------------|
| 20. $8 + x = 5$ | 21. $6y = -12$ | 22. $9 = 6s$ |
| 23. $p - 7 = 9$ | 24. $\frac{z}{5} = 5$ | 25. $8.4 = -1.2r$ |

Unpacking the Standards

The information below “unpacks” the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

California Standard	Academic Vocabulary	Chapter Concept
 1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning. (Lessons 2-1, 2-3, 2-6, 2-7, Extension) (Lab 2-3)	inductive reach a conclusion based on examples deductive reach a conclusion based on laws	You study examples and then write a conclusion based on the pattern you observe. You use the laws of logic to decide if your conjecture is true. You also learn how to write two-column, flowchart, and paragraph proofs to verify your conjectures.
 2.0 Students write geometric proofs, including proofs by contradiction. (Lessons 2-6, 2-7) (Lab 2-6)	geometric relating to the laws and methods of geometry	You review properties from Algebra and then use these properties to write algebraic proofs. You also learn how to write geometric proofs using definitions, postulates, and theorems from Geometry.
 3.0 Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement. (Lessons 2-1, 2-2, 2-4)	valid(ity) legal argument statements that support or are against something	You write the inverse, converse, and contrapositive of a conditional statement. You also write biconditional statements. Then you try to find a counterexample to see if these statements are false.

Reading Strategy: Read and Interpret a Diagram

A diagram is an informational tool. To correctly read a diagram, you must know what you can and cannot assume based on what you see in it.

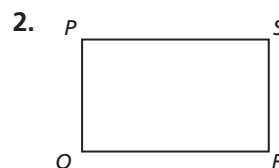
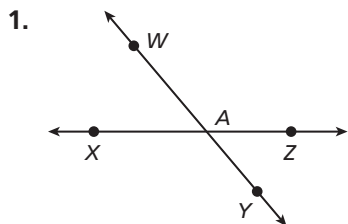
What You CAN Assume	What You CANNOT Assume
<ul style="list-style-type: none"> ✓ Collinear points ✓ Betweenness of points ✓ Coplanar points ✓ Straight angles and lines ✓ Adjacent angles ✓ Linear pairs of angles ✓ Vertical angles 	<ul style="list-style-type: none"> ✗ Measures of segments ✗ Measures of angles ✗ Congruent segments ✗ Congruent angles ✗ Right angles

If a diagram includes labeled information, such as an angle measure or a right angle mark, treat this information as given.

What You See	
What You CAN Assume	What You CANNOT Assume
<ul style="list-style-type: none"> ✓ Points A, B, and C are collinear. ✓ Points A, B, C, and D are coplanar. ✓ B is between A and C. ✓ \overleftrightarrow{AC} is a line. ✓ $\angle ABD$ and $\angle CBD$ are adjacent angles. ✓ $\angle ABD$ and $\angle CBD$ form a linear pair. 	<ul style="list-style-type: none"> ✗ $\angle CBD$ is acute. ✗ $\angle ABD$ is obtuse. ✗ $\overline{AB} \cong \overline{BC}$

Try This

List what you can and cannot assume from each diagram.



2-1

Using Inductive Reasoning to Make Conjectures

Objectives

Use inductive reasoning to identify patterns and make conjectures.

Find counterexamples to disprove conjectures.

Vocabulary

inductive reasoning
conjecture
counterexample

Who uses this?

Biologists use inductive reasoning to develop theories about migration patterns.

Biologists studying the migration patterns of California gray whales developed two theories about the whales' route across Monterey Bay. The whales either swam directly across the bay or followed the shoreline.



EXAMPLE 1 Identifying a Pattern



California Standards

1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.

Also covered: **3.0**

Find the next item in each pattern.

- A** Monday, Wednesday, Friday, ...
Alternating days of the week make up the pattern.
The next day is Sunday.
- B** 3, 6, 9, 12, 15, ...
Multiples of 3 make up the pattern. The next multiple is 18.
- C** \leftarrow , \nearrow , \uparrow , ...
In this pattern, the figure rotates 45° clockwise each time.
The next figure is \nearrow .



1. Find the next item in the pattern 0.4, 0.04, 0.004, ...

When several examples form a pattern and you assume the pattern will continue, you are applying *inductive reasoning*. **Inductive reasoning** is the process of reasoning that a rule or statement is true because specific cases are true. You may use inductive reasoning to draw a conclusion from a pattern. A statement you believe to be true based on inductive reasoning is called a **conjecture**.

EXAMPLE 2 Making a Conjecture

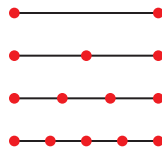
Complete each conjecture.

- A** The product of an even number and an odd number is ?.
List some examples and look for a pattern.
 $(2)(3) = 6$ $(2)(5) = 10$ $(4)(3) = 12$ $(4)(5) = 20$
The product of an even number and an odd number is even.

Complete each conjecture.

B The number of segments formed by n collinear points is ?.

Draw a segment. Mark points on the segment, and count the number of individual segments formed. Be sure to include overlapping segments.



Points	Segments
2	1
3	$2 + 1 = 3$
4	$3 + 2 + 1 = 6$
5	$4 + 3 + 2 + 1 = 10$

The number of segments formed by n collinear points is the sum of the whole numbers less than n .



2. Complete the conjecture: The product of two odd numbers is ?.

EXAMPLE 3 *Biology Application*

To learn about the migration behavior of California gray whales, biologists observed whales along two routes. For seven days they counted the numbers of whales seen along each route. Make a conjecture based on the data.



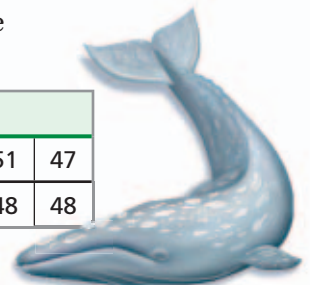
Numbers of Whales Each Day							
Direct Route	1	3	0	2	1	1	0
Shore Route	7	9	5	8	8	6	7

More whales were seen along the shore route each day. The data supports the conjecture that most California gray whales migrate along the shoreline.



3. Make a conjecture about the lengths of male and female whales based on the data.

Average Whale Lengths						
Length of Female (ft)	49	51	50	48	51	47
Length of Male (ft)	47	45	44	46	48	48



To show that a conjecture is always true, you must prove it. To show that a conjecture is false, you have to find only one example in which the conjecture is not true. This case is called a **counterexample**. A counterexample can be a drawing, a statement, or a number.



Inductive Reasoning
1. Look for a pattern
2. Make a conjecture.
3. Prove the conjecture or find a counterexample.

EXAMPLE 4 Finding a Counterexample

Show that each conjecture is false by finding a counterexample.

A For all positive numbers n , $\frac{1}{n} \leq n$.

Pick positive values for n and substitute them into the equation to see if the conjecture holds.

Let $n = 1$. Since $\frac{1}{n} = 1$ and $1 \leq 1$, the conjecture holds.

Let $n = 2$. Since $\frac{1}{n} = \frac{1}{2}$ and $\frac{1}{2} \leq 2$, the conjecture holds.

Let $n = \frac{1}{2}$. Since $\frac{1}{n} = \frac{1}{\frac{1}{2}} = 2$ and $2 \not\leq \frac{1}{2}$, the conjecture is false.

$n = \frac{1}{2}$ is a counterexample.

B For any three points in a plane, there are three different lines that contain two of the points.



Draw three collinear points.

If the three points are collinear, the conjecture is false.

C The temperature in Abilene, Texas, never exceeds 100°F during the spring months (March, April, and May).

Monthly High Temperatures (°F) in Abilene, Texas											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
88	89	97	99	107	109	110	107	106	103	92	89

The temperature in May was 107°F, so the conjecture is false.



Show that each conjecture is false by finding a counterexample.

4a. For any real number x , $x^2 \geq x$.

4b. Supplementary angles are adjacent.

4c. The radius of every planet in the solar system is less than 50,000 km.



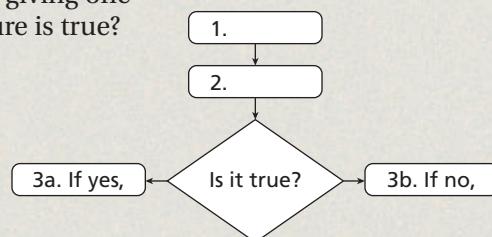
Planets' Diameters (km)

Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
4880	12,100	12,800	6790	143,000	121,000	51,100	49,500

THINK AND DISCUSS

1. Can you prove a conjecture by giving one example in which the conjecture is true? Explain your reasoning.

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe the steps of the inductive reasoning process.





GUIDED PRACTICE

1. **Vocabulary** Explain why a *conjecture* may be true or false.

SEE EXAMPLE 1 Find the next item in each pattern.

p. 74

2. March, May, July, ...

3. $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \dots$ 4. $| \circ |, \overline{\overline{\circ}}, | \circ | \circ |, \dots$

SEE EXAMPLE 2 Complete each conjecture.

p. 74

5. The product of two even numbers is ?.6. A rule in terms of n for the sum of the first n odd positive integers is ?.

SEE EXAMPLE 3

p. 75

7. **Biology** A laboratory culture contains 150 bacteria. After twenty minutes, the culture contains 300 bacteria. After one hour, the culture contains 1200 bacteria. Make a conjecture about the rate at which the bacteria increases.

SEE EXAMPLE 4

p. 76

Show that each conjecture is false by finding a counterexample.

8. Kennedy is the youngest U.S. president to be inaugurated.

9. Three points on a plane always form a triangle.

10. For any real number x , if $x^2 \geq 1$, then $x \geq 1$.

President	Age at Inauguration
Washington	57
T. Roosevelt	42
Truman	60
Kennedy	43
Clinton	46

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
11–13	1
14–15	2
16	3
17–19	4

11–13

1

14–15

2

16

3

17–19

4

Find the next item in each pattern.

11. 8 A.M., 11 A.M., 2 P.M., ...

12. 75, 64, 53, ...

13. $\triangle, \square, \diamond, \dots$

Complete each conjecture.

14. A rule in terms of n for the sum of the first n even positive integers is ?.15. The number of nonoverlapping segments formed by n collinear points is ?.

Extra Practice

Skills Practice p. 56

Application Practice p. S29

16. **Industrial Arts** About 5% of the students at Lincoln High School usually participate in the robotics competition. There are 526 students in the school this year. Make a conjecture about the number of students who will participate in the robotics competition this year.

Show that each conjecture is false by finding a counterexample.

17. If $1 - y > 0$, then $0 < y < 1$.18. For any real number x , $x^3 \geq x^2$.

19. Every pair of supplementary angles includes one obtuse angle.

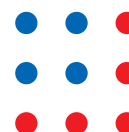
Make a conjecture about each pattern. Write the next two items.

20. 2, 4, 16, ...

21. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$

22. -3, 6, -9, 12, ...

23. Draw a square of dots. Make a conjecture about the number of dots needed to increase the size of the square from $n \times n$ to $(n + 1) \times (n + 1)$.



Determine if each conjecture is true. If not, write or draw a counterexample.

24. Points X , Y , and Z are coplanar.
25. If n is an integer, then $-n$ is positive.
26. In a triangle with one right angle, two of the sides are congruent.
27. If \overrightarrow{BD} bisects $\angle ABC$, then $m\angle ABD = m\angle CBD$.

28. **Estimation** The Westside High School band is selling coupon books to raise money for a trip. The table shows the amount of money raised for the first four days of the sale. If the pattern continues, estimate the amount of money raised during the sixth day.

Day	Money Raised (\$)
1	146.25
2	195.75
3	246.25
4	295.50

29. Write each fraction in the pattern $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \dots$ as a repeating decimal. Then write a description of the fraction pattern and the resulting decimal pattern.

30. **Math History** Remember that a prime number is a whole number greater than 1 that has exactly two factors, itself and 1. Goldbach's conjecture states that every even number greater than 2 can be written as the sum of two primes. For example, $4 = 2 + 2$. Write the next five even numbers as the sum of two primes.

31. The pattern 1, 1, 2, 3, 5, 8, 13, 21, ... is known as the *Fibonacci sequence*. Find the next three terms in the sequence and write a conjecture for the pattern.

32. Look at a monthly calendar and pick any three squares in a row—across, down, or diagonal. Make a conjecture about the number in the middle.

12	13	14
19	20	21
26	27	28

33. Make a conjecture about the value of $2n - 1$ when n is an integer.

34. **Critical Thinking** The turnaround date for migrating gray whales occurs when the number of northbound whales exceeds the number of southbound whales. Make a conjecture about the turnaround date, based on the table below. What factors might affect the validity of your conjecture in the future?

Migration Direction of Gray Whales							
	Feb. 16	Feb. 17	Feb. 18	Feb. 19	Feb. 20	Feb. 21	Feb. 22
Southbound	0	2	3	0	1	1	0
Northbound	0	0	2	5	3	2	1



35. **Write About It** Explain why a true conjecture about even numbers does not necessarily hold for all numbers. Give an example to support your answer.



Math History



Goldbach first stated his conjecture in a letter to Leonhard Euler in 1742. Euler, a Swiss mathematician who published over 800 papers, replied, "I consider [the conjecture] a theorem which is quite true, although I cannot demonstrate it."

CONCEPT CONNECTION



36. This problem will prepare you for the Concept Connection on page 102.
- a. For how many hours did the Mock Turtle do lessons on the third day?
 - b. On what day did the Mock Turtle do 1 hour of lessons?

"And how many hours a day did you do lessons?" said Alice, in a hurry to change the subject.

"Ten hours the first day," said the Mock Turtle: "nine the next, and so on."



37. Which of the following conjectures is false?
- (A) If x is odd, then $x + 1$ is even.
 (B) The sum of two odd numbers is even.
 (C) The difference of two even numbers is positive.
 (D) If x is positive, then $-x$ is negative.
38. A student conjectures that if x is a prime number, then $x + 1$ is not prime. Which of the following is a counterexample?
- (F) $x = 11$ (G) $x = 6$ (H) $x = 3$ (J) $x = 2$
39. The class of 2004 holds a reunion each year. In 2005, 87.5% of the 120 graduates attended. In 2006, 90 students went, and in 2007, 75 students went. About how many students do you predict will go to the reunion in 2010?
- (A) 12 (B) 15 (C) 24 (D) 30

CHALLENGE AND EXTEND

40. **Multi-Step** Make a table of values for the rule $x^2 + x + 11$ when x is an integer from 1 to 8. Make a conjecture about the type of number generated by the rule. Continue your table. What value of x generates a counterexample?
41. **Political Science** Presidential elections are held every four years. U.S. senators are elected to 6-year terms, but only $\frac{1}{3}$ of the Senate is up for election every two years. If $\frac{1}{3}$ of the Senate is elected during a presidential election year, how many years must pass before these same senate seats are up for election during another presidential election year?
42. **Physical Fitness** Rob is training for the President's Challenge physical fitness program. During his first week of training, Rob does 15 sit-ups each day. He will add 20 sit-ups to his daily routine each week. His goal is to reach 150 sit-ups per day.
- Make a table of the number of sit-ups Rob does each week from week 1 through week 10.
 - During which week will Rob reach his goal?
 - Write a conjecture for the number of sit-ups Rob does during week n .
43. **Construction** Draw \overline{AB} . Then construct point C so that it is not on \overline{AB} and is the same distance from A and B . Construct \overline{AC} and \overline{BC} . Compare $m\angle CAB$ and $m\angle CBA$ and compare AC and CB . Make a conjecture.



SPIRAL REVIEW

Determine if the given point is a solution to $y = 3x - 5$. (*Previous course*)

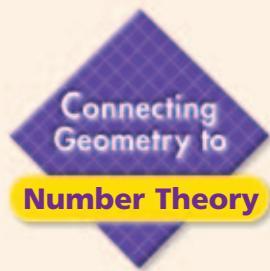
44. $(1, 8)$ 45. $(-2, -11)$ 46. $(3, 4)$ 47. $(-3.5, 0.5)$

Find the perimeter or circumference of each of the following. Leave answers in terms of x . (*Lesson 1-5*)

48. a square whose area is x^2 49. a rectangle with dimensions x and $4x - 3$
 50. a triangle with side lengths of $x + 2$ 51. a circle whose area is $9\pi x^2$

A triangle has vertices $(-1, -1)$, $(0, 1)$, and $(4, 0)$. Find the coordinates for the vertices of the image of the triangle after each transformation. (*Lesson 1-7*)

52. $(x, y) \rightarrow (x, y + 2)$ 53. $(x, y) \rightarrow (x + 4, y - 1)$



Venn Diagrams

See Skills Bank pages 553 and 581

Recall that in a Venn diagram, ovals are used to represent each set. The ovals can overlap if the sets share common elements.

The real number system contains an infinite number of subsets. The following chart shows some of them. Other examples of subsets are even numbers, multiples of 3, and numbers less than 6.

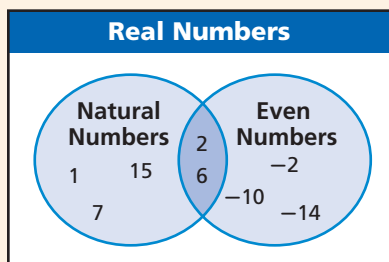
Set	Description	Examples
Natural numbers	The counting numbers	1, 2, 3, 4, 5, ...
Whole numbers	The set of natural numbers and 0	0, 1, 2, 3, 4, ...
Integers	The set of whole numbers and their opposites	..., -2, -1, 0, 1, 2, ...
Rational numbers	The set of numbers that can be written as a ratio of integers	$-\frac{3}{4}$, 5, -2, 0.5, 0
Irrational numbers	The set of numbers that cannot be written as a ratio of integers	π , $\sqrt{10}$, $8 + \sqrt{2}$

Example

Draw a Venn diagram to show the relationship between the set of even numbers and the set of natural numbers.

The set of even numbers includes all numbers that are divisible by 2. This includes natural numbers such as 2, 4, and 6. But even numbers such as -4 and -10 are not natural numbers.

So the set of even numbers includes some, but not all, elements in the set of natural numbers. Similarly, the set of natural numbers includes some, but not all, even numbers.



Draw a rectangle to represent all real numbers.

Draw overlapping ovals to represent the sets of even and natural numbers. You may write individual elements in each region.

Try This

Draw a Venn diagram to show the relationship between the given sets.

1. natural numbers, whole numbers
2. odd numbers, whole numbers
3. irrational numbers, integers

2-2

Conditional Statements



Objectives

Identify, write, and analyze the truth value of conditional statements.

Write the inverse, converse, and contrapositive of a conditional statement.

Vocabulary

conditional statement
hypothesis
conclusion
truth value
negation
converse
inverse
contrapositive
logically equivalent
statements

Why learn this?

To identify a species of butterfly, you must know what characteristics one butterfly species has that another does not.

It is thought that the viceroy butterfly mimics the bad-tasting monarch butterfly to avoid being eaten by birds. By comparing the appearance of the two butterfly species, you can make the following conjecture:

If a butterfly has a curved black line on its hind wing, then it is a viceroy.

Conditional Statements

DEFINITION	SYMBOLS	VENN DIAGRAM
A conditional statement is a statement that can be written in the form "if p , then q ."	$p \rightarrow q$	
The hypothesis is the part p of a conditional statement following the word <i>if</i> .		
The conclusion is the part q of a conditional statement following the word <i>then</i> .		



By phrasing a conjecture as an if-then statement, you can quickly identify its hypothesis and conclusion.

EXAMPLE 1 Identifying the Parts of a Conditional Statement

Writing Math

"If p , then q " can also be written as "if p , q ," " q , if p ," " p implies q ," and " p only if q ."

Identify the hypothesis and conclusion of each conditional.

- A** If a butterfly has a curved black line on its hind wing, then it is a viceroy.
Hypothesis: A butterfly has a curved black line on its hind wing.
Conclusion: The butterfly is a Viceroy.
- B** A number is an integer if it is a natural number.
Hypothesis: A number is a natural number.
Conclusion: The number is an integer.



California Standards

3.0 Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement.



- Identify the hypothesis and conclusion of the statement "A number is divisible by 3 if it is divisible by 6."

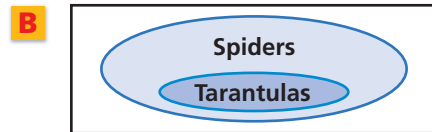
Many sentences without the words *if* and *then* can be written as conditionals. To do so, identify the sentence's hypothesis and conclusion by figuring out which part of the statement depends on the other.

EXAMPLE 2 Writing a Conditional Statement

Write a conditional statement from each of the following.

- A** The midpoint M of a segment bisects the segment.
The **midpoint M of a segment** *Identify the hypothesis and conclusion.*
bisects the segment.

Conditional: If M is the midpoint of a segment,
then M bisects the segment.



The **inner** oval represents the **hypothesis**, and the **outer** oval represents the **conclusion**.

Conditional: If an animal is a tarantula, then it is a spider.



2. Write a conditional statement from the sentence
“Two angles that are complementary are acute.”

A conditional statement has a **truth value** of either true (T) or false (F).
It is false only when the hypothesis is true and the conclusion is false.
Consider the conditional “If I get paid, I will take you to the movie.”
If I don’t get paid, I haven’t broken my promise. So the statement is still true.

To show that a conditional statement is false, you need to find only one counterexample where the hypothesis is true and the conclusion is false.

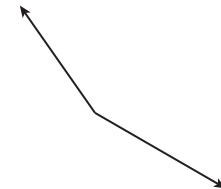
EXAMPLE 3 Analyzing the Truth Value of a Conditional Statement

Determine if each conditional is true. If false, give a counterexample.

- A** If today is Sunday, then tomorrow is Monday.
When the hypothesis is true, the conclusion is also true because Monday follows Sunday. So the conditional is true.

- B** If an angle is obtuse, then it has a measure of 100° .
You can draw an obtuse angle whose measure is not 100° . In this case, the hypothesis is true, but the conclusion is false. Since you can find a counterexample, the conditional is false.

- C** If an odd number is divisible by 2, then 8 is a perfect square.
An odd number is never divisible by 2, so the hypothesis is false. The number 8 is not a perfect square, so the conclusion is false. However, the conditional is true because the hypothesis is false.



Remember!

If the hypothesis is false, the conditional statement is true, regardless of the truth value of the conclusion.



3. Determine if the conditional “If a number is odd, then it is divisible by 3” is true. If false, give a counterexample.

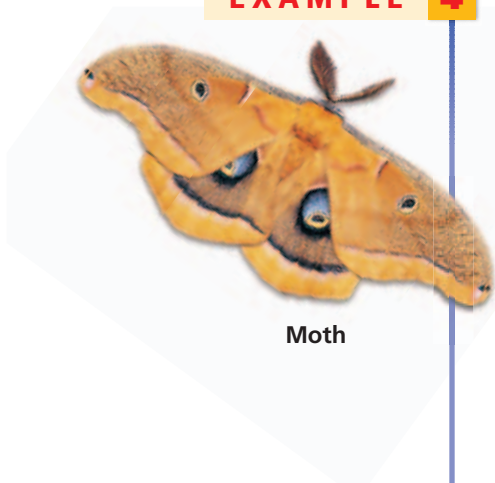
The **negation** of statement p is “not p ,” written as $\sim p$. The negation of the statement “ M is the midpoint of \overline{AB} ” is “ M is *not* the midpoint of \overline{AB} .”
The negation of a true statement is false, and the negation of a false statement is true. Negations are used to write related conditional statements.



Related Conditionals

DEFINITION	SYMBOLS
A conditional is a statement that can be written in the form "If p , then q ."	$p \rightarrow q$
The converse is the statement formed by exchanging the hypothesis and conclusion.	$q \rightarrow p$
The inverse is the statement formed by negating the hypothesis and the conclusion.	$\sim p \rightarrow \sim q$
The contrapositive is the statement formed by both exchanging and negating the hypothesis and conclusion.	$\sim q \rightarrow \sim p$

EXAMPLE 4 Biology Application



Moth

Write the converse, inverse, and contrapositive of the conditional statement. Use the photos to find the truth value of each.

If an insect is a butterfly, then it has four wings.

If **an insect is a butterfly**, then **it has four wings**.

Converse: If **an insect has four wings**, then **it is a butterfly**.

A moth also is an insect with four wings. So the converse is false.

Inverse: If **an insect is not a butterfly**, then **it does not have four wings**.

A moth is not a butterfly, but it has four wings. So the inverse is false.

Contrapositive: If **an insect does not have four wings**, then **it is not a butterfly**.

Butterflies must have four wings. So the contrapositive is true.



Butterfly



4. Write the converse, inverse, and contrapositive of the conditional statement "If an animal is a cat, then it has four paws." Find the truth value of each.

Helpful Hint

The logical equivalence of a conditional and its contrapositive is known as the Law of Contrapositive.

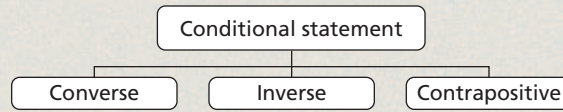
In the example above, the conditional statement and its contrapositive are both true, and the converse and inverse are both false. Related conditional statements that have the same truth value are called **logically equivalent statements**. A conditional and its contrapositive are logically equivalent, and so are the converse and inverse.

Statement	Example	Truth Value
Conditional	If $m\angle A = 95^\circ$, then $\angle A$ is obtuse.	T
Converse	If $\angle A$ is obtuse, then $m\angle A = 95^\circ$.	F
Inverse	If $m\angle A \neq 95^\circ$, then $\angle A$ is not obtuse.	F
Contrapositive	If $\angle A$ is not obtuse, then $m\angle A \neq 95^\circ$.	T

However, the converse of a true conditional is not necessarily false. All four related conditionals can be true, or all four can be false, depending on the statement.

THINK AND DISCUSS

1. If a conditional statement is false, what are the truth values of its hypothesis and conclusion?
2. What is the truth value of a conditional whose hypothesis is false?
3. Can a conditional statement and its converse be logically equivalent? Support your answer with an example.
4. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write the definition and give an example.



2-2

Exercises



California Standards

3.0, 7SDAP1.1, 7MR1.1



go.hrw.com

Homework Help Online

KEYWORD: MG7 2-2

Parent Resources Online

KEYWORD: MG7 Parent

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. The ___?___ of a *conditional statement* is formed by exchanging the hypothesis and conclusion. (*converse, inverse, or contrapositive*)
2. A *conditional* and its *contrapositive* are ___?___ because they have the same truth value. (*logically equivalent or converses*)

SEE EXAMPLE 1

p. 81

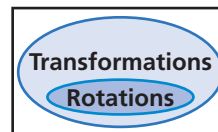
1. Identify the hypothesis and conclusion of each conditional.
 3. If a person is at least 16 years old, then the person can drive a car.
 4. A figure is a parallelogram if it is a rectangle.
 5. The statement $a - b < a$ implies that b is a positive number.

SEE EXAMPLE 2

p. 82

2. Write a conditional statement from each of the following.
 6. Eighteen-year-olds are eligible to vote.
 7. $\left(\frac{a}{b}\right)^2 < \frac{a}{b}$ when $0 < a < b$.

8.



SEE EXAMPLE 3

p. 82

3. Determine if each conditional is true. If false, give a counterexample.
 9. If three points form the vertices of a triangle, then they lie in the same plane.
 10. If $x > y$, then $|x| > |y|$.
 11. If the season is spring, then the month is March.

SEE EXAMPLE 4

p. 83

4. **Travel** Write the converse, inverse, and contrapositive of the following conditional statement. Find the truth value of each.

If Brielle drives at exactly 30 mi/h, then she travels 10 mi in 20 min.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
13–15	1
16–18	2
19–21	3
22–23	4

Extra Practice

Skills Practice p. S6
Application Practice p. S29

Identify the hypothesis and conclusion of each conditional.

13. If an animal is a tabby, then it is a cat.
14. Four angles are formed if two lines intersect.
15. If 8 ounces of cereal cost \$2.99, then 16 ounces of cereal cost \$5.98.

Write a conditional statement from each sentence.

16. You should monitor the heart rate of a patient who is ill.
17. After three strikes, the batter is out.
18. Congruent segments have equal measures.

Determine if each conditional is true. If false, give a counterexample.

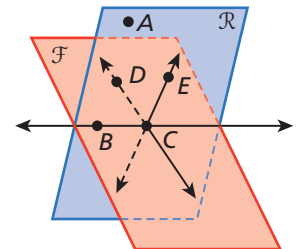
19. If you subtract -2 from -6 , then the result is -4 .
20. If two planes intersect, then they intersect in exactly one point.
21. If a cat is a bird, then today is Friday.

Write the converse, inverse, and contrapositive of each conditional statement. Find the truth value of each.

22. **Probability** If the probability of an event is 0.1, then the event is unlikely to occur.
23. **Meteorology** If freezing rain is falling, then the air temperature is 32°F or less. (*Hint: The freezing point of water is 32°F .*)

Find the truth value of each statement.

- | | |
|--|---|
| <ol style="list-style-type: none"> 24. E lies in plane \mathcal{R}. 26. C, E, and D are coplanar. 28. B and E are collinear. | <ol style="list-style-type: none"> 25. \overleftrightarrow{CD} lies in plane \mathcal{F}. 27. Plane \mathcal{F} contains \overleftrightarrow{ED}. 29. \overleftrightarrow{BC} contains \mathcal{F} and \mathcal{R}. |
|--|---|



Draw a Venn diagram.

- | | |
|--|--|
| <ol style="list-style-type: none"> 30. All integers are rational numbers. 32. All rectangles are quadrilaterals. | <ol style="list-style-type: none"> 31. All natural numbers are real. 33. Plane is an undefined term. |
|--|--|

Write a conditional statement from each Venn diagram.

- | | | |
|--|--|--|
| <ol style="list-style-type: none"> 34. | <ol style="list-style-type: none"> 35. | <ol style="list-style-type: none"> 36. |
|--|--|--|

CONCEPT CONNECTION



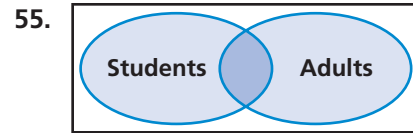
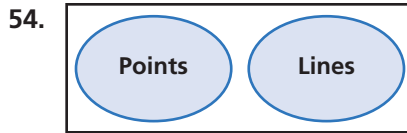
37. This problem will prepare you for the Concept Connection on page 102.
 - a. Identify the hypothesis and conclusion in the Duchess's statement.
 - b. Rewrite the Duchess's claim as a conditional statement.

"Tut, tut, child!" said the Duchess. "Everything's got a moral, if only you can find it." And she squeezed herself up closer to Alice's side as she spoke.



CHALLENGE AND EXTEND

For each Venn diagram, write two statements beginning with *Some*, *All*, or *No*.



56. Given: If a figure is a square, then it is a rectangle. Figure *A* is not a rectangle.
Conclusion: Figure *A* is not a square.

- Draw a Venn diagram to represent the given conditional statement. Use the Venn diagram to explain why the conclusion is valid.
- Write the contrapositive of the given conditional. How can you use the contrapositive to justify the conclusion?

57. **Multi-Step** How many true conditionals can you write using the statements below?
 p : n is an integer. q : n is a whole number. r : n is a natural number.

SPIRAL REVIEW

Write a rule to describe each relationship. (*Previous course*)

58.

x	-8	4	7	9
y	-5	7	10	12

59.

x	-3	-1	0	4
y	-5	-1	1	9

60.

x	-2	0	4	6
y	-9	-4	6	11

Determine whether each statement is true or false. If false, explain why. (*Lesson 1-4*)

- If two angles are complementary and congruent, then the measure of each is 45° .
- A pair of acute angles can be supplementary.
- A linear pair of angles is also a pair of supplementary angles.

Find the next item in each pattern. (*Lesson 2-1*)

64. 1, 13, 131, 1313, ...

65. $2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \dots$

66. $x, 2x^2, 3x^3, 4x^4, \dots$

Career Path



Stephanie Poulin
Desktop Publisher
Daily Reporter

go.hrw.com
Career Resources Online

KEYWORD: MG7 Career

Q: What high school math classes did you take?

A: I took three years of math: Pre-Algebra, Algebra, and Geometry.

Q: What training do you need to be a desktop publisher?

A: Most of my training was done on the job. The computer science and typing classes I took in high school have been helpful.

Q: How do you use math?

A: Part of my job is to make sure all the text, charts, and photographs are formatted to fit the layout of each page. I have to manipulate things by comparing ratios, calculating areas, and using estimation.

Q: What future plans do you have?

A: My goal is to start my own business as a freelance graphic artist.

2-3

Using Deductive Reasoning to Verify Conjectures

Objective

Apply the Law of Detachment and the Law of Syllogism in logical reasoning.

Vocabulary

deductive reasoning

Why learn this?

You can use inductive and deductive reasoning to decide whether a common myth is accurate.

You learned in Lesson 2-1 that one counterexample is enough to disprove a conjecture. But to prove that a conjecture is true, you must use *deductive reasoning*. **Deductive reasoning** is the process of using logic to draw conclusions from given facts, definitions, and properties.

EXAMPLE 1 Media Application



California Standards

1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and **inductive and deductive reasoning**.

Urban legends and modern myths spread quickly through the media. Many Web sites and television shows are dedicated to confirming or disproving such myths. Is each conclusion a result of inductive or deductive reasoning?

A There is a myth that toilets and sinks drain in opposite directions in the Southern and Northern Hemispheres. However, if you were to observe sinks draining in the two hemispheres, you would see that this myth is false. Since the conclusion is based on a pattern of observation, it is a result of inductive reasoning.

B There is a myth that you should not touch a baby bird that has fallen from its nest because the mother bird will disown the baby if she detects human scent. However, biologists have shown that birds cannot detect human scent. Therefore, the myth cannot be true. The conclusion is based on logical reasoning from scientific research. It is a result of deductive reasoning.



1. There is a myth that an eelskin wallet will demagnetize credit cards because the skin of the electric eels used to make the wallet holds an electric charge. However, eelskin products are not made from electric eels. Therefore, the myth cannot be true. Is this conclusion a result of inductive or deductive reasoning?

In deductive reasoning, if the given facts are true and you apply the correct logic, then the conclusion must be true. The Law of Detachment is one valid form of deductive reasoning.



Law of Detachment

If $p \rightarrow q$ is a true statement and p is true, then q is true.

EXAMPLE 2 Verifying Conjectures by Using the Law of Detachment

Determine if each conjecture is valid by the Law of Detachment.

A Given: If two segments are congruent, then they have the same length.
 $\overline{AB} \cong \overline{XY}$.

Conjecture: $AB = XY$

Identify the **hypothesis** and **conclusion** in the given conditional.

If **two segments are congruent**, then **they have the same length**.

The given statement $\overline{AB} \cong \overline{XY}$ matches the hypothesis of a true conditional. By the Law of Detachment $AB = XY$. The conjecture is valid.

B Given: If you are tardy 3 times, you must go to detention.
Shea is in detention.

Conjecture: Shea was tardy at least 3 times.

Identify the **hypothesis** and **conclusion** in the given conditional.

If **you are tardy 3 times**, **you must go to detention**.

The given statement “**Shea is in detention**” matches the conclusion of a true conditional. But this does not mean the hypothesis is true. Shea could be in detention for another reason. The conjecture is not valid.



2. Determine if the conjecture is valid by the Law of Detachment.

Given: If a student passes his classes, the student is eligible to play sports. Ramon passed his classes.

Conjecture: Ramon is eligible to play sports.

Another valid form of deductive reasoning is the Law of Syllogism. It allows you to draw conclusions from two conditional statements when the conclusion of one is the hypothesis of the other.



Law of Syllogism

If $p \rightarrow q$ and $q \rightarrow r$ are true statements, then $p \rightarrow r$ is a true statement.

EXAMPLE 3 Verifying Conjectures by Using the Law of Syllogism

Determine if each conjecture is valid by the Law of Syllogism.

A Given: If $m\angle A < 90^\circ$, then $\angle A$ is acute. If $\angle A$ is acute, then it is not a right angle.

Conjecture: If $m\angle A < 90^\circ$, then it is not a right angle.

Let p , q , and r represent the following.

p : The measure of an angle is less than 90° .

q : The angle is acute.

r : The angle is not a right angle.

You are given that $p \rightarrow q$ and $q \rightarrow r$. Since q is the conclusion of the first conditional and the hypothesis of the second conditional, you can conclude that $p \rightarrow r$. The conjecture is valid by the Law of Syllogism.

Caution!

It is possible to arrive at a true conclusion by applying invalid logical reasoning, as in Example 3B.

Determine if each conjecture is valid by the Law of Syllogism.

- B** Given: If a number is divisible by 4, then it is divisible by 2.
If a number is even, then it is divisible by 2.

Conjecture: If a number is divisible by 4, then it is even.

Let x , y , and z represent the following.

x : A number is divisible by 4.

y : A number is divisible by 2.

z : A number is even.

You are given that $x \rightarrow y$ and $z \rightarrow y$. The Law of Syllogism cannot be used to draw a conclusion since y is the conclusion of both conditionals. Even though the conjecture $x \rightarrow z$ is true, the logic used to draw the conclusion is not valid.



3. Determine if the conjecture is valid by the Law of Syllogism.

Given: If an animal is a mammal, then it has hair.

If an animal is a dog, then it is a mammal.

Conjecture: If an animal is a dog, then it has hair.

EXAMPLE 4 Applying the Laws of Deductive Reasoning

Draw a conclusion from the given information.

- A** Given: If a team wins 10 games, then they play in the finals. If a team plays in the finals, then they travel to Boston. The Ravens won 10 games.

Conclusion: The Ravens will travel to Boston.

- B** Given: If two angles form a linear pair, then they are adjacent.
If two angles are adjacent, then they share a side. $\angle 1$ and $\angle 2$ form a linear pair.

Conclusion: $\angle 1$ and $\angle 2$ share a side.



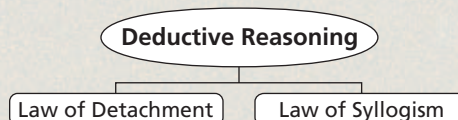
4. Draw a conclusion from the given information.

Given: If a polygon is a triangle, then it has three sides.

If a polygon has three sides, then it is not a quadrilateral. Polygon P is a triangle.

THINK AND DISCUSS

1. Could "A square has exactly two sides" be the conclusion of a valid argument? If so, what do you know about the truth value of the given information?
2. Explain why writing conditional statements as symbols might help you evaluate the validity of an argument.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Write each law in your own words and give an example of each.





GUIDED PRACTICE

1. **Vocabulary** Explain how *deductive reasoning* differs from inductive reasoning.

SEE EXAMPLE 1

p. 88

- Does each conclusion use inductive or deductive reasoning?

2. At Bell High School, students must take Biology before they take Chemistry. Sam is in Chemistry, so Marcia concludes that he has taken Biology.
3. A detective learns that his main suspect was out of town the day of the crime. He concludes that the suspect is innocent.

SEE EXAMPLE 2

p. 89

- Determine if each conjecture is valid by the Law of Detachment.

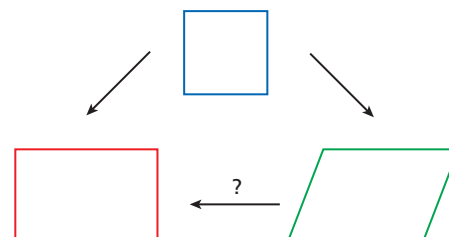
4. Given: If you want to go on a field trip, you must have a signed permission slip. Zola has a signed permission slip.
Conjecture: Zola wants to go on a field trip.
5. Given: If the side lengths of a rectangle are 3 ft and 4 ft, then its area is 12 ft^2 . A rectangle has side lengths of 3 ft and 4 ft.
Conjecture: The area of the rectangle is 12 ft^2 .

SEE EXAMPLE 3

p. 89

- Determine if each conjecture is valid by the Law of Syllogism.

6. Given: If you fly from Texas to California, you travel from the central to the Pacific time zone. If you travel from the central to the Pacific time zone, then you gain two hours.
Conjecture: If you fly from Texas to California, you gain two hours.
7. Given: If a figure is a **square**, then the figure is a **rectangle**. If a figure is a **square**, then it is a **parallelogram**.
Conjecture: If a figure is a **parallelogram**, then it is a **rectangle**.



SEE EXAMPLE 4

p. 90

8. Draw a conclusion from the given information. Given: If you leave your car lights on overnight, then your car battery will drain. If your battery is drained, your car might not start. Alex left his car lights on last night.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
9–10	1
11	2
12	3
13	4

Extra Practice

Skills Practice p. 56

Application Practice p. 529

- Does each conclusion use inductive or deductive reasoning?

9. The sum of the angle measures of a triangle is 180° . Two angles of a triangle measure 40° and 60° , so Kandy concludes that the third angle measures 80° .
10. All of the students in Henry's Geometry class are juniors. Alexander takes Geometry, but has another teacher. Henry concludes that Alexander is also a junior.
11. Determine if the conjecture is valid by the Law of Detachment. Given: If one integer is odd and another integer is even, their product is even. The product of two integers is 24.
Conjecture: One of the two integers is odd.

12. **Science** Determine if the conjecture is valid by the Law of Syllogism.
 Given: If an element is an alkali metal, then it reacts with water. If an element is in the first column of the periodic table, then it is an alkali metal.
 Conjecture: If an element is in the first column of the periodic table, then it reacts with water.
13. Draw a conclusion from the given information.
 Given: If Dakota watches the news, she is informed about current events.
 If Dakota knows about current events, she gets better grades in Social Studies.
 Dakota watches the news.
14. **Technology** Joseph downloads a file in 18 minutes with a dial-up modem. How long would it take to download the file with a Cheetah-Net cable modem?



LINK

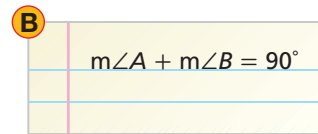
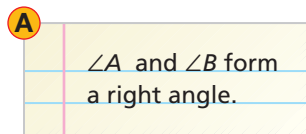
Recreation



The Top Thrill Dragster is 420 feet tall and includes a 400-foot vertical drop. It twists 270° as it drops. It is one of 16 roller coasters at Cedar Point amusement park.

Recreation Use the true statements below for Exercises 15–18. Determine whether each conclusion is valid.

- I. The Top Thrill Dragster is at Cedar Point amusement park in Sandusky, OH.
 - II. Carter and Mary go to Cedar Point.
 - III. The Top Thrill Dragster roller coaster reaches speeds of 120 mi/h.
 - IV. When Carter goes to an amusement park, he rides all the roller coasters.
15. Carter went to Sandusky, OH.
16. Mary rode the Top Thrill Dragster.
17. Carter rode a roller coaster that travels 120 mi/h.
18. Mary rode a roller coaster that travels 120 mi/h.
19. **Critical Thinking** Is the argument below a valid application of the Law of Syllogism? Is the conclusion true? Explain your answers.
 If $3 - x < 5$, then $x < -2$. If $x < -2$, then $-5x > 10$. Thus, if $3 - x < 5$, then $-5x > 10$.
20. **///ERROR ANALYSIS///** Below are two conclusions. Which is incorrect? Explain the error.
 If two angles are complementary, their measures add to 90° . If an angle measures 90° , then it is a right angle. $\angle A$ and $\angle B$ are complementary.



21. **Write About It** Write one example of a real-life logical argument that uses the Law of Detachment and one that uses the Law of Syllogism. Explain why the conclusions are valid.

CONCEPT CONNECTION



22. This problem will prepare you for the Concept Connection on page 102.
 When Alice meets the Pigeon in Wonderland, the Pigeon thinks she is a serpent. The Pigeon reasons that serpents eat eggs, and Alice confirms that she has eaten eggs.
- a. Write “Serpents eat eggs” as a conditional statement.
 - b. Is the Pigeon’s conclusion that Alice is a serpent valid? Explain your reasoning.



23. The Supershots scored over 75 points in each of ten straight games. The newspaper predicts that they will score more than 75 points tonight. Which form of reasoning is this conclusion based on?
- (A) Deductive reasoning, because the conclusion is based on logic
 (B) Deductive reasoning, because the conclusion is based on a pattern
 (C) Inductive reasoning, because the conclusion is based on logic
 (D) Inductive reasoning, because the conclusion is based on a pattern
24. \overrightarrow{HF} bisects $\angle EHG$. Which conclusion is NOT valid?
- (F) $E, F,$ and G are coplanar. (H) $\overline{EF} \cong \overline{FG}$
 (G) $\angle EHF \cong \angle FHG$ (J) $m\angle EHF = m\angle FHG$
-
25. **Gridded Response** If Whitney plays a low G on her piano, the frequency of the note is 24.50 hertz. The frequency of a note doubles with each octave. What is the frequency in hertz of a G note that is 3 octaves above low G?

CHALLENGE AND EXTEND

26. **Political Science** To be eligible to hold the office of the president of the United States, a person must be at least 35 years old, be a natural-born U.S. citizen, and have been a U.S. resident for at least 14 years. Given this information, what conclusion, if any, can be drawn from the statements below? Explain your reasoning.
- Andre is not eligible to be the president of the United States.
 Andre has lived in the United States for 16 years.
27. **Multi-Step** Consider the two conditional statements below.
 If you live in San Diego, then you live in California.
 If you live in California, then you live in the United States.
- Draw a conclusion from the given conditional statements.
 - Write the contrapositive of each conditional statement.
 - Draw a conclusion from the two contrapositives.
 - How does the conclusion in part **a** relate to the conclusion in part **c**?
28. If Cassie goes to the skate park, Hanna and Amy will go. If Hanna or Amy goes to the skate park, then Marc will go. If Marc goes to the skate park, then Dallas will go. If only two of the five people went to the skate park, who were they?

SPIRAL REVIEW

Simplify each expression. (*Previous course*)

29. $2(x + 5)$ 30. $(4y + 6) - (3y - 5)$ 31. $(3c + 4c) + 2(-7c + 7)$

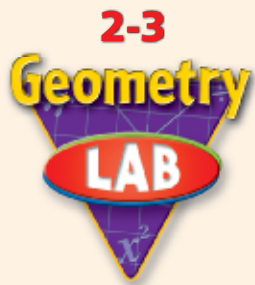
Find the coordinates of the midpoint of the segment connecting each pair of points.

(*Lesson 1-6*)

32. $(1, 2)$ and $(4, 5)$ 33. $(-3, 6)$ and $(0, 1)$ 34. $(-2.5, 9)$ and $(2.5, -3)$

Identify the hypothesis and conclusion of each conditional statement. (*Lesson 2-2*)

35. If the fire alarm rings, then everyone should exit the building.
 36. If two different lines intersect, then they intersect at exactly one point.
 37. The statement $\overline{AB} \cong \overline{CD}$ implies that $AB = CD$.



Solve Logic Puzzles

In Lesson 2-3, you used deductive reasoning to analyze the truth values of conditional statements. Now you will learn some methods for diagramming conditional statements to help you solve logic puzzles.

Use with Lesson 2-3



California Standards

1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.

Activity 1

Bonnie, Cally, Daphne, and Fiona own a bird, cat, dog, and fish. No girl has a type of pet that begins with the same letter as her name. Bonnie is allergic to animal fur. Daphne feeds Fiona's bird when Fiona is away. Make a table to determine who owns which animal.

- 1 Since no girl has a type of pet that starts with the same letter as her name, place an X in each box along the diagonal of the table.

	Bird	Cat	Dog	Fish
Bonnie	X			
Cally		X		
Daphne			X	
Fiona				X

- 2 Bonnie cannot have a cat or dog because of her allergy. So she must own the fish, and no other girl can have the fish.

	Bird	Cat	Dog	Fish
Bonnie	X	X	X	✓
Cally		X		X
Daphne			X	X
Fiona				X

- 3 Fiona owns the bird, so place a check in Fiona's row, in the bird column. Place an X in the remaining boxes in the same column and row.

	Bird	Cat	Dog	Fish
Bonnie	X	X	X	✓
Cally	X	X		X
Daphne	X		X	X
Fiona	✓	X	X	X

- 4 Therefore, Daphne owns the cat, and Cally owns the dog.

	Bird	Cat	Dog	Fish
Bonnie	X	X	X	✓
Cally	X	X	✓	X
Daphne	X	✓	X	X
Fiona	✓	X	X	X

Try This

- After figuring out that Fiona owns the bird in Step 3, why can you place an X in every other box in that row and column?
- Ally, Emily, Misha, and Tracy go to a dance with Danny, Frank, Jude, and Kian. Ally and Frank are siblings. Jude and Kian are roommates. Misha does not know Kian. Emily goes with Kian's roommate. Tracy goes with Ally's brother. Who went to the dance with whom?

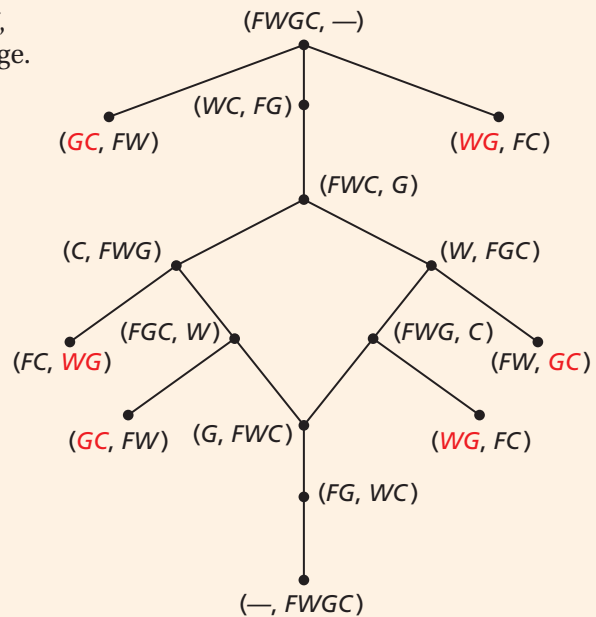
	Danny	Frank	Jude	Kian
Ally				
Emily				
Misha				
Tracy				

Activity 2

A farmer has a goat, a wolf, and a cabbage. He wants to transport all three from one side of a river to the other. He has a boat, but it has only enough room for the farmer and one thing. The wolf will eat the goat if they are left alone together, and the goat will eat the cabbage if they are left alone. How can the farmer get everything to the other side of the river?

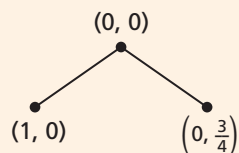
You can use a *network* to solve this kind of puzzle. A **network** is a diagram of *vertices* and *edges*, also known as a graph. An **edge** is a curve or a segment that joins two *vertices* of the graph. A **vertex** is a point on the graph.

- Let F represent the farmer, W represent the wolf, G represent the goat, and C represent the cabbage. Use an ordered pair to represent what is on each side of the river. The first ordered pair is $(FWGC, -)$, and the desired result is $(-, FWGC)$.
- Draw a vertex and label it with the first ordered pair. Then draw an edge and vertex for each possible trip the farmer could make across the river. If at any point a path results in an unworkable combination of things, no more edges can be drawn from that vertex.
- From each workable vertex, continue to draw edges and vertices that represent the next trip across the river. When you get to a vertex for $(-, FWGC)$, the network is complete.
- Use the network to write out the solution in words.



Try This

- What combinations are unworkable? Why?
- How many solutions are there to the farmer's transport problem? How many steps does each solution take?
- What is the advantage of drawing a complete solution network rather than working out one solution with a diagram?
- Madeline has two measuring cups—a 1-cup measuring cup and a $\frac{3}{4}$ -cup measuring cup. Neither cup has any markings on it. How can Madeline get exactly $\frac{1}{2}$ cup of flour in the larger measuring cup? Complete the network below to solve the problem.



2-4

Biconditional Statements and Definitions

Objective

Write and analyze biconditional statements.

Vocabulary

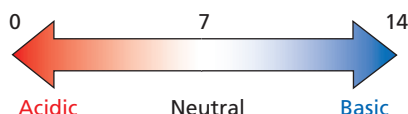
biconditional statement
definition
polygon
triangle
quadrilateral

Who uses this?

A gardener can plan the color of the hydrangeas she plants by checking the pH of the soil.



The pH of a solution is a measure of the concentration of hydronium ions in the solution. If a solution has a pH less than 7, it is an acid. Also, if a solution is an acid, it has a pH less than 7.



Writing Math

The biconditional “ p if and only if q ” can also be written as “ p iff q ” or $p \leftrightarrow q$.

When you combine a conditional statement and its converse, you create a *biconditional statement*. A **biconditional statement** is a statement that can be written in the form “ p if and only if q .” This means “if p , then q ” and “if q , then p .”

$$p \leftrightarrow q \text{ means } p \rightarrow q \text{ and } q \rightarrow p$$

So you can define an acid with the following biconditional statement: A solution is an acid if and only if it has a pH less than 7.

EXAMPLE 1 Identifying the Conditionals within a Biconditional Statement

California Standards

3.0 Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement.

Write the conditional statement and converse within each biconditional.

- A** Two angles are congruent if and only if their measures are equal.

Let p and q represent the following.

p : Two angles are congruent.

q : Two angle measures are equal.

The two parts of the biconditional $p \leftrightarrow q$ are $p \rightarrow q$ and $q \rightarrow p$.

Conditional: If **two angles are congruent**, then **their measures are equal**.

Converse: If **two angle measures are equal**, then **the angles are congruent**.

- B** A solution is a base \leftrightarrow it has a pH greater than 7.

Let x and y represent the following.

x : A solution is a base.

y : A solution has a pH greater than 7.

The two parts of the biconditional $x \leftrightarrow y$ are $x \rightarrow y$ and $y \rightarrow x$.

Conditional: If **a solution is a base**, then **it has a pH greater than 7**.

Converse: If **a solution has a pH greater than 7**, then **it is a base**.



Write the conditional statement and converse within each biconditional.

- 1a. An angle is acute iff its measure is greater than 0° and less than 90° .
- 1b. Cho is a member if and only if he has paid the \$5 dues.

EXAMPLE 2 Writing a Biconditional Statement

For each conditional, write the converse and a biconditional statement.

A If $2x + 5 = 11$, then $x = 3$.

Converse: If $x = 3$, then $2x + 5 = 11$.

Biconditional: $2x + 5 = 11$ if and only if $x = 3$.

B If a point is a midpoint, then it divides the segment into two congruent segments.

Converse: If a point divides a segment into two congruent segments, then the point is a midpoint.

Biconditional: A point is a midpoint if and only if it divides the segment into two congruent segments.



For each conditional, write the converse and a biconditional statement.

2a. If the date is July 4th, then it is Independence Day.

2b. If points lie on the same line, then they are collinear.

For a biconditional statement to be true, both the conditional statement and its converse must be true. If either the conditional or the converse is false, then the biconditional statement is false.

EXAMPLE 3 Analyzing the Truth Value of a Biconditional Statement

Determine if each biconditional is true. If false, give a counterexample.

A A square has a side length of 5 if and only if it has an area of 25.

Conditional: If a square has a side length of 5, then it has an area of 25. *The conditional is true.*

Converse: If a square has an area of 25, then it has a side length of 5. *The converse is true.*

Since the conditional and its converse are true, the biconditional is true.

B The number n is a positive integer $\leftrightarrow 2n$ is a natural number.

Conditional: If n is a positive integer, then $2n$ is a natural number. *The conditional is true.*

Converse: If $2n$ is a natural number, then n is a positive integer. *The converse is false.*

If $2n = 1$, then $n = \frac{1}{2}$, which is not an integer. Because the converse is false, the biconditional is false.



Determine if each biconditional is true. If false, give a counterexample.

3a. An angle is a right angle iff its measure is 90° .

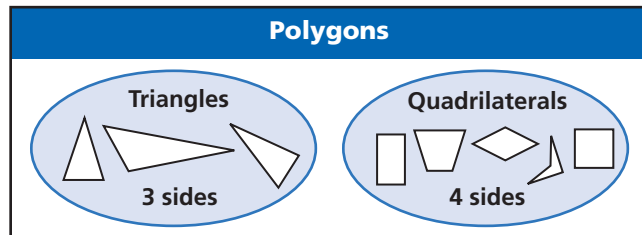
3b. $y = -5 \leftrightarrow y^2 = 25$

In geometry, biconditional statements are used to write *definitions*. A **definition** is a statement that describes a mathematical object and can be written as a true biconditional. Most definitions in the glossary are not written as biconditional statements, but they can be. The “if and only if” is implied.

In the glossary, a **polygon** is defined as a closed plane figure formed by three or more line segments. Each segment intersects exactly two other segments only at their endpoints, and no two segments with a common endpoint are collinear.

Polygons	Not Polygons

A **triangle** is defined as a three-sided polygon, and a **quadrilateral** is a four-sided polygon.



A good, precise definition can be used forward and backward. For example, if a figure is a quadrilateral, then it is a four-sided polygon. If a figure is a four-sided polygon, then it is a quadrilateral. To make sure a definition is precise, it helps to write it as a biconditional statement.

EXAMPLE 4 Writing Definitions as Biconditional Statements

Helpful Hint

Think of definitions as being reversible. Postulates, however, are not necessarily true when reversed.

Write each definition as a biconditional.

- A** A triangle is a three-sided polygon.
A figure is a triangle if and only if it is a three-sided polygon.
- B** A segment bisector is a ray, segment, or line that divides a segment into two congruent segments.
A ray, segment, or line is a segment bisector if and only if it divides a segment into two congruent segments.

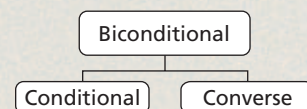


Write each definition as a biconditional.

- 4a. A quadrilateral is a four-sided polygon.
4b. The measure of a straight angle is 180° .

THINK AND DISCUSS

- How do you determine if a biconditional statement is true or false?
- Compare a triangle and a quadrilateral.
- GET ORGANIZED** Copy and complete the graphic organizer. Use the definition of a polygon to write a conditional, converse, and biconditional in the appropriate boxes.





GUIDED PRACTICE

1. **Vocabulary** How is a *biconditional statement* different from a conditional statement?

SEE EXAMPLE 1

p. 96

Write the conditional statement and converse within each biconditional.

2. Perry can paint the entire living room if and only if he has enough paint.
3. Your medicine will be ready by 5 P.M. if and only if you drop your prescription off by 8 A.M.

SEE EXAMPLE 2

p. 97

For each conditional, write the converse and a biconditional statement.

4. If a student is a sophomore, then the student is in the tenth grade.
5. If two segments have the same length, then they are congruent.

SEE EXAMPLE 3

p. 97

Multi-Step Determine if each biconditional is true. If false, give a counterexample.

6. $xy = 0 \leftrightarrow x = 0$ or $y = 0$.
7. A figure is a quadrilateral if and only if it is a polygon.

SEE EXAMPLE 4

p. 98

Write each definition as a biconditional.

8. Parallel lines are two coplanar lines that never intersect.
9. A hummingbird is a tiny, brightly colored bird with narrow wings, a slender bill, and a long tongue.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
10–12	1
13–15	2
16–17	3
18–19	4

Extra Practice

Skills Practice p. S6

Application Practice p. S29

Write the conditional statement and converse within each biconditional.

10. Three points are coplanar if and only if they lie in the same plane.
11. A parallelogram is a rectangle if and only if it has four right angles.
12. A lunar eclipse occurs if and only if Earth is between the Sun and the Moon.

For each conditional, write the converse and a biconditional statement.

13. If today is Saturday or Sunday, then it is the weekend.
14. If Greg has the fastest time, then he wins the race.
15. If a triangle contains a right angle, then it is a right triangle.

Multi-Step Determine if each biconditional is true. If false, give a counterexample.

16. Felipe is a swimmer if and only if he is an athlete.
17. The number $2n$ is even if and only if n is an integer.

Write each definition as a biconditional.

18. A circle is the set of all points in a plane that are a fixed distance from a given point.
19. A catcher is a baseball player who is positioned behind home plate and who catches throws from the pitcher.

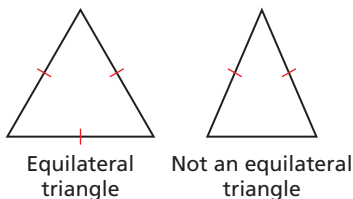


Algebra Determine if a true biconditional can be written from each conditional statement. If not, give a counterexample.

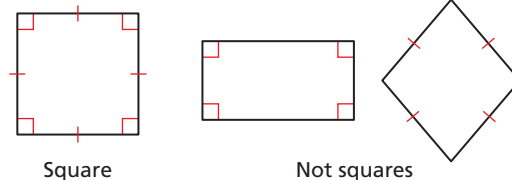
20. If $a = b$, then $|a| = |b|$. 21. If $3x - 2 = 13$, then $\frac{4}{5}x + 8 = 12$.
22. If $y^2 = 64$, then $3y = 24$. 23. If $x > 0$, then $x^2 > 0$.

Use the diagrams to write a definition for each figure.

24.



25.



26. **Biology** White blood cells are cells that defend the body against invading organisms by engulfing them or by releasing chemicals called *antibodies*. Write the definition of a white blood cell as a biconditional statement.

Explain why the given statement is not a definition.

27. An automobile is a vehicle that moves along the ground.
28. A calculator is a machine that performs computations with numbers.
29. An angle is a geometric object formed by two rays.

Chemistry Use the table for Exercises 30–32. Determine if a true biconditional statement can be written from each conditional.

30. If a solution has a pH of 4, then it is tomato juice.
31. If a solution is bleach, then its pH is 13.
32. If a solution has a pH greater than 7, then it is not battery acid.

pH	Examples
0	Battery Acid
4	Acid rain, tomato juice
6	Saliva
8	Sea water
13	Bleach, oven cleaner
14	Drain cleaner

Complete each statement to form a true biconditional.

33. The circumference of a circle is 10π if and only if its radius is ? .
34. Four points in a plane form a ? if and only if no three of them are collinear.
35. **Critical Thinking** Write the definition of a biconditional statement as a biconditional statement. Use the conditional and converse within the statement to explain why your biconditional is true.
36. **Write About It** Use the definition of an angle bisector to explain what is meant by the statement “A good definition is reversible.”

LINK

Biology

White blood cells live less than a few weeks. A drop of blood can contain anywhere from 7000 to 25,000 white blood cells.

CONCEPT CONNECTION



37. This problem will prepare you for the Concept Connection on page 102.
- a. Write “I say what I mean” and “I mean what I say” as conditionals.
- b. Explain why the biconditional statement implied by Alice is false.

“Then you should say what you mean,” the March Hare went on.

“I do,” Alice hastily replied; “at least—at least I mean what I say—that’s the same thing, you know.”

38. Which is a counterexample for the biconditional "An angle measures 80° if and only if the angle is acute"?
- (A) $m\angle S = 60^\circ$ (B) $m\angle S = 115^\circ$ (C) $m\angle S = 90^\circ$ (D) $m\angle S = 360^\circ$
39. Which biconditional is equivalent to the spelling phrase "I before E except after C"?
- (F) The letter I comes before E if and only if I follows C.
 (G) The letter E comes before I if and only if E follows C.
 (H) The letter E comes before I if and only if E comes before C.
 (J) The letter I comes before E if and only if I comes before C.
40. Which conditional statement can be used to write a true biconditional?
- (A) If a number is divisible by 4, then it is even.
 (B) If a ratio compares two quantities measured in different units, the ratio is a rate.
 (C) If two angles are supplementary, then they are adjacent.
 (D) If an angle is right, then it is not acute.
41. **Short Response** Write the two conditional statements that make up the biconditional "You will get a traffic ticket if and only if you are speeding." Is the biconditional true or false? Explain your answer.

CHALLENGE AND EXTEND

42. **Critical Thinking** Describe what the Venn diagram of a true biconditional statement looks like. How does this support the idea that a definition can be written as a true biconditional?
43. Consider the conditional "If an angle measures 105° , then the angle is obtuse."
- Write the inverse of the conditional statement.
 - Write the converse of the inverse.
 - How is the converse of the inverse related to the original conditional?
 - What is the truth value of the biconditional statement formed by the inverse of the original conditional and the converse of the inverse? Explain.
44. Suppose A , B , C , and D are coplanar, and A , B , and C are not collinear. What is the truth value of the biconditional formed from the true conditional "If $m\angle ABD + m\angle DBC = m\angle ABC$, then D is in the interior of $\angle ABC$ "? Explain.
45. Find a counterexample for " n is divisible by 4 if and only if n^2 is even."

SPIRAL REVIEW

Describe how the graph of each function differs from the graph of the parent function $y = x^2$. (*Previous course*)

46. $y = \frac{1}{2}x^2 + 5$ 47. $y = -2x^2 - 1$ 48. $y = (x - 2)(x + 2)$

A transformation maps S onto T and X onto Y . Name each of the following. (*Lesson 1-7*)

49. the image of S 50. the image of X 51. the preimage of T

Determine if each conjecture is true. If not, give a counterexample. (*Lesson 2-1*)

52. If $n \geq 0$, then $\frac{n}{2} > 0$. 53. If x is prime, then $x + 2$ is also prime.
54. The vertices of the image of a figure under the translation $(x, y) \rightarrow (x + 0, y + 0)$ have the same coordinates as the preimage.

CONCEPT CONNECTION



Inductive and Deductive Reasoning

Rhyme or Reason

Alice's Adventures in Wonderland originated as a story told by Charles Lutwidge Dodgson (Lewis Carroll) to three young traveling companions. The story is famous for its wordplay and logical absurdities.



1. When Alice first meets the Cheshire Cat, she asks what sort of people live in Wonderland. The Cat explains that everyone in Wonderland is mad. What conjecture might the Cat make since Alice, too, is in Wonderland?

2. **"I don't much care where—" said Alice.**
"Then it doesn't matter which way you go," said the Cat.
"—so long as I get *somewhere*," Alice added as an explanation.
"Oh, you're sure to do that," said the Cat, "if you only walk long enough."



Write the conditional statement implied by the Cat's response to Alice.

3. **"Well, then," the Cat went on, "you see a dog growls when it's angry, and wags its tail when it's pleased. Now I growl when I'm pleased, and wag my tail when I'm angry. Therefore I'm mad."**



Is the Cat's conclusion valid by the Law of Detachment or the Law of Syllogism? Explain your reasoning.

4. **"You might just as well say," added the Dormouse, who seemed to be talking in his sleep, "that 'I breathe when I sleep' is the same thing as 'I sleep when I breathe'!"**



Write a biconditional statement from the Dormouse's example. Explain why the biconditional statement is false.

Quiz for Lessons 2-1 Through 2-4

2-1 Using Inductive Reasoning to Make Conjectures

Find the next item in each pattern.

1. 1, 10, 18, 25, ... 2. July, May, March, ... 3. $\frac{1}{8}, -\frac{1}{4}, \frac{1}{2}, \dots$ 4. |, †, ‡, ...

5. A biologist recorded the following data about the weight of male lions in a wildlife park in Africa. Use the table to make a conjecture about the average weight of a male lion.
6. Complete the conjecture “The sum of two negative numbers is ?.”
7. Show that the conjecture “If an even number is divided by 2, then the result is an even number” is false by finding a counterexample.

ID Number	Weight (lb)
A1902SM	387.2
A1904SM	420.5
A1920SM	440.6
A1956SM	398.7
A1974SM	415.0

2-2 Conditional Statements

8. Identify the hypothesis and conclusion of the conditional statement “An angle is obtuse if its measure is 107° .”

Write a conditional statement from each of the following.

9. A whole number is an integer. 10.  11. The diagonals of a square are congruent.

Determine if each conditional is true. If false, give a counterexample.

12. If an angle is acute, then it has a measure of 30° .
13. If $9x - 11 = 2x + 3$, then $x = 2$.
14. Write the converse, inverse, and contrapositive of the statement “If a number is even, then it is divisible by 4.” Find the truth value of each.

2-3 Using Deductive Reasoning to Verify Conjectures

15. Determine if the following conjecture is valid by the Law of Detachment.
Given: If Sue finishes her science project, she can go to the movie. Sue goes to the movie.
Conjecture: Sue finished her science project.
16. Use the Law of Syllogism to draw a conclusion from the given information.
Given: If one angle of a triangle is 90° , then the triangle is a right triangle. If a triangle is a right triangle, then its acute angle measures are complementary.

2-4 Biconditional Statements and Definitions

17. For the conditional “If two angles are supplementary, the sum of their measures is 180° ,” write the converse and a biconditional statement.
18. Determine if the biconditional “ $\sqrt{x} = 4$ if and only if $x = 16$ ” is true. If false, give a counterexample.

2-5

Algebraic Proof



Objectives

Review properties of equality and use them to write algebraic proofs.

Identify properties of equality and congruence.

Vocabulary

proof

Who uses this?

Game designers and animators solve equations to simulate motion. (See Example 2.)

A **proof** is an argument that uses logic, definitions, properties, and previously proven statements to show that a conclusion is true.

If you've ever solved an equation in Algebra, then you've already done a proof! An algebraic proof uses algebraic properties such as the properties of equality and the Distributive Property.

Remember!

The Distributive Property states that $a(b + c) = ab + ac$.



California Standards

Preparation for **2.0**
Students write geometric proofs, including proofs by contradiction.

Properties of Equality

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $ac = bc$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then b can be substituted for a in any expression.

As you learned in Lesson 2-3, if you start with a true statement and each logical step is valid, then your conclusion is valid.

An important part of writing a proof is giving justifications to show that every step is valid. For each justification, you can use a definition, postulate, property, or a piece of information that is given.

EXAMPLE 1 Solving an Equation in Algebra

Solve the equation $-5 = 3n + 1$. Write a justification for each step.

$-5 = 3n + 1$	Given equation
$\frac{-1}{-1} \quad \frac{-1}{-1}$	Subtraction Property of Equality
$-6 = 3n$	Simplify.
$\frac{-6}{3} = \frac{3n}{3}$	Division Property of Equality
$-2 = n$	Simplify.
$n = -2$	Symmetric Property of Equality



1. Solve the equation $\frac{1}{2}t = -7$. Write a justification for each step.

EXAMPLE

2

Problem-Solving Application



To simulate the motion of an object in a computer game, the designer uses the formula $sr = 3.6p$ to find the number of pixels the object must travel during each second of animation. In the formula, s is the desired speed of the object in kilometers per hour, r is the scale of pixels per meter, and p is the number of pixels traveled per second.

The graphics in a game are based on a scale of 6 pixels per meter. The designer wants to simulate a vehicle moving at 75 km/h. How many pixels must the vehicle travel each second? Solve the equation for p and justify each step.



1 Understand the Problem

The answer will be the number of pixels traveled per second.

List the important information:

- $sr = 3.6p$
- $s = 75$ km/h
- p : pixels traveled per second
- $r = 6$ pixels per meter

2 Make a Plan

Substitute the given information into the formula and solve.

3 Solve

$sr = 3.6p$	Given equation
$(75)(6) = 3.6p$	Substitution Property of Equality
$450 = 3.6p$	Simplify.
$\frac{450}{3.6} = \frac{3.6p}{3.6}$	Division Property of Equality
$125 = p$	Simplify.
$p = 125$ pixels	Symmetric Property of Equality

4 Look Back


Check your answer by substituting it back into the original formula.

$$sr = 3.6p$$

$$(75)(6) = 3.6(125)$$

$$450 = 450 \quad \checkmark$$

Helpful Hint


 AB represents the length of \overline{AB} , so you can think of AB as a variable representing a number.



2. What is the temperature in degrees Celsius C when it is 86°F ? Solve the equation $C = \frac{5}{9}(F - 32)$ for C and justify each step.

Like algebra, geometry also uses numbers, variables, and operations. For example, segment lengths and angle measures are numbers. So you can use these same properties of equality to write algebraic proofs in geometry.

EXAMPLE 3 Solving an Equation in Geometry

Write a justification for each step.

$$KM = KL + LM$$

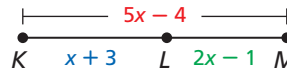
$$5x - 4 = (x + 3) + (2x - 1)$$

$$5x - 4 = 3x + 2$$

$$2x - 4 = 2$$

$$2x = 6$$

$$x = 3$$



Segment Addition Postulate
 Substitution Property of Equality
 Simplify.
 Subtraction Property of Equality
 Addition Property of Equality
 Division Property of Equality



3. Write a justification for each step.

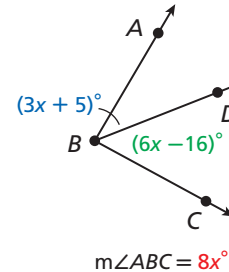
$$m\angle ABC = m\angle ABD + m\angle DBC$$

$$8x^\circ = (3x + 5)^\circ + (6x - 16)^\circ$$

$$8x = 9x - 11$$

$$-x = -11$$

$$x = 11$$



You learned in Chapter 1 that segments with equal lengths are congruent and that angles with equal measures are congruent. So the Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence.



Properties of Congruence

SYMBOLS	EXAMPLE
Reflexive Property of Congruence figure $A \cong$ figure A (Reflex. Prop. of \cong)	$\overline{EF} \cong \overline{EF}$
Symmetric Property of Congruence If figure $A \cong$ figure B , then figure $B \cong$ figure A . (Sym. Prop. of \cong)	If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.
Transitive Property of Congruence If figure $A \cong$ figure B and figure $B \cong$ figure C , then figure $A \cong$ figure C . (Trans. Prop. of \cong)	If $\overline{PQ} \cong \overline{RS}$ and $\overline{RS} \cong \overline{TU}$, then $\overline{PQ} \cong \overline{TU}$.

EXAMPLE 4 Identifying Properties of Equality and Congruence

Remember!

Numbers are equal ($=$) and figures are congruent (\cong).

Identify the property that justifies each statement.

- A** $m\angle 1 = m\angle 1$ Reflex. Prop. of $=$
- B** $\overline{XY} \cong \overline{VW}$, so $\overline{VW} \cong \overline{XY}$. Sym. Prop. of \cong
- C** $\angle ABC \cong \angle ABC$ Reflex. Prop. of \cong
- D** $\angle 1 \cong \angle 2$, and $\angle 2 \cong \angle 3$. So $\angle 1 \cong \angle 3$. Trans. Prop. of \cong



Identify the property that justifies each statement.

- 4a. $DE = GH$, so $GH = DE$. 4b. $94^\circ = 94^\circ$
- 4c. $0 = a$, and $a = x$. So $0 = x$. 4d. $\angle A \cong \angle Y$, so $\angle Y \cong \angle A$.

THINK AND DISCUSS

- Tell what property you would use to solve the equation $\frac{k}{6} = 3.5$.
- Explain when to use a congruence symbol instead of an equal sign.
- GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of the property, using the correct symbol.

Property	Equality	Congruence
Reflexive		
Symmetric		
Transitive		



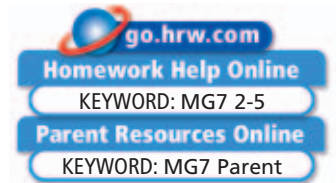
2-5

Exercises



California Standards

Preparation for **2.0**;
1.0, **16.0**, **7AF1.1**,
7AF4.0, **7AF4.1**,
7AF4.2, **1A4.0**,
1A5.0



GUIDED PRACTICE

- Vocabulary** Write the definition of *proof* in your own words.

SEE EXAMPLE 1

p. 104

- Multi-Step** Solve each equation. Write a justification for each step.

$$2. y + 1 = 5$$

$$3. t - 3.2 = -8.3$$

$$4. 2p - 30 = -4p + 6$$

$$5. \frac{x+3}{-2} = 8$$

$$6. \frac{1}{2}n = \frac{3}{4}$$

$$7. 0 = 2(r - 3) + 4$$

SEE EXAMPLE 2

p. 105

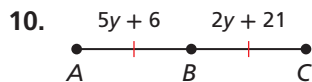
- Nutrition** Amy's favorite breakfast cereal has 102 Calories per serving. The equation $C = 9f + 90$ relates the grams of fat f in one serving to the Calories C in one serving. How many grams of fat are in one serving of the cereal? Solve the equation for f and justify each step.

- Movie Rentals** The equation $C = \$5.75 + \$0.89m$ relates the number of movie rentals m to the monthly cost C of a movie club membership. How many movies did Elias rent this month if his membership cost \$11.98? Solve the equation for m and justify each step.

SEE EXAMPLE 3

p. 106

- Write a justification for each step.



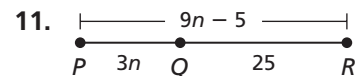
$$AB = BC$$

$$5y + 6 = 2y + 21$$

$$3y + 6 = 21$$

$$3y = 15$$

$$y = 5$$



$$PQ + QR = PR$$

$$3n + 25 = 9n - 5$$

$$25 = 6n - 5$$

$$30 = 6n$$

$$5 = n$$

SEE EXAMPLE 4

p. 106

- Identify the property that justifies each statement.

$$12. \overline{AB} \cong \overline{AB}$$

$$13. m\angle 1 = m\angle 2, \text{ and } m\angle 2 = m\angle 4. \text{ So } m\angle 1 = m\angle 4.$$

$$14. x = y, \text{ so } y = x.$$

$$15. \overline{ST} \cong \overline{YZ}, \text{ and } \overline{YZ} \cong \overline{PR}. \text{ So } \overline{ST} \cong \overline{PR}.$$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
16–21	1
22	2
23–24	3
25–28	4

Extra Practice

Skills Practice p. 57

Application Practice p. 529

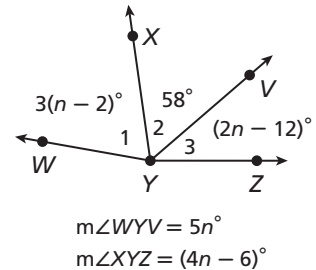
Multi-Step Solve each equation. Write a justification for each step.

16. $5x - 3 = 4(x + 2)$ 17. $1.6 = 3.2n$ 18. $\frac{z}{3} - 2 = -10$
 19. $-(h + 3) = 72$ 20. $9y + 17 = -19$ 21. $\frac{1}{2}(p - 16) = 13$

22. **Ecology** The equation $T = 0.03c + 0.05b$ relates the numbers of cans c and bottles b collected in a recycling rally to the total dollars T raised. How many cans were collected if \$147 was raised and 150 bottles were collected? Solve the equation for c and justify each step.

Write a justification for each step.

23. $m\angle XYZ = m\angle 2 + m\angle 3$ 24. $m\angle WYV = m\angle 1 + m\angle 2$
 $4n - 6 = 58 + (2n - 12)$ $5n = 3(n - 2) + 58$
 $4n - 6 = 2n + 46$ $5n = 3n - 6 + 58$
 $2n - 6 = 46$ $5n = 3n + 52$
 $2n = 52$ $2n = 52$
 $n = 26$ $n = 26$

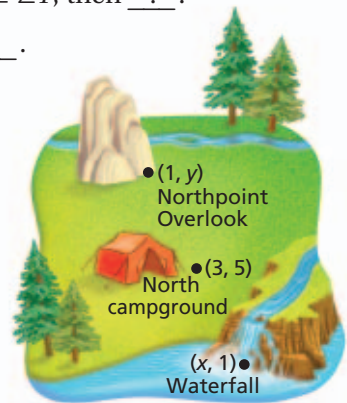


Identify the property that justifies each statement.

25. $\overline{KL} \cong \overline{PR}$, so $\overline{PR} \cong \overline{KL}$. 26. $412 = 412$
 27. If $a = b$ and $b = 0$, then $a = 0$. 28. figure $A \cong$ figure A
 29. **Estimation** Round the numbers in the equation $2(3.1x - 0.87) = 94.36$ to the nearest whole number and estimate the solution. Then solve the equation, justifying each step. Compare your estimate to the exact solution.

Use the indicated property to complete each statement.

30. Reflexive Property of Equality: $3x - 1 = \underline{\quad?}$
 31. Transitive Property of Congruence: If $\angle A \cong \angle X$ and $\angle X \cong \angle T$, then $\underline{\quad?}$.
 32. Symmetric Property of Congruence: If $\overline{BC} \cong \overline{NP}$, then $\underline{\quad?}$.
 33. **Recreation** The north campground is midway between the Northpoint Overlook and the waterfall. Use the midpoint formula to find the values of x and y , and justify each step.



34. **Business** A computer repair technician charges \$35 for each job plus \$21 per hour of labor and 110% of the cost of parts. The total charge for a 3-hour job was \$169.50. What was the cost of parts for this job? Write and solve an equation and justify each step in the solution.
 35. **Finance** Morgan spent a total of \$1,733.65 on her car last year. She spent \$92.50 on registration, \$79.96 on maintenance, and \$983 on insurance. She spent the remaining money on gas. She drove a total of 10,820 miles.
 a. How much on average did the gas cost per mile? Write and solve an equation and justify each step in the solution.
 b. **What if...?** Suppose Morgan's car averages 32 miles per gallon of gas. How much on average did Morgan pay for a gallon of gas?
 36. **Critical Thinking** Use the definition of segment congruence and the properties of equality to show that all three properties of congruence are true for segments.

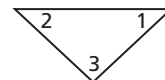


37. This problem will prepare you for the Concept Connection on page 126. Recall from Algebra 1 that the Multiplication and Division Properties of Inequality tell you to reverse the inequality sign when multiplying or dividing by a negative number.
- Solve the inequality $x + 15 \leq 63$ and write a justification for each step.
 - Solve the inequality $-2x > 36$ and write a justification for each step.

38. **Write About It** Compare the conclusion of a deductive proof and a conjecture based on inductive reasoning.

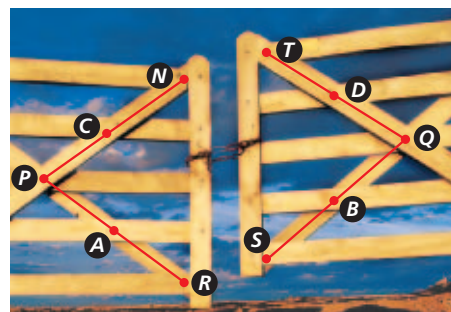


39. Which could NOT be used to justify the statement $\overline{AB} \cong \overline{CD}$?
- Definition of congruence
 - Reflexive Property of Congruence
 - Symmetric Property of Congruence
 - Transitive Property of Congruence
40. A club membership costs \$35 plus \$3 each time t the member uses the pool. Which equation represents the total cost C of the membership?
- $35 = C + 3t$
 - $C + 35 = 3t$
 - $C = 35 + 3t$
 - $C = 35t + 3$
41. Which statement is true by the Reflexive Property of Equality?
- $x = 35$
 - $\overline{CD} = \overline{CD}$
 - $\overline{RT} \cong \overline{TR}$
 - $CD = CD$
42. **Gridded Response** In the triangle, $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$. If $m\angle 3 = 2m\angle 1$ and $m\angle 1 = m\angle 2$, find $m\angle 3$ in degrees.



CHALLENGE AND EXTEND

43. In the gate, $PA = QB$, $QB = RA$, and $PA = 18$ in. Find PR , and justify each step.
44. **Critical Thinking** Explain why there is no Addition Property of Congruence.
45. **Algebra** Justify each step in the solution of the inequality $7 - 3x > 19$.



SPIRAL REVIEW

46. The members of a high school band have saved \$600 for a trip. They deposit the money in a savings account. What additional information is needed to find the amount of interest the account earns during a 3-month period? (*Previous course*)

Use a compass and straightedge to construct each of the following. (*Lesson 1-2*)

47. \overline{JK} congruent to \overline{MN}



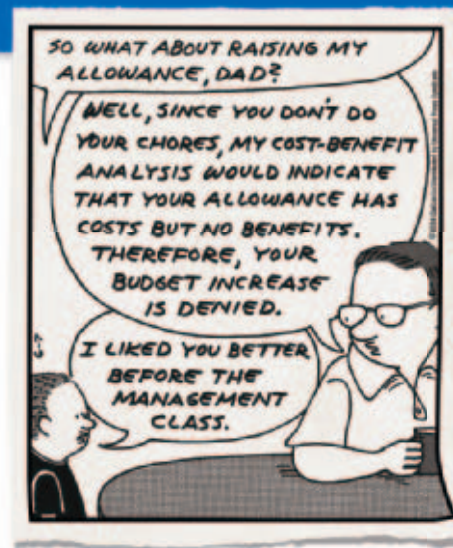
48. a segment bisector of \overline{JK}

Identify whether each conclusion uses inductive or deductive reasoning. (*Lesson 2-3*)

49. A triangle is obtuse if one of its angles is obtuse. Jacob draws a triangle with two acute angles and one obtuse angle. He concludes that the triangle is obtuse.
50. Tonya studied 3 hours for each of her last two geometry tests. She got an A on both tests. She concludes that she will get an A on the next test if she studies for 3 hours.

2-6

Geometric Proof



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Objectives

Write two-column proofs.

Prove geometric theorems by using deductive reasoning.

Vocabulary

theorem
two-column proof

Who uses this?

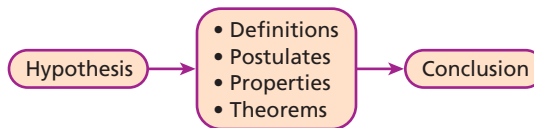
To persuade your parents to increase your allowance, your argument must be presented logically and precisely.

When writing a geometric proof, you use deductive reasoning to create a chain of logical steps that move from the hypothesis to the conclusion of the conjecture you are proving. By proving that the conclusion is true, you have proven that the original conjecture is true.

California Standards

2.0 Students write geometric proofs, including proofs by contradiction.

Also covered: **1.0**



When writing a proof, it is important to justify each logical step with a reason. You can use symbols and abbreviations, but they must be clear enough so that anyone who reads your proof will understand them.

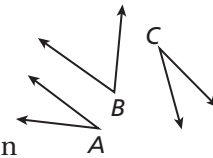
EXAMPLE 1 Writing Justifications

Helpful Hint

When a justification is based on more than the previous step, you can note this after the reason, as in Example 1 Step 5.

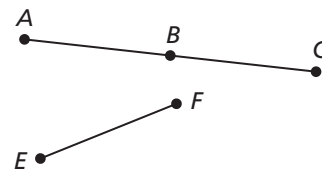
Write a justification for each step, given that $\angle A$ and $\angle B$ are complementary and $\angle A \cong \angle C$.

- | | |
|---|-------------------------------------|
| 1. $\angle A$ and $\angle B$ are complementary. | Given information |
| 2. $m\angle A + m\angle B = 90^\circ$ | Def. of comp. \triangle |
| 3. $\angle A \cong \angle C$ | Given information |
| 4. $m\angle A = m\angle C$ | Def. of $\cong \triangle$ |
| 5. $m\angle C + m\angle B = 90^\circ$ | Subst. Prop. of = Steps 2, 4 |
| 6. $\angle C$ and $\angle B$ are complementary. | Def. of comp. \triangle |



1. Write a justification for each step, given that B is the midpoint of \overline{AC} and $\overline{AB} \cong \overline{EF}$.

- B is the midpoint of \overline{AC} .
- $\overline{AB} \cong \overline{BC}$
- $\overline{AB} \cong \overline{EF}$
- $\overline{BC} \cong \overline{EF}$



A **theorem** is any statement that you can prove. Once you have proven a theorem, you can use it as a reason in later proofs.



Theorem

THEOREM	HYPOTHESIS	CONCLUSION
2-6-1 Linear Pair Theorem If two angles form a linear pair, then they are supplementary.	$\angle A$ and $\angle B$ form a linear pair.	$\angle A$ and $\angle B$ are supplementary.



Theorem

THEOREM	HYPOTHESIS	CONCLUSION
2-6-2 Congruent Supplements Theorem If two angles are supplementary to the same angle (or to two congruent angles), then the two angles are congruent.	$\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	$\angle 1 \cong \angle 3$

A geometric proof begins with *Given* and *Prove* statements, which restate the hypothesis and conclusion of the conjecture. In a **two-column proof**, you list the steps of the proof in the left column. You write the matching reason for each step in the right column.

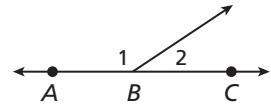
EXAMPLE 2 Completing a Two-Column Proof

Fill in the blanks to complete a two-column proof of the Linear Pair Theorem.

Given: $\angle 1$ and $\angle 2$ form a linear pair.

Prove: $\angle 1$ and $\angle 2$ are supplementary.

Proof:



Writing Math

Since there is no other substitution property, the Substitution Property of Equality is often written as "Substitution" or "Subst."

Statements	Reasons
1. $\angle 1$ and $\angle 2$ form a linear pair.	1. Given
2. \overrightarrow{BA} and \overrightarrow{BC} form a line.	2. Def. of lin. pair
3. $m\angle ABC = 180^\circ$	3. Def. of straight \angle
4. a. <u> ?</u>	4. \angle Add. Post.
5. b. <u> ?</u>	5. Subst. <i>Steps 3, 4</i>
6. $\angle 1$ and $\angle 2$ are supplementary.	6. c. <u> ?</u>

Use the existing statements and reasons in the proof to fill in the blanks.

- a. $m\angle 1 + m\angle 2 = m\angle ABC$ *The \angle Add. Post. is given as the reason.*
 b. $m\angle 1 + m\angle 2 = 180^\circ$ *Substitute 180° for $m\angle ABC$.*
 c. Def. of supp. \triangleq *The measures of supp. \triangleq add to 180° by def.*

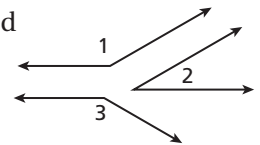


2. Fill in the blanks to complete a two-column proof of one case of the Congruent Supplements Theorem.

Given: $\angle 1$ and $\angle 2$ are supplementary, and $\angle 2$ and $\angle 3$ are supplementary.

Prove: $\angle 1 \cong \angle 3$

Proof:



Statements	Reasons
1. a. <u> ?</u>	1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$ $m\angle 2 + m\angle 3 = 180^\circ$	2. Def. of supp. \triangleq
3. b. <u> ?</u>	3. Subst.
4. $m\angle 2 = m\angle 2$	4. Reflex. Prop. of =
5. $m\angle 1 = m\angle 3$	5. c. <u> ?</u>
6. d. <u> ?</u>	6. Def. of $\cong \triangleq$

Before you start writing a proof, you should plan out your logic. Sometimes you will be given a plan for a more challenging proof. This plan will detail the major steps of the proof for you.



Theorems

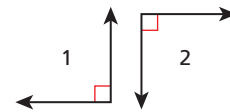
THEOREM	HYPOTHESIS	CONCLUSION
2-6-3 Right Angle Congruence Theorem All right angles are congruent.	$\angle A$ and $\angle B$ are right angles.	$\angle A \cong \angle B$
2-6-4 Congruent Complements Theorem If two angles are complementary to the same angle (or to two congruent angles), then the two angles are congruent.	$\angle 1$ and $\angle 2$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	$\angle 1 \cong \angle 3$

EXAMPLE 3 Writing a Two-Column Proof from a Plan

Use the given plan to write a two-column proof of the Right Angle Congruence Theorem.

Given: $\angle 1$ and $\angle 2$ are right angles.

Prove: $\angle 1 \cong \angle 2$



Plan: Use the definition of a right angle to write the measure of each angle.

Then use the Transitive Property and the definition of congruent angles.

Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are right angles.	1. Given
2. $m\angle 1 = 90^\circ$, $m\angle 2 = 90^\circ$	2. Def. of rt. \angle
3. $m\angle 1 = m\angle 2$	3. Trans. Prop. of =
4. $\angle 1 \cong \angle 2$	4. Def. of $\cong \angle$

Helpful Hint

If a diagram for a proof is not provided, draw your own and mark the given information on it. But do not mark the information in the Prove statement on it.

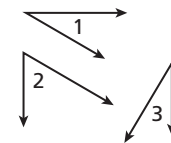


3. Use the given plan to write a two-column proof of one case of the Congruent Complements Theorem.

Given: $\angle 1$ and $\angle 2$ are complementary, and $\angle 2$ and $\angle 3$ are complementary.

Prove: $\angle 1 \cong \angle 3$

Plan: The measures of complementary angles add to 90° by definition. Use substitution to show that the sums of both pairs are equal. Use the Subtraction Property and the definition of congruent angles to conclude that $\angle 1 \cong \angle 3$.



The Proof Process

1. Write the conjecture to be proven.
2. Draw a diagram to represent the hypothesis of the conjecture.
3. State the given information and mark it on the diagram.
4. State the conclusion of the conjecture in terms of the diagram.
5. Plan your argument and prove the conjecture.

THINK AND DISCUSS

- Which step in a proof should match the Prove statement?
- Why is it important to include every logical step in a proof?
- List four things you can use to justify a step in a proof.
- GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe the steps of the proof process.



1. → 2. → 3. → 4. → 5.

2-6

Exercises



California Standards

1.0, 2.0, 4.0,
7AF4.1



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Homework Help Online
KEYWORD: MG7 2-6
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

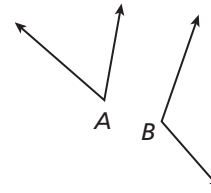
Vocabulary Apply the vocabulary from this lesson to answer each question.

1. In a *two-column proof*, you list the ? in the left column and the ? in the right column. (*statements* or *reasons*)

2. A ? is a statement you can prove. (*postulate* or *theorem*)

3. Write a justification for each step, given that $m\angle A = 60^\circ$ and $m\angle B = 2m\angle A$.

- $m\angle A = 60^\circ$, $m\angle B = 2m\angle A$
- $m\angle B = 2(60^\circ)$
- $m\angle B = 120^\circ$
- $m\angle A + m\angle B = 60^\circ + 120^\circ$
- $m\angle A + m\angle B = 180^\circ$
- $\angle A$ and $\angle B$ are supplementary.



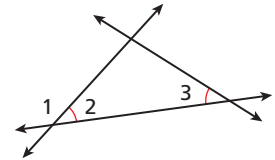
4. Fill in the blanks to complete the two-column proof.

Given: $\angle 2 \cong \angle 3$

Prove: $\angle 1$ and $\angle 3$ are supplementary.

Proof:

Statements	Reasons
1. $\angle 2 \cong \angle 3$	1. Given
2. $m\angle 2 = m\angle 3$	2. a. <u> ? </u>
3. b. <u> ? </u>	3. Lin. Pair Thm.
4. $m\angle 1 + m\angle 2 = 180^\circ$	4. Def. of supp. \triangle
5. $m\angle 1 + m\angle 3 = 180^\circ$	5. c. <u> ? </u> Steps 2, 4
6. d. <u> ? </u>	6. Def. of supp. \triangle



SEE EXAMPLE 1
p. 110

SEE EXAMPLE 2
p. 111

SEE EXAMPLE 3
p. 112

5. Use the given plan to write a two-column proof.

Given: X is the midpoint of \overline{AX} , and Y is the midpoint of \overline{XB} .

Prove: $\overline{AX} \cong \overline{YB}$

Plan: By the definition of midpoint, $\overline{AX} \cong \overline{XY}$, and $\overline{XY} \cong \overline{YB}$.

Use the Transitive Property to conclude that $\overline{AX} \cong \overline{YB}$.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
6	1
7–8	2
9–10	3

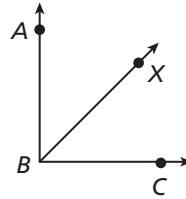
Extra Practice

Skills Practice p. 57

Application Practice p. 529

6. Write a justification for each step, given that \overrightarrow{BX} bisects $\angle ABC$ and $m\angle XBC = 45^\circ$.

1. \overrightarrow{BX} bisects $\angle ABC$.
2. $\angle ABX \cong \angle XBC$
3. $m\angle ABX = m\angle XBC$
4. $m\angle XBC = 45^\circ$
5. $m\angle ABX = 45^\circ$
6. $m\angle ABX + m\angle XBC = m\angle ABC$
7. $45^\circ + 45^\circ = m\angle ABC$
8. $90^\circ = m\angle ABC$
9. $\angle ABC$ is a right angle.

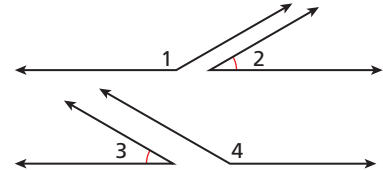


Fill in the blanks to complete each two-column proof.

7. **Given:** $\angle 1$ and $\angle 2$ are supplementary, and $\angle 3$ and $\angle 4$ are supplementary.
 $\angle 2 \cong \angle 3$

Prove: $\angle 1 \cong \angle 4$

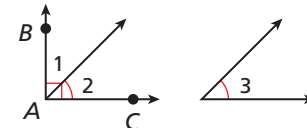
Proof:



Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary.	1. Given
2. a. <u> ?</u>	2. Def. of supp. \sphericalangle
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3. b. <u> ?</u>
4. $\angle 2 \cong \angle 3$	4. Given
5. $m\angle 2 = m\angle 3$	5. Def. of $\cong \sphericalangle$
6. c. <u> ?</u>	6. Subtr. Prop. of = Steps 3, 5
7. $\angle 1 \cong \angle 4$	7. d. <u> ?</u>

8. **Given:** $\angle BAC$ is a right angle. $\angle 2 \cong \angle 3$
Prove: $\angle 1$ and $\angle 3$ are complementary.

Proof:

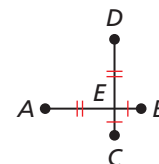


Statements	Reasons
1. $\angle BAC$ is a right angle.	1. Given
2. $m\angle BAC = 90^\circ$	2. a. <u> ?</u>
3. b. <u> ?</u>	3. \angle Add. Post.
4. $m\angle 1 + m\angle 2 = 90^\circ$	4. Subst. Steps 2, 3
5. $\angle 2 \cong \angle 3$	5. Given
6. c. <u> ?</u>	6. Def. of $\cong \sphericalangle$
7. $m\angle 1 + m\angle 3 = 90^\circ$	7. d. <u> ?</u> Steps 4, 6
8. e. <u> ?</u>	8. Def. of comp. \sphericalangle

Use the given plan to write a two-column proof.

9. **Given:** $\overline{BE} \cong \overline{CE}$, $\overline{DE} \cong \overline{AE}$
Prove: $\overline{AB} \cong \overline{CD}$

Plan: Use the definition of congruent segments to write the given information in terms of lengths. Then use the Segment Addition Postulate to show that $AB = CD$ and thus $\overline{AB} \cong \overline{CD}$.

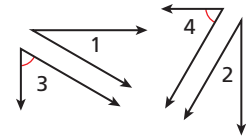


Use the given plan to write a two-column proof.

10. **Given:** $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary. $\angle 3 \cong \angle 4$

Prove: $\angle 1 \cong \angle 2$

Plan: Since $\angle 1$ and $\angle 3$ are complementary and $\angle 2$ and $\angle 4$ are complementary, both pairs of angle measures add to 90° . Use substitution to show that the sums of both pairs are equal. Since $\angle 3 \cong \angle 4$, their measures are equal. Use the Subtraction Property of Equality and the definition of congruent angles to conclude that $\angle 1 \cong \angle 2$.



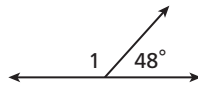
Engineering



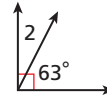
The Oresund Bridge, at 7845 meters long, is the world's longest single bridge. It carries both rail and car traffic between Denmark and Sweden.

Find each angle measure.

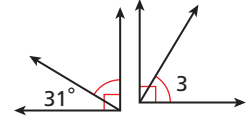
11. $m\angle 1$



12. $m\angle 2$



13. $m\angle 3$



14. **Engineering** The Oresund Bridge, which connects the countries of Denmark and Sweden, was completed in 1999. If $\angle 1 \cong \angle 2$, which theorem can you use to conclude that $\angle 3 \cong \angle 4$?

15. **Critical Thinking** Explain why there are two cases to consider when proving the Congruent Supplements Theorem and the Congruent Complements Theorem.

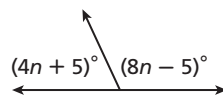


Tell whether each statement is sometimes, always, or never true.

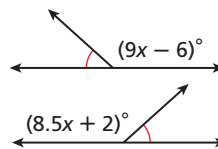
16. An angle and its complement are congruent.
 17. A pair of right angles forms a linear pair.
 18. An angle and its complement form a right angle.
 19. A linear pair of angles is complementary.

Algebra Find the value of each variable.

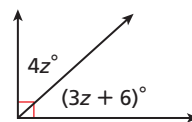
- 20.



- 21.



- 22.



23. **Write About It** How are a theorem and a postulate alike? How are they different?

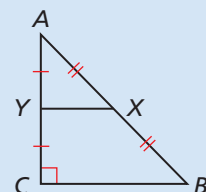
CONCEPT CONNECTION



24. This problem will prepare you for the Concept Connection on page 126.

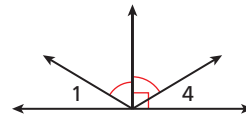
Sometimes you may be asked to write a proof without a specific statement of the Given and Prove information being provided for you. For each of the following situations, use the triangle to write a Given and Prove statement.

- The segment connecting the midpoints of two sides of a triangle is half as long as the third side.
- The acute angles of a right triangle are complementary.
- In a right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse.



25. Which theorem justifies the conclusion that $\angle 1 \cong \angle 4$?

- (A) Linear Pair Theorem
- (B) Congruent Supplements Theorem
- (C) Congruent Complements Theorem
- (D) Right Angle Congruence Theorem



26. What can be concluded from the statement $m\angle 1 + m\angle 2 = 180^\circ$?

- (F) $\angle 1$ and $\angle 2$ are congruent.
- (G) $\angle 1$ and $\angle 2$ are supplementary.
- (H) $\angle 1$ and $\angle 2$ are complementary.
- (J) $\angle 1$ and $\angle 2$ form a linear pair.

27. Given: Two angles are complementary. The measure of one angle is 10° less than the measure of the other angle. Conclusion: The measures of the angles are 85° and 95° . Which statement is true?

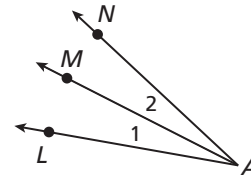
- (A) The conclusion is correct because 85° is 10° less than 95° .
- (B) The conclusion is verified by the first statement given.
- (C) The conclusion is invalid because the angles are not congruent.
- (D) The conclusion is contradicted by the first statement given.

CHALLENGE AND EXTEND

28. Write a two-column proof.

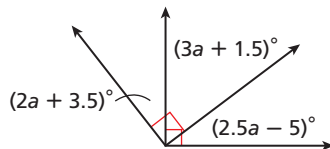
Given: $m\angle LAN = 30^\circ$, $m\angle 1 = 15^\circ$

Prove: \overrightarrow{AM} bisects $\angle LAN$.

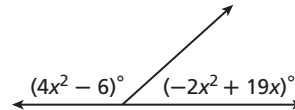


Multi-Step Find the value of the variable and the measure of each angle.

29.



30.



SPIRAL REVIEW

The table shows the number of tires replaced by a repair company during one week, classified by the mileage on the tires when they were replaced.

Use the table for Exercises 31 and 32. (*Previous course*)

- 31. What percent of the tires had mileage between 40,000 and 49,999 when replaced?
- 32. If the company replaces twice as many tires next week, about how many tires would you expect to have lasted between 80,000 and 89,999 miles?

Mileage on Replaced Tires	
Mileage	Tires
40,000–49,999	60
50,000–59,999	82
60,000–69,999	54
70,000–79,999	40
80,000–89,999	14

Sketch a figure that shows each of the following. (*Lesson 1-1*)

33. Through any two collinear points, there is more than one plane containing them.

34. A pair of opposite rays forms a line.

Identify the property that justifies each statement. (*Lesson 2-5*)

35. $\overline{JK} \cong \overline{KL}$, so $\overline{KL} \cong \overline{JK}$.

36. If $m = n$ and $n = p$, then $m = p$.

Design Plans for Proofs

Sometimes the most challenging part of writing a proof is planning the logical steps that will take you from the Given statement to the Prove statement. Like working a jigsaw puzzle, you can start with any piece. Write down everything you know from the Given statement. If you don't see the connection right away, start with the Prove statement and work backward. Then connect the pieces into a logical order.

Use with Lesson 2-6

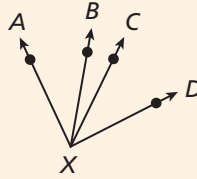
California Standards
 **2.0** Students write geometric proofs, including proofs by contradiction.

Activity

Prove the Common Angles Theorem.

Given: $\angle AXB \cong \angle CXD$

Prove: $\angle AXC \cong \angle BXD$



- Start by considering the difference in the Given and Prove statements. How does $\angle AXB$ compare to $\angle AXC$? How does $\angle CXD$ compare to $\angle BXD$?
In both cases, $\angle BXC$ is combined with the first angle to get the second angle.
- The situation involves combining adjacent angle measures, so list any definitions, properties, postulates, and theorems that might be helpful.
Definition of congruent angles, Angle Addition Postulate, properties of equality, and Reflexive, Symmetric, and Transitive Properties of Congruence

- Start with what you are given and what you are trying to prove and then work toward the middle.

$\angle AXB \cong \angle CXD$	<i>The first reason will be "Given."</i>
$m\angle AXB = m\angle CXD$	<i>Def. of $\cong \triangle$</i>
???	???
$m\angle AXC = m\angle BXD$???
$\angle AXC \cong \angle BXD$	<i>The last statement will be the Prove statement.</i>

- Based on Step 1, $\angle BXC$ is the missing piece in the middle of the logical flow. So write down what you know about $\angle BXC$.

$\angle BXC \cong \angle BXC$	<i>Reflex. Prop. of \cong</i>
$m\angle BXC = m\angle BXC$	<i>Reflex. Prop. of =</i>

- Now you can see that the Angle Addition Postulate needs to be used to complete the proof.

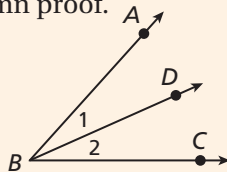
$m\angle AXB + m\angle BXC = m\angle AXC$	<i>\angle Add. Post.</i>
$m\angle BXC + m\angle CXD = m\angle BXD$	<i>\angle Add. Post.</i>

- Use the pieces to write a complete two-column proof of the Common Angles Theorem.

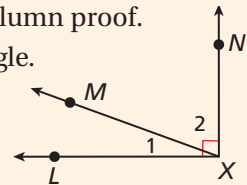
Try This

- Describe how a plan for a proof differs from the actual proof.
- Write a plan and a two-column proof.
- Write a plan and a two-column proof.

Given: \overrightarrow{BD} bisects $\angle ABC$.
Prove: $2m\angle 1 = m\angle ABC$



Given: $\angle LXN$ is a right angle.
Prove: $\angle 1$ and $\angle 2$ are complementary.



2-7

Flowchart and Paragraph Proofs



Objectives

Write flowchart and paragraph proofs.
Prove geometric theorems by using deductive reasoning.

Vocabulary

flowchart proof
paragraph proof

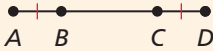
Why learn this?

Flowcharts make it easy to see how the steps of a process are linked together.

A second style of proof is a **flowchart proof**, which uses boxes and arrows to show the structure of the proof. The steps in a flowchart proof move from left to right or from top to bottom, shown by the arrows connecting each box. The justification for each step is written below the box.



Theorem 2-7-1 Common Segments Theorem

THEOREM	HYPOTHESIS	CONCLUSION
Given collinear points $A, B, C,$ and D arranged as shown, if $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$. 	$\overline{AB} \cong \overline{CD}$	$\overline{AC} \cong \overline{BD}$

EXAMPLE 1 Reading a Flowchart Proof

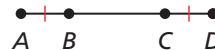
California Standards

2.0 Students write geometric proofs, including proofs by contradiction.
Also covered: **1.0**

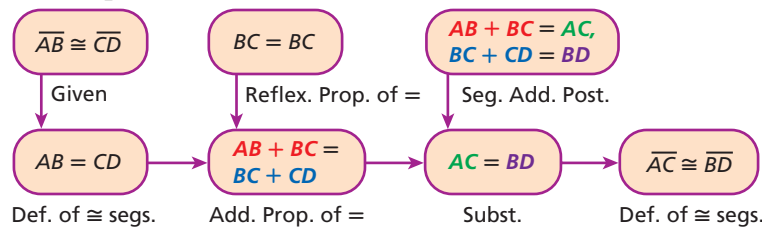
Use the given flowchart proof to write a two-column proof of the Common Segments Theorem.

Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$



Flowchart proof:



Two-column proof:

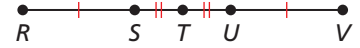
Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $AB = CD$	2. Def. of \cong segs.
3. $BC = BC$	3. Reflex. Prop. of =
4. $AB + BC = BC + CD$	4. Add. Prop. of =
5. $AB + BC = AC, BC + CD = BD$	5. Seg. Add. Post.
6. $AC = BD$	6. Subst.
7. $\overline{AC} \cong \overline{BD}$	7. Def. of \cong segs.



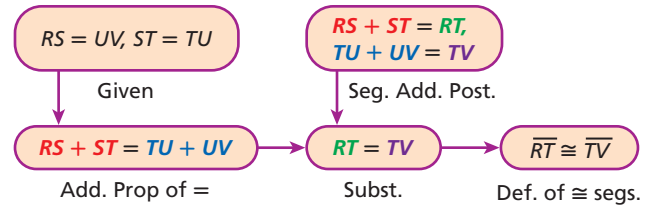
1. Use the given flowchart proof to write a two-column proof.

Given: $RS = UV, ST = TU$

Prove: $\overline{RT} \cong \overline{TV}$



Flowchart proof:

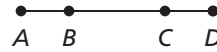


EXAMPLE 2 Writing a Flowchart Proof

Use the given two-column proof to write a flowchart proof of the Converse of the Common Segments Theorem.

Given: $\overline{AC} \cong \overline{BD}$

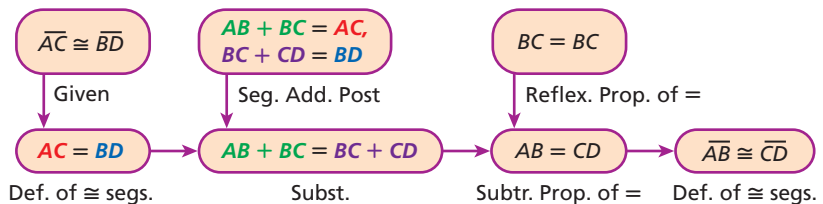
Prove: $\overline{AB} \cong \overline{CD}$



Two-column proof:

Statements	Reasons
1. $\overline{AC} \cong \overline{BD}$	1. Given
2. $AC = BD$	2. Def. of \cong segs.
3. $AB + BC = AC, BC + CD = BD$	3. Seg. Add. Post.
4. $AB + BC = BC + CD$	4. Subst. Steps 2, 3
5. $BC = BC$	5. Reflex. Prop. of =
6. $AB = CD$	6. Subtr. Prop. of =
7. $\overline{AB} \cong \overline{CD}$	7. Def. of \cong segs.

Flowchart proof:



Helpful Hint

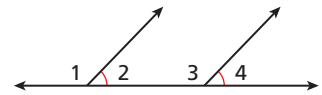
Like the converse of a conditional statement, the converse of a theorem is found by switching the hypothesis and conclusion.



2. Use the given two-column proof to write a flowchart proof.

Given: $\angle 2 \cong \angle 4$

Prove: $m\angle 1 = m\angle 3$



Two-column proof:

Statements	Reasons
1. $\angle 2 \cong \angle 4$	1. Given
2. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary.	2. Lin. Pair Thm.
3. $\angle 1 \cong \angle 3$	3. \cong Supps. Thm.
4. $m\angle 1 = m\angle 3$	4. Def. of $\cong \angle$

A **paragraph proof** is a style of proof that presents the steps of the proof and their matching reasons as sentences in a paragraph. Although this style of proof is less formal than a two-column proof, you still must include every step.



Theorems

THEOREM	HYPOTHESIS	CONCLUSION
2-7-2 Vertical Angles Theorem Vertical angles are congruent.	$\angle A$ and $\angle B$ are vertical angles.	$\angle A \cong \angle B$
2-7-3 If two congruent angles are supplementary, then each angle is a right angle. ($\cong \triangle$ supp. \rightarrow rt. \triangle)	$\angle 1 \cong \angle 2$ $\angle 1$ and $\angle 2$ are supplementary.	$\angle 1$ and $\angle 2$ are right angles.

EXAMPLE 3 Reading a Paragraph Proof

Use the given paragraph proof to write a two-column proof of the Vertical Angles Theorem.

Given: $\angle 1$ and $\angle 3$ are vertical angles.
Prove: $\angle 1 \cong \angle 3$



Paragraph proof: $\angle 1$ and $\angle 3$ are vertical angles, so they are formed by intersecting lines. Therefore $\angle 1$ and $\angle 2$ are a linear pair, and $\angle 2$ and $\angle 3$ are a linear pair. By the Linear Pair Theorem, $\angle 1$ and $\angle 2$ are supplementary, and $\angle 2$ and $\angle 3$ are supplementary. So by the Congruent Supplements Theorem, $\angle 1 \cong \angle 3$.

Two-column proof:

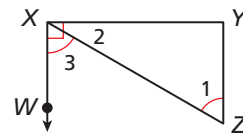
Statements	Reasons
1. $\angle 1$ and $\angle 3$ are vertical angles.	1. Given
2. $\angle 1$ and $\angle 3$ are formed by intersecting lines.	2. Def. of vert. \triangle
3. $\angle 1$ and $\angle 2$ are a linear pair. $\angle 2$ and $\angle 3$ are a linear pair.	3. Def. of lin. pair
4. $\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	4. Lin. Pair Thm.
5. $\angle 1 \cong \angle 3$	5. \cong Supps. Thm.



3. Use the given paragraph proof to write a two-column proof.

Given: $\angle WXY$ is a right angle. $\angle 1 \cong \angle 3$
Prove: $\angle 1$ and $\angle 2$ are complementary.

Paragraph proof: Since $\angle WXY$ is a right angle, $m\angle WXY = 90^\circ$ by the definition of a right angle. By the Angle Addition Postulate, $m\angle WXY = m\angle 2 + m\angle 3$. By substitution, $m\angle 2 + m\angle 3 = 90^\circ$. Since $\angle 1 \cong \angle 3$, $m\angle 1 = m\angle 3$ by the definition of congruent angles. Using substitution, $m\angle 2 + m\angle 1 = 90^\circ$. Thus by the definition of complementary angles, $\angle 1$ and $\angle 2$ are complementary.



Student to Student

Writing a Proof



Claire Jeffords
Riverbend High School

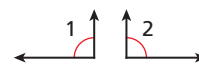
When I have to write a proof and I don't see how to start, I look at what I'm supposed to be proving and see if it makes sense. If it does, I ask myself why. Sometimes this helps me to see what the reasons in the proof might be. If all else fails, I just start writing down everything I know based on the diagram and the given statement. By brainstorming like this, I can usually figure out the steps of the proof. You can even write each thing on a separate piece of paper and arrange the pieces of paper like a flowchart.

EXAMPLE 4 Writing a Paragraph Proof

Use the given two-column proof to write a paragraph proof of Theorem 2-7-3.

Given: $\angle 1$ and $\angle 2$ are supplementary. $\angle 1 \cong \angle 2$

Prove: $\angle 1$ and $\angle 2$ are right angles.



Two-column proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 1 \cong \angle 2$	1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$	2. Def. of supp. \sphericalangle
3. $m\angle 1 = m\angle 2$	3. Def. of $\cong \sphericalangle$ Step 1
4. $m\angle 1 + m\angle 1 = 180^\circ$	4. Subst. Steps 2, 3
5. $2m\angle 1 = 180^\circ$	5. Simplification
6. $m\angle 1 = 90^\circ$	6. Div. Prop. of =
7. $m\angle 2 = 90^\circ$	7. Trans. Prop. of = Steps 3, 6
8. $\angle 1$ and $\angle 2$ are right angles.	8. Def. of rt. \sphericalangle

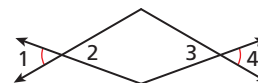
Paragraph proof: $\angle 1$ and $\angle 2$ are supplementary, so $m\angle 1 + m\angle 2 = 180^\circ$ by the definition of supplementary angles. They are also congruent, so their measures are equal by the definition of congruent angles. By substitution, $m\angle 1 + m\angle 1 = 180^\circ$, so $m\angle 1 = 90^\circ$ by the Division Property of Equality. Because $m\angle 1 = m\angle 2$, $m\angle 2 = 90^\circ$ by the Transitive Property of Equality. So both are right angles by the definition of a right angle.



4. Use the given two-column proof to write a paragraph proof.

Given: $\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$



Two-column proof:

Statements	Reasons
1. $\angle 1 \cong \angle 4$	1. Given
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	2. Vert. \sphericalangle Thm.
3. $\angle 2 \cong \angle 4$	3. Trans. Prop. of \cong Steps 1, 2
4. $\angle 2 \cong \angle 3$	4. Trans. Prop. of \cong Steps 2, 3

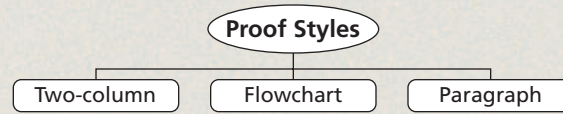
THINK AND DISCUSS

1. Explain why there might be more than one correct way to write a proof.
2. Describe the steps you take when writing a proof.



3. GET ORGANIZED

Copy and complete the graphic organizer. In each box, describe the proof style in your own words.



2-7

Exercises



California Standards

1.0, 2.0, 4.0,
7AF1.0, 7AF4.1, 1A9.0



go.hrw.com

Homework Help Online

KEYWORD: MG7 2-7

Parent Resources Online

KEYWORD: MG7 Parent

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. In a ? proof, the logical order is represented by arrows that connect each step.
(*flowchart* or *paragraph*)
2. The steps and reasons of a ? proof are written out in sentences.
(*flowchart* or *paragraph*)

SEE EXAMPLE 1

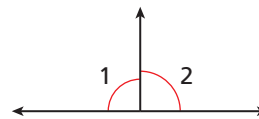
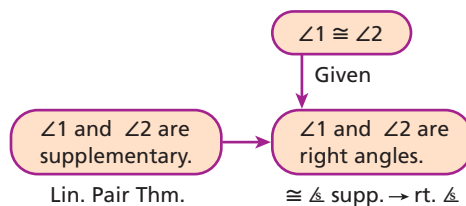
p. 118

3. Use the given flowchart proof to write a two-column proof.

Given: $\angle 1 \cong \angle 2$

Prove: $\angle 1$ and $\angle 2$ are right angles.

Flowchart proof:



SEE EXAMPLE 2

p. 119

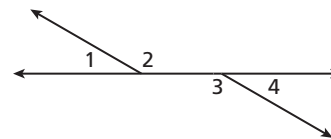
4. Use the given two-column proof to write a flowchart proof.

Given: $\angle 2$ and $\angle 4$ are supplementary.

Prove: $m\angle 2 = m\angle 3$

Two-column proof:

Statements	Reasons
1. $\angle 2$ and $\angle 4$ are supplementary.	1. Given
2. $\angle 3$ and $\angle 4$ are supplementary.	2. Lin. Pair Thm.
3. $\angle 2 \cong \angle 3$	3. \cong Supps. Thm. <i>Steps 1, 2</i>
4. $m\angle 2 = m\angle 3$	4. Def. of $\cong \angle$



SEE EXAMPLE 3

p. 120

5. Use the given paragraph proof to write a two-column proof.

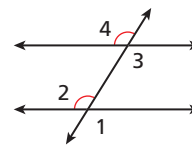
Given: $\angle 2 \cong \angle 4$

Prove: $\angle 1 \cong \angle 3$

Paragraph proof:

By the Vertical Angles Theorem, $\angle 1 \cong \angle 2$, and $\angle 3 \cong \angle 4$.

It is given that $\angle 2 \cong \angle 4$. By the Transitive Property of Congruence, $\angle 1 \cong \angle 4$, and thus $\angle 1 \cong \angle 3$.



SEE EXAMPLE 4

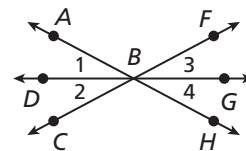
p. 121

6. Use the given two-column proof to write a paragraph proof.

Given: \overrightarrow{BD} bisects $\angle ABC$.

Prove: \overrightarrow{BG} bisects $\angle FBH$.

Two-column proof:



Statements	Reasons
1. \overrightarrow{BD} bisects $\angle ABC$.	1. Given
2. $\angle 1 \cong \angle 2$	2. Def. of \angle bisector
3. $\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$	3. Vert. \angle Thm.
4. $\angle 4 \cong \angle 2$	4. Trans. Prop. of \cong <i>Steps 2, 3</i>
5. $\angle 4 \cong \angle 3$	5. Trans. Prop. of \cong <i>Steps 3, 4</i>
6. \overrightarrow{BG} bisects $\angle FBH$.	6. Def. of \angle bisector

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
7	1
8	2
9	3
10	4

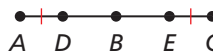
Extra Practice

Skills Practice p. S7
Application Practice p. S29

7. Use the given flowchart proof to write a two-column proof.

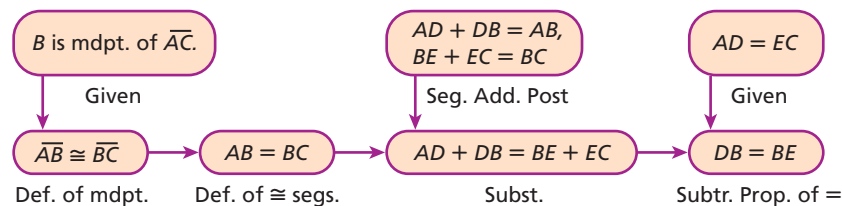
Given: B is the midpoint of \overline{AC} .

$AD = EC$



Prove: $DB = BE$

Flowchart proof:

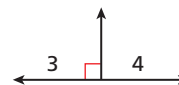


8. Use the given two-column proof to write a flowchart proof.

Given: $\angle 3$ is a right angle.

Prove: $\angle 4$ is a right angle.

Two-column proof:



Statements	Reasons
1. $\angle 3$ is a right angle.	1. Given
2. $m\angle 3 = 90^\circ$	2. Def. of rt. \angle
3. $\angle 3$ and $\angle 4$ are supplementary.	3. Lin. Pair Thm.
4. $m\angle 3 + m\angle 4 = 180^\circ$	4. Def. of supp. \angle
5. $90^\circ + m\angle 4 = 180^\circ$	5. Subst. <i>Steps 2, 4</i>
6. $m\angle 4 = 90^\circ$	6. Subtr. Prop. of =
7. $\angle 4$ is a right angle.	7. Def. of rt. \angle

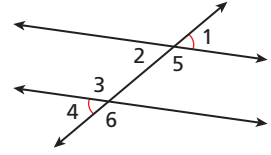
9. Use the given paragraph proof to write a two-column proof.

Given: $\angle 1 \cong \angle 4$

Prove: $\angle 2$ and $\angle 3$ are supplementary.

Paragraph proof:

$\angle 4$ and $\angle 3$ form a linear pair, so they are supplementary by the Linear Pair Theorem. Therefore, $m\angle 4 + m\angle 3 = 180^\circ$. Also, $\angle 1$ and $\angle 2$ are vertical angles, so $\angle 1 \cong \angle 2$ by the Vertical Angles Theorem. It is given that $\angle 1 \cong \angle 4$. So by the Transitive Property of Congruence, $\angle 4 \cong \angle 2$, and by the definition of congruent angles, $m\angle 4 = m\angle 2$. By substitution, $m\angle 2 + m\angle 3 = 180^\circ$, so $\angle 2$ and $\angle 3$ are supplementary by the definition of supplementary angles.



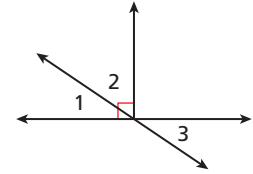
10. Use the given two-column proof to write a paragraph proof.

Given: $\angle 1$ and $\angle 2$ are complementary.

Prove: $\angle 2$ and $\angle 3$ are complementary.

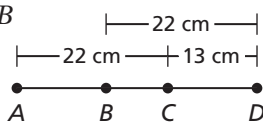
Two-column proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are complementary.	1. Given
2. $m\angle 1 + m\angle 2 = 90^\circ$	2. Def. of comp. \triangle
3. $\angle 1 \cong \angle 3$	3. Vert. \triangle Thm.
4. $m\angle 1 = m\angle 3$	4. Def. of $\cong \triangle$
5. $m\angle 3 + m\angle 2 = 90^\circ$	5. Subst. <i>Steps 2, 4</i>
6. $\angle 2$ and $\angle 3$ are complementary.	6. Def. of comp. \triangle

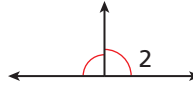


Find each measure and name the theorem that justifies your answer.

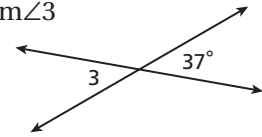
11. AB



12. $m\angle 2$

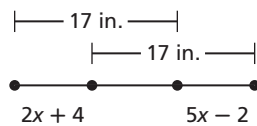


13. $m\angle 3$

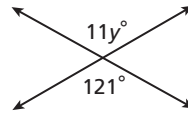


Algebra Find the value of each variable.

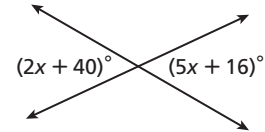
- 14.



- 15.



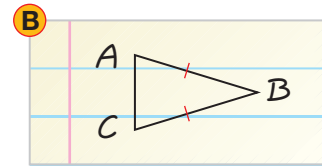
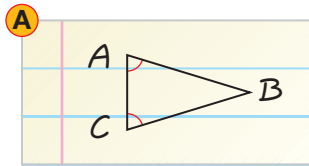
- 16.



17. **/// ERROR ANALYSIS ///** Below are two drawings for the given proof. Which is incorrect? Explain the error.

Given: $\overline{AB} \cong \overline{BC}$

Prove: $\angle A \cong \angle C$



CONCEPT CONNECTION

DANGER

18. This problem will prepare you for the Concept Connection on page 126. Rearrange the pieces to create a flowchart proof.

$m\angle 1 + m\angle 2 = 180^\circ$

Def. of supp. \triangle

$m\angle 1 = 117^\circ$

Subtr. Prop. of =

$\angle 1$ and $\angle 2$ are supplementary.

Lin. Pair Thm.

$m\angle 2 = 63^\circ$

Given

$m\angle 1 + 63^\circ = 180^\circ$

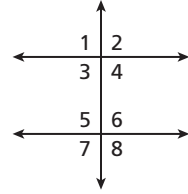
Subst.

19. **Critical Thinking** Two lines intersect, and one of the angles formed is a right angle. Explain why all four angles are congruent.
20. **Write About It** Which style of proof do you find easiest to write? to read?



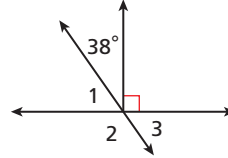
21. Which pair of angles in the diagram must be congruent?

- (A) $\angle 1$ and $\angle 5$ (C) $\angle 5$ and $\angle 8$
 (B) $\angle 3$ and $\angle 4$ (D) None of the above



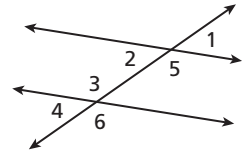
22. What is the measure of $\angle 2$?

- (F) 38° (H) 128°
 (G) 52° (J) 142°



23. Which statement is NOT true if $\angle 2$ and $\angle 6$ are supplementary?

- (A) $m\angle 2 + m\angle 6 = 180^\circ$
 (B) $\angle 2$ and $\angle 3$ are supplementary.
 (C) $\angle 1$ and $\angle 6$ are supplementary.
 (D) $m\angle 1 + m\angle 4 = 180^\circ$



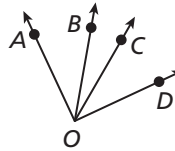
CHALLENGE AND EXTEND

24. **Textiles** Use the woven pattern to write a flowchart proof.

Given: $\angle 1 \cong \angle 3$
 Prove: $m\angle 4 + m\angle 5 = m\angle 6$

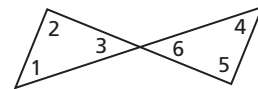
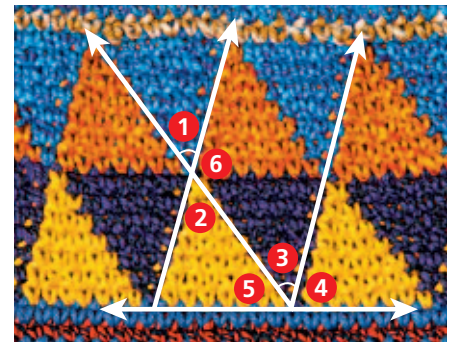
25. Write a two-column proof.

Given: $\angle AOC \cong \angle BOD$
 Prove: $\angle AOB \cong \angle COD$

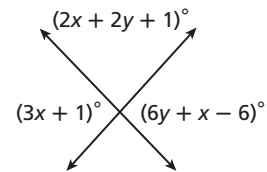


26. Write a paragraph proof.

Given: $\angle 2$ and $\angle 5$ are right angles.
 $m\angle 1 + m\angle 2 + m\angle 3 = m\angle 4 + m\angle 5 + m\angle 6$
 Prove: $\angle 1 \cong \angle 4$



27. **Multi-Step** Find the value of each variable and the measures of all four angles.



SPIRAL REVIEW

Solve each system of equations. Check your solution. (*Previous course*)

28.
$$\begin{cases} y = 2x + 14 \\ y = -6x + 18 \end{cases}$$

29.
$$\begin{cases} 7x - y = -33 \\ 3x + y = -7 \end{cases}$$

30.
$$\begin{cases} 2x + y = 8 \\ -x + 3y = 10 \end{cases}$$

Use a protractor to draw an angle with each of the following measures. (*Lesson 1-3*)

31. 125°

32. 38°

33. 94°

34. 175°

For each conditional, write the converse and a biconditional statement. (*Lesson 2-4*)

35. If a positive integer has more than two factors, then it is a composite number.

36. If a quadrilateral is a trapezoid, then it has exactly one pair of parallel sides.

CONCEPT CONNECTION



Mathematical Proof

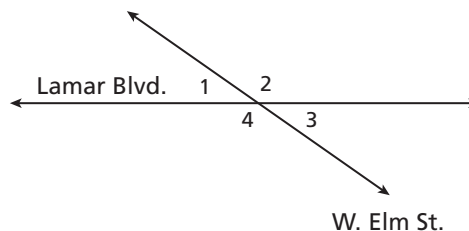
Intersection Inspection According to the U.S. Department of Transportation, it is ideal for two intersecting streets to form four 90° angles. If this is not possible, roadways should meet at an angle of 75° or greater for maximum safety and visibility.



1. Write a compound inequality to represent the range of measures an angle in an intersection should have.
2. Suppose that an angle in an intersection meets the guidelines specified by the U.S. Department of Transportation. Find the range of measures for the adjacent angle in the intersection.

The intersection of West Elm Street and Lamar Boulevard has a history of car accidents. The Southland neighborhood association is circulating a petition to have the city reconstruct the intersection. A surveyor measured the intersection, and one of the angles measures 145° .

3. Given that $m\angle 2 = 145^\circ$, write a two-column proof to show that $m\angle 1$ and $m\angle 3$ are less than 75° .
4. Write a paragraph proof to justify the argument that the intersection of West Elm Street and Lamar Boulevard should be reconstructed.



Quiz for Lessons 2-5 Through 2-7

2-5 Algebraic Proof

Solve each equation. Write a justification for each step.

1. $m - 8 = 13$

2. $4y - 1 = 27$

3. $-\frac{x}{3} = 2$

Identify the property that justifies each statement.

4. $m\angle XYZ = m\angle PQR$, so $m\angle PQR = m\angle XYZ$.

5. $\overline{AB} \cong \overline{AB}$

6. $\angle 4 \cong \angle A$, and $\angle A \cong \angle 1$. So $\angle 4 \cong \angle 1$.

7. $k = 7$, and $m = 7$. So $k = m$.

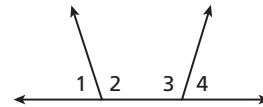
2-6 Geometric Proof

8. Fill in the blanks to complete the two-column proof.

Given: $m\angle 1 + m\angle 3 = 180^\circ$

Prove: $\angle 1 \cong \angle 4$

Proof:



Statements	Reasons
1. $m\angle 1 + m\angle 3 = 180^\circ$	1. a. <u> ?</u>
2. b. <u> ?</u>	2. Def. of supp. \sphericalangle
3. $\angle 3$ and $\angle 4$ are supplementary.	3. Lin. Pair Thm.
4. $\angle 3 \cong \angle 3$	4. c. <u> ?</u>
5. d. <u> ?</u>	5. \cong Supps. Thm.

9. Use the given plan to write a two-column proof of the Symmetric Property of Congruence.

Given: $\overline{AB} \cong \overline{EF}$

Prove: $\overline{EF} \cong \overline{AB}$



Plan: Use the definition of congruent segments to write $\overline{AB} \cong \overline{EF}$ as a statement of equality. Then use the Symmetric Property of Equality to show that $EF = AB$. So $\overline{EF} \cong \overline{AB}$ by the definition of congruent segments.

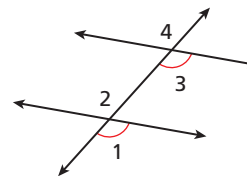
2-7 Flowchart and Paragraph Proofs

Use the given two-column proof to write the following.

Given: $\angle 1 \cong \angle 3$

Prove: $\angle 2 \cong \angle 4$

Proof:



Statements	Reasons
1. $\angle 1 \cong \angle 3$	1. Given
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	2. Vert. \sphericalangle Thm.
3. $\angle 2 \cong \angle 3$	3. Trans. Prop. of \cong
4. $\angle 2 \cong \angle 4$	4. Trans. Prop. of \cong

10. a flowchart proof

11. a paragraph proof

EXTENSION

Introduction to Symbolic Logic

Objectives

Analyze the truth value of conjunctions and disjunctions.

Construct truth tables to determine the truth value of logical statements.

Vocabulary

compound statement
conjunction
disjunction
truth table



Symbolic logic is used by computer programmers, mathematicians, and philosophers to analyze the truth value of statements, independent of their actual meaning.

A **compound statement** is created by combining two or more statements. Suppose p and q each represent a statement. Two compound statements can be formed by combining p and q : a *conjunction* and a *disjunction*.

Compound Statements

TERM	WORDS	SYMBOLS	EXAMPLE
Conjunction	A compound statement that uses the word <i>and</i>	p AND q $p \wedge q$	Pat is a band member AND Pat plays tennis.
Disjunction	A compound statement that uses the word <i>or</i>	p OR q $p \vee q$	Pat is a band member OR Pat plays tennis.

A conjunction is true only when all of its parts are true. A disjunction is true if any one of its parts is true.

EXAMPLE 1 Analyzing Truth Values of Conjunctions and Disjunctions

California Standards

1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.

Use p , q , and r to find the truth value of each compound statement.

p : Washington, D.C., is the capital of the United States.

q : The day after Monday is Tuesday.

r : California is the largest state in the United States.

A $q \vee r$

Since q is true, the disjunction is true.

B $r \wedge p$

Since r is false, the conjunction is false.



Use the information given above to find the truth value of each compound statement.

1a. $r \vee p$

1b. $p \wedge q$

A table that lists all possible combinations of truth values for a statement is called a **truth table**. A truth table shows you the truth value of a compound statement, based on the possible truth values of its parts.

Caution!

Make sure you include all possible combinations of truth values for each piece of the compound statement.

p	q	$p \rightarrow q$	$p \wedge q$	$p \vee q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	F

EXAMPLE**2****Constructing Truth Tables for Compound Statements**

Construct a truth table for the compound statement $\sim u \wedge (v \vee w)$.

Since u , v , and w can each be either true or false, the truth table will have $(2)(2)(2) = 8$ rows.

Remember!

The negation (\sim) of a statement has the opposite truth value.

u	v	w	$\sim u$	$v \vee w$	$\sim u \wedge (v \vee w)$
T	T	T	F	T	F
T	T	F	F	T	F
T	F	T	F	T	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	F	F



2. Construct a truth table for the compound statement $\sim u \wedge \sim v$.

EXTENSION**Exercises**

Use p , q , and r to find the truth value of each compound statement.

p : The day after Friday is Sunday.

q : $\frac{1}{2} = 0.5$

r : If $-4x - 2 = 10$, then $x = 3$.

1. $r \wedge q$
2. $r \vee p$
3. $p \vee r$
4. $q \wedge \sim q$
5. $\sim q \vee q$
6. $q \vee r$

Construct a truth table for each compound statement.

7. $s \wedge \sim t$
8. $\sim u \vee t$
9. $\sim u \vee (s \wedge t)$

Use a truth table to show that the two statements are logically equivalent.

10. $p \rightarrow q$; $\sim q \rightarrow \sim p$
11. $q \rightarrow p$; $\sim p \rightarrow \sim q$
12. A biconditional statement can be written as $(p \rightarrow q) \wedge (q \rightarrow p)$. Construct a truth table for this compound statement.
13. DeMorgan's Laws state that $\sim(p \wedge q) = \sim p \vee \sim q$ and that $\sim(p \vee q) = \sim p \wedge \sim q$.
 - a. Use truth tables to show that both statements are true.
 - b. If you think of disjunction and conjunction as inverse operations, DeMorgan's Laws are similar to which algebraic property?
14. The Law of Disjunctive Inference states that if $p \vee q$ is true and p is false, then q must be true.
 - a. Construct a truth table for $p \vee q$.
 - b. Use the truth table to explain why the Law of Disjunctive Inference is true.



For a complete list of the postulates and theorems in this chapter, see p. S82.

Vocabulary

biconditional statement	96	definition	97	paragraph proof	120
conclusion	81	flowchart proof	118	polygon	98
conditional statement	81	hypothesis	81	proof	104
conjecture	74	inductive reasoning	74	quadrilateral	98
contrapositive	83	inverse	83	theorem	110
converse	83	logically equivalent statements	83	triangle	98
counterexample	75	negation	82	truth value	82
deductive reasoning	88			two-column proof	111

Complete the sentences below with vocabulary words from the list above.

1. A statement you can prove and then use as a reason in later proofs is a(n) ? .
2. ? is the process of using logic to draw conclusions from given facts, definitions, and properties.
3. A(n) ? is a case in which a conjecture is not true.
4. A statement you believe to be true based on inductive reasoning is called a(n) ? .

2-1 Using Inductive Reasoning to Make Conjectures (pp. 74–79)



1.0, 3.0

EXAMPLES

- Find the next item in the pattern below.



The red square moves in the counterclockwise direction. The next figure is .

- Complete the conjecture “The sum of two odd numbers is ? .”

List some examples and look for a pattern.

$$1 + 1 = 2 \quad 3 + 5 = 8 \quad 7 + 11 = 18$$

The sum of two odd numbers is even.

- Show that the conjecture “For all non-zero integers, $-x < x$ ” is false by finding a counterexample.

Pick positive and negative values for x and substitute to see if the conjecture holds.

Let $n = 3$. Since $-3 < 3$, the conjecture holds.

Let $n = -5$. Since $-(-5)$ is 5 and $5 \not< -5$, the conjecture is false.

$n = -5$ is a counterexample.

EXERCISES

Make a conjecture about each pattern. Write the next two items.



6. $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \dots$



Complete each conjecture.

8. The sum of an even number and an odd number is ? .
9. The square of a natural number is ? .

Determine if each conjecture is true. If not, write or draw a counterexample.

10. All whole numbers are natural numbers.
11. If C is the midpoint of \overline{AB} , then $\overline{AC} \cong \overline{BC}$.
12. If $2x + 3 = 15$, then $x = 6$.
13. There are 28 days in February.
14. Draw a triangle. Construct the bisectors of each angle of the triangle. Make a conjecture about where the three angle bisectors intersect.

2-2 Conditional Statements (pp. 81–87)

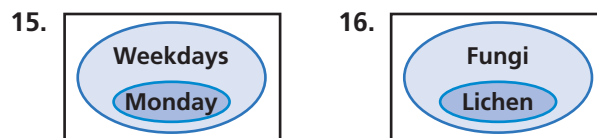


EXAMPLES

- Write a conditional statement from the sentence “A rectangle has congruent diagonals.”
If a figure is a rectangle, then it has congruent diagonals.
- Write the inverse, converse, and contrapositive of the conditional statement “If $m\angle 1 = 35^\circ$, then $\angle 1$ is acute.” Find the truth value of each.
Converse: If $\angle 1$ is acute, then $m\angle 1 = 35^\circ$.
Not all acute angles measure 35° , so this is false.
Inverse: If $m\angle 1 \neq 35^\circ$, then $\angle 1$ is not acute.
You can draw an acute angle that does not measure 35° , so this is false.
Contrapositive: If $\angle 1$ is not acute, then $m\angle 1 \neq 35^\circ$. An angle that measures 35° must be acute. So this statement is true.

EXERCISES

Write a conditional statement from each Venn diagram.



Determine if each conditional is true. If false, give a counterexample.

- If two angles are adjacent, then they have a common ray.
- If you multiply two irrational numbers, the product is irrational.

Write the converse, inverse, and contrapositive of each conditional statement. Find the truth value of each.

- If $\angle X$ is a right angle, then $m\angle X = 90^\circ$.
- If x is a whole number, then $x = 2$.

2-3 Using Deductive Reasoning to Verify Conjectures (pp. 88–93)



EXAMPLES

- Determine if the conjecture is valid by the Law of Detachment or the Law of Syllogism.
Given: If $5c = 8y$, then $2w = -15$. If $5c = 8y$, then $x = 17$.
Conjecture: If $2w = -15$, then $x = 17$.
Let p be $5c = 8y$, q be $2w = -15$, and r be $x = 17$.
Using symbols, the given information is written as $p \rightarrow q$ and $p \rightarrow r$. Neither the Law of Detachment nor the Law of Syllogism can be applied. The conjecture is not valid.
- Draw a conclusion from the given information.
Given: If two points are distinct, then there is one line through them. A and B are distinct points.
Let p be the hypothesis: two points are distinct.
Let q be the conclusion: there is one line through the points.
The statement “ A and B are distinct points” matches the hypothesis, so you can conclude that there is one line through A and B .

EXERCISES

Use the true statements below to determine whether each conclusion is true or false.

Sue is a member of the swim team. When the team practices, Sue swims. The team begins practice when the pool opens. The pool opens at 8 A.M. on weekdays and at 12 noon on Saturday.

- The swim team practices on weekdays only.
- Sue swims on Saturdays.
- Swim team practice starts at the same time every day.

Use the following information for Exercises 24–26.

The expression $2.15 + 0.07x$ gives the cost of a long-distance phone call, where x is the number of minutes after the first minute.

If possible, draw a conclusion from the given information. If not possible, explain why.

- The cost of Sara’s long-distance call is \$2.57.
- Paulo makes a long-distance call that lasts ten minutes.
- Asa’s long-distance phone bill for the month is \$19.05.

2-4 Biconditional Statements and Definitions (pp. 96–101)



EXAMPLES

- For the conditional “If a number is divisible by 10, then it ends in 0”, write the converse and a biconditional statement.

Converse: If a number ends in 0, then it is divisible by 10.

Biconditional: A number is divisible by 10 if and only if it ends in 0.

- Determine if the biconditional “The sides of a triangle measure 3, 7, and 15 if and only if the perimeter is 25” is true. If false, give a counterexample.

Conditional: If the sides of a triangle measure 3, 7, and 15, then the perimeter is 25. True.

Converse: If the perimeter of a triangle is 25, then its sides measure 3, 7, and 15. False; a triangle with side lengths of 6, 10, and 9 also has a perimeter of 25.

Therefore the biconditional is false.

EXERCISES

Determine if a true biconditional can be written from each conditional statement. If not, give a counterexample.

- If $3 - \frac{2x}{5} = 2$, then $x = \frac{5}{2}$.
- If $x < 0$, then the value of x^4 is positive.
- If a segment has endpoints at $(1, 5)$ and $(-3, 1)$, then its midpoint is $(-1, 3)$.
- If the measure of one angle of a triangle is 90° , then the triangle is a right triangle.

Complete each statement to form a true biconditional.

- Two angles are $\underline{\quad ? \quad}$ if and only if the sum of their measures is 90° .
- $x^3 > 0$ if and only if x is $\underline{\quad ? \quad}$.
- Trey can travel 100 miles in less than 2 hours if and only if his average speed is $\underline{\quad ? \quad}$.
- The area of a square is equal to s^2 if and only if the perimeter of the square is $\underline{\quad ? \quad}$.

2-5 Algebraic Proof (pp. 104–109)



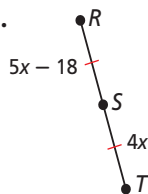
EXAMPLES

- Solve the equation $5x - 3 = -18$. Write a justification for each step.

$$\begin{array}{ll}
 5x - 3 = -18 & \text{Given} \\
 \underline{+3} \quad \underline{+3} & \text{Add. Prop. of =} \\
 5x = -15 & \text{Simplify.} \\
 \underline{\frac{5x}{5}} = \underline{\frac{-15}{5}} & \text{Div. Prop. of =} \\
 x = -3 & \text{Simplify.}
 \end{array}$$

- Write a justification for each step.

$$\begin{array}{ll}
 RS = ST & \text{Given} \\
 5x - 18 = 4x & \text{Subst. Prop. of =} \\
 x - 18 = 0 & \text{Subtr. Prop. of =} \\
 x = 18 & \text{Add. Prop. of =}
 \end{array}$$



Identify the property that justifies each statement.

- $\angle X \cong \angle 2$, so $\angle 2 \cong \angle X$.
Symmetric Property of Congruence
- If $m\angle 2 = 180^\circ$ and $m\angle 3 = 180^\circ$, then $m\angle 2 = m\angle 3$.
Transitive Property of Equality

EXERCISES

Solve each equation. Write a justification for each step.

$$35. \frac{m}{-5} + 3 = -4.5 \qquad 36. -47 = 3x - 59$$

Identify the property that justifies each statement.

- $a + b = a + b$
- If $\angle RST \cong \angle ABC$, then $\angle ABC \cong \angle RST$.
- $2x = 9$, and $y = 9$. So $2x = y$.

Use the indicated property to complete each statement.

- Reflex. Prop. of \cong : figure $ABCD \cong \underline{\quad ? \quad}$
- Sym. Prop. of $=$: If $m\angle 2 = m\angle 5$, then $\underline{\quad ? \quad}$.
- Trans. Prop. of \cong : If $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \cong \overline{EF}$, then $\underline{\quad ? \quad}$.
- Kim borrowed money at an annual simple interest rate of 6% to buy a car. How much did she borrow if she paid \$4200 in interest over the life of the 4-year loan? Solve the equation $I = Prt$ for P and justify each step.

EXAMPLES

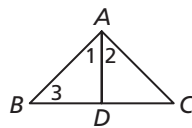
- Write a justification for each step, given that $m\angle 2 = 2m\angle 1$.

1. $\angle 1$ and $\angle 2$ supp.	Lin. Pair Thm.
2. $m\angle 1 + m\angle 2 = 180^\circ$	Def. of supp. \triangle
3. $m\angle 2 = 2m\angle 1$	Given
4. $m\angle 1 + 2m\angle 1 = 180^\circ$	Subst. <i>Steps 2, 3</i>
5. $3m\angle 1 = 180^\circ$	Simplify
6. $m\angle 1 = 60^\circ$	Div. Prop. of $=$

- Use the given plan to write a two-column proof.

Given: \overline{AD} bisects $\angle BAC$.
 $\angle 1 \cong \angle 3$

Prove: $\angle 2 \cong \angle 3$



Plan: Use the definition of angle bisector to show that $\angle 1 \cong \angle 2$. Use the Transitive Property to conclude that $\angle 2 \cong \angle 3$.

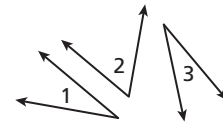
Two-column proof:

Statements	Reasons
1. \overline{AD} bisects $\angle BAC$.	1. Given
2. $\angle 1 \cong \angle 2$	2. Def. of \angle bisector
3. $\angle 1 \cong \angle 3$	3. Given
4. $\angle 2 \cong \angle 3$	4. Trans. Prop. of \cong

EXERCISES

- 44. Write a justification for each step, given that $\angle 1$ and $\angle 2$ are complementary, and $\angle 1 \cong \angle 3$.

1. $\angle 1$ and $\angle 2$ comp.	
2. $m\angle 1 + m\angle 2 = 90^\circ$	
3. $\angle 1 \cong \angle 3$	
4. $m\angle 1 = m\angle 3$	
5. $m\angle 3 + m\angle 2 = 90^\circ$	
6. $\angle 3$ and $\angle 2$ comp.	

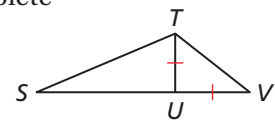


- 45. Fill in the blanks to complete the two-column proof.

Given: $\overline{TU} \cong \overline{UV}$

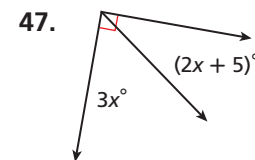
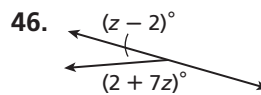
Prove: $SU + TU = SV$

Two-column proof:



Statements	Reasons
1. $\overline{TU} \cong \overline{UV}$	1. a. <u> ?</u>
2. b. <u> ?</u>	2. Def. of \cong segs.
3. c. <u> ?</u>	3. Seg. Add. Post.
4. $SU + TU = SV$	4. d. <u> ?</u>

Find the value of each variable.

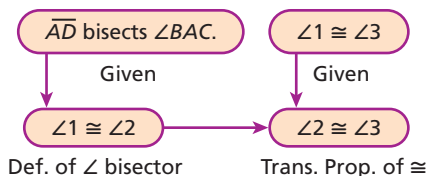


2-7 Flowchart and Paragraph Proofs (pp. 118–125)

EXAMPLES

Use the two-column proof in the example for Lesson 2-6 above to write each of the following.

- a flowchart proof



- a paragraph proof

Since \overline{AD} bisects $\angle BAC$, $\angle 1 \cong \angle 2$ by the definition of angle bisector. It is given that $\angle 1 \cong \angle 3$. Therefore, $\angle 2 \cong \angle 3$ by the Transitive Property of Congruence.

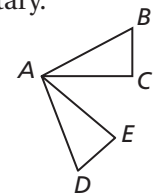
EXERCISES

Use the given plan to write each of the following.

Given: $\angle ADE$ and $\angle DAE$ are complementary.
 $\angle ADE$ and $\angle BAC$ are complementary.

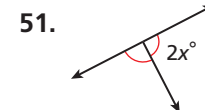
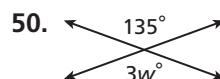
Prove: $\angle DAC \cong \angle BAE$

Plan: Use the Congruent Complements Theorem to show that $\angle DAE \cong \angle BAC$. Since $\angle CAE \cong \angle CAE$, $\angle DAC \cong \angle BAE$ by the Common Angles Theorem.




- 48. a flowchart proof
- 49. a paragraph proof

Find the value of each variable and name the theorem that justifies your answer.



Find the next item in each pattern.

1.  , ...
2. 405, 135, 45, 15, ...
3. Complete the conjecture “The sum of two even numbers is ? .”
4. Show that the conjecture “All complementary angles are adjacent” is false by finding a counterexample.
5. Identify the hypothesis and conclusion of the conditional statement “The show is cancelled if it rains.”
6. Write a conditional statement from the sentence “Parallel lines do not intersect.”

Determine if each conditional is true. If false, give a counterexample.

7. If two lines intersect, then they form four right angles.
8. If a number is divisible by 10, then it is divisible by 5.

Use the conditional “If you live in the United States, then you live in Kentucky” for Items 9–11. Write the indicated type of statement and determine its truth value.

9. converse
10. inverse
11. contrapositive
12. Determine if the following conjecture is valid by the Law of Detachment.
Given: If it is colder than 50°F, Tom wears a sweater. It is 46°F today.
Conjecture: Tom is wearing a sweater.
13. Use the Law of Syllogism to draw a conclusion from the given information.
Given: If a figure is a square, then it is a quadrilateral. If a figure is a quadrilateral, then it is a polygon. Figure $ABCD$ is a square.
14. Write the conditional statement and converse within the biconditional “Chad will work on Saturday if and only if he gets paid overtime.”
15. Determine if the biconditional “ B is the midpoint of \overline{AC} iff $AB = BC$ ” is true. If false, give a counterexample.

Solve each equation. Write a justification for each step.

16. $8 - 5s = 1$
17. $0.4t + 3 = 1.6$
18. $38 = -3w + 2$

Identify the property that justifies each statement.

19. If $2x = y$ and $y = 7$, then $2x = 7$.
20. $m\angle DEF = m\angle DEF$
21. $\angle X \cong \angle P$, and $\angle P \cong \angle D$. So $\angle X \cong \angle D$.
22. If $\overline{ST} \cong \overline{XY}$, then $\overline{XY} \cong \overline{ST}$.

Use the given plan to write a proof in each format.

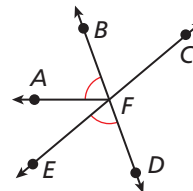
Given: $\angle AFB \cong \angle EFD$

Prove: \overrightarrow{FB} bisects $\angle AFC$.

Plan: Since vertical angles are congruent, $\angle EFD \cong \angle BFC$.

Use the Transitive Property to conclude that $\angle AFB \cong \angle BFC$.

Thus \overrightarrow{FB} bisects $\angle AFC$ by the definition of angle bisector.



23. two-column proof
24. paragraph proof
25. flowchart proof

COLLEGE ENTRANCE EXAM PRACTICE

FOCUS ON SAT MATHEMATICS SUBJECT TESTS

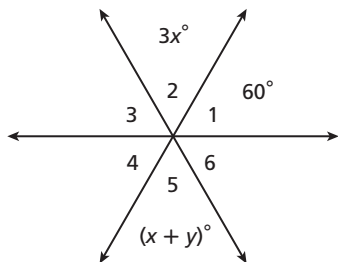
Some colleges require that you take the SAT Subject Tests. There are two math subject tests—Level 1 and Level 2. Take the Mathematics Subject Test Level 1 when you have completed three years of college-prep mathematics courses.



On SAT Mathematics Subject Test questions, you receive one point for each correct answer, but you lose a fraction of a point for each incorrect response. Guess only when you can eliminate at least one of the answer choices.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. In the figure below, $m\angle 1 = m\angle 2$. What is the value of y ?



Note: Figure not drawn to scale.

- (A) 10 (B) 30
 (C) 40 (D) 50
 (E) 60
2. The statement “I will cancel my appointment if and only if I have a conflict” is true. Which of the following can be concluded?
- I. If I have a conflict, then I will cancel my appointment.
 II. If I do not cancel my appointment, then I do not have a conflict.
 III. If I cancel my appointment, then I have a conflict.
- (A) I only (B) II only
 (C) III only (D) I and III
 (E) I, II, and III

3. What is the contrapositive of the statement “If it is raining, then the football team will win”?

- (A) If it is not raining, then the football team will not win.
 (B) If it is raining, then the football team will not win.
 (C) If the football team wins, then it is raining.
 (D) If the football team does not win, then it is not raining.
 (E) If it is not raining, then the football team will win.

4. Given the points $D(1, 5)$ and $E(-2, 3)$, which conclusion is NOT valid?

- (A) The midpoint of \overline{DE} is $(-\frac{1}{2}, 4)$.
 (B) D and E are collinear.
 (C) The distance between D and E is $\sqrt{5}$.
 (D) $\overline{DE} \cong \overline{ED}$
 (E) D and E are distinct points.

5. For all integers x , what conclusion can be drawn about the value of the expression $\frac{x^2}{2}$?

- (A) The value is negative.
 (B) The value is not negative.
 (C) The value is even.
 (D) The value is odd.
 (E) The value is not a whole number.

Gridded Response: Record Your Answer

When responding to a gridded-response test item, you must fill out the grid on your answer sheet correctly, or the item will be marked as incorrect.

EXAMPLE 1

Gridded Response: Solve the equation $25 - 2(3x - 4) = 13$.

1	0	/	3	
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<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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The value of x is $\frac{20}{6}$, $\frac{10}{3}$, $3\frac{1}{3}$, or $3.\bar{3}$.

- Mixed numbers and repeating decimals cannot be gridded, so you must grid the answer as $\frac{20}{6}$ or $\frac{10}{3}$.
- Using a pencil, write your answer in the answer boxes at the top of the grid.
- Put only one digit or symbol in each box. On some grids, the fraction bar and decimal point have a designated column.
- Do not leave a blank box in the middle of an answer.
- For each digit or symbol, shade the bubble that is in the same column as the digit or symbol in the answer box.

EXAMPLE 2

Gridded Response: The perimeter of a rectangle is 90 in. The width of the rectangle is 18 in. Find the length of the rectangle in feet.

	2	.	2	5
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<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

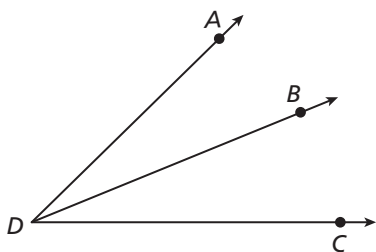
The length of the rectangle is 27 inches, but the problem asks for the measurement in feet.

27 inches = 2.25 or $\frac{9}{4}$ feet

- Using a pencil, write your answer in the answer boxes at the top of the grid.
- Put only one digit or symbol in each box. On some grids, the fraction bar and the decimal point have a designated column.
- Do not leave a blank box in the middle of an answer.
- For each digit or symbol, shade the bubble that is in the same column as the digit or symbol in the answer box.

CUMULATIVE ASSESSMENT, CHAPTERS 1–2
Multiple Choice

Use the figure below for Items 1 and 2. In the figure, \overline{DB} bisects $\angle ADC$.



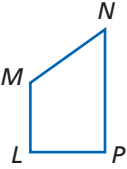
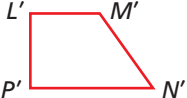
- Which best describes the intersection of $\angle ADB$ and $\angle BDC$?
 - Exactly one ray
 - Exactly one point
 - Exactly one angle
 - Exactly one segment
- Which expression is equal to the measure of $\angle ADC$?
 - $2(m\angle ADB)$
 - $90^\circ - m\angle BDC$
 - $180^\circ - 2(m\angle ADC)$
 - $m\angle BDC - m\angle ADB$
- What is the inverse of the statement, "If a polygon has 8 sides, then it is an octagon"?
 - If a polygon is an octagon, then it has 8 sides.
 - If a polygon is not an octagon, then it does not have 8 sides.
 - If an octagon has 8 sides, then it is a polygon.
 - If a polygon does not have 8 sides, then it is not an octagon.
- Lily conjectures that if a number is divisible by 15, then it is also divisible by 9. Which of the following is a counterexample?

<input type="radio"/> F 45	<input type="radio"/> H 60
<input type="radio"/> G 50	<input type="radio"/> J 72
- A diagonal of a polygon connects nonconsecutive vertices. The table shows the number of diagonals in a polygon with n sides.

Number of Sides	Number of Diagonals
4	2
5	5
6	9
7	14

If the pattern continues, how many diagonals does a polygon with 8 sides have?

<input type="radio"/> A 17	<input type="radio"/> C 20
<input type="radio"/> B 19	<input type="radio"/> D 21
- Which type of transformation maps figure $LMNP$ onto figure $L'M'N'P'$?

<input type="radio"/> F Reflection	<input type="radio"/> H Translation
<input type="radio"/> G Rotation	<input type="radio"/> J None of these
- Miyoko went jogging on July 25, July 28, July 31, and August 3. If this pattern continues, when will Miyoko go jogging next?

<input type="radio"/> A August 5	<input type="radio"/> C August 7
<input type="radio"/> B August 6	<input type="radio"/> D August 8
- Congruent segments have equal measures. A segment bisector divides a segment into two congruent segments. \overline{XY} intersects \overline{DE} at X and bisects \overline{DE} . Which conjecture is valid?

<input type="radio"/> F $m\angle YXD = m\angle YXE$
<input type="radio"/> G Y is between D and E .
<input type="radio"/> H $DX = XE$
<input type="radio"/> J $DE = YE$

9. Which statement is true by the Symmetric Property of Congruence?
- (A) $\overline{ST} \cong \overline{ST}$
- (B) $15 + MN = MN + 15$
- (C) If $\angle P \cong \angle Q$, then $\angle Q \cong \angle P$.
- (D) If $\angle D \cong \angle E$ and $\angle E \cong \angle F$, then $\angle D \cong \angle F$.



To find a counterexample for a biconditional statement, write the conditional statement and converse it contains. Then try to find a counterexample for one of these statements.

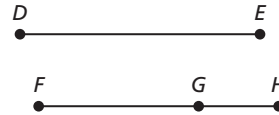
10. Which is a counterexample for the following biconditional statement?
A pair of angles is supplementary if and only if the angles form a linear pair.
- (F) The measures of supplementary angles add to 180° .
- (G) A linear pair of angles is supplementary.
- (H) Complementary angles do not form a linear pair.
- (J) Two supplementary angles are not adjacent.
11. K is between J and L . The distance between J and K is 3.5 times the distance between K and L . If $JK = 14$, what is JL ?
- (A) 10.5 (C) 24.5
- (B) 18 (D) 49
12. What is the length of the segment connecting the points $(-7, -5)$ and $(5, -2)$?
- (F) $\sqrt{13}$ (H) $3\sqrt{17}$
- (G) $\sqrt{53}$ (J) $\sqrt{193}$

Gridded Response

13. A segment has an endpoint at $(5, -2)$. The midpoint of the segment is $(2, 2)$. What is the length of the segment?
14. $\angle P$ measures 30° more than the measure of its supplement. What is the measure of $\angle P$ in degrees?
15. The perimeter of a square field is 1.6 kilometers. What is the area of the field in square kilometers?

Short Response

16. Solve the equation $2(AB) + 16 = 24$ to find the length of segment AB . Write a justification for each step.
17. Use the given two-column proof to write a flowchart proof.



Given: $\overline{DE} \cong \overline{FH}$

Prove: $DE = FG + GH$

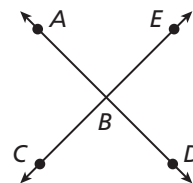
Two-column proof:

Statements	Reasons
1. $\overline{DE} \cong \overline{FH}$	1. Given
2. $DE = FH$	2. Def. of \cong segs.
3. $FG + GH = FH$	3. Seg. Add. Post.
4. $DE = FG + GH$	4. Subst.

18. Consider the following conditional statement.
If two angles are complementary, then the angles are acute.
- a. Determine if the conditional is true or false. If false, give a counterexample.
- b. Write the converse of the conditional statement.
- c. Determine whether the converse is true or false. If false, give a counterexample.

Extended Response

19. The figure below shows the intersection of two lines.

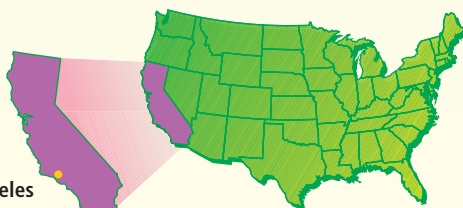


- a. Name the linear pairs of angles in the figure. What conclusion can you make about each pair? Explain your reasoning.
- b. Name the pairs of vertical angles in the figure. What conclusion can you make about each pair? Explain your reasoning.
- c. Suppose $m\angle EBD = 90^\circ$. What are the measures of the other angles in the figure? Write a two-column proof to support your answer.



Problem Solving on Location

CALIFORNIA



Los Angeles

★ Los Angeles Marathon

Every year in March, runners participate in a 26.2 mile marathon through the heart of Los Angeles. The race is one of the largest single-day running events and includes an estimated 23,000 runners and more than one million spectators. It offers on-course live performances including 10 main entertainment centers and more than 85 entertainment sites.

Choose one or more strategies to solve each problem.

1. During the marathon, a runner maintains a steady pace and completes the first 2.6 miles in 20 minutes. After 1 hour 20 minutes, she has completed 10.4 miles. Make a conjecture about the runner's average speed in miles per hour. How long do you expect it to take her to complete the marathon?
2. Suppose sports drink stations are available every 2 miles and medical stations are available every 3 miles and both are at the end of the course. At how many points is there both a sports drink station and a medical station?
3. An entertainment center is located at mile 22. The location of the center is about 1.2 times the distance from the start of the race to a motivating tunnel created for the runners. What is the distance from the starting point to the tunnel? Round to the nearest tenth.





Problem Solving Strategies

- Draw a Diagram
- Make a Model
- Guess and Test
- Work Backward
- Find a Pattern
- Make a Table
- Solve a Simpler Problem
- Use Logical Reasoning
- Use a Venn Diagram
- Make an Organized List

★ California's Waterfalls

California's State Parks are crisscrossed by hiking trails that lead to more than 24 waterfalls, which range from shallow cascades to the spectacular 640-foot-tall Feather Falls.

Choose one or more strategies to solve each problem.

1. A hiker made a round-trip hike of at least 3 miles and saw a waterfall that is less than 100 feet tall. Which waterfalls might the hiker have visited?
2. A travel brochure includes the following statements about California's waterfalls. Determine if each statement is true or false. If false, explain why.
 - a. If your round-trip hike is greater than 6 miles, you will be rewarded with an incredible view of a waterfall that is more than 600 feet tall.
 - b. If you haven't been to Codfish Falls, then you haven't seen a waterfall that is at least 100 feet tall.
 - c. If you don't want to hike more than 2 miles but want to see a 70-foot-tall waterfall, then you should visit Pfeiffer Falls.
3. Malibu Creek State Park has three waterfalls. The first falls is called the Rindge Dam Waterfall and is one mile closer to the start of the trail than the second falls, Century Lake Falls. The third falls, Malibu Lake Falls, is 2.3 miles farther than Century Lake Falls. If the total distance of all three trails is 10.3 miles, what is the length of each trail?

California Waterfalls		
Waterfall	Height (ft)	Trail Length, One Way (mi)
Berry Creek Falls	70	5.0
Codfish Falls	100	1.2
Feather Falls	640	3.4
Jamison Falls	70	1.0
Pfeiffer Falls	60	1.0
Sempervirens Falls	25	1.5



Berry Creek Falls
Boulder Creek, California