



MEA 2013-2014
Teacher: Claudia Valle
Start Date:

Course: Geometry A

Student: _____

Completed Date:

Unit 6: Polygons and Quadrilaterals

Objectives: Students will understand how to identify various types of quadrilaterals. Students will understand how to use properties of special quadrilaterals to solve real-life problems.

Essential Questions: What methods are used to prove a quadrilateral is a parallelogram? How can you identify special quadrilaterals based on limited information?

TEKS Standards: G.2.A, G.2.B, G.3.B, G.3.E, G.5.B, G.7.A, G.7.B, G.7.C, G.9.B

Geometry

(2) Geometric structure. The student analyzes geometric relationships in order to make and verify conjectures. The student is expected to:

(A) use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships; and

(B) make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

(3) Geometric structure. The student applies logical reasoning to justify and prove mathematical statements. The student is expected to:

(B) construct and justify statements about geometric figures and their properties;

(E) use deductive reasoning to prove a statement.

(5) Geometric patterns. The student uses a variety of representations to describe geometric relationships and solve problems. The student is expected to:

LESSON
6-1

Reteach

Properties and Attributes of Polygons *continued*

The **Polygon Angle Sum Theorem** states that the sum of the interior angle measures of a convex polygon with n sides is $(n - 2)180^\circ$.

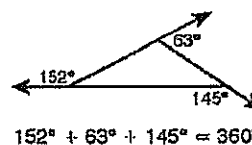
Convex Polygon	Number of Sides	Sum of Interior Angle Measures: $(n - 2)180^\circ$
quadrilateral	4	$(4 - 2)180^\circ = 360^\circ$
hexagon	6	$(6 - 2)180^\circ = 720^\circ$
decagon	10	$(10 - 2)180^\circ = 1440^\circ$

If a polygon is a regular polygon, then you can divide the sum of the interior angle measures by the number of sides to find the measure of each interior angle.

Regular Polygon	Number of Sides	Sum of Interior Angle Measures	Measure of Each Interior Angle
quadrilateral	4	360°	$360^\circ \div 4 = 90^\circ$
hexagon	6	720°	$720^\circ \div 6 = 120^\circ$
decagon	10	1440°	$1440^\circ \div 10 = 144^\circ$

The **Polygon External Angle Sum Theorem** states that the sum of the exterior angle measures, one angle at each vertex, of a convex polygon is 360° .

The measure of each exterior angle of a regular polygon with n exterior angles is $360^\circ \div n$. So the measure of each exterior angle of a regular decagon is $360^\circ \div 10 = 36^\circ$.



Find the sum of the interior angle measures of each convex polygon.

7. pentagon

8. octagon

9. nonagon

Find the measure of each interior angle of each regular polygon.

Round to the nearest tenth if necessary.

10. pentagon

11. heptagon

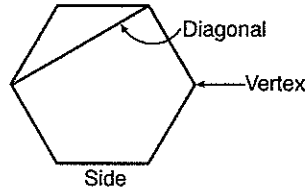
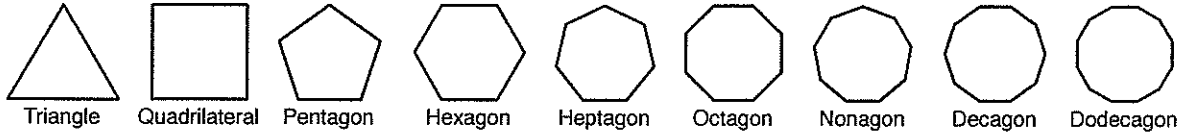
12. 15-gon

Find the measure of each exterior angle of each regular polygon.

13. quadrilateral

14. octagon

LESSON
6-1 **Reading Strategies**
Understanding Vocabulary



1. How many sides does a pentagon have? _____
2. Give some examples of pentagons in real life.

3. How many vertices does a quadrilateral have? _____
4. How does the number of vertices of a polygon compare to the number of sides of the same polygon?

5. What is the name of a polygon with eight sides? _____
6. How many diagonals can be drawn from one vertex of a hexagon? _____

concave—any part of a diagonal contains points in the exterior of the polygon

convex—no diagonal contains points in the exterior of the polygon

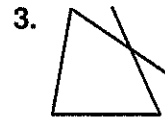
Draw an example of each polygon.

7. convex heptagon

8. concave quadrilateral

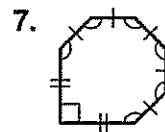
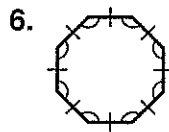
LESSON **Practice B**
6-1 **Properties and Attributes of Polygons**

Tell whether each figure is a polygon. If it is a polygon, name it by the number of its sides.



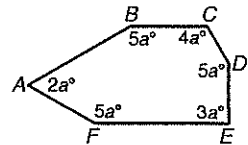
4. For a polygon to be regular, it must be both equiangular and equilateral.
 Name the only type of polygon that must be regular if it is equiangular. _____

Tell whether each polygon is regular or irregular. Then tell whether it is concave or convex.

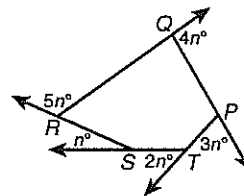


8. Find the sum of the interior angle measures of a 14-gon. _____

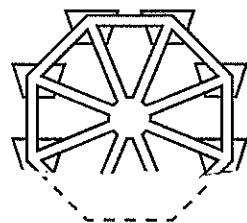
9. Find the measure of each interior angle of hexagon $ABCDEF$.



10. Find the value of n in pentagon $PQRST$.



Before electric or steam power, a common way to power machinery was with a waterwheel. The simplest form of waterwheel is a series of paddles on a frame partially submerged in a stream. The current in the stream pushes the paddles forward and turns the frame. The power of the turning frame can then be used to drive machinery to saw wood or grind grain. The waterwheel shown has a frame in the shape of a regular octagon.



11. Find the measure of one interior angle of the waterwheel. _____

12. Find the measure of one exterior angle of the waterwheel. _____

LESSON
6-1

Practice C

Properties and Attributes of Polygons

Find the sum of the interior angle measures of each n -gon.

1. 52-gon

2. 102-gon

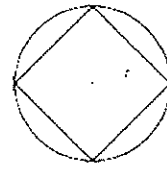
3. 1002-gon

4. Do you believe there is an upper limit to the sum of the interior angle measures in n -gons? Explain your reasoning.

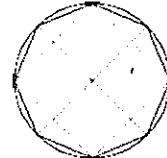
5. A polygon is convex if no part of a diagonal lies in the exterior of the polygon. Write an alternative definition for *convex* based on interior angles.

Any regular polygon can be inscribed in a circle. For Exercises 6–9, find the length of a side of the regular polygon in terms of r , the radius of the circle. Give the lengths in simplest radical form.

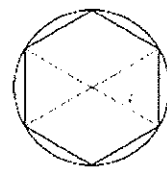
6. regular quadrilateral (square)



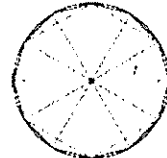
7. regular octagon (*Hint: The dotted lines show a square.*)



8. regular hexagon



9. regular dodecagon (*Hint: The dotted lines show a regular hexagon.*)



7. $x = 12\sqrt{3}$; $y = 36$

8. $x = 11\sqrt{3}$; $y = 22\sqrt{3}$

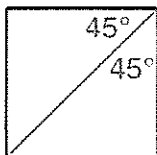
Challenge

1. $7\sqrt{2}$
2. $18 + 6\sqrt{3}$
3. $86 + 18\sqrt{2}$
4. $72 + 36\sqrt{6}$
5. $a = 7, b = 4\sqrt{3}, c = 11, d = 0$
6. $w = 13, x = 13, y = 20, z = 20 + 13\sqrt{3}$

Problem Solving

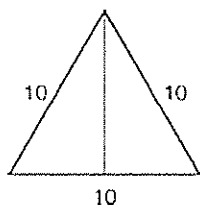
1. $12\sqrt{3}$ in. or about 20.8 in.
2. $32\sqrt{3}$ in. or about 55.4 in.
3. $3\sqrt{2}$ in. or about 4.2 in.
4. $\frac{8\sqrt{3}}{3}$ in. or about 4.6 in.
5. $14\sqrt{3}$ in.
6. $19\sqrt{2}$ cm
7. B
8. F

Reading Strategies



$10^2; 10^2; 200; 10$

1. $s\sqrt{2}$
2. 60°
- 3.



4. Students should say that they have created two 30° - 60° - 90° triangles.
5. The altitude is $5\sqrt{3}$.

LESSON 6-1

Practice A

1. B
2. C
3. A
4. not a polygon
5. polygon; octagon
6. not a polygon
7. regular; convex
8. irregular; concave

9. irregular; convex
10. 720°
11. 120°
12. 120°
13. 60°

Practice B

1. polygon; nonagon
2. not a polygon
3. not a polygon
4. triangle
5. irregular; concave
6. regular; convex
7. irregular; convex
8. 2160°
9. $m\angle A = 60^\circ; m\angle B = m\angle D = m\angle F = 150^\circ; m\angle C = 120^\circ; m\angle E = 90^\circ$
10. 24
11. 135°
12. 45°

Practice C

1. 90008
2. 18,0008
3. 180,0008
4. Possible answer: No; a convex polygon may have any number of sides. As the number of sides increases, so does the sum of the interior angle measures. So the sum has no upper limit.
5. Possible answer: A polygon is convex if each interior angle and the interior of the polygon together contain all points of the polygon.
6. $r\sqrt{2}$
7. $r\sqrt{2 - \sqrt{2}}$
8. r
9. $r\sqrt{2 - \sqrt{3}}$

Reteach

1. polygon; pentagon
2. polygon; heptagon
3. not a polygon
4. irregular; convex
5. regular; convex
6. irregular; concave
7. 540°
8. 1080°
9. 1260°
10. 108°
11. 128.6°
12. 156°
13. 90°
14. 45°

TEKS G.5.B



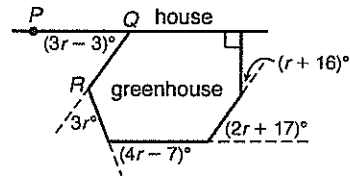
LESSON
6-1

Problem Solving

Properties and Attributes of Polygons

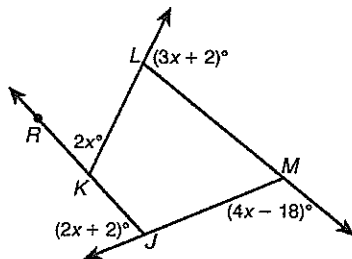
1. A campground site is in the shape of a convex quadrilateral. Three sides of the campground form two right angles. The third interior angle measures 10° less than the fourth angle. Find the measure of each interior angle.
2. A pentagon has two exterior angles that measure $(3x)^\circ$, two exterior angles that measure $(2x + 22)^\circ$, and an exterior angle that measures $(x + 41)^\circ$. If all of these angles have different vertices, what are the measures of the exterior angles of the pentagon?

3. The top view of a hexagonal greenhouse is shown at the right. What is the measure of $\angle PQR$, the acute angle formed by the house and the greenhouse?

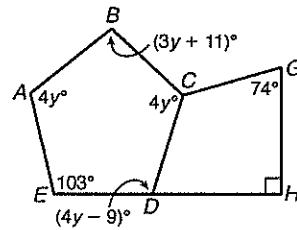


Choose the best answer.

4. A figure is an equiangular 18-gon. What is the measure of each exterior angle of the polygon?
 A 10°
 B 18°
 C 20°
 D 36°
5. Three interior angles of a convex heptagon measure 125° , and two of the interior angles measure 143° . Which are possible measures for the other two interior angles of the heptagon?
 F 48° and 48° H 100° and 116°
 G 39° and 100° J 89° and 150°
6. Find the measure of $\angle RKL$.
7. What is the measure of $\angle GCD$?



- A 34° C 86°
 B 68° D 148°



- F 123° H 73°
 G 116° J 29°

TEKS G.5.B



LESSON
6-1

Problem Solving

Properties and Attributes of Polygons

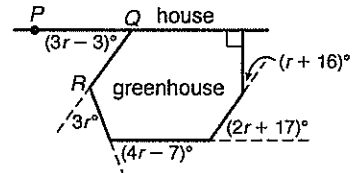
1. A campground site is in the shape of a convex quadrilateral. Three sides of the campground form two right angles. The third interior angle measures 10° less than the fourth angle. Find the measure of each interior angle.

$90^\circ, 90^\circ, 85^\circ, 95^\circ$

2. A pentagon has two exterior angles that measure $(3x)^\circ$, two exterior angles that measure $(2x + 22)^\circ$, and an exterior angle that measures $(x + 41)^\circ$. If all of these angles have different vertices, what are the measures of the exterior angles of the pentagon?

$75^\circ, 75^\circ, 72^\circ, 72^\circ, 66^\circ$

3. The top view of a hexagonal greenhouse is shown at the right. What is the measure of $\angle PQR$, the acute angle formed by the house and the greenhouse?



54°

Choose the best answer.

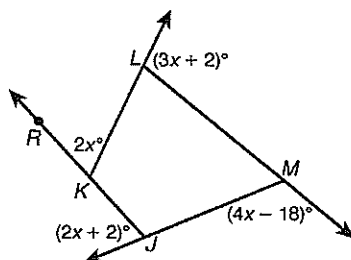
4. A figure is an equiangular 18-gon. What is the measure of each exterior angle of the polygon?

- A 10°
 B 18°
 C 20°
 D 36°

5. Three interior angles of a convex heptagon measure 125° , and two of the interior angles measure 143° . Which are possible measures for the other two interior angles of the heptagon?

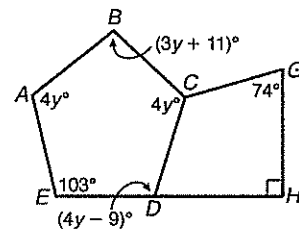
- F 48° and 48° H 100° and 116°
 G 39° and 100° J 89° and 150°

6. Find the measure of $\angle RKL$.



- A 34° C 86°
 B 68° D 148°

7. What is the measure of $\angle GCD$?

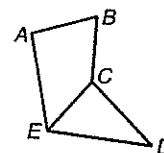


- F 123° H 73°
 G 116° J 29°

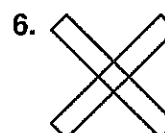
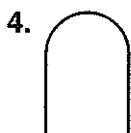
LESSON **6-1** **Practice A**
Properties and Attributes of Polygons

Match each vocabulary term on the left with a part of polygon *ABCDE* on the right.

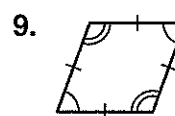
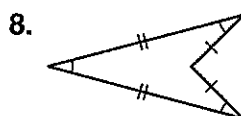
- | | | |
|----------------------------|-------|--------------------|
| 1. a diagonal | _____ | A. point <i>D</i> |
| 2. a side of the polygon | _____ | B. \overline{CE} |
| 3. a vertex of the polygon | _____ | C. \overline{CD} |



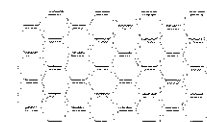
A polygon is a closed flat figure made of straight segments that do not cross each other. Tell whether each figure is a polygon. If it is a polygon, name it by the number of its sides.



A regular polygon has all sides congruent and all angles congruent. Tell whether each polygon is regular or irregular. A concave polygon has a pair of sides that make a "cave" in the polygon. Tell whether each polygon is concave or convex.



Honeybees store their honey in honeycombs. The honeycomb is made of many small wax compartments that are perfect regular hexagons.



10. Use the Polygon Angle Sum Theorem to find the sum of the interior angle measures of a regular hexagon. _____
11. Find the measure of one interior angle of a regular hexagon. (Hint: Divide the answer to Exercise 10 by the number of sides.) _____
12. Use the Polygon Exterior Angle Sum Theorem to find the sum of the exterior angle measures, one exterior angle at each vertex, of a regular hexagon. _____
13. Find the measure of one exterior angle of a regular hexagon. (Hint: Divide the answer to Exercise 12 by the number of sides.) _____

LESSON 6-1 Practice A
Properties and Attributes of Polygons

Match each vocabulary term on the left with a part of polygon $ABCDE$ on the right.

- a diagonal _____ A. point D
- a side of the polygon _____ B. \overline{CE}
- a vertex of the polygon _____ C. \overline{CD}

A polygon is a closed flat figure made of straight segments that do not cross each other. Tell whether each figure is a polygon. If it is a polygon, name it by the number of its sides.

- _____
- _____
- _____

A regular polygon has all sides congruent and all angles congruent. Tell whether each polygon is regular or irregular. A concave polygon has a pair of sides that make a "cave" in the polygon. Tell whether each polygon is concave or convex.

- _____
- _____
- _____

Honeybees store their honey in honeycombs. The honeycomb is made of many small wax compartments that are perfect regular hexagons.

- Use the Polygon Angle Sum Theorem to find the sum of the interior angle measures of a regular hexagon. _____
- Find the measure of one interior angle of a regular hexagon. (Hint: Divide the answer to Exercise 10 by the number of sides.) _____
- Use the Polygon Exterior Angle Sum Theorem to find the sum of the exterior angle measures, one exterior angle at each vertex, of a regular hexagon. _____
- Find the measure of one exterior angle of a regular hexagon. (Hint: Divide the answer to Exercise 12 by the number of sides.) _____

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LESSON 6-1 Practice B
Properties and Attributes of Polygons

Tell whether each figure is a polygon. If it is a polygon, name it by the number of its sides.

- _____
- _____
- _____

- For a polygon to be regular, it must be both equiangular and equilateral. Name the only type of polygon that must be regular if it is equiangular. _____

Tell whether each polygon is regular or irregular. Then tell whether it is concave or convex.

- _____
- _____
- _____

- Find the sum of the interior angle measures of a 14-gon. _____
- Find the measure of each interior angle of hexagon $ABCDEF$. _____
- Find the value of n in pentagon $QRST$. _____

Before electric or steam power, a common way to power machinery was with a waterwheel. The simplest form of waterwheel is a series of paddles on a frame partially submerged in a stream. The current in the stream pushes the paddles forward and turns the frame. The power of the turning frame can then be used to drive machinery to saw wood or grind grain. The waterwheel shown has a frame in the shape of a regular octagon.

- Find the measure of one interior angle of the waterwheel. _____
- Find the measure of one exterior angle of the waterwheel. _____

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LESSON 6-1 Practice C
Properties and Attributes of Polygons

Find the sum of the interior angle measures of each n -gon.

- 52-gon _____
- 102-gon _____
- 1002-gon _____

- Do you believe there is an upper limit to the sum of the interior angle measures in n -gons? Explain your reasoning. _____
- A polygon is convex if no part of a diagonal lies in the exterior of the polygon. Write an alternative definition for *convex* based on interior angles. _____

Any regular polygon can be inscribed in a circle. For Exercises 6–9, find the length of a side of the regular polygon in terms of r , the radius of the circle. Give the lengths in simplest radical form.

- regular quadrilateral (square) _____
- regular octagon (Hint: The dotted lines show a square.) _____
- regular hexagon _____
- regular dodecagon (Hint: The dotted lines show a regular hexagon.) _____

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LESSON 6-1 Reteach
Properties and Attributes of Polygons

The parts of a polygon are named on the quadrilateral below.

You can name a polygon by the number of its sides.

A regular polygon has all sides congruent and all angles congruent. A polygon is *convex* if all its diagonals lie in the interior of the polygon. A polygon is *concave* if all or part of at least one diagonal lies outside the polygon.

Number of Sides	Polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
n	n -gon

Types of Polygons		
regular, convex 	irregular, convex 	irregular, concave

Tell whether each figure is a polygon. If it is a polygon, name it by the number of sides.

- _____
- _____
- _____

Tell whether each polygon is regular or irregular. Then tell whether it is concave or convex.

- _____
- _____
- _____

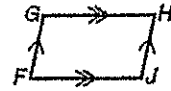
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LESSON
6-2

Reteach

Properties of Parallelograms

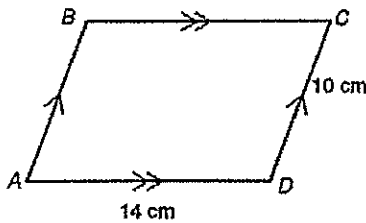
A parallelogram is a quadrilateral with two pairs of parallel sides. All parallelograms, such as $\square FGHJ$, have the following properties.



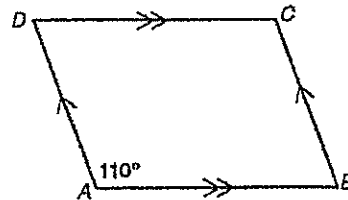
Properties of Parallelograms	
<p>Opposite sides are congruent.</p> $\overline{FG} \cong \overline{HJ}$ $\overline{GH} \cong \overline{JF}$	<p>Opposite angles are congruent.</p> $\angle F \cong \angle H$ $\angle G \cong \angle J$
<p>Consecutive angles are supplementary.</p> $m\angle F + m\angle G = 180^\circ$ $m\angle G + m\angle H = 180^\circ$ $m\angle H + m\angle J = 180^\circ$ $m\angle J + m\angle F = 180^\circ$	<p>The diagonals bisect each other.</p> $\overline{FP} \cong \overline{HP}$ $\overline{GP} \cong \overline{JP}$

Find each measure.

1. AB



2. $m\angle D$



Find each measure in $\square LMNP$.

3. ML

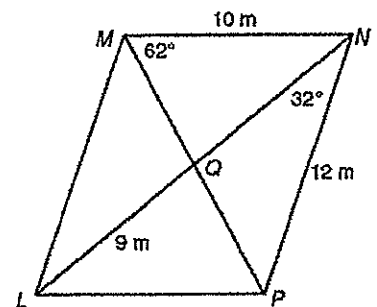
4. LP

5. $m\angle LPM$

6. LN

7. $m\angle MLN$

8. QN



LESSON
6-2

Reteach

Properties of Parallelograms *continued*

You can use properties of parallelograms to find measures.

WXYZ is a parallelogram. Find $m\angle X$.

$m\angle W + m\angle X = 180^\circ$

$(7x + 15) + 4x = 180^\circ$

$11x + 15 = 180$

$11x = 165$

$x = 15$

$m\angle X = (4x)^\circ = [4(15)]^\circ = 60^\circ$

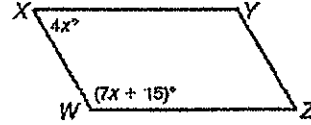
If a quadrilateral is a \square , then cons. \sphericalangle are supp.

Substitute the given values.

Combine like terms.

Subtract 15° from both sides.

Divide both sides by 11.



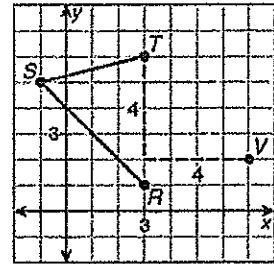
If you know the coordinates of three vertices of a parallelogram, you can use slope to find the coordinates of the fourth vertex.

Three vertices of $\square RSTV$ are $R(3, 1)$, $S(-1, 5)$, and $T(3, 6)$. Find the coordinates of V .

Since opposite sides must be parallel, the rise and the run from S to R must be the same as the rise and the run from T to V .

From S to R , you go down 4 units and right 4 units. So, from T to V , go down 4 units and right 4 units. Vertex V is at $V(7, 2)$.

You can use the slope formula to verify that $\overline{ST} \parallel \overline{RV}$.



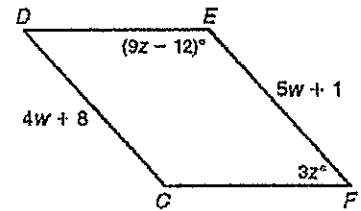
CDEF is a parallelogram. Find each measure.

9. CD

10. EF

11. $m\angle F$

12. $m\angle E$



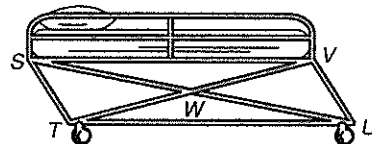
The coordinates of three vertices of a parallelogram are given. Find the coordinates of the fourth vertex.

13. $\square ABCD$ with $A(0, 6)$, $B(5, 8)$, $C(5, 5)$

14. $\square KLMN$ with $K(-4, 7)$, $L(3, 6)$, $M(5, 3)$

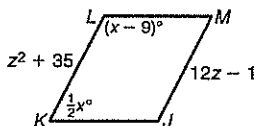
LESSON **Practice B**
6-2 **Properties of Parallelograms**

A gurney is a wheeled cot or stretcher used in hospitals. Many gurneys are made so that the base will fold up for easy storage in an ambulance. When partially folded, the base forms a parallelogram. In $\square STUV$, $VU = 91$ centimeters, $UW = 108.8$ centimeters, and $m\angle TSV = 57^\circ$. Find each measure.



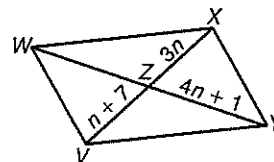
- | | | |
|------------------|------------------|------------------|
| 1. SW | 2. TS | 3. US |
| _____ | _____ | _____ |
| 4. $m\angle SVU$ | 5. $m\angle STU$ | 6. $m\angle TUV$ |
| _____ | _____ | _____ |

$JKLM$ is a parallelogram. Find each measure.



- | | | |
|----------------|----------------|---------|
| 7. $m\angle L$ | 8. $m\angle K$ | 9. MJ |
| _____ | _____ | _____ |

$VWXY$ is a parallelogram. Find each measure.

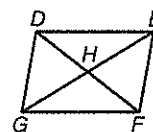


- | | |
|----------|----------|
| 10. VX | 11. XZ |
| _____ | _____ |
| 12. ZW | 13. WY |
| _____ | _____ |

14. Three vertices of $\square ABCD$ are $B(-3, 3)$, $C(2, 7)$, and $D(5, 1)$. Find the coordinates of vertex A .

Write a two-column proof.

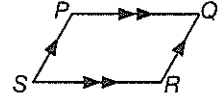
15. **Given:** $DEFG$ is a parallelogram.
Prove: $m\angle DHG = m\angle EDH + m\angle FGH$



6-2 Study Guide and Intervention

Parallelograms

Sides and Angles of Parallelograms A quadrilateral with both pairs of opposite sides parallel is a **parallelogram**. Here are four important properties of parallelograms.



	If $PQRS$ is a parallelogram, then
If a quadrilateral is a parallelogram, then its opposite sides are congruent.	$\overline{PQ} \cong \overline{SR}$ and $\overline{PS} \cong \overline{QR}$
If a quadrilateral is a parallelogram, then its opposite angles are congruent.	$\angle P \cong \angle R$ and $\angle S \cong \angle Q$
If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.	$\angle P$ and $\angle S$ are supplementary; $\angle S$ and $\angle R$ are supplementary; $\angle R$ and $\angle Q$ are supplementary; $\angle Q$ and $\angle P$ are supplementary.
If a parallelogram has one right angle, then it has four right angles.	If $m\angle P = 90$, then $m\angle Q = 90$, $m\angle R = 90$, and $m\angle S = 90$.

If $ABCD$ is a parallelogram, find the value of each variable.

\overline{AB} and \overline{CD} are opposite sides, so $\overline{AB} \cong \overline{CD}$.

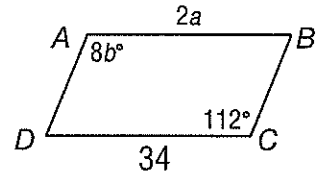
$$2a = 34$$

$$a = 17$$

$\angle A$ and $\angle C$ are opposite angles, so $\angle A \cong \angle C$.

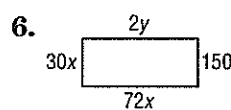
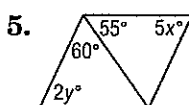
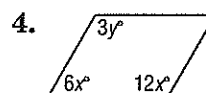
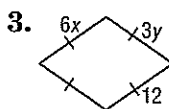
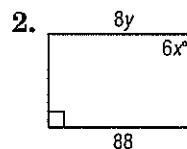
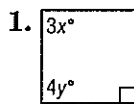
$$8b = 112$$

$$b = 14$$



Exercises

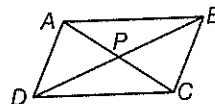
Find the value of each variable.



6-2 Study Guide and Intervention *(continued)*

Parallelograms

Diagonals of Parallelograms Two important properties of parallelograms deal with their diagonals.



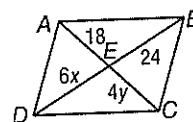
	If $ABCD$ is a parallelogram, then
If a quadrilateral is a parallelogram, then its diagonals bisect each other.	$AP = PC$ and $DP = PB$
If a quadrilateral is a parallelogram, then each diagonal separates the parallelogram into two congruent triangles.	$\triangle ACD \cong \triangle CAB$ and $\triangle ADB \cong \triangle CBD$

Find the value of x and y in parallelogram $ABCD$.

The diagonals bisect each other, so $AE = CE$ and $DE = BE$.

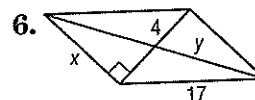
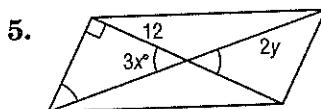
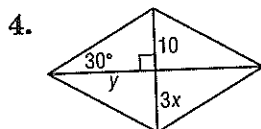
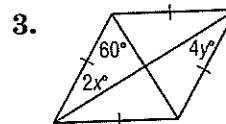
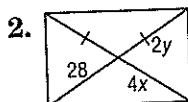
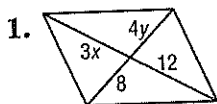
$$6x = 24 \quad 4y = 18$$

$$x = 4 \quad y = 4.5$$



Exercises

Find the value of each variable.



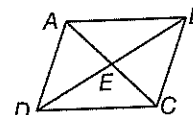
COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of $\square ABCD$ with the given vertices.

7. $A(3, 6)$, $B(5, 8)$, $C(3, -2)$, and $D(1, -4)$ 8. $A(-4, 3)$, $B(2, 3)$, $C(-1, -2)$, and $D(-7, -2)$

9. **PROOF** Write a paragraph proof of the following.

Given: $\square ABCD$

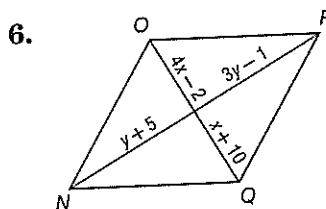
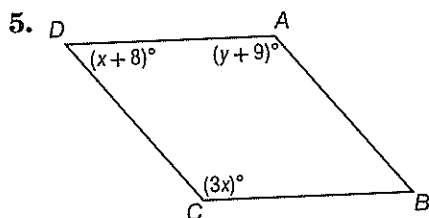
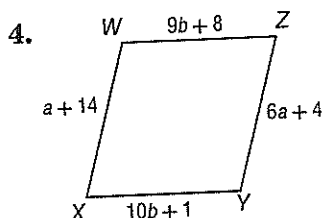
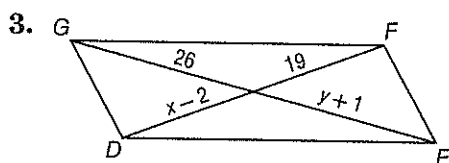
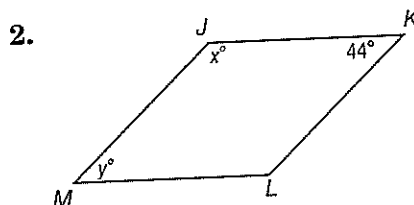
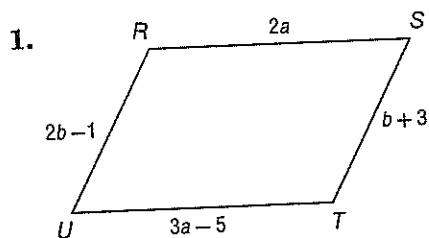
Prove: $\triangle AED \cong \triangle BEC$



6-2 Skills Practice

Parallelograms

ALGEBRA Find the value of each variable.



COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of $\square HJKL$ with the given vertices.

7. $H(1, 1), J(2, 3), K(6, 3), L(5, 1)$

8. $H(-1, 4), J(3, 3), K(3, -2), L(-1, -1)$

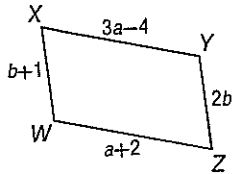
9. **PROOF** Write a paragraph proof of the theorem *Consecutive angles in a parallelogram are supplementary.*

6-2 Practice

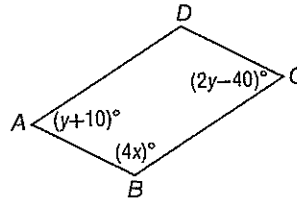
Parallelograms

ALGEBRA Find the value of each variable.

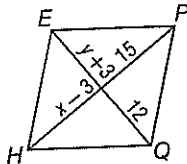
1.



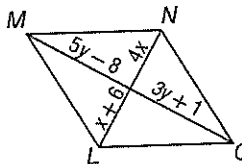
2.



3.



4.



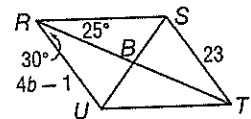
ALGEBRA Use $\square RSTU$ to find each measure or value.

5. $m\angle RST =$ _____

6. $m\angle STU =$ _____

7. $m\angle TUR =$ _____

8. $b =$ _____



COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of $\square PRYZ$ with the given vertices.

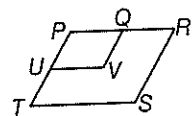
9. $P(2, 5), R(3, 3), Y(-2, -3), Z(-3, -1)$

10. $P(2, 3), R(1, -2), Y(-5, -7), Z(-4, -2)$

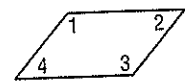
11. PROOF Write a paragraph proof of the following.

Given: $\square PRST$ and $\square PQVU$

Prove: $\angle V \cong \angle S$



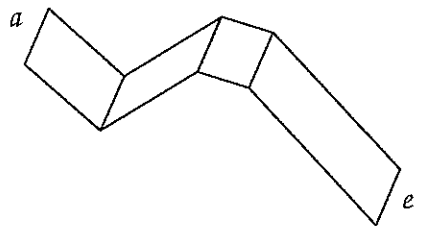
12. CONSTRUCTION Mr. Rodriguez used the parallelogram at the right to design a herringbone pattern for a paving stone. He will use the paving stone for a sidewalk. If $m\angle 1$ is 130, find $m\angle 2, m\angle 3,$ and $m\angle 4$.



6-2 Word Problem Practice

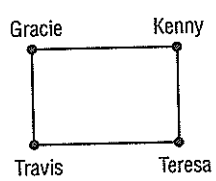
Parallelograms

1. **WALKWAY** A walkway is made by adjoining four parallelograms as shown.

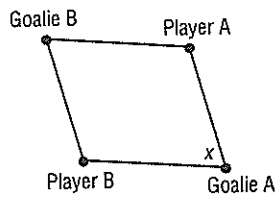


Are the end segments a and e parallel to each other? Explain.

2. **DISTANCE** Four friends live at the four corners of a block shaped like a parallelogram. Gracie lives 3 miles away from Kenny. How far apart do Teresa and Travis live from each other?



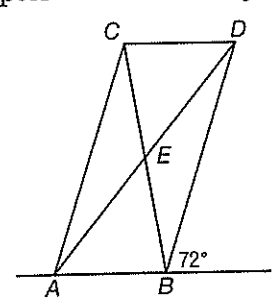
3. **SOCCER** Four soccer players are located at the corners of a parallelogram. Two of the players in opposite corners are the goalies. In order for goalie A to be able to see the three others, she must be able to see a certain angle x in her field of vision.



What angle does the other goalie have to be able to see in order to keep an eye on the other three players?

4. **VENN DIAGRAMS** Make a Venn diagram showing the relationship between squares, rectangles, and parallelograms.

5. **SKYSCRAPERS** On vacation, Tony's family took a helicopter tour of the city. The pilot said the newest building in the city was the building with this top view. He told Tony that the exterior angle by the front entrance is 72° . Tony wanted to know more about the building, so he drew this diagram and used his geometry skills to learn a few more things. The front entrance is next to vertex B .



- What are the measures of the four angles of the parallelogram?
- How many pairs of congruent triangles are there in the figure? What are they?

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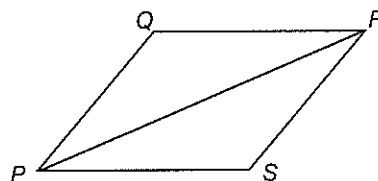
6-2 Enrichment

Diagonals of Parallelograms

In some drawings the diagonal of a parallelogram appears to be the angle bisector of both opposite angles. When might that be true?

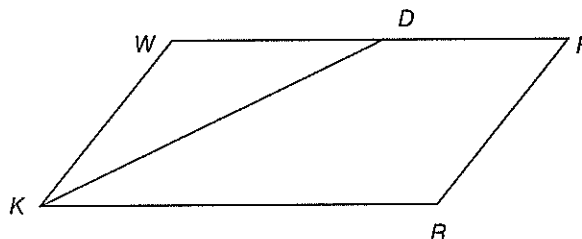
1. **Given:** Parallelogram $PQRS$ with diagonal \overline{PR} .
 \overline{PR} is an angle bisector of $\angle QPS$ and $\angle QRS$.

What type of parallelogram is $PQRS$? Justify your answer.



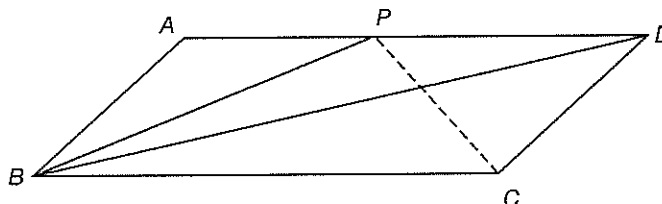
2. **Given:** Parallelogram $WPRK$ with angle bisector \overline{KD} , $DP = 5$, and $WD = 7$.

Find WK and KR .



3. Refer to Exercise 2. Write a statement about parallelogram $WPRK$ and angle bisector \overline{KD} .

4. **Given:** Parallelogram $ABCD$ with diagonal \overline{BD} and angle bisector \overline{BP} .
 $PD = 5$, $BP = 6$, and $CP = 6$.
 The perimeter of triangle PCD is 15.
 Find AB and BC .

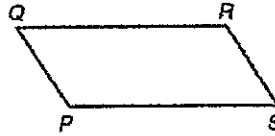


LESSON
6-3

Reteach

Conditions for Parallelograms

You can use the following conditions to determine whether a quadrilateral such as $PQRS$ is a parallelogram.

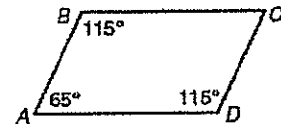


Conditions for Parallelograms	
<p style="text-align: center;"> $\overline{QR} \parallel \overline{SP}$ $\overline{QR} \cong \overline{SP}$ </p> <p>If one pair of opposite sides is \parallel and \cong, then $PQRS$ is a parallelogram.</p>	<p style="text-align: center;"> $\overline{QR} \cong \overline{SP}$ $\overline{PQ} \cong \overline{RS}$ </p> <p>If both pairs of opposite sides are \cong, then $PQRS$ is a parallelogram.</p>
<p style="text-align: center;"> $\angle P \cong \angle R$ $\angle Q \cong \angle S$ </p> <p>If both pairs of opposite angles are \cong, then $PQRS$ is a parallelogram.</p>	<p style="text-align: center;"> $\overline{PT} \cong \overline{RT}$ $\overline{QT} \cong \overline{ST}$ </p> <p>If the diagonals bisect each other, then $PQRS$ is a parallelogram.</p>

A quadrilateral is also a parallelogram if one of the angles is supplementary to both of its consecutive angles.

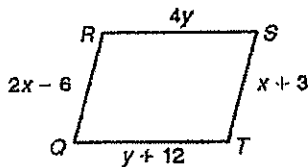
$65^\circ + 115^\circ = 180^\circ$, so $\angle A$ is supplementary to $\angle B$ and $\angle D$.

Therefore, $ABCD$ is a parallelogram.

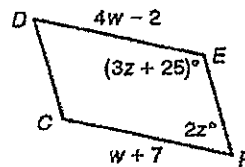


Show that each quadrilateral is a parallelogram for the given values. Explain.

1. Given: $x = 9$ and $y = 4$



2. Given: $w = 3$ and $z = 31$



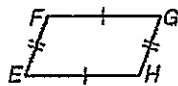
LESSON
6-3

Reteach

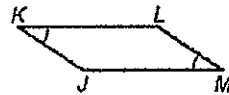
Conditions for Parallelograms *continued*

You can show that a quadrilateral is a parallelogram by using any of the conditions listed below.

Conditions for Parallelograms
<ul style="list-style-type: none"> • Both pairs of opposite sides are parallel (definition). • One pair of opposite sides is parallel and congruent. • Both pairs of opposite sides are congruent. • Both pairs of opposite angles are congruent. • The diagonals bisect each other. • One angle is supplementary to both its consecutive angles.



EFGH must be a parallelogram because both pairs of opposite sides are congruent.



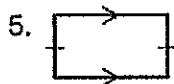
JKLM may not be a parallelogram because none of the sets of conditions for a parallelogram is met.

Determine whether each quadrilateral must be a parallelogram.

Justify your answer.









Show that the quadrilateral with the given vertices is a parallelogram by using the given definition or theorem.

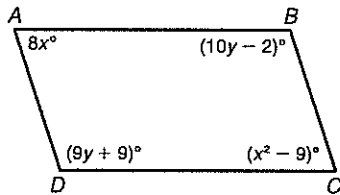
7. $J(-2, -2), K(-3, 3), L(1, 5), M(2, 0)$
Both pairs of opposite sides are parallel.

8. $N(5, 1), P(2, 7), Q(6, 9), R(9, 3)$
Both pairs of opposite sides are congruent.

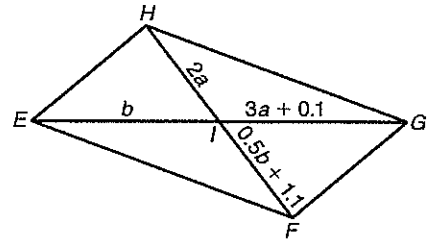
LESSON **Practice B**
6-3 **Conditions for Parallelograms**

For Exercises 1 and 2, determine whether the figure is a parallelogram for the given values of the variables. Explain your answers.

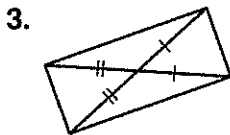
1. $x = 9$ and $y = 11$

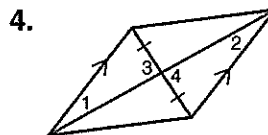


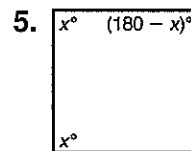
2. $a = 4.3$ and $b = 13$



Determine whether each quadrilateral must be a parallelogram. Justify your answers.







Use the given method to determine whether the quadrilateral with the given vertices is a parallelogram.

6. Find the slopes of all four sides: $J(-4, -1)$, $K(-7, -4)$, $L(2, -10)$, $M(5, -7)$

7. Find the lengths of all four sides: $P(2, 2)$, $Q(1, -3)$, $R(-4, 2)$, $S(-3, 7)$

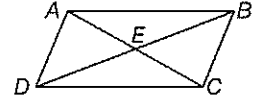
8. Find the slopes and lengths of one pair of opposite sides:

$T(\frac{3}{2}, -2)$, $U(\frac{3}{2}, 4)$, $V(-\frac{1}{2}, 0)$, $W(-\frac{1}{2}, -6)$

6-3 Study Guide and Intervention

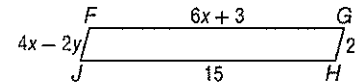
Tests for Parallelograms

Conditions for Parallelograms There are many ways to establish that a quadrilateral is a parallelogram.



If:	If:
both pairs of opposite sides are parallel,	$\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$,
both pairs of opposite sides are congruent,	$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$,
both pairs of opposite angles are congruent,	$\angle ABC \cong \angle ADC$ and $\angle DAB \cong \angle BCD$,
the diagonals bisect each other,	$\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$,
one pair of opposite sides is congruent and parallel,	$\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$, or $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$,
then: the figure is a parallelogram.	then: ABCD is a parallelogram.

Example Find x and y so that $FGHJ$ is a parallelogram.

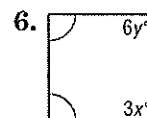
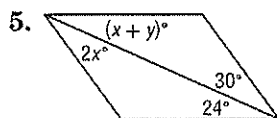
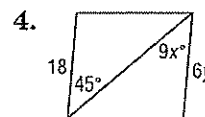
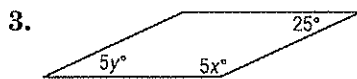
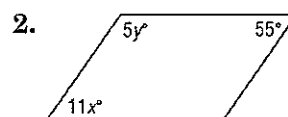
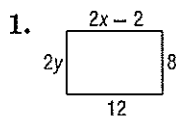


$FGHJ$ is a parallelogram if the lengths of the opposite sides are equal.

$$\begin{aligned}
 6x + 3 &= 15 & 4x - 2y &= 2 \\
 6x &= 12 & 4(2) - 2y &= 2 \\
 x &= 2 & 8 - 2y &= 2 \\
 & & -2y &= -6 \\
 & & y &= 3
 \end{aligned}$$

Exercises

Find x and y so that the quadrilateral is a parallelogram.



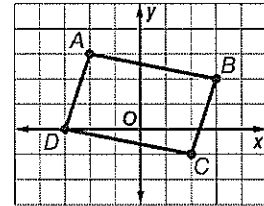
6-3 Study Guide and Intervention *(continued)*

Tests for Parallelograms

Parallelograms on the Coordinate Plane On the coordinate plane, the Distance, Slope, and Midpoint Formulas can be used to test if a quadrilateral is a parallelogram.

Example Determine whether $ABCD$ is a parallelogram.

The vertices are $A(-2, 3)$, $B(3, 2)$, $C(2, -1)$, and $D(-3, 0)$.



Method 1: Use the Slope Formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

$$\text{slope of } \overline{AD} = \frac{3 - 0}{-2 - (-3)} = \frac{3}{1} = 3 \quad \text{slope of } \overline{BC} = \frac{2 - (-1)}{3 - 2} = \frac{3}{1} = 3$$

$$\text{slope of } \overline{AB} = \frac{2 - 3}{3 - (-2)} = -\frac{1}{5} \quad \text{slope of } \overline{CD} = \frac{-1 - 0}{2 - (-3)} = -\frac{1}{5}$$

Since opposite sides have the same slope, $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$. Therefore, $ABCD$ is a parallelogram by definition.

Method 2: Use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$$AB = \sqrt{(-2 - 3)^2 + (3 - 2)^2} = \sqrt{25 + 1} \text{ or } \sqrt{26}$$

$$CD = \sqrt{(2 - (-3))^2 + (-1 - 0)^2} = \sqrt{25 + 1} \text{ or } \sqrt{26}$$

$$AD = \sqrt{(-2 - (-3))^2 + (3 - 0)^2} = \sqrt{1 + 9} \text{ or } \sqrt{10}$$

$$BC = \sqrt{(3 - 2)^2 + (2 - (-1))^2} = \sqrt{1 + 9} \text{ or } \sqrt{10}$$

Since both pairs of opposite sides have the same length, $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$. Therefore, $ABCD$ is a parallelogram by Theorem 6.9.

Exercises

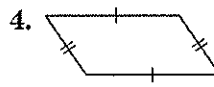
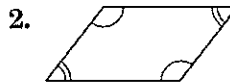
Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

- $A(0, 0)$, $B(1, 3)$, $C(5, 3)$, $D(4, 0)$;
Slope Formula
- $D(-1, 1)$, $E(2, 4)$, $F(6, 4)$, $G(3, 1)$;
Slope Formula
- $R(-1, 0)$, $S(3, 0)$, $T(2, -3)$, $U(-3, -2)$;
Distance Formula
- $A(-3, 2)$, $B(-1, 4)$, $C(2, 1)$, $D(0, -1)$;
Distance and Slope Formulas
- $S(-2, 4)$, $T(-1, -1)$, $U(3, -4)$, $V(2, 1)$;
Distance and Slope Formulas
- $F(3, 3)$, $G(1, 2)$, $H(-3, 1)$, $I(-1, 4)$;
Midpoint Formula
- A parallelogram has vertices $R(-2, -1)$, $S(2, 1)$, and $T(0, -3)$. Find all possible coordinates for the fourth vertex.

6-3 Skills Practice

Tests for Parallelograms

Determine whether each quadrilateral is a parallelogram. Justify your answer.



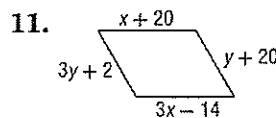
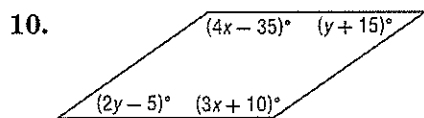
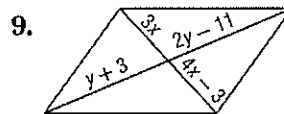
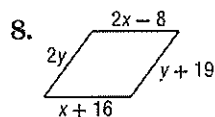
COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

5. $P(0, 0)$, $Q(3, 4)$, $S(7, 4)$, $Y(4, 0)$; Slope Formula

6. $S(-2, 1)$, $R(1, 3)$, $T(2, 0)$, $Z(-1, -2)$; Distance and Slope Formulas

7. $W(2, 5)$, $R(3, 3)$, $Y(-2, -3)$, $N(-3, 1)$; Midpoint Formula

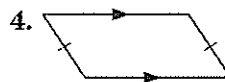
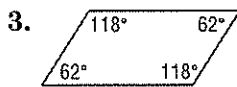
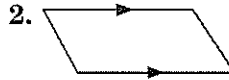
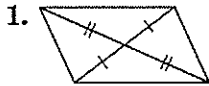
ALGEBRA Find x and y so that each quadrilateral is a parallelogram.



6-3 Practice

Tests for Parallelograms

Determine whether each quadrilateral is a parallelogram. Justify your answer.

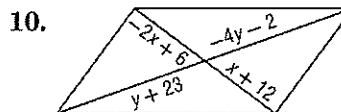
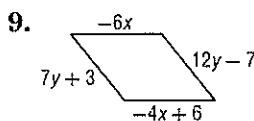
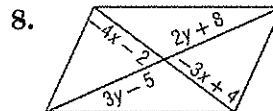
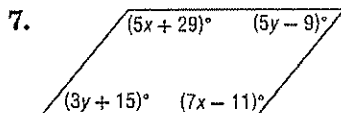


COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

5. $P(-5, 1)$, $S(-2, 2)$, $F(-1, -3)$, $T(2, -2)$; Slope Formula

6. $R(-2, 5)$, $O(1, 3)$, $M(-3, -4)$, $Y(-6, -2)$; Distance and Slope Formulas

ALGEBRA Find x and y so that the quadrilateral is a parallelogram.



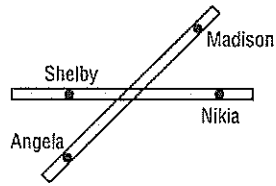
11. **TILE DESIGN** The pattern shown in the figure is to consist of congruent parallelograms. How can the designer be certain that the shapes are parallelograms?



6-3 Word Problem Practice

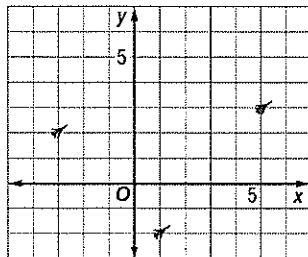
Tests for Parallelograms

1. **BALANCING** Nikia, Madison, Angela, and Shelby are balancing themselves on an "X"-shaped floating object. To balance themselves, they want to make themselves the vertices of a parallelogram.

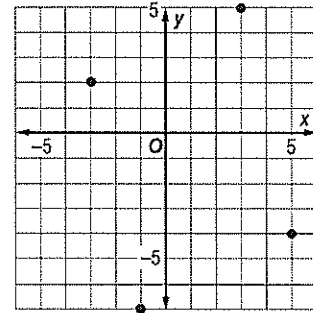


In order to achieve this, do all four of them have to be the same distance from the center of the object? Explain.

2. **COMPASSES** Two compass needles placed side by side on a table are both 2 inches long and point due north. Do they form the sides of a parallelogram?
3. **FORMATION** Four jets are flying in formation. Three of the jets are shown in the graph. If the four jets are located at the vertices of a parallelogram, what are the three possible locations of the missing jet?



4. **STREET LAMPS** When a coordinate plane is placed over the Harrisville town map, the four street lamps in the center are located as shown. Do the four lamps form the vertices of a parallelogram? Explain.



5. **PICTURE FRAME** Aaron is making a wooden picture frame in the shape of a parallelogram. He has two pieces of wood that are 3 feet long and two that are 4 feet long.
- If he connects the pieces of wood at their ends to each other, in what order must he connect them to make a parallelogram?
 - How many different parallelograms could he make with these four lengths of wood?
 - Explain something Aaron might do to specify precisely the shape of the parallelogram.

6-3 Enrichment**Tests for Parallelograms**

By definition, a quadrilateral is a parallelogram *if and only if* both pairs of opposite sides are parallel. What conditions other than both pairs of opposite sides parallel will guarantee that a quadrilateral is a parallelogram? In this activity, several possibilities will be investigated by drawing quadrilaterals to satisfy certain conditions. Remember that any test that seems to work is not guaranteed to work unless it can be formally proven.

Complete.

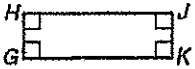
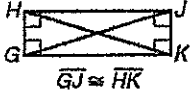
1. Draw a quadrilateral with one pair of opposite sides congruent.
Must it be a parallelogram?
2. Draw a quadrilateral with both pairs of opposite sides congruent.
Must it be a parallelogram?
3. Draw a quadrilateral with one pair of opposite sides parallel and the other pair of opposite sides congruent. Must it be a parallelogram?
4. Draw a quadrilateral with one pair of opposite sides both parallel and congruent. Must it be a parallelogram?
5. Draw a quadrilateral with one pair of opposite angles congruent.
Must it be a parallelogram?
6. Draw a quadrilateral with both pairs of opposite angles congruent.
Must it be a parallelogram?
7. Draw a quadrilateral with one pair of opposite sides parallel and one pair of opposite angles congruent. Must it be a parallelogram?

LESSON
6-4

Reteach

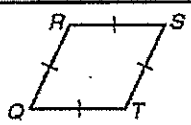
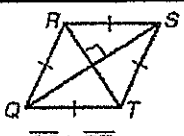
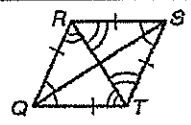
Properties of Special Parallelograms

A **rectangle** is a quadrilateral with four right angles. A rectangle has the following properties.

Properties of Rectangles	
 <p style="text-align: center;">$GHJK$ is a parallelogram.</p> <p>If a quadrilateral is a rectangle, then it is a parallelogram.</p>	 <p style="text-align: center;">$\overline{GJ} \cong \overline{HK}$</p> <p>If a parallelogram is a rectangle, then its diagonals are congruent.</p>

Since a rectangle is a parallelogram, a rectangle also has all the properties of parallelograms.

A **rhombus** is a quadrilateral with four congruent sides. A rhombus has the following properties.

Properties of Rhombuses		
 <p style="text-align: center;">$QRST$ is a parallelogram.</p> <p>If a quadrilateral is a rhombus, then it is a parallelogram.</p>	 <p style="text-align: center;">$\overline{QS} \perp \overline{RT}$</p> <p>If a parallelogram is a rhombus, then its diagonals are perpendicular.</p>	 <p style="text-align: center;">$\angle RQS \cong \angle SQT$</p> <p>If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.</p>

Since a rhombus is a parallelogram, a rhombus also has all the properties of parallelograms.

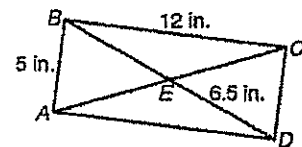
$ABCD$ is a rectangle. Find each length.

1. BD

2. CD

3. AC

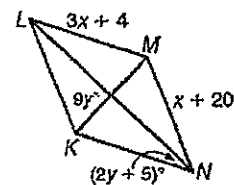
4. AE



$KLMN$ is a rhombus. Find each measure.

5. KL

6. $m\angle MNK$

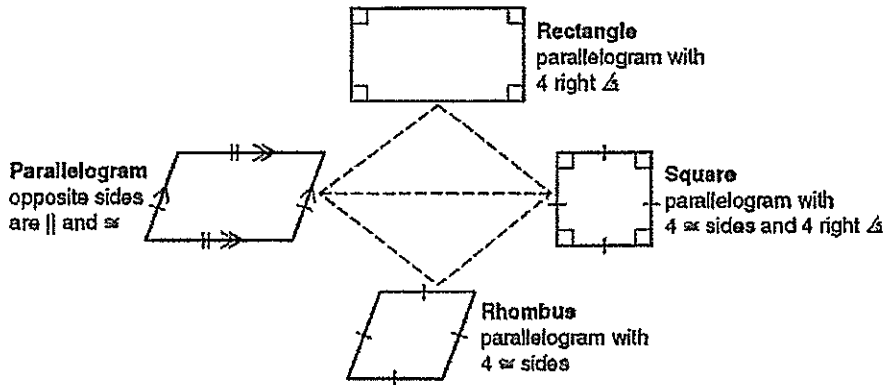


LESSON
6-4

Reteach

Properties of Special Parallelograms *continued*

A square is a quadrilateral with four right angles and four congruent sides.
A square is a parallelogram, a rectangle, and a rhombus.



Show that the diagonals of square $HJKL$ are congruent perpendicular bisectors of each other.

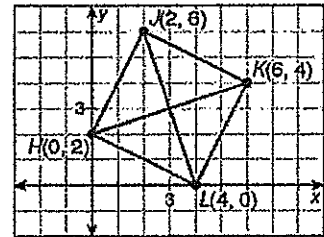
Step 1

Show that $\overline{HK} \cong \overline{JL}$.

$$HK = \sqrt{(6-0)^2 + (4-2)^2} = 2\sqrt{10}$$

$$JL = \sqrt{(4-2)^2 + (0-6)^2} = 2\sqrt{10}$$

$$HK = JL = 2\sqrt{10}, \text{ so } \overline{HK} \cong \overline{JL}.$$



Step 2

Show that $\overline{HK} \perp \overline{JL}$.

$$\text{slope of } \overline{HK} = \frac{4-2}{6-0} = \frac{1}{3} \quad \text{slope of } \overline{JL} = \frac{0-6}{4-2} = -3$$

Since the product of the slopes is -1 , $\overline{HK} \perp \overline{JL}$.

Step 3

Show that \overline{HK} and \overline{JL} bisect each other by comparing their midpoints.

$$\text{midpoint of } \overline{HK} = (3, 3) \quad \text{midpoint of } \overline{JL} = (3, 3)$$

Since they have the same midpoint, \overline{HK} and \overline{JL} bisect each other.

The vertices of square $ABCD$ are $A(-1, 0)$, $B(-4, 5)$, $C(1, 8)$, and $D(4, 3)$.

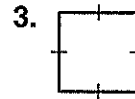
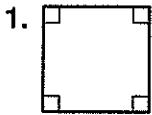
Show that each of the following is true.

7. The diagonals are congruent.

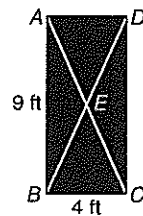
8. The diagonals are perpendicular bisectors of each other.

LESSON 6-4 Practice B
Properties of Special Parallelograms

Tell whether each figure must be a rectangle, rhombus, or square based on the information given. Use the most specific name possible.

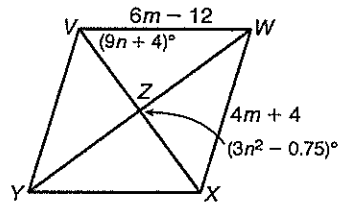


A modern artist's sculpture has rectangular faces. The face shown here is 9 feet long and 4 feet wide. Find each measure in simplest radical form. (*Hint: Use the Pythagorean Theorem.*)



4. $DC =$ _____ 5. $AD =$ _____
 6. $DB =$ _____ 7. $AE =$ _____

$VWXY$ is a rhombus. Find each measure.



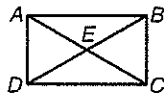
8. $XY =$ _____
 9. $m\angle YVW =$ _____
 10. $m\angle VYX =$ _____
 11. $m\angle XYZ =$ _____

12. The vertices of square $JKLM$ are $J(-2, 4)$, $K(-3, -1)$, $L(2, -2)$, and $M(3, 3)$. Find each of the following to show that the diagonals of square $JKLM$ are congruent perpendicular bisectors of each other.

- $JL =$ _____ $KM =$ _____
 slope of $\overline{JL} =$ _____ slope of $\overline{KM} =$ _____
 midpoint of $\overline{JL} =$ (_____, _____) midpoint of $\overline{KM} =$ (_____, _____)

Write a paragraph proof.

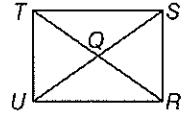
13. **Given:** $ABCD$ is a rectangle.
Prove: $\angle EDC \cong \angle ECD$



6-4 Study Guide and Intervention

Rectangles

Properties of Rectangles A **rectangle** is a quadrilateral with four right angles. Here are the properties of rectangles.



A rectangle has all the properties of a parallelogram.

- Opposite sides are parallel.
- Opposite angles are congruent.
- Opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Also:

- All four angles are right angles. $\angle UTS, \angle TSR, \angle SRU,$ and $\angle RUT$ are right angles.
- The diagonals are congruent. $\overline{TR} \cong \overline{US}$

Example 1 Quadrilateral *RUTS* above is a rectangle. If $US = 6x + 3$ and $RT = 7x - 2$, find x .

The diagonals of a rectangle are congruent, so $US = RT$.

$$\begin{aligned} 6x + 3 &= 7x - 2 \\ 3 &= x - 2 \\ 5 &= x \end{aligned}$$

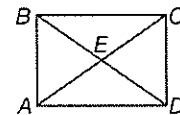
Example 2 Quadrilateral *RUTS* above is a rectangle. If $m\angle STR = 8x + 3$ and $m\angle UTR = 16x - 9$, find $m\angle STR$.

$\angle UTS$ is a right angle, so $m\angle STR + m\angle UTR = 90$.

$$\begin{aligned} 8x + 3 + 16x - 9 &= 90 \\ 24x - 6 &= 90 \\ 24x &= 96 \\ x &= 4 \\ m\angle STR &= 8x + 3 = 8(4) + 3 \text{ or } 35 \end{aligned}$$

Exercises

Quadrilateral *ABCD* is a rectangle.



1. If $AE = 36$ and $CE = 2x - 4$, find x .
2. If $BE = 6y + 2$ and $CE = 4y + 6$, find y .
3. If $BC = 24$ and $AD = 5y - 1$, find y .
4. If $m\angle BEA = 62$, find $m\angle BAC$.
5. If $m\angle AED = 12x$ and $m\angle BEC = 10x + 20$, find $m\angle AED$.
6. If $BD = 8y - 4$ and $AC = 7y + 3$, find BD .
7. If $m\angle DBC = 10x$ and $m\angle ACB = 4x^2 - 6$, find $m\angle ACB$.
8. If $AB = 6y$ and $BC = 8y$, find BD in terms of y .

6-4 Study Guide and Intervention *(continued)*

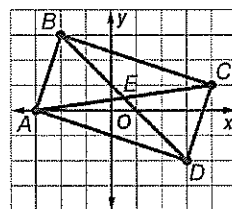
Rectangles

Prove that Parallelograms Are Rectangles The diagonals of a rectangle are congruent, and the converse is also true.

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

In the coordinate plane you can use the Distance Formula, the Slope Formula, and properties of diagonals to show that a figure is a rectangle.

Example Quadrilateral $ABCD$ has vertices $A(-3, 0)$, $B(-2, 3)$, $C(4, 1)$, and $D(3, -2)$. Determine whether $ABCD$ is a rectangle.



Method 1: Use the Slope Formula.

$$\text{slope of } \overline{AB} = \frac{3 - 0}{-2 - (-3)} = \frac{3}{1} \text{ or } 3 \quad \text{slope of } \overline{AD} = \frac{-2 - 0}{3 - (-3)} = \frac{-2}{6} \text{ or } -\frac{1}{3}$$

$$\text{slope of } \overline{CD} = \frac{-2 - 1}{3 - 4} = \frac{-3}{-1} \text{ or } 3 \quad \text{slope of } \overline{BC} = \frac{1 - 3}{4 - (-2)} = \frac{-2}{6} \text{ or } -\frac{1}{3}$$

Opposite sides are parallel, so the figure is a parallelogram. Consecutive sides are perpendicular, so $ABCD$ is a rectangle.

Method 2: Use the Distance Formula.

$$AB = \sqrt{(-3 - (-2))^2 + (0 - 3)^2} \text{ or } \sqrt{10}$$

$$BC = \sqrt{(-2 - 4)^2 + (3 - 1)^2} \text{ or } \sqrt{40}$$

$$CD = \sqrt{(4 - 3)^2 + (1 - (-2))^2} \text{ or } \sqrt{10}$$

$$AD = \sqrt{(-3 - 3)^2 + (0 - (-2))^2} \text{ or } \sqrt{40}$$

Opposite sides are congruent, thus $ABCD$ is a parallelogram.

$$AC = \sqrt{(-3 - 4)^2 + (0 - 1)^2} \text{ or } \sqrt{50}$$

$$BD = \sqrt{(-2 - 3)^2 + (3 - (-2))^2} \text{ or } \sqrt{50}$$

$ABCD$ is a parallelogram with congruent diagonals, so $ABCD$ is a rectangle.

Exercises

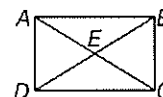
COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

- $A(-3, 1)$, $B(-3, 3)$, $C(3, 3)$, $D(3, 1)$; Distance Formula
- $A(-3, 0)$, $B(-2, 3)$, $C(4, 5)$, $D(3, 2)$; Slope Formula
- $A(-3, 0)$, $B(-2, 2)$, $C(3, 0)$, $D(2, -2)$; Distance Formula
- $A(-1, 0)$, $B(0, 2)$, $C(4, 0)$, $D(3, -2)$; Distance Formula

6-4 Skills Practice

Rectangles

ALGEBRA Quadrilateral $ABCD$ is a rectangle.

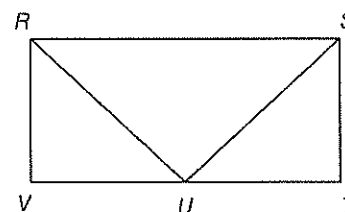


- If $AC = 2x + 13$ and $DB = 4x - 1$, find DB .
- If $AC = x + 3$ and $DB = 3x - 19$, find AC .
- If $AE = 3x + 3$ and $EC = 5x - 15$, find AC .
- If $DE = 6x - 7$ and $AE = 4x + 9$, find DB .
- If $m\angle DAC = 2x + 4$ and $m\angle BAC = 3x + 1$, find $m\angle BAC$.
- If $m\angle BDC = 7x + 1$ and $m\angle ADB = 9x - 7$, find $m\angle BDC$.
- If $m\angle ABD = 7x - 31$ and $m\angle CDB = 4x + 5$, find $m\angle ABD$.
- If $m\angle BAC = x + 3$ and $m\angle CAD = x + 15$, find $m\angle BAC$.

9. PROOF: Write a two-column proof.

Given: $RSTV$ is a rectangle and U is the midpoint of \overline{VT} .

Prove: $\triangle RUV \cong \triangle SUT$



Statements	Reasons

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

- $P(-3, -2)$, $Q(-4, 2)$, $R(2, 4)$, $S(3, 0)$; Slope Formula
- $J(-6, 3)$, $K(0, 6)$, $L(2, 2)$, $M(-4, -1)$; Distance Formula
- $T(4, 1)$, $U(3, -1)$, $X(-3, 2)$, $Y(-2, 4)$; Distance Formula

6-4 Practice

Rectangles

ALGEBRA Quadrilateral $RSTU$ is a rectangle.

1. If $UZ = x + 21$ and $ZS = 3x - 15$, find US .

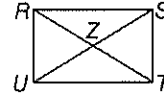
2. If $RZ = 3x + 8$ and $ZS = 6x - 28$, find UZ .

3. If $RT = 5x + 8$ and $RZ = 4x + 1$, find ZT .

4. If $m\angle SUT = 3x + 6$ and $m\angle RUS = 5x - 4$, find $m\angle SUT$.

5. If $m\angle SRT = x + 9$ and $m\angle UTR = 2x - 44$, find $m\angle UTR$.

6. If $m\angle RSU = x + 41$ and $m\angle TUS = 3x + 9$, find $m\angle RSU$.



Quadrilateral $GHJK$ is a rectangle. Find each measure if $m\angle 1 = 37$.

7. $m\angle 2$

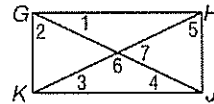
8. $m\angle 3$

9. $m\angle 4$

10. $m\angle 5$

11. $m\angle 6$

12. $m\angle 7$



COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

13. $B(-4, 3)$, $G(-2, 4)$, $H(1, -2)$, $L(-1, -3)$; Slope Formula

14. $N(-4, 5)$, $O(6, 0)$, $P(3, -6)$, $Q(-7, -1)$; Distance Formula

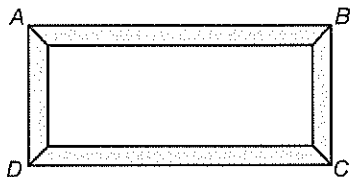
15. $C(0, 5)$, $D(4, 7)$, $E(5, 4)$, $F(1, 2)$; Slope Formula

16. **LANDSCAPING** Huntington Park officials approved a rectangular plot of land for a Japanese Zen garden. Is it sufficient to know that opposite sides of the garden plot are congruent and parallel to determine that the garden plot is rectangular? Explain.

6-4 Word Problem Practice

Rectangles

1. **FRAMES** Jalen makes the rectangular frame shown.

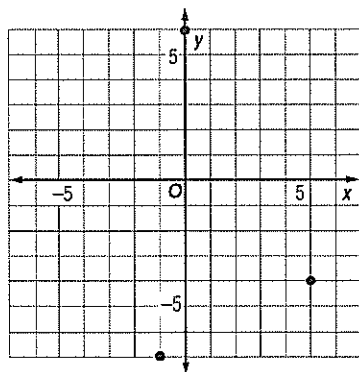


In order to make sure that it is a rectangle, Jalen measures the distances BD and AC . How should these two distances compare if the frame is a rectangle?

2. **BOOKSHELVES** A bookshelf consists of two vertical planks with five horizontal shelves. Are each of the four sections for books rectangles? Explain.

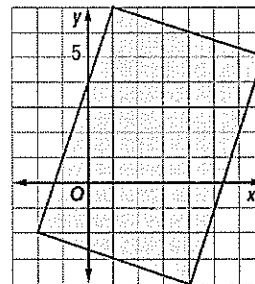


3. **LANDSCAPING** A landscaper is marking off the corners of a rectangular plot of land. Three of the corners are in place as shown.

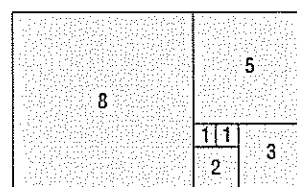


What are the coordinates of the fourth corner?

4. **SWIMMING POOLS** Antonio is designing a swimming pool on a coordinate grid. Is it a rectangle? Explain.



5. **PATTERNS** Veronica made the pattern shown out of 7 rectangles with four equal sides. The side length of each rectangle is written inside the rectangle.



- How many rectangles can be formed using the lines in this figure?
- If Veronica wanted to extend her pattern by adding another rectangle with 4 equal sides to make a larger rectangle, what are the possible side lengths of rectangles that she can add?

6-4 Enrichment

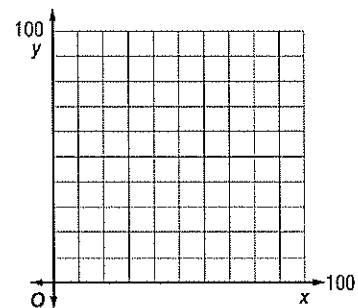
Constant Perimeter

Douglas wants to fence a rectangular region of his back yard for his dog. He bought 200 feet of fence.

- Complete the table to show the dimensions of five different rectangular pens that would use the entire 200 feet of fence. Then find the area of each rectangular pen.
- Do all five of the rectangular pens have the same area? If not, which one has the larger area?

Perimeter	Length	Width	Area
200	80		
200	70		
200	60		
200	50		
200	45		

- Write a rule for finding the dimensions of a rectangle with the largest possible area for a given perimeter.
- Let x represent the length of a rectangle and y the width. Write the formula for all rectangles with a perimeter of 200. Then graph this relationship on the coordinate plane at the right.

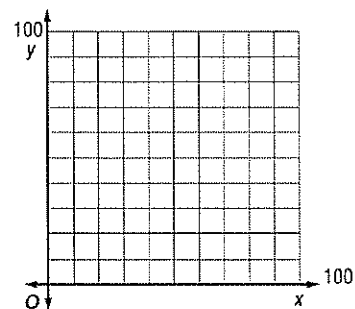


Julio read that a dog the size of his new pet, Bennie, should have at least 100 square feet in his pen. Before going to the store to buy fence, Julio made a table to determine the dimensions for Bennie's rectangular pen.

- Complete the table to find five possible dimensions of a rectangular fenced area of 100 square feet.
- Julio wants to save money by purchasing the least number feet of fencing to enclose the 100 square feet. What will be the dimensions of the completed pen?

Area	Length	Width	How much fence to buy
100			
100			
100			
100			
100			

- Write a rule for finding the dimensions of a rectangle with the least possible perimeter for a given area.
- For length x and width y , write a formula for the area of a rectangle with an area of 100 square feet. Then graph the formula.



6-4 Graphing Calculator Activity**TI-Nspire: Exploring Rectangles**

A quadrilateral with four right angles is a **rectangle**. The TI-Nspire can be used to explore some of the characteristics of a rectangle. Use the following steps to draw a rectangle.

Step 1 Set up the calculator in the correct mode.

- Choose **Graphs & Geometry** from the Home Menu.
- From the View menu, choose **4: Hide Axis**

Step 2 Draw the rectangle

- From the **8: Shapes** menu choose **3: Rectangle**.
- Click once to define the corner of the rectangle. Then move and click again. The side of the rectangle is now defined. Move perpendicularly to draw the rectangle. Click to anchor the shape.

Step 3 Measure the lengths of the sides of the rectangle.

- From the **7: Measurement** menu choose **1: Length** (Note that when you scroll over the rectangle, the value now shown is the perimeter of the rectangle.)
- Select each endpoint of a segment of the rectangle. Then click or press Enter to anchor the length of the segment in the work area.
- Repeat for the other sides of the rectangle.

Exercises

1. What appears to be true about the opposite sides of the rectangle?
2. Draw the diagonals of the rectangle using **5: Segment** from the **6: Points and Lines** Menu. Click on two opposite vertices to draw the diagonal. Repeat to draw the other diagonal.
 - a. Measure each diagonal using the measurement tool. What do you observe?
 - b. What is true about the triangles formed by the sides of the rectangle and a diagonal? Justify your conclusion.
3. Press **Clear** three times and select **Yes** to clear the screen. Repeat the steps and draw another rectangle. Do the relationships that you found for the first rectangle you drew hold true for this rectangle?

6-4 Geometer's Sketchpad Activity

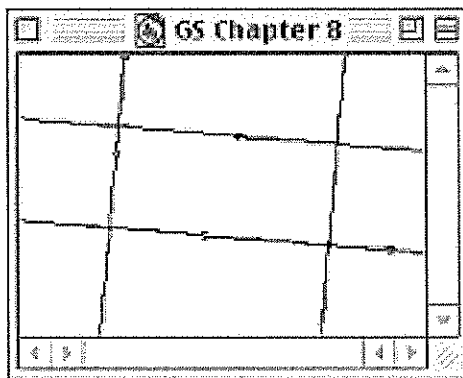
Exploring Rectangles

A quadrilateral with four right angles is a **rectangle**. The Geometer's Sketchpad is a useful tool for exploring some of the characteristics of a rectangle. Use the following steps to draw a rectangle.

Step 1 Use the Line tool to draw a line anywhere on the screen.

Step 2 Use the Point tool to draw a point that is not on the line. To draw a line perpendicular to the first line you drew, select the first line and the point. Then choose **Perpendicular Line** from the **Construct** menu.

Step 3 Use the Point tool to draw a point that is not on either of the lines you have drawn. Repeat the procedure in Step 2 to draw lines perpendicular to the two lines you have drawn.



A rectangle is formed by the segments whose endpoints are the points of intersection of the lines.

Exercises

Use the measuring capabilities of The Geometer's Sketchpad to explore the characteristics of a rectangle.


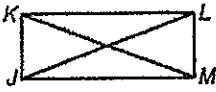
- What appears to be true about the opposite sides of the rectangle that you drew? Make a conjecture and then measure each side to check your conjecture.
- Draw the diagonals of the rectangle by using the Selection Arrow tool to choose two opposite vertices. Then choose **Segment** from the **Construct** menu to draw the diagonal. Repeat to draw the other diagonal.
 - Measure each diagonal. What do you observe?
 - What is true about the triangles formed by the sides of the rectangle and a diagonal? Justify your conclusion.
- Choose **New Sketch** from the **File** menu and follow steps 1–3 to draw another rectangle. Do the relationships you found for the first rectangle you drew hold true for this rectangle also?

LESSON
6-5

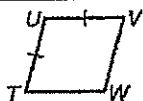


Reteach

Conditions for Special Parallelograms

You can use the following conditions to determine whether a parallelogram is a rectangle.

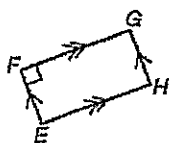
	 $\overline{JL} \cong \overline{KM}$
If one angle is a right angle, then $\square JKLM$ is a rectangle.	If the diagonals are congruent, then $\square JKLM$ is a rectangle.

You can use the following conditions to determine whether a parallelogram is a rhombus.

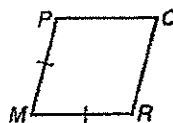
		
If one pair of consecutive sides are congruent, then $\square TUVW$ is a rhombus.	If the diagonals are perpendicular, then $\square TUVW$ is a rhombus.	If one diagonal bisects a pair of opposite angles, then $\square TUVW$ is a rhombus.

Determine whether the conclusion is valid. If not, tell what additional information is needed to make it valid.

1. $EFGH$ is a rectangle.



2. $MPQR$ is a rhombus.



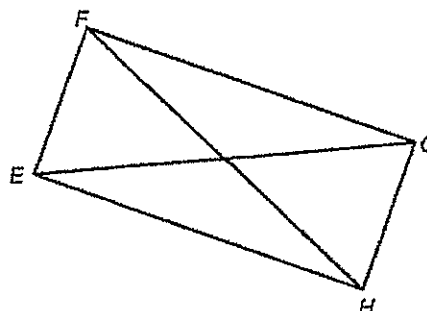
For Exercises 3 and 4, use the figure to determine whether the conclusion is valid. If not, tell what additional information is needed to make it valid.

3. **Given:** $\overline{EF} \parallel \overline{GH}$, $\overline{HE} \parallel \overline{FG}$, $\overline{EG} \cong \overline{FH}$

Conclusion: $EFGH$ is a rectangle.

4. **Given:** $m\angle EFG = 90^\circ$

Conclusion: $EFGH$ is a rectangle.



LESSON
6-5

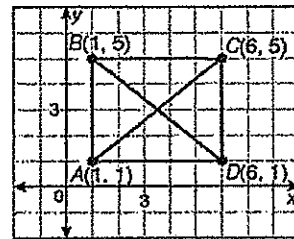
Reteach

Conditions for Special Parallelograms *continued*

You can identify special parallelograms in the coordinate plane by examining their diagonals.

If the Diagonals are the Parallelogram is a
congruent	rectangle
perpendicular	rhombus
congruent and perpendicular	square

Use the diagonals to determine whether parallelogram $ABCD$ is a rectangle, rhombus, or square. Give all the names that apply.



Step 1 Find AC and BD to determine whether $ABCD$ is a rectangle.

$$AC = \sqrt{(6-1)^2 + (5-1)^2} = \sqrt{41}$$

$$BD = \sqrt{(6-1)^2 + (1-5)^2} = \sqrt{41}$$

Since $\sqrt{41} = \sqrt{41}$, the diagonals are congruent. So $ABCD$ is a rectangle.

Step 2 Find the slopes of \overline{AC} and \overline{BD} to determine whether $ABCD$ is a rhombus.

$$\text{slope of } \overline{AC} = \frac{5-1}{6-1} = \frac{4}{5}$$

$$\text{slope of } \overline{BD} = \frac{1-5}{6-1} = -\frac{4}{5}$$

Since $\left(\frac{4}{5}\right)\left(-\frac{4}{5}\right) \neq -1$, the diagonals are not perpendicular. So $ABCD$ is not a rhombus and cannot be a square.

Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

5. $V(3, 0), W(6, 4), X(11, 4), Y(8, 0)$

6. $L(1, 2), M(3, 5), N(6, 3), P(4, 0)$

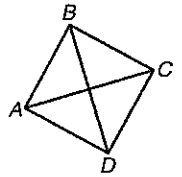
7. $H(1, 3), J(10, 6), K(12, 0), L(3, -3)$

8. $E(-4, 3), F(-1, 2), G(-2, -1), H(-5, 0)$

LESSON **Practice B**
6-5 **Conditions for Special Parallelograms**

1. On the National Mall in Washington, D.C., a reflecting pool lies between the Lincoln Memorial and the World War II Memorial. The pool has two 2300-foot-long sides and two 150-foot-long sides. Tell what additional information you need to know in order to determine whether the reflecting pool is a rectangle. (*Hint: Remember that you have to show it is a parallelogram first.*)

Use the figure for Exercises 2–5. Determine whether each conclusion is valid. If not, tell what additional information is needed to make it valid.



2. **Given:** \overline{AC} and \overline{BD} bisect each other. $\overline{AC} \cong \overline{BD}$
Conclusion: $ABCD$ is a square.

3. **Given:** $\overline{AC} \perp \overline{BD}$, $\overline{AB} \cong \overline{BC}$
Conclusion: $ABCD$ is a rhombus.

4. **Given:** $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$, $m\angle ADB = m\angle ABD = 45^\circ$
Conclusion: $ABCD$ is a square.

5. **Given:** $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \cong \overline{BC}$, $\overline{AC} \cong \overline{BD}$
Conclusion: $ABCD$ is a rectangle.

Find the lengths and slopes of the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all names that apply.

6. $E(-2, -4)$, $F(0, -1)$, $G(-3, 1)$, $H(-5, -2)$

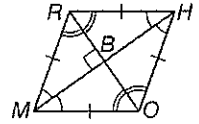
$EG =$ _____	$FH =$ _____
slope of $\overline{EG} =$ _____	slope of $\overline{FH} =$ _____
7. $P(-1, 3)$, $Q(-2, 5)$, $R(0, 4)$, $S(1, 2)$

$PR =$ _____	$QS =$ _____
slope of $\overline{PR} =$ _____	slope of $\overline{QS} =$ _____

6-5 Study Guide and Intervention

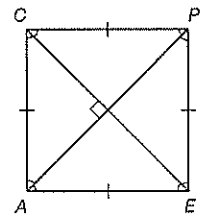
Rhombi and Squares

Properties of Rhombi and Squares A **rhombus** is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.



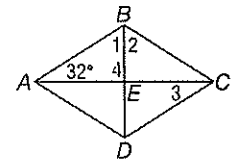
The diagonals are perpendicular.	$\overline{MH} \perp \overline{RO}$
Each diagonal bisects a pair of opposite angles.	\overline{MH} bisects $\angle RMO$ and $\angle RHO$. \overline{RO} bisects $\angle MRH$ and $\angle MOH$.

A **square** is a parallelogram with four congruent sides and four congruent angles. A square is both a rectangle and a rhombus; therefore, all properties of parallelograms, rectangles, and rhombi apply to squares.



Example In rhombus $ABCD$, $m\angle BAC = 32$. Find the measure of each numbered angle.

$ABCD$ is a rhombus, so the diagonals are perpendicular and $\triangle ABE$ is a right triangle. Thus $m\angle 4 = 90$ and $m\angle 1 = 90 - 32$ or 58 . The diagonals in a rhombus bisect the vertex angles, so $m\angle 1 = m\angle 2$. Thus, $m\angle 2 = 58$.

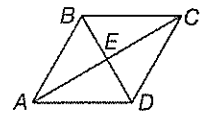


A rhombus is a parallelogram, so the opposite sides are parallel. $\angle BAC$ and $\angle 3$ are alternate interior angles for parallel lines, so $m\angle 3 = 32$.

Exercises

Quadrilateral $ABCD$ is a rhombus. Find each value or measure.

- If $m\angle ABD = 60$, find $m\angle BDC$.
- If $AE = 8$, find AC .
- If $AB = 26$ and $BD = 20$, find AE .
- Find $m\angle CEB$.
- If $m\angle CBD = 58$, find $m\angle ACB$.
- If $AE = 3x - 1$ and $AC = 16$, find x .
- If $m\angle CDB = 6y$ and $m\angle ACB = 2y + 10$, find y .
- If $AD = 2x + 4$ and $CD = 4x - 4$, find x .



6-5 Study Guide and Intervention *(continued)*

Rhombi and Squares

Conditions for Rhombi and Squares The theorems below can help you prove that a parallelogram is a rectangle, rhombus, or square.

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.

If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.

If a quadrilateral is both a rectangle and a rhombus, then it is a square.

Example Determine whether parallelogram $ABCD$ with vertices $A(-3, -3)$, $B(1, 1)$, $C(5, -3)$, $D(1, -7)$ is a rhombus, a rectangle, or a square.

$$AC = \sqrt{(-3 - 5)^2 + ((-3 - (-3)))^2} = \sqrt{64} = 8$$

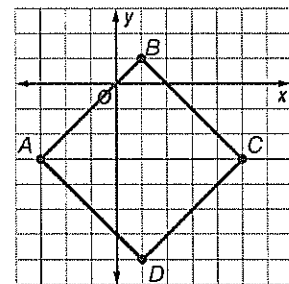
$$BD = \sqrt{(1 - 1)^2 + (-7 - 1)^2} = \sqrt{64} = 8$$

The diagonals are the same length; the figure is a rectangle.

$$\text{Slope of } \overline{AC} = \frac{-3 - (-3)}{-3 - 5} = \frac{0}{-8} = 8 \quad \text{The line is horizontal.}$$

$$\text{Slope of } \overline{BD} = \frac{1 - (-7)}{1 - 1} = \frac{8}{0} = \text{undefined} \quad \text{The line is vertical.}$$

Since a horizontal and vertical line are perpendicular, the diagonals are perpendicular. Parallelogram $ABCD$ is a square which is also a rhombus and a rectangle.

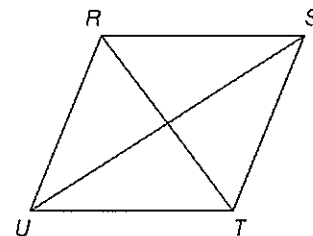


Exercises

Given each set of vertices, determine whether $\square ABCD$ is a rhombus, rectangle, or square. List all that apply. Explain.

- $A(0, 2)$, $B(2, 4)$, $C(4, 2)$, $D(2, 0)$
- $A(-2, 1)$, $B(-1, 3)$, $C(3, 1)$, $D(2, -1)$
- $A(-2, -1)$, $B(0, 2)$, $C(2, -1)$, $D(0, -4)$
- $A(-3, 0)$, $B(-1, 3)$, $C(5, -1)$, $D(3, -4)$

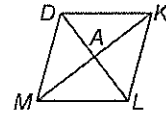
5. **PROOF** Write a two-column proof.
Given: Parallelogram $RSTU$. $\overline{RS} \cong \overline{ST}$
Prove: $RSTU$ is a rhombus.



6-5 Skills Practice

Rhombi and Squares

ALGEBRA Quadrilateral $DKLM$ is a rhombus.

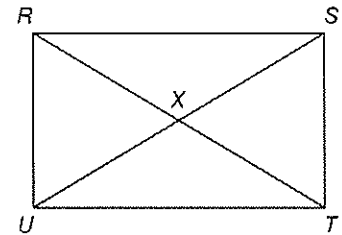


- If $DK = 8$, find KL .
- If $m\angle DML = 82$ find $m\angle DKM$.
- If $m\angle KAL = 2x - 8$, find x .
- If $DA = 4x$ and $AL = 5x - 3$, find DL .
- If $DA = 4x$ and $AL = 5x - 3$, find AD .
- If $DM = 5y + 2$ and $DK = 3y + 6$, find KL .
- PROOF** Write a two-column proof.

Given: $RSTU$ is a parallelogram.

$$\overline{RX} \cong \overline{TX} \cong \overline{SX} \cong \overline{UX}$$

Prove: $RSTU$ is a rectangle.



Statements	Reasons

COORDINATE GEOMETRY Given each set of vertices, determine whether $\square QRST$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.

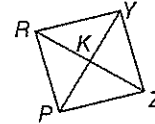
- $Q(3, 5), R(3, 1), S(-1, 1), T(-1, 5)$
- $Q(-5, 12), R(5, 12), S(-1, 4), T(-11, 4)$
- $Q(-6, -1), R(4, -6), S(2, 5), T(-8, 10)$
- $Q(2, -4), R(-6, -8), S(-10, 2), T(-2, 6)$

6-5 Practice

Rhombi and Squares

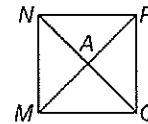
$PRYZ$ is a rhombus. If $RK = 5$, $RY = 13$ and $m\angle YRZ = 67$, find each measure.

1. KY
2. PK
3. $m\angle YKZ$
4. $m\angle PZR$



$MNPQ$ is a rhombus. If $PQ = 3\sqrt{2}$ and $AP = 3$, find each measure.

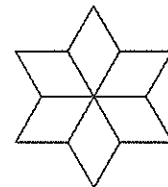
5. AQ
6. $m\angle APQ$
7. $m\angle MNP$
8. PM



COORDINATE GEOMETRY Given each set of vertices, determine whether $\square BEFG$ is a rhombus, a rectangle, or a square. List all that apply. Explain.

9. $B(-9, 1)$, $E(2, 3)$, $F(12, -2)$, $G(1, -4)$
10. $B(1, 3)$, $E(7, -3)$, $F(1, -9)$, $G(-5, -3)$
11. $B(-4, -5)$, $E(1, -5)$, $F(-2, -1)$, $G(-7, -1)$

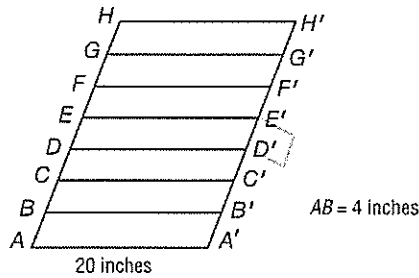
12. **TESSELLATIONS** The figure is an example of a tessellation. Use a ruler or protractor to measure the shapes and then name the quadrilaterals used to form the figure.



6-5 Word Problem Practice

Rhombi and Squares

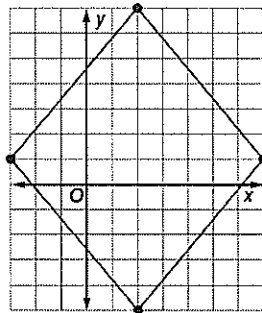
1. **TRAY RACKS** A tray rack looks like a parallelogram from the side. The levels for the trays are evenly spaced.



What two labeled points form a rhombus with base AA' ?

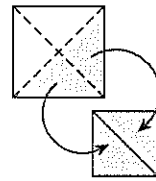
2. **SLICING** Charles cuts a rhombus along both diagonals. He ends up with four congruent triangles. Classify these triangles as *acute*, *obtuse*, or *right*.

3. **WINDOWS** The edges of a window are drawn in the coordinate plane.

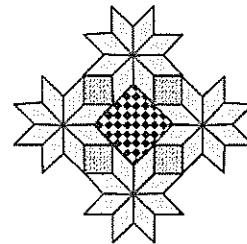


Determine whether the window is a square or a rhombus.

4. **SQUARES** Mackenzie cut a square along its diagonals to get four congruent right triangles. She then joined two of them along their long sides. Show that the resulting shape is a square.



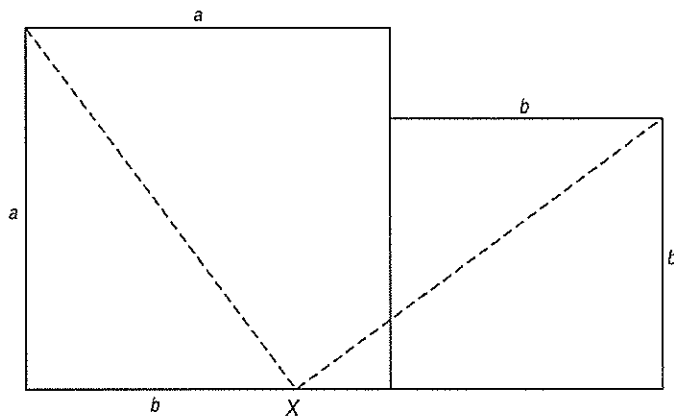
5. **DESIGN** Tatianna made the design shown. She used 32 congruent rhombi to create the flower-like design at each corner.



- What are the angles of the corner rhombi?
- What kinds of quadrilaterals are the dotted and checkered figures?

6-5 Enrichment**Creating Pythagorean Puzzles**

By drawing two squares and cutting them in a certain way, you can make a puzzle that demonstrates the Pythagorean Theorem. A sample puzzle is shown. You can create your own puzzle by following the instructions below.



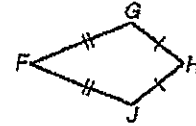
- Carefully construct a square and label the length of a side as a . Then construct a smaller square to the right of it and label the length of a side as b , as shown in the figure above. The bases should be adjacent and collinear.
- Mark a point X that is b units from the left edge of the larger square. Then draw the segments from the upper left corner of the larger square to point X , and from point X to the upper right corner of the smaller square.
- Cut out and rearrange your five pieces to form a larger square. Draw a diagram to show your answer.
- Verify that the length of each side is equal to $\sqrt{a^2 + b^2}$.

LESSON
6-6

Reteach

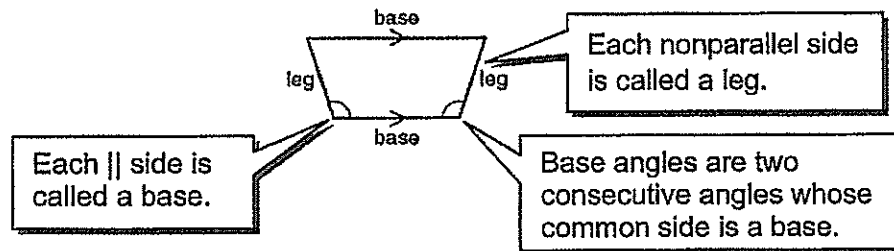
Properties of Kites and Trapezoids

A kite is a quadrilateral with exactly two pairs of congruent consecutive sides. If a quadrilateral is a kite, such as $FGHJ$, then it has the following properties.



Properties of Kites	
<p>The diagonals are perpendicular.</p>	$\overline{FH} \perp \overline{GJ}$
<p>Exactly one pair of opposite angles is congruent.</p>	$\angle G \cong \angle J$

A trapezoid is a quadrilateral with exactly one pair of parallel sides. If the legs of a trapezoid are congruent, the trapezoid is an **isosceles trapezoid**.



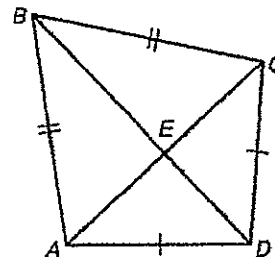
Isosceles Trapezoid Theorems
<ul style="list-style-type: none"> • In an isosceles trapezoid, each pair of base angles is congruent. • If a trapezoid has one pair of congruent base angles, then it is isosceles. • A trapezoid is isosceles if and only if its diagonals are congruent.

In kite $ABCD$, $m\angle BCD = 98^\circ$, and $m\angle ADE = 47^\circ$. Find each measure.

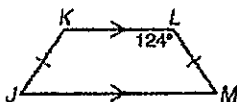
- $m\angle DAE$

- $m\angle BCE$

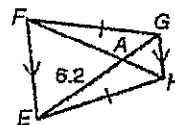
- $m\angle ABC$



4. Find $m\angle J$ in trapezoid $JKLM$.



5. In trapezoid $EFGH$, $FH = 9$. Find AG .



LESSON
6-6

Reteach

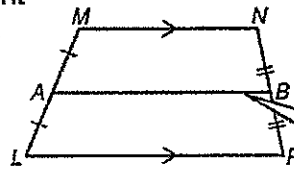
Properties of Kites and Trapezoids *continued*

Trapezoid Midsegment Theorem

The **midsegment of a trapezoid** is the segment whose endpoints are the midpoints of the legs.

- The midsegment of a trapezoid is parallel to each base. $\overline{AB} \parallel \overline{MN}$ and $\overline{AB} \parallel \overline{LP}$
- The length of the midsegment is one-half the sum of the length of the bases.

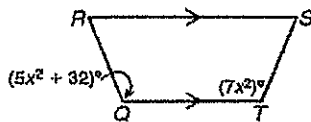
$$AB = \frac{1}{2}(MN + LP)$$



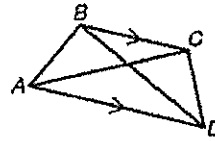
\overline{AB} is the midsegment of $LMNP$.

Find each value so that the trapezoid is isosceles.

6. Find the value of x .

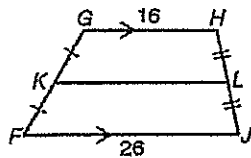


7. $AC = 2z + 9$, $BD = 4z - 3$. Find the value of z .

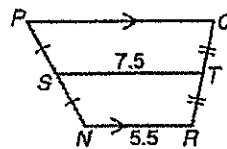


Find each length.

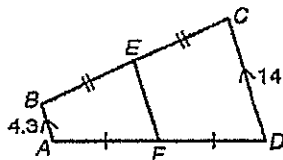
8. KL



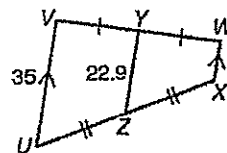
9. PQ



10. EF



11. WX



LESSON 5-4 Reteach
Properties and Attributes of Polygons continued

The Polygon Angle Sum Theorem states that the sum of the interior angle measures of a convex polygon with n sides is $(n - 2)180^\circ$.

Convex Polygon	Number of Sides	Sum of Interior Angle Measures: $(n - 2)180^\circ$
quadrilateral	4	$(4 - 2)180^\circ = 360^\circ$
hexagon	6	$(6 - 2)180^\circ = 720^\circ$
decagon	10	$(10 - 2)180^\circ = 1440^\circ$

If a polygon is a regular polygon, then you can divide the sum of the interior angle measures by the number of sides to find the measure of each interior angle.

Regular Polygon	Number of Sides	Sum of Interior Angle Measures	Measure of Each Interior Angle
quadrilateral	4	360°	$360^\circ \div 4 = 90^\circ$
hexagon	6	720°	$720^\circ \div 6 = 120^\circ$
decagon	10	1440°	$1440^\circ \div 10 = 144^\circ$

The Polygon External Angle Sum Theorem states that the sum of the exterior angle measures, one angle at each vertex, of a convex polygon is 360° .

The measure of each exterior angle of a regular polygon with n exterior angles is $360^\circ \div n$. So the measure of each exterior angle of a regular decagon is $360^\circ \div 10 = 36^\circ$.

$152^\circ + 63^\circ + 145^\circ = 360^\circ$

Find the sum of the interior angle measures of each convex polygon.

7. pentagon 540° 8. octagon 1080° 9. nonagon 1260°

Find the measure of each interior angle of each regular polygon. Round to the nearest tenth if necessary.

10. pentagon 108° 11. heptagon 128.6° 12. 15-gon 156°

Find the measure of each exterior angle of each regular polygon.

13. quadrilateral 90° 14. octagon 45°

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LESSON 5-5 Challenge
Dissections

In the exercises on this page, you will explore a fascinating branch of mathematics that is called *dissection theory*.

1. Carefully trace the four figures at the right onto a sheet of paper. Cut them out. Arrange the figures so that together they form a square. Sketch the arrangement in the blank space at the right.

When you dissect a geometric figure, you cut it into two or more parts. The puzzle pieces in Exercise 1 were formed by dissecting a square into four congruent polygons. The figures at the right show three other dissections.

2. Show four additional ways to dissect a square into four congruent polygons. (The polygons may be either convex or concave.)

Answers will vary.

3. Show four ways to dissect an equilateral triangle into three congruent polygons.

Answers will vary.

4. Show four ways to dissect a regular pentagon into five congruent polygons.

Answers will vary.

5. Describe a general technique for dissecting any regular n -gon into n congruent polygons.

Descriptions will vary.

6. The figure at the right is a 4-by-4 grid of squares. Making cuts only along the grid lines, find all possible ways to dissect the grid into two congruent parts. Sketch your dissections on a separate sheet of paper.

There are six possible dissections.

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LESSON 5-4 Problem Solving
Properties and Attributes of Polygons

1. A campground site is in the shape of a convex quadrilateral. Three sides of the campground form two right angles. The third interior angle measures 10° less than the fourth angle. Find the measure of each interior angle.

90°, 90°, 85°, 95°

2. A pentagon has two exterior angles that measure $(3x)^\circ$, two exterior angles that measure $(2x + 22)^\circ$, and an exterior angle that measures $(x + 41)^\circ$. If all of these angles have different vertices, what are the measures of the exterior angles of the pentagon?

75°, 75°, 72°, 72°, 66°

3. The top view of a hexagonal greenhouse is shown at the right. What is the measure of $\angle PQR$, the acute angle formed by the house and the greenhouse?

54°

Choose the best answer.

4. A figure is an equiangular 18-gon. What is the measure of each exterior angle of the polygon?

A 10° B 18°
 C 20° D 36°

5. Three interior angles of a convex heptagon measure 125° , and two of the interior angles measure 143° . Which are possible measures for the other two interior angles of the heptagon?

F 48° and 48° H 100° and 116°
 G 39° and 100° J 89° and 150°

6. Find the measure of $\angle AKL$.

A 34° C 86°
 B 68° D 148°

7. What is the measure of $\angle GCD$?

F 123° H 73°
 G 116° J 29°

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LESSON 5-5 Reading Strategies
Understanding Vocabulary

Triangle Quadrilateral Pentagon Hexagon Heptagon Octagon Nonagon Decagon Dodecagon

1. How many sides does a pentagon have? five

2. Give some examples of pentagons in real life.

Sample answer: pedestrian crossing street signs, front faces of barns

3. How many vertices does a quadrilateral have? four

4. How does the number of vertices of a polygon compare to the number of sides of the same polygon?

There is an equal number of sides and vertices in polygons.

5. What is the name of a polygon with eight sides? octagon

6. How many diagonals can be drawn from one vertex of a hexagon? three

concave—any part of a diagonal contains points in the exterior of the polygon
convex—no diagonal contains points in the exterior of the polygon

Draw an example of each polygon.

7. convex heptagon 8. concave quadrilateral

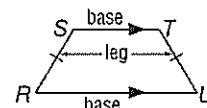
Sample answer: Sample answer:

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6-6 Study Guide and Intervention

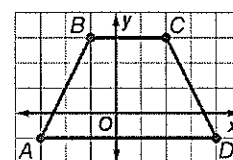
Trapezoids and Kites

Properties of Trapezoids A trapezoid is a quadrilateral with exactly one pair of parallel sides. The **midsegment** or **median** of a trapezoid is the segment that connects the midpoints of the legs of the trapezoid. Its measure is equal to one-half the sum of the lengths of the bases. If the legs are congruent, the trapezoid is an **isosceles trapezoid**. In an isosceles trapezoid both pairs of **base angles** are congruent and the diagonals are congruent.



$STUR$ is an isosceles trapezoid.
 $\overline{SR} \cong \overline{TU}$; $\angle R \cong \angle U$, $\angle S \cong \angle T$

Example The vertices of $ABCD$ are $A(-3, -1)$, $B(-1, 3)$, $C(2, 3)$, and $D(4, -1)$. Show that $ABCD$ is a trapezoid and determine whether it is an isosceles trapezoid.



$$\text{slope of } \overline{AB} = \frac{3 - (-1)}{-1 - (-3)} = \frac{4}{2} = 2$$

$$\text{slope of } \overline{AD} = \frac{-1 - (-1)}{4 - (-3)} = \frac{0}{7} = 0$$

$$\text{slope of } \overline{BC} = \frac{3 - 3}{2 - (-1)} = \frac{0}{3} = 0$$

$$\text{slope of } \overline{CD} = \frac{-1 - 3}{4 - 2} = \frac{-4}{2} = -2$$

$$AB = \sqrt{(-3 - (-1))^2 + (-1 - 3)^2}$$

$$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$$

$$CD = \sqrt{(2 - 4)^2 + (3 - (-1))^2}$$

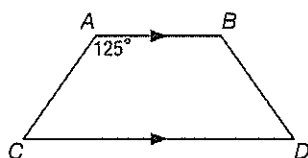
$$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$$

Exactly two sides are parallel, \overline{AD} and \overline{BC} , so $ABCD$ is a trapezoid. $AB = CD$, so $ABCD$ is an isosceles trapezoid.

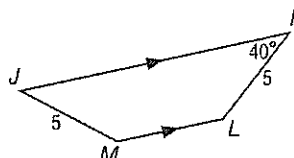
Exercises

Find each measure.

1. $m\angle D$



2. $m\angle L$



COORDINATE GEOMETRY For each quadrilateral with the given vertices, verify that the quadrilateral is a trapezoid and determine whether the figure is an isosceles trapezoid.

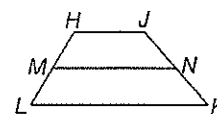
3. $A(-1, 1)$, $B(3, 2)$, $C(1, -2)$, $D(-2, -1)$

4. $J(1, 3)$, $K(3, 1)$, $L(3, -2)$, $M(-2, 3)$

For trapezoid $HJKL$, M and N are the midpoints of the legs.

5. If $HJ = 32$ and $LK = 60$, find MN .

6. If $HJ = 18$ and $MN = 28$, find LK .



6-6 Study Guide and Intervention *(continued)*

Trapezoids and Kites

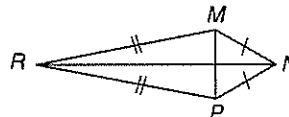
Properties of Kites A kite is a quadrilateral with exactly two pairs of consecutive congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.

The diagonals of a kite are perpendicular.

For kite $RMNP$, $\overline{MP} \perp \overline{RN}$

In a kite, exactly one pair of opposite angles is congruent.

For kite $RMNP$, $\angle M \cong \angle P$



Example 1 If $WXYZ$ is a kite, find $m\angle Z$.

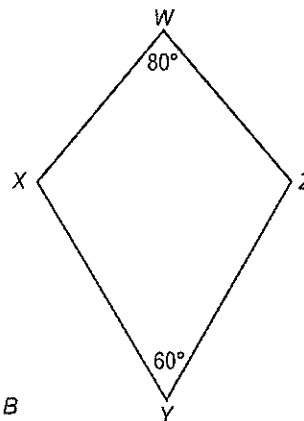
The measures of $\angle Y$ and $\angle W$ are not congruent, so $\angle X \cong \angle Z$.

$$m\angle X + m\angle Y + m\angle Z + m\angle W = 360$$

$$m\angle X + 60 + m\angle Z + 80 = 360$$

$$m\angle X + m\angle Z = 220$$

$$m\angle X = 110, m\angle Z = 110$$



Example 2 If $ABCD$ is a kite, find BC .

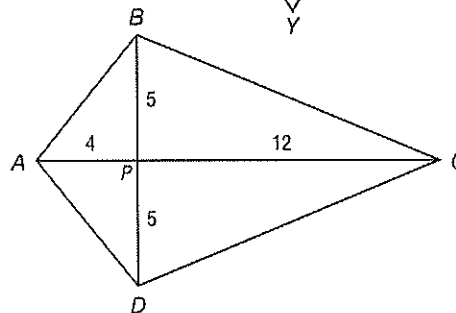
The diagonals of a kite are perpendicular. Use the Pythagorean Theorem to find the missing length.

$$BP^2 + PC^2 = BC^2$$

$$5^2 + 12^2 = BC^2$$

$$169 = BC^2$$

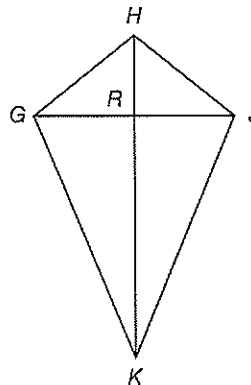
$$13 = BC$$



Exercises

If $GHJK$ is a kite, find each measure.

- Find $m\angle JRK$.
- If $RJ = 3$ and $RK = 10$, find JK .
- If $m\angle GHJ = 90$ and $m\angle GKJ = 110$, find $m\angle HGK$.
- If $HJ = 7$, find HG .
- If $HG = 7$ and $GR = 5$, find HR .
- If $m\angle GHJ = 52$ and $m\angle GKJ = 95$, find $m\angle HGK$.

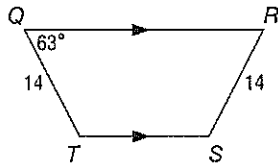


6-6 Skills Practice

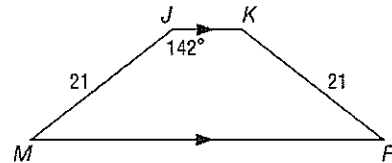
Trapezoids and Kites

ALGEBRA Find each measure.

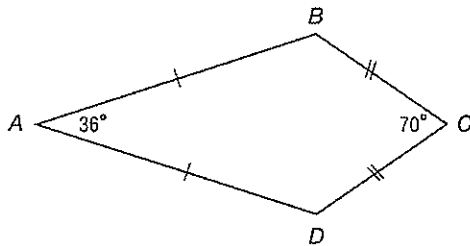
1. $m\angle S$



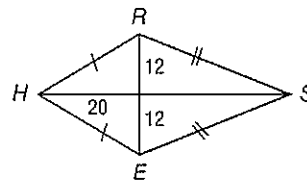
2. $m\angle M$



3. $m\angle D$

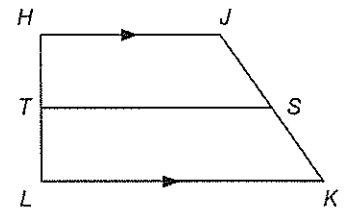


4. RH



ALGEBRA For trapezoid $HJKL$, T and S are midpoints of the legs.

5. If $HJ = 14$ and $LK = 42$, find TS .
6. If $LK = 19$ and $TS = 15$, find HJ .
7. If $HJ = 7$ and $TS = 10$, find LK .
8. If $KL = 17$ and $JH = 9$, find ST .



COORDINATE GEOMETRY $EFGH$ is a quadrilateral with vertices $E(1, 3)$, $F(5, 0)$, $G(8, -5)$, $H(-4, 4)$.

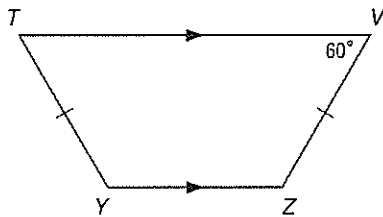
9. Verify that $EFGH$ is a trapezoid.
10. Determine whether $EFGH$ is an isosceles trapezoid. Explain.

6-6 Practice

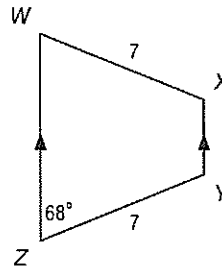
Trapezoids and Kites

Find each measure.

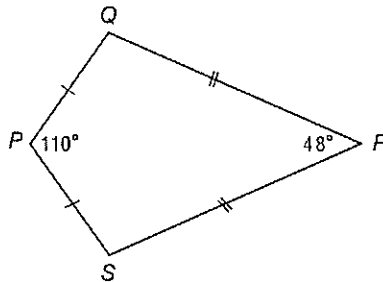
1. $m\angle T$



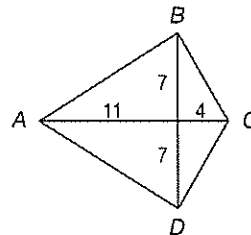
2. $m\angle Y$



3. $m\angle Q$

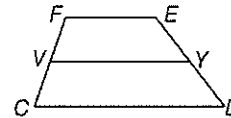


4. BC



ALGEBRA For trapezoid $FEDC$, V and Y are midpoints of the legs.

- If $FE = 18$ and $VY = 28$, find CD .
- If $m\angle F = 140$ and $m\angle E = 125$, find $m\angle D$.



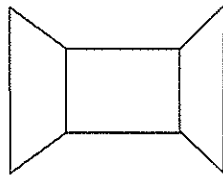
COORDINATE GEOMETRY $RSTU$ is a quadrilateral with vertices $R(-3, -3)$, $S(5, 1)$, $T(10, -2)$, $U(-4, -9)$.

- Verify that $RSTU$ is a trapezoid.
- Determine whether $RSTU$ is an isosceles trapezoid. Explain.
- CONSTRUCTION** A set of stairs leading to the entrance of a building is designed in the shape of an isosceles trapezoid with the longer base at the bottom of the stairs and the shorter base at the top. If the bottom of the stairs is 21 feet wide and the top is 14 feet wide, find the width of the stairs halfway to the top.
- DESK TOPS** A carpenter needs to replace several trapezoid-shaped desktops in a classroom. The carpenter knows the lengths of both bases of the desktop. What other measurements, if any, does the carpenter need?

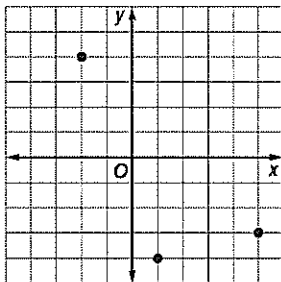
6-6 Word Problem Practice

Trapezoids and Kites

1. **PERSPECTIVE** Artists use different techniques to make things appear to be 3-dimensional when drawing in two dimensions. Kevin drew the walls of a room. In real life, all of the walls are rectangles. In what shape did he draw the side walls to make them appear 3-dimensional?



2. **PLAZA** In order to give the feeling of spaciousness, an architect decides to make a plaza in the shape of a kite. Three of the four corners of the plaza are shown on the coordinate plane. If the fourth corner is in the first quadrant, what are its coordinates?

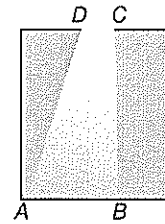


3. **AIRPORTS** A simplified drawing of the reef runway complex at Honolulu International Airport is shown below.



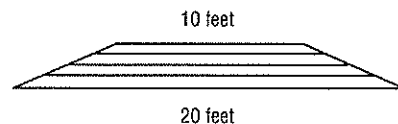
How many trapezoids are there in this image?

4. **LIGHTING** A light outside a room shines through the door and illuminates a trapezoidal region $ABCD$ on the floor.



Under what circumstances would trapezoid $ABCD$ be isosceles?

5. **RISERS** A riser is designed to elevate a speaker. The riser consists of 4 trapezoidal sections that can be stacked one on top of the other to produce trapezoids of varying heights.



All of the stages have the same height. If all four stages are used, the width of the top of the riser is 10 feet.

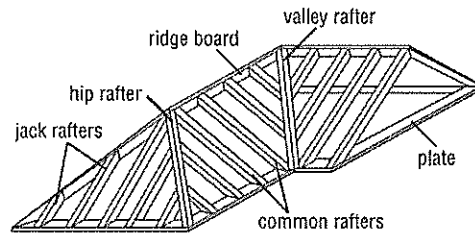
- If only the bottom two stages are used, what is the width of the top of the resulting riser?
- What would be the width of the riser if the bottom three stages are used?

6-6 Enrichment

Quadrilaterals in Construction

Quadrilaterals are often used in construction work.

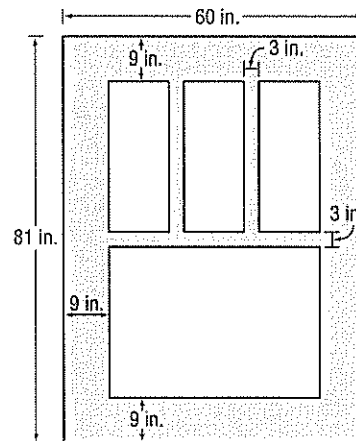
1. The diagram at the right represents a roof frame and shows many quadrilaterals. Find the following shapes in the diagram and shade in their edges.



Roof Frame

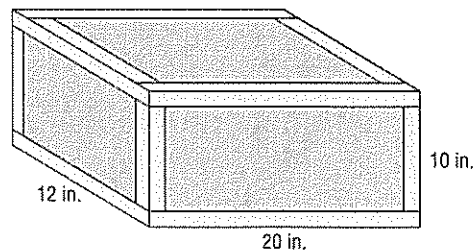
- isosceles triangle
- scalene triangle
- rectangle
- rhombus
- trapezoid (not isosceles)
- isosceles trapezoid

2. The figure at the right represents a window. The wooden part between the panes of glass is 3 inches wide. The frame around the outer edge is 9 inches wide. The outside measurements of the frame are 60 inches by 81 inches. The height of the top and bottom panes is the same. The top three panes are the same size.



- How wide is the bottom pane of glass?
- How wide is each top pane of glass?
- How high is each pane of glass?

3. Each edge of this box has been reinforced with a piece of tape. The box is 10 inches high, 20 inches wide, and 12 inches deep. What is the length of the tape that has been used?



6 Student Recording Sheet

SCORE _____

Use this recording sheet with pages 452–453 of the Student Edition.

Multiple Choice

Read each question. Then fill in the correct answer.

1. A B C D

4. F G H J

7. A B C D

2. F G H J

5. A B C D

3. A B C D

6. F G H J

Short Response/Gridded Response

Record your answer in the blank.

For gridded response questions, also enter your answer in the grid by writing each number or symbol in a box. Then fill in the corresponding circle for that number or symbol.

8. _____ (grid in)

8.

	0	0	0	
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

12.

	0	0	0	
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

9. _____

10. _____

11. _____

12. _____ (grid in)

13. _____

Extended Response

Record your answers for Question 14 on the back of this paper.

6 Rubric for Scoring Extended Response

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended-response questions require the student to show work.
- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is *not* considered a fully correct response.
- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 14 Rubric

Score	Specific Criteria
4	Students correctly determine the quadrilateral in part a is a parallelogram since opposite sides are congruent. Students correctly determine the quadrilateral in part b does not contain sufficient information to prove it is a parallelogram. The two horizontal sides must also be congruent. Students correctly determine that the quadrilateral in part c is a parallelogram since both pairs of opposite angles are congruent.
3	A generally correct solution, but may contain minor flaws in reasoning or computation.
2	A partially correct interpretation and/or solution to the problem.
1	A correct solution with no evidence or explanation.
0	An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.

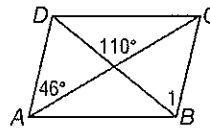
6 Chapter 6 Quiz 1

SCORE _____

(Lessons 6-1 and 6-2)

- Find the sum of the measures of the interior angles of a convex 70-gon. 1. _____
- The measure of each interior angle of a regular polygon is 172. Find the number of sides in the polygon. 2. _____
- The measure of each exterior angle of a regular polygon is 18. Find the number of sides in the polygon. 3. _____
- Given parallelogram $ABCD$ with $C(5, 4)$, find the coordinates of A if the diagonals \overline{AC} and \overline{BD} intersect at $(2, 7)$. 4. _____
- MULTIPLE CHOICE** Find $m\angle 1$ in parallelogram $ABCD$. 5. _____

- A 64 C 46
B 58 D 36

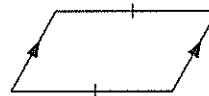


6 Chapter 6 Quiz 2

SCORE _____

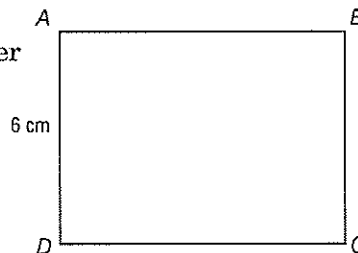
(Lesson 6-3)

- Determine whether this quadrilateral is a parallelogram. Justify your answer. 1. _____



For Questions 2–4, write *true* or *false*.

- A quadrilateral with two pairs of parallel sides is always a parallelogram. 2. _____
- The diagonals of a parallelogram are always perpendicular. 3. _____
- The slope of \overline{AB} and \overline{CD} is $\frac{3}{5}$ and the slope of \overline{BC} and \overline{AD} is $-\frac{5}{3}$. $ABCD$ is a parallelogram. 4. _____
- Refer to parallelogram $ABCD$. If $AB = 8$ cm, what is the perimeter of the parallelogram? 5. _____



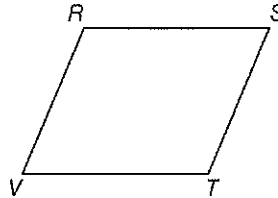
6 Chapter 6 Quiz 3

SCORE _____

(Lessons 6-4 and 6-5)

1. **MULTIPLE CHOICE** $RSTV$ is a rhombus. Which of the following statements is NOT true?

- A $\overline{RV} \cong \overline{TS}$
- B $\overline{RV} \perp \overline{TS}$
- C $\overline{RS} \parallel \overline{TV}$
- D $\angle R \cong \angle T$

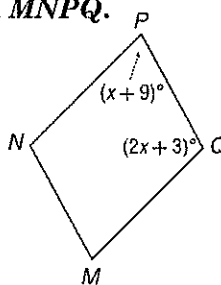


1. _____

For Questions 2 and 3, refer to trapezoid $MNPQ$.

2. Find $m\angle M$.

3. Find $m\angle Q$.



2. _____

3. _____

4. *True or false.* A quadrilateral that is a rectangle and a rhombus is a square.

4. _____

5. $\square ABCD$ has vertices $A(4, 0)$, $B(0, 4)$, $C(-4, 0)$, and $D(0, -4)$. Determine whether $ABCD$ is a *rectangle*, *rhombus*, or *square*. List all that apply.

5. _____

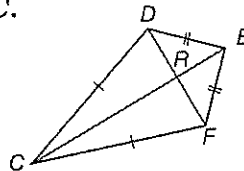
6 Chapter 6 Quiz 4

SCORE _____

(Lesson 6-6)

For Questions 1 and 2, refer to kite $DEFC$.

- 1. If $m\angle DCF = 34$ and $m\angle DEF = 90$, find $m\angle CDE$.
- 2. If $DR = 5$ and $RE = 5$, find FE .

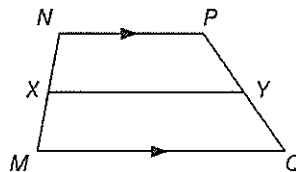


1. _____

2. _____

For Questions 3 and 4, refer to trapezoid $NPQM$ where X and Y are midpoints of the sides.

- 3. If $MQ = 15$ and $XY = 10$, find NP .
- 4. If $NP = 13$ and $MQ = 18$, find XY .



3. _____

4. _____

5. If $CDEF$ is a trapezoid with vertices $C(0, 2)$, $D(2, 4)$, $E(7, 3)$, and $F(1, -3)$, how can you prove that it is an isosceles trapezoid?

5. _____

6 Chapter 6 Mid-Chapter Test

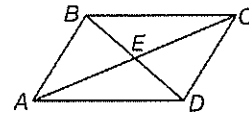
SCORE _____

(Lessons 6-1 through 6-3)

Part I Write the letter for the correct answer in the blank at the right of each question.

1. Find the measure of each exterior angle of a regular 56-gon. Round to the nearest tenth.
 A 3.2 B 6.4 C 173.6 D 9720 1. _____

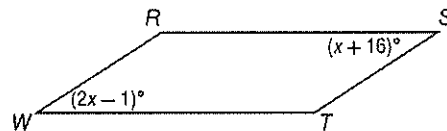
2. Given $BE = 2x + 6$ and $ED = 5x - 12$ in parallelogram $ABCD$, find BD .
 F 6 H 18
 G 12 J 36



2. _____

3. If the slope of \overline{PQ} is $\frac{2}{3}$ and the slope of \overline{QR} is $-\frac{1}{2}$, find the slope of \overline{SR} so that $PQRS$ is a parallelogram.
 A $\frac{2}{3}$ B $\frac{3}{2}$ C $-\frac{1}{2}$ D 2 3. _____

4. Find $m\angle W$ in parallelogram $RSTW$.
 F 17 H 55
 G 33 J 125

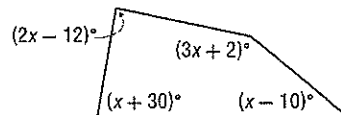


4. _____

5. Find the sum of the measures of the interior angles of a convex 48-gon.
 A 172.5 B 360 C 8280 D 8640 5. _____

Part II

6. Find x .

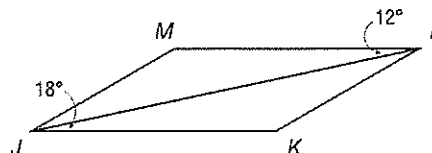


6. _____

7. $ABCD$ is a parallelogram with $m\angle A = 138$. Find $m\angle B$. 7. _____

8. Determine whether $ABCD$ is a parallelogram if $AB = 6$, $BC = 12$, $CD = 6$, and $DA = 12$. Justify your answer. 8. _____

9. In parallelogram $MLKJ$, find $m\angle MLK$ and $m\angle LKJ$.



9. _____

10. $XYWZ$ is a quadrilateral with vertices $W(1, -4)$, $X(-4, 2)$, $Y(1, -1)$, and $Z(-2, -3)$. Determine if the quadrilateral is a parallelogram. Use slope to justify your answer. 10. _____

6 Chapter 6 Vocabulary Test

SCORE _____

base	legs	rhombus
base angle	midsegment of a trapezoid	square
diagonal	parallelogram	trapezoid
isosceles trapezoid	rectangle	

Choose from the terms above to complete each sentence.

1. A quadrilateral with only one pair of opposite sides parallel and the other pair of opposite sides congruent is a(n) _____. 1. _____
2. A quadrilateral with two pairs of opposite sides parallel is a(n) _____. 2. _____
3. A quadrilateral with only one pair of opposite sides parallel is a(n) _____. 3. _____
4. A quadrilateral that is both a rectangle and a rhombus is a(n) _____. 4. _____
5. A quadrilateral with four congruent sides is a(n) _____. 5. _____

Write whether each sentence is *true* or *false*. If false, replace the underlined word or number to make a true sentence.

6. A quadrilateral with four right angles is a trapezoid. 6. _____
7. A quadrilateral with two pairs of congruent consecutive sides is a kite. 7. _____

Choose the correct term to complete each sentence.

8. Segments that join opposite vertices in a quadrilateral are called (*medians, diagonals*). 8. _____
9. The segment joining the midpoints of the nonparallel sides of a trapezoid is called the (*median, diagonal*). 9. _____

Define each term in your own words.

10. base angles of an isosceles trapezoid 10. _____
11. legs of a trapezoid 11. _____

6 Chapter 6 Test, Form 1

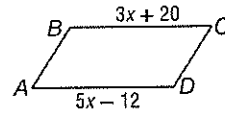
Write the letter for the correct answer in the blank at the right of each question.

1. Find the sum of the measures of the interior angles of a convex 30-gon.
 A 5400 B 5040 C 360 D 168 1. _____

2. Find the sum of the measures of the exterior angles of a convex 21-gon.
 F 21 G 180 H 360 J 3420 2. _____

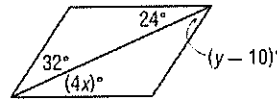
3. If the measure of each interior angle of a regular polygon is 108, find the measure of each exterior angle.
 A 18 B 72 C 90 D 108 3. _____

4. For parallelogram $ABCD$, find the value of x .
 F 4 H 16 G 10.25 J 21.5 4. _____

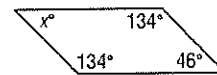


5. Which of the following is a property of a parallelogram?
 A The diagonals are congruent. C The diagonals are perpendicular.
 B The diagonals bisect the angles. D The diagonals bisect each other. 5. _____

6. Find the values of x and y so that $ABCD$ will be a parallelogram.
 F $x = 6, y = 42$
 G $x = 6, y = 22$
 H $x = 20, y = 42$
 J $x = 20, y = 22$ 6. _____



7. Find the value of x so that this quadrilateral is a parallelogram.
 A 44 C 90
 B 46 D 134 7. _____



8. Parallelogram $ABCD$ has vertices $A(0, 0)$, $B(2, 4)$, and $C(10, 4)$. Find the coordinates of D .
 F $D(8, 0)$ G $D(10, 0)$ H $D(0, 4)$ J $D(10, 8)$ 8. _____

9. Which of the following is a property of all rectangles?
 A four congruent sides C diagonals are perpendicular
 B diagonals bisect the angles D four right angles 9. _____

10. $ABCD$ is a rectangle with diagonals \overline{AC} and \overline{BD} . If $AC = 2x + 10$ and $BD = 56$, find the value of x .
 F 23 G 33 H 78 J 122 10. _____

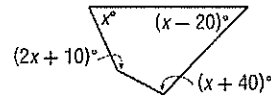
11. $ABCD$ is a rectangle with $B(-5, 0)$, $C(7, 0)$ and $D(7, 3)$. Find the coordinates of A .
 A $A(-5, 7)$ B $A(3, 5)$ C $A(-5, 3)$ D $A(7, -3)$ 11. _____

6 Chapter 6 Test, Form 2A

Write the letter for the correct answer in the blank at the right of each question.

1. Find the sum of the measures of the interior angles of a convex 45-gon.
 A 8100 B 7740 C 360 D 172 1. _____

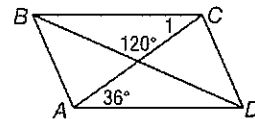
2. Find the value of x .
 F 30 H 102
 G 66 J 138 2. _____



3. Find the sum of the measures of the exterior angles of a convex 39-gon.
 A 39 B 90 C 180 D 360 3. _____

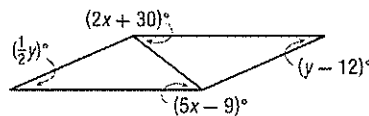
4. Which of the following is a property of a parallelogram?
 F Each pair of opposite sides is congruent.
 G Only one pair of opposite angles is congruent.
 H Each pair of opposite angles is supplementary.
 J There are four right angles. 4. _____

5. For parallelogram $ABCD$, find $m\angle 1$.
 A 60 C 36
 B 54 D 18 5. _____

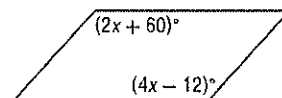


6. $ABCD$ is a parallelogram with diagonals intersecting at E . If $AE = 3x + 12$ and $EC = 27$, find the value of x .
 F 5 G 17 H 27 J 47 6. _____

7. Find the values of x and y so that this quadrilateral is a parallelogram.
 A $x = 13, y = 24$ C $x = 7, y = 24$
 B $x = 13, y = 6$ D $x = 7, y = 6$ 7. _____



8. Find the value of x so that this quadrilateral is a parallelogram.
 F 12 H 36
 G 24 J 132 8. _____



9. Parallelogram $ABCD$ has vertices $A(8, 2)$, $B(6, -4)$, and $C(-5, -4)$. Find the coordinates of D .
 A $D(-5, 2)$ B $D(-3, 2)$ C $D(-2, 2)$ D $D(-4, 8)$ 9. _____

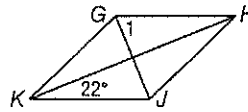
10. $ABCD$ is a rectangle. If $AC = 5x + 2$ and $BD = x + 22$, find the value of x .
 F 5 G 6 H 11 J 26 10. _____

11. Which of the following is true for all rectangles?
 A The diagonals are perpendicular.
 B The diagonals bisect the angles.
 C The consecutive sides are congruent.
 D The consecutive sides are perpendicular. 11. _____

6 Chapter 6 Test, Form 2A (continued)

12. $ABCD$ is a rectangle with $B(-4, 6)$, $C(-4, 2)$, and $D(10, 2)$. Find the coordinates of A .
- F $A(6, 4)$ G $A(10, 4)$ H $A(2, 6)$ J $A(10, 6)$ 12. _____

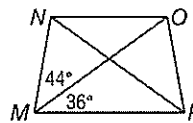
13. For rhombus $GHJK$, find $m\angle 1$.
- A 22 C 68
B 44 D 90



14. The diagonals of square $ABCD$ intersect at E . If $AE = 2x + 6$ and $BD = 6x - 10$, find AC .
- F 11 G 28 H 56 J 90 14. _____

15. $ABCD$ is an isosceles trapezoid with $A(10, -1)$, $B(8, 3)$, and $C(-1, 3)$. Find the coordinates of D .
- A $D(-3, -1)$ B $D(-10, -11)$ C $D(-1, 8)$ D $D(-3, 3)$ 15. _____

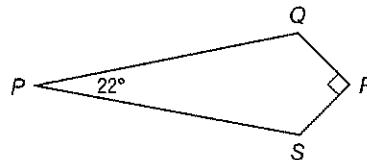
16. For isosceles trapezoid $MNOP$, find $m\angle MNP$.
- F 44 H 80
G 64 J 116



17. The length of one base of a trapezoid is 19 inches and the length of the median is 16 inches. Find the length of the other base.
- A 35 in. B 19 in. C 17.5 in. D 13 in. 17. _____

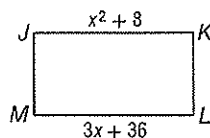
18. Judith built a fence to surround her property. On a coordinate plane, the four corners of the fence are located at $(-16, 1)$, $(-6, 5)$, $(4, 1)$, and $(-6, -3)$. Which of the following most accurately describes the shape of Judith's fence?
- F square H rhombus
G rectangle J trapezoid 18. _____

19. For kite $PQRS$, find $m\angle S$.
- A 248 C 112
B 68 D 124



20. $ABCD$ is a parallelogram with coordinates $A(4, 2)$, $B(4, -1)$, $C(-2, -1)$, and $D(-2, 2)$. To prove that $ABCD$ is a rectangle, you would plot the parallelogram on a coordinate plane and then find which of the following?
- F measures of the angles H slopes of the diagonals
G lengths of the diagonals J midpoints of the diagonals 20. _____

Bonus Find the possible value(s) of x in rectangle $JKLM$.



B: _____

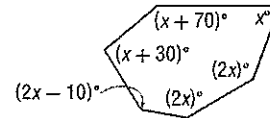
6 Chapter 6 Test, Form 2B

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

1. Find the sum of the measures of the interior angles of a convex 50-gon.
 A 9000 B 8640 C 360 D 172.8 1. _____

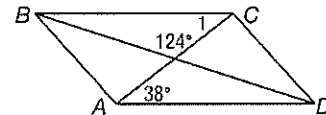
2. Find the value of x .
 F 16 H 50
 G 34 J 70 2. _____



3. Find the sum of the measures of the exterior angles of a convex 65-gon.
 A 5.54 B 90 C 180 D 360 3. _____

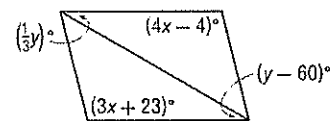
4. Which of the following is a property of all parallelograms?
 F Each pair of opposite angles is congruent.
 G Only one pair of opposite sides is congruent.
 H Each pair of opposite angles is supplementary.
 J There are four right angles. 4. _____

5. For parallelogram $ABCD$, find $m\angle 1$.
 A 19 C 52
 B 38 D 56 5. _____

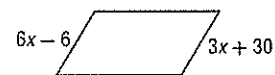


6. $ABCD$ is a parallelogram with diagonals intersecting at E . If $AE = 4x - 8$ and $EC = 36$, find the value of x .
 F 7 G 11 H 15.5 J 38 6. _____

7. Find the values of the values of x and y so that the quadrilateral is a parallelogram.
 A $x = 27, y = 90$ C $x = 13, y = 90$
 B $x = 27, y = 40$ D $x = 13, y = 40$ 7. _____



8. Find the value of x so that the quadrilateral is a parallelogram.
 F $7\frac{1}{3}$ H 12
 G 8 J 66 8. _____



9. $ABCD$ is a parallelogram with $A(5, 4)$, $B(-1, -2)$, and $C(8, -2)$. Find the coordinates of D .
 A $D(-5, 4)$ B $D(8, 2)$ C $D(14, 4)$ D $D(4, 1)$ 9. _____

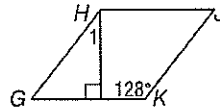
10. $ABCD$ is a rectangle. If $AB = 7x - 6$ and $CD = 5x + 30$, find the value of x .
 F $5\frac{1}{3}$ G 12 H 13 J 18 10. _____

11. Which of the following is true for all rectangles?
 A The diagonals are perpendicular.
 B The consecutive angles are supplementary.
 C The opposite sides are supplementary.
 D The opposite angles are complementary. 11. _____

6 Chapter 6 Test, Form 2B (continued)

12. $ABCD$ is a rectangle with $B(-7, 3)$, $C(5, 3)$, and $D(5, -8)$. Find the coordinates of A .
F $A(-8, -7)$ **G** $A(-7, -8)$ **H** $A(-5, -3)$ **J** $A(-8, -5)$ 12. _____

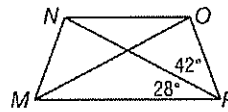
13. For rhombus $GHJK$, find $m\angle 1$.
A 90 **C** 52
B 64 **D** 38



14. The diagonals of square $ABCD$ intersect at E . If $AE = 3x - 4$ and $BD = 10x - 48$, find AC .
F 90 **G** 52 **H** 26 **J** 10 14. _____

15. $ABCD$ is an isosceles trapezoid with $A(0, -1)$, $B(-2, 3)$, and $D(6, -1)$. Find the coordinates of C .
A $C(6, 1)$ **B** $C(9, 4)$ **C** $C(2, 3)$ **D** $C(8, 3)$ 15. _____

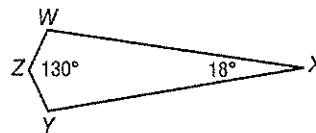
16. For isosceles trapezoid $MNOP$, find $m\angle MNP$.
F 42 **H** 82
G 70 **J** 98



17. The length of one base of a trapezoid is 19 meters and the length of the median is 23 meters. Find the length of the other base.
A 15 m **B** 21 m **C** 27 m **D** 42 m 17. _____

18. On a coordinate plane, the four corners of Ronald's garden are located at $(0, 2)$, $(4, 6)$, $(8, 2)$, and $(4, -2)$. Which of the following most accurately describes the shape of Ronald's garden?
F square **H** rhombus
G rectangle **J** trapezoid 18. _____

19. For kite $WXYZ$, find $m\angle W$.
A 106 **C** 212
B 148 **D** 360



20. $ABCD$ is a parallelogram with coordinates $A(4, 2)$, $B(3, -1)$, $C(-1, -1)$, and $D(-1, 2)$. To prove that $ABCD$ is a rhombus, you would plot the parallelogram on a coordinate plane and then find which of the following?
F measures of the angles **H** slopes of the diagonals
G lengths of the diagonals **J** midpoints of the diagonals 20. _____

Bonus The sum of the measures of the interior angles of a convex polygon is ten times the sum of the measures of its exterior angles. Find the number of sides of the polygon.

B: _____

6 Chapter 6 Test, Form 2C

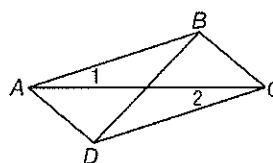
SCORE _____

1. What is the sum of the interior angles of an octagonal box? 1. _____

2. A convex pentagon has interior angles with measures $(5x - 12)^\circ$, $(2x + 100)^\circ$, $(4x + 16)^\circ$, $(6x + 15)^\circ$, and $(3x + 41)^\circ$. Find the value of x . 2. _____

3. If the measure of each interior angle of a regular polygon is 171, find the number of sides in the polygon. 3. _____

4. In parallelogram $ABCD$, $m\angle 1 = x + 12$, and $m\angle 2 = 6x - 18$. Find $m\angle 1$.



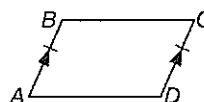
4. _____

5. Find the measure of each exterior angle of a regular 45-gon. 5. _____

6. In parallelogram $ABCD$, $m\angle A = 58$. Find $m\angle B$. 6. _____

7. Find the coordinates of the intersection of the diagonals of parallelogram $XYZW$ with vertices $X(2, 2)$, $Y(3, 6)$, $Z(10, 6)$, and $W(9, 2)$. 7. _____

8. Determine whether $ABCD$ is a parallelogram. Justify your answer.

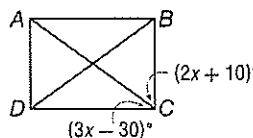


8. _____

9. Determine whether the quadrilateral with vertices $A(5, 7)$, $B(1, -2)$, $C(-6, -3)$, and $D(2, 5)$ is a parallelogram. Use the slope formula. 9. _____

10. For quadrilateral $ABCD$, the slope of \overline{AB} is $\frac{1}{4}$, the slope of \overline{BC} is $-\frac{2}{3}$, and the slope of \overline{CD} is $\frac{1}{4}$. Find the slope of \overline{DA} so that $ABCD$ will be a parallelogram. 10. _____

11. Given rectangle $ABCD$, find the value of x .



11. _____

12. $ABCD$ is a parallelogram and $\overline{AC} \cong \overline{BD}$. Determine whether $ABCD$ is a rectangle. Justify your answer. 12. _____

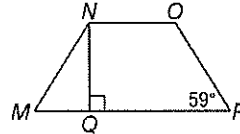
13. $ABCD$ is a rhombus with diagonals intersecting at E . If $m\angle ABC$ is three times $m\angle BAD$, find $m\angle EBC$. 13. _____

6 Chapter 6 Test, Form 2C *(continued)*

14. $TUVW$ is a square with $U(10, 2)$, $V(8, 8)$, and $W(2, 6)$. Find the coordinates of T .

14. _____

15. For isosceles trapezoid $MNOP$, find $m\angle MNQ$.



15. _____

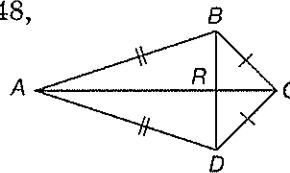
16. $ABCD$ is a quadrilateral with vertices $A(8, 3)$, $B(6, 7)$, $C(-1, 5)$, and $D(-6, -1)$. Determine whether $ABCD$ is a trapezoid. Justify your answer.

16. _____

17. The length of the median of trapezoid $EFGH$ is 13 feet. If the bases have lengths $2x + 4$ and $10x - 50$, find x .

17. _____

18. $ABCD$ is a kite. If $RC = 10$, and $BD = 48$, find CD .



18. _____

For Questions 19–25, write *true* or *false*.

19. A rectangle is always a parallelogram.

19. _____

20. The diagonals of a rhombus are always perpendicular.

20. _____

21. The diagonals of a square always bisect each other.

21. _____

22. A trapezoid always has two congruent sides.

22. _____

23. The median of a trapezoid is always parallel to the bases.

23. _____

24. A kite has exactly two congruent angles.

24. _____

25. If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rectangle.

25. _____

Bonus In parallelogram $ABCD$, $AB = 2x - 7$, $BC = x + 3y$, $CD = x + y$, and $AD = 2x - y - 1$. Find the values of x and y .

B: _____

6 Chapter 6 Test, Form 2D

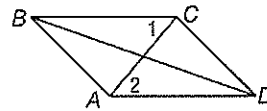
SCORE _____

1. Bruce is building a tabletop in the shape of an octagon. Find the sum of the external angles of the tabletop. 1. _____

2. A convex octagon has interior angles with measures $(x + 55)^\circ$, $(3x + 20)^\circ$, $4x^\circ$, $(4x - 10)^\circ$, $(6x - 55)^\circ$, $(3x + 52)^\circ$, $3x^\circ$, and $(2x + 30)^\circ$. Find the value of x . 2. _____

3. If the measure of each interior angle of a regular polygon is 176 find the number of sides in the polygon. 3. _____

4. In parallelogram $ABCD$, $m\angle 1 = x + 25$, and $m\angle 2 = 2x$. Find $m\angle 2$.



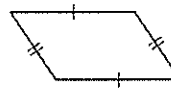
4. _____

5. Find the measure of each exterior angle of a regular 100-gon. 5. _____

6. In parallelogram $ABCD$, $m\angle A = 63$. Find $m\angle B$. 6. _____

7. Find the coordinates of the intersection of the diagonals of parallelogram $XYZW$ with vertices $X(3, 0)$, $Y(3, 8)$, $Z(-2, 6)$, and $W(-2, -2)$. 7. _____

8. Determine whether this quadrilateral is a parallelogram. Justify your answer.

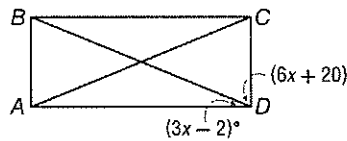


8. _____

9. Determine whether a quadrilateral with vertices $A(5, 7)$, $B(1, -1)$, $C(-6, -3)$, and $D(-2, 5)$ is a parallelogram. Use the slope formula. 9. _____

10. If the slope of \overline{AB} is $\frac{1}{2}$, the slope of \overline{BC} is -4 , and the slope of \overline{CD} is $\frac{1}{2}$, find the slope of \overline{DA} so that $ABCD$ is a parallelogram. 10. _____

11. For rectangle $ABCD$, find the value of x .



11. _____

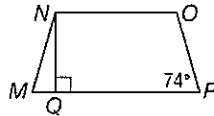
12. $ABCD$ is a parallelogram and $m\angle A = 90$. Determine whether $ABCD$ is a rectangle. Justify your answer. 12. _____

6 Chapter 6 Test, Form 2D (continued)

13. $ABCD$ is a rhombus with diagonals intersecting at E . If $m\angle ABC$ is four times $m\angle BAD$, find $m\angle EBC$. 13. _____

14. $PQRS$ is a square with $Q(-2, 8)$, $R(5, 7)$, and $S(4, 0)$. Find the coordinates of P . 14. _____

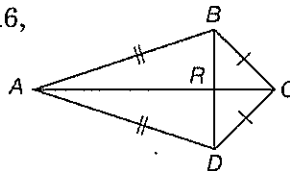
15. For isosceles trapezoid $MNOP$, find $m\angle MNQ$. 15. _____



16. $ABCD$ is a quadrilateral with $A(8, 21)$, $B(10, 27)$, $C(26, 26)$, and $D(18, 2)$. Determine whether $ABCD$ is a trapezoid. Justify your answer. 16. _____

17. The length of the median of trapezoid $EFGH$ is 17 centimeters. If the bases have lengths $2x + 4$ and $8x - 50$, find the value of x . 17. _____

18. For kite $ABCD$, if $RA = 15$, and $BD = 16$, find AD . 18. _____



For Questions 19–25, write *true* or *false*.

19. A parallelogram always has four right angles. 19. _____

20. The diagonals of a rhombus always bisect the angles. 20. _____

21. A rhombus is always a square. 21. _____

22. A rectangle is always a square. 22. _____

23. The diagonals of an isosceles trapezoid are always congruent. 23. _____

24. The median of a trapezoid always bisects the angles. 24. _____

25. The diagonals of a kite are always perpendicular. 25. _____

Bonus The measure of each interior angle of a regular polygon is 24 more than 38 times the measure of each exterior angle. Find the number of sides of the polygon. B: _____

6 Chapter 6 Test, Form 3

SCORE _____

1. The sum of the interior angles of an animal pen is 900° .
How many sides does the pen have?

1. _____

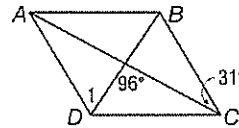
2. A convex hexagon has interior angles with measures x° , $(5x - 103)^\circ$, $(2x + 60)^\circ$, $(7x - 31)^\circ$, $(6x - 6)^\circ$, and $(9x - 100)^\circ$.
Find the value of x and the measure of each angle.

2. _____

3. Find the measure of each exterior angle of a regular $2x$ -gon.

3. _____

4. For parallelogram $ABCD$, find $m\angle 1$.

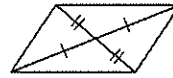


4. _____

5. $ABCD$ is a parallelogram with diagonals that intersect each other at E . If $AE = x^2$ and $EC = 6x - 8$, find all possible values of AC .

5. _____

6. Determine whether the quadrilateral is a parallelogram. Justify your answer.

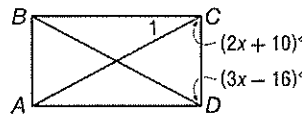


6. _____

7. For quadrilateral $ABCD$, the slope of \overline{AB} is $\frac{2}{3}$ and the slope of \overline{BC} is -2 . Find the slopes of \overline{CD} and \overline{DA} so that $ABCD$ will be a parallelogram.

7. _____

8. In rectangle $ABCD$, find $m\angle 1$.



8. _____

9. The diagonals of rhombus $ABCD$ intersect at E . If $m\angle BAE = \frac{2}{3}(m\angle ABE)$, find $m\angle BCD$.

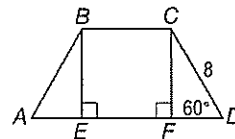
9. _____

10. The diagonals of square $ABCD$ intersect at E . If $AE = 2$, find the perimeter of $ABCD$.

10. _____

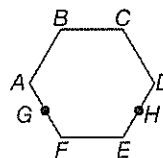
11. For isosceles trapezoid $ABCD$, find AE .

11. _____



12. Points G and H are midpoints of \overline{AF} and \overline{DE} in regular hexagon $ABCDEF$. If $AB = 6$ find GH .

12. _____



13. The vertices of trapezoid $ABCD$ are $A(10, -1)$, $B(6, 6)$, $C(-2, 6)$, and $D(-8, -1)$.
Find the length of the median.

13. _____

6 Chapter 6 Test, Form 3 (continued)

14. Determine whether the quadrilateral $ABCD$ with vertices $A(0, -1)$, $B(-4, -3)$, $C(-5, 1)$, $D(1, 7)$ is a kite. Justify your answer.

14. _____

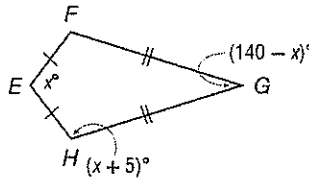
15. Determine whether the quadrilateral $ABCD$ with vertices $A(6, 2)$, $B(2, 10)$, $C(-6, 6)$, and $D(-2, -2)$ is a rectangle. Justify your answer.

15. _____

16. Determine whether quadrilateral $ABCD$ with vertices $A(1, 6)$, $B(7, 6)$, $C(2, -3)$, and $D(-4, -3)$ is a parallelogram. Use the distance formula.

16. _____

17. Find the value of x in kite $EFGH$.



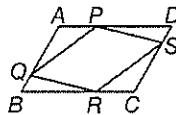
17. _____

For Questions 18 and 19, complete the two-column proof by supplying the missing information for each corresponding location.

Given: $ABCD$ is a parallelogram.

$$\overline{BQ} \cong \overline{DS}, \overline{PA} \cong \overline{RC}$$

Prove: $PQRS$ is a parallelogram.



Statements

Reasons

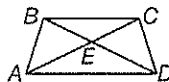
1. $ABCD$ is a \square .
2. $\overline{AD} \cong \overline{CB}$
3. $\overline{PA} \cong \overline{RC}$
4. $\overline{PD} \cong \overline{RB}$
5. $\overline{AB} \cong \overline{CD}$
6. $\overline{BQ} \cong \overline{DS}$
7. $\overline{AQ} \cong \overline{CS}$
8. $\angle B \cong \angle D, \angle A \cong \angle C$
9. $\triangle QBR \cong \triangle SDP, \triangle PAQ \cong \triangle RCS$
10. $\overline{QP} \cong \overline{RS}, \overline{QR} \cong \overline{PS}$
11. $PQRS$ is a parallelogram.

1. Given
2. (Question 18)
3. Given
4. Seg. Sub. Prop.
5. Opp. sides of a \square are \cong .
6. Given
7. Seg. Sub. Prop.
8. Opp. \angle s of a \square are \cong .
9. SAS
10. CPCTC
11. (Question 19)

18. _____

19. _____

20. In isosceles trapezoid $ABCD$, $AE = 2x + 5$, $EC = 3x - 12$, and $BD = 4x + 20$. Find the value of x .



20. _____

Bonus If three of the interior angles of a convex hexagon each measure 140, a fourth angle measures 84, and the measure of the fifth angle is 3 times the measure of the sixth angle, find the measure of the sixth angle.

B: _____

6 Chapter 6 Extended-Response Test

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1.
 - a. Draw a regular convex polygon and a convex polygon that is not regular, each with the same number of sides.

 - b. Label the measures of each exterior angle on your figures.

 - c. Find the sum of the exterior angles for each figure. What conjecture can be made?

2. Draw a rectangle. Connect the midpoints of the consecutive sides. What type of quadrilateral is formed? How do you know?

3. Draw an example to show why *one pair of opposite sides congruent and the other pair of opposite sides parallel* is not sufficient to form a parallelogram.

4.
 - a. Name a property that is true for a square and not always true for a rectangle.

 - b. Name a property that is true for a square and not always true for a rhombus.

 - c. Name a property that is true for a rectangle and not always true for a parallelogram.

6 Standardized Test Practice

SCORE _____

(Chapters 1–6)

Part 1: Multiple Choice

Instructions: Fill in the appropriate circle for the best answer.

1. Find the coordinates of X if $V(0.5, 5)$ is the midpoint of \overline{UX} with $U(15, 21)$. (Lesson 1-3)

A $(-14, -11)$ C $(0, 0)$
 B $(7.75, 22.5)$ D $(15.5, -5)$

1. Ⓐ Ⓑ Ⓒ Ⓓ

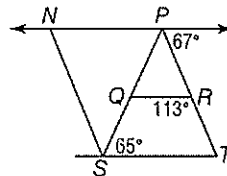
2. Which of the following are possible measures for vertical angles G and H ? (Lesson 2-8)

F $m\angle G = 125$ and $m\angle H = 55$
 G $m\angle G = 125$ and $m\angle H = 125$
 H $m\angle G = 55$ and $m\angle H = 45$
 J $m\angle G = 55$ and $m\angle H = 152.5$

2. Ⓕ Ⓖ Ⓗ Ⓙ

3. Determine which lines are parallel.

(Lesson 3-5)
 A $\overleftrightarrow{NS} \parallel \overleftrightarrow{PT}$ C $\overleftrightarrow{QR} \parallel \overleftrightarrow{ST}$
 B $\overleftrightarrow{NP} \parallel \overleftrightarrow{ST}$ D $\overleftrightarrow{NP} \parallel \overleftrightarrow{QR}$



3. Ⓐ Ⓑ Ⓒ Ⓓ

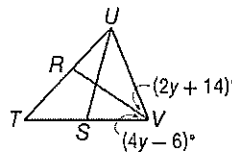
4. Find the coordinates of B , the midpoint of \overline{AC} , if $A(2a, b)$ and $C(0, 2b)$. (Lesson 4-8)

F $(2a, 2b)$ G (a, b) H $(a, \frac{3}{2}b)$ J $(\frac{3}{2}a, b)$

4. Ⓕ Ⓖ Ⓗ Ⓙ

5. If \overline{RV} is an angle bisector, find $m\angle UVT$. (Lesson 5-1)

A 10 C 68
 B 34 D 136



5. Ⓐ Ⓑ Ⓒ Ⓓ

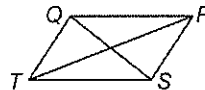
6. Find the slope of the line that passes through points $A(-7, 14)$ and $B(5, -2)$. (Lesson 3-3)

F $-\frac{4}{3}$ G $-\frac{3}{4}$ H $\frac{3}{4}$ J $\frac{4}{3}$

6. Ⓕ Ⓖ Ⓗ Ⓙ

7. Which statement ensures that quadrilateral $QRST$ is a parallelogram?

(Lesson 6-3)
 A $\angle Q \cong \angle S$ C $\overline{QT} \parallel \overline{RS}$
 B $\overline{QR} \cong \overline{TS}$ and $\overline{QR} \parallel \overline{TS}$ D $m\angle Q + m\angle S = 180$



7. Ⓐ Ⓑ Ⓒ Ⓓ

6 Standardized Test Practice *(continued)*

8. What is the equation of the line that contains $(-12, 9)$ and is perpendicular to the line $y = \frac{2}{3}x + 5$? (Lesson 3-4)

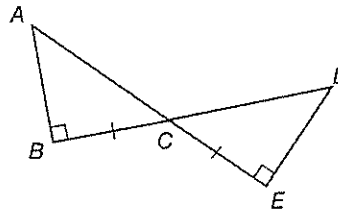
- F $y = -\frac{3}{2}x - 9$ H $y = -\frac{2}{3}x - 1$
 G $y = \frac{3}{2}x - 1$ J $y = \frac{2}{3}x + 17$

8. Ⓕ Ⓖ Ⓗ Ⓙ

9. Which of the following theorems can be used to prove $\triangle ABC \cong \triangle DEC$?

(Lesson 4-5)

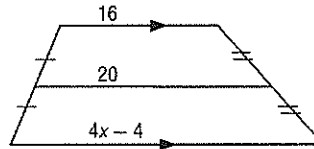
- A SSS C SAS
 B AAS D ASA



9. Ⓐ Ⓑ Ⓒ Ⓓ

10. What is the value of x ? (Lesson 6-6)

- F 2 H 5.5
 G 4 J 7



10. Ⓕ Ⓖ Ⓗ Ⓙ

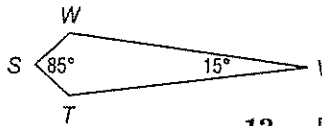
11. For $\triangle ABC$, $AB = 6$ and $BC = 17$. Which of the following is a possible length for AC ? (Lesson 5-3)

- A 5 B 9 C 13 D 24

11. Ⓐ Ⓑ Ⓒ Ⓓ

12. What is $m\angle T$ in kite $STVW$?

- F 100 H 95
 G 130 J 260



12. Ⓕ Ⓖ Ⓗ Ⓙ

Part 2: Gridded Response

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

13. If $\triangle UVW$ is an isosceles triangle, $\overline{UV} \cong \overline{WU}$, $UV = 16b - 40$, $VW = 6b$, and $WU = 10b + 2$, find the value of b . (Lesson 4-1)

13.

	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪

14. Find the sum of the measures of the interior angles for a convex heptagon. (Lesson 6-1)

14.

	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪
⓪	⓪	⓪	⓪	⓪

6 Standardized Test Practice *(continued)*

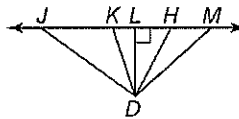
Part 3: Short Response

Instructions: Place your answers in the space provided.

15. A polygon has six congruent sides. Lines containing two of its sides contain points in its interior. Name the polygon by its number of sides, and then classify it as *convex* or *concave* and *regular* or *irregular*. (Lesson 1-6) 15. _____

16. If $\overline{RT} \cong \overline{QM}$ and $RT = 88.9$ centimeters, find QM . (Lesson 2-7) 16. _____

17. Which segment is the shortest segment from D to \overleftrightarrow{JM} ? (Lesson 5-2)



17. _____

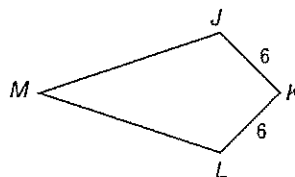
18. If $\triangle ABC \cong \triangle WXY$, $AB = 72$, $BC = 65$, $CA = 13$, $XY = 7x - 12$, and $WX = 19y + 34$, find the values of x and y . (Lesson 4-3) 18. _____

19. Freda bought two bells for just over \$90 before tax. State the assumption you would make to write an indirect proof to show that at least one of the bells costs more than \$45. (Lesson 5-4) 19. _____

20. The area of the base of a cylinder is 5 cm^2 and the height of the cylinder is 8 cm. Find the volume of the cylinder. (Lesson 1-7) 20. _____

21. $JKLM$ is a kite. Complete each statement. (Lesson 6-6)

- a. $\overline{MJ} \cong$ _____
 b. $\overline{MK} \perp$ _____
 c. $m\angle L = m\angle$ _____



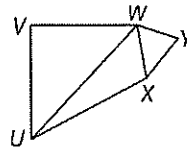
21. a. _____
 b. _____
 c. _____

6 Unit 2 Test

SCORE _____

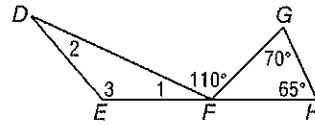
(Chapters 4–6)

1. Use a protractor to classify $\triangle UVW$, $\triangle UWX$, and $\triangle XWY$ as *acute*, *equiangular*, *obtuse*, or *right*.



1. _____

2. In the figure, $\angle 1 \cong \angle 2$. Find the measures of the numbered angles.



2. _____

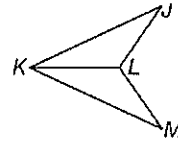
3. Name the corresponding congruent sides for $\triangle AFP \cong \triangle STX$.

3. _____

4. Determine whether $\triangle ABC \cong \triangle PQR$ given $A(2, -7)$, $B(5, 3)$, $C(-4, 6)$, $P(8, -1)$, $Q(11, 9)$, and $R(2, 12)$.

4. _____

5. In the figure, \overline{LK} bisects $\angle JKM$ and $\angle KLJ \cong \angle KLM$. Determine which theorem or postulate can be used to prove that $\triangle JKL \cong \triangle MKL$.

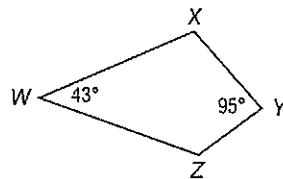


5. _____

6. Triangle ABC is isosceles with $AB = BC$. Name a pair of congruent angles in this triangle.

6. _____

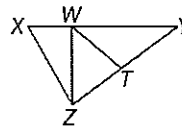
7. For kite $WXYZ$, find $m\angle Z$.



7. _____

For Questions 8 and 9, refer to the figure.

8. Find the value of a and $m\angle ZWT$ if \overline{ZW} is an altitude of $\triangle XYZ$, $m\angle ZWT = 3a + 5$, and $m\angle TWY = 5a + 13$.



8. _____

9. Determine which angle has the greatest measure: $\angle YWZ$, $\angle WZY$, or $\angle ZYW$.

9. _____

10. Mr. Ramirez bought a stove and a dishwasher for just over \$1206. State the assumption you would make to start an indirect proof to show that at least one of the appliances cost more than \$603.

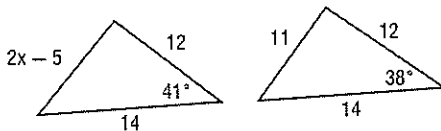
10. _____

6 Unit 2 Test *(continued)*

11. Determine whether 128 feet, 136 feet, and 245 feet can be the lengths of the sides of a triangle.

11. _____

12. Write an inequality to describe the possible values of x .

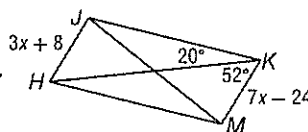


12. _____

13. The measure of an interior angle of a regular polygon is 140. Find the number of sides in the polygon.

13. _____

14. For parallelogram $JKMH$, find $m\angle JHK$, $m\angle HMK$, and the value of x .



14. _____

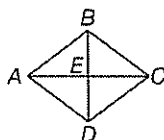
15. Determine whether the vertices of quadrilateral $DEFG$ form a parallelogram given $D(-3, 5)$, $E(3, 6)$, $F(-1, 0)$, and $G(6, 1)$.

15. _____

16. For rectangle $WXYZ$ with diagonals \overline{WY} and \overline{XZ} , $WY = 3d + 4$ and $XZ = 4d - 1$, find the value of d .

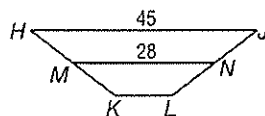
16. _____

17. If $m\angle BEC = 9z + 45$ in rhombus $ABCD$, find the value of z .



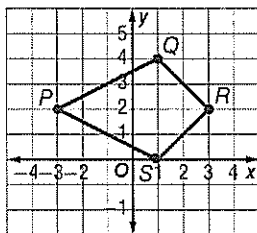
17. _____

18. In trapezoid $HJLK$, M and N are midpoints of the legs. Find KL .



18. _____

19. Prove that quadrilateral $PQRS$ is NOT a parallelogram.



19. _____

NAME _____ DATE _____ PERIOD _____

6 Anticipation Guide

Quadrilaterals

STEP 1 Before you begin *Chapter 6*

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. A triangle has no diagonals.	A
	2. A diagonal of a polygon is a segment joining the midpoints of two sides of the polygon.	D
	3. The sum of the measures of the angles in a polygon can be determined by subtracting 2 from the number of sides and multiplying the result by 180.	A
	4. For a quadrilateral to be a parallelogram it must have two pairs of parallel sides.	A
	5. The diagonals of a parallelogram are congruent.	D
	6. If you know that one pair of opposite sides of a quadrilateral is both parallel and congruent, then you know the quadrilateral is a parallelogram.	A
	7. If a quadrilateral is a rectangle, then all four angles are congruent.	A
	8. The diagonals of a rhombus are congruent.	D
	9. The properties of a rhombus are not true for a square.	D
	10. A trapezoid has only one pair of parallel sides.	A
	11. The median of a trapezoid is perpendicular to the bases.	D
	12. An isosceles trapezoid has exactly one pair of congruent sides.	A

STEP 2 After you complete *Chapter 6*

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

Answers (Anticipation Guide and Lesson 6-1)

NAME _____ DATE _____ PERIOD _____

6-1 Study Guide and Intervention

Angles of Polygons

Polygon Interior Angles Sum The segments that connect the nonconsecutive vertices of a polygon are called **diagonals**. Drawing all of the diagonals from one vertex of an n -gon separates the polygon into $n - 2$ triangles. The sum of the measures of the interior angles of the polygon can be found by adding the measures of the interior angles of those $n - 2$ triangles.

Polygon Interior Angle Sum Theorem The sum of the interior angle measures of an n -sided convex polygon is $(n - 2) \cdot 180$.

EXAMPLE A convex polygon has 13 sides. Find the sum of the measures of the interior angles.

$$\begin{aligned} (n - 2) \cdot 180 &= (13 - 2) \cdot 180 \\ &= 11 \cdot 180 \\ &= 1980 \end{aligned}$$

EXAMPLE The measure of an interior angle of a regular polygon is 120. Find the number of sides.

$$\begin{aligned} \text{The number of sides is } n, \text{ so the sum of the} \\ \text{measures of the interior angles is } 120n. \\ 120n &= (n - 2) \cdot 180 \\ 120n &= 180n - 360 \\ -60n &= -360 \\ n &= 6 \end{aligned}$$

Exercises

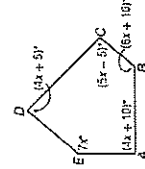
Find the sum of the measures of the interior angles of each convex polygon.

- decagon
1440
- 16-gon
2520
- 30-gon
5040
- octagon
1080
- 12-gon
1800
- 85-gon
5940

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

- 150
12
- 160
18
- 144
11
- 105
8
- 175
72
- 135
12
- 108
8

13. Find the value of x .



NAME _____ DATE _____ PERIOD _____

6-1 Study Guide and Intervention *(continued)*

Angles of Polygons

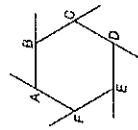
Polygon Exterior Angles Sum There is a simple relationship among the exterior angles of a convex polygon.

Polygon Exterior Angle Sum Theorem The sum of the exterior angle measures of a convex polygon, one angle at each vertex, is 360.

Example Find the sum of the measures of the exterior angles, one at each vertex, of a convex 27-gon.

For any convex polygon, the sum of the measures of its exterior angles, one at each vertex, is 360.

Example Find the measure of each exterior angle of regular hexagon $ABCDEF$.



The sum of the measures of the exterior angles is 360 and a hexagon has 6 angles. If n is the measure of each exterior angle, then

$$6n = 360$$

$$n = 60$$

The measure of each exterior angle of a regular hexagon is 60.

Exercises

Find the sum of the measures of the exterior angles of each convex polygon.

- | | | | | | |
|------------|-----|---------------|------|-------------|-----|
| 1. decagon | 360 | 2. 16-gon | 360 | 3. 36-gon | 360 |
| 4. 12-gon | 30 | 5. hexagon | 60 | 6. 20-gon | 18 |
| 7. 40-gon | 9 | 8. heptagon | 51.4 | 9. 12-gon | 30 |
| 10. 24-gon | 15 | 11. dodecagon | 30 | 12. octagon | 45 |

Chapter 6

6

Glencoe Geometry

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NAME _____ DATE _____ PERIOD _____

6-1 Skills Practice

Angles of Polygons

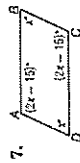
Find the sum of the measures of the interior angles of each convex polygon.

- nonagon **1260**
- heptagon **900**
- decagon **1440**

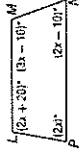
The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

- 108 **5**
- 120 **6**
- 150 **6**
- 120 **5**

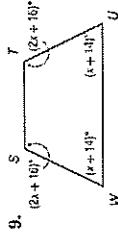
Find the measure of each interior angle.



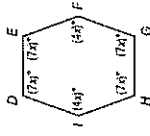
- $m\angle A = 115$, $m\angle B = 65$,
 $m\angle C = 115$, $m\angle D = 65$



- $m\angle L = 100$, $m\angle M = 110$,
 $m\angle N = 70$, $m\angle P = 80$



- $m\angle S = 116$, $m\angle T = 116$,
 $m\angle W = 64$, $m\angle U = 64$



- $m\angle D = 140$, $m\angle E = 140$,
 $m\angle F = 80$, $m\angle G = 80$,
 $m\angle H = 140$, $m\angle G = 140$

Find the measures of each interior angle of each regular polygon.

- quadrilateral **90, 90**
- pentagon **108, 72**
- dodecagon **150, 30**

Find the measures of each exterior angle of each regular polygon.

- octagon **45**
- nonagon **40**
- 12-gon **30**

Chapter 6

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Glencoe Geometry

Lesson 6-1

NAME _____ DATE _____ PERIOD _____

6-1 Practice

Angles of Polygons

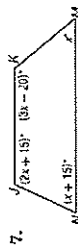
Find the sum of the measures of the interior angles of each convex polygon.

- 1. 11-gon **1620**
- 2. 14-gon **2160**
- 3. 17-gon **2700**

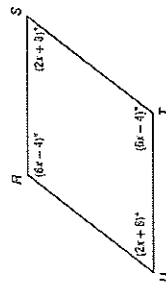
The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

- 4. 144° **10**
- 5. 156° **15**
- 6. 160° **18**

Find the measure of each interior angle.



$m\angle J = 115$, $m\angle K = 130$,
 $m\angle M = 50$, $m\angle N = 65$



$m\angle R = 128$, $m\angle S = 52$
 $m\angle T = 128$, $m\angle U = 52$

Find the measures of an exterior angle and an interior angle given the number of sides of each regular polygon. Round to the nearest tenth, if necessary.

- 9. 16 **157.5, 22.5**
- 10. 24 **165, 15**
- 11. 30 **168, 12**
- 12. 14 **163.6, 16.4**
- 13. 22 **171, 9**

15. **CRYSTALLOGRAPHY** Crystals are classified according to seven crystal systems. The basis of the classification is the shapes of the faces of the crystal. Turquoise belongs to the triclinic system. Each of the six faces of turquoise is in the shape of a parallelogram. Find the sum of the measures of the interior angles of one such face. **360**

NAME _____ DATE _____ PERIOD _____

6-1 Word Problem Practice

Angles of Polygons

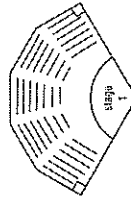
1. **ARCHITECTURE** In the Uffizi gallery in Florence, Italy, there is a room built by Buonataenti called the Tribune (*La Tribuna* in Italian). This room is shaped like a regular octagon.



What angle do consecutive walls of the Tribune make with each other?
 135°

2. **BOXES** Jasmine is designing boxes she will use to ship her jewelry. She wants to shape the box like a regular polygon. In order for the boxes to pack tightly, she decides to use a regular polygon that has the property that the measure of its interior angles is half the measure of its exterior angles. What regular polygon should she use?
an equilateral triangle

3. **THEATER** A theater floor plan is shown in the figure. The upper five sides are part of a regular dodecagon.



Find $m\angle 1$.
120

Answers (Lesson 6-1)

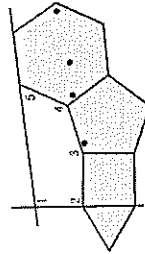
Lesson 6-1

4. **ARCHEOLOGY** Archeologists unearthed parts of two adjacent walls of an ancient castle.



Before it was unearthed, they knew from ancient texts that the castle was shaped like a regular polygon, but nobody knew how many sides it had. Some said 6, others 8, and some even said 100. From the information in the figure, how many sides did the castle really have?
15

5. **POLYGON PATH** In Ms. Kicketts' math class, students made a "polygon path" that consists of regular polygons of 3, 4, 5, and 6 sides joined together as shown.



a. Find $m\angle 2$ and $m\angle 5$.
90 and 60

b. Find $m\angle 3$ and $m\angle 4$.
162 and 132

c. What is $m\angle 1$?
96

NAME _____

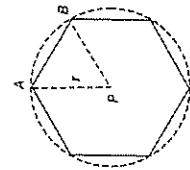
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6-1 Enrichment

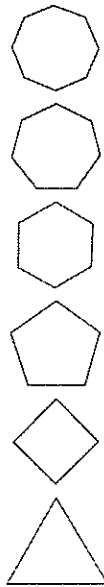
Central Angles of Regular Polygons

You have learned about the interior and exterior angles of a polygon. Regular polygons also have central angles. A central angle is measured from the center of the polygon.



The center of a polygon is the point equidistant from all of the vertices of the polygon, just as the center of a circle is the point equidistant from all of the points on the circle. The central angle is the angle drawn with the vertex at the center of the circle and the sides of angle drawn through consecutive vertices of the polygon. One of the central angles that can be drawn in this regular hexagon is $\angle APB$. You may remember from making circle graphs that there are 360° around the center of a circle.

1. By using logic or by drawing sketches, find the measure of the central angle of each regular polygon.



120 90 72 60 about 51.43 45

2. Make a conjecture about how the measure of a central angle of a regular polygon relates to the measures of the interior angles and exterior angles of a regular polygon.

The measure of the exterior angle equals the measure of the central angle. The central angle is supplementary to interior angle.

3. **CHALLENGE** In obtuse $\triangle ABC$, \overline{BC} is the longest side, \overline{AC} is also a side of a 21-sided regular polygon, \overline{AB} is also a side of a 28-sided regular polygon. The 21-sided regular polygon and the 28-sided regular polygon have the same center point P . Find n if \overline{BC} is a side of a n -sided regular polygon that has center point P .

(Hint: Sketch a circle with center P and place acute points A , B , and C on the circle.)

$n = 12$

Chapter 6

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Glencoe Geometry

Answers (Lesson 6-1 and Lesson 6-2)

NAME _____

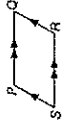
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PERIOD _____

6-2 Study Guide and Intervention

Parallelograms

Sides and Angles of Parallelograms A quadrilateral with both pairs of opposite sides parallel is a parallelogram. Here are four important properties of parallelograms.



If $PQRS$ is a parallelogram, then	If $PQRS$ is a parallelogram, then
$\overline{PQ} \cong \overline{SR}$ and $\overline{PS} \cong \overline{QR}$	If a quadrilateral is a parallelogram, then its opposite sides are congruent.
$\angle P \cong \angle R$ and $\angle S \cong \angle Q$	If a quadrilateral is a parallelogram, then its opposite angles are congruent.
$\angle P$ and $\angle S$ are supplementary; $\angle S$ and $\angle R$ are supplementary; $\angle R$ and $\angle Q$ are supplementary; $\angle Q$ and $\angle P$ are supplementary.	If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.
If $m\angle P = 90$, then $m\angle Q = 90$, $m\angle R = 90$, and $m\angle S = 90$.	If a parallelogram has one right angle, then it has four right angles.

Lesson 6-2

If $ABCD$ is a parallelogram, find the value of each variable.

\overline{AB} and \overline{CD} are opposite sides, so $\overline{AB} \cong \overline{CD}$.

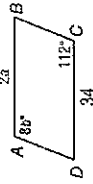
$2a = 34$

$a = 17$

$\angle A$ and $\angle C$ are opposite angles, so $\angle A \cong \angle C$.

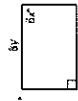
$8b = 112$

$b = 14$



Exercises

Find the value of each variable.

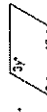


$x = 30; y = 22.5$

$x = 15; y = 11$



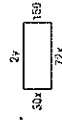
$x = 2; y = 4$



$x = 10; y = 40$



$x = 13; y = 32.5$



$x = 5; y = 180$

Chapter 6

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Glencoe Geometry

NAME _____ DATE _____ PERIOD _____

6-2 Study Guide and Intervention (continued)

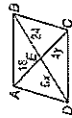
Parallelograms

Diagonals of Parallelograms Two important properties of parallelograms deal with their diagonals.



If a quadrilateral is a parallelogram, then its diagonals bisect each other.	If $ABCD$ is a parallelogram, then $AP = PC$ and $DP = PB$
If a quadrilateral is a parallelogram, then each diagonal separates the parallelogram into two congruent triangles.	$\triangle ACD \cong \triangle CAB$ and $\triangle ADB \cong \triangle CBD$

Find the value of x and y in parallelogram $ABCD$.



The diagonals bisect each other, so $AE = CE$ and $DE = BE$.

$$6x = 24$$

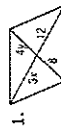
$$4y = 18$$

$$x = 4$$

$$y = 4.5$$

Exercises

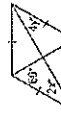
Find the value of each variable.



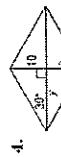
$x = 4; y = 2$



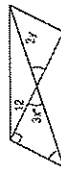
$x = 7; y = 14$



$x = 15; y = 7.5$



$x = 3\frac{1}{3}; y = 10\sqrt{3}$



$x = 15; y = 6\sqrt{2}$

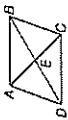


$x = 15; y = \sqrt{241}$

COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of $\square ABCD$ with the given vertices.

7. $A(3, 6), B(5, 8), C(3, -2)$, and $D(1, -4)$ 8. $A(-4, 3), B(2, 3), C(-1, -2)$, and $D(-7, -2)$
 (3, 2) (-2.5, 0.5)

9. **PROOF** Write a paragraph proof of the following.



Given: $\square ABCD$

Prove: $\triangle AED \cong \triangle BEC$

Diagonals of a parallelogram bisect each other, so $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$. Opposite sides of a parallelogram are congruent, therefore $\overline{AD} \cong \overline{BC}$. Because corresponding parts of the two triangles are congruent, the triangles are congruent by SSS.

Answers (Lesson 6-2)

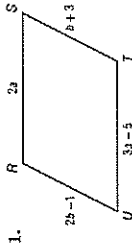
Lesson 6-2

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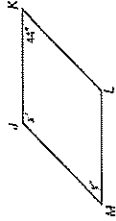
6-2 Skills Practice

Parallelograms

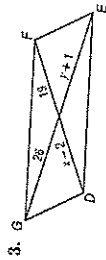
ALGEBRA Find the value of each variable.



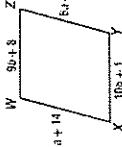
$a = 5, b = 4$



$x = 136, y = 44$

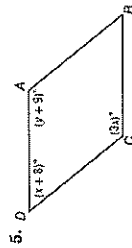


G

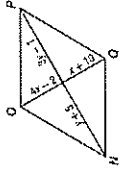


$x = 21, y = 25$

$a = 2, b = 7$



$x = 43, y = 120$



$x = 4, y = 3$

COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of $\square HJKL$ with the given vertices.

7. $H(1, 1), J(2, 3), K(6, 3), L(5, 1)$ 8. $H(-1, 4), J(3, 3), K(3, -2), L(-1, -1)$
 (3.5, 2) (1, 1)

9. **PROOF** Write a paragraph proof of the theorem. *Consecutive angles in a parallelogram are supplementary.*

Given: $\square ABCD$

Prove: $\angle A$ and $\angle B$ are supplementary.

$\angle B$ and $\angle C$ are supplementary.

$\angle C$ and $\angle D$ are supplementary.

$\angle D$ and $\angle A$ are supplementary.



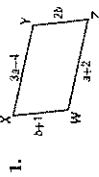
Proof: We are given $\square ABCD$, so we know that $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DA}$ by the definition of a parallelogram. We also know that if two parallel lines are cut by a transversal, then consecutive interior angles are supplementary. So, $\angle A$ and $\angle B$, $\angle B$ and $\angle C$, $\angle C$ and $\angle D$, and $\angle D$ and $\angle A$ are pairs of supplementary angles.

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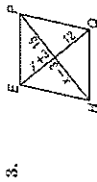
6-2 Practice

Parallelograms

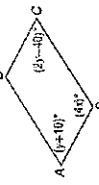
ALGEBRA Find the value of each variable.



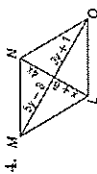
$a = 3, b = 1$



$x = 18, y = 9$



$x = 30°, y = 50°$



$x = 2, y = 4.5$



6. $m\angle STU = 55$
8. $b = 6$

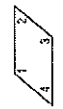
COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of $\square PRYZ$ with the given vertices.

- 9. $P(2, 5), R(3, 3), Y(-2, -3), Z(-3, -1)$
- 10. $P(-2, 3), R(1, -2), Y(-5, -7), Z(-4, -2)$



11. PROOF Write a paragraph proof of the following.
Given: $\square PRST$ and $\square PQVU$
Prove: $\angle V \cong \angle S$

Proof: We are given $\square PRST$ and $\square PQVU$. Since opposite angles of a parallelogram are congruent, $\angle P \cong \angle V$ and $\angle P \cong \angle S$. Since congruence of angles is transitive, $\angle V \cong \angle S$ by the Transitive Property of congruence.



12. CONSTRUCTION Mr. Rodriguez used the parallelogram at the right to design a herringbone pattern for a paving stone. He will use the paving stone for a sidewalk. If $m\angle 1$ is 130, find $m\angle 2, m\angle 3,$ and $m\angle 4$.
50, 130, 50

Answers (Lesson 6-2)

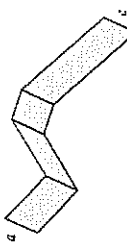
Lesson 6-2

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6-2 Word Problem Practice

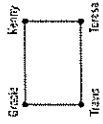
Parallelograms

1. WALKWAY A walkway is made by adjoining four parallelograms as shown.



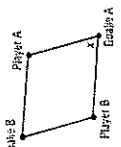
Are the end segments a and c parallel to each other? Explain.
Yes. Opposite sides of a parallelogram are parallel and the parallel property is transitive.

2. DISTANCE Four friends live at the four corners of a block shaped like a parallelogram. Gracie lives 3 miles away from Kenny. How far apart do Teresa and Travis live from each other?



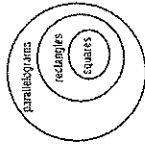
3 mi

3. SOCCER Four soccer players are located at the corners of a parallelogram. Two of the players in opposite corners are the goalies. In order for goalie A to be able to see the three others, she must be able to see a certain angle x in her field of vision.

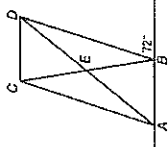


What angle does the other goalie have to be able to see in order to keep an eye on the other three players?
He or she also has to see angle x .

4. VENN DIAGRAMS Make a Venn diagram showing the relationship between squares, rectangles, and parallelograms.



5. SKYSCRAPERS On vacation, Tony's family took a helicopter tour of the city. The pilot said the newest building in the city was the building with this top view. He told Tony that the exterior angle by the front entrance is 79° . Tony wanted to know more about the building, so he drew this diagram and used his geometry skills to learn a few more things. The front entrance is next to vertex B .



- a. What are the measures of the four angles of the parallelogram?
72, 72, 108, 108
- b. How many pairs of congruent triangles are there in the figure? What are they?
4 pairs;
 $\triangle ABE \cong \triangle DCE,$
 $\triangle ABC \cong \triangle DCB,$
 $\triangle ACE \cong \triangle DBE,$ and
 $\triangle ABD \cong \triangle DCA$

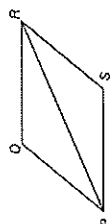
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6-2 Enrichment

Diagonals of Parallelograms

In some drawings the diagonal of a parallelogram appears to be the angle bisector of both opposite angles. When might that be true?

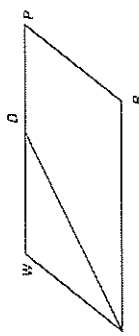
1. Given: Parallelogram $PQRS$ with diagonal \overline{PR} .
 \overline{PR} is an angle bisector of $\angle QPS$ and $\angle QRS$.
 What type of parallelogram is $PQRS$? Justify your answer.



$\angle QRS \cong \angle QPS$ opposite angles of a parallelogram

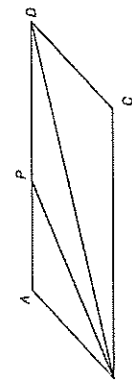
- $\angle QPR \cong \angle RPS$ definition of angle bisector
 - $\angle QRP \cong \angle SRP$ definition of angle bisector
 - $\angle QPR \cong \angle PRS$ alternate interior angles
 - $\angle RPS \cong \angle PRS$ Transitive Property
 - $\overline{SP} \cong \overline{SR}$ Isosceles Triangle Theorem
- So all sides are \cong and $QRPS$ is a rhombus

2. Given: Parallelogram $WPRK$ with angle bisector \overline{KD} , $DP = 5$, and $WD = 7$.
 Find WK and KR .
 $WK = 7$, $KR = 12$



3. Refer to Exercise 2. Write a statement about parallelogram $WPRK$ and angle bisector \overline{KD} .
Sample answer: Parallelogram $WPRK$ is not a rhombus and \overline{KD} is not a diagonal.

4. Given: Parallelogram $ABCD$ with diagonal \overline{BD} and angle bisector \overline{BP} .
 $PD = 5$, $BP = 6$, and $CP = 6$.
 The perimeter of triangle PDC is 15.
 Find AB and BC .
 $AB = 4$, $BC = 9$



Answers (Lesson 6-2 and Lesson 6-3)

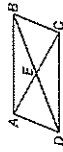
Lesson 6-3

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6-3 Study Guide and Intervention

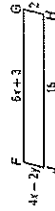
Tests for Parallelograms

Conditions for Parallelograms There are many ways to establish that a quadrilateral is a parallelogram.



If:	It:
both pairs of opposite sides are parallel.	$\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$,
both pairs of opposite sides are congruent.	$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$,
both pairs of opposite angles are congruent.	$\angle ABC \cong \angle ADC$ and $\angle DAB \cong \angle BCD$,
the diagonals bisect each other.	$\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$,
one pair of opposite sides is congruent and parallel.	$\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$, or $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$,
then: the figure is a parallelogram.	then: $ABCD$ is a parallelogram.

EXAMPLE Find x and y so that $FGEH$ is a parallelogram.

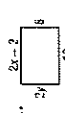



$FGEH$ is a parallelogram if the lengths of the opposite sides are equal.


$$\begin{aligned} 4x - 2y &= 2 \\ 6x + 3 &= 15 \\ 4(2) - 2y &= 2 \\ 6x &= 12 \\ x &= 2 \\ 8 - 2y &= 2 \\ -2y &= -6 \\ y &= 3 \end{aligned}$$

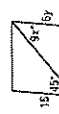
Exercises

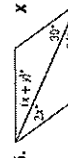
Find x and y so that the quadrilateral is a parallelogram.


1.  $x = 7$, $y = 4$

2.  $x = 5$, $y = 25$

3.  $x = 31$, $y = 5$

4.  $x = 5$, $y = 3$

5.  $x = 15$, $y = 9$

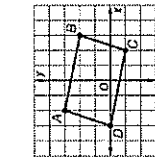
6.  $x = 30$, $y = 15$

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6-3 Study Guide and Intervention (continued)

Tests for Parallelograms

Parallelograms on the Coordinate Plane On the coordinate plane, the Distance, Slope, and Midpoint Formulas can be used to test if a quadrilateral is a parallelogram.



Determine whether ABCD is a parallelogram.

The vertices are $A(-2, 3)$, $B(3, 2)$, $C(2, -1)$, and $D(-3, 0)$.

Method 1: Use the Slope Formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

$$\text{slope of } \overline{AD} = \frac{3 - 0}{-2 - (-3)} = \frac{3}{1} = 3 \quad \text{slope of } \overline{BC} = \frac{2 - (-1)}{3 - 2} = \frac{3}{1} = 3$$

$$\text{slope of } \overline{AB} = \frac{2 - 3}{3 - (-2)} = \frac{-1}{5} = -\frac{1}{5} \quad \text{slope of } \overline{CD} = \frac{-1 - 0}{2 - (-3)} = \frac{-1}{5} = -\frac{1}{5}$$

Since opposite sides have the same slope, $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$. Therefore, ABCD is a parallelogram by definition.

Method 2: Use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$$AB = \sqrt{(-2 - 3)^2 + (3 - 2)^2} = \sqrt{25 + 1} = \sqrt{26}$$

$$CD = \sqrt{(2 - (-3))^2 + (-1 - 0)^2} = \sqrt{25 + 1} = \sqrt{26}$$

$$AD = \sqrt{(-2 - (-3))^2 + (3 - 0)^2} = \sqrt{1 + 9} = \sqrt{10}$$

$$BC = \sqrt{(3 - 2)^2 + (2 - (-1))^2} = \sqrt{1 + 9} = \sqrt{10}$$

Since both pairs of opposite sides have the same length, $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$. Therefore, ABCD is a parallelogram by Theorem 6.9.

Exercises

Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

See students' work

1. $A(0, 0)$, $B(1, 3)$, $C(5, 3)$, $D(4, 0)$;
Slope Formula
yes

2. $D(-1, 1)$, $E(2, 4)$, $F(6, 4)$, $G(3, 1)$;
Slope Formula
yes

3. $R(-1, 0)$, $S(3, 0)$, $T(2, -3)$, $U(-3, -2)$;
Distance Formula
no

4. $A(-3, 2)$, $B(-1, 4)$, $C(2, 1)$, $D(0, -1)$;
Distance and Slope Formulas
yes

5. $S(-2, 4)$, $T(-1, -1)$, $U(3, -4)$, $V(2, 1)$;
Distance and Slope Formulas
yes

6. $F(3, 3)$, $G(1, 2)$, $H(-3, 1)$, $K(-1, 4)$;
Midpoint Formula
no

7. A parallelogram has vertices $R(-2, -1)$, $S(2, 1)$, and $T(0, -3)$. Find all possible coordinates for the fourth vertex: $(4, -1)$, $(0, 3)$, or $(-4, -5)$

Chapter 6

Glencoe Geometry

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NAME _____ DATE _____ PERIOD _____

6-3 Skills Practice

Tests for Parallelograms

Determine whether each quadrilateral is a parallelogram. Justify your answer.



Yes; a pair of opposite sides is parallel and congruent.



Yes; both pairs of opposite angles are congruent.



No; none of the tests for parallelograms is fulfilled.



Yes; both pairs of opposite sides are congruent.

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

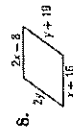
See students' graphs.

5. $P(0, 0)$, $Q(3, 4)$, $S(7, 4)$, $T(4, 0)$; Slope Formula
Yes; the slope of \overline{PQ} and \overline{QS} are equal and the slope of \overline{PQ} and \overline{TS} are equal, so $\overline{PQ} \parallel \overline{QS}$ and $\overline{PQ} \parallel \overline{TS}$. Opposite sides are parallel.

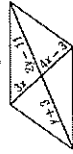
6. $S(-2, 1)$, $R(1, 3)$, $T(2, 0)$, $Z(-1, -2)$; Distance and Slope Formulas
Yes; $\overline{SR} = \overline{ZT}$ and the slopes of \overline{SR} and \overline{ZT} are equal, so one pair of opposite sides is parallel and congruent.

7. $W(2, 5)$, $R(3, 3)$, $Y(-2, -3)$, $M(-3, 1)$; Midpoint Formula
No; the midpoints of the diagonals are not the same point.

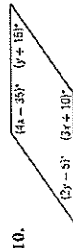
ALGEBRA Find x and y so that each quadrilateral is a parallelogram.



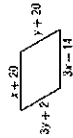
$x = 24$, $y = 19$



$x = 3$, $y = 14$



$x = 45$, $y = 20$



$x = 17$, $y = 9$

Chapter 6

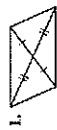
Glencoe Geometry

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6-3 Practice

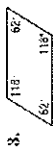
Tests for Parallelograms

Determine whether each quadrilateral is a parallelogram. Justify your answer.



Yes; the diagonals bisect each other.

No; none of the tests for parallelograms is fulfilled.



Yes; both pairs of opposite angles are congruent.

No; none of the tests for parallelograms is fulfilled.

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

See students' work

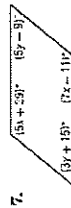
5. $P(-5, 1), S(-2, 2), T(2, -2);$ Slope Formula

yes

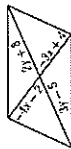
6. $R(-2, 5), O(1, 3), M(-3, -4), Y(-6, -2);$ Distance and Slope Formulas

yes

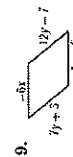
ALGEBRA Find x and y so that the quadrilateral is a parallelogram.



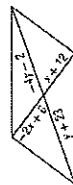
$x = 20, y = 12$



$x = -6, y = 13$



$x = -3, y = 2$



$x = -2, y = -5$

11. TILE DESIGN The pattern shown in the figure is to consist of congruent parallelograms. How can the designer be certain that the shapes are parallelograms?



Sample answer: Confirm that both pairs of opposite Δ are \cong .

Answers (Lesson 6-3)

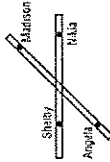
LESSON 6-3

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6-3 Word Problem Practice

Tests for Parallelograms

1. **BALANCING** Nikia, Madison, Angela, and Shelby are balancing themselves on an "X"-shaped floating object. To balance themselves, they want to make themselves the vertices of a parallelogram.

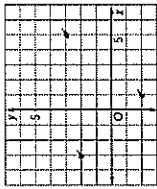


In order to achieve this, do all four of them have to be the same distance from the center of the object? Explain.

No, Madison and Angela have to be the same distance and Nikia and Shelby have to be the same distance but Nikia and Shelby's distance from the center does not have to be equal to Madison and Angela's distance.

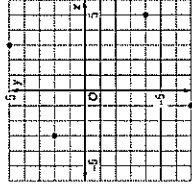
2. **COMPASSES** Two compass needles placed side by side on a table are both 2 inches long and point due north. Do they form the sides of a parallelogram? **yes**

3. **FORMATION** Four jets are flying in formation. Three of the jets are shown in the graph. If the four jets are located at the vertices of a parallelogram, what are the three possible locations of the missing jet?



(1, 7), (9, -1), (-7, -3)

4. **STREET LAMPS** When a coordinate plane is placed over the Harrisville town map, the four street lamps in the center are located as shown. Do the four lamps form the vertices of a parallelogram? Explain.



Yes; the lengths of the opposite sides are congruent.

5. **PICTURE FRAME** Aaron is making a wooden picture frame in the shape of a parallelogram. He has two pieces of wood that are 3 feet long and two that are 4 feet long.

a. If he connects the pieces of wood at their ends to each other, in what order must he connect them to make a parallelogram?

He must alternate the lengths 3, 4, 3 or 4, 3, 4, 3.

b. How many different parallelograms could he make with these four lengths of wood? **infinitely many**

c. Explain something Aaron might do to specify precisely the shape of the parallelogram.

Sample answer: He could specify the length of a diagonal.

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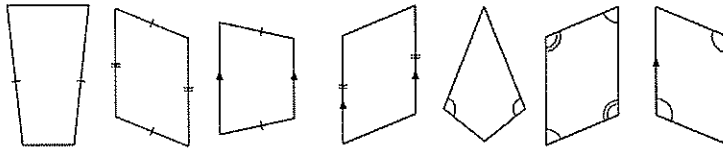
6-3 Enrichment

Tests for Parallelograms

By definition, a quadrilateral is a parallelogram if and only if both pairs of opposite sides are parallel. What conditions other than both pairs of opposite sides parallel will guarantee that a quadrilateral is a parallelogram? In this activity, several possibilities will be investigated by drawing quadrilaterals to satisfy certain conditions. Remember that any test that seems to work is not guaranteed to work unless it can be formally proven.

Complete.

1. Draw a quadrilateral with one pair of opposite sides congruent.
Must it be a parallelogram? **no**
2. Draw a quadrilateral with both pairs of opposite sides congruent.
Must it be a parallelogram? **yes**
3. Draw a quadrilateral with one pair of opposite sides parallel and the other pair of opposite sides congruent. Must it be a parallelogram? **no**
4. Draw a quadrilateral with one pair of opposite sides both parallel and congruent. Must it be a parallelogram? **yes**
5. Draw a quadrilateral with one pair of opposite angles congruent.
Must it be a parallelogram? **no**
6. Draw a quadrilateral with both pairs of opposite angles congruent.
Must it be a parallelogram? **yes**
7. Draw a quadrilateral with one pair of opposite sides parallel and one pair of opposite angles congruent. Must it be a parallelogram?
yes



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Answers (Lesson 6-3 and Lesson 6-4)

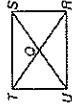
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6-4 Study Guide and Intervention

Rectangles

Properties of Rectangles A rectangle is a quadrilateral with four right angles. Here are the properties of rectangles.

- A rectangle has all the properties of a parallelogram.
- Opposite sides are parallel.
- Opposite angles are congruent.
- Opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.



Also:

- All four angles are right angles.
 - The diagonals are congruent.
- $\angle UTS, \angle TSR, \angle SRU,$ and $\angle RUT$ are right angles.
 $\overline{TR} \cong \overline{SU}$

EXAMPLE 1

Quadrilateral $RUTS$ above is a rectangle. If $US = 6x + 3$ and $RT = 7x - 2$, find x .

The diagonals of a rectangle are congruent, so $US = RT$.

$$\begin{aligned} 6x + 3 &= 7x - 2 \\ 3 &= x - 2 \\ 5 &= x \end{aligned}$$

EXAMPLE 2

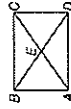
Quadrilateral $RUTS$ above is a rectangle. If $m\angle STR = 8x + 3$ and $m\angle UTR = 16x - 9$, find $m\angle STR$.

$\angle UTS$ is a right angle, so $m\angle STR + m\angle UTR = 90$.

$$\begin{aligned} 8x + 3 + 16x - 9 &= 90 \\ 24x - 6 &= 90 \\ 24x &= 96 \\ x &= 4 \\ m\angle STR &= 8x + 3 = 8(4) + 3 \text{ or } 35 \end{aligned}$$

Exercises

Quadrilateral $ABCD$ is a rectangle.



1. If $AE = 36$ and $CE = 2x - 4$, find x . **20**
2. If $BE = 6y + 2$ and $CE = 4y + 6$, find y . **2**
3. If $BC = 24$ and $AD = 5y - 1$, find y . **5**
4. If $m\angle BEA = 62^\circ$, find $m\angle BAC$. **59**
5. If $m\angle AED = 12x$ and $m\angle BEC = 10x + 20$, find $m\angle AED$. **120**
6. If $BD = 8y - 4$ and $AC = 7y + 3$, find BD . **52**
7. If $m\angle DBC = 10x$ and $m\angle ACB = 4x^\circ - 6$, find $m\angle ACB$. **30**
8. If $AB = 6y$ and $BC = 8y$, find BD in terms of y . **$10y$**

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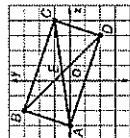
6-4 Study Guide and Intervention *(continued)*

Rectangles

Prove that Parallelograms Are Rectangles The diagonals of a rectangle are congruent, and the converse is also true.

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

In the coordinate plane, you can use the Distance Formula, the Slope Formula, and properties of diagonals to show that a figure is a rectangle.



Example Quadrilateral $ABCD$ has vertices $A(-3, 0)$, $B(-2, 3)$, $C(4, 1)$, and $D(3, -2)$. Determine whether $ABCD$ is a rectangle.

Method 1: Use the Slope Formula.

$$\text{slope of } \overline{AB} = \frac{3-0}{-2-(-3)} = \frac{3}{1} = 3 \text{ or } \frac{3}{1} \text{ or } 3$$

$$\text{slope of } \overline{AD} = \frac{-2-0}{3-(-3)} = \frac{-2}{6} = -\frac{1}{3} \text{ or } -\frac{1}{3}$$

$$\text{slope of } \overline{BC} = \frac{1-3}{4-(-2)} = \frac{-2}{6} = -\frac{1}{3} \text{ or } -\frac{1}{3}$$

$$\text{slope of } \overline{CD} = \frac{-2-1}{3-4} = \frac{-3}{-1} = 3 \text{ or } 3$$

Opposite sides are parallel, so the figure is a parallelogram. Consecutive sides are perpendicular, so $ABCD$ is a rectangle.

Method 2: Use the Distance Formula.

$$AB = \sqrt{(-3 - (-2))^2 + (0 - 3)^2} = \sqrt{1 + 9} = \sqrt{10}$$

$$BC = \sqrt{(-2 - 4)^2 + (3 - 1)^2} = \sqrt{36 + 4} = \sqrt{40}$$

$$CD = \sqrt{(4 - 3)^2 + (1 - (-2))^2} = \sqrt{1 + 9} = \sqrt{10}$$

$$AD = \sqrt{(3 - (-3))^2 + (0 - (-2))^2} = \sqrt{36 + 4} = \sqrt{40}$$

Opposite sides are congruent, thus $ABCD$ is a parallelogram.

$$AC = \sqrt{(-3 - 4)^2 + (0 - 1)^2} = \sqrt{49 + 1} = \sqrt{50}$$

$$BD = \sqrt{(-2 - 3)^2 + (3 - (-2))^2} = \sqrt{25 + 25} = \sqrt{50}$$

$ABCD$ is a parallelogram with congruent diagonals, so $ABCD$ is a rectangle.

Exercises

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

See students' work

- $A(-3, 1)$, $B(-3, 3)$, $C(3, 3)$, $D(3, 1)$; Distance Formula
Yes; $AB = 2$, $BC = 6$, $CD = 6$, $DA = 2$, $AC = \sqrt{40}$, $BD = \sqrt{40}$, opposite sides and diagonals are congruent.
- $A(-3, 0)$, $B(-2, 3)$, $C(4, 5)$, $D(3, 2)$; Slope Formula
No; slope of $AB = 3$, slope of $BC = \frac{1}{3}$, slopes show that two consecutive sides are not perpendicular.
- $A(-3, 0)$, $B(-2, 2)$, $C(3, 0)$, $D(2, -2)$; Distance Formula
No; $AC = 6$, $BD = \sqrt{32}$, diagonals are not congruent.
- $A(-1, 0)$, $B(0, 2)$, $C(4, 0)$, $D(3, -2)$; Distance Formula
Yes; $AB = \sqrt{5}$, $BC = \sqrt{20}$, $CD = \sqrt{5}$, $DA = \sqrt{20}$, $AC = 5$, $BD = 5$, opposite sides and diagonals are congruent.

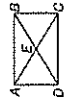
Answers (Lesson 6-4)

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6-4 Skills Practice

Rectangles

ALGEBRA Quadrilateral $ABCD$ is a rectangle.



- If $AC = 2x + 13$ and $DB = 4x - 1$, find DB . 27
- If $AC = x + 3$ and $DB = 3x - 19$, find AC . 14
- If $AE = 3x + 3$ and $EC = 5x - 15$, find AC . 60
- If $DE = 6x - 7$ and $AE = 4x + 9$, find DB . 82
- If $m\angle DAC = 2x + 4$ and $m\angle BAC = 3x + 1$, find $m\angle BAC$. 52
- If $m\angle BDC = 7x + 1$ and $m\angle ADB = 6x - 7$, find $m\angle BDC$. 43
- If $m\angle ABD = 7x - 31$ and $m\angle CDB = 4x + 5$, find $m\angle ABD$. 53
- If $m\angle BAC = x + 3$ and $m\angle CAD = x + 15$, find $m\angle BAC$. 39

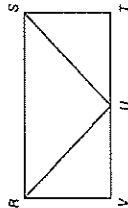
9. PROOF: Write a two-column proof.

Given: $RSTV$ is a rectangle and U is the midpoint of \overline{VT} .

Prove: $\triangle RUV \cong \triangle SUT$

Proof

Statements	Reasons
1. $RSTV$ is a rectangle.	1. Given
2. $\angle V$ and $\angle T$ are right angles.	2. Definition of rectangle
3. $\angle V \cong \angle T$	3. All rt \angle are \cong .
4. U is the midpoint of \overline{VT} .	4. Given
5. $\overline{UV} \cong \overline{TU}$	5. Definition of midpoint
6. $\overline{VR} \cong \overline{TS}$	6. Opp sides of \square are congruent.
7. $\triangle RUV \cong \triangle SUT$	7. SAS



Lesson 6-4

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

See students' graphs.

- $P(-3, -2)$, $Q(-4, 2)$, $R(2, 4)$, $S(3, 0)$; Slope Formula
No; Sample answer: Angles are not right angles.
- $J(-6, 3)$, $K(0, 6)$, $L(2, 2)$, $M(-4, -1)$; Distance Formula
Yes; Sample answer: Both pairs of opposite sides are congruent and diagonals are congruent.
- $T(4, 1)$, $U(3, -1)$, $X(-3, 2)$, $Y(-2, 4)$; Distance Formula
Yes; Sample answer: Both pairs of opposite sides are congruent and the diagonals are congruent.

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6-4 Practice

Rectangles

ALGEBRA Quadrilateral $RSTU$ is a rectangle.

1. If $UZ = x + 21$ and $ZS = 3x - 15$, find US . **78**

2. If $RZ = 3x + 8$ and $ZS = 6x - 28$, find UZ . **44**

3. If $RT = 5x + 8$ and $RZ = 4x + 1$, find ZT . **9**

4. If $m\angle SUT = 3x + 6$ and $m\angle RUS = 5x - 4$, find $m\angle SUT$. **39**

5. If $m\angle SRT = x + 9$ and $m\angle UTR = 2x - 34$, find $m\angle UTR$. **62**

6. If $m\angle RSU = x + 41$ and $m\angle TUS = 3x + 9$, find $m\angle RSU$. **57**

Quadrilateral $GHIK$ is a rectangle. Find each measure if $m\angle 1 = 37$.

7. $m\angle 2$ **53**

8. $m\angle 3$ **37**

9. $m\angle 4$ **37**

10. $m\angle 5$ **53**

11. $m\angle 6$ **106**

12. $m\angle 7$ **74**

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

See students' work

13. $B(-4, 3)$, $C(-2, 4)$, $I(1, -2)$, $L(-1, -3)$; Slope Formula

Yes; sample answer: Opposite sides are parallel and consecutive sides are perpendicular.

14. $N(-4, 5)$, $O(6, 0)$, $P(3, -6)$, $Q(-7, -1)$; Distance Formula

Yes; sample answer: Opposite sides are congruent and diagonals are congruent.

15. $C(0, 5)$, $D(4, 7)$, $E(5, 4)$, $F(1, 2)$; Slope Formula

No; sample answer: Diagonals are not congruent.

16. **LANDSCAPING** Huntington Park officials approved a rectangular plot of land for a Japanese Zen garden. Is it sufficient to know that opposite sides of the garden plot are congruent and parallel to determine that the garden plot is rectangular? Explain.

No; if you only know that opposite sides are congruent and parallel, the most you can conclude is that the plot is a parallelogram.

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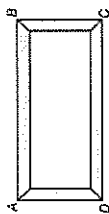
Glencoe Geometry

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6-4 Word Problem Practice

Rectangles

1. **FRAMES** Jalen makes the rectangular frame shown.

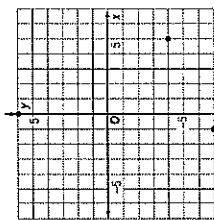


In order to make sure that it is a rectangle, Jalen measures the distances BD and AC . How should these two distances compare if the frame is a rectangle? **They should be equal.**



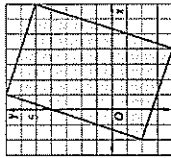
2. **BOOKSHELVES** A bookshelf consists of two vertical planks with five horizontal shelves. Are each of the four sections for books rectangles? Explain. **Yes; each of the four angles of each rectangle is created by a straight horizontal line and a straight vertical line, so each has four 90° angles.**

3. **LANDSCAPING** A landscaper is marking off the corners of a rectangular plot of land. Three of the corners are in place as shown.



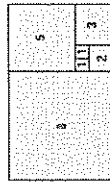
What are the coordinates of the fourth corner? **(-6, 3)**

4. **SWIMMING POOLS** Antonio is designing a swimming pool on a coordinate grid. Is it a rectangle? Explain.



Yes. Sample answer: The slope of the long sides is 3 and the slope of the short sides is 1/3, so each pair of sides is parallel. Also, the long and short sides have slopes that are perpendicular to each other, making the four corners right angles.

5. **PATTERNS** Veronica made the pattern shown out of 7 rectangles with four equal sides. The side length of each rectangle is written inside the rectangle.



a. How many rectangles can be formed using the lines in this figure? **11**

b. If Veronica wanted to extend her pattern by adding another rectangle with 4 equal sides to make a larger rectangle, what are the possible side lengths of rectangles that she can add? **8 and 13 units**

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Lesson 6-4

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6-4 Enrichment

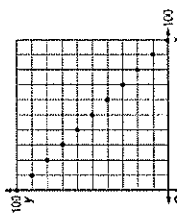
Constant Perimeter

Douglas wants to fence a rectangular region of his back yard for his dog. He bought 200 feet of fence.

- Complete the table to show the dimensions of five different rectangular pens that would use the entire 200 feet of fence. Then find the area of each rectangular pen.
- Do all five of the rectangular pens have the same area? If not, which one has the larger area?

Perimeter	Length	Width	Area
200	80	20	1600
200	70	30	2100
200	60	40	2400
200	50	50	2500
200	45	55	2475

- No, the 50 x 50 pen has the largest area. Write a rule for finding the dimensions of a rectangle with the largest possible area for a given perimeter.
- The rectangle with the largest area for a given perimeter is a square.
- Let x represent the length of a rectangle and y the width. Write the formula for all rectangles with a perimeter of 200. Then graph this relationship on the coordinate plane at the right.



$2x + 2y = 200$ or $x + y = 100$

Julio read that a dog the size of his new pet, Bennie, should have at least 100 square feet in his pen. Before going to the store to buy fence, Julio made a table to determine the dimensions for Bennie's rectangular pen.

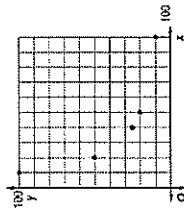
- Complete the table to find five possible dimensions of a rectangular fenced area of 100 square feet.
- See table for sample answers.

Area	Length	Width	How much fence to buy
100	1	100	202
100	2	50	104
100	4	25	58
100	5	20	50
100	10	10	40

- Write a rule for finding the dimensions of a rectangle with the least possible perimeter for a given area.
- The rectangle with the least possible perimeter for a given area is a square.

For length x and width y , write a formula for the area of a rectangle with an area of 100 square feet. Then graph the formula.

$xy = 100$



Answers (Lesson 6-4)

Lesson 6-4

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6-4 Graphing Calculator Activity

TI-Nspire: Exploring Rectangles

A quadrilateral with four right angles is a rectangle. The TI-Nspire can be used to explore some of the characteristics of a rectangle. Use the following steps to draw a rectangle.

- Step 1** Set up the calculator in the correct mode.
- Choose **Graphs & Geometry** from the Home Menu.
 - From the View menu, choose **4: Hide Axis**
- Step 2** Draw the rectangle
- From the **8: Shapes** menu choose **3: Rectangle**.
 - Click once to define the corner of the rectangle. Then move and click again. The side of the rectangle is now defined. Move perpendicularly to draw the rectangle. Click to anchor the shape.
- Step 3** Measure the lengths of the sides of the rectangle.
- From the **7: Measurement** menu choose **1: Length** (Note that when you scroll over the rectangle, the value now shown is the perimeter of the rectangle.)
 - Select each endpoint of a segment of the rectangle. Then click or press **Enter** to anchor the length of the segment in the work area.
 - Repeat for the other sides of the rectangle.

Exercises

- What appears to be true about the opposite sides of the rectangle?
The opposite sides of a rectangle are congruent.
- Draw the diagonals of the rectangle using **5: Segment** from the **6: Points and Lines** Menu. Click on two opposite vertices to draw the diagonal. Repeat to draw the other diagonal.
 - Measure each diagonal using the measurement tool. What do you observe?
 - What is true about the triangles formed by the sides of the rectangle and a diagonal? Justify your conclusion.
 - The diagonals of a rectangle are congruent.**
 - The triangles are congruent by SSS.**

- Press **Clear** three times and select **Yes** to clear the screen. Repeat the steps and draw another rectangle. Do the relationships that you found for the first rectangle you drew hold true for this rectangle?

Yes, the opposite sides are congruent and the diagonals are congruent.

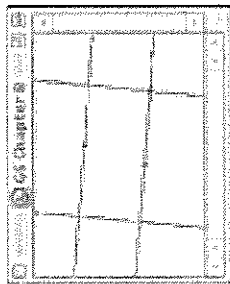
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6-4 Geometer's Sketchpad Activity

Exploring Rectangles

A quadrilateral with four right angles is a rectangle. The Geometer's Sketchpad is a useful tool for exploring some of the characteristics of a rectangle. Use the following steps to draw a rectangle.

- Step 1** Use the Line tool to draw a line anywhere on the screen.
- Step 2** Use the Point tool to draw a point that is not on the line. To draw a line perpendicular to the first line you drew, select the first line and the point. Then choose **Perpendicular Line** from the Construct menu.
- Step 3** Use the Point tool to draw a point that is not on either of the lines you have drawn. Repeat the procedure in Step 2 to draw lines perpendicular to the two lines you have drawn.



A rectangle is formed by the segments whose endpoints are the points of intersection of the lines.

Exercises

Use the measuring capabilities of The Geometer's Sketchpad to explore the characteristics of a rectangle.

1. What appears to be true about the opposite sides of the rectangle that you drew? Make a conjecture and then measure each side to check your conjecture.
The opposite sides of a rectangle are congruent.
2. Draw the diagonals of the rectangle by using the Selection Arrow tool to choose two opposite vertices. Then choose **Segment** from the Construct menu to draw the diagonal. Repeat to draw the other diagonal.
 - a. Measure each diagonal. What do you observe?
The diagonals of a rectangle are congruent.
 - b. What is true about the triangles formed by the sides of the rectangle and a diagonal? Justify your conclusion.
The triangles are congruent by SSS.

3. Choose **New Sketch** from the File menu and follow steps 1–3 to draw another rectangle. Do the relationships you found for the first rectangle you drew hold true for this rectangle also?
Yes, the opposite sides are congruent and the diagonals are congruent.

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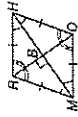
Answers (Lesson 6-4 and Lesson 6-5)

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6-5 Study Guide and Intervention

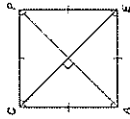
Rhombi and Squares

Properties of Rhombi and Squares A rhombus is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.



The diagonals are perpendicular.	$\overline{EF} \perp \overline{FO}$
Each diagonal bisects a pair of opposite angles.	\overline{EH} bisects $\angle RHO$ and $\angle RHO$, \overline{FO} bisects $\angle MRP$ and $\angle MRP$.

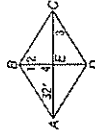
A square is a parallelogram with four congruent sides and four congruent angles. A square is both a rectangle and a rhombus; therefore, all properties of parallelograms, rectangles, and rhombi apply to squares.



Example 1 In rhombus $ABCD$, $m\angle BAC = 32$. Find the measure of each numbered angle.

$ABCD$ is a rhombus, so the diagonals are perpendicular and $\triangle ABE$ is a right triangle. Thus $m\angle 4 = 90$ and $m\angle 1 = 90 - 32$ or 58 . The diagonals in a rhombus bisect the vertex angles, so $m\angle 1 = m\angle 2$. Thus, $m\angle 2 = 58$.

A rhombus is a parallelogram, so the opposite sides are parallel. $\angle BAC$ and $\angle 3$ are alternate interior angles for parallel lines, so $m\angle 3 = 32$.



Exercises

Quadrilateral $ABCD$ is a rhombus. Find each value or measure.



1. If $m\angle ABD = 60$, find $m\angle BDC$. **60**
2. If $AE = 8$, find AC . **16**
3. If $AB = 26$ and $BD = 20$, find AE . **24**
4. Find $m\angle CEB$. **90**
5. If $m\angle CBD = 55$, find $m\angle ACB$. **32**
6. If $AE = 3x - 1$ and $AC = 16$, find x . **3**
7. If $m\angle CDB = 6y$ and $m\angle ACB = 2y + 10$, find y . **10**
8. If $AD = 2x + 4$ and $CD = 4x - 4$, find x . **4**

Chapter 6

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Glencoe Geometry

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6-5 Study Guide and Intervention (continued)

Rhombi and Squares

Conditions for Rhombi and Squares The theorems below can help you prove that a parallelogram is a rectangle, rhombus, or square.

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.
 If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.

If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.

If a quadrilateral is both a rectangle and a rhombus, then it is a square.

Example 1 Determine whether parallelogram $ABCD$ with vertices $A(-3, -3)$, $B(1, 1)$, $C(5, -3)$, $D(1, -7)$ is a rhombus, a rectangle, or a square.

$$AC = \sqrt{(-3 - 5)^2 + (-3 - (-3))^2} = \sqrt{64} = 8$$

$$BD = \sqrt{(1 - 1)^2 + (-7 - 1)^2} = \sqrt{64} = 8$$

The diagonals are the same length; the figure is a rectangle.

$$\text{Slope of } \overline{AC} = \frac{-3 - (-3)}{-3 - 5} = \frac{0}{-8} = 0$$

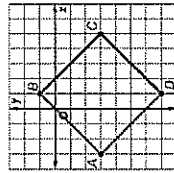
$$\text{Slope of } \overline{BD} = \frac{1 - (-7)}{1 - 1} = \frac{8}{0} = \text{undefined}$$

The line is horizontal.

The line is vertical.

Since a horizontal and vertical line are perpendicular, the diagonals are perpendicular.

Parallelogram $ABCD$ is a square which is also a rhombus and a rectangle.



Exercises

Given each set of vertices, determine whether $\square ABCD$ is a rhombus, rectangle, or square. List all that apply. Explain.

1. $A(0, 2)$, $B(2, 4)$, $C(4, 2)$, $D(2, 0)$

Rectangle, rhombus, square; the four sides are \cong and consecutive sides are \perp .

3. $A(-2, -1)$, $B(0, 2)$, $C(2, -1)$, $D(0, -4)$

Rhombus; the four sides are \cong and consecutive sides are not \perp .

5. PROOF Write a two-column proof.

Given: Parallelogram $RSTU$, $\overline{RS} \cong \overline{ST}$

Prove: $RSTU$ is a rhombus.

Statements	Reasons
1. $RSTU$ is a parallelogram	1. Given
$\overline{RS} \cong \overline{ST}$	
2. $\overline{RS} \cong \overline{UT}$, $\overline{RU} \cong \overline{ST}$	2. Definition of a parallelogram
3. $\overline{UT} \cong \overline{RS} \cong \overline{ST} \cong \overline{RU}$	3. Substitution
4. $RSTU$ is a rhombus	4. Definition of a rhombus

Answers (Lesson 6-5)

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6-5 Skills Practice

Rhombi and Squares

ALGEBRA Quadrilateral $DKLM$ is a rhombus.

1. If $DK = 8$, find KL . **8**

2. If $m\angle DML = 82$ find $m\angle DKM$. **41**

3. If $m\angle KAL = 2x - 8$, find x . **49**

4. If $DA = 4x$ and $AL = 5x - 3$, find DL . **24**

5. If $DA = 4x$ and $AL = 5x - 3$, find AD . **12**

6. If $DM = 5y + 2$ and $DK = 3y + 6$, find KL . **12**

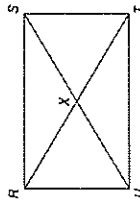
7. PROOF Write a two-column proof.

Given: $RSTU$ is a parallelogram.

$\overline{RX} \cong \overline{TX} \cong \overline{SX} \cong \overline{UX}$

Prove: $RSTU$ is a rectangle.

Proof



Reasons

- | Statements | Reasons |
|---|--|
| 1. $RSTU$ is a parallelogram. | 1. Given |
| $\overline{RX} \cong \overline{TX} \cong \overline{SX} \cong \overline{UX}$ | |
| 2. $RX = TX = SX = UX$ | 2. Definition of congruent segments |
| 3. $RX + XT = RT$, $SX + XU = SU$ | 3. Seg Add Postulate |
| 4. $RX + XT = SX$ | 4. Substitution Property |
| 5. $RT = SU$ | 5. Transitive Property |
| 6. $\overline{RT} \cong \overline{SU}$ | 6. Definition of \cong segment |
| 7. $RSTU$ is a rectangle. | 7. If diagonals \cong , the figure is a rectangle. |

COORDINATE GEOMETRY Given each set of vertices, determine whether $\square QRST$ is a rhombus, a rectangle, or a square. List all that apply. Explain.

8. $Q(3, 5)$, $R(3, 1)$, $S(-1, 1)$, $T(-1, 5)$

Rhombus, rectangle, square; all sides are congruent and the diagonals are perpendicular and congruent.

9. $Q(-5, 12)$, $R(5, 12)$, $S(-1, 4)$, $T(-11, 4)$

Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.

10. $Q(-6, -1)$, $R(4, -6)$, $S(2, 5)$, $T(-8, 10)$

Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.

11. $Q(2, -4)$, $R(-6, -8)$, $S(-10, 2)$, $T(-2, 6)$

None; opposite sides are congruent, but the diagonals are neither congruent nor perpendicular.

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6-5 Practice

Rhombi and Squares

PRYZ is a rhombus. If $RK = 5$, $RY = 13$ and $m\angle YRZ = 67$, find each measure.

1. KY 12

2. PK 12

3. $m\angle YKZ$ 90

4. $m\angle PZR$ 67



MNPQ is a rhombus. If $PQ = 3\sqrt{2}$ and $AP = 3$, find each measure.

5. AQ 3

6. $m\angle APQ$ 45

7. $m\angle MNP$ 90

8. PM 6



COORDINATE GEOMETRY Given each set of vertices, determine whether $\square BBFG$ is a rhombus, a rectangle, or a square. List all that apply. Explain.

9. $B(-9, 1)$, $E(2, 3)$, $F(12, -2)$, $G(1, -4)$

Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.

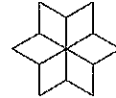
10. $B(1, 3)$, $E(7, -3)$, $F(1, -9)$, $G(-5, -3)$

Rhombus, rectangle, square; all sides are congruent and the diagonals are perpendicular and congruent.

11. $B(-4, -5)$, $E(1, -5)$, $F(-2, -1)$, $G(-7, -1)$

Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.

12. TESSELLATIONS The figure is an example of a tessellation. Use a ruler or protractor to measure the shapes and then name the quadrilaterals used to form the figure.



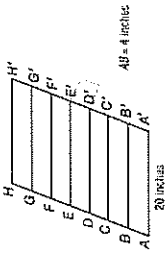
The figure consists of 6 congruent rhombi.

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6-5 Word Problem Practice

Rhombi and Squares

1. TRAY RACKS A tray rack looks like a parallelogram from the side. The levels for the trays are evenly spaced.



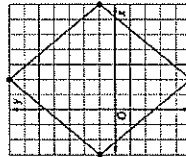
What two labeled points form a rhombus with base AA' ?

F and F'

2. SLICING Charles cuts a rhombus along both diagonals. He ends up with four congruent triangles. Classify these triangles as *acute*, *obtuse*, or *right*.

right triangles

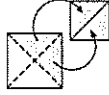
3. WINDOWS The edges of a window are drawn in the coordinate plane.



Determine whether the window is a square or a rhombus.

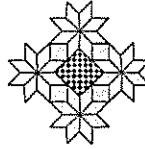
rhombus

4. SQUARES Maekenzie cut a square along its diagonals to get four congruent right triangles. She then joined two of them along their long sides. Show that the resulting shape is a square.



Sample answer: The diagonals of a square create four congruent right isosceles triangles. When two congruent right isosceles triangles are joined along their hypotenuses, the result is a quadrilateral with 4 equal sides and 4 right angle corners (because $45 + 45 = 90$), making it a square.

5. DESIGN Tatianna made the design shown. She used 32 congruent rhombi to create the flower-like design at each corner.



a. What are the angles of the corner rhombi? **45, 135, 45, 135**

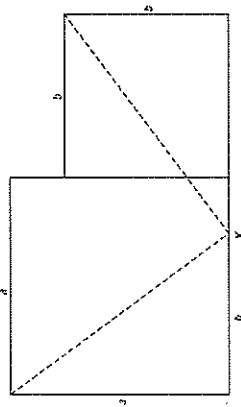
b. What kinds of quadrilaterals are the dotted and checkered figures? **They are all squares.**

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6-5 Enrichment

Creating Pythagorean Puzzles

By drawing two squares and cutting them in a certain way, you can make a puzzle that demonstrates the Pythagorean Theorem. A sample puzzle is shown. You can create your own puzzle by following the instructions below.



See students' work. A sample answer is shown.

- Carefully construct a square and label the length of a side as a . Then construct a smaller square to the right of it and label the length of a side as b , as shown in the figure above. The bases should be adjacent and collinear.
- Mark a point X that is b units from the left edge of the larger square. Then draw the segments from the upper left corner of the larger square to point X , and from point X to the upper right corner of the smaller square.
- Cut out and rearrange your five pieces to form a larger square. Draw a diagram to show your answer.
- Verify that the length of each side is equal to $\sqrt{a^2 + b^2}$.

Chapter 6

Glencoe Geometry

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Answers (Lesson 6-5 and Lesson 6-6)

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6-6 Study Guide and Intervention

Trapezoids and Kites

Properties of Trapezoids A trapezoid is a quadrilateral with exactly one pair of parallel sides. The midsegment or median of a trapezoid is the segment that connects the midpoints of the legs of the trapezoid. Its measure is equal to one-half the sum of the lengths of the bases. If the legs are congruent, the trapezoid is an isosceles trapezoid. In an isosceles trapezoid both pairs of base angles are congruent and the diagonals are congruent.



$STUR$ is an isosceles trapezoid.
 $\overline{SR} \cong \overline{TU}$; $\angle R \cong \angle U$; $\angle S \cong \angle T$

Example 2

The vertices of $ABCD$ are $A(-3, -1)$, $B(-1, 3)$, $C(2, 3)$, and $D(4, -1)$. Show that $ABCD$ is a trapezoid and determine whether it is an isosceles trapezoid.

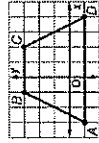
$$\text{slope of } \overline{AB} = \frac{3 - (-1)}{-1 - (-3)} = \frac{4}{2} = 2$$

$$\text{slope of } \overline{AD} = \frac{3 - (-1)}{4 - (-3)} = \frac{4}{7} \neq 2$$

$$\text{slope of } \overline{BC} = \frac{3 - 3}{2 - (-1)} = \frac{0}{3} = 0$$

$$\text{slope of } \overline{CD} = \frac{-1 - 3}{4 - 2} = \frac{-4}{2} = -2$$

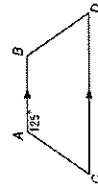
Exactly two sides are parallel, \overline{AD} and \overline{BC} , so $ABCD$ is a trapezoid. $AB = CD$, so $ABCD$ is an isosceles trapezoid.



Exercises

Find each measure.

1. $m\angle D$ 55



2. $m\angle L$ 140



COORDINATE GEOMETRY For each quadrilateral with the given vertices, verify that the quadrilateral is a trapezoid and determine whether the figure is an isosceles trapezoid.

3. $A(-1, 1)$, $B(3, 2)$, $C(1, -2)$, $D(-2, -1)$

$\overline{AD} \parallel \overline{BC}$, $\overline{AB} \nparallel \overline{CD}$, $ABCD$ is a trapezoid but not an isosceles trapezoid because $AB = \sqrt{17}$, $CD = \sqrt{10}$.

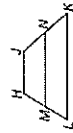
4. $J(1, 3)$, $K(3, 1)$, $L(3, -2)$, $M(-2, 3)$

$\overline{JK} \parallel \overline{LM}$, $\overline{JM} \nparallel \overline{KL}$, $JKLM$ is an isosceles trapezoid because $\overline{JM} = \overline{KL} = 3$.

For trapezoid $IJKL$, M and N are the midpoints of the legs.

5. If $HM = 32$ and $LK = 60$, find MN . 46

6. If $HM = 18$ and $MN = 28$, find LK . 38



Chapter 6

Glencoe Geometry

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6-6 Study Guide and Intervention *(continued)*

Trapezoids and Kites

Properties of Kites A kite is a quadrilateral with exactly two pairs of consecutive congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.

The diagonals of a kite are perpendicular.

For kite $RAMP$, $\overline{MP} \perp \overline{RN}$

In a kite, exactly one pair of opposite angles is congruent.

For kite $RAMP$, $\angle M \cong \angle P$

EXAMPLE If $WXYZ$ is a kite, find $m\angle Z$.

The measures of $\angle Y$ and $\angle W$ are not congruent, so $\angle X \cong \angle Z$.

$$m\angle X + m\angle Y + m\angle Z + m\angle W = 360$$

$$m\angle X + 60 + m\angle Z + 80 = 360$$

$$m\angle X + m\angle Z = 220$$

$$m\angle X = 110, m\angle Z = 110$$

EXAMPLE If $ABCD$ is a kite, find BC .

The diagonals of a kite are perpendicular. Use the Pythagorean Theorem to find the missing length.

$$BP^2 + PC^2 = BC^2$$

$$5^2 + 12^2 = BC^2$$

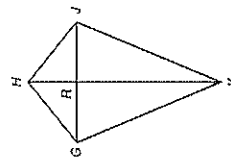
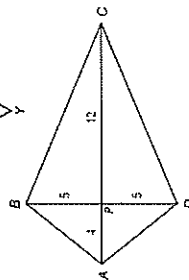
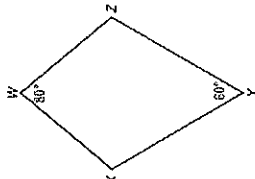
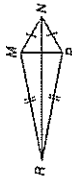
$$169 = BC^2$$

$$13 = BC$$

Exercises

If $GHIK$ is a kite, find each measure.

1. Find $m\angle HIK$. **90**
2. If $RI = 3$ and $RK = 10$, find JK . **$\sqrt{109}$**
3. If $m\angle GHI = 80$ and $m\angle GKI = 110$, find $m\angle HIK$. **80**
4. If $HI = 7$, find HG . **7**
5. If $HG = 7$ and $GR = 5$, find HR . **$\sqrt{24} = 2\sqrt{6}$**
6. If $m\angle GHI = 52$ and $m\angle GKI = 95$, find $m\angle HIK$. **106.5**



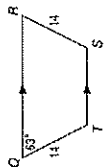
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6-6 Skills Practice

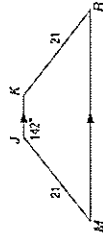
Trapezoids and Kites

ALGEBRA Find each measure.

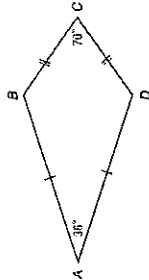
1. $m\angle S$ **117**



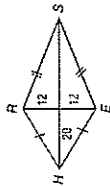
2. $m\angle M$ **38**



3. $m\angle D$ **127**



4. RH **$\sqrt{544} = 4\sqrt{34}$**



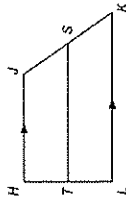
ALGEBRA For trapezoid $HJKL$, T and S are midpoints of the legs.

5. If $HJ = 14$ and $LK = 42$, find TS . **28**

6. If $LK = 19$ and $TS = 15$, find HJ . **11**

7. If $HJ = 7$ and $TS = 10$, find LK . **13**

8. If $KL = 17$ and $JH = 9$, find ST . **13**



COORDINATE GEOMETRY $EFGH$ is a quadrilateral with vertices $E(1, 3)$, $F(5, 0)$, $G(8, -5)$, $H(-4, 4)$.

9. Verify that $EFGH$ is a trapezoid.

$\overline{EF} \parallel \overline{GH}$, but $\overline{HE} \not\parallel \overline{FG}$

10. Determine whether $EFGH$ is an isosceles trapezoid. Explain.

not isosceles; $EH = \sqrt{26}$ and $FG = \sqrt{34}$

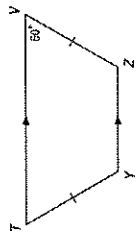
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6-6 Practice

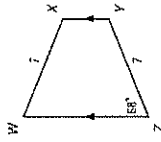
Trapezoids and Kites

Find each measure.

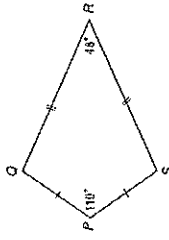
1. $m\angle T$ 60



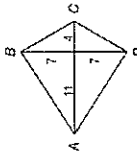
2. $m\angle Y$ 112



3. $m\angle Q$ 101



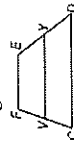
4. BC $\sqrt{65}$



ALGEBRA For trapezoid $FEDC$, V and Y are midpoints of the legs.

5. If $FE = 18$ and $VY = 23$, find CD . 38

6. If $m\angle F = 140$ and $m\angle E = 125$, find $m\angle D$. 55



COORDINATE GEOMETRY $RSTU$ is a quadrilateral with vertices $R(-3, -3)$, $S(5, 1)$, $T(10, -2)$, $U(-4, -9)$.

7. Verify that $RSTU$ is a trapezoid. $RS \parallel TU$

8. Determine whether $RSTU$ is an isosceles trapezoid. Explain.

not isosceles; $RU = \sqrt{37}$ and $ST = \sqrt{34}$

9. CONSTRUCTION A set of stairs leading to the entrance of a building is designed in the shape of an isosceles trapezoid with the longer base at the bottom of the stairs and the shorter base at the top. If the bottom of the stairs is 21 feet wide and the top is 14 feet wide, find the width of the stairs halfway to the top. 17.5 ft

10. DESK TOPS A carpenter needs to replace several trapezoid-shaped desktops in a classroom. The carpenter knows the lengths of both bases of the desktop. What other measurements, if any, does the carpenter need?

Sample answer: the measures of the base angles

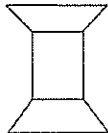
Answers (Lesson 6-6)

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6-6 Word Problem Practice

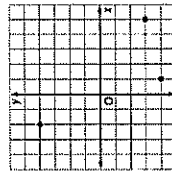
Trapezoids and Kites

1. PERSPECTIVE Artists use different techniques to make things appear to be 3-dimensional when drawing in two dimensions. Kevin drew the walls of a room. In real life, all of the walls are rectangles. In what shape did he draw the side walls to make them appear 3-dimensional?

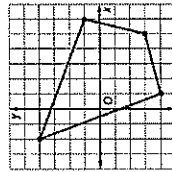


trapezoids

2. PLAZA In order to give the feeling of spaciousness, an architect decides to make a plaza in the shape of a kite. Three of the four corners of the plaza are shown on the coordinate plane. If the fourth corner is in the first quadrant, what are its coordinates?



(6, 1)

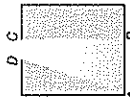


3. AIRPORTS A simplified drawing of the reef runway complex at Honolulu International Airport is shown below.



How many trapezoids are there in this image? 5

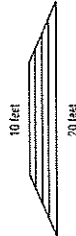
4. LIGHTING A light outside a room shines through the door and illuminates a trapezoidal region $ABCD$ on the floor.



Under what circumstances would trapezoid $ABCD$ be isosceles?

When the light source is an equal distance from C and D , shining straight through the door.

5. RISERS A riser is designed to elevate a speaker. The riser consists of 4 trapezoidal sections that can be stacked one on top of the other to produce trapezoids of varying heights.



All of the stages have the same height. If all four stages are used, the width of the top of the riser is 10 feet.

a. If only the bottom two stages are used, what is the width of the top of the resulting riser? 15 ft

b. What would be the width of the riser if the bottom three stages are used? 12.5 ft

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Answers (Lesson 6-6)

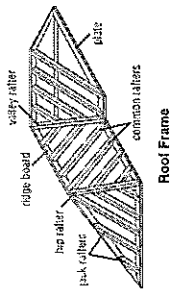
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6-6 Enrichment

Quadrilaterals in Construction

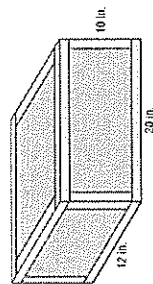
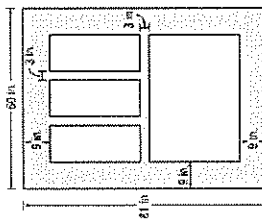
Quadrilaterals are often used in construction work.

1. The diagram at the right represents a roof frame and shows many quadrilaterals. Find the following shapes in the diagram, and shade in their edges. **See students' work.**



- isosceles triangle
- scalene triangle
- rectangle
- rhombus
- trapezoid (not isosceles)
- isosceles trapezoid

2. The figure at the right represents a window. The wooden part between the panes of glass is 3 inches wide. The frame around the outer edge is 9 inches wide. The outside measurements of the frame are 60 inches by 81 inches. The height of the top and bottom panes is the same. The top three panes are the same size.
- How wide is the bottom pane of glass? **42 in.**
 - How wide is each top pane of glass? **12 in.**
 - How high is each pane of glass? **30 in.**



3. Each edge of this box has been reinforced with a piece of tape. The box is 10 inches high, 20 inches wide, and 12 inches deep. What is the length of the tape that has been used? **168 in.**

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Chapter 6 Assessment Answer Key

Quiz 1 (Lessons 6-1 and 6-2) Page 45

1. 12,240
2. 45
3. 20
4. (-1, 10)
5. A

Quiz 2 (Lesson 6-3) Page 45

1. No; none of the tests for \square s are fulfilled.
2. true
3. false
4. true
5. 28 cm

Quiz 3 (Lessons 6-4 and 6-5) Page 46

1. B
2. 65
3. 115
4. true
5. rectangle, rhombus, square

Quiz 4 (Lesson 6-6) Page 46

1. 118
2. $\sqrt{50} = 5\sqrt{2}$
3. 5
4. 15.5
5. Use the distance formula to show $CF = DE$.

Mid-Chapter Test Page 47

1. B
2. J
3. A
4. G
5. C
6. 50
7. 42
8. yes; opp. sides are \cong to each other
9. 30, 150
10. No; slope $\overline{XY} = -\frac{3}{5}$ and slope of $\overline{WZ} = -\frac{1}{3}$, so opposite sides are not parallel.

Chapter 6 Assessment Answer Key

Vocabulary Test

Page 48

1. isosceles trapezoid
2. parallelogram
3. trapezoid
4. square
5. rhombus
6. false, rectangle
7. false; rhombus
8. diagonals
9. median
Sample answer:
angles formed by the
base and one of the
legs of the trapezoid
10. the nonparallel
sides of a
trapezoid
11.

Form 1

Page 49

1. B
2. H
3. B
4. H
5. D
6. F
7. B
8. F
9. D
10. F
11. C

Page 50

12. H
 13. B
 14. J
 15. A
 16. G
 17. A
 18. J
 19. A
 20. G
- $x = 7,$
 B: $m\angle WYZ = 41$

Chapter 6 Assessment Answer Key

Form 2A

Page 51

1. B

2. G

3. D

4. F

5. C

6. F

7. A

8. H

9. B

10. F

11. D

Page 52

12. J

13. C

14. H

15. A

16. G

17. D

18. H

19. D

20. G

B: 7 or -4

Form 2B

Page 53

1. B

2. J

3. D

4. F

5. B

6. G

7. A

8. H

9. C

10. J

11. B

Page 54

12. G

13. D

14. G

15. D

16. H

17. C

18. F

19. A

20. H

B: 22

Chapter 6 Assessment Answer Key

Form 2C

Page 55

1. 1080

2. 19

3. 40

4. 18

5. 8

6. 122

7. (6, 4)

Yes; \overline{AB} and \overline{CD} are \parallel and \cong .

8.

No; the slopes are $\frac{9}{4}$, $\frac{1}{7}$, 1, and $\frac{2}{3}$.

Thus, $ABCD$ does not have \parallel sides.

9.

10. $-\frac{2}{3}$

11. 22

Yes; if the diagonals of a \square are \cong , then the \square is a rectangle.

12.

13. 67.5

Page 56

14. (4, 0)

15. 31

Yes; $ABCD$ has only one pair of opposite sides \parallel ,

16. \overline{BC} and \overline{AD} .

17. 6

18. 26

19. true

20. true

21. true

22. false

23. true

24. true

25. false

B: $x = 9, y = 2$

Chapter 6 Assessment Answer Key

Form 2D

Page 57

1. 360

2. 38

3. 90

4. 50

5. 3.6

6. 117

7. $(\frac{1}{2}, 3)$

8. Yes; Both pairs of opp. sides are \cong .

9. $ABCD$ has two pairs of \parallel sides, $\overline{AB} \parallel \overline{CD}$ and $\overline{BD} \parallel \overline{DA}$; it is a \square .

10. -4

11. 8

12. One rt. \angle means that the other \triangle will be rt. \triangle . If all 4 \triangle are rt. \triangle , the \square is a rectangle.

Page 58

13. 72

14. $(-3, 1)$

15. 16

16. Yes; $ABCD$ has only one pair of opp. sides \parallel , AB and BD .

17. 8

18. 17

19. false

20. true

21. false

22. false

23. true

24. false

25. true

B: 90

Chapter 6 Assessment Answer Key

Form 3

Page 59

1. 7

2. 30; 30, 47, 120,
179, 174, and 170

3. $\frac{180}{x}$

4. 65

5. 8 or 32

6. Yes; the diagonals
bisect each other.

7. slope of $\overline{CD} = \frac{2}{3}$;
slope of $\overline{DA} = -2$.

8. 28

9. 72

10. $8\sqrt{2}$

11. 4

12. 9

13. 13

Page 60

No; two pairs of
congruent consecutive
sides do not exist.

14. $CD = \sqrt{72}, DA = \sqrt{65},$
 $AB = \sqrt{20}, BC = \sqrt{17}$

15. Yes; $\overline{AB} \perp \overline{BC},$
 $\overline{BC} \perp \overline{CD}, \overline{CD} \perp \overline{AD},$

so opp \triangle are \cong , and
all \triangle are rt. \triangle .

16. $ABCD$ has 2 pairs of
opp. sides \cong , \overline{AB}
 $\cong \overline{CD}$ and $\overline{BC} \cong \overline{DA},$
so $ABCD$ is a \square .

17. 105

18. Opp. sides of a
 \square are \cong .

19. If both pairs of
opp. sides of a
quad. are \cong ,
then the quad.
is a \square .

20. 27

B: 54

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Chapter 6 Assessment Answer Key

Extended-Response Test, Page 61 Scoring Rubric

Score	General Description	Specific Criteria
4	Superior A correct solution that is supported by well-developed, accurate explanations	<ul style="list-style-type: none"> Shows thorough understanding of the concepts of <i>angles of polygons, properties of parallelograms, rectangles, rhombi, squares, and trapezoids</i>. Uses appropriate strategies to solve problems. Computations are correct. Written explanations are exemplary. Graphs and figures are accurate and appropriate. Goes beyond requirements of some or all problems.
3	Satisfactory A generally correct solution, but may contain minor flaws in reasoning or computation	<ul style="list-style-type: none"> Shows an understanding of the concepts of <i>angles of polygons, properties of parallelograms, rectangles, rhombi, squares, and trapezoids</i>. Uses appropriate strategies to solve problems. Computations are mostly correct. Written explanations are effective. Graphs and figures are mostly accurate and appropriate. Satisfies all requirements of problems.
2	Nearly Satisfactory A partially correct interpretation and/or solution to the problem	<ul style="list-style-type: none"> Shows an understanding of most of the concepts of <i>angles of polygons, properties of parallelograms, rectangles, rhombi, squares, and trapezoids</i>. May not use appropriate strategies to solve problems. Computations are mostly correct. Written explanations are satisfactory. Graphs and figures are mostly accurate. Satisfies the requirements of most of the problems.
1	Nearly Unsatisfactory A correct solution with no supporting evidence or explanation	<ul style="list-style-type: none"> Final computation is correct. No written explanations or work shown to substantiate the final computation. Graphs and figures may be accurate but lack detail or explanation. Satisfies minimal requirements of some of the problems.
0	Unsatisfactory An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	<ul style="list-style-type: none"> Shows little or no understanding of most of the concepts of <i>angles of polygons, properties of parallelograms, rectangles, rhombi, squares, and trapezoids</i>. Does not use appropriate strategies to solve problems. Computations are incorrect. Written explanations are unsatisfactory. Graphs and figures are inaccurate or inappropriate. Does not satisfy requirements of problems. No answer may be given.

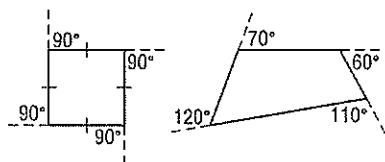
Chapter 6 Assessment Answer Key

Extended-Response Test, Page 61

Sample Answers

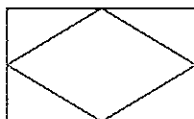
In addition to the scoring rubric found on page A30, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. a. Any type of convex polygon can be drawn as long as one is regular and one is not regular and both have the same number of sides.



- b. Check to be sure that the exterior angles have been properly drawn and accurately measured.
- c. $4(90) = 360$; $120 + 70 + 60 + 110 = 360$; The sum of the exterior angles of each figure should be 360° . The sum of the exterior angles of both the regular convex polygon and the irregular convex polygon is 360° .

2. The student should draw a rectangle and join the midpoints of consecutive sides. The figure formed inside is a rhombus. Since all four small triangles can be proved to be congruent by SAS, the four sides of the interior quadrilateral are congruent by CPCTC, making it a rhombus.



3. The student should draw an isosceles trapezoid with one pair of opposite sides parallel and the other pair of opposite sides congruent, as in the figure below.



4. a. Possible properties:
A square has four congruent sides and a rectangle may not.
A square has perpendicular diagonals and a rectangle may not.
The diagonals of a square bisect the angles and those in a rectangle may not.
- b. Possible properties:
A square has four right angles and a rhombus may not.
The diagonals of a square are congruent and those of a rhombus may not be.
- c. Possible properties:
A rectangle has four right angles and a parallelogram may not.
The diagonals of a rectangle are congruent and those of a parallelogram may not be.

Chapter 6 Assessment Answer Key

Standardized Test Practice (continued)

Page 64

15. hexagon;
concave;
irregular

16. 88.9 cm

17. \overline{DL}

18. $x = 11, y = 2$

19. Assume that
neither bell cost
more than \$45.

20. 40 cm^3

21. a. \overline{ML}

b. \overline{JL}

c. J

Chapter 6 Assessment Answer Key

Unit 2 Test

Page 65

1. right; obtuse;
acute
2. $m\angle 1 = 25;$
 $m\angle 2 = 25;$
 $m\angle 3 = 130$
3. $\overline{AF} \cong \overline{ST},$
 $\overline{FP} \cong \overline{TX},$
 $\overline{PA} \cong \overline{XS}$
4. yes
5. ASA Postulate
6. $\angle A \cong \angle C$
7. 111
8. $a = 9;$
 $m\angle ZWT = 32$
9. $\angle YWZ$
10. Neither
appliance cost
more than \$603.

Page 66

11. yes
12. $\frac{7}{2} < x < \frac{31}{2}$
13. 9
14. $m\angle JHK = 52;$
 $m\angle HMK = 108,$
and $x = 8$
15. No; opp. side are
not \parallel .
16. 5
17. 5
18. 11
19. In a parallelogram,
opposite sides are
congruent. Using
the distance
formula, $PQ = \sqrt{20},$
 $PS = \sqrt{20}, QR = \sqrt{8},$
 $RS = \sqrt{8}.$

Chapter 6 Assessment Answer Key

Standardized Test Practice (continued)

Page 64

15. hexagon;
concave;
irregular

16. 88.9 cm

17. \overline{DL}

18. $x = 11, y = 2$

19. Assume that
neither bell cost
more than \$45.

20. 40 cm^3

21. a. \overline{ML}

b. \overline{JL}

c. J