CHAPTER 1

Foundations for Geometry

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Chapter Project Online
KEYWORD: MG7 ChProj

Representations of points, lines, and planes can be seen in the Los Angeles skyline.

Skyline
Los Angeles, CA

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. coordinate
2. metric system of measurement
3. expression
4. order of operations

A. a mathematical phrase that contains operations, numbers, and/or variables

B. the measurement system often used in the United States

C. one of the numbers of an ordered pair that locates a point on a coordinate graph

D. a list of rules for evaluating expressions

E. a decimal system of weights and measures that is used universally in science and commonly throughout the world

Measure with Customary and Metric Units

For each object tell which is the better measurement.

5. length of an unsharpened pencil
$7\frac{1}{2}$ in. or $9\frac{3}{4}$ in.
6. the diameter of a quarter
1 m or $2\frac{1}{2}$ cm
7. length of a soccer field
100 yd or 40 yd
8. height of a classroom
5 ft or 10 ft
9. height of a student's desk
30 in. or 4 ft
10. length of a dollar bill
15.6 cm or 35.5 cm

Combine Like Terms

Simplify each expression.

11. $-y + 3y - 6y + 12y$
12. $63 + 2x - 7 - 4x$
13. $-5 - 9 - 7x + 6x$
14. $24 - 3y + y + 7$

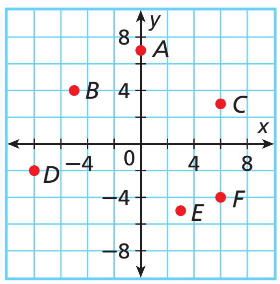
Evaluate Expressions

Evaluate each expression for the given value of the variable.

15. $x + 3x + 7x$ for $x = -5$
16. $5p + 10$ for $p = 78$
17. $2a - 8a$ for $a = 12$
18. $3n - 3$ for $n = 16$

Ordered Pairs

Write the ordered pair for each point.



19. A
20. B
21. C
22. D
23. E
24. F

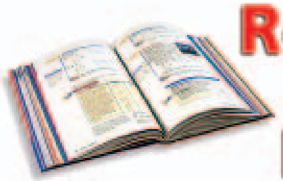
CHAPTER 1

Unpacking the Standards

The information below "unpacks" the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

California Standard	Academic Vocabulary	Chapter Concept
1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning. (Lesson 1-1)	**demonstrate** show **identifying** seeing and being able to name what something is	You begin to see how terms and basic facts can be used to develop geometric arguments.
8.0 Students know, derive, **and solve problems involving the perimeter, circumference, area,** volume, lateral area, and surface area **of common geometric figures.** (Lesson 1-5)	**solve** find the value of a variable that makes the left side of an equation equal to the right side of the equation ***Example:*** $2x = 6$ $2(3) = 6$ The value that makes $2x = 6$ true is **3**.	You learn basic formulas so you can solve problems involving the perimeter and area of triangles, quadrilaterals, and circles.
16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line. (Lessons 1-2, 1-3)	**basic** most important or fundamental; used as a starting point	You are introduced to constructions to help you see how geometry is organized. You learn about length, midpoints, congruence, angles, and bisectors.
22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, **including rotations, translations, and reflections.** (Lesson 1-7) (Lab 1-7)	**effect** outcome **rigid motions** movements of a figure that do not change its shape	You learn how to identify and graph reflections, rotations, and translations of basic geometric figures.

Standard 15.0 is also covered in this chapter. To see this standard unpacked, go to Chapter 5, p. 298.



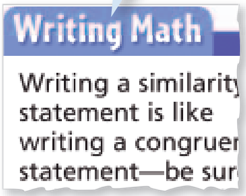
Reading and Writing Math

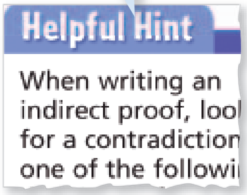
CHAPTER 1

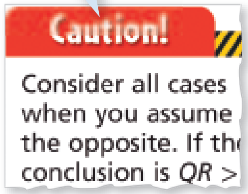
Reading Strategy: Use Your Book for Success

Understanding how your textbook is organized will help you locate and use helpful information.

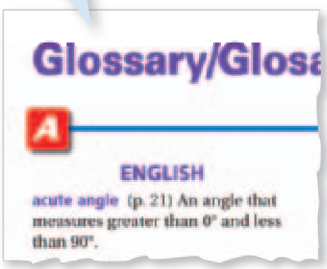
As you read through an example problem, pay attention to the notes in the **margin.** These notes highlight key information about the concept and will help you to avoid common mistakes.



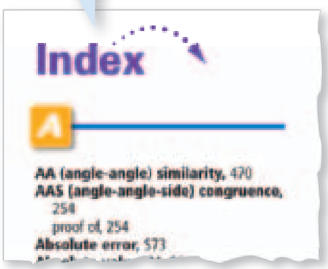




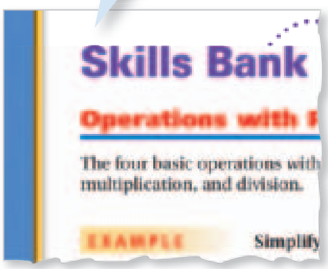
The **Glossary** is found in the back of your textbook. Use it when you need a definition of an unfamiliar word or phrase.



The **Index** is located at the end of your textbook. If you need to locate the page where a particular concept is explained, use the **Index** to find the corresponding page number.



The **Skills Bank** is located in the back of your textbook. Look in the **Skills Bank** for help with math topics that were taught in previous courses, such as the order of operations.

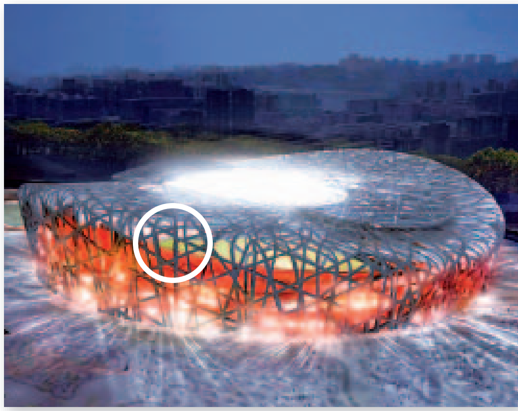


Try This

Use your textbook for the following problems.

1. Use the index to find the page where *right angle* is defined.
2. What formula does the Know-It Note on the first page of Lesson 1-6 refer to?
3. Use the glossary to find the definition of *congruent segments.*
4. In what part of the textbook can you find help for solving equations?

1-1 Understanding Points, Lines, and Planes



Objectives

Identify, name, and draw points, lines, segments, rays, and planes.

Apply basic facts about points, lines, and planes.

Vocabulary

undefined term
point
line
plane
collinear
coplanar
segment
endpoint
ray
opposite rays
postulate

California Standards

1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.

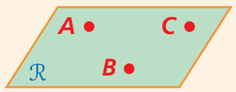
Who uses this?

Architects use representations of points, lines, and planes to create models of buildings. Interwoven segments were used to model the beams of Beijing's National Stadium for the 2008 Olympics.

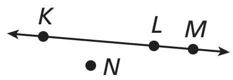
The most basic figures in geometry are **undefined terms**, which cannot be defined by using other figures. The undefined terms *point*, *line*, and *plane* are the building blocks of geometry.

Know it! Note

Undefined Terms

TERM	NAME	DIAGRAM
A **point** names a location and has no size. It is represented by a dot.	A capital letter point P	
A **line** is a straight path that has no thickness and extends forever.	A lowercase letter or two points on the line line ℓ, $\overleftrightarrow{XY}$ or $\overleftrightarrow{YX}$	
A **plane** is a flat surface that has no thickness and extends forever.	A script capital letter or three points not on a line plane $\mathcal{R}$ or plane ABC	

Points that lie on the same line are **collinear**. K, L, and M are collinear. K, L, and N are *noncollinear*. Points that lie in the same plane are **coplanar**. Otherwise they are *noncoplanar*.



EXAMPLE 1 Naming Points, Lines, and Planes

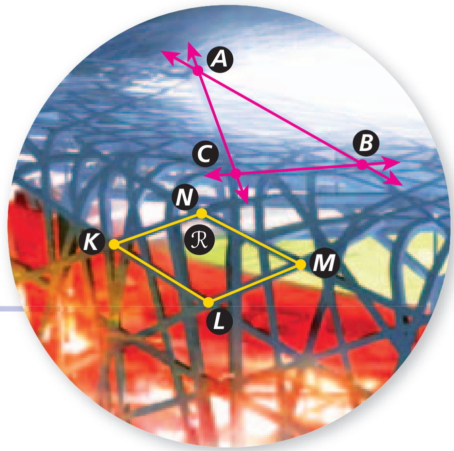
Refer to the design in the roof of Beijing's National Stadium.

A **Name four coplanar points.**

K, L, M, and N all lie in plane $\mathcal{R}$.

B **Name three lines.**

$\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, and $\overleftrightarrow{CA}$.



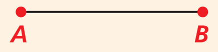
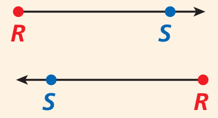
Helpful Hint

A plane may be named by any three noncollinear points on that plane. Plane *ABC* may also be named *BCA*, *CAB*, *CBA*, *ACB*, or *BAC*.

CHECK IT OUT!

1. Use the diagram to name two planes.

Segments and Rays

DEFINITION	NAME	DIAGRAM
A **segment**, or line segment, is the part of a line consisting of two points and all points between them.	The two endpoints $\overline{AB}$ or $\overline{BA}$	
An **endpoint** is a point at one end of a segment or the starting point of a *ray.*	A capital letter C and D	
A **ray** is a part of a line that starts at an endpoint and extends forever in one direction.	Its endpoint and any other point on the ray $\overrightarrow{RS}$	
Opposite rays are two rays that have a common endpoint and form a line.	The common endpoint and any other point on each ray $\overrightarrow{EF}$ and $\overrightarrow{EG}$	

EXAMPLE 2 Drawing Segments and Rays

Draw and label each of the following.

A a segment with endpoints U and V

B opposite rays with a common endpoint Q

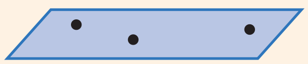
CHECK IT OUT! **2.** Draw and label a ray with endpoint M that contains N.

A **postulate**, or *axiom,* is a statement that is accepted as true without proof. Postulates about points, lines, and planes help describe geometric properties.

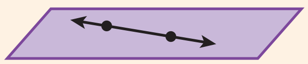
Postulates Points, Lines, and Planes

1-1-1 Through any two points there is exactly one line.

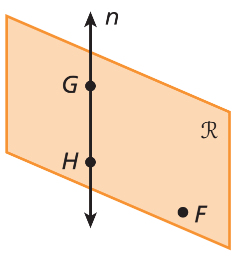
1-1-2 Through any three noncollinear points there is exactly one plane containing them.



1-1-3 If two points lie in a plane, then the line containing those points lies in the plane.



EXAMPLE 3 Identifying Points and Lines in a Plane



Name a line that passes through two points.

There is exactly one line n passing through G and H.

CHECK IT OUT! **3.** Name a plane that contains three noncollinear points.

Recall that a system of equations is a set of two or more equations containing two or more of the same variables. The coordinates of the solution of the system satisfy all equations in the system. These coordinates also locate the point where all the graphs of the equations in the system *intersect*.

An *intersection* is the set of all points that two or more figures have in common. The next two postulates describe intersections involving lines and planes.

Know it! Note

Postulates **Intersection of Lines and Planes**

1-1-4 If two lines intersect, then they intersect in exactly one point.

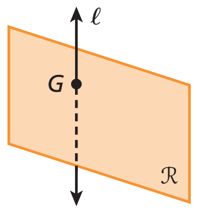
1-1-5 If two planes intersect, then they intersect in exactly one line.

Use a dashed line to show the hidden parts of any figure that you are drawing. A dashed line will indicate the part of the figure that is not seen.

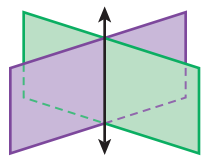
EXAMPLE 4 Representing Intersections

Sketch a figure that shows each of the following.

A **A line intersects a plane, but does not lie in the plane.**



B **Two planes intersect in one line.**



CHECK IT OUT!

4. Sketch a figure that shows two lines intersect in one point in a plane, but only one of the lines lies in the plane.

THINK AND DISCUSS

1. Explain why any two points are collinear.

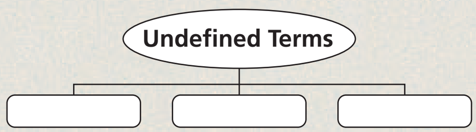
2. Which postulate explains the fact that two straight roads cannot cross each other more than once?

3. Explain why points and lines may be coplanar even when the plane containing them is not drawn.

4. Name all the possible lines, segments, and rays for the points A and B. Then give the maximum number of planes that can be determined by these points.

Know it! Note

5. **GET ORGANIZED** Copy and complete the graphic organizer below. In each box, name, describe, and illustrate one of the undefined terms.



1-1 Exercises

California Standards
1.0, 6SDAP1.1

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KEYWORD: MG7 1-1
Parent Resources Online
KEYWORD: MG7 Parent

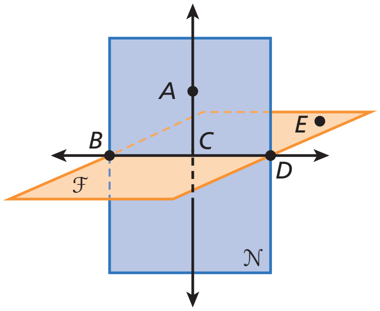
GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. Give an example from your classroom of three *collinear* points.
2. Make use of the fact that *endpoint* is a compound of *end* and *point* and name the *endpoint* of $\overrightarrow{ST}$.

SEE EXAMPLE 1
p. 6

Use the figure to name each of the following.



3. five points
4. two lines
5. two planes
6. point on $\overleftrightarrow{BD}$

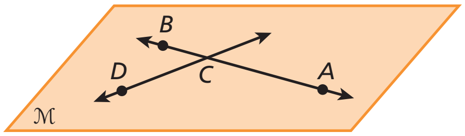
SEE EXAMPLE 2
p. 7

Draw and label each of the following.

7. a segment with endpoints M and N
8. a ray with endpoint F that passes through G

SEE EXAMPLE 3
p. 7

Use the figure to name each of the following.



9. a line that contains A and C
10. a plane that contains A, D, and C

SEE EXAMPLE 4
p. 8

Sketch a figure that shows each of the following.

11. three coplanar lines that intersect in a common point
12. two lines that do not intersect

PRACTICE AND PROBLEM SOLVING

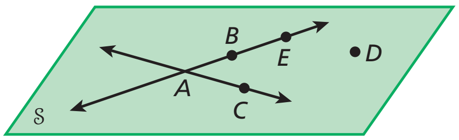
Independent Practice

For Exercises	See Example
13–15	1
16–17	2
18–19	3
20–21	4

Extra Practice

Skills Practice p. S4
Application Practice p. S28

Use the figure to name each of the following.

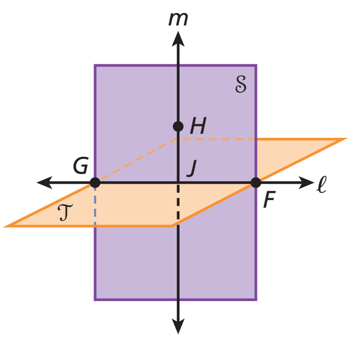


13. three collinear points
14. four coplanar points
15. a plane containing E

Draw and label each of the following.

16. a line containing X and Y
17. a pair of opposite rays that both contain R

Use the figure to name each of the following.



18. two points and a line that lie in plane $\mathcal{T}$
19. two planes that contain ℓ

Sketch a figure that shows each of the following.

20. a line that intersects two nonintersecting planes
21. three coplanar lines that intersect in three different points

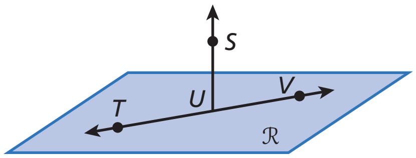
CONCEPT CONNECTION

22. This problem will prepare you for the Concept Connection on page 34. Name an object at the archaeological site shown that is represented by each of the following.
    a. a point
    b. a segment
    c. a plane

Draw each of the following.

23. plane $\mathcal{H}$ containing two lines that intersect at M
24. $\overleftrightarrow{ST}$ intersecting plane $\mathcal{M}$ at R

Use the figure to name each of the following.



25. the intersection of $\overleftrightarrow{TV}$ and $\overrightarrow{US}$
26. the intersection of $\overrightarrow{US}$ and plane $\mathcal{R}$
27. the intersection of $\overline{TU}$ and $\overline{UV}$

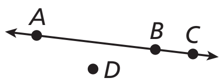
Write the postulate that justifies each statement.

28. The line connecting two dots on a sheet of paper lies on the same sheet of paper as the dots.
29. If two ants are walking in straight lines but in different directions, their paths cannot cross more than once.
30. **Critical Thinking** Is it possible to draw three points that are noncoplanar? Explain.

Tell whether each statement is sometimes, always, or never true. Support your answer with a sketch.

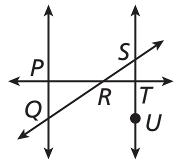
31. If two planes intersect, they intersect in a straight line.
32. If two lines intersect, they intersect at two different points.
33. $\overleftrightarrow{AB}$ is another name for $\overleftrightarrow{BA}$.
34. If two rays share a common endpoint, then they form a line.
35. **Art** Pointillism is a technique in which tiny dots of complementary colors are combined to form a picture. Which postulate ensures that a line connecting two of these points also lies in the plane containing the points?

36. **Probability** Three of the labeled points are chosen at random. What is the probability that they are collinear?



37. Campers often use a cooking stove with three legs. Which postulate explains why they might prefer this design to a stove that has four legs?
38. **Write About It** Explain why three coplanar lines may have zero, one, two, or three points of intersection. Support your answer with a sketch.

STANDARDIZED TEST PREP



39. Which of the following is a set of noncollinear points?

Ⓐ *P, R, T* Ⓒ *P, Q, R*

Ⓑ *Q, R, S* Ⓓ *S, T, U*

40. What is the greatest number of intersection points four coplanar lines can have?

Ⓕ 6 Ⓗ 2

Ⓖ 4 Ⓙ 0

41. Two flat walls meet in the corner of a classroom. Which postulate best describes this situation?

Ⓐ Through any three noncollinear points there is exactly one plane.

Ⓑ If two points lie in a plane, then the line containing them lies in the plane.

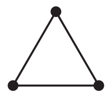
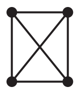
Ⓒ If two lines intersect, then they intersect in exactly one point.

Ⓓ If two planes intersect, then they intersect in exactly one line.

42. **Gridded Response** What is the greatest number of planes determined by four noncollinear points?

CHALLENGE AND EXTEND

Use the table for Exercises 43–45.

Figure			
Number of Points	2	3	4
Maximum Number of Segments	1	3	

43. What is the maximum number of segments determined by 4 points?

44. **Multi-Step** Extend the table. What is the maximum number of segments determined by 10 points?

45. Write a formula for the maximum number of segments determined by n points.

46. **Critical Thinking** Explain how rescue teams could use two of the postulates from this lesson to locate a distress signal.

SPIRAL REVIEW

47. The combined age of a mother and her twin daughters is 58 years. The mother was 25 years old when the twins were born. Write and solve an equation to find the age of each of the three people. *(Previous course)*

Determine whether each set of ordered pairs is a function. *(Previous course)*

48. $\{(0, 1), (1, -1), (5, -1), (-1, 2)\}$ **49.** $\{(3, 8), (10, 6), (9, 8), (10, -6)\}$

Find the mean, median, and mode for each set of data. *(Previous course)*

50. 0, 6, 1, 3, 5, 2, 7, 10 **51.** 0.47, 0.44, 0.4, 0.46, 0.44

1-2 Technology LAB

Use with Lesson 1-2

Explore Properties Associated with Points

The two endpoints of a segment determine its length. Other points on the segment are *between* the endpoints. Only one of these points is the *midpoint* of the segment. In this lab, you will use geometry software to measure lengths of segments and explore properties of points on segments.

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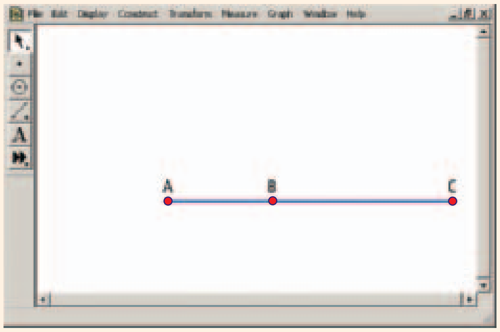
Lab Resources Online
KEYWORD: MG7 Lab1

Activity

1. Construct a segment and label its endpoints A and C.

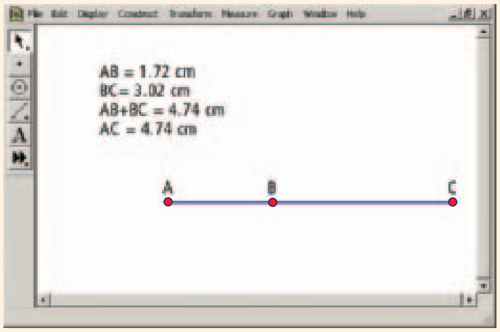
2. Create point B on $\overline{AC}$.



3. Measure the distances from A to B and from B to C. Use the Calculate tool to calculate the sum of AB and BC.

4. Measure the length of $\overline{AC}$. What do you notice about this length compared with the measurements found in Step 3?

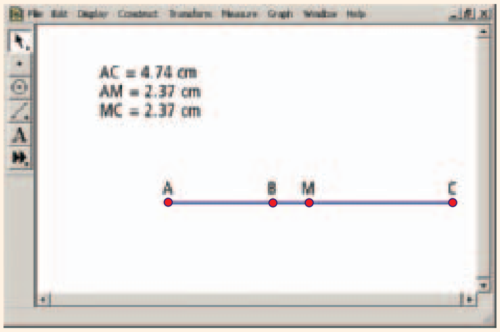
5. Drag point B along $\overline{AC}$. Drag one of the endpoints of $\overline{AC}$. What relationships do you think are true about the three measurements?



6. Construct the midpoint of $\overline{AC}$ and label it M.

7. Measure $\overline{AM}$ and $\overline{MC}$. What relationships do you think are true about the lengths of $\overline{AC}$, $\overline{AM}$, and $\overline{MC}$? Use the Calculate tool to confirm your findings.

8. How many midpoints of $\overline{AC}$ exist?



Try This

1. Repeat the activity with a new segment. Drag each of the points in your figure (the endpoints, the point on the segment, and the midpoint). Write down any relationships you observe about the measurements.
2. Create a point D not on $\overline{AC}$. Measure $\overline{AD}$, $\overline{DC}$, and $\overline{AC}$. Does $AD + DC = AC$? What do you think has to be true about D for the relationship to always be true?

1-2 Measuring and Constructing Segments

Objectives

Use length and midpoint of a segment.

Construct midpoints and congruent segments.

Vocabulary

coordinate
distance
length
congruent segments
construction
between
midpoint
bisect
segment bisector

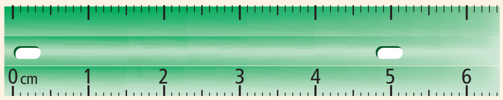
Why learn this?

You can measure a segment to calculate the distance between two locations. Maps of a race are used to show the distance between stations on the course. (See Example 4.)

A ruler can be used to measure the distance between two points. A point corresponds to one and only one number on the ruler. This number is called a **coordinate**. The following postulate summarizes this concept.

Postulate 1-2-1 Ruler Postulate

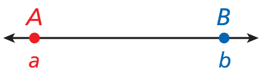
The points on a line can be put into a one-to-one correspondence with the real numbers.



California Standards

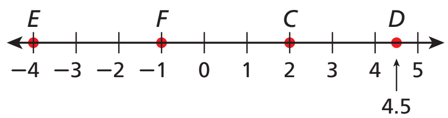
16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line.

The **distance** between any two points is the absolute value of the difference of the coordinates. If the coordinates of points A and B are a and b, then the distance between A and B is $|a - b|$ or $|b - a|$. The distance between A and B is also called the **length** of $\overline{AB}$, or AB.



$AB = |a - b| = |b - a|$

EXAMPLE 1 Finding the Length of a Segment



Find each length.

A DC

$$DC = |4.5 - 2|$$
$$= |2.5|$$
$$= 2.5$$

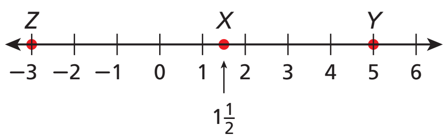
B EF

$$EF = |-4 - (-1)|$$
$$= |-4 + 1|$$
$$= |-3|$$
$$= 3$$

CHECK IT OUT! **Find each length.**

1a. XY

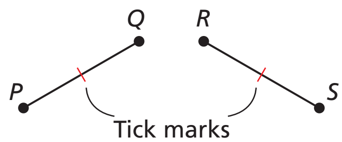
1b. XZ



Caution!

PQ represents a number, while $\overline{PQ}$ represents a geometric figure. Be sure to use equality for numbers ($PQ = RS$) and congruence for figures ($\overline{PQ} \cong \overline{RS}$).

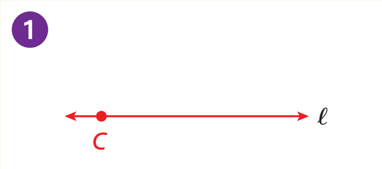
Congruent segments are segments that have the same length. In the diagram, $PQ = RS$, so you can write $\overline{PQ} \cong \overline{RS}$. This is read as "segment PQ is congruent to segment RS." *Tick marks* are used in a figure to show congruent segments.



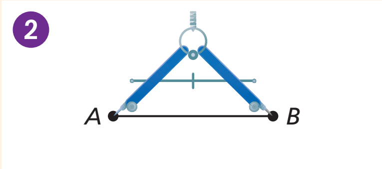
You can make a sketch or measure and draw a segment. These may not be exact. A **construction** is a way of creating a figure that is more precise. One way to make a geometric construction is to use a compass and straightedge.

Construction Congruent Segment

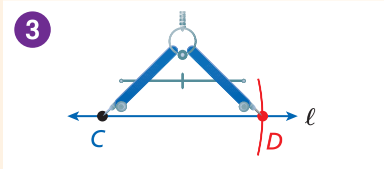
Construct a segment congruent to $\overline{AB}$.



Draw ℓ. Choose a point on ℓ and label it C.



Open the compass to distance AB.



Place the point of the compass at C and make an arc through ℓ. Find the point where the arc and ℓ intersect and label it D.

$\overline{CD} \cong \overline{AB}$

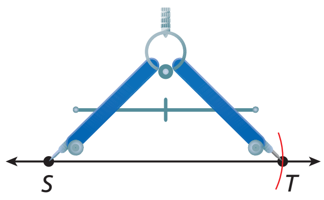
EXAMPLE 2 Copying a Segment

Sketch, draw, and construct a segment congruent to $\overline{MN}$.

Step 1 Estimate and sketch.
Estimate the length of $\overline{MN}$ and sketch $\overline{PQ}$ approximately the same length.

Step 2 Measure and draw.
Use a ruler to measure $\overline{MN}$. MN appears to be 3.1 cm. Use a ruler and draw $\overline{XY}$ to have length 3.1 cm.

Step 3 Construct and compare.
Use a compass and straightedge to construct $\overline{ST}$ congruent to $\overline{MN}$.



A ruler shows that $\overline{PQ}$ and $\overline{XY}$ are approximately the same length as $\overline{MN}$, but $\overline{ST}$ is precisely the same length.

CHECK IT OUT! **2.** Sketch, draw, and construct a segment congruent to $\overline{JK}$.

In order for you to say that a point B is **between** two points A and C, all three of the points must lie on the same line, and $AB + BC = AC$.

Know it! Note

Postulate 1-2-2 Segment Addition Postulate

If B is between A and C, then $AB + BC = AC$.

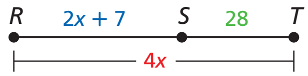
EXAMPLE 3 Using the Segment Addition Postulate

Algebra

A **B is between A and C, $AC = 14$, and $BC = 11.4$. Find AB.**

$AC = AB + BC$	*Seg. Add. Post.*
$14 = AB + 11.4$	*Substitute 14 for AC and 11.4 for BC.*
$\underline{-11.4} \quad \underline{-11.4}$	*Subtract 11.4 from both sides.*
$2.6 = AB$	*Simplify.*

B **S is between R and T. Find RT.**

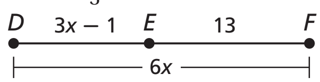


$RT = RS + ST$	*Seg. Add. Post.*
$4x = (2x + 7) + 28$	*Substitute the given values.*
$4x = 2x + 35$	*Simplify.*
$\underline{-2x} \quad \underline{-2x}$	*Subtract 2x from both sides.*
$2x = 35$	*Simplify.*
$\frac{2x}{2} = \frac{35}{2}$	*Divide both sides by 2.*
$x = \frac{35}{2}$, or 17.5	*Simplify.*
$RT = 4x$	
$= 4(17.5) = 70$	*Substitute 17.5 for x.*

CHECK IT OUT!

3a. Y is between X and Z, $XZ = 3$, and $XY = 1\frac{1}{3}$. Find YZ.

3b. E is between D and F. Find DF.

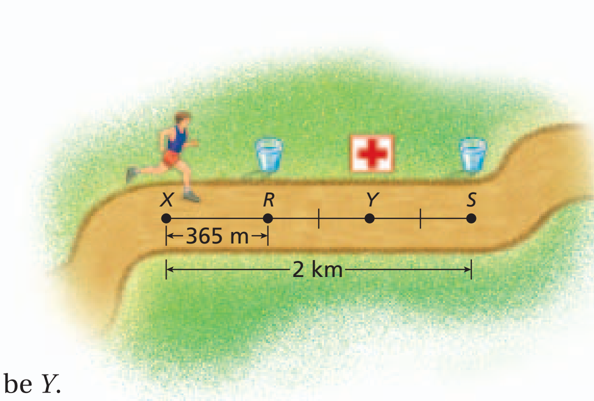


The **midpoint** M of $\overline{AB}$ is the point that **bisects**, or divides, the segment into two congruent segments. If M is the midpoint of $\overline{AB}$, then $AM = MB$. So if $AB = 6$, then $AM = 3$ and $MB = 3$.

EXAMPLE 4 *Recreation Application*

Algebra

The map shows the route for a race. You are 365 m from drink station R and 2 km from drink station S. The first-aid station is located at the midpoint of the two drink stations. How far are you from the first-aid station?



Let your current location be X and the location of the first-aid station be Y.

$XR + RS = XS$	*Seg. Add. Post.*
$365 + RS = 2000$	*Substitute 365 for XR and 2000 for XS.*
$\underline{-365} \quad \underline{-365}$	*Subtract 365 from both sides.*
$RS = 1635$	*Simplify.*
$RY = 817.5$	*Y is the mdpt. of $\overline{RS}$, so $RY = \frac{1}{2}RS$.*

$XY = XR + RY$	
$= 365 + 817.5 = 1182.5$ m	*Substitute 365 for XR and 817.5 for RY.*

You are 1182.5 m from the first-aid station.

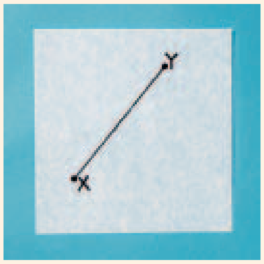
CHECK IT OUT!

4. What is the distance to a drink station located at the midpoint between your current location and the first-aid station?

A **segment bisector** is any ray, segment, or line that intersects a segment at its midpoint. It divides the segment into two equal parts at its midpoint.

Construction Segment Bisector

1

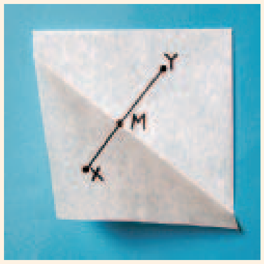


Draw $\overline{XY}$ on a sheet of paper.

2

Fold the paper so that Y is on top of X.

3

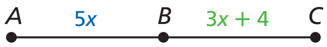


Unfold the paper. The line represented by the crease bisects $\overline{XY}$. Label the midpoint M.

$XM = MY$

EXAMPLE 5 Using Midpoints to Find Lengths

Algebra



B is the midpoint of $\overline{AC}$, $AB = 5x$, and $BC = 3x + 4$. Find AB, BC, and AC.

Step 1 Solve for x.

$AB = BC$ — *B is the mdpt. of $\overline{AC}$.*

$5x = 3x + 4$ — *Substitute $5x$ for AB and $3x + 4$ for BC.*

$\underline{-3x} \quad \underline{-3x}$ — *Subtract $3x$ from both sides.*

$2x = 4$ — *Simplify.*

$\frac{2x}{2} = \frac{4}{2}$ — *Divide both sides by 2.*

$x = 2$ — *Simplify.*

Step 2 Find AB, BC, and AC.

$AB = 5x = 5(2) = 10$

$BC = 3x + 4 = 3(2) + 4 = 10$

$AC = AB + BC = 10 + 10 = 20$

CHECK IT OUT!

5. S is the midpoint of $\overline{RT}$, $RS = -2x$, and $ST = -3x - 2$. Find RS, ST, and RT.

THINK AND DISCUSS

1. Suppose R is the midpoint of $\overline{ST}$. Explain how SR and ST are related.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. Make a sketch and write an equation to describe each relationship.

	***B* is between *A* and *C*.**	***B* is the midpoint of $\overline{AC}$.**
Sketch		
Equation		

1-2 Exercises

California Standards
16.0, 7NS1.2, 7NS2.5, 7AF1.2, 7AF4.1, 7MR1.2

go.hrw.com
Homework Help Online
KEYWORD: MG7 1-2
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

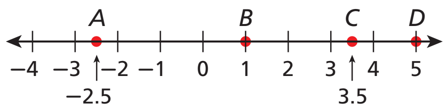
Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. Line ℓ bisects $\overline{XY}$ at M and divides $\overline{XY}$ into two equal parts. Name a pair of congruent segments.

2. __?__ is the amount of space between two points on a line. It is always expressed as a nonnegative number. (*distance* or *midpoint*)

SEE EXAMPLE 1 p. 13

Find each length.



3. AB **4.** BC

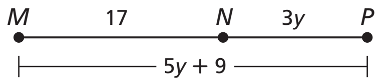
SEE EXAMPLE 2 p. 14

5. Sketch, draw, and construct a segment congruent to $\overline{RS}$.

SEE EXAMPLE 3 p. 15

6. B is between A and C, $AC = 15.8$, and $AB = 9.9$. Find BC.

7. Find MP.



SEE EXAMPLE 4 p. 15

8. **Travel** If a picnic area is located at the midpoint between Sacramento and Oakland, find the distance to the picnic area from the road sign.

SEE EXAMPLE 5 p. 16

9. **Multi-Step** K is the midpoint of $\overline{JL}$, $JL = 4x - 2$, and $JK = 7$. Find x, KL, and JL.

10. E bisects $\overline{DF}$, $DE = 2y$, and $EF = 8y - 3$. Find DE, EF, and DF.

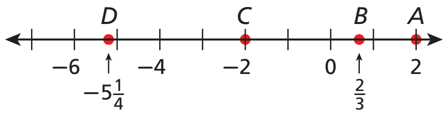
PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
11–12	1
13	2
14–15	3
16	4
17–18	5

Extra Practice
Skills Practice p. S4
Application Practice p. S28

Find each length.

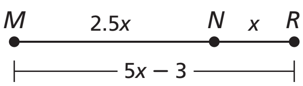


11. DB **12.** CD

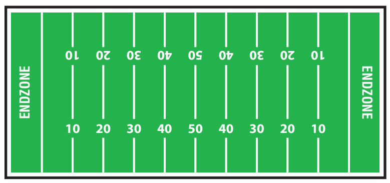
13. Sketch, draw, and construct a segment twice the length of $\overline{AB}$.

14. D is between C and E, $CE = 17.1$, and $DE = 8$. Find CD.

15. Find MN.



16. **Sports** During a football game, a quarterback standing at the 9-yard line passes the ball to a receiver at the 24-yard line. The receiver then runs with the ball halfway to the 50-yard line. How many total yards (passing plus running) did the team gain on the play?

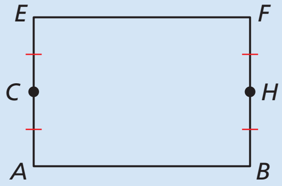


17. **Multi-Step** E is the midpoint of $\overline{DF}$, $DE = 2x + 4$, and $EF = 3x - 1$. Find DE, EF, and DF.

18. Q bisects $\overline{PR}$, $PQ = 3y$, and $PR = 42$. Find y and QR.

CONCEPT CONNECTION

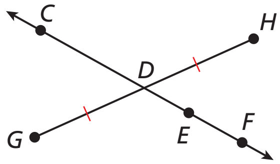
19. This problem will prepare you for the Concept Connection on page 34. Archaeologists at Valley Forge were eager to find what remained of the winter camp that soldiers led by George Washington called home for several months. The diagram represents one of the restored log cabins.



a. How is C related to $\overline{AE}$?

b. If $AC = 7$ ft, $EF = 2(AC) + 2$, and $AB = 2(EF) - 16$, what are AB and EF?

Use the diagram for Exercises 20–23.



20. $GD = 4\frac{2}{3}$. Find GH.

21. $\overline{CD} \cong \overline{DF}$, E bisects $\overline{DF}$, and $CD = 14.2$. Find EF.

22. $GH = 4x - 1$, and $DH = 8$. Find x.

23. $\overline{GH}$ bisects $\overline{CF}$, $CF = 2y - 2$, and $CD = 3y - 11$. Find CD.

Tell whether each statement is sometimes, always, or never true. Support each of your answers with a sketch.

24. Two segments that have the same length must be congruent.

25. If M is between A and B, then M bisects $\overline{AB}$.

26. If Y is between X and Z, then X, Y, and Z are collinear.

27. **ERROR ANALYSIS** Below are two statements about the midpoint of $\overline{AB}$. Which is incorrect? Explain the error.

A
M is the mdpt. of $\overline{AB}$.
Therefore $AM \cong MB$.

B
M is the mdpt. of $\overline{AB}$.
Therefore $AM = MB$.

28. **Carpentry** A carpenter has a wooden dowel that is 72 cm long. She wants to cut it into two pieces so that one piece is 5 times as long as the other. What are the lengths of the two pieces?

29. The coordinate of M is 2.5, and $MN = 4$. What are the possible coordinates for N?

30. Draw three collinear points where E is between D and F. Then write an equation using these points and the Segment Addition Postulate.

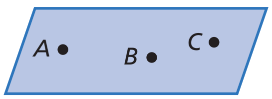
Suppose S is between R and T. Use the Segment Addition Postulate to solve for each variable.

31. $RS = 7y - 4$
$ST = y + 5$
$RT = 28$

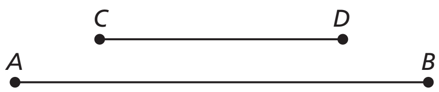
32. $RS = 3x + 1$
$ST = \frac{1}{2}x + 3$
$RT = 18$

33. $RS = 2z + 6$
$ST = 4z - 3$
$RT = 5z + 12$

34. **Write About It** In the diagram, B is not between A and C. Explain.



35. **Construction** Use a compass and straightedge to construct a segment whose length is $AB + CD$.

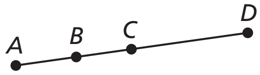


STANDARDIZED TEST PREP

36. Q is between P and R. S is between Q and R, and R is between Q and T. $PT = 34$, $QR = 8$, and $PQ = SQ = SR$. What is the length of $\overline{RT}$?

(A) 9 (B) 10 (C) 18 (D) 22

37. C is the midpoint of $\overline{AD}$. B is the midpoint of $\overline{AC}$. $BC = 12$. What is the length of $\overline{AD}$?

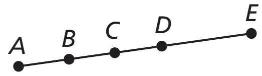


(F) 12 (G) 24 (H) 36 (J) 48

38. Which expression correctly states that $\overline{XY}$ is congruent to $\overline{VW}$?

(A) $XY \cong VW$ (B) $\overline{XY} \cong \overline{VW}$ (C) $\overline{XY} = \overline{VW}$ (D) $XY = VW$

39. A, B, C, D, and E are collinear points. $AE = 34$, $BD = 16$, and $AB = BC = CD$. What is the length of $\overline{CE}$?



(F) 10 (G) 16 (H) 18 (J) 24

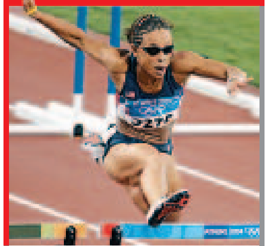
CHALLENGE AND EXTEND

40. HJ is twice JK. J is between H and K. If $HJ = 4x$ and $HK = 78$, find JK.

41. A, D, N, and X are collinear points. D is between N and A. $NA + AX = NX$. Draw a diagram that represents this information.

LINK

Sports



Joanna Hayes, of the United States, clears a hurdle on her way to winning the gold medal in the women's 100 m hurdles during the 2004 Olympic Games.

Sports **Use the following information for Exercises 42 and 43.**

The table shows regulation distances between hurdles in women's and men's races. In both the women's and men's events, the race consists of a straight track with 10 equally spaced hurdles.

Event	Distance of Race	Distance from Start to First Hurdle	Distance Between Hurdles	Distance from Last Hurdle to Finish
Women's	100 m	13.00 m	8.50 m	
Men's	110 m	13.72 m	9.14 m	

42. Find the distance from the last hurdle to the finish line for the women's race.

43. Find the distance from the last hurdle to the finish line for the men's race.

44. **Critical Thinking** Given that J, K, and L are collinear and that K is between J and L, is it possible that $JK = JL$? If so, draw an example. If not, explain.

SPIRAL REVIEW

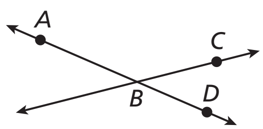
Evaluate each expression. *(Previous course)*

45. $|20 - 8|$ **46.** $|-9 + 23|$ **47.** $-|4 - 27|$

Simplify each expression. *(Previous course)*

48. $8a - 3(4 + a) - 10$ **49.** $x + 2(5 - 2x) - (4 + 5x)$

Use the figure to name each of the following. *(Lesson 1-1)*



50. two lines that contain B

51. two segments containing D

52. three collinear points

53. a ray with endpoint C

1-3 Measuring and Constructing Angles

Objectives

Name and classify angles.

Measure and construct angles and angle bisectors.

Vocabulary

angle
vertex
interior of an angle
exterior of an angle
measure
degree
acute angle
right angle
obtuse angle
straight angle
congruent angles
angle bisector

California Standards

16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line.

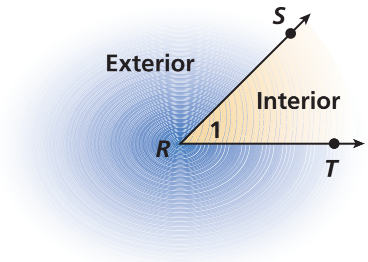
Who uses this?

Surveyors use angles to help them measure and map the earth's surface. (See Exercise 27.)

A transit is a tool for measuring angles. It consists of a telescope that swivels horizontally and vertically. Using a transit, a surveyor can measure the *angle* formed by his or her location and two distant points.

An **angle** is a figure formed by two rays, or sides, with a common endpoint called the **vertex** (plural: *vertices*). You can name an angle several ways: by its vertex, by a point on each ray and the vertex, or by a number.

The set of all points between the sides of the angle is the **interior of an angle**. The **exterior of an angle** is the set of all points outside the angle.



Angle Name

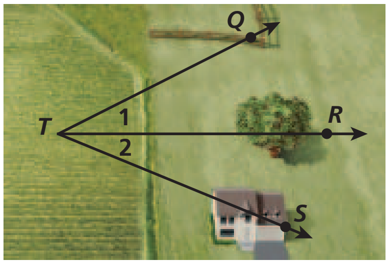
$\angle R$, $\angle SRT$, $\angle TRS$, or $\angle 1$

You cannot name an angle just by its vertex if the point is the vertex of more than one angle. In this case, you must use all three points to name the angle, and the middle point is always the vertex.

EXAMPLE 1 Naming Angles

A surveyor recorded the angles formed by a transit (point *T*) and three distant points, *Q*, *R*, and *S*. Name three of the angles.

$\angle QTR$, $\angle QTS$, and $\angle RTS$



CHECK IT OUT!

1. Write the different ways you can name the angles in the diagram.

The **measure** of an angle is usually given in degrees. Since there are 360° in a circle, one **degree** is $\frac{1}{360}$ of a circle. When you use a protractor to measure angles, you are applying the following postulate.

Know it! Note

Postulate 1-3-1 Protractor Postulate

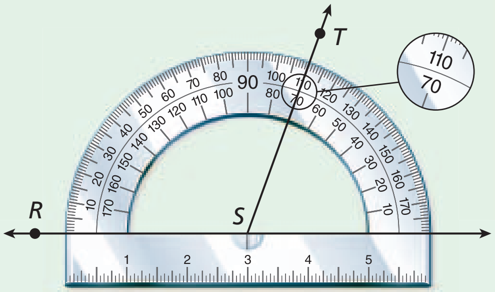
Given $\overleftrightarrow{AB}$ and a point O on $\overleftrightarrow{AB}$, all rays that can be drawn from O can be put into a one-to-one correspondence with the real numbers from 0 to 180.

Student to Student

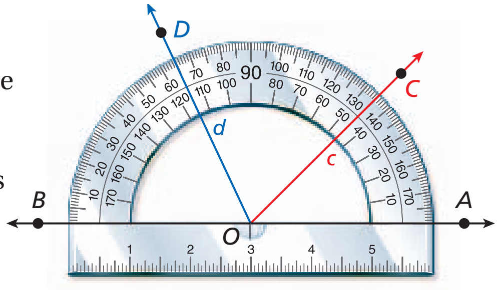
Using a Protractor

José Muñoz
Lincoln High School

Most protractors have two sets of numbers around the edge. When I measure an angle and need to know which number to use, I first ask myself whether the angle is acute, right, or obtuse. For example, ∠RST looks like it is obtuse, so I know its measure must be 110°, not 70°.

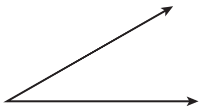
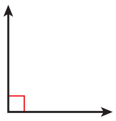
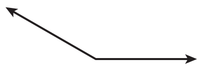


You can use the Protractor Postulate to help you classify angles by their measure. The measure of an angle is the absolute value of the difference of the real numbers that the rays correspond with on a protractor. If $\overrightarrow{OC}$ corresponds with c and $\overrightarrow{OD}$ corresponds with d, $m\angle DOC = |d - c|$ or $|c - d|$.



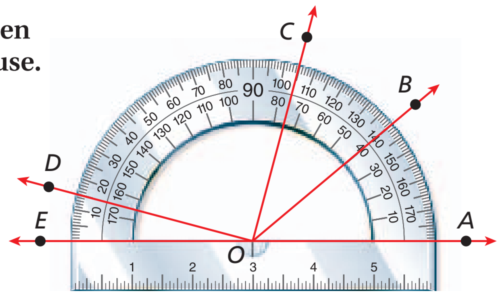
Know it! Note

Types of Angles

Acute Angle	Right Angle	Obtuse Angle	Straight Angle
			
Measures greater than 0° and less than 90°	Measures 90°	Measures greater than 90° and less than 180°	Formed by two opposite rays and meaures 180°

EXAMPLE 2 Measuring and Classifying Angles

Find the measure of each angle. Then classify each as acute, right, or obtuse.



A $\angle AOD$

$m\angle AOD = 165°$

$\angle AOD$ is obtuse.

B $\angle COD$

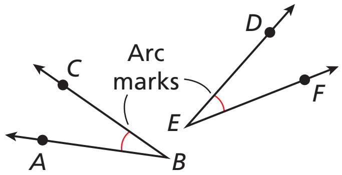
$m\angle COD = |165 - 75| = 90°$

$\angle COD$ is a right angle.

CHECK IT OUT! **Use the diagram to find the measure of each angle. Then classify each as acute, right, or obtuse.**

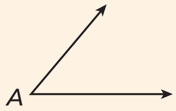
2a. $\angle BOA$ **2b.** $\angle DOB$ **2c.** $\angle EOC$

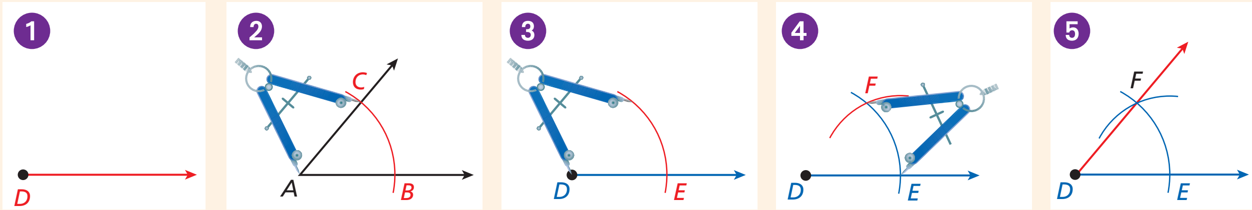
Congruent angles are angles that have the same measure. In the diagram, m∠*ABC* = m∠*DEF*, so you can write ∠*ABC* ≅ ∠*DEF*. This is read as "angle *ABC* is congruent to angle *DEF*." ***Arc marks*** are used to show that the two angles are congruent.



Construction Congruent Angle

Construct an angle congruent to ∠*A*.





1. Use a straightedge to draw a ray with endpoint ***D***.
2. Place the compass point at *A* and draw an arc that intersects both sides of ∠*A*. Label the intersection points ***B*** and ***C***.
3. Using the same compass setting, place the compass point at ***D*** and draw an arc that intersects the ray. Label the intersection ***E***.
4. Place the compass point at *B* and open it to the distance *BC*. Place the point of the compass at ***E*** and draw an arc. Label its intersection with the first arc ***F***.
5. Use a straightedge to draw $\overrightarrow{DF}$.

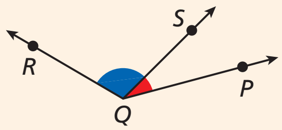
   ∠*D* ≅ ∠*A*

The Angle Addition Postulate is very similar to the Segment Addition Postulate that you learned in the previous lesson.

Know it! Note

Postulate 1-3-2 Angle Addition Postulate

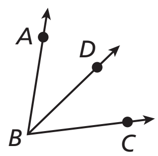
If *S* is in the interior of ∠*PQR*, then m∠*PQS* + m∠*SQR* = m∠*PQR*.
(∠ Add. Post.)



EXAMPLE 3 Using the Angle Addition Postulate

Algebra

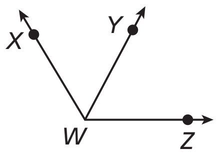
m∠*ABD* = 37° and m∠*ABC* = 84°. Find m∠*DBC*.



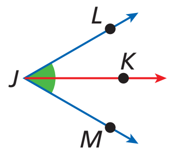
m∠*ABC* = m∠*ABD* + m∠*DBC*	*∠ Add. Post.*
84° = 37° + m∠*DBC*	*Substitute the given values.*
−37 −37	*Subtract 37 from both sides.*
47° = m∠*DBC*	*Simplify.*

CHECK IT OUT!

3. m∠*XWZ* = 121° and m∠*XWY* = 59°. Find m∠*YWZ*.

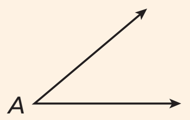


An **angle bisector** is a ray that divides an angle into two congruent angles. $\overrightarrow{JK}$ bisects $\angle LJM$; thus $\angle LJK \cong \angle KJM$.

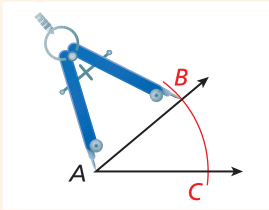


Construction Angle Bisector

Construct the bisector of ∠A.

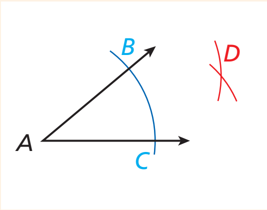


1



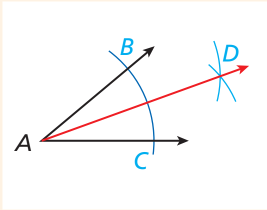
Place the point of the compass at *A* and draw an arc. Label its points of intersection with ∠*A* as ***B*** and ***C***.

2



Without changing the compass setting, draw intersecting arcs from *B* and *C*. Label the intersection of the arcs as ***D***.

3

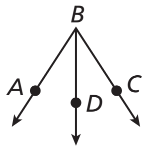


Use a straightedge to draw $\overrightarrow{AD}$. $\overrightarrow{AD}$ bisects ∠*A*.

EXAMPLE 4 Finding the Measure of an Angle

Algebra

$\overrightarrow{BD}$ bisects $\angle ABC$, $m\angle ABD = (6x + 3)°$, and $m\angle DBC = (8x - 7)°$. Find $m\angle ABD$.



Step 1 Find x.

$m\angle ABD = m\angle DBC$		*Def. of ∠ bisector*
$(6x + 3)° = (8x - 7)°$		*Substitute the given values.*
$\underline{\quad +7} \quad \underline{\quad +7}$		*Add 7 to both sides.*
$6x + 10 = 8x$		*Simplify.*
$\underline{-6x} \quad \underline{-6x}$		*Subtract 6x from both sides.*
$10 = 2x$		*Simplify.*
$\frac{10}{2} = \frac{2x}{2}$		*Divide both sides by 2.*
$5 = x$		*Simplify.*

Step 2 Find $m\angle ABD$.

$m\angle ABD = 6x + 3$	
$= 6(5) + 3$	*Substitute 5 for x.*
$= 33°$	*Simplify.*

CHECK IT OUT!

Find the measure of each angle.

4a. $\overrightarrow{QS}$ bisects $\angle PQR$, $m\angle PQS = (5y - 1)°$, and $m\angle PQR = (8y + 12)°$. Find $m\angle PQS$.

4b. $\overrightarrow{JK}$ bisects $\angle LJM$, $m\angle LJK = (-10x + 3)°$, and $m\angle KJM = (-x + 21)°$. Find $m\angle LJM$.

THINK AND DISCUSS

1. Explain why any two right angles are congruent.
2. $\overrightarrow{BD}$ bisects $\angle ABC$. How are $m\angle ABC$, $m\angle ABD$, and $m\angle DBC$ related?
3. **GET ORGANIZED** Copy and complete the graphic organizer. In the cells sketch, measure, and name an example of each angle type.

	Diagram	Measure	Name
Acute Angle			
Right Angle			
Obtuse Angle			
Straight Angle			

1-3 Exercises

California Standards
16.0, 7NS1.2, 7AF4.1, 7MG1.1, 7MG2.4, 7MR2.1, 1A4.0

go.hrw.com
Homework Help Online
KEYWORD: MG7 1-3
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

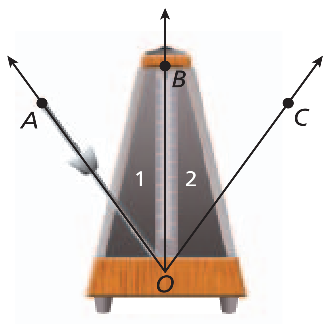
Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. $\angle A$ is an acute angle. $\angle O$ is an obtuse angle. $\angle R$ is a right angle. Put $\angle A$, $\angle O$, and $\angle R$ in order from least to greatest by measure.

2. Which point is the vertex of $\angle BCD$? Which rays form the sides of $\angle BCD$?

SEE EXAMPLE 1
p. 20

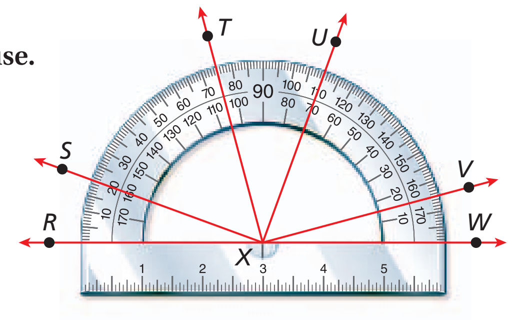
3. **Music** Musicians use a metronome to keep time as they play. The metronome's needle swings back and forth in a fixed amount of time. Name all of the angles in the diagram.



SEE EXAMPLE 2
p. 21

Use the protractor to find the measure of each angle. Then classify each as acute, right, or obtuse.

4. $\angle VXW$
5. $\angle TXW$
6. $\angle RXU$



SEE EXAMPLE 3
p. 22

L is in the interior of $\angle JKM$. Find each of the following.

7. $m\angle JKM$ if $m\angle JKL = 42°$ and $m\angle LKM = 28°$
8. $m\angle LKM$ if $m\angle JKL = 56.4°$ and $m\angle JKM = 82.5°$

SEE EXAMPLE 4
p. 23

Multi-Step **$\overrightarrow{BD}$ bisects $\angle ABC$. Find each of the following.**

9. $m\angle ABD$ if $m\angle ABD = (6x + 4)°$ and $m\angle DBC = (8x - 4)°$
10. $m\angle ABC$ if $m\angle ABD = (5y - 3)°$ and $m\angle DBC = (3y + 15)°$

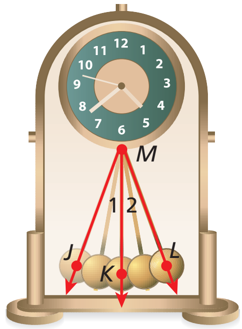
Independent Practice	
For Exercises	See Example
11	1
12–14	2
15–16	3
17–18	4

Extra Practice

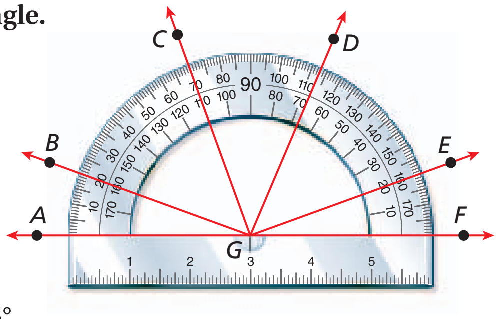
Skills Practice p. S4
Application Practice p. S28

PRACTICE AND PROBLEM SOLVING

11. Physics Pendulum clocks have been used since 1656 to keep time. The pendulum swings back and forth once or twice per second. Name all of the angles in the diagram.



Use the protractor to find the measure of each angle. Then classify each as acute, right, or obtuse.



12. $\angle CGE$ **13.** $\angle BGD$ **14.** $\angle AGB$

***T* is in the interior of $\angle RSU$. Find each of the following.**

15. $m\angle RSU$ if $m\angle RST = 38°$ and $m\angle TSU = 28.6°$

16. $m\angle RST$ if $m\angle TSU = 46.7°$ and $m\angle RSU = 83.5°$

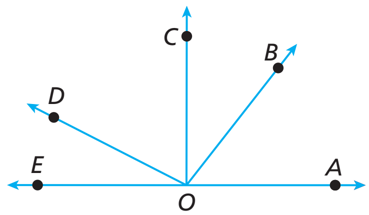
Multi-Step $\overrightarrow{SP}$ **bisects $\angle RST$. Find each of the following.**

17. $m\angle RST$ if $m\angle RSP = (3x - 2)°$ and $m\angle PST = (9x - 26)°$

18. $m\angle RSP$ if $m\angle RST = \frac{5}{2}y°$ and $m\angle PST = (y + 5)°$

Estimation Use the following information for Exercises 19–22.

Assume the corner of a sheet of paper is a right angle. Use the corner to estimate the measure and classify each angle in the diagram.



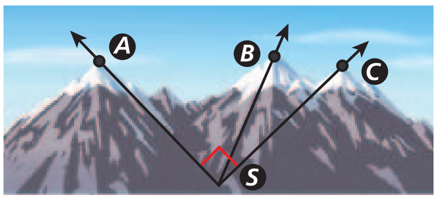
19. $\angle BOA$ **20.** $\angle COA$

21. $\angle EOD$ **22.** $\angle EOB$

Use a protractor to draw an angle with each of the following measures.

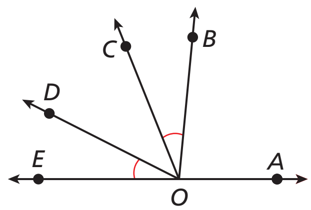
23. 33° **24.** 142° **25.** 90° **26.** 168°

27. Surveying A surveyor at point *S* discovers that the angle between peaks *A* and *B* is 3 times as large as the angle between peaks *B* and *C*. The surveyor knows that $\angle ASC$ is a right angle. Find $m\angle ASB$ and $m\angle BSC$.



28. Math History As far back as the 5th century B.C., mathematicians have been fascinated by the problem of trisecting an angle. It is possible to construct an angle with $\frac{1}{4}$ the measure of a given angle. Explain how to do this.

Find the value of *x*.



29. $m\angle AOC = 7x - 2$, $m\angle DOC = 2x + 8$, $m\angle EOD = 27$

30. $m\angle AOB = 4x - 2$, $m\angle BOC = 5x + 10$, $m\angle COD = 3x - 8$

31. $m\angle AOB = 6x + 5$, $m\angle BOC = 4x - 2$, $m\angle AOC = 8x + 21$

32. Multi-Step *Q* is in the interior of right $\angle PRS$. If $m\angle PRQ$ is 4 times as large as $m\angle QRS$, what is $m\angle PRQ$?

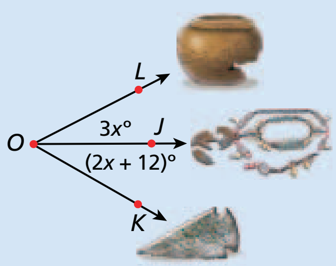
CONCEPT CONNECTION

33. This problem will prepare you for the Concept Connection on page 34. An archaeologist standing at O looks for clues on where to dig for artifacts.

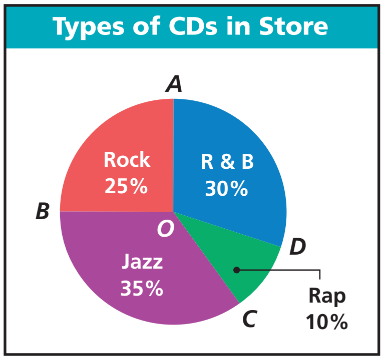
a. What value of x will make the angle between the pottery and the arrowhead measure 57°?

b. What value of x makes $\angle LOJ \cong \angle JOK$?

c. What values of x make $\angle LOK$ an acute angle?



Data Analysis **Use the circle graph for Exercises 34–36.**



34. Find m$\angle AOB$, m$\angle BOC$, m$\angle COD$, and m$\angle DOA$. Classify each angle as acute, right, or obtuse.

35. What if...? Next year, the music store will use some of the shelves currently holding jazz music to double the space for rap. What will m$\angle COD$ and m$\angle BOC$ be next year?

36. Suppose a fifth type of music, salsa, is added. If the space is divided equally among the five types, what will be the angle measure for each type of music in the circle graph?

37. Critical Thinking Can an obtuse angle be congruent to an acute angle? Why or why not?

38. The measure of an obtuse angle is $(5x + 45)^\circ$. What is the largest value for x?

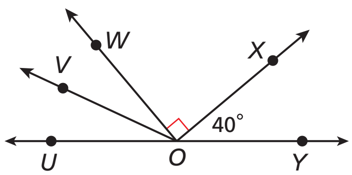
39. Write About It $\overrightarrow{FH}$ bisects $\angle EFG$. Use the Angle Addition Postulate to explain why m$\angle EFH = \frac{1}{2}$m$\angle EFG$.

40. Multi-Step Use a protractor to draw a 70° angle. Then use a compass and straightedge to bisect the angle. What do you think will be the measure of each angle formed? Use a protractor to support your answer.

STANDARDIZED TEST PREP

41. m$\angle UOW = 50°$, and $\overrightarrow{OV}$ bisects $\angle UOW$. What is m$\angle VOY$?

Ⓐ 25° Ⓑ 65° Ⓒ 130° Ⓓ 155°



42. What is m$\angle UOX$?

Ⓕ 50° Ⓖ 115° Ⓗ 140° Ⓙ 165°

43. $\overrightarrow{BD}$ bisects $\angle ABC$, m$\angle ABC = (4x + 5)^\circ$, and m$\angle ABD = (3x - 1)^\circ$. What is the value of x?

Ⓐ 2.2 Ⓑ 3 Ⓒ 3.5 Ⓓ 7

44. If an angle is bisected and then 30° is added to the measure of the bisected angle, the result is the measure of a right angle. What is the measure of the original angle?

Ⓕ 30° Ⓖ 60° Ⓗ 75° Ⓙ 120°

45. Short Response If an obtuse angle is bisected, are the resulting angles acute or obtuse? Explain.

CHALLENGE AND EXTEND

46. Find the measure of the angle formed by the hands of a clock when it is 7:00.

47. $\overrightarrow{QS}$ bisects $\angle PQR$, $m\angle PQR = (x^2)°$, and $m\angle PQS = (2x + 6)°$. Find all the possible measures for $\angle PQR$.

48. For more precise measurements, a degree can be divided into 60 minutes, and each minute can be divided into 60 seconds. An angle measure of 42 degrees, 30 minutes, and 10 seconds is written as 42°30′10″. Subtract this angle measure from the measure 81°24′15″.

49. If 1 degree equals 60 minutes and 1 minute equals 60 seconds, how many seconds are in 2.25 degrees?

50. $\angle ABC \cong \angle DBC$. $m\angle ABC = \left(\frac{3x}{2} + 4\right)°$ and $m\angle DBC = \left(2x - 27\frac{1}{4}\right)°$. Is $\angle ABD$ a straight angle? Explain.

SPIRAL REVIEW

51. What number is 64% of 35?

52. What percent of 280 is 33.6? *(Previous course)*

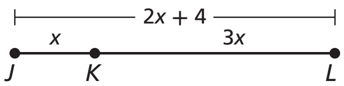
Sketch a figure that shows each of the following. *(Lesson 1-1)*

53. a line that contains $\overline{AB}$ and $\overrightarrow{CB}$

54. two different lines that intersect $\overline{MN}$

55. a plane and a ray that intersect only at Q

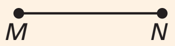
Find the length of each segment. *(Lesson 1-2)*

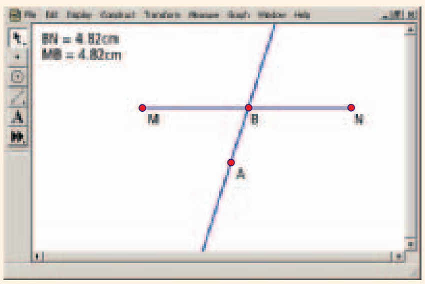


56. $\overline{JK}$ **57.** $\overline{KL}$ **58.** $\overline{JL}$

Using Technology — Segment and Angle Bisectors

1. Construct the bisector of $\overline{MN}$.





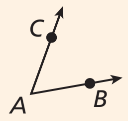
a. Draw $\overline{MN}$ and construct the midpoint B.

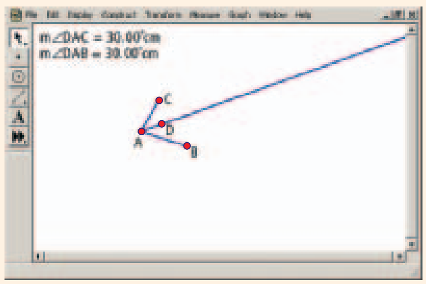
b. Construct a point A not on the segment.

c. Construct bisector $\overleftrightarrow{AB}$ and measure $\overline{MB}$ and $\overline{NB}$.

d. Drag M and N and observe MB and NB.

2. Construct the bisector of $\angle BAC$.



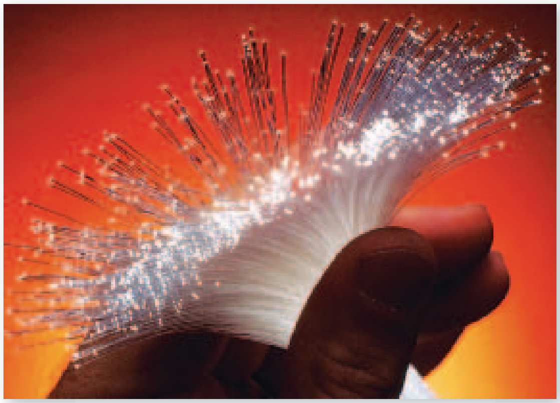


a. Draw $\angle BAC$.

b. Construct the angle bisector $\overrightarrow{AD}$ and measure $\angle DAC$ and $\angle DAB$.

c. Drag the angle and observe $m\angle DAB$ and $m\angle DAC$.

1-4 Pairs of Angles



Objectives

Identify adjacent, vertical, complementary, and supplementary angles.

Find measures of pairs of angles.

Vocabulary

adjacent angles
linear pair
complementary angles
supplementary angles
vertical angles

Who uses this?

Scientists use properties of angle pairs to design fiber-optic cables. (See Example 4.)

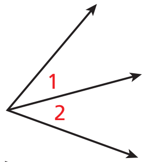
A fiber-optic cable is a strand of glass as thin as a human hair. Data can be transmitted over long distances by bouncing light off the inner walls of the cable.

Many pairs of angles have special relationships. Some relationships are because of the measurements of the angles in the pair. Other relationships are because of the positions of the angles in the pair.

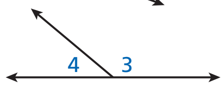
Know it! Note

Pairs of Angles

Adjacent angles are two angles in the same plane with a common vertex and a common side, but no common interior points. $\angle 1$ and $\angle 2$ are adjacent angles.



A **linear pair** of angles is a pair of adjacent angles whose noncommon sides are opposite rays. $\angle 3$ and $\angle 4$ form a linear pair.



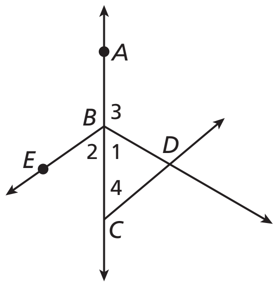
EXAMPLE 1

Identifying Angle Pairs

California Standards

Preparation for 13.0 Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior **angles.**

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.



A $\angle 1$ and $\angle 2$

$\angle 1$ and $\angle 2$ have a common vertex, B, a common side, $\overrightarrow{BC}$, and no common interior points. Therefore $\angle 1$ and $\angle 2$ are only adjacent angles.

B $\angle 2$ and $\angle 4$

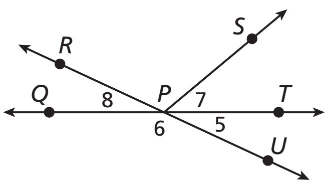
$\angle 2$ and $\angle 4$ share $\overline{BC}$ but do not have a common vertex, so $\angle 2$ and $\angle 4$ are not adjacent angles.

C $\angle 1$ and $\angle 3$

$\angle 1$ and $\angle 3$ are adjacent angles. Their noncommon sides, $\overrightarrow{BC}$ and $\overrightarrow{BA}$, are opposite rays, so $\angle 1$ and $\angle 3$ also form a linear pair.

CHECK IT OUT!

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.



1a. $\angle 5$ and $\angle 6$

1b. $\angle 7$ and $\angle SPU$

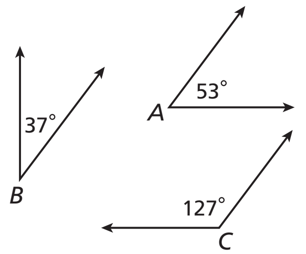
1c. $\angle 7$ and $\angle 8$

Complementary and Supplementary Angles

Know it! Note

Complementary angles are two angles whose measures have a sum of 90°. $\angle A$ and $\angle B$ are complementary.

Supplementary angles are two angles whose measures have a sum of 180°. $\angle A$ and $\angle C$ are supplementary.



You can find the complement of an angle that measures $x°$ by subtracting its measure from 90°, or $(90 - x)°$. You can find the supplement of an angle that measures $x°$ by subtracting its measure from 180°, or $(180 - x)°$.

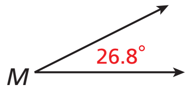
EXAMPLE 2 Finding the Measures of Complements and Supplements

Find the measure of each of the following.

A complement of $\angle M$

$(90 - x)°$

$90° - 26.8° = 63.2°$

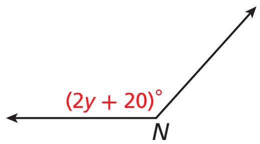


B supplement of $\angle N$

$(180 - x)°$

$180° - (2y + 20)° = 180° - 2y - 20$

$= (160 - 2y)°$

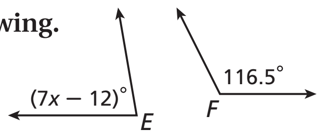


CHECK IT OUT!

Find the measure of each of the following.

2a. complement of $\angle E$

2b. supplement of $\angle F$



EXAMPLE 3 Using Complements and Supplements to Solve Problems

Algebra

An angle measures 3 degrees less than twice the measure of its complement. Find the measure of its complement.

Step 1 Let $m\angle A = x°$. Then $\angle B$, its complement, measures $(90 - x)°$.

Step 2 Write and solve an equation.

$m\angle A = 2m\angle B - 3$

$x = 2(90 - x) - 3$ *Substitute x for m∠A and 90 − x for m∠B.*

$x = 180 - 2x - 3$ *Distrib. Prop.*

$x = 177 - 2x$ *Combine like terms.*

$\underline{+2x} \quad \underline{\quad +2x}$ *Add 2x to both sides.*

$3x = 177$ *Simplify.*

$\frac{3x}{3} = \frac{177}{3}$ *Divide both sides by 3.*

$x = 59$ *Simplify.*

The measure of the complement, $\angle B$, is $(90 - 59)° = 31°$.

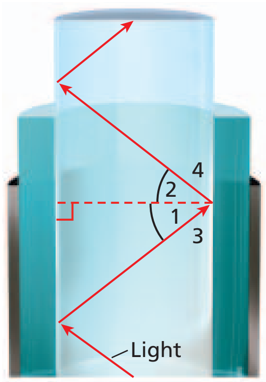
CHECK IT OUT!

3. An angle's measure is 12° more than $\frac{1}{2}$ the measure of its supplement. Find the measure of the angle.

EXAMPLE 4 Problem-Solving Application

PROBLEM SOLVING

Light passing through a fiber optic cable reflects off the walls in such a way that $\angle 1 \cong \angle 2$. $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary.
If $m\angle 1 = 38°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.



1 Understand the Problem

The **answers** are the measures of $\angle 2$, $\angle 3$, and $\angle 4$.
List the important information:

- $\angle 1 \cong \angle 2$
- $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary.
- $m\angle 1 = 38°$

2 Make a Plan

If $\angle 1 \cong \angle 2$, then $m\angle 1 = m\angle 2$.
If $\angle 3$ and $\angle 1$ are complementary, then $m\angle 3 = (90 - 38)°$.
If $\angle 4$ and $\angle 2$ are complementary, then $m\angle 4 = (90 - 38)°$.

3 Solve

By the Transitive Property of Equality, if $m\angle 1 = 38°$ and $m\angle 1 = m\angle 2$, then $m\angle 2 = 38°$. Since $\angle 3$ and $\angle 1$ are complementary, $m\angle 3 = 52°$. Similarly, since $\angle 2$ and $\angle 4$ are complementary, $m\angle 4 = 52°$.

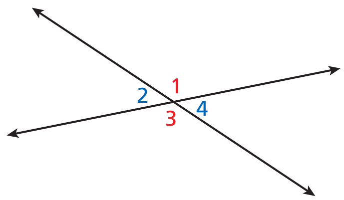
4 Look Back

The answer makes sense because $38° + 52° = 90°$, so $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary. Thus $m\angle 2 = 38°$, $m\angle 3 = 52°$, and $m\angle 4 = 52°$.

CHECK IT OUT!

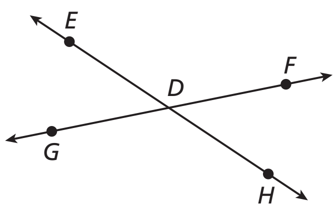
4. **What if...?** Suppose $m\angle 3 = 27.6°$. Find $m\angle 1$, $m\angle 2$, and $m\angle 4$.

Another angle pair relationship exists between two angles whose sides form two pairs of opposite rays. **Vertical angles** are two nonadjacent angles formed by two intersecting lines. $\angle 1$ and $\angle 3$ are vertical angles, as are $\angle 2$ and $\angle 4$.



EXAMPLE 5 Identifying Vertical Angles

Name one pair of vertical angles.
Do they appear to have the same measure?
Check by measuring with a protractor.



$\angle EDF$ and $\angle GDH$ are vertical angles and appear to have the same measure.

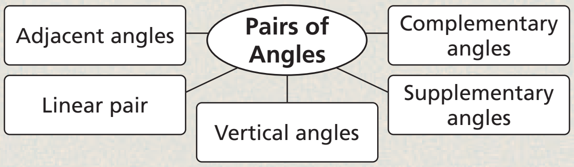
Check $m\angle EDF \approx m\angle GDH \approx 135°$.

CHECK IT OUT!

5. Name another pair of vertical angles. Do they appear to have the same measure? Check by measuring with a protractor.

THINK AND DISCUSS

1. Explain why any two right angles are supplementary.
2. Is it possible for a pair of vertical angles to also be adjacent? Explain.
3. **GET ORGANIZED** Copy and complete the graphic organizer below. In each box, draw a diagram and write a definition of the given angle pair.



1-4 Exercises

California Standards
Preparation for 13.0; 7NS1.2, 7AF1.1, 7AF4.1, 1A4.0

go.hrw.com
Homework Help Online
KEYWORD: MG7 1-4
Parent Resources Online
KEYWORD: MG7 Parent

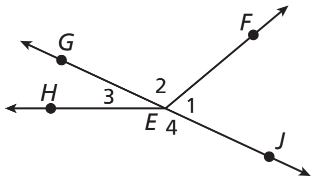
GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. An angle measures $x°$. What is the measure of its *complement*? What is the measure of its *supplement*?
2. $\angle ABC$ and $\angle CBD$ are *adjacent angles*. Which side do the angles have in common?

SEE EXAMPLE 1 p. 28

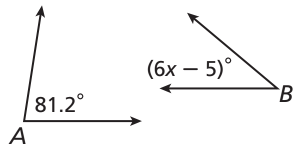
Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.



3. $\angle 1$ and $\angle 2$
4. $\angle 1$ and $\angle 3$
5. $\angle 2$ and $\angle 4$
6. $\angle 2$ and $\angle 3$

SEE EXAMPLE 2 p. 29

Find the measure of each of the following.



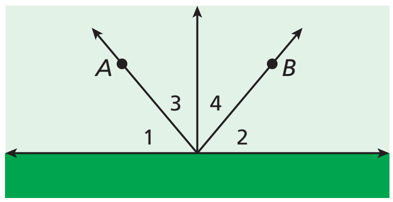
7. supplement of $\angle A$
8. complement of $\angle A$
9. supplement of $\angle B$
10. complement of $\angle B$

SEE EXAMPLE 3 p. 29

11. **Multi-Step** An angle's measure is 6 degrees more than 3 times the measure of its complement. Find the measure of the angle.

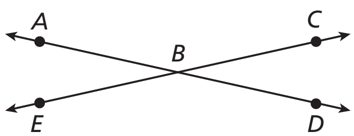
SEE EXAMPLE 4 p. 30

12. **Landscaping** A sprinkler swings back and forth between A and B in such a way that $\angle 1 \cong \angle 2$. $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary. If $m\angle 1 = 47.5°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.



SEE EXAMPLE 5 p. 30

13. Name each pair of vertical angles.



Independent Practice	
For Exercises	See Example
14–17	1
18–21	2
22	3
23	4
24	5

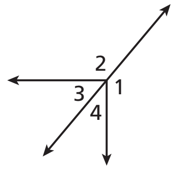
Extra Practice

Skills Practice p. S4

Application Practice p. S28

PRACTICE AND PROBLEM SOLVING

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.



14. $\angle 1$ and $\angle 4$ **15.** $\angle 2$ and $\angle 3$

16. $\angle 3$ and $\angle 4$ **17.** $\angle 3$ and $\angle 1$

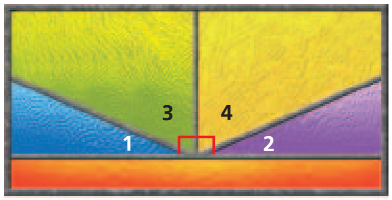
Given $m\angle A = 56.4°$ and $m\angle B = (2x - 4)°$, find the measure of each of the following.

18. supplement of $\angle A$ **19.** complement of $\angle A$

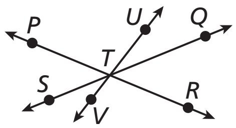
20. supplement of $\angle B$ **21.** complement of $\angle B$

22. Multi-Step An angle's measure is 3 times the measure of its complement. Find the measure of the angle and the measure of its complement.

23. Art In the stained glass pattern, $\angle 1 \cong \angle 2$. $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary. If $m\angle 1 = 22.3°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.



24. Name the pairs of vertical angles.



25. Probability The angle measures 30°, 60°, 120°, and 150° are written on slips of paper. You choose two slips of paper at random. What is the probability that the angle measures are supplementary?

Multi-Step **$\angle ABD$ and $\angle BDE$ are supplementary. Find the measures of both angles.**

26. $m\angle ABD = 5x°$, $m\angle BDE = (17x - 18)°$

27. $m\angle ABD = (3x + 12)°$, $m\angle BDE = (7x - 32)°$

28. $m\angle ABD = (12x - 12)°$, $m\angle BDE = (3x + 48)°$

Multi-Step **$\angle ABD$ and $\angle BDC$ are complementary. Find the measures of both angles.**

29. $m\angle ABD = (5y + 1)°$, $m\angle BDC = (3y - 7)°$

30. $m\angle ABD = (4y + 5)°$, $m\angle BDC = (4y + 8)°$

31. $m\angle ABD = (y - 30)°$, $m\angle BDC = 2y°$

32. Critical Thinking Explain why an angle that is supplementary to an acute angle must be an obtuse angle.

CONCEPT CONNECTION

33. This problem will prepare you for the Concept Connection on page 34. H is in the interior of $\angle JAK$. $m\angle JAH = (3x - 8)°$, and $m\angle KAH = (x + 2)°$. Draw a picture of each relationship. Then find the measure of each angle.

a. $\angle JAH$ and $\angle KAH$ are complementary angles.

b. $\angle JAH$ and $\angle KAH$ form a linear pair.

c. $\angle JAH$ and $\angle KAH$ are congruent angles.

Determine whether each statement is true or false. If false, explain why.

34. If an angle is acute, then its complement must be greater than its supplement.

35. A pair of vertical angles may also form a linear pair.

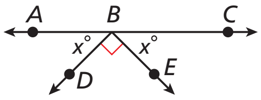
36. If two angles are supplementary and congruent, the measure of each angle is 90°.

37. If a ray divides an angle into two complementary angles, then the original angle is a right angle.

38. **Write About It** Describe a situation in which two angles are both congruent and complementary. Explain.

STANDARDIZED TEST PREP

39. What is the value of x in the diagram?



Ⓐ 15 Ⓑ 30 Ⓒ 45 Ⓓ 90

40. The ratio of the measures of two complementary angles is 1:2. What is the measure of the larger angle? (*Hint:* Let x and $2x$ represent the angle measures.)

Ⓕ 30° Ⓖ 45° Ⓗ 60° Ⓙ 120°

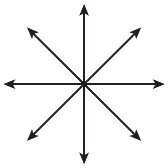
41. $m\angle A = 3y$, and $m\angle B = 2m\angle A$. Which value of y makes $\angle A$ supplementary to $\angle B$?

Ⓐ 10 Ⓑ 18 Ⓒ 20 Ⓓ 36

42. The measures of two supplementary angles are in the ratio 7:5. Which value is the measure of the smaller angle? (*Hint:* Let $7x$ and $5x$ represent the angle measures.)

Ⓕ 37.5 Ⓖ 52.5 Ⓗ 75 Ⓙ 105

CHALLENGE AND EXTEND



43. How many pairs of vertical angles are in the diagram?

44. The supplement of an angle is 4 more than twice its complement. Find the measure of the angle.

45. An angle's measure is twice the measure of its complement. The larger angle is how many degrees greater than the smaller angle?

46. The supplement of an angle is 36° less than twice the supplement of the complement of the angle. Find the measure of the supplement.

SPIRAL REVIEW

Solve each equation. Check your answer. *(Previous course)*

47. $4x + 10 = 42$

48. $5m - 9 = m + 4$

49. $2(y + 3) = 12$

50. $-(d + 4) = 18$

Y is between X and Z, $XY = 3x + 1$, $YZ = 2x - 2$, and $XZ = 84$. Find each of the following. *(Lesson 1-2)*

51. x

52. XY

53. YZ

$\overrightarrow{XY}$ bisects $\angle WYZ$. Given $m\angle WYX = 26°$, find each of the following. *(Lesson 1-3)*

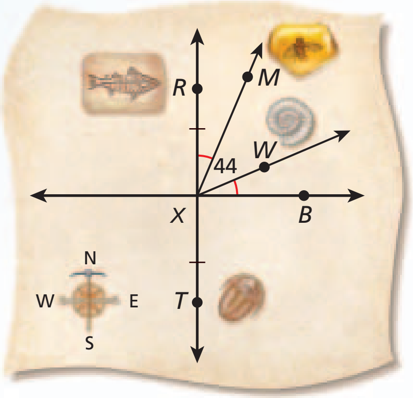
54. $m\angle XYZ$

55. $m\angle WYZ$

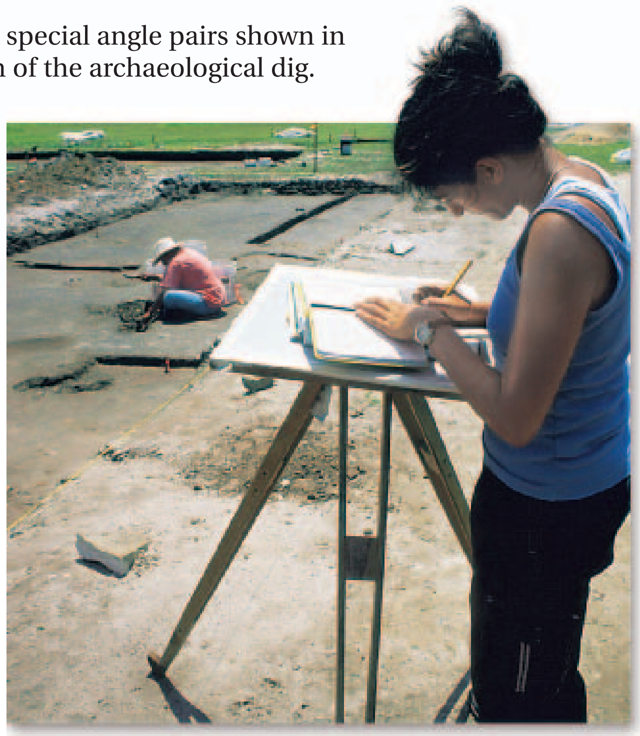
CONCEPT CONNECTION

Euclidean and Construction Tools

Can You Dig It? A group of college and high school students participated in an archaeological dig. The team discovered four fossils. To organize their search, Sierra used a protractor and ruler to make a diagram of where different members of the group found fossils. She drew the locations based on the location of the campsite. The campsite is located at X on $\overleftrightarrow{XB}$. The four fossils were found at R, T, W, and M.



1. Are the locations of the campsite at X and the fossils at R and T collinear or noncollinear?
2. How is X related to $\overline{RT}$? If $RX = 10x - 6$ and $XT = 3x + 8$, what is the distance between the locations of the fossils at R and T?
3. $\angle RXB$ and $\angle BXT$ are right angles. Find the measure of each angle formed by the locations of the fossils and the campsite. Then classify each angle by its measure.
4. Identify the special angle pairs shown in the diagram of the archaeological dig.



READY TO GO ON?

CHAPTER 1

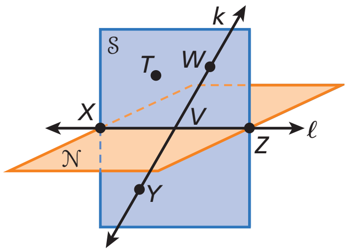
SECTION 1A

Quiz for Lessons 1-1 Through 1-4

1-1 Understanding Points, Lines, and Planes

Draw and label each of the following.

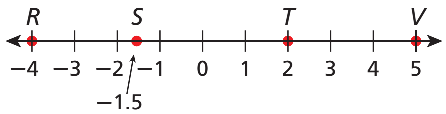
1. a segment with endpoints X and Y
2. a ray with endpoint M that passes through P
3. three coplanar lines intersecting at a point
4. two points and a line that lie in a plane



Use the figure to name each of the following.

5. three coplanar points
6. two lines
7. a plane containing T, V, and X
8. a line containing V and Z

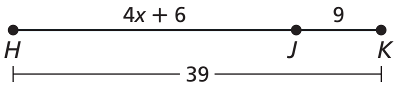
1-2 Measuring and Constructing Segments



Find the length of each segment.

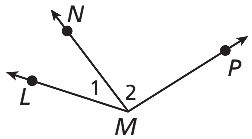
9. $\overline{SV}$
10. $\overline{TR}$
11. $\overline{ST}$
12. Sketch, draw, and construct a segment congruent to $\overline{CD}$.

13. The diagram represents a straight highway with three towns, Henri, Joaquin, and Kenard. Find the distance from Henri H to Joaquin J.



14. Q is the midpoint of $\overline{PR}$, $PQ = 2z$, and $PR = 8z - 12$. Find z, PQ, and PR.

1-3 Measuring and Constructing Angles

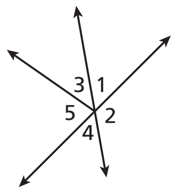


15. Name all the angles in the diagram.

Classify each angle by its measure.

16. $m\angle PVQ = 21°$
17. $m\angle RVT = 96°$
18. $m\angle PVS = 143°$
19. $\overrightarrow{RS}$ bisects $\angle QRT$, $m\angle QRS = (3x + 8)°$, and $m\angle SRT = (9x - 4)°$. Find $m\angle SRT$.
20. Use a protractor and straightedge to draw a 130° angle. Then bisect the angle.

1-4 Pairs of Angles



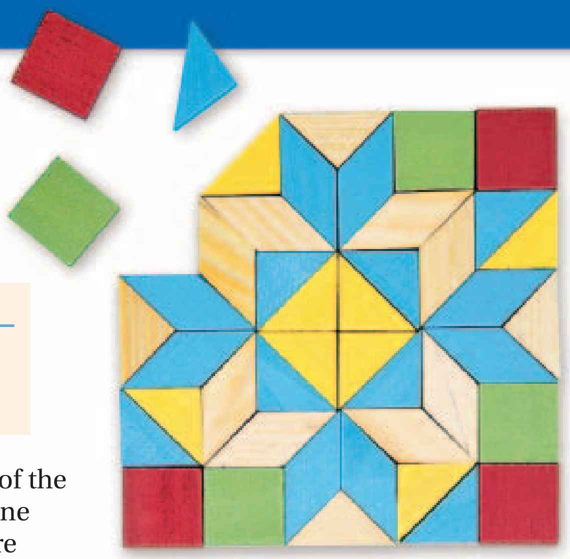
Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

21. $\angle 1$ and $\angle 2$
22. $\angle 4$ and $\angle 5$
23. $\angle 3$ and $\angle 4$

If $m\angle T = (5x - 10)°$, find the measure of each of the following.

24. supplement of $\angle T$
25. complement of $\angle T$

1-5 Using Formulas in Geometry



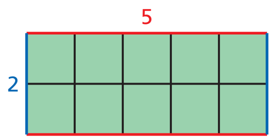
Objective
Apply formulas for perimeter, area, and circumference.

Vocabulary
perimeter
area
base
height
diameter
radius
circumference
pi

Why learn this?
Puzzles use geometric-shaped pieces. Formulas help determine the amount of materials needed. (See Exercise 6.)

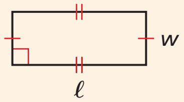
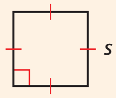
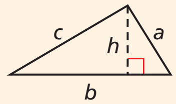
The **perimeter** P of a plane figure is the sum of the side lengths of the figure. The **area** A of a plane figure is the number of nonoverlapping square units of a given size that exactly cover the figure.

area = 2 units × 5 units
= 10 square units



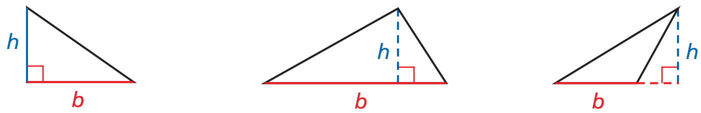
Know it! Note

Perimeter and Area

RECTANGLE	SQUARE	TRIANGLE
		
$P = 2\ell + 2w$ or $2(\ell + w)$	$P = 4s$	$P = a + b + c$
$A = \ell w$	$A = s^2$	$A = \frac{1}{2}bh$ or $\frac{bh}{2}$

California Standards
8.0 Students know, derive, **and solve problems involving the perimeter, circumference, area,** volume, lateral area, and surface area **of common geometric figures.**

The **base** b can be any side of a triangle. The **height** h is a segment from a vertex that forms a right angle with a line containing the base. The height may be a side of the triangle or in the interior or the exterior of the triangle.



EXAMPLE 1 Finding Perimeter and Area

Find the perimeter and area of each figure.

A rectangle in which $\ell = 17$ cm and $w = 5$ cm

$P = 2\ell + 2w$
$= 2(17) + 2(5)$
$= 34 + 10 = 44$ cm

$A = \ell w$
$= (17)(5) = 85 \text{ cm}^2$

B triangle in which $a = 8$, $b = (x + 1)$, $c = 4x$, and $h = 6$

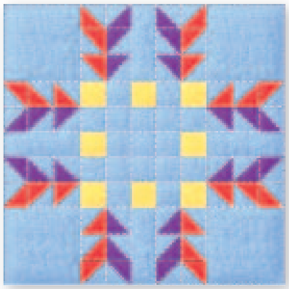
$P = a + b + c$
$= 8 + (x + 1) + 4x$
$= 5x + 9$

$A = \frac{1}{2}bh$
$= \frac{1}{2}(x + 1)(6) = 3x + 3$

Remember!
Perimeter is expressed in linear units, such as inches (in.) or meters (m). Area is expressed in square units, such as square centimeters (cm^2).

CHECK IT OUT! 1. Find the perimeter and area of a square with $s = 3.5$ in.

EXAMPLE 2 Crafts Application



The Texas Treasures quilt block includes 24 purple triangles. The base and height of each triangle are about 3 in. Find the approximate amount of fabric used to make the 24 triangles.

The area of one triangle is

$$A = \frac{1}{2}bh = \frac{1}{2}(3)(3) = 4\frac{1}{2}\text{ in}^2.$$

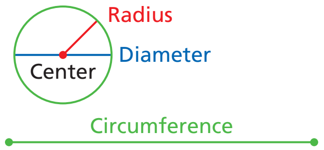
The total area of the 24 triangles is

$$24\left(4\frac{1}{2}\right) = 108\text{ in}^2.$$

CHECK IT OUT!

2. Find the amount of fabric used to make the four rectangles. Each rectangle has a length of $6\frac{1}{2}$ in. and a width of $2\frac{1}{2}$ in.

In a circle a **diameter** is a segment that passes through the center of the circle and whose endpoints are on the circle. A **radius** of a circle is a segment whose endpoints are the center of the circle and a point on the circle. The **circumference** of a circle is the distance around the circle.



Know it! Note

Circumference and Area of a Circle

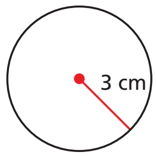
The circumference C of a circle is given by the formula $C = \pi d$ or $C = 2\pi r$.

The area A of a circle is given by the formula $A = \pi r^2$.

The ratio of a circle's circumference to its diameter is the same for all circles. This ratio is represented by the Greek letter **π (pi)**. The value of π is irrational. Pi is often approximated as 3.14 or $\frac{22}{7}$.

EXAMPLE 3 Finding the Circumference and Area of a Circle

Find the circumference and area of the circle.



$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(3) = 6\pi \\ &\approx 18.8\text{ cm} \end{aligned}$$

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(3)^2 = 9\pi \\ &\approx 28.3\text{ cm}^2 \end{aligned}$$

CHECK IT OUT!

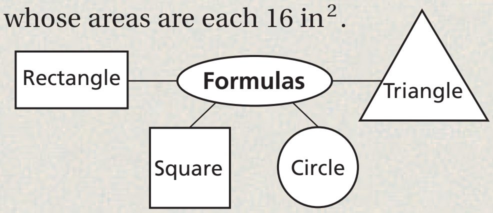
3. Find the circumference and area of a circle with radius 14 m.

THINK AND DISCUSS

1. Describe three different figures whose areas are each 16 in^2.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each shape, write the formula for its area and perimeter.



1-5 Exercises

California Standards
8.0, 10.0, 7NS1.2, 7AF1.1, 7AF4.1, 7MG2.1, 7MG2.2, 1A4.0

go.hrw.com
Homework Help Online
KEYWORD: MG7 1-5
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

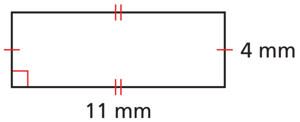
1. Explain how the concepts of *perimeter* and *circumference* are related.

2. For a rectangle, length and width are sometimes used in place of __?__. (*base and height* or *radius and diameter*)

SEE EXAMPLE 1
p. 36

Find the perimeter and area of each figure.

3.



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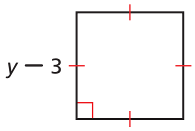

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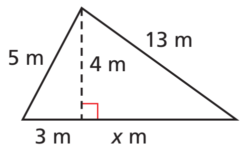
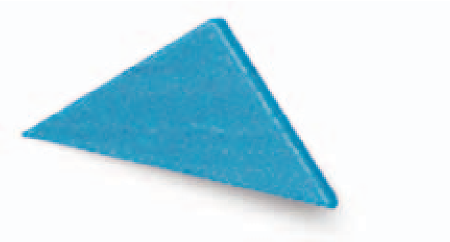


SEE EXAMPLE 2
p. 37

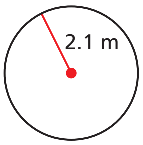
6. **Manufacturing** A puzzle contains a triangular piece with a base of 3 in. and a height of 4 in. A manufacturer wants to make 80 puzzles. Find the amount of wood used if each puzzle contains 20 triangular pieces.



SEE EXAMPLE 3
p. 37

Find the circumference and area of each circle. Use the π key on your calculator. Round to the nearest tenth.

7.



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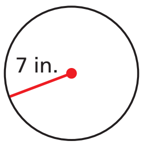

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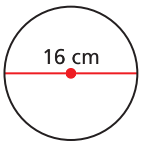


PRACTICE AND PROBLEM SOLVING

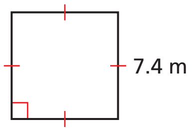
Independent Practice

For Exercises	See Example
10–12	1
13	2
14–16	3

Extra Practice
Skills Practice p. S5
Application Practice p. S28

Find the perimeter and area of each figure.

10.



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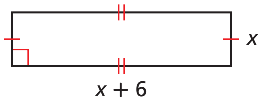
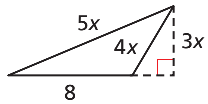
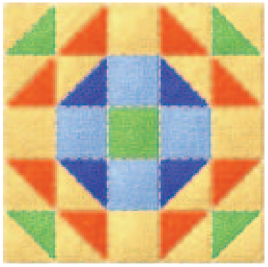


12.



13. **Crafts** The quilt pattern includes 32 small triangles. Each has a base of 3 in. and a height of 1.5 in. Find the amount of fabric used to make the 32 triangles.



Find the circumference and area of each circle with the given radius or diameter. Use the π key on your calculator. Round to the nearest tenth.

14. $r = 12$ m **15.** $d = 12.5$ ft **16.** $d = \frac{1}{2}$ mi

Find the area of each of the following.

17. square whose sides are 9.1 yd in length

18. square whose sides are $(x + 1)$ in length

19. triangle whose base is $5\frac{1}{2}$ in. and whose height is $2\frac{1}{4}$ in.

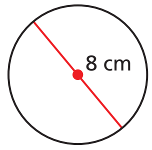
Given the area of each of the following figures, find each unknown measure.

20. The area of a triangle is 6.75 m^2. If the base of the triangle is 3 m, what is the height of the triangle?

21. A rectangle has an area of 347.13 cm^2. If the length is 20.3 cm, what is the width of the rectangle?

22. The area of a circle is 64π. Find the radius of the circle.

23. **/// ERROR ANALYSIS ///** Below are two statements about the area of the circle. Which is incorrect? Explain the error.



A

$$A = \pi r^2$$
$$= \pi(8)^2$$
$$= 64\pi \text{ cm}^2$$

B

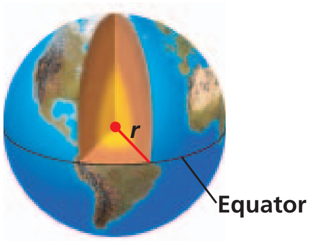
$$A = \pi r^2$$
$$= \pi(4)^2$$
$$= 16\pi \text{ cm}^2$$

Find the area of each circle. Leave answers in terms of π.

24. circle with a diameter of 28 m

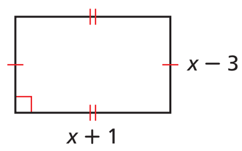
25. circle with a radius of $3y$

26. **Geography** The radius r of the earth at the equator is approximately 3964 mi. Find the distance around the earth at the equator. Use the π key on your calculator and round to the nearest mile.



27. **Critical Thinking** Explain how the formulas for the perimeter and area of a square may be derived from the corresponding formulas for a rectangle.

28. Find the perimeter and area of a rectangle whose length is $(x + 1)$ and whose width is $(x - 3)$. Express your answer in terms of x.

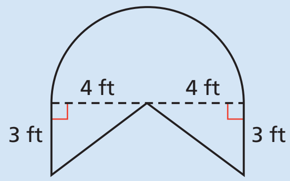


29. **Multi-Step** If the height h of a triangle is 3 inches less than the length of the base b, and the area A of the triangle is 19 times the length of the base, find b and h.

Concept Connection

30. This problem will prepare you for the Concept Connection on page 58.

A landscaper is to install edging around a garden. The edging costs $1.39 for each 24-inch-long strip. The landscaper estimates it will take 4 hours to install the edging.



a. If the total cost is $120.30, what is the cost of the material purchased?

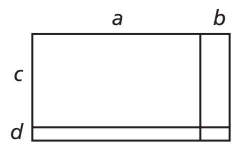
b. What is the charge for labor?

c. What is the area of the semicircle to the nearest tenth?

d. What is the area of each triangle?

e. What is the total area of the garden to the nearest foot?

31. **Algebra** The large rectangle has length $a + b$ and width $c + d$. Therefore, its area is $(a + b)(c + d)$.



a. Find the area of each of the four small rectangles in the figure. Then find the sum of these areas. Explain why this sum must be equal to the product $(a + b)(c + d)$.

b. Suppose $b = d = 1$. Write the area of the large rectangle as a product of its length and width. Then find the sum of the areas of the four small rectangles. Explain why this sum must be equal to the product $(a + 1)(c + 1)$.

c. Suppose $b = d = 1$ and $a = c$. Write the area of the large rectangle as a product of its length and width. Then find the sum of the areas of the four small rectangles. Explain why this sum must be equal to the product $(a + 1)^2$.

32. **Sports** The table shows the minimum and maximum dimensions for rectangular soccer fields used in international matches. Find the difference in area of the largest possible field and the smallest possible field.

	Minimum	Maximum
Length	100 m	110 m
Width	64 m	75 m

Find the value of each missing measure of a triangle.

33. $b = 2$ ft; $h = \square$ ft; $A = 28$ ft^2

34. $b = \square$ ft; $h = 22.6$ yd; $A = 282.5$ yd^2

Find the area of each rectangle with the given base and height.

35. 9.8 ft; 2.7 ft

36. 4 mi 960 ft; 440 ft

37. 3 yd 12 ft; 11 ft

Find the perimeter of each rectangle with the given base and height.

38. 21.4 in.; 7.8 in.

39. 4 ft 6 in.; 6 in.

40. 2 yd 8 ft; 6 ft

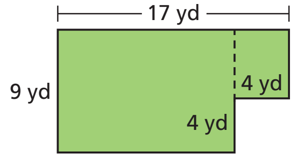
Find the diameter of the circle with the given measurement. Leave answers in terms of π.

41. $C = 14$

42. $A = 100\pi$

43. $C = 50\pi$

44. A skate park consists of a two adjacent rectangular regions as shown. Find the perimeter and area of the park.



45. **Critical Thinking** Explain how you would measure a triangular piece of paper if you wanted to find its area.

46. **Write About It** A student wrote in her journal, "To find the perimeter of a rectangle, add the length and width together and then double this value." Does her method work? Explain.

STANDARDIZED TEST PREP

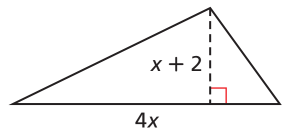
47. Manda made a circular tabletop that has an area of 452 in^2. Which is closest to the radius of the tabletop?

(A) 9 in.   (B) 12 in.   (C) 24 in.   (D) 72 in.

48. A piece of wire 48 m long is bent into the shape of a rectangle whose length is twice its width. Find the length of the rectangle.

(F) 8 m   (G) 16 m   (H) 24 m   (J) 32 m

49. Which equation best represents the area A of the triangle?



Ⓐ $A = 2x^2 + 4x$

Ⓑ $A = 4x(x + 2)$

Ⓒ $A = 2x^2 + 2$

Ⓓ $A = 4x^2 + 8$

50. Ryan has a 30 ft piece of string. He wants to use the string to lay out the boundary of a new flower bed in his garden. Which of these shapes would use all the string?

Ⓕ A circle with a radius of about 37.2 in.

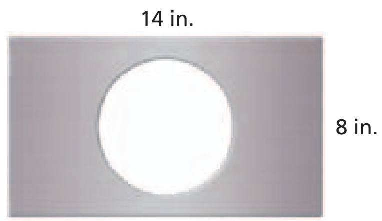
Ⓖ A rectangle with a length of 6 ft and a width of 5 ft

Ⓗ A triangle with each side 9 ft long

Ⓙ A square with each side 90 in. long

CHALLENGE AND EXTEND

51. A circle with a 6 in. diameter is stamped out of a rectangular piece of metal as shown. Find the area of the remaining piece of metal. Use the π key on your calculator and round to the nearest tenth.



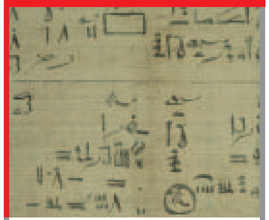
52. **a.** Solve $P = 2\ell + 2w$ for w.

b. Use your result from part **a** to find the width of a rectangle that has a perimeter of 9 ft and a length of 3 ft.

53. Find all possible areas of a rectangle whose sides are natural numbers and whose perimeter is 12.

LINK

Math History



The Ahmes Papyrus is an ancient Egyptian source of information about mathematics. A page of the Ahmes Papyrus is about 1 foot wide and 18 feet long.

Source: scholars.nus.edu.sg

54. **Estimation** The Ahmes Papyrus dates from approximately 1650 B.C.E. Lacking a precise value for π, the author assumed that the area of a circle with a diameter of 9 units had the same area as a square with a side length of 8 units. By what percent did the author overestimate or underestimate the actual area of the circle?

55. **Multi-Step** The width of a painting is $\frac{4}{5}$ the measure of the length of the painting. If the area is 320 in^2, what are the length and width of the painting?

SPIRAL REVIEW

Determine the domain and range of each function. *(Previous course)*

56. $\{(2, 4), (-5, 8), (-3, 4)\}$

57. $\{(4, -2), (-2, 8), (16, 0)\}$

Name the geometric figure that each item suggests. *(Lesson 1-1)*

58. the wall of a classroom

59. the place where two walls meet

60. Marion has a piece of fabric that is 10 yd long. She wants to cut it into 2 pieces so that one piece is 4 times as long as the other. Find the lengths of the two pieces. *(Lesson 1-2)*

61. Suppose that A, B, and C are collinear points. B is the midpoint of $\overline{AC}$. The coordinate of A is -8, and the coordinate of B is -2.5. What is the coordinate of C? *(Lesson 1-2)*

62. An angle's measure is 9 degrees more than 2 times the measure of its supplement. Find the measure of the angle. *(Lesson 1-4)*

Connecting Geometry to

Algebra

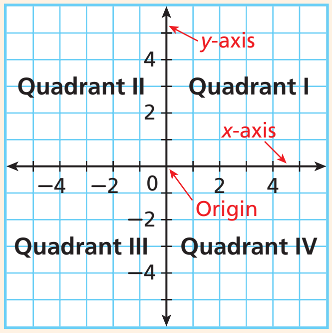
See Skills Bank page S56

Graphing in the Coordinate Plane

California Standards

Preparation for 17.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

The *coordinate plane* is used to name and locate points. Points in the coordinate plane are named by ordered pairs of the form (x, y). The first number is the x-coordinate. The second number is the y-coordinate. The x-axis and y-axis intersect at the origin, forming right angles. The axes separate the coordinate plane into four regions, called quadrants, numbered with Roman numerals placed counterclockwise.



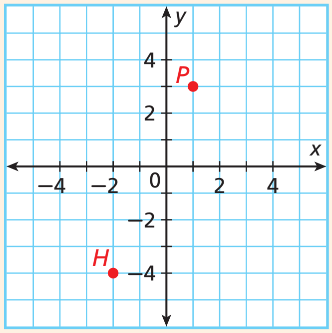
Examples

1 Name the coordinates of *P*.

Starting at the origin $(0, 0)$, you count 1 unit to the right. Then count 3 units up. So the coordinates of P are $(1, 3)$.

2 Plot and label $H(-2, -4)$ on a coordinate plane. Name the quadrant in which it is located.

Start at the origin $(0, 0)$ and move 2 units left. Then move 4 units down. Draw a dot and label it H. H is in Quadrant III.



You can also use a coordinate plane to locate places on a map.

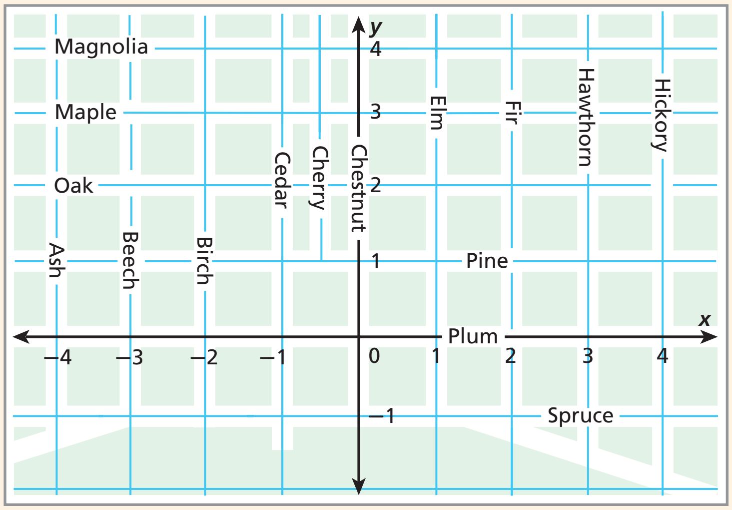
Try This

Name the coordinates of the point where the following streets intersect.

1. Chestnut and Plum
2. Magnolia and Chestnut
3. Oak and Hawthorn
4. Plum and Cedar

Name the streets that intersect at the given points.

5. $(-3, -1)$
6. $(4, -1)$
7. $(1, 3)$
8. $(-2, 1)$



1-6 Midpoint and Distance in the Coordinate Plane

Objectives
Develop and apply the formula for midpoint.

Use the Distance Formula and the Pythagorean Theorem to find the distance between two points.

Vocabulary
coordinate plane
leg
hypotenuse

Why learn this?
You can use a coordinate plane to help you calculate distances. (See Example 5.)

Major League baseball fields are laid out according to strict guidelines. Once you know the dimensions of a field, you can use a coordinate plane to find the distance between two of the bases.

A **coordinate plane** is a plane that is divided into four regions by a horizontal line (x-axis) and a vertical line (y-axis). The location, or coordinates, of a point are given by an ordered pair (x, y).

You can find the midpoint of a segment by using the coordinates of its endpoints. Calculate the average of the x-coordinates and the average of the y-coordinates of the endpoints.

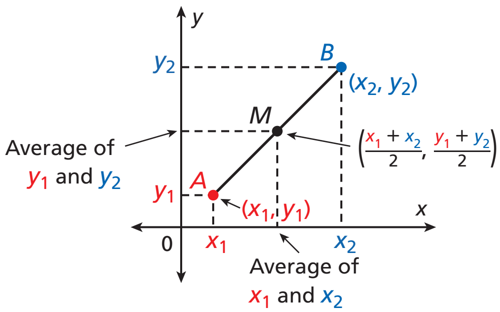
Know it! Note

California Standards
Preparation for 17.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.
Also covered: **15.0**

Midpoint Formula

The midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is found by

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



EXAMPLE 1

Finding the Coordinates of a Midpoint

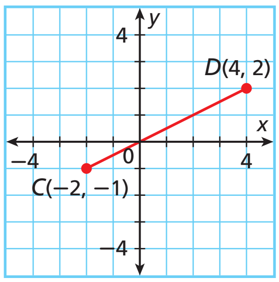
Helpful Hint
To make it easier to picture the problem, plot the segment's endpoints on a coordinate plane.

Find the coordinates of the midpoint of $\overline{CD}$ with endpoints $C(-2, -1)$ and $D(4, 2)$.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\frac{-2 + 4}{2}, \frac{-1 + 2}{2} = \left(\frac{2}{2}, \frac{1}{2}\right)$$

$$= \left(1, \frac{1}{2}\right)$$



CHECK IT OUT!

1. Find the coordinates of the midpoint of $\overline{EF}$ with endpoints $E(-2, 3)$ and $F(5, -3)$.

EXAMPLE 2 Finding the Coordinates of an Endpoint

Algebra

M is the midpoint of $\overline{AB}$. A has coordinates $(2, 2)$, and M has coordinates $(4, -3)$. Find the coordinates of B.

Step 1 Let the coordinates of B equal (x, y).

Step 2 Use the Midpoint Formula: $(4, -3) = \left(\frac{2+x}{2}, \frac{2+y}{2}\right)$.

Step 3 Find the x-coordinate. Find the y-coordinate.

Find the x-coordinate.		Find the y-coordinate.
$4 = \frac{2+x}{2}$	*Set the coordinates equal.*	$-3 = \frac{2+y}{2}$
$2(4) = 2\left(\frac{2+x}{2}\right)$	*Multiply both sides by 2.*	$2(-3) = 2\left(\frac{2+y}{2}\right)$
$8 = 2 + x$	*Simplify.*	$-6 = 2 + y$
$-2 \quad -2$	*Subtract 2 from both sides.*	$-2 \quad -2$
$6 = x$	*Simplify.*	$-8 = y$

The coordinates of B are $(6, -8)$.

CHECK IT OUT! **2.** S is the midpoint of $\overline{RT}$. R has coordinates $(-6, -1)$, and S has coordinates $(-1, 1)$. Find the coordinates of T.

The Ruler Postulate can be used to find the distance between two points on a number line. The Distance Formula is used to calculate the distance between two points in a coordinate plane.

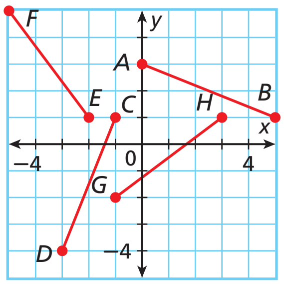
Know it! Note — Distance Formula

In a coordinate plane, the distance d between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

EXAMPLE 3 Using the Distance Formula

Find AB and CD. Then determine if $\overline{AB} \cong \overline{CD}$.



Step 1 Find the coordinates of each point.

$A(0, 3)$, $B(5, 1)$, $C(-1, 1)$, and $D(-3, -4)$

Step 2 Use the Distance Formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(5-0)^2 + (1-3)^2} = \sqrt{5^2 + (-2)^2} = \sqrt{25 + 4} = \sqrt{29}$$

$$CD = \sqrt{[-3-(-1)]^2 + (-4-1)^2} = \sqrt{(-2)^2 + (-5)^2} = \sqrt{4 + 25} = \sqrt{29}$$

Since $AB = CD$, $\overline{AB} \cong \overline{CD}$.

CHECK IT OUT! **3.** Find EF and GH. Then determine if $\overline{EF} \cong \overline{GH}$.

You can also use the Pythagorean Theorem to find the distance between two points in a coordinate plane. You will learn more about the Pythagorean Theorem in Chapter 5.

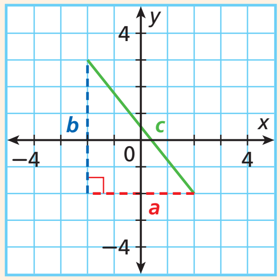
In a right triangle, the two sides that form the right angle are the **legs**. The side across from the right angle that stretches from one leg to the other is the **hypotenuse**. In the diagram, a and b are the lengths of the shorter sides, or legs, of the right triangle. The longest side is called the hypotenuse and has length c.

Know it! Note

Theorem 1-6-1 **Pythagorean Theorem**

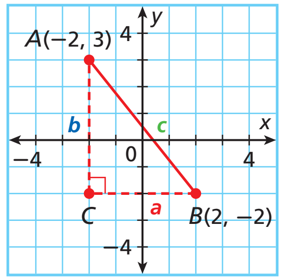
In a right triangle, the sum of the squares of the lengths of the *legs* is equal to the square of the length of the *hypotenuse*.

$$a^2 + b^2 = c^2$$



EXAMPLE 4 Finding Distances in the Coordinate Plane

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, from A to B.



Method 1

Use the Distance Formula. Substitute the values for the coordinates of A and B into the Distance Formula.

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[2 - (-2)]^2 + (-2 - 3)^2} \\ &= \sqrt{4^2 + (-5)^2} \\ &= \sqrt{16 + 25} \\ &= \sqrt{41} \\ &\approx 6.4 \end{aligned}$$

Method 2

Use the Pythagorean Theorem. Count the units for sides a and b.

$a = 4$ and $b = 5$.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 4^2 + 5^2 \\ &= 16 + 25 \\ &= 41 \\ c &= \sqrt{41} \\ c &\approx 6.4 \end{aligned}$$

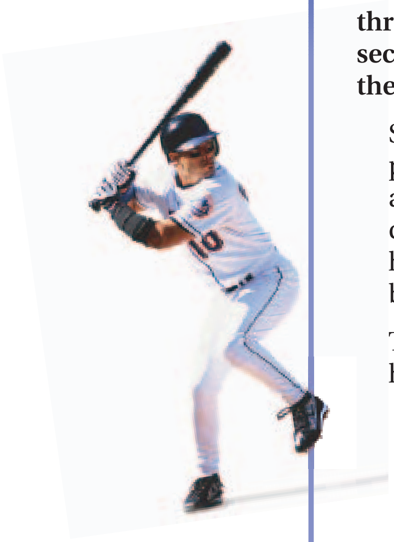
CHECK IT OUT!

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, from R to S.

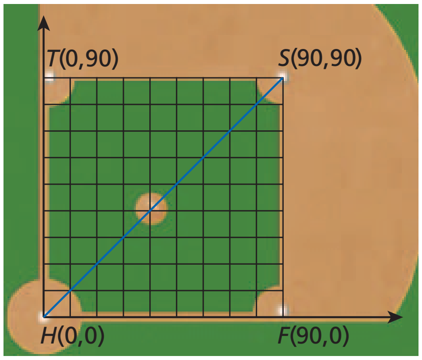
4a. $R(3, 2)$ and $S(-3, -1)$

4b. $R(-4, 5)$ and $S(2, -1)$

EXAMPLE 5 *Sports Application*



The four bases on a baseball field form a square with 90 ft sides. When a player throws the ball from home plate to second base, what is the distance of the throw, to the nearest tenth?



Set up the field on a coordinate plane so that home plate H is at the origin, first base F has coordinates $(90, 0)$, second base S has coordinates $(90, 90)$, and third base T has coordinates $(0, 90)$.

The distance HS from home plate to second base is the length of the hypotenuse of a right triangle.

$$
\begin{aligned}
HS &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(90 - 0)^2 + (90 - 0)^2} \\
&= \sqrt{90^2 + 90^2} \\
&= \sqrt{8100 + 8100} \\
&= \sqrt{16{,}200} \\
&\approx 127.3 \text{ ft}
\end{aligned}
$$

CHECK IT OUT!

5. The center of the pitching mound has coordinates $(42.8, 42.8)$. When a pitcher throws the ball from the center of the mound to home plate, what is the distance of the throw, to the nearest tenth?

THINK AND DISCUSS

1. Can you exchange the coordinates (x_1, y_1) and (x_2, y_2) in the Midpoint Formula and still find the correct midpoint? Explain.

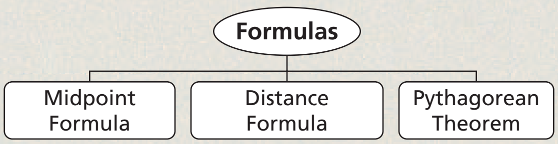
2. A right triangle has sides lengths of r, s, and t. Given that $s^2 + t^2 = r^2$, which variables represent the lengths of the legs and which variable represents the length of the hypotenuse?

3. Do you always get the same result using the Distance Formula to find distance as you do when using the Pythagorean Theorem? Explain your answer.

4. Why do you think that most cities are laid out in a rectangular grid instead of a triangular or circular grid?

Know it! Note

5. GET ORGANIZED Copy and complete the graphic organizer below. In each box, write a formula. Then make a sketch that will illustrate the formula.



1-6 Exercises

California Standards
Preparation for 17.0; 15.0, 7NS1.2, 7AF4.2, 7MG2.1, 7MG3.2, 1A2.0

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Homework Help Online
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Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

1. **Vocabulary** The __?__ is the side of a right triangle that is directly across from the right angle. (*hypotenuse* or *leg*)

SEE EXAMPLE 1 p. 43

Find the coordinates of the midpoint of each segment.

2. $\overline{AB}$ with endpoints $A(4, -6)$ and $B(-4, 2)$
3. $\overline{CD}$ with endpoints $C(0, -8)$ and $D(3, 0)$

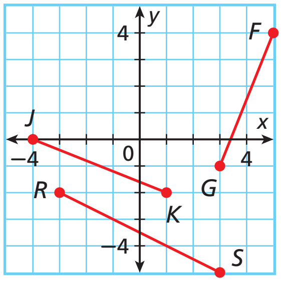
SEE EXAMPLE 2 p. 44

4. M is the midpoint of $\overline{LN}$. L has coordinates $(-3, -1)$, and M has coordinates $(0, 1)$. Find the coordinates of N.
5. B is the midpoint of $\overline{AC}$. A has coordinates $(-3, 4)$, and B has coordinates $\left(-1\frac{1}{2}, 1\right)$. Find the coordinates of C.

SEE EXAMPLE 3 p. 44

Multi-Step **Find the length of the given segments and determine if they are congruent.**

6. $\overline{JK}$ and $\overline{FG}$
7. $\overline{JK}$ and $\overline{RS}$



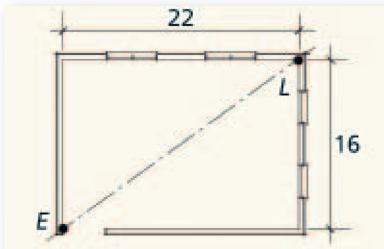
SEE EXAMPLE 4 p. 45

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, between each pair of points.

8. $A(1, -2)$ and $B(-4, -4)$
9. $X(-2, 7)$ and $Y(-2, -8)$
10. $V(2, -1)$ and $W(-4, 8)$

SEE EXAMPLE 5 p. 46

11. **Architecture** The plan for a rectangular living room shows electrical wiring will be run in a straight line from the entrance E to a light L at the opposite corner of the room. What is the length of the wire to the nearest tenth?



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12–13	1
14–15	2
16–17	3
18–20	4
21	5

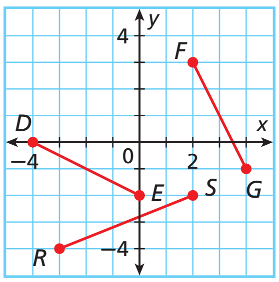
Extra Practice
Skills Practice p. S5
Application Practice p. S28

Find the coordinates of the midpoint of each segment.

12. $\overline{XY}$ with endpoints $X(-3, -7)$ and $Y(-1, 1)$
13. $\overline{MN}$ with endpoints $M(12, -7)$ and $N(-5, -2)$
14. M is the midpoint of $\overline{QR}$. Q has coordinates $(-3, 5)$, and M has coordinates $(7, -9)$. Find the coordinates of R.
15. D is the midpoint of $\overline{CE}$. E has coordinates $(-3, -2)$, and D has coordinates $\left(2\frac{1}{2}, 1\right)$. Find the coordinates of C.

Multi-Step **Find the length of the given segments and determine if they are congruent.**

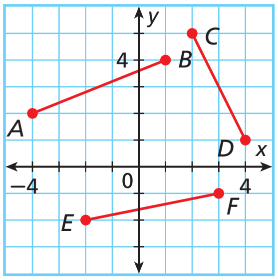
16. $\overline{DE}$ and $\overline{FG}$
17. $\overline{DE}$ and $\overline{RS}$



Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, between each pair of points.

18. $U(0, 1)$ and $V(-3, -9)$ **19.** $M(10, -1)$ and $N(2, -5)$ **20.** $P(-10, 1)$ and $Q(5, 5)$

21. Consumer Application Televisions and computer screens are usually advertised based on the length of their diagonals. If the height of a computer screen is 11 in. and the width is 14 in., what is the length of the diagonal? Round to the nearest inch.



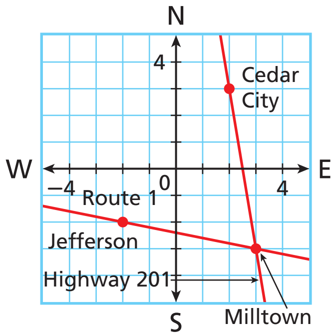
22. Multi-Step Use the Distance Formula to order $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ from shortest to longest.

23. Use the Pythagorean Theorem to find the distance from A to E. Round to the nearest hundredth.

24. X has coordinates $(a, 3a)$, and Y has coordinates $(-5a, 0)$. Find the coordinates of the midpoint of (XY).

25. Describe a shortcut for finding the midpoint of a segment when one of its endpoints has coordinates (a, b) and the other endpoint is the origin.

On the map, each square of the grid represents 1 square mile. Find each distance to the nearest tenth of a mile.



26. Find the distance along Highway 201 from Cedar City to Milltown.

27. A car breaks down on Route 1, at the midpoint between Jefferson and Milltown. A tow truck is sent out from Jefferson. How far does the truck travel to reach the car?

LINK

History

The construction of the Forbidden City lasted for 14 years. It began in 1406 with an estimated workforce of 200,000 men.

Source: www.wikipedia.com

28. History The Forbidden City in Beijing, China, is the world's largest palace complex. Surrounded by a wall and a moat, the rectangular complex is 960 m long and 750 m wide. Find the distance, to the nearest meter, from one corner of the complex to the opposite corner.

29. Critical Thinking Give an example of a line segment with midpoint $(0, 0)$.

The coordinates of the vertices of $\triangle ABC$ are $A(1, 4)$, $B(-2, -1)$, and $C(-3, -2)$.

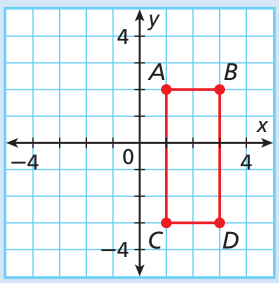
30. Find the perimeter of $\triangle ABC$ to the nearest tenth.

31. The height h to side $\overline{BC}$ is $\sqrt{2}$, and b is the length of $\overline{BC}$. What is the area of $\triangle ABC$?

32. Write About It Explain why the Distance Formula is not needed to find the distance between two points that lie on a horizontal or a vertical line.

CONCEPT CONNECTION

33. This problem will prepare you for the Concept Connection on page 58. Tania uses a coordinate plane to map out plans for landscaping a rectangular patio area. On the plan, one square represents 2 feet. She plans to plant a tree at the midpoint of $\overline{AC}$. How far from each corner of the patio does she plant the tree? Round to the nearest tenth.

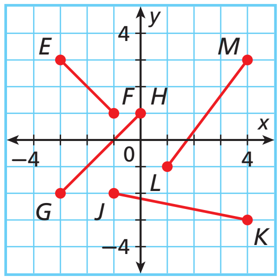


STANDARDIZED TEST PREP

34. Which segment has a length closest to 4 units?

Ⓐ $\overline{EF}$ Ⓒ $\overline{JK}$

Ⓑ $\overline{GH}$ Ⓓ $\overline{LM}$



35. Find the distance, to the nearest tenth, between the midpoints of $\overline{LM}$ and $\overline{JK}$.

Ⓕ 1.8 Ⓗ 4.0

Ⓖ 3.6 Ⓙ 5.3

36. What are the coordinates of the midpoint of a line segment that connects the points $(7, -3)$ and $(-5, 6)$?

Ⓐ $\left(6, -4\frac{1}{2}\right)$ Ⓒ $\left(2, \frac{1}{2}\right)$

Ⓑ $(2, 3)$ Ⓓ $\left(1, 1\frac{1}{2}\right)$

37. A coordinate plane is placed over the map of a town. A library is located at $(-5, 1)$, and a museum is located at $(3, 5)$. What is the distance, to the nearest tenth, from the library to the museum?

Ⓕ 4.5 Ⓖ 5.7 Ⓗ 6.3 Ⓙ 8.9

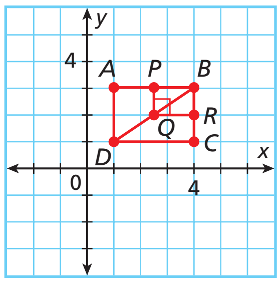
CHALLENGE AND EXTEND

38. Use the diagram to find the following.

a. P is the midpoint of $\overline{AB}$, and R is the midpoint of $\overline{BC}$. Find the coordinates of Q.

b. Find the area of rectangle $PBRQ$.

c. Find DB. Round to the nearest tenth.



39. The coordinates of X are $(a - 5, -2a)$. The coordinates of Y are $(a + 1, 2a)$. If the distance between X and Y is 10, find the value of a.

40. Find two points on the y-axis that are a distance of 5 units from $(4, 2)$.

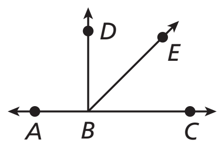
41. Given $\angle ACB$ is a right angle of $\triangle ABC$, $AC = x$, and $BC = y$, find AB in terms of x and y.

SPIRAL REVIEW

Determine if the ordered pair $(-1, 4)$ satisfies each function. *(Previous course)*

42. $y = 3x - 1$ **43.** $f(x) = 5 - x^2$ **44.** $g(x) = x^2 - x + 2$

$\overrightarrow{BD}$ bisects straight angle ABC, and $\overrightarrow{BE}$ bisects $\angle CBD$. Find the measure of each angle and classify it as acute, right, or obtuse. *(Lesson 1-3)*



45. $\angle ABD$ **46.** $\angle CBE$ **47.** $\angle ABE$

Find the area of each of the following. *(Lesson 1-5)*

48. square whose perimeter is 20 in.

49. triangle whose height is 2 ft and whose base is twice its height

50. rectangle whose length is x and whose width is $(4x + 5)$

1-7 Transformations in the Coordinate Plane

Objectives
Identify reflections, rotations, and translations.

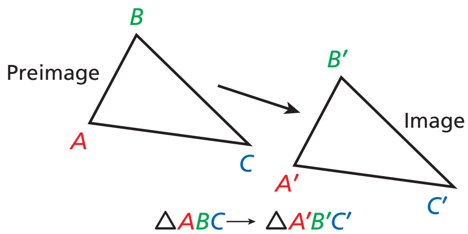
Graph transformations in the coordinate plane.

Vocabulary
transformation
preimage
image
reflection
rotation
translation

Who uses this?
Artists use transformations to create decorative patterns. (See Example 4.)

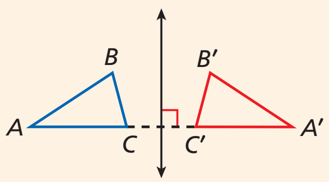
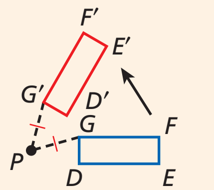
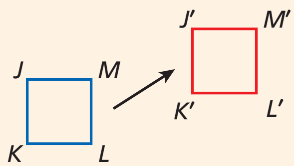
The Alhambra, a 13th-century palace in Granada, Spain, is famous for the geometric patterns that cover its walls and floors. To create a variety of designs, the builders based the patterns on several different *transformations*.

A **transformation** is a change in the position, size, or shape of a figure. The original figure is called the **preimage**. The resulting figure is called the **image**. A transformation *maps* the preimage to the image. Arrow notation (→) is used to describe a transformation, and primes (′) are used to label the image.



Know it! Note

Transformations

REFLECTION	ROTATION	TRANSLATION
		
A **reflection** (or *flip*) is a transformation across a line, called the line of reflection. Each point and its image are the same distance from the line of reflection.	A **rotation** (or *turn*) is a transformation about a point P, called the center of rotation. Each point and its image are the same distance from P.	A **translation** (or *slide*) is a transformation in which all the points of a figure move the same distance in the same direction.

Math Builders
For more on transformations, see the Transformation Builder on page MB2.

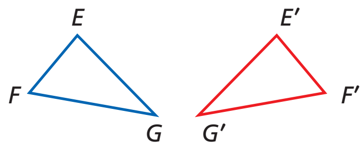
EXAMPLE 1

Identifying Transformations

California Standards
🔑 **22.0 Students know the effect of rigid motions on figures in the coordinate plane** and space, **including rotations, translations, and reflections.**

Identify the transformation. Then use arrow notation to describe the transformation.

A

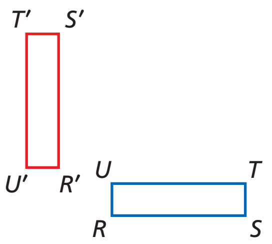


The transformation cannot be a translation because each point and its image are not in the same position.

The transformation is a reflection. $\triangle EFG \rightarrow \triangle E'F'G'$

Identify the transformation. Then use arrow notation to describe the transformation.

B

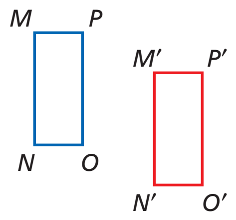


The transformation cannot be a reflection because each point and its image are not the same distance from a line of reflection.

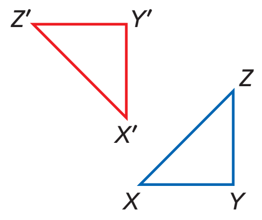
The transformation is a 90° rotation. $RSTU \rightarrow R'S'T'U'$

CHECK IT OUT! **Identify each transformation. Then use arrow notation to describe the transformation.**

1a.

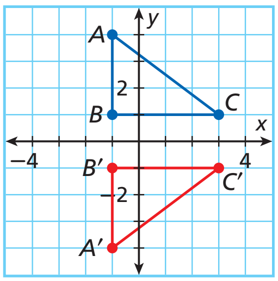


1b.



EXAMPLE 2 Drawing and Identifying Transformations

A figure has vertices at $A(-1, 4)$, $B(-1, 1)$, and $C(3, 1)$. After a transformation, the image of the figure has vertices at $A'(-1, -4)$, $B'(-1, -1)$, and $C'(3, -1)$. Draw the preimage and image. Then identify the transformation.



Plot the points. Then use a straightedge to connect the vertices.

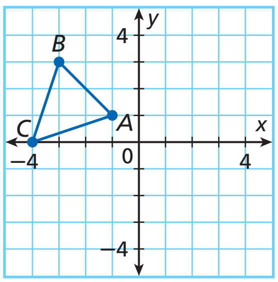
The transformation is a reflection across the x-axis because each point and its image are the same distance from the x-axis.

CHECK IT OUT! **2.** A figure has vertices at $E(2, 0)$, $F(2, -1)$, $G(5, -1)$, and $H(5, 0)$. After a transformation, the image of the figure has vertices at $E'(0, 2)$, $F'(1, 2)$, $G'(1, 5)$, and $H'(0, 5)$. Draw the preimage and image. Then identify the transformation.

To find coordinates for the image of a figure in a translation, add a to the x-coordinates of the preimage and add b to the y-coordinates of the preimage. Translations can also be described by a rule such as $(x, y) \rightarrow (x + a, y + b)$.

EXAMPLE 3 Translations in the Coordinate Plane

Find the coordinates for the image of $\triangle ABC$ after the translation $(x, y) \rightarrow (x + 3, y - 4)$. Draw the image.



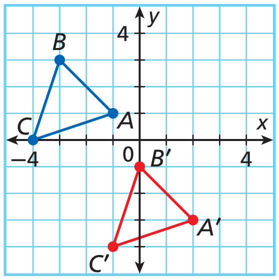
Step 1 Find the coordinates of $\triangle ABC$.

The vertices of $\triangle ABC$ are $A(-1, 1)$, $B(-3, 3)$, and $C(-4, 0)$.

Step 2 Apply the rule to find the vertices of the image.

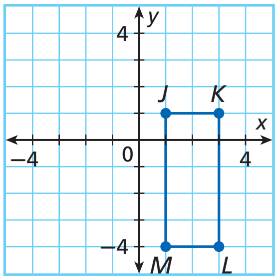
$A'(-1+3, 1-4) = A'(2, -3)$
$B'(-3+3, 3-4) = B'(0, -1)$
$C'(-4+3, 0-4) = C'(-1, -4)$

Step 3 Plot the points. Then finish drawing the image by using a straightedge to connect the vertices.



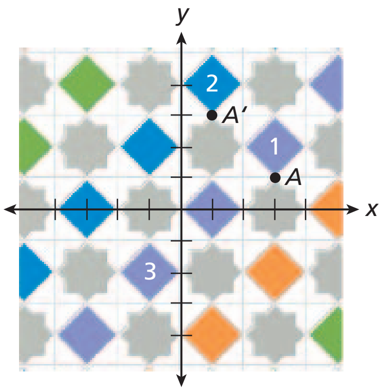
CHECK IT OUT!

3. Find the coordinates for the image of *JKLM* after the translation $(x, y) \rightarrow (x-2, y+4)$. Draw the image.



EXAMPLE 4 *Art History Application*

The pattern shown is similar to a pattern on a wall of the Alhambra. Write a rule for the translation of square 1 to square 2.



Step 1 Choose 2 points

Choose a point A on the preimage and a corresponding point A' on the image. A has coordinates $(3, 1)$, and A' has coordinates $(1, 3)$.

Step 2 Translate

To translate A to A', 2 units are subtracted from the x-coordinate and 2 units are added to the y-coordinate. Therefore, the translation rule is $(x, y) \rightarrow (x-2, y+2)$.

CHECK IT OUT!

4. Use the diagram to write a rule for the translation of square 1 to square 3.

THINK AND DISCUSS

Know it! Note

1. Explain how to recognize a reflection when given a figure and its image.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, sketch an example of each transformation.

Transformations

Reflection | Rotation | Translation

1-7 Exercises

California Standards
15.0, 22.0, 7NS1.2, 7MG3.2, 1A2.0

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Homework Help Online
KEYWORD: MG7 1-7
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

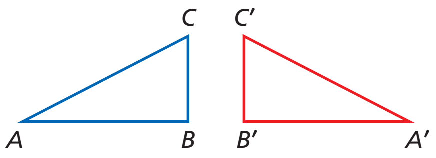
1. Given the transformation $\triangle XYZ \rightarrow \triangle X'Y'Z'$, name the preimage and image of the transformation.

2. The types of transformations of geometric figures in the coordinate plane can be described as a slide, a flip, or a turn. What are the other names used to identify these transformations?

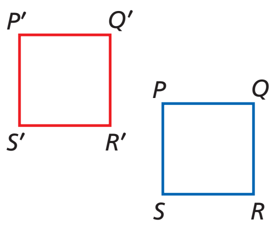
SEE EXAMPLE 1
p. 50

Identify each transformation. Then use arrow notation to describe the transformation.

3.



4.



SEE EXAMPLE 2
p. 51

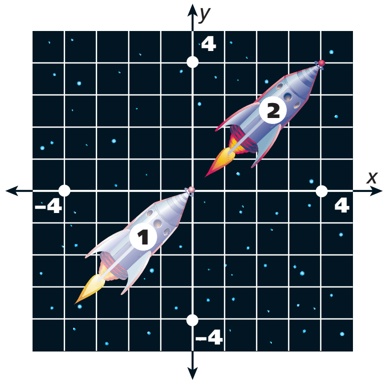
5. A figure has vertices at $A(-3, 2)$, $B(-1, -1)$, and $C(-4, -2)$. After a transformation, the image of the figure has vertices at $A'(3, 2)$, $B'(1, -1)$, and $C'(4, -2)$. Draw the preimage and image. Then identify the transformation.

SEE EXAMPLE 3
p. 51

6. **Multi-Step** The coordinates of the vertices of $\triangle DEF$ are $D(2, 3)$, $E(1, 1)$, and $F(4, 0)$. Find the coordinates for the image of $\triangle DEF$ after the translation $(x, y) \rightarrow (x - 3, y - 2)$. Draw the preimage and image.

SEE EXAMPLE 4
p. 52

7. **Animation** In an animated film, a simple scene can be created by translating a figure against a still background. Write a rule for the translation that maps the rocket from position 1 to position 2.



PRACTICE AND PROBLEM SOLVING

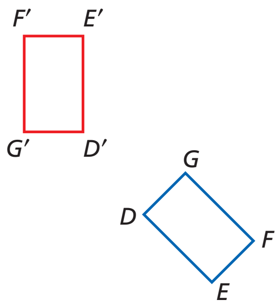
Independent Practice

For Exercises	See Example
8–9	1
10	2
11	3
12	4

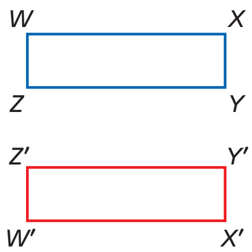
Extra Practice
Skills Practice p. S5
Application Practice p. S28

Identify each transformation. Then use arrow notation to describe the transformation.

8.



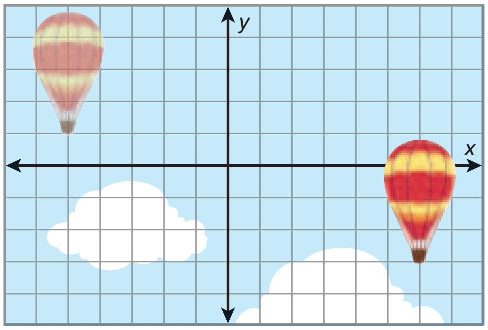
9.



10. A figure has vertices at $J(-2, 3)$, $K(0, 3)$, $L(0, 1)$, and $M(-2, 1)$. After a transformation, the image of the figure has vertices at $J'(2, 1)$, $K'(4, 1)$, $L'(4, -1)$, and $M'(2, -1)$. Draw the preimage and image. Then identify the transformation.

11. **Multi-Step** The coordinates of the vertices of rectangle $ABCD$ are $A(-4, 1)$, $B(1, 1)$, $C(1, -2)$, and $D(-4, -2)$. Find the coordinates for the image of rectangle $ABCD$ after the translation $(x, y) \rightarrow (x + 3, y - 2)$. Draw the preimage and the image.

12. **Travel** Write a rule for the translation that maps the descent of the hot air balloon.



Which transformation is suggested by each of the following?

13. mountain range and its image on a lake

14. straight line path of a band marching down a street

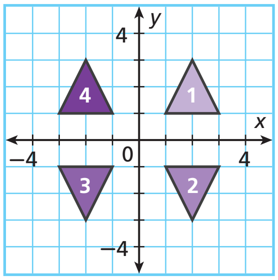
15. wings of a butterfly

Given points $F(3, 5)$, $G(-1, 4)$, and $H(5, 0)$, draw $\triangle FGH$ and its reflection across each of the following lines.

16. the x-axis

17. the y-axis

18. Find the vertices of one of the triangles on the graph. Then use arrow notation to write a rule for translating the other three triangles.



A transformation maps A onto B and C onto D.

19. Name the image of A.

20. Name the preimage of B.

21. Name the image of C.

22. Name the preimage of D.

23. Find the coordinates for the image of $\triangle RST$ with vertices $R(1, -4)$, $S(-1, -1)$, and $T(-5, 1)$ after the translation $(x, y) \rightarrow (x - 2, y - 8)$.

24. **Critical Thinking** Consider the translations $(x, y) \rightarrow (x + 5, y + 3)$ and $(x, y) \rightarrow (x + 10, y + 5)$. Compare the two translations.

Graph each figure and its image after the given translation.

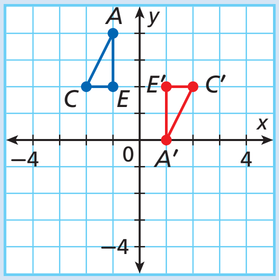
25. $\overline{MN}$ with endpoints $M(2, 8)$ and $N(-3, 4)$ after the translation $(x, y) \rightarrow (x + 2, y - 5)$

26. $\overline{KL}$ with endpoints $K(-1, 1)$ and $L(3, -4)$ after the translation $(x, y) \rightarrow (x - 4, y + 3)$

27. **Write About It** Given a triangle in the coordinate plane, explain how to draw its image after the translation $(x, y) \rightarrow (x + 1, y + 1)$.

CONCEPT CONNECTION

28. This problem will prepare you for the Concept Connection on page 58. Greg wants to rearrange the triangular pattern of colored stones on his patio. What combination of transformations could he use to transform $\triangle CAE$ to the image on the coordinate plane?

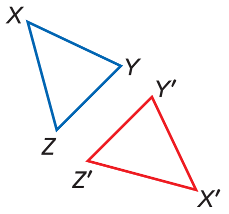


STANDARDIZED TEST PREP

29. Which type of transformation maps $\triangle XYZ$ to $\triangle X'Y'Z'$?

Ⓐ Reflection Ⓒ Translation

Ⓑ Rotation Ⓓ Not here



30. $\triangle DEF$ has vertices at $D(-4, 2)$, $E(-3, -3)$, and $F(1, 4)$. Which of these points is a vertex of the image of $\triangle DEF$ after the translation $(x, y) \rightarrow (x - 2, y + 1)$?

Ⓕ $(-2, 1)$ Ⓗ $(-5, -2)$

Ⓖ $(3, 3)$ Ⓙ $(-6, -1)$

31. Consider the translation $(1, 4) \rightarrow (-2, 3)$. What number was added to the x-coordinate?

Ⓐ -3 Ⓑ -1 Ⓒ 1 Ⓓ 7

32. Consider the translation $(-5, -7) \rightarrow (-2, -1)$. What number was added to the y-coordinate?

Ⓕ -3 Ⓖ 3 Ⓗ 6 Ⓙ 8

CHALLENGE AND EXTEND

33. $\triangle RST$ with vertices $R(-2, -2)$, $S(-3, 1)$, and $T(1, 1)$ is translated by $(x, y) \rightarrow (x - 1, y + 3)$. Then the image, $\triangle R'S'T'$, is translated by $(x, y) \rightarrow (x + 4, y - 1)$, resulting in $\triangle R''S''T''$.
    a. Find the coordinates for the vertices of $\triangle R''S''T''$.
    b. Write a rule for a single translation that maps $\triangle RST$ to $\triangle R''S''T''$.

34. Find the angle through which the minute hand of a clock rotates over a period of 12 minutes.

35. A triangle has vertices $A(1, 0)$, $B(5, 0)$, and $C(2, 3)$. The triangle is rotated 90° counterclockwise about the origin. Draw and label the image of the triangle.

Determine the coordinates for the reflection image of any point $A(x, y)$ across the given line.

36. x-axis

37. y-axis

SPIRAL REVIEW

Use factoring to find the zeros of each function. *(Previous course)*

38. $y = x^2 + 12x + 35$

39. $y = x^2 + 3x - 18$

40. $y = x^2 - 18x + 81$

41. $y = x^2 - 3x + 2$

Given $m\angle A = 76.1°$, find the measure of each of the following. *(Lesson 1-4)*

42. supplement of $\angle A$

43. complement of $\angle A$

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, between each pair of points. *(Lesson 1-6)*

44. $(2, 3)$ and $(4, 6)$

45. $(-1, 4)$ and $(0, 8)$

46. $(-3, 7)$ and $(-6, -2)$

47. $(5, 1)$ and $(-1, 3)$

1-7 Technology LAB

Use with Lesson 1-7

Explore Transformations

A transformation is a movement of a figure from its original position (preimage) to a new position (image). In this lab, you will use geometry software to perform transformations and explore their properties.

California Standards

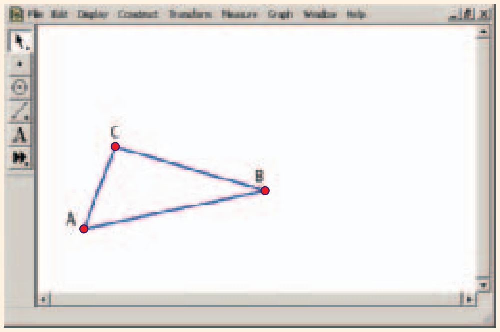
22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, **including rotations, translations,** and reflections.

go.hrw.com
Lab Resources Online
KEYWORD: MG7 Lab1

Activity 1

1. Construct a triangle using the segment tool. Use the text tool to label the vertices *A*, *B*, and *C*.

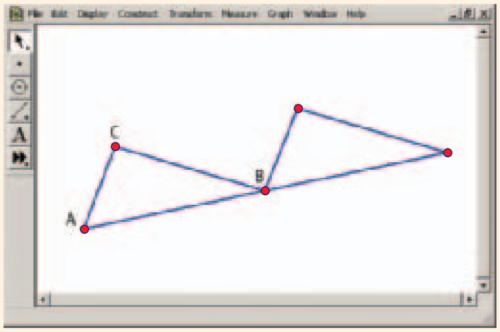
2. Select points *A* and *B* in that order. Choose Mark Vector from the Transform menu.



3. Select △*ABC* by clicking on all three segments of the triangle.

4. Choose Translate from the Transform menu, using *Marked* as the translation vector. What do you notice about the relationship between your preimage and its image?

5. What happens when you drag a vertex or a side of △*ABC*?

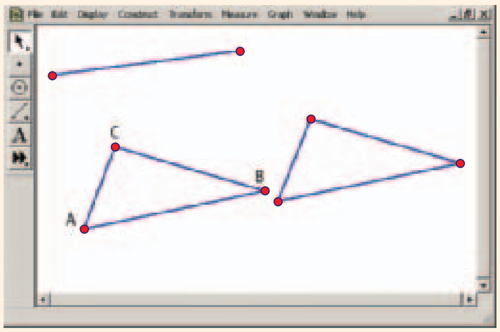


Try This

For Problems 1 and 2 choose New Sketch from the File menu.

1. Construct a triangle and a segment outside the triangle. Mark this segment as a translation vector as you did in Step 2 of Activity 1. Use Step 4 of Activity 1 to translate the triangle. What happens when you drag an endpoint of the new segment?

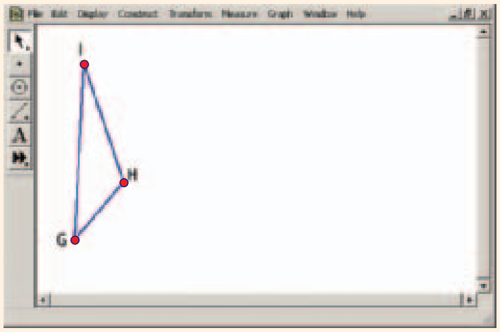
2. Instead of translating by a marked vector, use *Rectangular* as the translation vector and translate by a horizontal distance of 1 cm and a vertical distance of 2 cm. Compare this method with the marked vector method. What happens when you drag a side or vertex of the triangle?



3. Select the angles and sides of the preimage and image triangles. Use the tools in the Measure menu to measure length, angle measure, perimeter, and area. What do you think is true about these two figures?

Activity 2

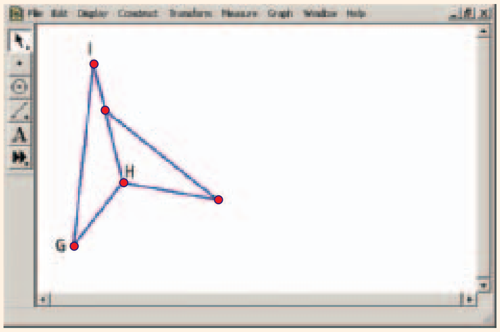
1. Construct a triangle. Label the vertices *G*, *H*, and *I*.



2. Select point *H* and choose Mark Center from the Transform menu.

3. Select $\angle GHI$ by selecting points *G*, *H*, and *I* in that order. Choose Mark Angle from the Transform menu.

4. Select the entire triangle $\triangle GHI$ by dragging a selection box around the figure.



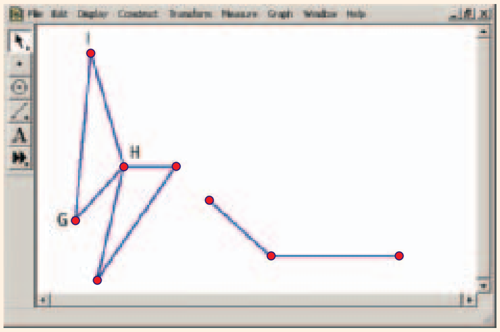
5. Choose Rotate from the Transform menu, using *Marked Angle* as the angle of rotation.

6. What happens when you drag a vertex or a side of $\triangle GHI$?

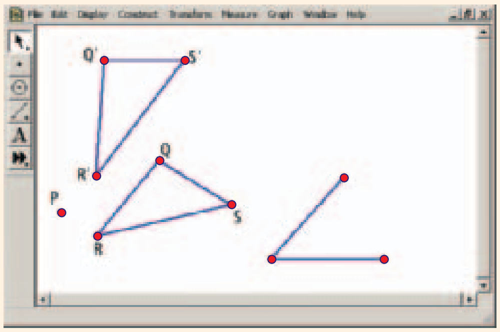
Try This

For Problems 4–6 choose New Sketch from the File menu.

4. Instead of selecting an angle of the triangle as the rotation angle, draw a new angle outside of the triangle. Mark this angle. Mark $\angle GHI$ as Center and rotate the triangle. What happens when you drag one of the points that form the rotation angle?



5. Construct $\triangle QRS$, a new rotation angle, and a point *P* not on the triangle. Mark *P* as the center and mark the angle. Rotate the triangle. What happens when you drag *P* outside, inside, or on the preimage triangle?



6. Instead of rotating by a marked angle, use *Fixed Angle* as the rotation method and rotate by a fixed angle measure of 30°. Compare this method with the marked angle method.

7. Using the fixed angle method of rotation, can you find an angle measure that will result in an image figure that exactly covers the preimage figure?

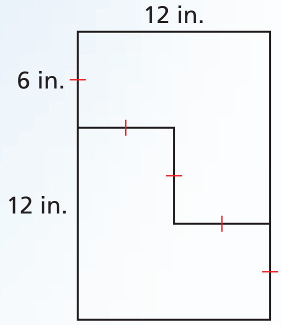
CHAPTER 1

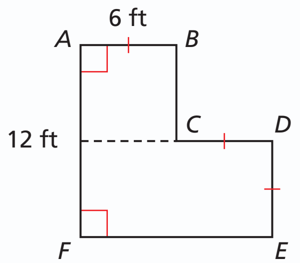
SECTION 1B

CONCEPT CONNECTION

Coordinate and Transformation Tools

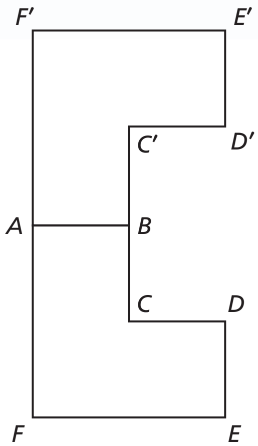
Pave the Way Julia wants to use L-shaped paving stones to pave a patio. Two stones will cover a 12 in. by 18 in. rectangle.





1. She drew diagram *ABCDEF* to represent the patio. Find the area and perimeter of the patio. How many paving stones would Julia need to purchase to pave the patio? If each stone costs $2.25, what is the total cost of the stones for the patio? Describe how you calculated your answer.

2. Julia plans to place a fountain at the midpoint of $\overline{AF}$. How far is the fountain from B, C, E, and F? Round to the nearest tenth.



3. Julia used a pair of paving stones to create another pattern for the patio. Describe the transformation she used to create the pattern. If she uses just one transformation, how many other patterns can she create using two stones? Draw all the possible combinations. Describe the transformation used to create each pattern.

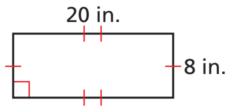
READY TO GO ON?

Quiz for Lessons 1-5 Through 1-7

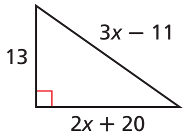
1-5 Using Formulas in Geometry

Find the perimeter and area of each figure.

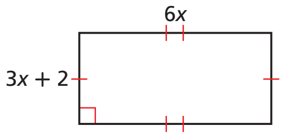
1.



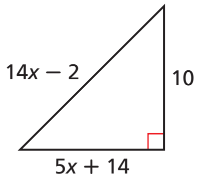
2.



3.



4.



5. Find the circumference and area of a circle with a radius of 6 m. Use the π key on your calculator and round to the nearest tenth.

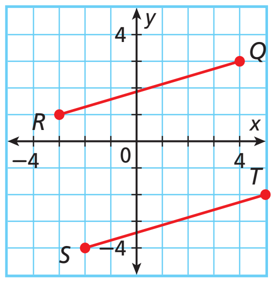
1-6 Midpoint and Distance in the Coordinate Plane

6. Find the coordinates for the midpoint of $\overline{XY}$ with endpoints $X(-4, 6)$ and $Y(3, 8)$.

7. J is the midpoint of $\overline{HK}$, H has coordinates $(6, -2)$, and J has coordinates $(9, 3)$. Find the coordinates of K.

8. Using the Distance Formula, find QR and ST to the nearest tenth. Then determine if $\overline{QR} \cong \overline{ST}$.

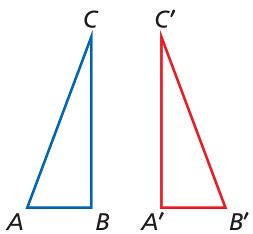
9. Using the Distance Formula and the Pythagorean Theorem, find the distance, to the nearest tenth, from $F(4, 3)$ to $G(-3, -2)$.



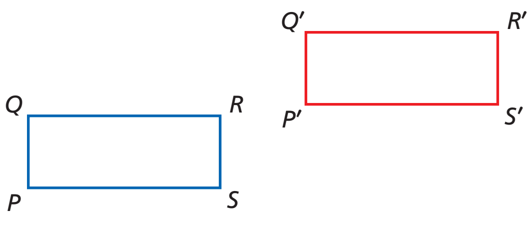
1-7 Transformations in the Coordinate Plane

Identify the transformation. Then use arrow notation to describe the transformation.

10.

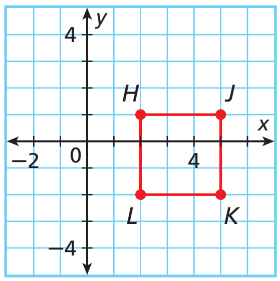


11.



12. A graphic designer used the translation $(x, y) \rightarrow (x - 3, y + 2)$ to transform square $HJKL$. Find the coordinates and graph the image of square $HJKL$.

13. A figure has vertices at $X(1, 1)$, $Y(3, 1)$, and $Z(3, 4)$. After a transformation, the image of the figure has vertices at $X'(-1, -1)$, $Y'(-3, -1)$, and $Z'(-3, -4)$. Graph the preimage and image. Then identify the transformation.



CHAPTER 1

Study Guide: Review

Know it! Note: For a complete list of the postulates and theorems in this chapter, see p. S82.

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ divides an angle into two congruent angles.
2. ___?___ are two angles whose measures have a sum of 90°.
3. The length of the longest side of a right triangle is called the ___?___.

1-1 Understanding Points, Lines, and Planes *(pp. 6–11)*

1.0

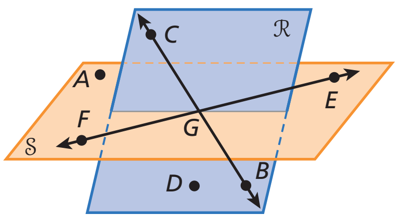
EXAMPLES

■ **Name the common endpoint of $\overrightarrow{SR}$ and $\overrightarrow{ST}$.**

$\overrightarrow{SR}$ and $\overrightarrow{ST}$ are opposite rays with common endpoint *S*.

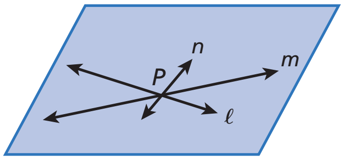
EXERCISES

Name each of the following.



4. four coplanar points
5. line containing *B* and *C*
6. plane that contains *A*, *G*, and *E*

■ **Draw and label three coplanar lines intersecting in one point.**



Draw and label each of the following.

7. line containing P and Q

8. pair of opposite rays both containing C

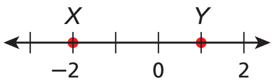
9. $\overleftrightarrow{CD}$ intersecting plane $\mathcal{P}$ at B

1-2 Measuring and Constructing Segments *(pp. 13–19)*

16.0

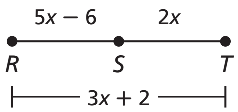
EXAMPLES

■ **Find the length of $\overline{XY}$.**



$XY = |-2 - 1|$
$= |-3| = 3$

■ **S is between R and T. Find RT.**

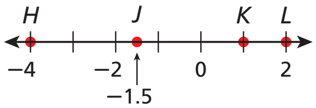


$RT = RS + ST$
$3x + 2 = 5x - 6 + 2x$
$3x + 2 = 7x - 6$
$x = 2$
$RT = 3(2) + 2 = 8$

EXERCISES

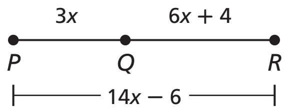
Find each length.

10. JL **11.** HK



12. Y is between X and Z, $XY = 13.8$, and $XZ = 21.4$. Find YZ.

13. Q is between P and R. Find PR.



14. U is the midpoint of $\overline{TV}$, $TU = 3x + 4$, and $UV = 5x - 2$. Find TU, UV, and TV.

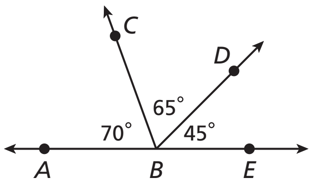
15. E is the midpoint of $\overline{DF}$, $DE = 9x$, and $EF = 4x + 10$. Find DE, EF, and DF.

1-3 Measuring and Constructing Angles *(pp. 20–27)*

16.0

EXAMPLES

■ **Classify each angle as acute, right, or obtuse.**



$\angle ABC$ acute;
$\angle CBD$ acute;
$\angle ABD$ obtuse;
$\angle DBE$ acute;
$\angle CBE$ obtuse

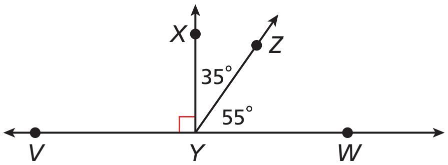
■ **$\overrightarrow{KM}$ bisects $\angle JKL$, $m\angle JKM = (3x + 4)°$, and $m\angle MKL = (6x - 5)°$. Find $m\angle JKL$.**

$3x + 4 = 6x - 5$ *Def. of ∠ bisector*
$3x + 9 = 6x$ *Add 5 to both sides.*
$9 = 3x$ *Subtract 3x from both sides.*
$x = 3$ *Divide both sides by 3.*

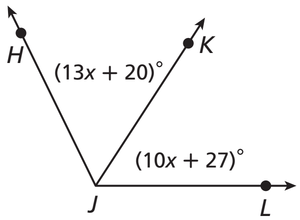
$m\angle JKL = 3x + 4 + 6x - 5$
$= 9x - 1$
$= 9(3) - 1 = 26°$

EXERCISES

16. Classify each angle as acute, right, or obtuse.



17. $m\angle HJL = 116°$. Find $m\angle HJK$.



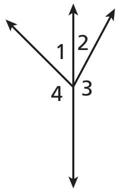
18. $\overrightarrow{NP}$ bisects $\angle MNQ$, $m\angle MNP = (6x - 12)°$, and $m\angle PNQ = (4x + 8)°$. Find $m\angle MNQ$.

1-4 Pairs of Angles *(pp. 28–33)*

Prep for **13.0**

EXAMPLES

■ **Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.**



$\angle 1$ and $\angle 2$ are only adjacent.

$\angle 2$ and $\angle 4$ are not adjacent.

$\angle 2$ and $\angle 3$ are adjacent and form a linear pair.

$\angle 1$ and $\angle 4$ are adjacent and form a linear pair.

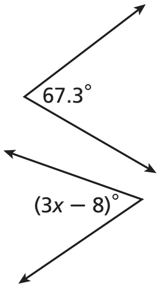
■ **Find the measure of the complement and supplement of each angle.**

$90 - 67.3 = 22.7°$

$180 - 67.3 = 112.7°$

$90 - (3x - 8) = (98 - 3x)°$

$180 - (3x - 8) = (188 - 3x)°$



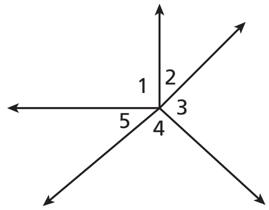
EXERCISES

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

19. $\angle 1$ and $\angle 2$

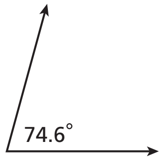
20. $\angle 3$ and $\angle 4$

21. $\angle 2$ and $\angle 5$

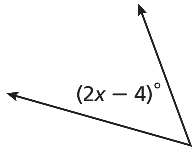


Find the measure of the complement and supplement of each angle.

22.



23.



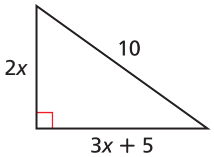
24. An angle measures 5 degrees more than 4 times its complement. Find the measure of the angle.

1-5 Using Formulas in Geometry *(pp. 36–41)*

8.0

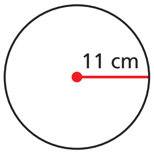
EXAMPLES

■ **Find the perimeter and area of the triangle.**



$$P = 2x + 3x + 5 + 10$$
$$= 5x + 15$$
$$A = \frac{1}{2}(3x + 5)(2x)$$
$$= 3x^2 + 5x$$

■ **Find the circumference and area of the circle to the nearest tenth.**

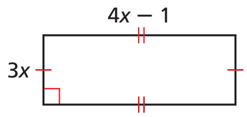


$$C = 2\pi r$$
$$= 2\pi(11)$$
$$= 22\pi$$
$$\approx 69.1 \text{ cm}$$
$$A = \pi r^2$$
$$= \pi(11)^2$$
$$= 121\pi$$
$$\approx 380.1 \text{ cm}^2$$

EXERCISES

Find the perimeter and area of each figure.

25.



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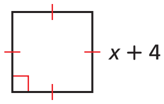

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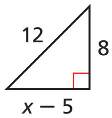

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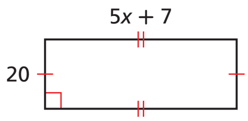
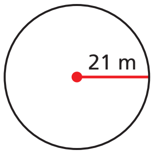
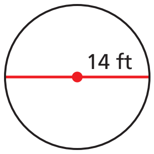


Find the circumference and area of each circle to the nearest tenth.

29.



30.



31. The area of a triangle is 102 m^2. The base of the triangle is 17 m. What is the height of the triangle?

1-6 Midpoint and Distance in the Coordinate Plane *(pp. 43–49)*

Prep for 17.0, 15.0

EXAMPLES

- **X is the midpoint of $\overline{CD}$. C has coordinates $(-4, 1)$, and X has coordinates $(3, -2)$. Find the coordinates of D.**

$$(3, -2) = \left(\frac{-4 + x}{2}, \frac{1 + y}{2}\right)$$

$$3 = \frac{-4 + x}{2} \qquad -2 = \frac{1 + y}{2}$$

$$6 = -4 + x \qquad -4 = 1 + y$$

$$10 = x \qquad -5 = y$$

The coordinates of D are $(10, -5)$.

- **Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, from $(1, 6)$ to $(4, 2)$.**

$$\begin{aligned} d &= \sqrt{(4-1)^2 + (2-6)^2} \\ &= \sqrt{3^2 + (-4)^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5.0 \end{aligned} \qquad \begin{aligned} c^2 &= a^2 + b^2 \\ &= 3^2 + 4^2 \\ &= 9 + 16 = 25 \\ c &= \sqrt{25} \\ &= 5.0 \end{aligned}$$

EXERCISES

Y is the midpoint of $\overline{AB}$. Find the missing coordinates of each point.

32. $A(3, 2)$; $B(-1, 4)$; $Y(\square, \square)$

33. $A(5, 0)$; $B(\square, \square)$; $Y(-2, 3)$

34. $A(\square, \square)$; $B(-4, 4)$; $Y(-2, 3)$

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, between each pair of points.

35. $X(-2, 4)$ and $Y(6, 1)$

36. $H(0, 3)$ and $K(-2, -4)$

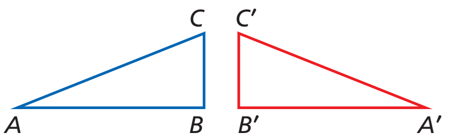
37. $L(-4, 2)$ and $M(3, -2)$

1-7 Transformations in the Coordinate Plane *(pp. 50–55)*

22.0

EXAMPLES

- **Identify the transformation. Then use arrow notation to describe the transformation.**



The transformation is a reflection.
$\triangle ABC \rightarrow \triangle A'B'C'$

- **The coordinates of the vertices of rectangle $HJKL$ are $H(2, -1)$, $J(5, -1)$, $K(5, -3)$, and $L(2, -3)$. Find the coordinates of the image of rectangle $HJKL$ after the translation $(x, y) \rightarrow (x - 4, y + 1)$.**

$H' = (2 - 4, -1 + 1) = H'(-2, 0)$

$J' = (5 - 4, -1 + 1) = J'(1, 0)$

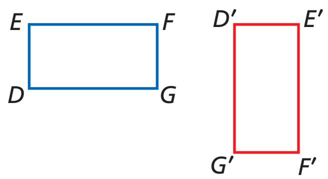
$K' = (5 - 4, -3 + 1) = K'(1, -2)$

$L' = (2 - 4, -3 + 1) = L'(-2, -2)$

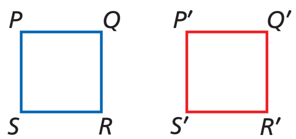
EXERCISES

Identify each transformation. Then use arrow notation to describe the transformation.

38.



39.

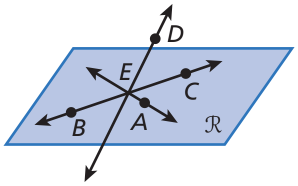


40. The coordinates for the vertices of $\triangle XYZ$ are $X(-5, -4)$, $Y(-3, -1)$, and $Z(-2, -2)$. Find the coordinates for the image of $\triangle XYZ$ after the translation $(x, y) \rightarrow (x + 4, y + 5)$.

CHAPTER 1

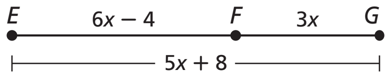
CHAPTER TEST

1. Draw and label plane $\mathcal{N}$ containing two lines that intersect at B.



Use the figure to name each of the following.

2. four noncoplanar points
3. line containing B and E
4. The coordinate of A is -3, and the coordinate of B is 0.5. Find AB.
5. E, F, and G represent mile markers along a straight highway. Find EF.



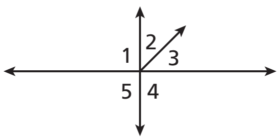
6. J is the midpoint of $\overline{HK}$. Find HJ, JK, and HK.

Classify each angle by its measure.

7. $m\angle LMP = 70°$
8. $m\angle QMN = 90°$
9. $m\angle PMN = 125°$
10. $\overrightarrow{TV}$ bisects $\angle RTS$. If the $m\angle RTV = (16x - 6)°$ and $m\angle VTS = (13x + 9)°$, what is the $m\angle RTV$?
11. An angle's measure is 5 degrees less than 3 times the measure of its supplement. Find the measure of the angle and its supplement.

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

12. $\angle 2$ and $\angle 3$
13. $\angle 4$ and $\angle 5$
14. $\angle 1$ and $\angle 4$



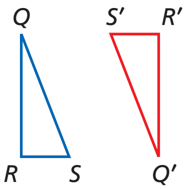
15. Find the perimeter and area of a rectangle with $b = 8$ ft and $h = 4$ ft.

Find the circumference and area of each circle to the nearest tenth.

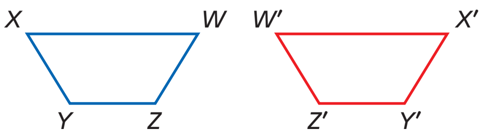
16. $r = 15$ m
17. $d = 25$ ft
18. $d = 2.8$ cm
19. Find the midpoint of the segment with endpoints $(-4, 6)$ and $(3, 2)$.
20. M is the midpoint of $\overline{LN}$. M has coordinates $(-5, 1)$, and L has coordinates $(2, 4)$. Find the coordinates of N.
21. Given $A(-5, 1)$, $B(-1, 3)$, $C(1, 4)$, and $D(4, 1)$, is $\overline{AB} \cong \overline{CD}$? Explain.

Identify each transformation. Then use arrow notation to describe the transformation.

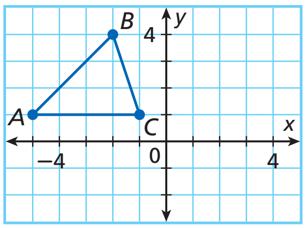
22.



23.



24. A designer used the translation $(x, y) \rightarrow (x + 3, y - 3)$ to transform a triangular-shaped pin ABC. Find the coordinates and draw the image of $\triangle ABC$.



College Entrance Exam Practice

FOCUS ON SAT

The SAT has three sections: Math, Critical Reading, and Writing. Your SAT scores show how you compare with other students. It can be used by colleges to determine admission and to award merit-based financial aid.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

HOT TIP! On SAT multiple-choice questions, you receive one point for each correct answer, but you lose a fraction of a point for each incorrect response. Guess only when you can eliminate at least one of the answer choices.

1. Points D, E, F, and G are on a line, in that order. If $DE = 2$, $FG = 5$, and $DF = 6$, what is the value of $EG(DG)$?

(A) 13

(B) 18

(C) 19

(D) 42

(E) 99

2. $\overrightarrow{QS}$ bisects $\angle PQR$, $m\angle PQR = (4x + 2)°$, and $m\angle SQR = (3x - 6)°$. What is the value of x?

(A) 1

(B) 4

(C) 7

(D) 10

(E) 19

3. A rectangular garden is enclosed by a brick border. The total length of bricks used to enclose the garden is 42 meters. If the length of the garden is twice the width, what is the area of the garden?

(A) 7 meters

(B) 14 meters

(C) 42 meters

(D) 42 square meters

(E) 98 square meters

4. What is the area of the square?

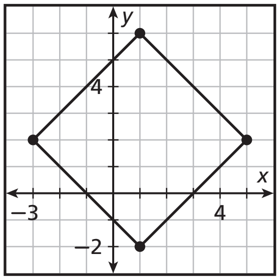
(A) 16

(B) 25

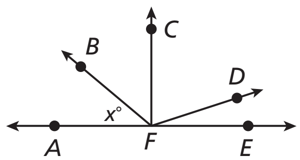
(C) 32

(D) 36

(E) 41



5. If $\angle BFD$ and $\angle AFC$ are right angles and $m\angle CFD = 72°$, what is the value of x?



Note: Figure not drawn to scale.

(A) 18

(B) 36

(C) 72

(D) 90

(E) 108

CHAPTER 1

STRATEGIES FOR SUCCESS

Multiple Choice: Work Backward

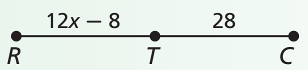
When you do not know how to solve a multiple-choice test item, use the answer choices and work the question backward. Plug in the answer choices to see which choice makes the question true.

EXAMPLE 1

T is the midpoint of $\overline{RC}$, $RT = 12x - 8$, and $TC = 28$. What is the value of x?

Ⓐ -4 Ⓒ 3

Ⓑ 2 Ⓓ 28



Since T is the midpoint of $\overline{RC}$, then $RT = RC$, or $12x - 8 = 28$.
Find what value of x makes the left side of the equation equal 28.

Try choice A: If $x = -4$, then $12x - 8 = 12(-4) - 8 = -56$.
This choice is not correct because length is always a positive number.

Try choice B: If $x = 2$, then $12x - 8 = 12(2) - 8 = 16$.
Since $16 \neq 28$, choice B is not the answer.

Try choice C: If $x = 3$, then $12x - 8 = 12(3) - 8 = 28$.

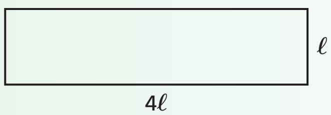
Since $28 = 28$, the correct answer is C, 3.

EXAMPLE 2

Joel used 6400 feet of fencing to make a rectangular horse pen. The width of the pen is 4 times as long as the length. What is the length of the horse pen?

Ⓕ 25 feet Ⓗ 640 feet

Ⓖ 480 feet Ⓙ 1600 feet



Use the formula $P = 2\ell + 2w$. $P = 6400$ and $w = 4\ell$. You can work backward to determine which answer choice is the most reasonable.

Try choice J: Use mental math. If $\ell = 1600$, then $4\ell = 6400$. This choice is not reasonable because the perimeter of the pen would then be far greater than 6400 feet.

Try choice F: Use mental math. If $\ell = 25$, then $4\ell = 100$. This choice is incorrect because the perimeter of the pen is 6400 ft, which is far greater than $2(25) + 2(100)$.

Try choice H: If $\ell = 640$, then $4\ell = 2560$. When you substitute these values into the perimeter formula, it makes a true statement.

The correct answer is H, 640 ft.

Read each test item and answer the questions that follow.

Item A

The measure of an angle is 3 times as great as that of its complement. Which value is the measure of the smaller angle?

(A) $22.5°$ (C) $63.5°$

(B) $27.5°$ (D) $67.5°$

1. Are there any definitions that you can use to solve this problem? If so, what are they?
2. Describe how to work backward to find the correct answer.

Item B

In a town's annual relay marathon race, the second runner of each team starts at mile marker 4 and runs to the halfway point of the 26-mile marathon. At that point the second runner passes the relay baton to the third runner of the team. How many total miles does the second runner of each team run?

(F) 4 miles (H) 9 miles

(G) 6.5 miles (J) 13 miles

3. Which answer choice should you plug in first? Why?
4. Describe, by working backward, how you know that choices F and G are not correct.

Item C

Consider the translation $(-2, 8) \rightarrow (8, -4)$. What number was added to the x-coordinate?

(A) -12 (C) 4

(B) -6 (D) 10

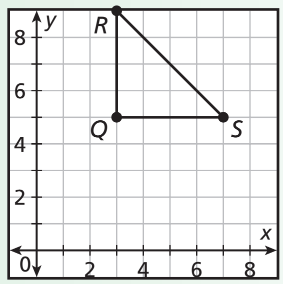
5. Which answer choice should you plug in first? Why?
6. Explain how to work the test question backward to determine the correct answer.

HOT TIP!

When you work a test question backward start with choice C. The choices are usually listed in order from least to greatest. If choice C is incorrect because it is too low, you do not need to plug in the smaller numbers.

Item D

$\triangle QRS$ has vertices at $Q(3, 5)$, $R(3, 9)$, and $S(7, 5)$. Which of these points is a vertex of the image of $\triangle QRS$ after the translation $(x, y) \rightarrow (x - 7, y - 6)$?



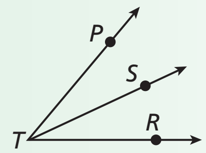
(F) $(-4, 3)$ (H) $(4, 1)$

(G) $(0, 0)$ (J) $(4, -3)$

7. Explain how to use mental math to find an answer that is NOT reasonable.
8. Describe, by working backward, how you can determine the correct answer.

Item E

$\overrightarrow{TS}$ bisects $\angle PTR$. If $m\angle PTS = (9x + 2)°$ and $m\angle STR = (x + 18)°$, what is the value of x?



(A) -10 (C) 2

(B) 0 (D) 20

9. Explain how to use mental math to find an answer that is NOT reasonable.
10. Describe how to use the answer choices to work backward to find which answer is reasonable.

CHAPTER 1

STANDARDIZED TEST PREP

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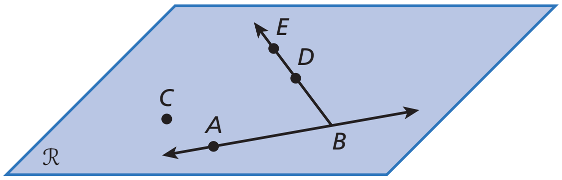
CUMULATIVE ASSESSMENT, CHAPTER 1

Multiple Choice

Use the diagram for Items 1–3.



1. Which points are collinear?
   - (A) *A*, *B*, and *C*
   - (B) *B*, *C*, and *D*
   - (C) *A*, *B*, and *E*
   - (D) *B*, *D*, and *E*

2. What is another name for plane ℛ?
   - (F) Plane 𝒞
   - (G) Plane *AB*
   - (H) Plane *ACE*
   - (J) Plane *BDE*

3. Use your protractor to find the approximate measure of $\angle ABD$.
   - (A) 123°
   - (B) 117°
   - (C) 77°
   - (D) 63°

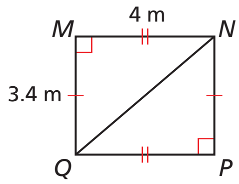
4. *S* is between *R* and *T*. The distance between *R* and *T* is 4 times the distance between *S* and *T*. If $RS = 18$, what is *RT*?
   - (F) 24
   - (G) 22.5
   - (H) 14.4
   - (J) 6

5. A ray bisects a straight angle into two congruent angles. Which term describes each of the congruent angles that are formed?
   - (A) Acute
   - (B) Obtuse
   - (C) Right
   - (D) Straight

6. Which expression states that $\overline{AB}$ is congruent to $\overline{CD}$?
   - (F) $AB \cong CD$
   - (G) $AB = CD$
   - (H) $\overline{AB} = \overline{CD}$
   - (J) $\overline{AB} \cong \overline{CD}$

7. The measure of an angle is 35°. What is the measure of its complement?
   - (A) 35°
   - (B) 45°
   - (C) 55°
   - (D) 145°

Use the diagram for Items 8–10.



8. Which of these angles is adjacent to $\angle MQN$?
   - (F) $\angle QMN$
   - (G) $\angle NPQ$
   - (H) $\angle QNP$
   - (J) $\angle PQN$

9. What is the area of $\triangle NQP$?
   - (A) 3.7 square meters
   - (B) 6.8 square meters
   - (C) 7.4 square meters
   - (D) 13.6 square meters

10. Which of the following pairs of angles are complementary?
    - (F) $\angle MNQ$ and $\angle QNP$
    - (G) $\angle NQP$ and $\angle QPN$
    - (H) $\angle MNP$ and $\angle QNP$
    - (J) $\angle QMN$ and $\angle NPQ$

11. *K* is the midpoint of $\overline{JL}$. *J* has coordinates $(2, -1)$, and *K* has coordinates $(-4, 3)$. What are the coordinates of *L*?
    - (A) $(3, -2)$
    - (B) $(1, -1)$
    - (C) $(-1, 1)$
    - (D) $(-10, 7)$

12. A circle with a diameter of 10 inches has a circumference equal to the perimeter of a square. To the nearest tenth, what is the length of each side of the square?
    - (F) 2.5 inches
    - (G) 3.9 inches
    - (H) 5.6 inches
    - (J) 7.9 inches

13. The map coordinates of a campground are $(1, 4)$, and the coordinates of a fishing pier are $(4, 7)$. Each unit on the map represents 1 kilometer. If Alejandro walks in a straight line from the campground to the pier, how many kilometers, to the nearest tenth, will he walk?
    - (A) 3.5 kilometers
    - (B) 4.2 kilometers
    - (C) 6.0 kilometers
    - (D) 12.1 kilometers

HOT TIP!

For many types of geometry problems, it may be helpful to draw a diagram and label it with the information given in the problem. This method is a good way of organizing the information and helping you decide how to solve the problem.

14. m∠R is 57°. What is the measure of its supplement?

Ⓕ 33° Ⓗ 123°

Ⓖ 43° Ⓙ 133°

15. What rule would you use to translate a triangle 4 units to the right?

Ⓐ $(x, y) \rightarrow (x + 4, y)$

Ⓑ $(x, y) \rightarrow (x - 4, y)$

Ⓒ $(x, y) \rightarrow (x, y + 4)$

Ⓓ $(x, y) \rightarrow (x, y - 4)$

16. If $\overline{WZ}$ bisects ∠XWY, which of the following statements is true?

Ⓕ $m\angle XWZ > m\angle YWZ$

Ⓖ $m\angle XWZ < m\angle YWZ$

Ⓗ $m\angle XWZ = m\angle YWZ$

Ⓙ $m\angle XWZ \cong m\angle YWZ$

17. The x- and y-axes separate the coordinate plane into four regions, called quadrants. If (c, d) is a point that is not on the axes, such that $c < 0$ and $d < 0$, which quadrant would contain point (c, d)?

Ⓐ I Ⓒ III

Ⓑ II Ⓓ IV

Gridded Response

18. The measure of ∠1 is 4 times the measure of its supplement. What is the measure, in degrees, of ∠1?

19. The exits for Market St. and Finch St. are 3.5 miles apart on a straight highway. The exit for King St. is at the midpoint between these two exits. How many miles apart are the King St. and Finch St. exits?

20. R has coordinates $(-4, 9)$. S has coordinates $(4, -6)$. What is RS?

21. If ∠A is a supplement of ∠B and is a right angle, then what is m∠B in degrees?

22. ∠C and ∠D are complementary. m∠C is 4 times m∠D. What is m∠C?

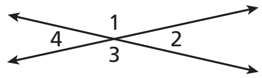
Short Response

23. △ABC has vertices $A(-2, 0)$, $B(0, 0)$, and $C(0, 3)$. The image of △ABC has vertices $A'(1, -4)$, $B'(3, -4)$, and $C'(3, -1)$.

a. Draw △ABC and its image △$A'B'C'$ on a coordinate plane.

b. Write a rule for the transformation of △ABC using arrow notation.

24. You are given the measure of ∠4. You also know the following angles are supplementary: ∠1 and ∠2, ∠2 and ∠3, and ∠1 and ∠4.

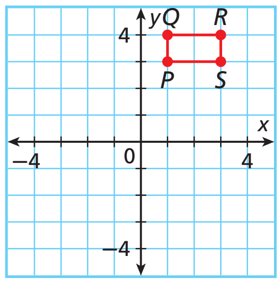


Explain how you can determine the measures of ∠1, ∠2, and ∠3.

25. Marian is making a circular tablecloth from a rectangular piece of fabric that measures 6 yards by 4 yards. What is the area of the largest circular piece that can be cut from the fabric? Leave your answer in terms of π. Show your work or explain in words how you found your answer.

Extended Response

26. Demara is creating a design using a computer illustration program. She begins by drawing the rectangle shown on the coordinate grid.



a. Demara translates rectangle $PQRS$ using the rule $(x, y) \rightarrow (x - 4, y - 6)$. On a copy of the coordinate grid, draw this translation and label each vertex.

b. Describe one way that Demara could have moved rectangle $PQRS$ to the same position in part **a** using a reflection and then a translation.

c. On the same coordinate grid, Demara reflects rectangle $PQRS$ across the x-axis. She draws a figure with vertices at $(1, -3)$, $(3, -3)$, $(3, -5)$, and $(1, -5)$. Did Demara reflect rectangle $PQRS$ correctly? Explain your answer.