11-2 Exponential Functions

Objectives

Evaluate exponential functions.

Identify and graph exponential functions.

11-2 Exponential Functions

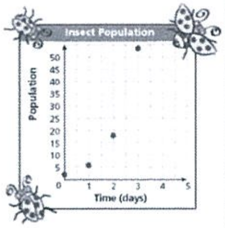
Vocabulary

Exponential function

11-2 Exponential Functions

The table and the graph show an insect population that increases over time.

Time (days)	Population	
0	2	
1	6	× 3
2	18	× 3
3	54	× 3



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A function rule that describes the pattern above is $f(x) = 2(3)^x$. This type of function, in which the independent variable appears in an exponent, is an **exponential function**. Notice that 2 is the starting population and 3 is the amount by which the population is multiplied each day.

Exponential Functions

An exponential function has the form $f(x) = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$.

11-2 Exponential Functions

Example 1A: Evaluating an Exponential Function

The function $f(x) = 500(1.035)^x$ models the amount of money in a certificate of deposit after x years. How much money will there be in 6 years?

$f(x) = 500(1.035)^x$ *Write the function.*

$f(6) = 500(1.035)^6$ *Substitute 6 for x.*

$= 500(1.229)$ *Evaluate 1.035^6.*

$= 614.63$ *Multiply.*

There will be $614.63 in 6 years.

11-2 Exponential Functions

Example 1B: Evaluating an Exponential Function

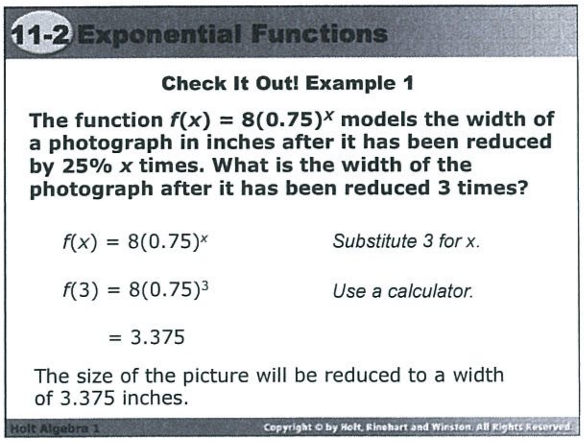
The function $f(x) = 200{,}000(0.98)^x$, where x is the time in years, models the population of a city. What will the population be in 7 years?

$f(x) = 200{,}000(0.98)^x$ *Substitute 7 for x.*

$f(7) = 200{,}000(0.98)^7$ *Use a calculator. Round to the nearest whole number.*

$\approx 173{,}625$

The population will be about 173,625 in 7 years.



11-2 Exponential Functions

Check It Out! Example 1

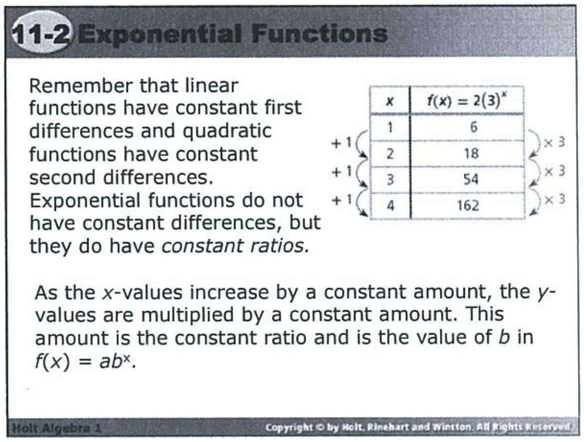
The function $f(x) = 8(0.75)^x$ models the width of a photograph in inches after it has been reduced by 25% x times. What is the width of the photograph after it has been reduced 3 times?

$f(x) = 8(0.75)^x$ *Substitute 3 for x.*

$f(3) = 8(0.75)^3$ *Use a calculator.*

$= 3.375$

The size of the picture will be reduced to a width of 3.375 inches.

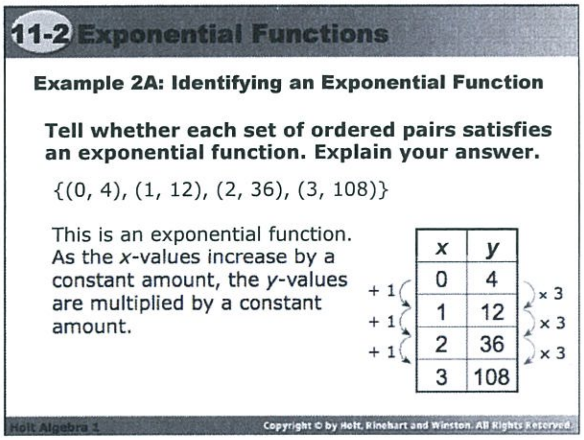


11-2 Exponential Functions

Remember that linear functions have constant first differences and quadratic functions have constant second differences. Exponential functions do not have constant differences, but they do have *constant ratios.*

	x	$f(x) = 2(3)^x$	
+ 1	1	6	× 3
+ 1	2	18	× 3
+ 1	3	54	× 3
	4	162	

As the x-values increase by a constant amount, the y-values are multiplied by a constant amount. This amount is the constant ratio and is the value of b in $f(x) = ab^x$.



11-2 Exponential Functions

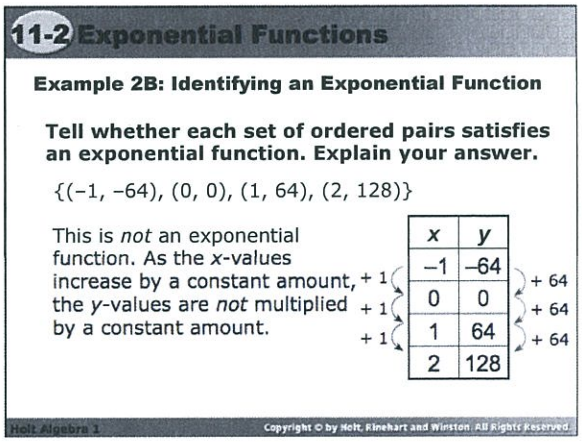
Example 2A: Identifying an Exponential Function

Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

{(0, 4), (1, 12), (2, 36), (3, 108)}

This is an exponential function. As the x-values increase by a constant amount, the y-values are multiplied by a constant amount.

	x	y	
+ 1	0	4	× 3
+ 1	1	12	× 3
+ 1	2	36	× 3
	3	108	



11-2 Exponential Functions

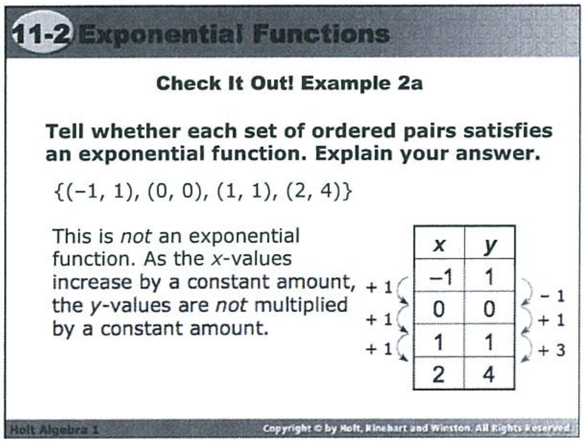
Example 2B: Identifying an Exponential Function

Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

{(–1, –64), (0, 0), (1, 64), (2, 128)}

This is *not* an exponential function. As the x-values increase by a constant amount, the y-values are *not* multiplied by a constant amount.

	x	y	
+ 1	–1	–64	+ 64
+ 1	0	0	+ 64
+ 1	1	64	+ 64
	2	128	



11-2 Exponential Functions

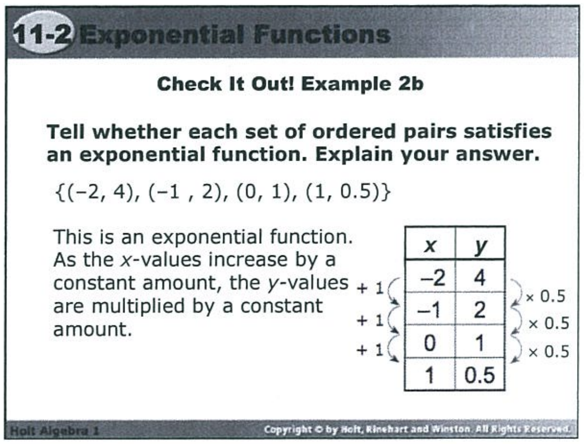
Check It Out! Example 2a

Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

{(–1, 1), (0, 0), (1, 1), (2, 4)}

This is *not* an exponential function. As the x-values increase by a constant amount, the y-values are *not* multiplied by a constant amount.

	x	y	
+ 1	–1	1	– 1
+ 1	0	0	+ 1
+ 1	1	1	+ 3
	2	4	



11-2 Exponential Functions

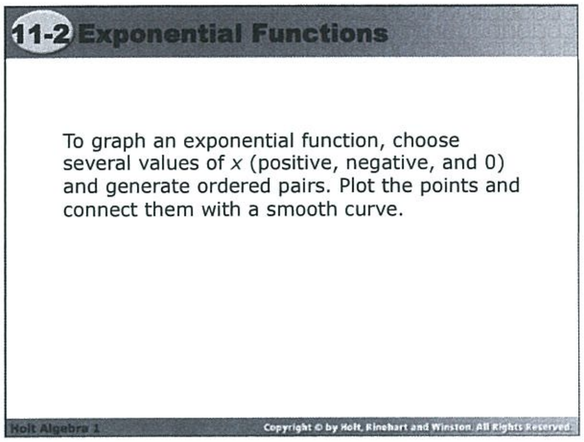
Check It Out! Example 2b

Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

{(–2, 4), (–1 , 2), (0, 1), (1, 0.5)}

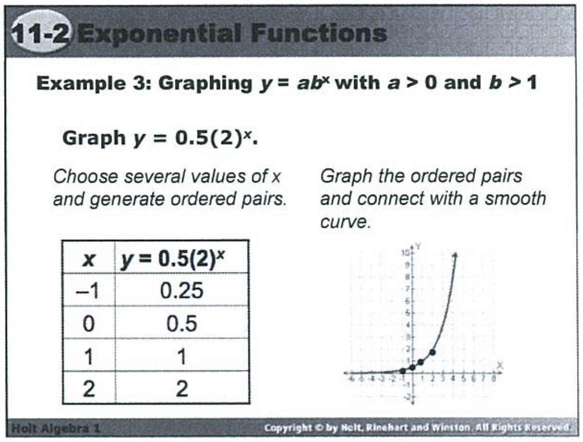
This is an exponential function. As the x-values increase by a constant amount, the y-values are multiplied by a constant amount.

	x	y	
+ 1	–2	4	× 0.5
+ 1	–1	2	× 0.5
+ 1	0	1	× 0.5
	1	0.5	



11-2 Exponential Functions

To graph an exponential function, choose several values of x (positive, negative, and 0) and generate ordered pairs. Plot the points and connect them with a smooth curve.



11-2 Exponential Functions

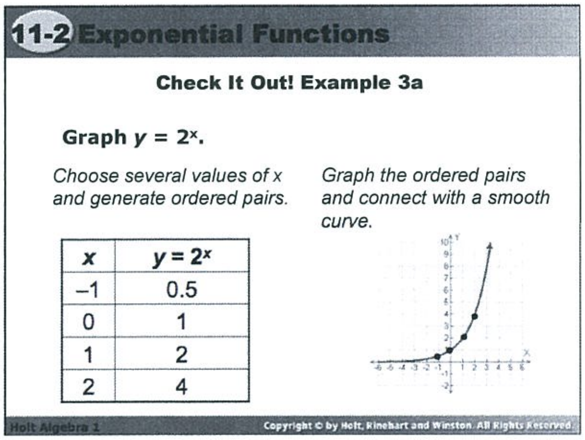
Example 3: Graphing $y = ab^x$ with $a > 0$ and $b > 1$

Graph $y = 0.5(2)^x$.

Choose several values of x and generate ordered pairs.

x	$y = 0.5(2)^x$
–1	0.25
0	0.5
1	1
2	2

Graph the ordered pairs and connect with a smooth curve.



11-2 Exponential Functions

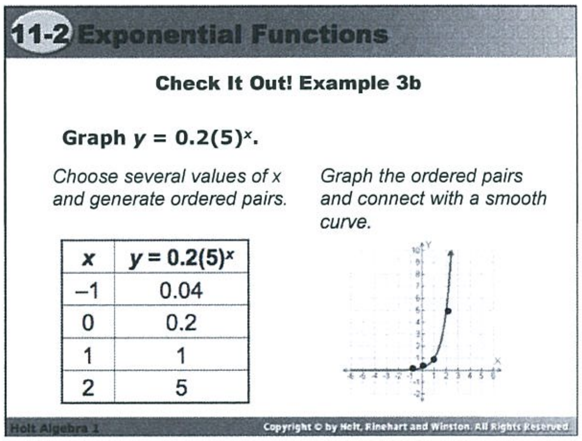
Check It Out! Example 3a

Graph $y = 2^x$.

Choose several values of x and generate ordered pairs.

x	$y = 2^x$
–1	0.5
0	1
1	2
2	4

Graph the ordered pairs and connect with a smooth curve.



11-2 Exponential Functions

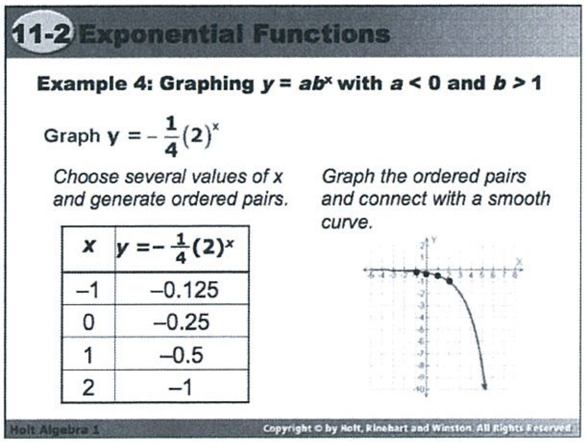
Check It Out! Example 3b

Graph $y = 0.2(5)^x$.

Choose several values of x and generate ordered pairs.

x	$y = 0.2(5)^x$
–1	0.04
0	0.2
1	1
2	5

Graph the ordered pairs and connect with a smooth curve.



11-2 Exponential Functions

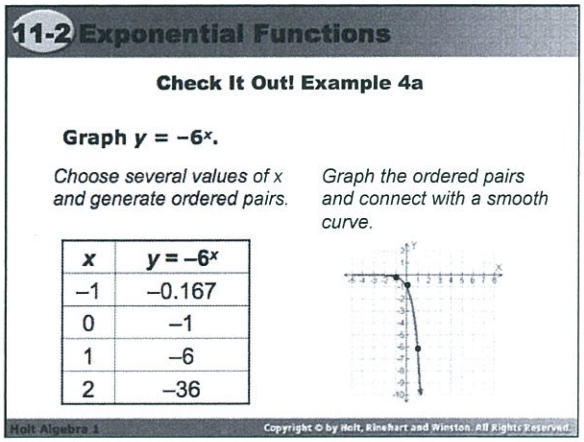
Example 4: Graphing $y = ab^x$ with $a < 0$ and $b > 1$

Graph $y = -\frac{1}{4}(2)^x$

Choose several values of x and generate ordered pairs.

x	$y = -\frac{1}{4}(2)^x$
–1	–0.125
0	–0.25
1	–0.5
2	–1

Graph the ordered pairs and connect with a smooth curve.



11-2 Exponential Functions

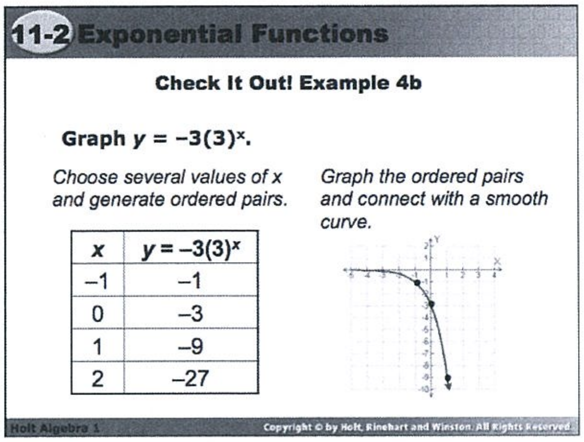
Check It Out! Example 4a

Graph $y = -6^x$.

Choose several values of x and generate ordered pairs.

x	$y = -6^x$
–1	–0.167
0	–1
1	–6
2	–36

Graph the ordered pairs and connect with a smooth curve.



11-2 Exponential Functions

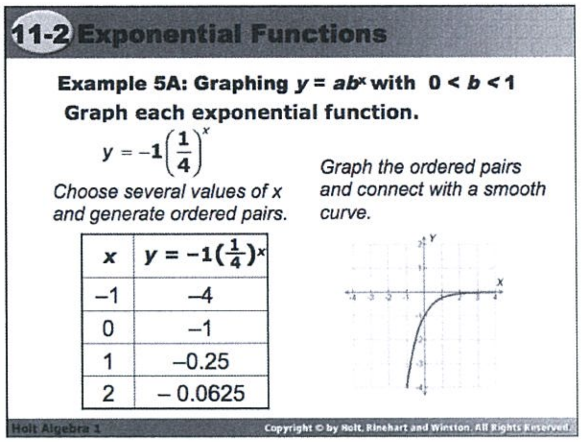
Check It Out! Example 4b

Graph $y = -3(3)^x$.

Choose several values of x and generate ordered pairs.

x	$y = -3(3)^x$
–1	–1
0	–3
1	–9
2	–27

Graph the ordered pairs and connect with a smooth curve.



11-2 Exponential Functions

Example 5A: Graphing $y = ab^x$ with $0 < b < 1$

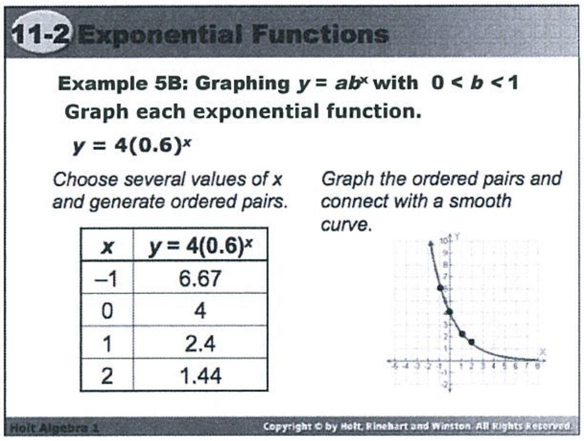
Graph each exponential function.

$y = -1\left(\frac{1}{4}\right)^x$

Choose several values of x and generate ordered pairs.

x	$y = -1\left(\frac{1}{4}\right)^x$
–1	–4
0	–1
1	–0.25
2	– 0.0625

Graph the ordered pairs and connect with a smooth curve.



11-2 Exponential Functions

Example 5B: Graphing $y = ab^x$ with $0 < b < 1$

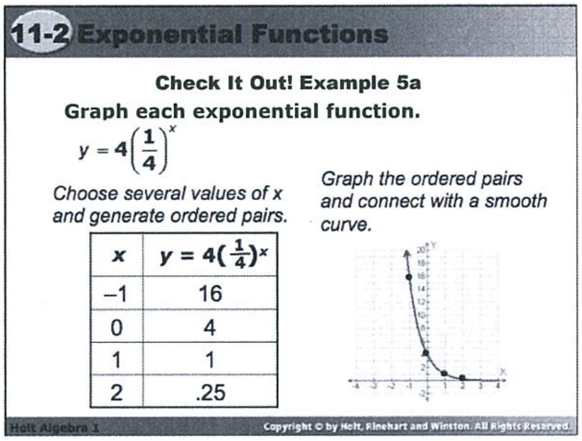
Graph each exponential function.

$y = 4(0.6)^x$

Choose several values of x and generate ordered pairs.

x	$y = 4(0.6)^x$
–1	6.67
0	4
1	2.4
2	1.44

Graph the ordered pairs and connect with a smooth curve.



11-2 Exponential Functions

Check It Out! Example 5a

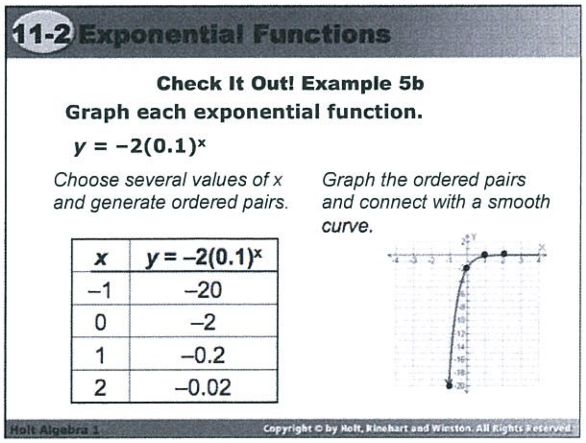
Graph each exponential function.

$y = 4\left(\frac{1}{4}\right)^x$

Choose several values of x and generate ordered pairs.

x	$y = 4\left(\frac{1}{4}\right)^x$
–1	16
0	4
1	1
2	.25

Graph the ordered pairs and connect with a smooth curve.



11-2 Exponential Functions

Check It Out! Example 5b

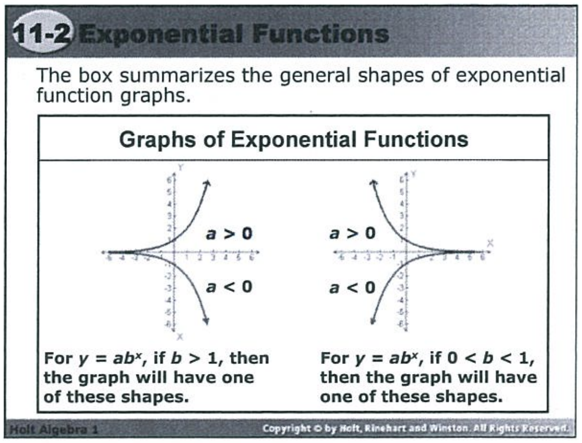
Graph each exponential function.

$y = -2(0.1)^x$

Choose several values of x and generate ordered pairs.

x	$y = -2(0.1)^x$
–1	–20
0	–2
1	–0.2
2	–0.02

Graph the ordered pairs and connect with a smooth curve.



11-2 Exponential Functions

The box summarizes the general shapes of exponential function graphs.

Graphs of Exponential Functions

For $y = ab^x$, if $b > 1$, then the graph will have one of these shapes.

For $y = ab^x$, if $0 < b < 1$, then the graph will have one of these shapes.

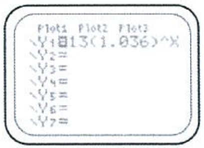
11-2 Exponential Functions

Example 6: *Application*

In 2000, each person in India consumed an average of 13 kg of sugar. Sugar consumption in India is projected to increase by 3.6% per year. At this growth rate the function $f(x) = 13(1.036)^x$ gives the average yearly amount of sugar, in kilograms, consumed per person x years after 2000. Using this model, in about what year will sugar consumption average about 18 kg per person?

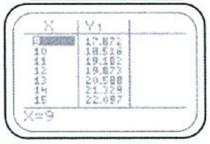
11-2 Exponential Functions

Example 6 Continued



```
Plot1 Plot2 Plot3
\Y1■13(1.036)^X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Enter the function into the Y = editor of a graphing calculator.



```
  X    | Y1     |
9      | 17.872 |
10     | 18.516 |
11     | 19.182 |
12     | 19.873 |
13     | 20.588 |
14     | 21.329 |
15     | 22.097 |
X=9
```

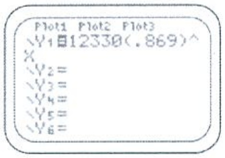
Press 2nd $\overset{\text{TABLE}}{\text{GRAPH}}$. *Use the arrow keys to find a y-value as close to 18 as possible. The corresponding x-value is 9.*

The average consumption will reach 18kg in 2009.

11-2 Exponential Functions

Check It Out! Example 6

An accountant uses $f(x) = 12{,}330(0.869)^x$, where x is the time in years since the purchase, to model the value of a car. When will the car be worth $2000?



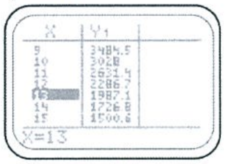
```
Plot1 Plot2 Plot3
\Y1■12330(.869)^
X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

Enter the function into the Y = editor of a graphing calculator.

11-2 Exponential Functions

Check It Out! Example 6 Continued

An accountant uses $f(x) = 12{,}330(0.869)^x$, is the time in years since the purchase, to model the value of a car. When will the car be worth $2000?



```
  X    | Y1     |
9      | 3484.5 |
10     | 3028   |
11     | 2631.4 |
12     | 2286.7 |
13     | 1987.1 |
14     | 1726.8 |
15     | 1500.6 |
X=13
```

Press 2nd $\overset{\text{TABLE}}{\text{GRAPH}}$. *Use the arrow keys to find a y-value as close to 2000 as possible. The corresponding x-value is 13.*

The value of the car will reach $2000 in about 13 years.

11-2 Exponential Functions

Lesson Quiz: Part I

Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

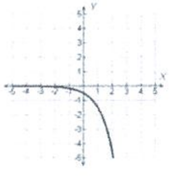
1. {(0, 0), (1, –2), (2, –16), (3, –54)}
No; for a constant change in *x*, *y* is not multiplied by the same value.

2. {(0,–5), (1, –2.5), (2, –1.25), (3, –0.625)}
Yes; for a constant change in *x*, *y* is multiplied by the same value.

11-2 Exponential Functions

Lesson Quiz: Part II

3. Graph $y = -0.5(3)^x$.



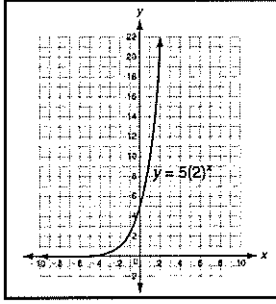
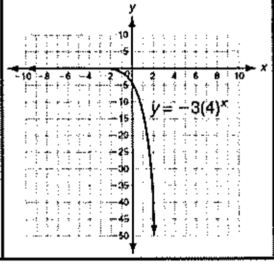
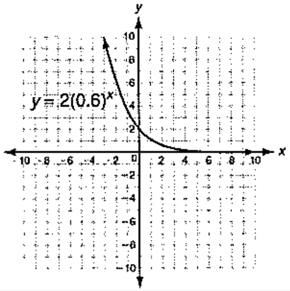
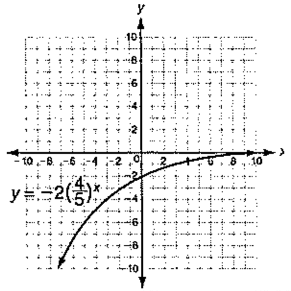
Name ______________________________ Date ___________ Class ___________

LESSON 11-2

Reteach

Exponential Functions (continued)

The graph of an exponential function is always a curve in two quadrants. $y = ab^x$

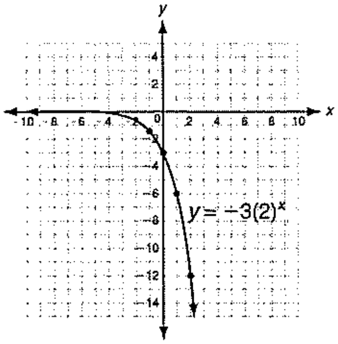
$a > 0$ and $b > 1$	$a < 0$ and $b > 1$	$a > 0$ and $0 < b < 1$	$a < 0$ and $0 < b < 1$
			

Graph $y = -3(2)^x$.

Create a table of ordered pairs. Plot the points.

Because $a < 0$ and $b > 1$, this graph should look similar to the second graph above.

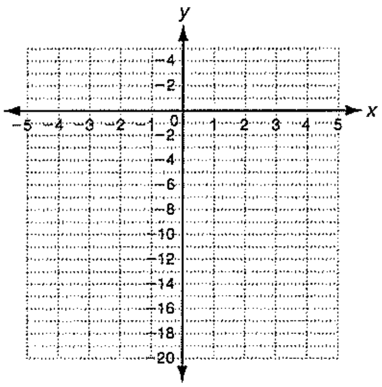
x	$y = -3(2)^x$	y
−1	$y = -3(2)^{-1}$	−1.5
0	$y = -3(2)^0$	−3
1	$y = -3(2)^1$	−6
2	$y = -3(2)^2$	−12



Graph each exponential function.

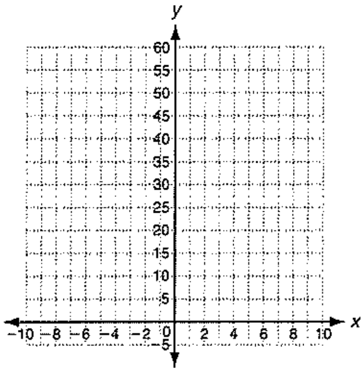
6. $y = -4(0.5)^x$

x	$y = -4(0.5)^x$	y
−2		
−1		
0		
1		



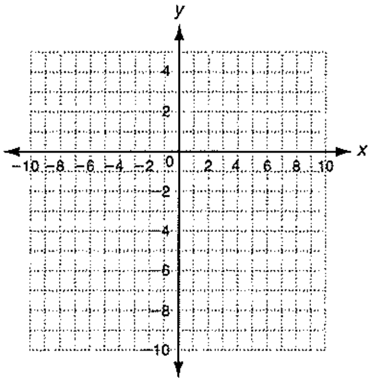
7. $y = 2(5)^x$

x	$y = 2(5)^x$	y
−1		
0		
1		
2		



8. $y = -1(2)^x$

x	$y = -1(2)^x$	y
−1		
0		
1		
2		



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LESSON **11-2**

Reteach

Exponential Functions

An **exponential function** has the independent variable as the exponent.

$y = 3^x$ and $y = -2(0.5)^x$ are exponential functions.

A set of ordered pairs satisfies an exponential function if the *y*-values are multiplied by a constant amount as the *x*-values change by a constant amount.

Tell whether the following ordered pairs satisfy an exponential function.

x	y
3	4
5	12
7	36
9	108

Think $4 \times ? = 12$.
Think $12 \times ? = 36$.
Think $36 \times ? = 108$.

The *x*-values increase by the constant amount 2.

Each *y*-value is multiplied by the constant amount 3.

This is an exponential function.

x	y
1	2
2	4
3	6
4	8

Think $2 \times ? = 4$.
Think $4 \times ? = 6$.
Think $6 \times ? = 8$.

The *x*-values increase by the constant amount 1.

The *y*-value is multiplied by 2, then 1.5, then$1.\overline{3}$. There is no constant ratio.

This is not an exponential function.

The population of a school can be described by the function $f(x) = 1500(1.02)^x$, where x represents the number of years since the school was built. What will be the population of the school in 12 years?

$f(x) = 1500(1.02)^x$

$f(12) = 1500(1.02)^{12}$ *Substitute 12 for x.*

≈ 1902 *Round number of people to the nearest whole number.*

Tell whether the ordered pairs satisfy an exponential function.

1.

x	y
−1	1.5
−2	3
−3	6
−4	12

____

2.

x	y
1	1
2	2
3	6
4	24

____

3.

x	y
−2	−2
−1	−10
0	−50
1	−250

____

4. If a rubber ball is dropped from a height of 10 feet, the function $f(x) = 20(0.6)^x$ gives the height in feet of each bounce, where x is the bounce number. What will be the height of the 5th bounce? Round to the nearest tenth of a foot. _______________

5. A population of pigs is expected to increase at a rate of 4% each year. If the original population is 1000, the function $f(x) = 1000(1.04)^x$ gives the population in x years. What will be the population in 12 years? _______________

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LESSON **11-2**

Practice A

Exponential Functions

1. If a superball is bounced from a height of 20 feet, the function $f(x) = 20(0.9)^x$ gives the height of the ball in feet of each bounce, where x is the bounce number. What will be the height of the 6th bounce? Round your answer to the nearest tenth of a foot. ____________

Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

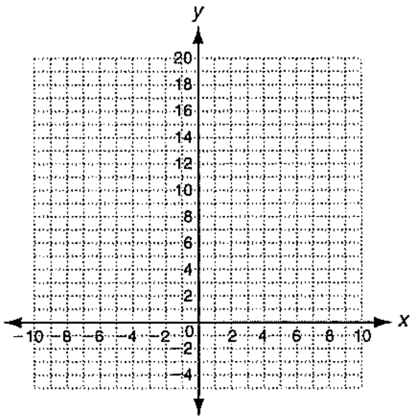
2. {(1, 10), (2, 20), (3, 40), (4, 80)} ______________________

3. {(1, 5), (2, 10), (3, 15), (4, 20)} ______________________

Graph each exponential function.

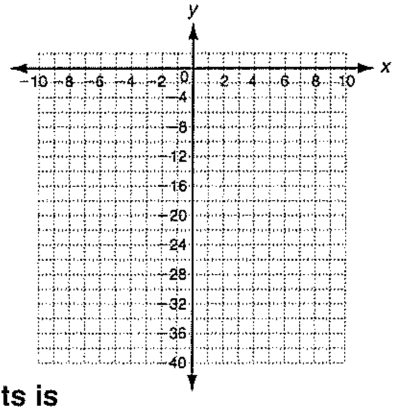
4. $y = 2(3)^x$

x	$y = 2(3)^x$	y	(x, y)
−2	$y = 2(3)^{-2}$		
−1	$y = 2(3)^{-1}$		
0	$y = 2(3)^0$		
1	$y = 2(3)^1$		
2	$y = 2(3)^2$		



5. $y = -2(4)^x$

x	$y = -2(4)^x$	y	(x, y)
−2			
−1			
0			
1			
2			



In the absence of predators, the natural growth rate of rabbits is 4% per year. A population begins with 100 rabbits. The function $f(x) = 100(1.04)^x$ gives the population of rabbits in x years.

6. How long will it take the population of rabbits to double? ____________

7. How long will it take the population of rabbits to reach 1000? ____________

Name ______________________ Date __________ Class __________

LESSON 11-2

Practice B

Exponential Functions

1. If a basketball is bounced from a height of 15 feet, the function $f(x) = 15(0.75)^x$ gives the height of the ball in feet of each bounce, where x is the bounce number. What will be the height of the 5th bounce? Round to the nearest tenth of a foot. ______________

Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

2. $\{(2, 4), (4, 8), (6, 16), (8, 32)\}$ ______________

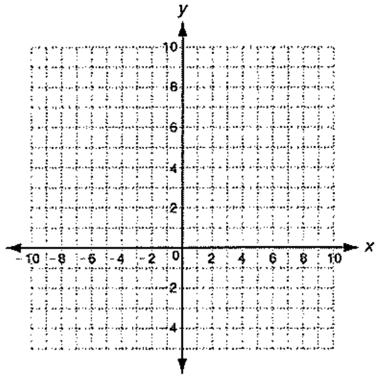
3. $\{(-2, 5), (-1, 10), (0, 15), (1, 20)\}$ ______________

4. $\{(1, 750), (2, 150), (3, 30), (4, 6)\}$ ______________

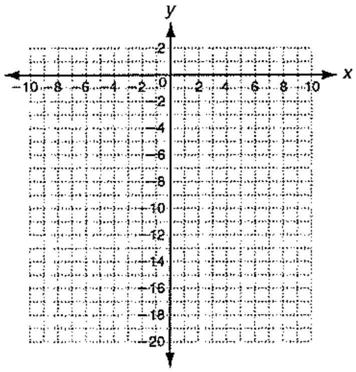
5. $\left\{\left(-5, \frac{1}{3}\right), (0, 1), (5, 3), (10, 9)\right\}$ ______________

Graph each exponential function.

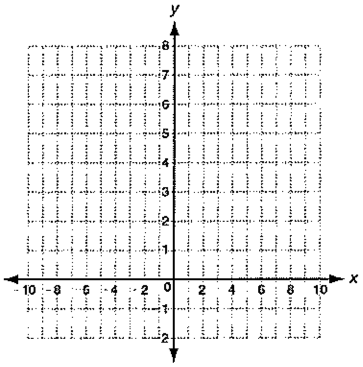
6. $y = 5(2)^x$



7. $y = -2(3)^x$



8. $y = 3\left(\frac{1}{2}\right)^x$



In the year 2000, the population of Virginia was about 7,400,000. Between the years 2000 and 2004, the population in Virginia grew at a rate of 5.4%. At this growth rate, the function $f(x) = 7{,}400{,}000(1.054)^x$ gives the population x years after 2000.

9. In what year will the population reach 15,000,000? ______________

10. In what year will the population reach 20,000,000? ______________

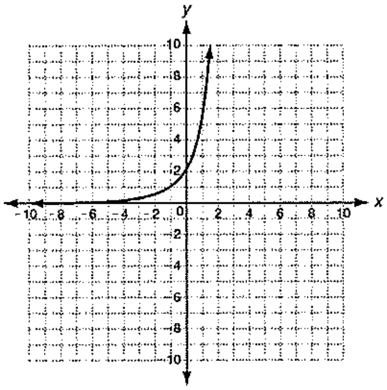
Name ______________________________ Date ____________ Class ____________

LESSON 11-2

Reading Strategies

Analyzing Information

An *exponential function* is always in the form $f(x) = ab^x$. And the graph of an exponential function is always the same general shape. But the values of a and b determine the quadrants the graph is in, and the "direction" in which the graph is facing. Analyze the information in the examples below.



$y = 2(3)^x$

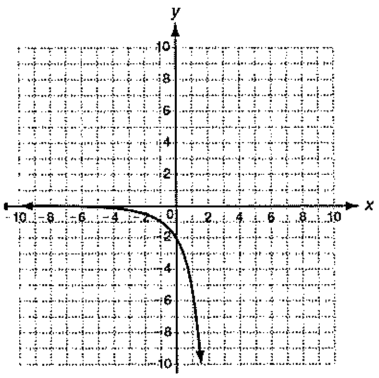
$a > 0$

$b > 1$

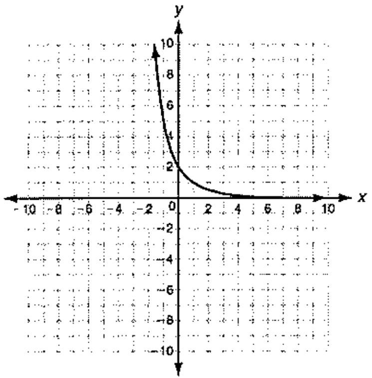
$y = -2(3)^x$

$a < 0$

$b > 1$



$y = ab^x$



$a > 0$

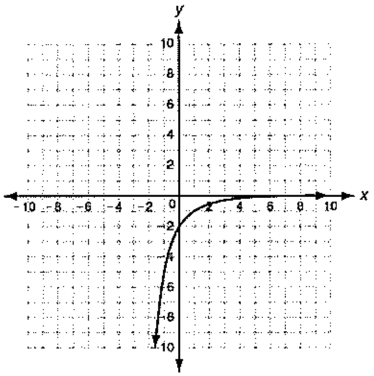
$0 < b < 1$

$y = 2\left(\frac{1}{3}\right)^x$

$a < 0$

$0 < b < 1$

$y = -2\left(\frac{1}{3}\right)^x$



Respond to each statement by writing *sometimes*, *always*, or *never*.

1. When a is negative, the graph of $y = ab^x$ is contained entirely in quadrants III and IV.

____

2. When b is a fraction, the graph of $y = ab^x$ is contained entirely in quadrants I and II.

____

3. The y-intercept of $y = ab^x$ is at a.

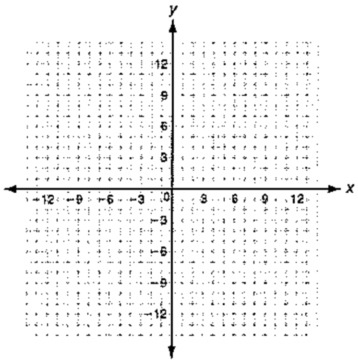
____

4. The graph of $y = ab^x$ is a straight line.

____

5. Complete the table of values and sketch the graph of $y = -3(2)^x$.

x	y
−1	
0	
1	
2	



11-3 Exponential Growth and Decay

Objective

Solve problems involving exponential growth and decay.

11-3 Exponential Growth and Decay

Vocabulary

exponential growth
compound interest
exponential decay
half-life

11-3 Exponential Growth and Decay

Exponential growth occurs when an quantity increases by the same rate r in each period t. When this happens, the value of the quantity at any given time can be calculated as a function of the rate and the original amount.

Exponential Growth

An exponential growth function has the form $y = a(1 + r)^t$, where $a > 0$.

y represents the final amount.
a represents the original amount.
r represents the rate of growth expressed as a decimal.
t represents time.

11-3 Exponential Growth and Decay

Example 1: Exponential Growth

The original value of a painting is $9,000 and the value increases by 7% each year. Write an exponential growth function to model this situation. Then find the painting's value in 15 years.

Step 1 Write the exponential growth function for this situation.

$y = a(1 + r)^t$ *Write the formula.*

$= 9000(1 + 0.07)^t$ *Substitute 9000 for a and 0.07 for r.*

$= 9000(1.07)^t$ *Simplify.*

11-3 Exponential Growth and Decay

Example 1 Continued

The original value of a painting is $9,000 and the value increases by 7% each year. Write an exponential growth function to model this situation. Then find the painting's value in 15 years.

Step 2 Find the value in 15 years.

$y = 9000(1.07)^t$

$= 9000(1 + 0.07)^{15}$ *Substitute 15 for t.*

$\approx 24{,}831.28$ *Use a calculator and round to the nearest hundredth.*

The value of the painting in 15 years is $24,831.28.

11-3 Exponential Growth and Decay

Check It Out! Example 1

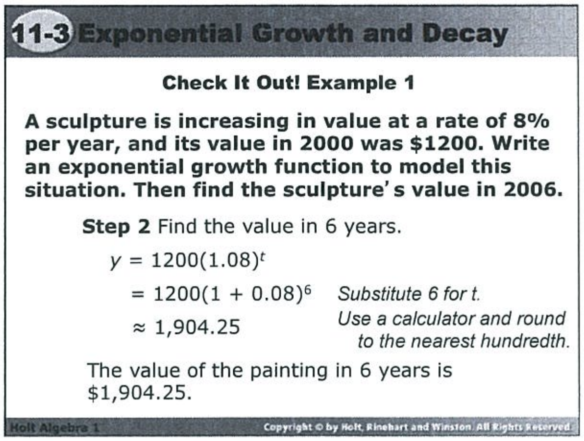
A sculpture is increasing in value at a rate of 8% per year, and its value in 2000 was $1200. Write an exponential growth function to model this situation. Then find the sculpture's value in 2006.

Step 1 Write the exponential growth function for this situation.

$y = a(1 + r)^t$ *Write the formula*

$= 1200(1 + 0.08)^6$ *Substitute 1200 for a and 0.08 for r.*

$= 1200(1.08)^t$ *Simplify.*



11-3 Exponential Growth and Decay

Check It Out! Example 1

A sculpture is increasing in value at a rate of 8% per year, and its value in 2000 was $1200. Write an exponential growth function to model this situation. Then find the sculpture's value in 2006.

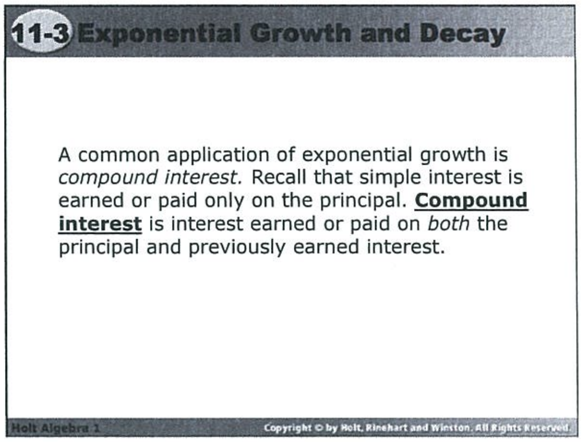
Step 2 Find the value in 6 years.

$y = 1200(1.08)^t$

$= 1200(1 + 0.08)^6$ *Substitute 6 for t.*

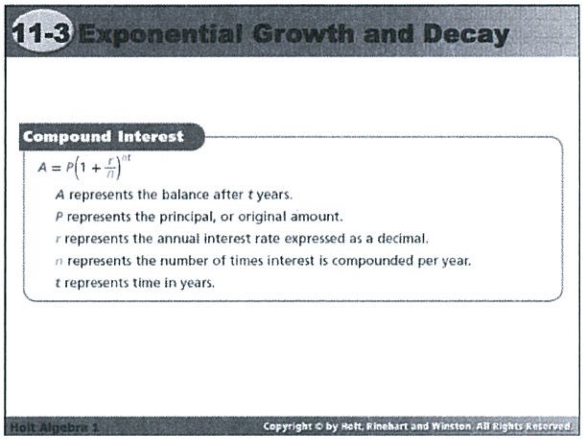
$\approx 1{,}904.25$ *Use a calculator and round to the nearest hundredth.*

The value of the painting in 6 years is $1,904.25.



11-3 Exponential Growth and Decay

A common application of exponential growth is *compound interest.* Recall that simple interest is earned or paid only on the principal. **Compound interest** is interest earned or paid on *both* the principal and previously earned interest.



11-3 Exponential Growth and Decay

Compound Interest

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

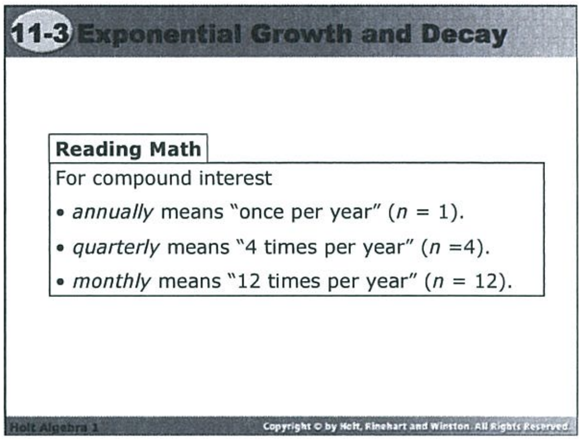
A represents the balance after *t* years.

P represents the principal, or original amount.

r represents the annual interest rate expressed as a decimal.

n represents the number of times interest is compounded per year.

t represents time in years.

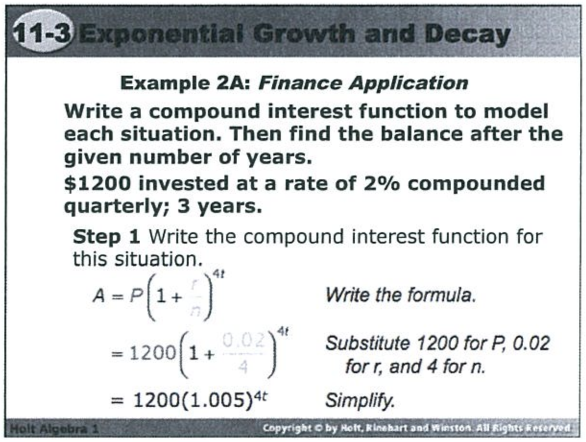


11-3 Exponential Growth and Decay

Reading Math

For compound interest

- *annually* means "once per year" ($n = 1$).
- *quarterly* means "4 times per year" ($n = 4$).
- *monthly* means "12 times per year" ($n = 12$).



11-3 Exponential Growth and Decay

Example 2A: *Finance Application*

Write a compound interest function to model each situation. Then find the balance after the given number of years.

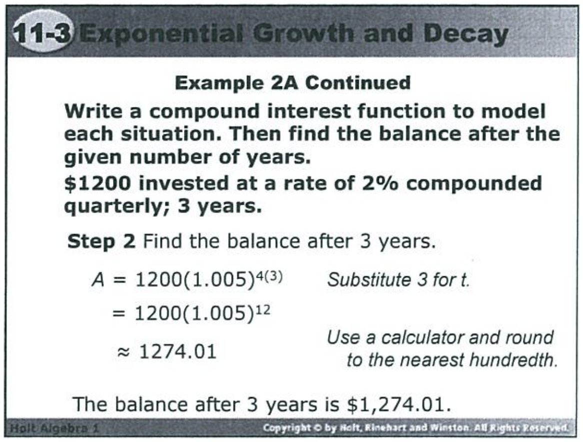
$1200 invested at a rate of 2% compounded quarterly; 3 years.

Step 1 Write the compound interest function for this situation.

$A = P\left(1 + \frac{r}{n}\right)^{4t}$ *Write the formula.*

$= 1200\left(1 + \frac{0.02}{4}\right)^{4t}$ *Substitute 1200 for P, 0.02 for r, and 4 for n.*

$= 1200(1.005)^{4t}$ *Simplify.*



11-3 Exponential Growth and Decay

Example 2A Continued

Write a compound interest function to model each situation. Then find the balance after the given number of years.

$1200 invested at a rate of 2% compounded quarterly; 3 years.

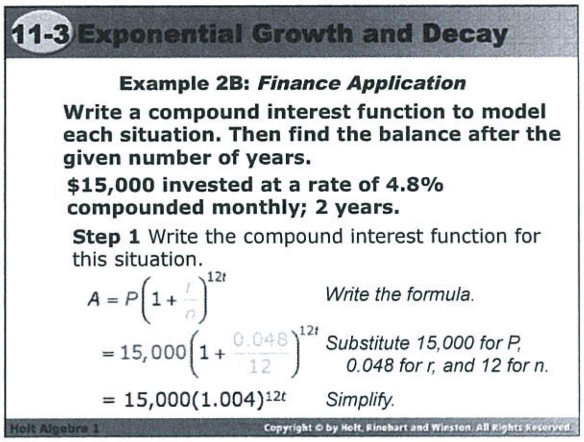
Step 2 Find the balance after 3 years.

$A = 1200(1.005)^{4(3)}$ *Substitute 3 for t.*

$= 1200(1.005)^{12}$

≈ 1274.01 *Use a calculator and round to the nearest hundredth.*

The balance after 3 years is $1,274.01.



11-3 Exponential Growth and Decay

Example 2B: *Finance Application*

Write a compound interest function to model each situation. Then find the balance after the given number of years.

$15,000 invested at a rate of 4.8% compounded monthly; 2 years.

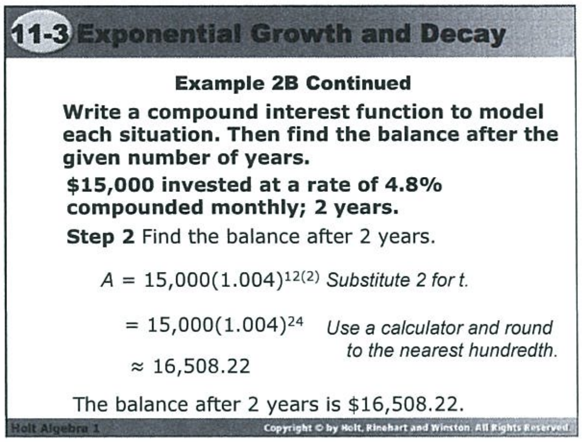
Step 1 Write the compound interest function for this situation.

$A = P\left(1 + \frac{r}{n}\right)^{12t}$ *Write the formula.*

$= 15{,}000\left(1 + \frac{0.048}{12}\right)^{12t}$ *Substitute 15,000 for P, 0.048 for r, and 12 for n.*

$= 15{,}000(1.004)^{12t}$ *Simplify.*

Holt Algebra 1



11-3 Exponential Growth and Decay

Example 2B Continued

Write a compound interest function to model each situation. Then find the balance after the given number of years.

$15,000 invested at a rate of 4.8% compounded monthly; 2 years.

Step 2 Find the balance after 2 years.

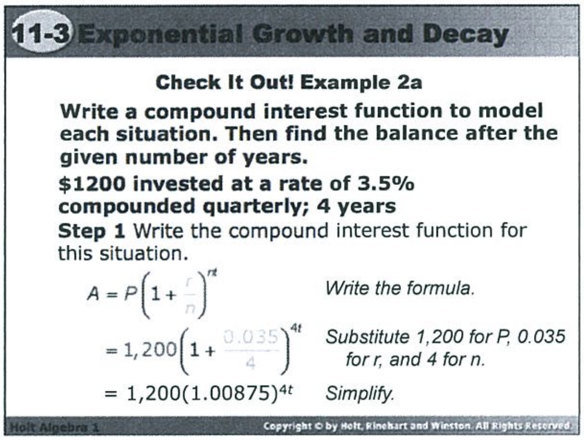
$A = 15{,}000(1.004)^{12(2)}$ *Substitute 2 for t.*

$= 15{,}000(1.004)^{24}$ *Use a calculator and round to the nearest hundredth.*

$\approx 16{,}508.22$

The balance after 2 years is $16,508.22.

Holt Algebra 1



11-3 Exponential Growth and Decay

Check It Out! Example 2a

Write a compound interest function to model each situation. Then find the balance after the given number of years.

$1200 invested at a rate of 3.5% compounded quarterly; 4 years

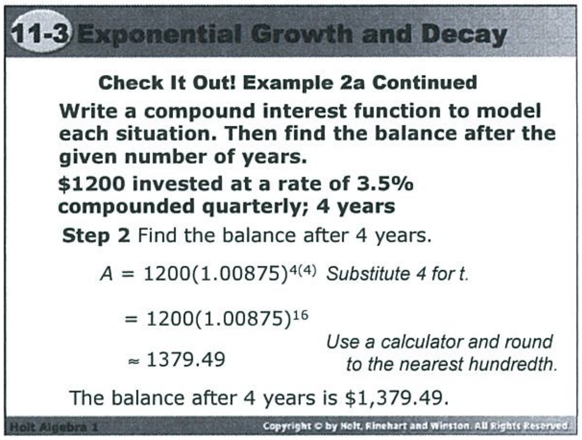
Step 1 Write the compound interest function for this situation.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ *Write the formula.*

$= 1{,}200\left(1 + \frac{0.035}{4}\right)^{4t}$ *Substitute 1,200 for P, 0.035 for r, and 4 for n.*

$= 1{,}200(1.00875)^{4t}$ *Simplify.*

Holt Algebra 1



11-3 Exponential Growth and Decay

Check It Out! Example 2a Continued

Write a compound interest function to model each situation. Then find the balance after the given number of years.

$1200 invested at a rate of 3.5% compounded quarterly; 4 years

Step 2 Find the balance after 4 years.

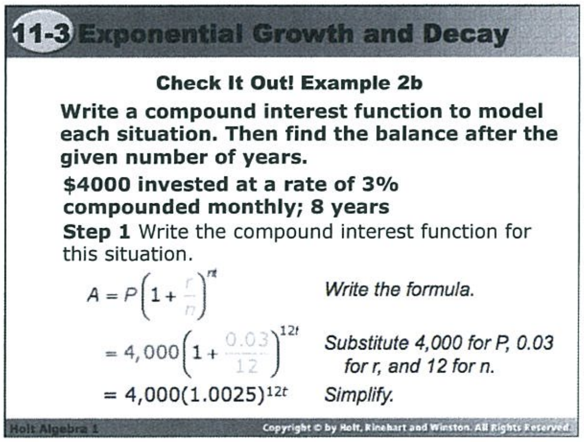
$A = 1200(1.00875)^{4(4)}$ *Substitute 4 for t.*

$= 1200(1.00875)^{16}$

≈ 1379.49 *Use a calculator and round to the nearest hundredth.*

The balance after 4 years is $1,379.49.

Holt Algebra 1



11-3 Exponential Growth and Decay

Check It Out! Example 2b

Write a compound interest function to model each situation. Then find the balance after the given number of years.

$4000 invested at a rate of 3% compounded monthly; 8 years

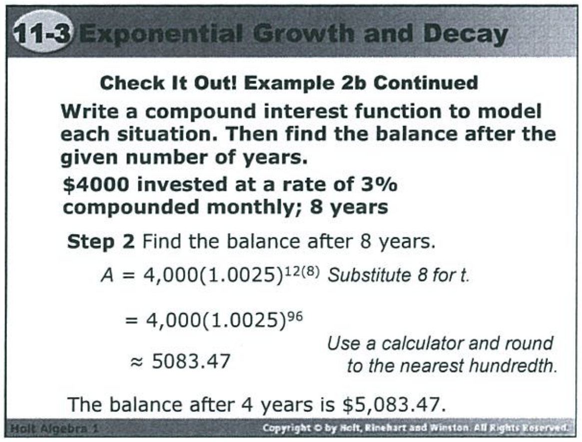
Step 1 Write the compound interest function for this situation.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ *Write the formula.*

$= 4{,}000\left(1 + \frac{0.03}{12}\right)^{12t}$ *Substitute 4,000 for P, 0.03 for r, and 12 for n.*

$= 4{,}000(1.0025)^{12t}$ *Simplify.*

Holt Algebra 1



11-3 Exponential Growth and Decay

Check It Out! Example 2b Continued

Write a compound interest function to model each situation. Then find the balance after the given number of years.

$4000 invested at a rate of 3% compounded monthly; 8 years

Step 2 Find the balance after 8 years.

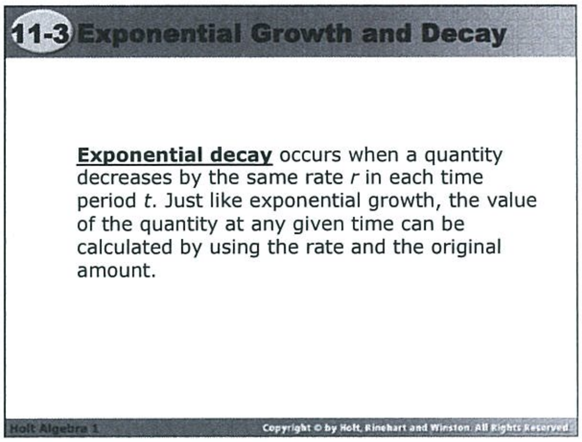
$A = 4{,}000(1.0025)^{12(8)}$ *Substitute 8 for t.*

$= 4{,}000(1.0025)^{96}$

≈ 5083.47 *Use a calculator and round to the nearest hundredth.*

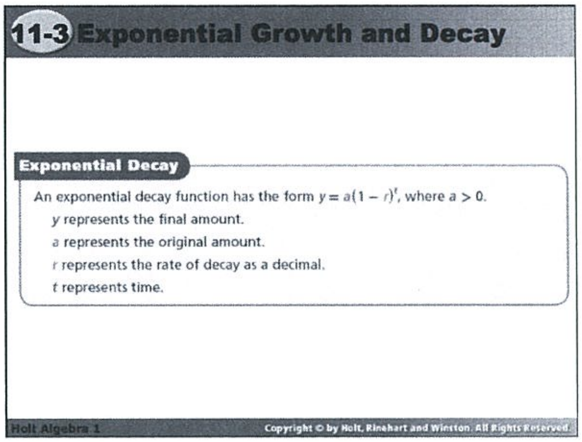
The balance after 4 years is $5,083.47.

Holt Algebra 1



11-3 Exponential Growth and Decay

Exponential decay occurs when a quantity decreases by the same rate r in each time period t. Just like exponential growth, the value of the quantity at any given time can be calculated by using the rate and the original amount.



11-3 Exponential Growth and Decay

Exponential Decay

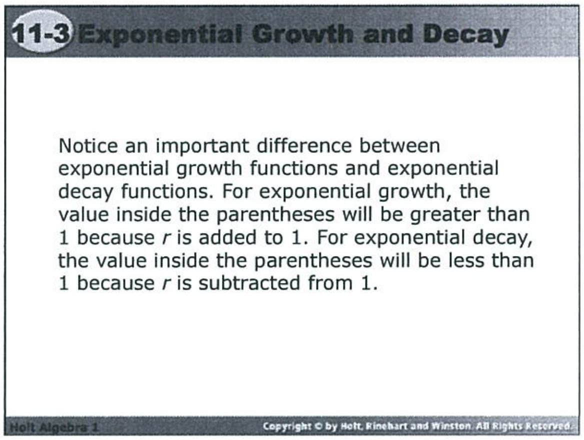
An exponential decay function has the form $y = a(1 - r)^t$, where $a > 0$.

y represents the final amount.

a represents the original amount.

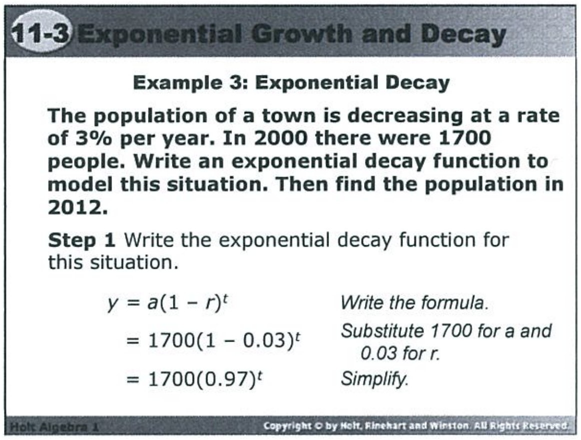
r represents the rate of decay as a decimal.

t represents time.



11-3 Exponential Growth and Decay

Notice an important difference between exponential growth functions and exponential decay functions. For exponential growth, the value inside the parentheses will be greater than 1 because r is added to 1. For exponential decay, the value inside the parentheses will be less than 1 because r is subtracted from 1.



11-3 Exponential Growth and Decay

Example 3: Exponential Decay

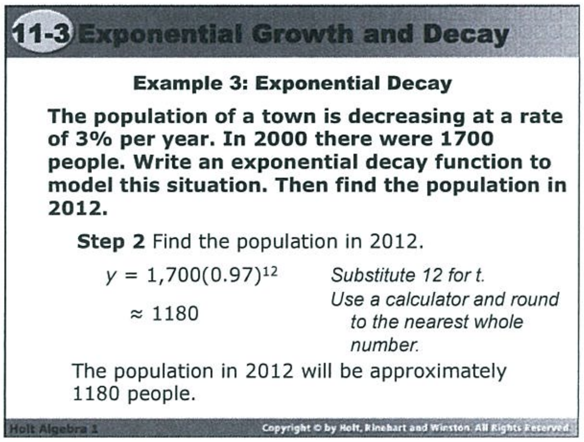
The population of a town is decreasing at a rate of 3% per year. In 2000 there were 1700 people. Write an exponential decay function to model this situation. Then find the population in 2012.

Step 1 Write the exponential decay function for this situation.

$y = a(1 - r)^t$ *Write the formula.*

$= 1700(1 - 0.03)^t$ *Substitute 1700 for a and 0.03 for r.*

$= 1700(0.97)^t$ *Simplify.*



11-3 Exponential Growth and Decay

Example 3: Exponential Decay

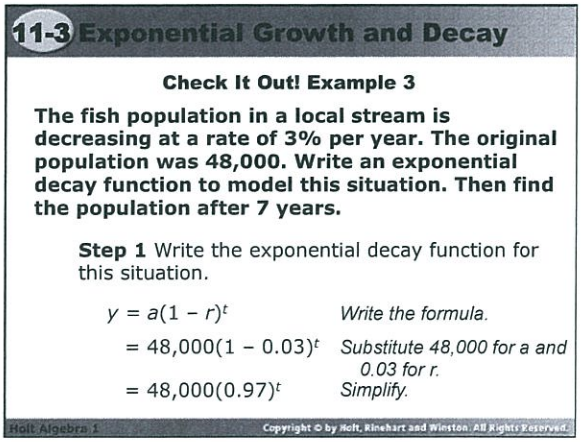
The population of a town is decreasing at a rate of 3% per year. In 2000 there were 1700 people. Write an exponential decay function to model this situation. Then find the population in 2012.

Step 2 Find the population in 2012.

$y = 1{,}700(0.97)^{12}$ *Substitute 12 for t.*

≈ 1180 *Use a calculator and round to the nearest whole number.*

The population in 2012 will be approximately 1180 people.



11-3 Exponential Growth and Decay

Check It Out! Example 3

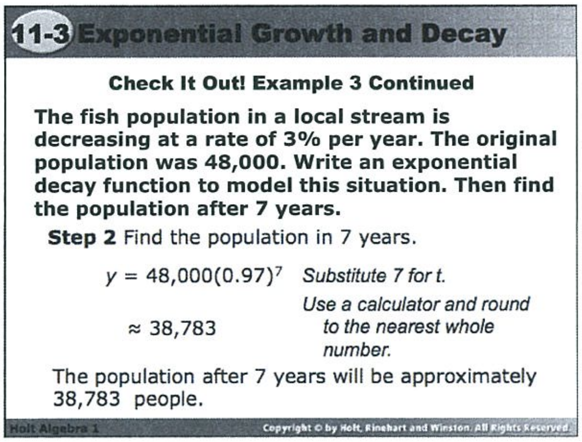
The fish population in a local stream is decreasing at a rate of 3% per year. The original population was 48,000. Write an exponential decay function to model this situation. Then find the population after 7 years.

Step 1 Write the exponential decay function for this situation.

$y = a(1 - r)^t$ *Write the formula.*

$= 48{,}000(1 - 0.03)^t$ *Substitute 48,000 for a and 0.03 for r.*

$= 48{,}000(0.97)^t$ *Simplify.*



11-3 Exponential Growth and Decay

Check It Out! Example 3 Continued

The fish population in a local stream is decreasing at a rate of 3% per year. The original population was 48,000. Write an exponential decay function to model this situation. Then find the population after 7 years.

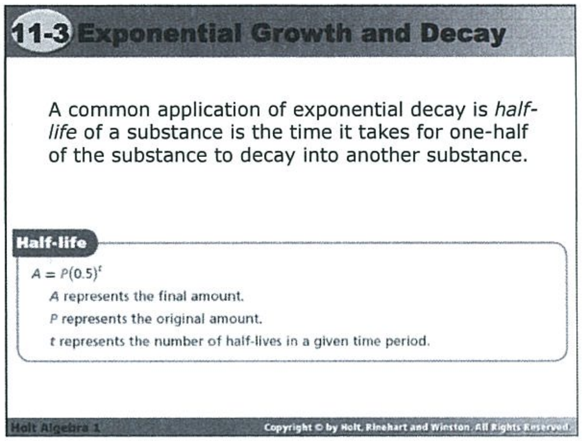
Step 2 Find the population in 7 years.

$y = 48,000(0.97)^7$ *Substitute 7 for t.*

$\approx 38,783$ *Use a calculator and round to the nearest whole number.*

The population after 7 years will be approximately 38,783 people.

Holt Algebra 1



11-3 Exponential Growth and Decay

A common application of exponential decay is *half-life* of a substance is the time it takes for one-half of the substance to decay into another substance.

Half-life

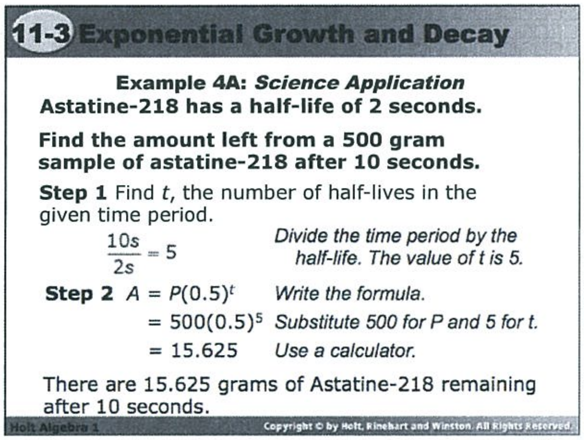
$A = P(0.5)^t$

A represents the final amount.

P represents the original amount.

t represents the number of half-lives in a given time period.

Holt Algebra 1



11-3 Exponential Growth and Decay

Example 4A: *Science Application*

Astatine-218 has a half-life of 2 seconds.

Find the amount left from a 500 gram sample of astatine-218 after 10 seconds.

Step 1 Find *t*, the number of half-lives in the given time period.

$\frac{10s}{2s} = 5$ *Divide the time period by the half-life. The value of t is 5.*

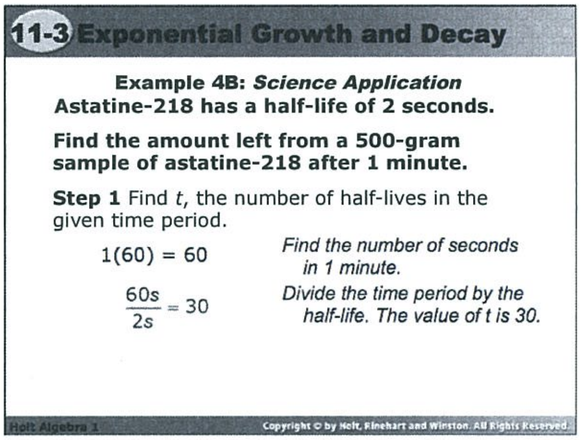
Step 2 $A = P(0.5)^t$ *Write the formula.*

$= 500(0.5)^5$ *Substitute 500 for P and 5 for t.*

$= 15.625$ *Use a calculator.*

There are 15.625 grams of Astatine-218 remaining after 10 seconds.

Holt Algebra 1



11-3 Exponential Growth and Decay

Example 4B: *Science Application*

Astatine-218 has a half-life of 2 seconds.

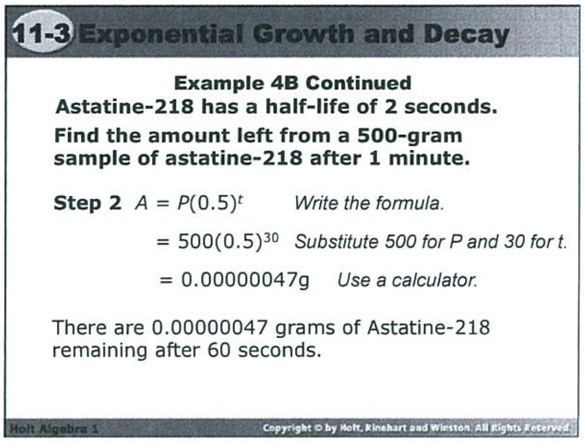
Find the amount left from a 500-gram sample of astatine-218 after 1 minute.

Step 1 Find *t*, the number of half-lives in the given time period.

$1(60) = 60$ *Find the number of seconds in 1 minute.*

$\frac{60s}{2s} = 30$ *Divide the time period by the half-life. The value of t is 30.*

Holt Algebra 1



11-3 Exponential Growth and Decay

Example 4B Continued

Astatine-218 has a half-life of 2 seconds.

Find the amount left from a 500-gram sample of astatine-218 after 1 minute.

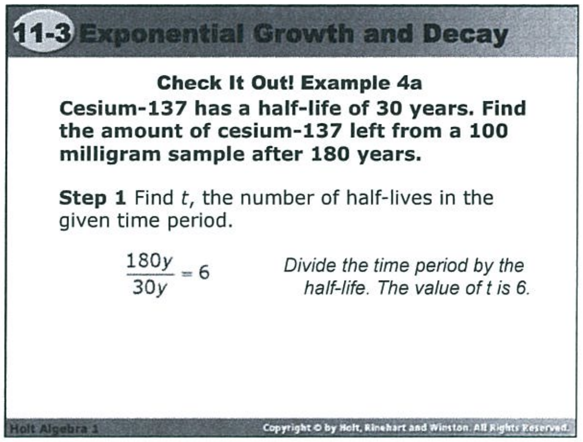
Step 2 $A = P(0.5)^t$ *Write the formula.*

$= 500(0.5)^{30}$ *Substitute 500 for P and 30 for t.*

$= 0.00000047g$ *Use a calculator.*

There are 0.00000047 grams of Astatine-218 remaining after 60 seconds.

Holt Algebra 1



11-3 Exponential Growth and Decay

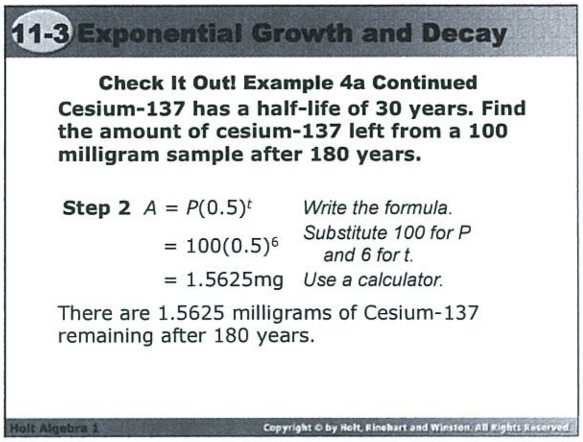
Check It Out! Example 4a

Cesium-137 has a half-life of 30 years. Find the amount of cesium-137 left from a 100 milligram sample after 180 years.

Step 1 Find *t*, the number of half-lives in the given time period.

$\frac{180y}{30y} = 6$ *Divide the time period by the half-life. The value of t is 6.*

Holt Algebra 1



11-3 Exponential Growth and Decay

Check It Out! Example 4a Continued

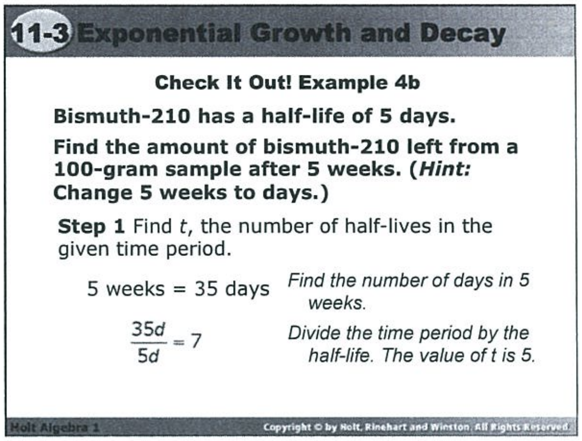
Cesium-137 has a half-life of 30 years. Find the amount of cesium-137 left from a 100 milligram sample after 180 years.

Step 2 $A = P(0.5)^t$ *Write the formula.*

$= 100(0.5)^6$ *Substitute 100 for P and 6 for t.*

$= 1.5625\text{mg}$ *Use a calculator.*

There are 1.5625 milligrams of Cesium-137 remaining after 180 years.



11-3 Exponential Growth and Decay

Check It Out! Example 4b

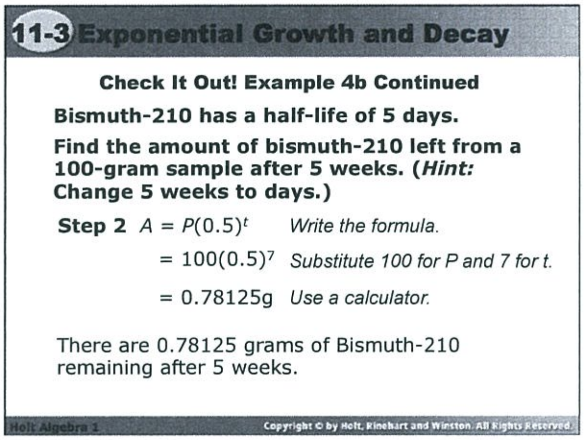
Bismuth-210 has a half-life of 5 days.

Find the amount of bismuth-210 left from a 100-gram sample after 5 weeks. (*Hint:* Change 5 weeks to days.)

Step 1 Find *t*, the number of half-lives in the given time period.

5 weeks = 35 days *Find the number of days in 5 weeks.*

$\frac{35d}{5d} = 7$ *Divide the time period by the half-life. The value of t is 5.*



11-3 Exponential Growth and Decay

Check It Out! Example 4b Continued

Bismuth-210 has a half-life of 5 days.

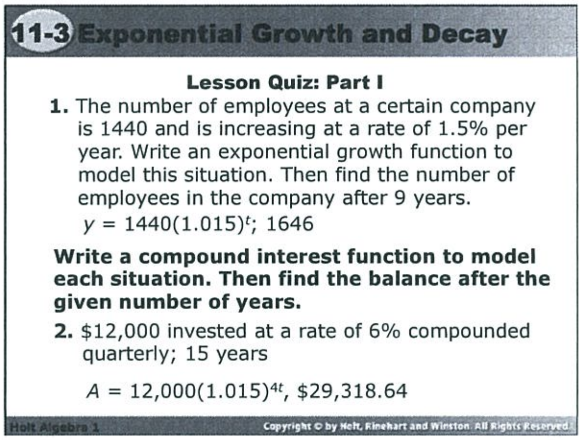
Find the amount of bismuth-210 left from a 100-gram sample after 5 weeks. (*Hint:* Change 5 weeks to days.)

Step 2 $A = P(0.5)^t$ *Write the formula.*

$= 100(0.5)^7$ *Substitute 100 for P and 7 for t.*

$= 0.78125\text{g}$ *Use a calculator.*

There are 0.78125 grams of Bismuth-210 remaining after 5 weeks.



11-3 Exponential Growth and Decay

Lesson Quiz: Part I

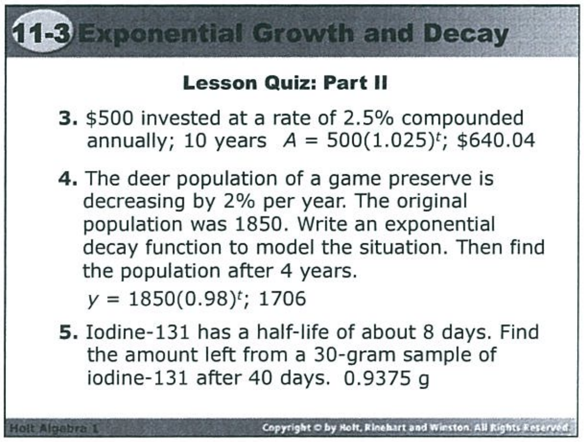
1. The number of employees at a certain company is 1440 and is increasing at a rate of 1.5% per year. Write an exponential growth function to model this situation. Then find the number of employees in the company after 9 years.

$y = 1440(1.015)^t$; 1646

Write a compound interest function to model each situation. Then find the balance after the given number of years.

2. $12,000 invested at a rate of 6% compounded quarterly; 15 years

$A = 12{,}000(1.015)^{4t}$, $29,318.64



11-3 Exponential Growth and Decay

Lesson Quiz: Part II

3. $500 invested at a rate of 2.5% compounded annually; 10 years $A = 500(1.025)^t$; $640.04

4. The deer population of a game preserve is decreasing by 2% per year. The original population was 1850. Write an exponential decay function to model the situation. Then find the population after 4 years.

$y = 1850(0.98)^t$; 1706

5. Iodine-131 has a half-life of about 8 days. Find the amount left from a 30-gram sample of iodine-131 after 40 days. 0.9375 g

Name ______________________ Date __________ Class __________

LESSON **11-3**

Reteach

Exponential Growth and Decay

In the exponential growth and decay formulas, y = final amount, a = original amount, r = rate of growth or decay, and t = time.

Exponential growth: $y = a(1 + r)^t$

The population of a city is increasing at a rate of 4% each year. In 2000 there were 236,000 people in the city. Write an exponential growth function to model this situation. Then find the population in 2009.

Step 1: Identify the variables.

$a = 236{,}000 \qquad r = 0.04$

Step 2: Substitute for a and r.

$y = a(1 + r)^t$

$y = 236{,}000(1 + 0.04)^t$

The exponential growth function is $y = 236{,}000(1.04)^t$.

Growth = greater than 1.

Step 3: Substitute for t.

$y = 236{,}000(1.04)^9$

$\approx 335{,}902$

The population will be about 335,902.

Exponential decay: $y = a(1 - r)^t$

The population of a city is decreasing at a rate of 6% each year. In 2000 there were 35,000 people in the city. Write an exponential decay function to model this situation. Then find the population in 2012.

Step 1: Identify the variables.

$a = 35{,}000 \qquad r = 0.06$

Step 2: Substitute for a and r.

$y = a(1 - r)^t$

$y = 35{,}000(1 - 0.06)^t$

The exponential decay function is $y = 35{,}000(0.94)^t$.

Decay = less than 1.

Step 3: Substitute for t.

$y = 35{,}000(0.94)^{12}$

$\approx 16{,}657$

The population will be about 16,657.

Write an exponential growth function to model each situation. Then find the value of the function after the given amount of time.

1. Annual sales at a company are $372,000 and increasing at a rate of 5% per year; 8 years $y = \boxed{}(\mathbf{1} + \boxed{})^{\Box}$ ______________

2. The population of a town is 4200 and increasing at a rate of 3% per year; 7 years ______________

Write an exponential decay function to model each situation. Then find the value of the function after the given amount of time.

3. Monthly car sales for a certain type of car are $350,000 and are decreasing at a rate of 3% per month; 6 months $y = \boxed{}(1 - \boxed{})^{\Box}$; ______________

4. An internet chat room has 1200 participants and is decreasing at a rate of 2% per year; 5 years ______________

Name ______________________ Date ____________ Class ____________

LESSON
11-3

Reteach

Exponential Growth and Decay (continued)

A special type of exponential growth involves finding compound interest.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

- where A is the total balance after t years
- P is the original amount
- r is the interest rate
- n is the number of times the interest is compounded in one year
- t is the number of years

Write a compound interest function to model $15,000 invested at a rate of 3% compounded quarterly. Then find the balance after 8 years.

$A = 15{,}000\left(1 + \frac{0.03}{4}\right)^{4t}$

$A = 15{,}000(1.0075)^{4t}$ *Compound interest function*

$A = 15{,}000(1.0075)^{4(8)}$ *Substitute 8 for t.*

$A = 15{,}000(1.0075)^{32}$

$\approx 19{,}051.67$

The balance after 8 years is $19,051.67.

A special type of exponential decay involves the half-life of substances.

$$A = P(0.5)^{t}$$

- where A is the final amount
- P is the original amount
- t is the number of half-lives in a given time period

Ismuth-212 has a half-life of approximately 60 seconds. Find the amount of Ismuth-212 left from a 25 gram sample after 300 seconds.

Step 1: Find t. $t = \frac{300}{60} = 5$

Step 2: Substitute for P and t.

$A = 25(0.5)^{5}$

$= 0.78125$

The amount after 300 s is 0.78125 g.

Write a compound interest function to model each situation. Then find the balance after the given number of years.

5. $17,000 invested at 3%, compounded annually; 6 years ______________________

____

6. $23,000 invested at 2%, compounded quarterly; 8 years ______________________

____

Write an exponential decay function to model each situation. Then find the value after the given amount of time.

7. A 30 gram sample of Iodine-131 has a half-life of about 8 days; 24 days ____________

8. A 40 gram sample of Sodium-24 has a half-life of 15 hours; 60 hours ____________

Name ______________________________ Date ____________ Class ____________

LESSON
11-3

Practice A

Exponential Growth and Decay

Write an exponential growth function to model each situation. Then find the value of the function after the given amount of time.

1. Annual sales for a clothing store are $270,000 and are increasing at a rate of 7% per year; 3 years

$y = \square(1 + \square)^{\square}$

$y =$ ____________________

2. The population of a school is 2200 and is increasing at a rate of 2%; 6 years

$y = \square(1 + \square)^{\square}$

$y \approx$ ____________________

3. The value of an antique vase is $200 and is increasing at a rate of 8%; 12 years

$y =$ ____________________

$y \approx$ ____________________

Write a compound interest function to model each situation. Then find the balance after the given number of years.

4. $20,000 invested at a rate of 3% compounded annually; 8 years.

$A = \square\left(1 + \frac{\square}{\square}\right)^{(\square)(\square)}$

$A \approx$ ____________________

5. $35,000 invested at a rate of 6% compounded monthly; 10 years

$A = \square\left(1 + \frac{\square}{\square}\right)^{(\square)(\square)}$

$A \approx$ ____________________

6. $35,000 invested at a rate of 8% compounded quarterly; 5 years

$A =$ ____________________

$A \approx$ ____________________

Write an exponential decay function to model each situation. Then find the value of the function after the given amount of time.

7. The population of a school is 800 and is decreasing at a rate of 2% per year; 4 years

$y = \square(1 - \square)^{\square}$

$y \approx$ ____________________

8. The bird population in a forest is about 2300 and decreasing at a rate of 4% per year; 10 years

$y = \square(1 - \square)^{\square}$

$y \approx$ ____________________

9. The half-life of strontium-90 is approximately 28 years. Find the amount of strontium-90 left from a 10 gram sample after 56 years.

$A = \square(0.5)^{\square}$

$A \approx$ ____________________

Name ______________________ Date __________ Class __________

LESSON **11-3**

Practice B

Exponential Growth and Decay

Write an exponential growth function to model each situation. Then find the value of the function after the given amount of time.

1. Annual sales for a fast food restaurant are $650,000 and are increasing at a rate of 4% per year; 5 years ____________ ____________

2. The population of a school is 800 students and is increasing at a rate of 2% per year; 6 years ____________ ____________

3. During a certain period of time, about 70 northern sea otters had an annual growth rate of 18%; 4 years ____________ ____________

Write a compound interest function to model each situation. Then find the balance after the given number of years.

4. $50,000 invested at a rate of 3% compounded monthly; 6 years ____________ ____________

5. $43,000 invested at a rate of 5% compounded annually; 3 years ____________ ____________

6. $65,000 invested at a rate of 6% compounded quarterly; 12 years ____________ ____________

Write an exponential decay function to model each situation. Then find the value of the function after the given amount of time.

7. The population of a town is 2500 and is decreasing at a rate of 3% per year; 5 years ____________ ____________

8. The value of a company's equipment is $25,000 and decreases at a rate of 15% per year; 8 years ____________ ____________

9. The half-life of Iodine-131 is approximately 8 days. Find the amount of Iodine-131 left from a 35 gram sample after 32 days. ____________

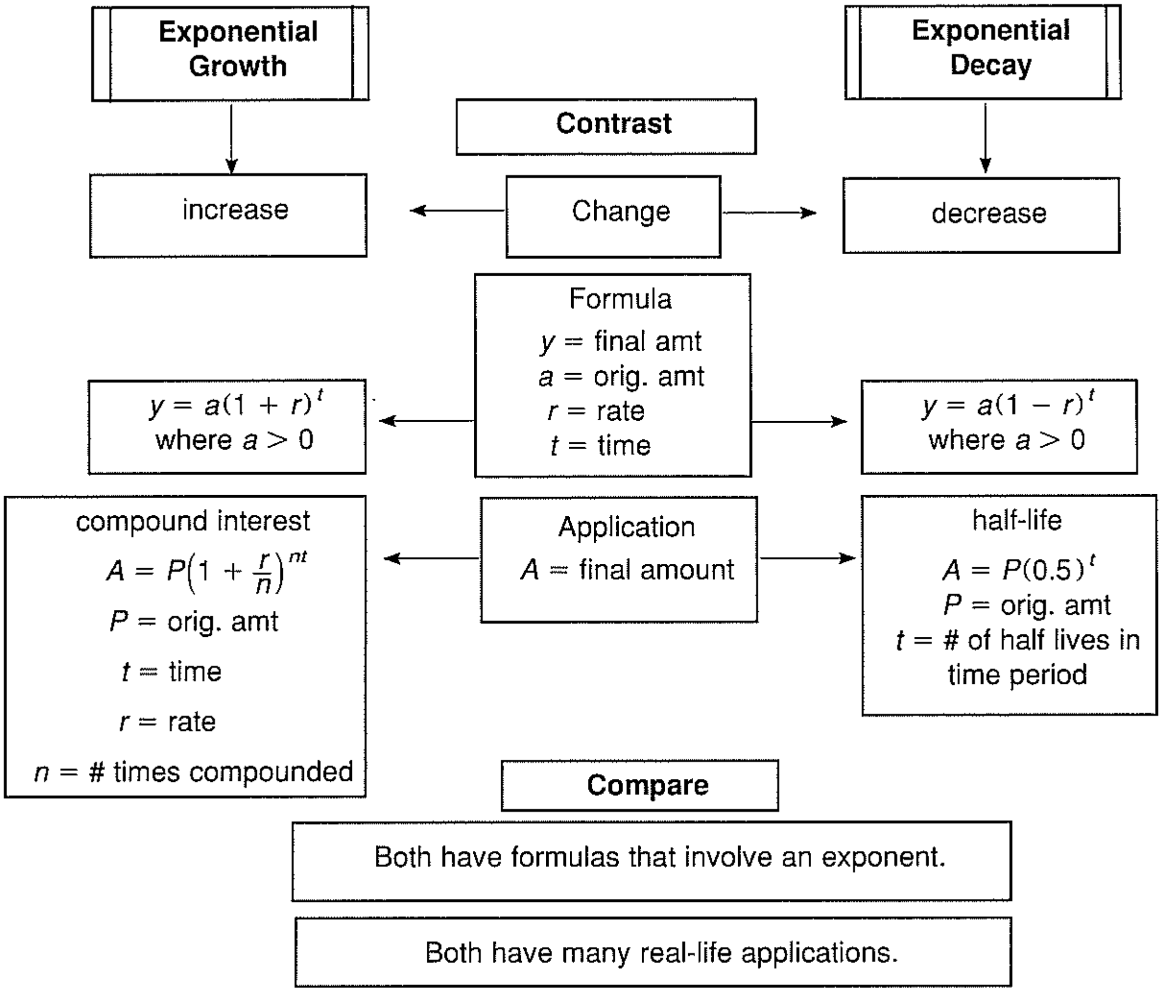
Name ______________________________ Date __________ Class __________

LESSON **11-3**

Reading Strategies

Compare and Contrast

The diagram below highlights important concepts of exponential growth and exponential decay.



For each situation: a. identify it as exponential growth or exponential decay, and b. use a formula to calculate the answer.

1a. The bird population in an wooded area is decreasing by 3% each year from 1250.

____

1b. Find the bird population after 6 years.

____

2a. A town's population was 3800 in 2005 and growing at a rate of 2% every year.

____

2b. Find the town's population in 2025.

____

3a. $800 is invested at a rate of 4% and is compounded monthly (12 times/year).

____

3b. Find the balance after 10 years.

____

11-4 Linear, Quadratic, and Exponential Models

Objectives

Compare linear, quadratic, and exponential models.

Given a set of data, decide which type of function models the data and write an equation to describe the function.

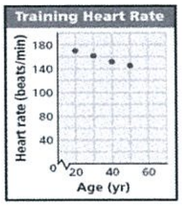
11-4 Linear, Quadratic, and Exponential Models

Look at the tables and graphs below. The data show three ways you have learned that variable quantities can be related. The relationships shown are linear, quadratic, and exponential.

Linear

Training Heart Rate

Age (yr)	Beats/min
20	170
30	161.5
40	153
50	144.5



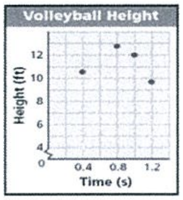
11-4 Linear, Quadratic, and Exponential Models

Look at the tables and graphs below. The data show three ways you have learned that variable quantities can be related. The relationships shown are linear, quadratic, and exponential.

Quadratic

Volleyball Height

Time (s)	Height (ft)
0.4	10.44
0.8	12.76
1	12
1.2	9.96



11-4 Linear, Quadratic, and Exponential Models

Look at the tables and graphs below. The data show three ways you have learned that variable quantities can be related. The relationships shown are linear, quadratic, and exponential.

Exponential

Volleyball Tournament

Round	Teams Left
1	16
2	8
3	4
4	2

11-4 Linear, Quadratic, and Exponential Models

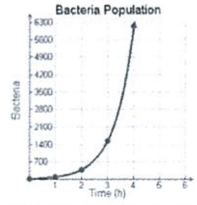
In the real world, people often gather data and then must decide what kind of relationship (if any) they think best describes their data.

11-4 Linear, Quadratic, and Exponential Models

Example 1A: Graphing Data to Choose a Model

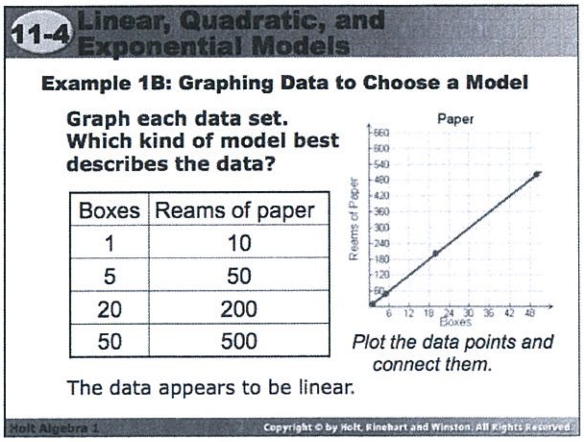
Graph each data set. Which kind of model best describes the data?

Time(h)	Bacteria
0	24
1	96
2	384
3	1536
4	6144



Plot the data points and connect them.

The data appear to be exponential.



11-4 Linear, Quadratic, and Exponential Models

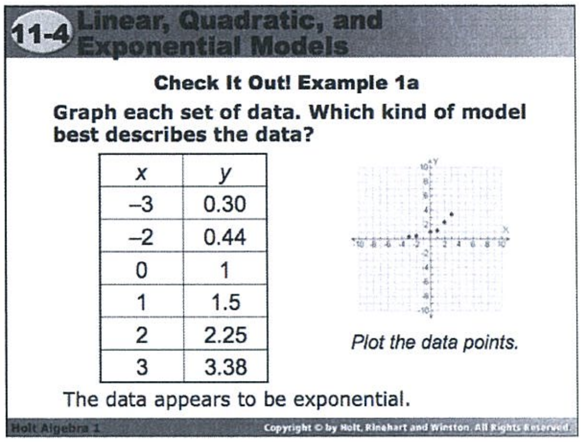
Example 1B: Graphing Data to Choose a Model

Graph each data set. Which kind of model best describes the data?

Boxes	Reams of paper
1	10
5	50
20	200
50	500

Plot the data points and connect them.

The data appears to be linear.



11-4 Linear, Quadratic, and Exponential Models

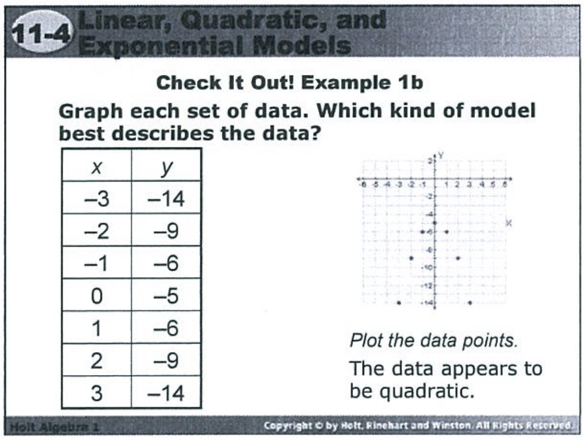
Check It Out! Example 1a

Graph each set of data. Which kind of model best describes the data?

x	*y*
–3	0.30
–2	0.44
0	1
1	1.5
2	2.25
3	3.38

Plot the data points.

The data appears to be exponential.



11-4 Linear, Quadratic, and Exponential Models

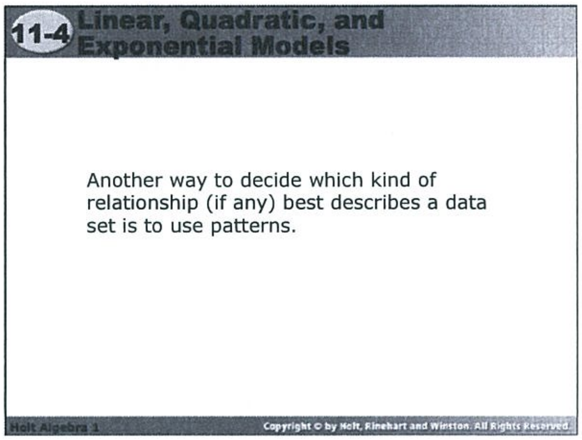
Check It Out! Example 1b

Graph each set of data. Which kind of model best describes the data?

x	*y*
–3	–14
–2	–9
–1	–6
0	–5
1	–6
2	–9
3	–14

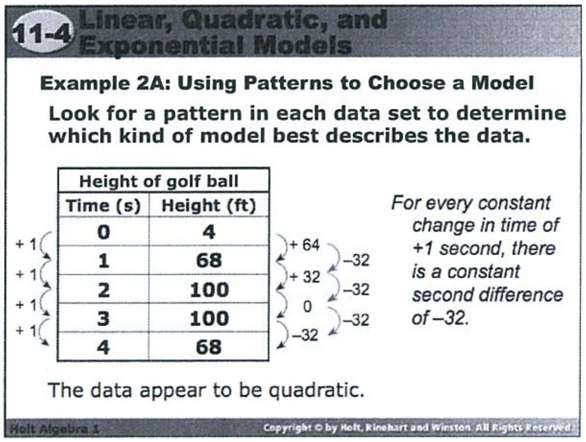
Plot the data points.

The data appears to be quadratic.



11-4 Linear, Quadratic, and Exponential Models

Another way to decide which kind of relationship (if any) best describes a data set is to use patterns.



11-4 Linear, Quadratic, and Exponential Models

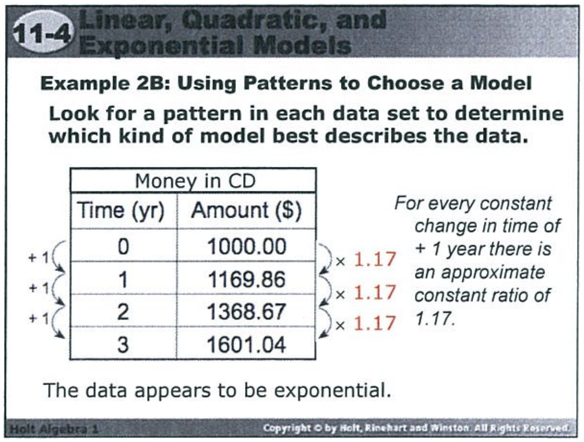
Example 2A: Using Patterns to Choose a Model

Look for a pattern in each data set to determine which kind of model best describes the data.

Height of golf ball	
Time (s)	**Height (ft)**
0	**4**
1	**68**
2	**100**
3	**100**
4	**68**

For every constant change in time of +1 second, there is a constant second difference of –32.

The data appear to be quadratic.



11-4 Linear, Quadratic, and Exponential Models

Example 2B: Using Patterns to Choose a Model

Look for a pattern in each data set to determine which kind of model best describes the data.

Money in CD	
Time (yr)	Amount ($)
0	1000.00
1	1169.86
2	1368.67
3	1601.04

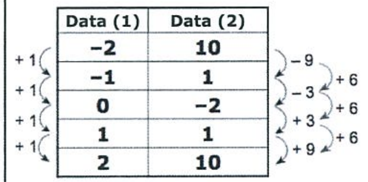
For every constant change in time of + 1 year there is an approximate constant ratio of 1.17.

The data appears to be exponential.

11-4 Linear, Quadratic, and Exponential Models

Check It Out! Example 2

Look for a pattern in the data set {(−2, 10), (−1, 1), (0, −2), (1, 1), (2, 10)} to determine which kind of model best describes the data.



Data (1)	Data (2)
−2	10
−1	1
0	−2
1	1
2	10

For every constant change of +1 there is a constant ratio of 6.

The data appear to be quadratic.

11-4 Linear, Quadratic, and Exponential Models

After deciding which model best fits the data, you can write a function. Recall the general forms of linear, quadratic, and exponential functions.

General Forms of Functions

LINEAR	QUADRATIC	EXPONENTIAL
$y = mx + b$	$y = ax^2 + bx + c$	$y = ab^x$

11-4 Linear, Quadratic, and Exponential Models

Example 3: *Problem-Solving Application*

PROBLEM SOLVING

Use the data in the table to describe how the number of people changes. Then write a function that models the data. Use your function to predict the number of people who received the e-mail after one week.

E-mail forwarding	
Time (Days)	Number of People Who Received the E-mail
0	8
1	56
2	392
3	2744

11-4 Linear, Quadratic, and Exponential Models

1 Understand the Problem

The answer will have three parts–a description, a function, and a prediction.

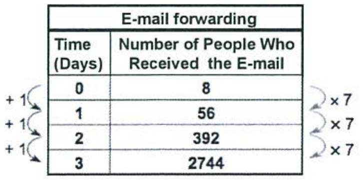
2 Make a Plan

Determine whether the data is linear, quadratic, or exponential. Use the general form to write a function. Then use the function to find the number of people after one year.

11-4 Linear, Quadratic, and Exponential Models

3 Solve

Step 1 Describe the situation in words.



E-mail forwarding	
Time (Days)	Number of People Who Received the E-mail
0	8
1	56
2	392
3	2744

Each day, the number of e-mails is multiplied by 7.

11-4 Linear, Quadratic, and Exponential Models

Step 2 Write the function.

There is a constant ratio of 7. The data appear to be exponential.

$y = ab^x$ *Write the general form of an exponential function.*

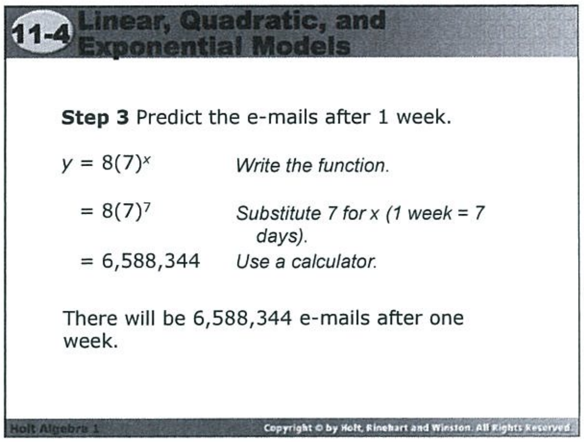
$y = a(7)^x$

$8 = a(7)^0$ *Choose an ordered pair from the table, such as (0, 8). Substitute for x and y.*

$8 = a(1)$ *Simplify $7^0 = 1$*

$8 = a$ *The value of a is 8.*

$y = 8(7)^x$ *Substitute 8 for a in $y = a(7)^x$.*

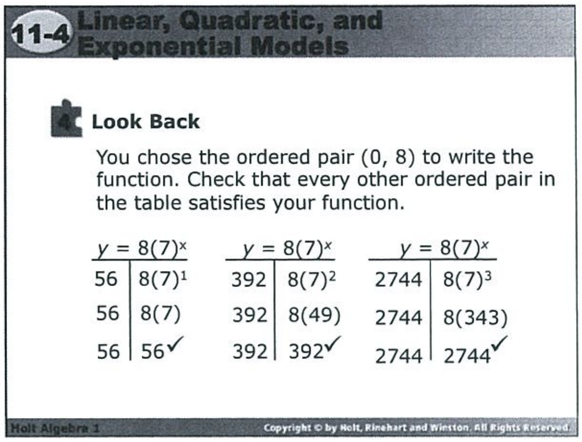


11-4 Linear, Quadratic, and Exponential Models

Step 3 Predict the e-mails after 1 week.

$y = 8(7)^x$	*Write the function.*
$= 8(7)^7$	*Substitute 7 for x (1 week = 7 days).*
$= 6,588,344$	*Use a calculator.*

There will be 6,588,344 e-mails after one week.

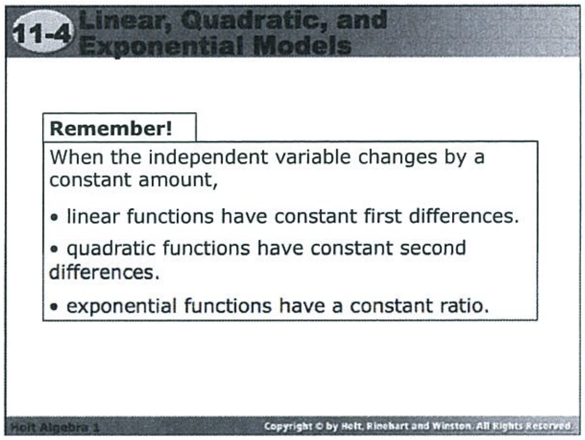


11-4 Linear, Quadratic, and Exponential Models

4 Look Back

You chose the ordered pair (0, 8) to write the function. Check that every other ordered pair in the table satisfies your function.

$$\begin{array}{r|l} \multicolumn{2}{c}{\underline{y = 8(7)^x}} \\ 56 & 8(7)^1 \\ 56 & 8(7) \\ 56 & 56\checkmark \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{\underline{y = 8(7)^x}} \\ 392 & 8(7)^2 \\ 392 & 8(49) \\ 392 & 392\checkmark \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{\underline{y = 8(7)^x}} \\ 2744 & 8(7)^3 \\ 2744 & 8(343) \\ 2744 & 2744\checkmark \end{array}$$

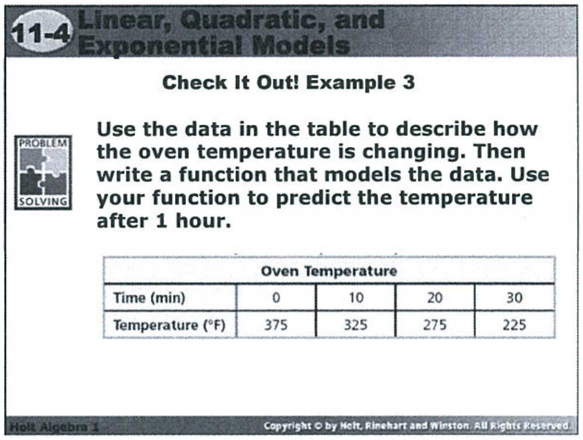


11-4 Linear, Quadratic, and Exponential Models

Remember!

When the independent variable changes by a constant amount,

- linear functions have constant first differences.
- quadratic functions have constant second differences.
- exponential functions have a constant ratio.

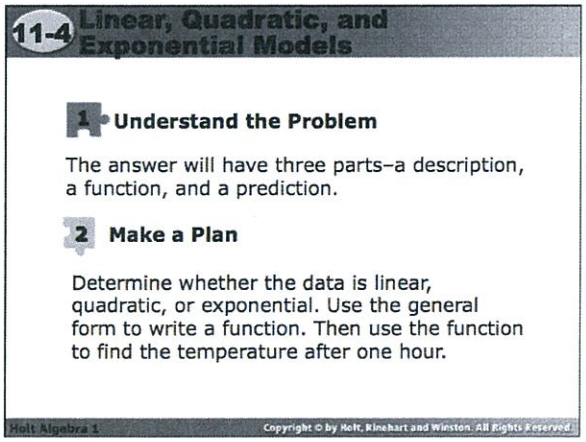


11-4 Linear, Quadratic, and Exponential Models

Check It Out! Example 3

Use the data in the table to describe how the oven temperature is changing. Then write a function that models the data. Use your function to predict the temperature after 1 hour.

Oven Temperature				
Time (min)	0	10	20	30
Temperature (°F)	375	325	275	225



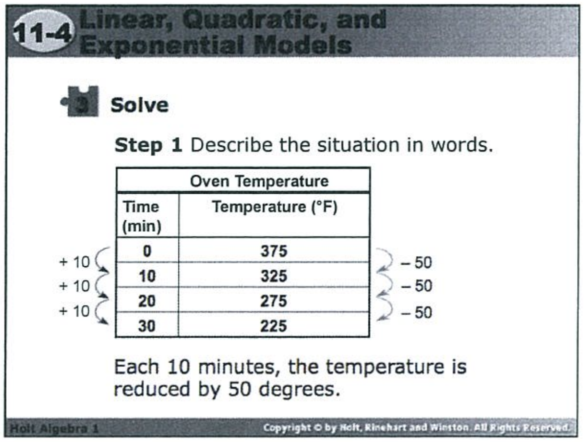
11-4 Linear, Quadratic, and Exponential Models

1 Understand the Problem

The answer will have three parts–a description, a function, and a prediction.

2 Make a Plan

Determine whether the data is linear, quadratic, or exponential. Use the general form to write a function. Then use the function to find the temperature after one hour.



11-4 Linear, Quadratic, and Exponential Models

3 Solve

Step 1 Describe the situation in words.

	Oven Temperature		
	Time (min)	Temperature (°F)	
	0	375	
+ 10	10	325	– 50
+ 10	20	275	– 50
+ 10	30	225	– 50

Each 10 minutes, the temperature is reduced by 50 degrees.

11-4 Linear, Quadratic, and Exponential Models

Step 2 Write the function.

There is a constant reduction of 50° each 10 minutes. The data appear to be linear.

$y = mx + b$	*Write the general form of a linear function.*
$y = -5(x) + b$	*The slope m is –50 divided by 10.*
$y = -5(0) + b$	*Choose an x value from the table, such as 0.*
$y = 0 + 375$	*The starting point is b which is 375.*
$y = 375$	

11-4 Linear, Quadratic, and Exponential Models

Step 3 Predict the temperature after 1 hour.

$y = -5x + 375$	*Write the function.*
$= 8(7)^7$	*Substitute 7 for x (1 week = 7 days).*
$= 6{,}588{,}344$	*Use a calculator.*

There will be 6,588,344 e-mails after one week.

11-4 Linear, Quadratic, and Exponential Models

Look Back

You chose the ordered pair (0, 375) to write the function. Check that every other ordered pair in the table satisfies your function.

$$\begin{array}{r|l} \multicolumn{2}{c}{\underline{y = -5(x) + 375}} \\ 375 & -5(0) + 375 \\ 375 & 0 + 375 \\ 375 & 375 \checkmark \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{\underline{y = -5(x) + 375}} \\ 325 & -5(10) + 375 \\ 325 & -50 + 375 \\ 325 & 325 \checkmark \end{array}$$

11-4 Linear, Quadratic, and Exponential Models

Look Back

You chose the ordered pair (0, 375) to write the function. Check that every other ordered pair in the table satisfies your function.

$$\begin{array}{r|l} \multicolumn{2}{c}{\underline{y = -5(x) + 375}} \\ 275 & -5(20) + 375 \\ 275 & -100 + 375 \\ 275 & 275 \checkmark \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{\underline{y = -5(x) + 375}} \\ 225 & -5(30) + 375 \\ 225 & -150 + 375 \\ 225 & 225 \checkmark \end{array}$$

11-4 Linear, Quadratic, and Exponential Models

Lesson Quiz: Part I

Which kind of model best describes each set of data?

1.

Time (s)	Height of Ball (ft)
0	200
1	184
2	136
3	56

quadratic

2.

Value of Townhouse	
Age (yr)	Value ($)
0	100,000
1	102,000
2	104,040
3	106,121

exponential

11-4 Linear, Quadratic, and Exponential Models

Lesson Quiz: Part II

3. Use the data in the table to describe how the amount of water is changing. Then write a function that models the data. Use your function to predict the amount of water in the pool after 3 hours.

Water in a Swimming Pool	
Time (min)	Amount of Water (gal)
10	327
20	342
30	357
40	372

Increasing by 15 gal every 10 min; $y = 1.5x + 312$; 582 gal

Name ______________________ Date __________ Class __________

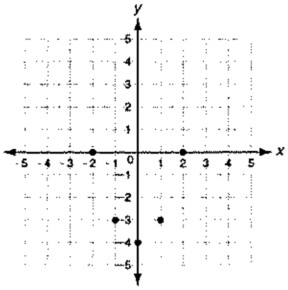
LESSON **11-4**

Reteach

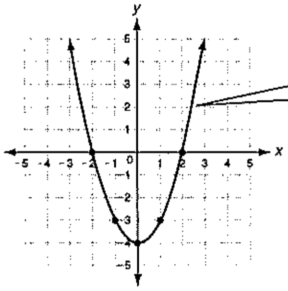
Linear, Quadratic, and Exponential Models

Graph to decide whether data is best modeled by a linear, quadratic or exponential function.

Graph (−2, 0), (−1, −3), (0, −4), (1, −3), (2, 0). What kind of model best describes the data?



Connect the points.



The data appear to be quadratic.

You can also look at patterns in data to determine the correct model.

Linear functions have constant 1st differences.

x	*y*
2	5
4	2
6	−1
8	−4

−3
−3
−3

Quadratic functions have constant 2nd differences.

x	*y*
1	−8
2	−5
3	0
4	7

+3 +5 +7
+2 +2

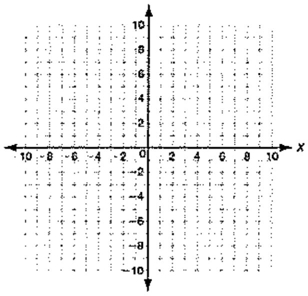
Exponential functions have a constant ratio.

x	*y*
0	−2
1	−8
2	−32
3	−128

×4
×4
×4

Graph each data set. Which kind of model best describes the data?

1. (−2, −4), (−1, −2), (0, 0), (1, 2), (2, 4)



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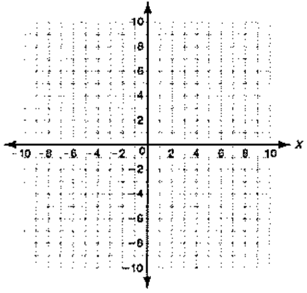
2. (−1, 4), (0, 2), (1, 1), $\left(2, \frac{1}{2}\right)$, $\left(3, \frac{1}{4}\right)$



____

Look for a pattern in each data set to determine which kind of model best describes the data.

3.

x	*y*
0	6
1	12
2	24
3	48

____

4.

x	*y*
0	10
1	18
2	28
3	40

____

5.

x	*y*
3	4
6	−2
9	−8
12	−14

____

Name ______________________________ Date __________ Class __________

LESSON 11-4

Reteach

Linear, Quadratic, and Exponential Models (continued)

After deciding which model fits best, you can write a function.

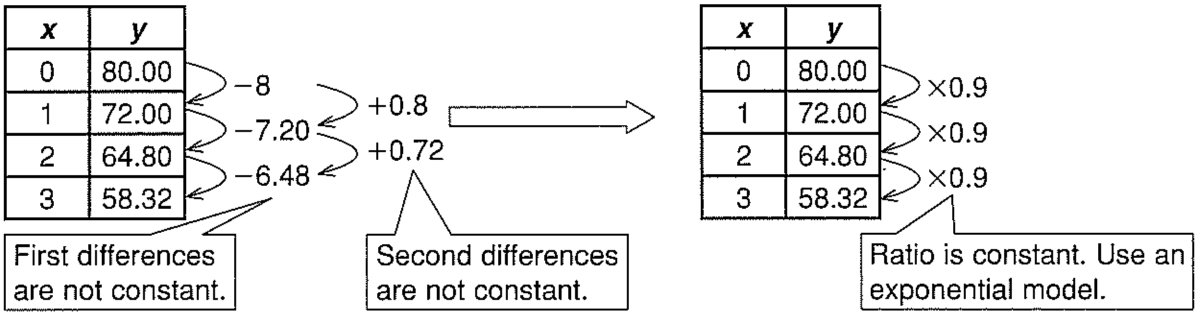
Linear	Quadratic	Exponential
$y = mx + b$	$y = ax^2 + bx + c$	$y = ab^x$

Use the data in the table to describe how the software's cost is changing. Then write a function to model the data.

Computer Software				
Year	0	1	2	3
Cost ($)	80.00	72.00	64.80	58.32

Step 1: Determine whether data is linear, quadratic, or exponential.

Check differences: Check ratio:



First differences are not constant.

Second differences are not constant.

Ratio is constant. Use an exponential model.

Step 2: Write the function.

Use $y = ab^x$

$y = a(0.9)^x$	*Substitute the constant ratio 0.9, for b.*
$80 = a(0.9)^0$	*Substitute the ordered pair (0, 80) for x and y.*
$80 = a(1)$	*Simplify $(0.9)^0$.*
$80 = a$	*The value of a is 80.*
$y = 80(0.9)^x$	*Write the function.*

Describe the model that best fits the data below. Then write a function to model the data.

6.

x	y
0	1
1	4
2	16
3	64

model: ____________________

function: ____________________

7.

x	y
0	7
1	10
2	13
3	16

model: ____________________

function: ____________________

Name ________________________ Date ____________ Class ____________

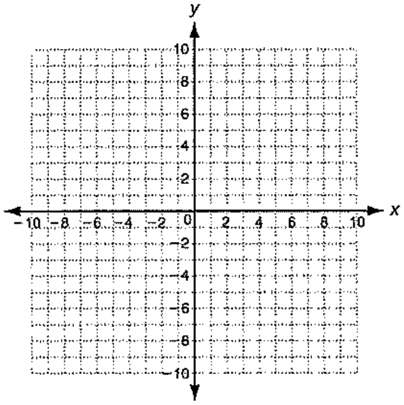
LESSON 11-4

Practice A

Linear, Quadratic, and Exponential Models

Graph each data set. Write *linear*, *quadratic*, or *exponential*.

1. $\{(0, -4), (1, -2), (2, 0), (3, 2), (4, 4)\}$



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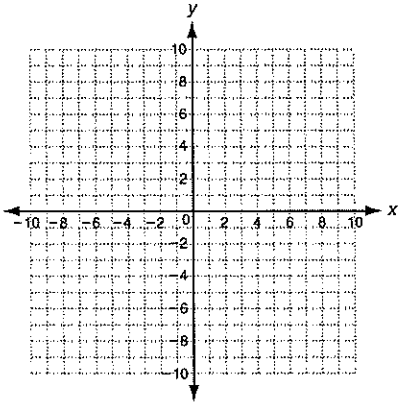
2. $\{(-2, -5), (-1, -8), (0, -9), (1, -8), (2, -5)\}$



____

Look for a pattern in each data set. Write *linear*, *quadratic*, or *exponential*.

3.

x	*y*
0	3
1	6
2	12
3	24

____

4.

x	*y*
−2	−10
−1	−8
0	−6
1	−4

____

5.

x	*y*
0	2
1	6
2	12
3	20

____

6. The data in the table show the price of apples at a local store over several years.

Year	1	2	3	4
Cost ($)	0.45	0.90	1.35	1.80

a. Which model best describes the data for apples? ______________________

b. Write the function that models the data for apples. ______________________

c. Predict the cost of apples in year 8. ______________________

7. The data in the table show the price of a game over several years.

Year	0	1	2	3
Cost ($)	5.00	6.00	7.20	8.64

a. Which model best describes the data for the game? ______________________

b. Write the function that models the data for the game. ______________________

c. Predict the cost of the game in year 7. Round the cost to the nearest cent. ______________________

Name ______________________ Date __________ Class __________

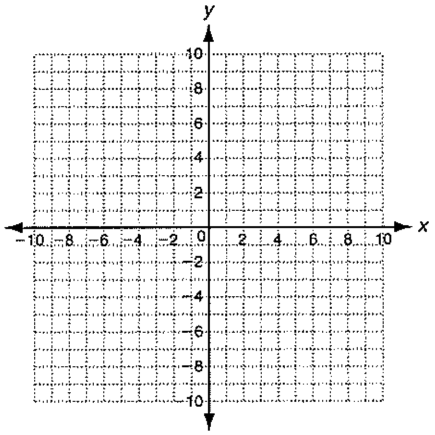
LESSON 11-4

Practice B

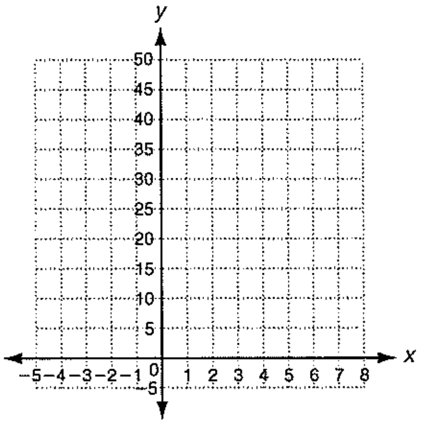
Linear, Quadratic, and Exponential Models

Graph each data set. Which kind of model best describes the data?

1. $\{(-2, 0), (-1, -3), (0, -4), (1, -3), (2, 0)\}$



2. $\{(0, 3), (1, 6), (2, 12), (3, 24), (4, 48)\}$



Look for a pattern in each data set to determine which kind of model best describes the data.

3. $\{(-5, 9), (-4, 0), (-3, -7), (-2, -12)\}$

4. $\{(-2, 9), (-1, 13), (0, 17), (1, 21)\}$

5. $\{(1, 4), (2, 6), (3, 9), (4, 13.5)\}$

6. $\{(0, 4), (2, 12), (4, 36), (6, 76)\}$

7. $\left\{(1, 17), \left(3, 8\frac{1}{2}\right), \left(5, 4\frac{1}{4}\right), \left(7, 2\frac{1}{8}\right)\right\}$

8. Use the data in the table to describe how the restaurant's sales are changing. Then write a function that models the data. Use your function to predict the amount of sales after 8 years.

Restaurant				
Year	0	1	2	3
Sales ($)	20,000	19,000	18,050	17,147.50

9. Use the data in the table to describe how the clothing store's sales are changing. Then write a function that models the data. Use your function to predict the amount of sales after 10 years.

Clothing Store				
Year	0	1	2	3
Sales ($)	15,000	15,750	16,500	17,250

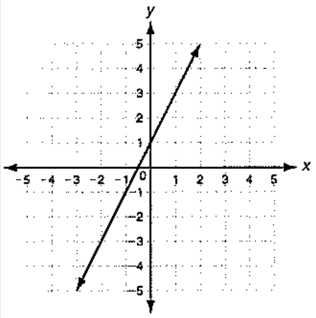
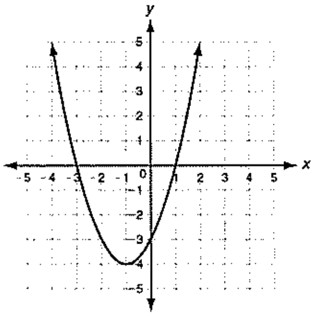
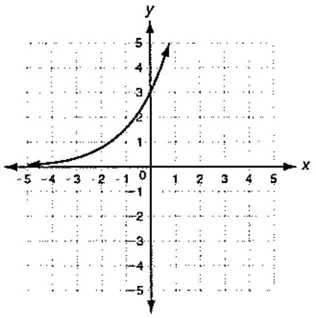
Name ______________________ Date ____________ Class ____________

LESSON 11-4

Reading Strategies

Use a Table

Use the table below to help you learn the characteristics of each data model and how to tell them apart.

Model	Linear	Quadratic	Exponential
Pattern (*y*-values)	constant first differences	constant second differences	constant ratios
Graph			
Equation	$y = mx + b$	$y = ax^2 + bx + c$	$y = ab^x$

Identify each of the following as *linear, quadratic,* or *exponential.*

1. $y = 6(2)^x$

____

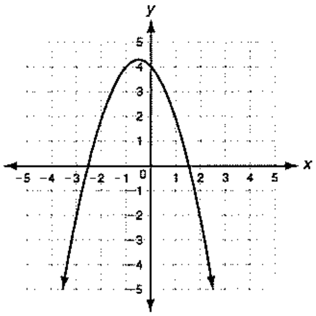
2. $y = 4x + 6$

____

3. $y = 2x^2 + 5x + 3$

____

4.



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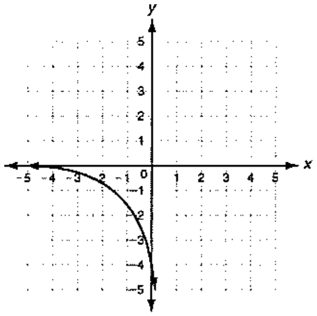
5.



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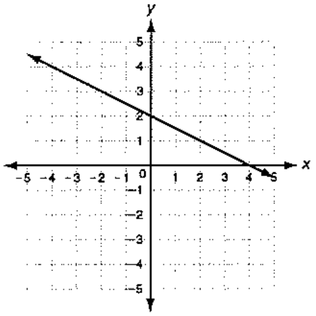
6.



____

7.

x	−6	−7	−8	−9	−10
y	17	20	23	26	29

____

8.

x	1	3	5	7	9
y	4	12	28	52	84

____

9.

x	5	4	3	2	1
y	128	64	32	16	8

____