MEA 2013-2014
Teacher: Claudia Valle
Start Date:

Course: Math Models B
Student: _____
Completed Date:

Unit 3: Financial Planning

Objectives: Students will use algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning.

Essential Questions: When should you start saving for your retirement?

TEKS Standards: M.1.A, M1.B, M.1.C, M.7.C

Mathematical Models with Applications

(1) The student uses a variety of strategies and approaches to solve both routine and non-routine problems. The student is expected to:

(A) compare and analyze various methods for solving a real-life problem;

(B) use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines; and

(C) select a method to solve a problem, defend the method, and justify the reasonableness of the results.

(7) The student uses algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning. The student is expected to:

(C) investigate and compare investment options including stocks, bonds, annuities, and retirement plans.

Turn In:

Assignment #	Activity	TEKS
18	Annuities, Retirement Plans, and Life Insurance	M.1.A, M1.B, M.1.C, M.7.C
19	Stocks, Bonds, and Mutual Funds	M.1.A, M1.B, M.1.C, M.7.C
20	Evaluating Financial Planning	M.1.A, M1.B, M.1.C, M.7.C
21	Unit 3 Test	M.1.A, M1.B, M.1.C, M.7.C

Notes

Annuities, Retirement Plans, and Life Insurance KEY

Good financial planning involves looking at your goals for the future and determining investments that will help you meet those goals. Two of the more secure methods are savings investments and annuities. An annuity is an account in which a fixed amount of money is paid in over a specified amount of time at a set interest rate.

Sample 1

A deposit of $500 is made into an annuity account at the beginning of every six-month period for three years. The account earns 6% compounded semi-annually.

a. How does this differ from the savings accounts analyzed previously?
In the savings account, one deposit was added in at the beginning which accrued interest. In the annuity account, money is added in periodically and the account also accrues interest.

b. Fill in the table below to model the growth of the annuity account over the three years.

Number of Compounding Periods	Deposited Amount	Beginning Balance	Amount Earning Interest	Interest Earned in Compounding Period	Ending Balance
1 (6 months)	$500	$500.00	$500.00	$15.00	$515.00
2 (12 months)	$500	$1015.00	$1015.00	$30.45	$1045.45
3 (18 months)	$500	$1545.45	$1545.45	$46.36	$1591.81
4 (24 months)	$500	$2091.81	$2091.81	$62.75	$2154.56
5 (30 months)	$500	$2654.56	$2654.56	$79.64	$2734.20
6 (36 months)	$500	$3234.20	$3234.20	$97.03	$3331.23

Sample 2

A deposit of $500 is made into an annuity account at the end of every six-month period for three years. The account earns 6% compounded semi-annually.

a. Fill in the table below to model the growth of the annuity account over the three years.

Number of Compounding Periods	Deposited Amount	Beginning Balance	Amount Earning Interest	Interest Earned in Compounding Period	Ending Balance
1 (6 months)	$500	$0	$0	$0	$500
2 (12 months)	$500	$500	$500	$15.00	$1015.00
3 (18 months)	$500	$1015.00	$1015.00	$30.45	$1545.45
4 (24 months)	$500	$1545.45	$1545.45	$46.36	$2091.81
5 (30 months)	$500	$2091.81	$2091.81	$62.75	$2654.56
6 (36 months)	$500	$2654.56	$2654.56	$79.64	$3234.20

b. How did changing from adding the deposit at the beginning to the end affect the ending balance? Answers will vary. Sample: The first annuity payment made at the beginning allows for one more period of interest accrued ($97.03).

Notes

Annuities, Retirement Plans, and Life Insurance KEY

An ordinary annuity is one in which the payment or deposit is made at the end of each period.
An annuity due is one in which the payment or deposit is made at the beginning of each period.

Formulas can simplify calculations and be used to find the future value of either type of annuity.

Future Value of an Ordinary Annuity	Future Value of an Annuity Due
$FV = P\left(\frac{(1+r)^n - 1}{r}\right)$	$FV = P(1+r)\left(\frac{(1+r)^n - 1}{r}\right)$

P = annuity payment or deposit
r = interest rate per period in decimal form
n = number of compounding periods

Sample 3
Annuity payments of $200 are made monthly for ten years at 8% compounded monthly. What are the predicted values at the end of ten years if using an ordinary annuity? What are the predicted values at the end of ten years if using an annuity due? How do they compare?

Future Value of an Ordinary Annuity	Future Value of an Annuity Due
$FV = 200\left(\frac{\left(1+\frac{.08}{12}\right)^{10\cdot 12} - 1}{\frac{.08}{12}}\right)$	$FV = 200\left(1+\frac{.08}{12}\right)\left(\frac{\left(1+\frac{.08}{12}\right)^{10\cdot 12} - 1}{\frac{.08}{12}}\right)$
FV = $36,589.21	**FV = $36,833.14**

The annuity due method would result in a higher yield, because it would have one additional interest payment.

Sample 4
How could the TVM Solver be used to calculate future values of annuities? Check your answers to Sample 3 and see if your conclusions are accurate.
N = 10*12 = 120
I% = 8
PV = 0 (Since the annuity begins with no money until the first deposit is made)
PMT = -200
FV = (Amount to be solved for)
P/Y = 12
C/Y = 12
PMT:END (use when calculating ordinary annuity), PMT:BEGIN (use when calculating annuity due)

swers do check.

Notes

Annuities, Retirement Plans, and Life Insurance KEY

Up to this point, the annuities investigated have started with a zero balance and increased in value. In some cases, annuities will begin with a present value and will decrease due to fixed payments that are made at specified time intervals while the account still collects a set interest.

Sample 5

Pat will be retiring from his job at the end of the year. He estimates that in addition to his social security and 401K at work, he will need $15,000 per year. He received an inheritance from his grandfather and would like to determine how much money he would have to put in an ordinary annuity at 5% compounded annually to receive payments of $15,000 per year for the next 20 years. Calculate the present value that he would have to deposit. How does it compare to how much money he will receive over the 20 years?

N = 20
I% = 5
PV = (Amount to be solved for) = **$186,933.16**
PMT = 15000
FV = 0 (At the end of 20 years, no money will be left in the account.)
P/Y = 1
C/Y = 1
PMT:END (use when calculating ordinary annuity)

He will have to put $186,933.16 into the annuity at the present. Over the next twenty years he will receive $300,000. That is approximately 160% of the original investment. If Pat has the money to invest, it would be well worth it.

In order to provide for the family in the event of death (terminal or critical illness), many people purchase life insurance. **Life insurance** is a contract between the insurer and the *policy owner* whereby a benefit is paid to the designated beneficiary of the insured. Different types of life insurance are available including term, whole life, universal life, and variable life.

Sample 6

A 35-year old woman wants to purchase a 20-year life insurance policy with a face value of $150,000. The annual rate is $6.25 per $1,000 of face value. What would be the annual premium?
(150000/1000)6.25 = $937.50 annual premium

If premiums are paid in intervals less than one year, a percentage of the annual premium is paid for each specified interval.

Semiannually – 54%
Quarterly – 28%
Monthly – 10%

What would be the premium for each semiannual payment? Quarterly payment? Monthly payment? How do these compare to the cost of one annual payment?

Semiannual premium – (937.50)(.54) = $506.25 Over annual by 2(506.25) – 937.50 = $75
Quarterly premium – (937.50)(.28) = $262.50 Over annual by 4(262.50) – 937.50 = $112.50
Monthly premium – (937.50)(.10) = $93.75 Over annual by 12(93.75) – 937.50 = $187.50

Annuities, Retirement Plans, and Life Insurance

Practice Problems

1. Paul's company will match him dollar-for-dollar up to 6% of his monthly salary to invest into an annuity. Paul makes $4,000 per month and invests the full 6% of his salary plus the company's matching fund. In 8 years, what will be the future value of an annuity due, if the annuity is at 4.5% annual interest compounded monthly?

   N =
   I% =
   PV =
   PMT =
   FV =
   P/Y =
   C/Y =

   Circle
   PMT: END BEGIN

2. When Jackson was born, his parents established a college fund annuity in which they deposit $150 at the end of each month. The annual interest is 5.5% compounded monthly. How much will be in his college account when he turns eighteen?

   N =
   I% =
   PV =
   PMT =
   FV =
   P/Y =
   C/Y =

   Circle
   PMT: END BEGIN

3. Marcia invests $2,500 into an IRA at the beginning of each year as part of her retirement plan. If she plans to retire in 20 years, how much will be in the IRA, if it earns 7.5% compounded annually?

   N =
   I% =
   PV =
   PMT =
   FV =
   P/Y =
   C/Y =

   Circle
   PMT: END BEGIN

4. The state must keep sufficient funds to cover all lottery winners. If a winner is to receive $50,000 per year for 20 years from the beginning day they win, how much money must the state have on deposit at 6% compounded annually in order to make the payments?

   N =
   I%
   PV =
   PMT =
   FV =
   P/Y =
   C/Y =

   Circle
   PMT: END BEGIN

5. Tomas purchased a $500,000, 20-year life insurance policy. The rate is $0.825 per $100 of face value. Tomas has decided to pay monthly at 8.75% of the annual rate. What will be his monthly payments? How much could he have saved by paying one annual premium?

Notes

Stocks, Bonds, and Mutual Funds KEY

Although savings and annuities are considered safe investments, some of the riskier investments can offer better returns over a longer period of time. These investments include stocks, bonds, and mutual funds. Certain factors can impact the value of these investments such as the success of companies, the economic and political climate, interest rates, taxes, and unemployment.

Stocks

Companies that go public can sell ownership in their company in the form of stock.

- A share of stock is a unit of ownership in a particular company.
- Dividends are amounts sometimes paid on a quarterly basis to shareholders from profits made by the company over the previous period.
- Most stocks are traded on the New York Stock Exchange (NYSE) or the National Association of Securities Dealer Automated Quotations (NASDAQ).
- Stock transactions can be done by an individual, especially now with online access; these transactions many times have a flat fee for each transaction. Stock transactions can also be carried out by a stockbroker who usually charges a commission fee.

Sample 1

52-WEEK		STOCK	DIV	YLD%	PE	VOL 100s	CLOSE	NET CHG
HIGH	LOW							
42.97	33.32	T	1.60	4.20	19.56	228,944	37.88	-0.04
[illegible].95	26.99	ETE	2.20	6.90	20.97	1,077	32.00	-0.02
[illegible]6.79	26.30	DIS	0.35	1.10	15.67	152,261	32.49	+0.17

Use the stock listings in the table above to answer the following questions:

a. How many shares of Disney (DIS) stock were traded on this day?
15,226,100 shares were traded.

b. What was the closing price of a share of Disney stock?
The closing price on this day was $32.49 per share.

c. Is the closing price of Disney stock higher or lower than the previous day?
The closing price for this day is 17¢ higher per share.

d. What was the price of a share of Disney stock on the previous day?
32.49 – 0.17 = $32.32 was the price of a share on the previous day.

e. What was the percent decrease between the high and low of Disney stock over the past 52-week period?
(36.79 – 26.30)/36.79 x 100 = 28.51% decrease

f. If you owned 25 shares of Disney stock, how much will you receive in dividends for the quarter?
(.35)(25) = $8.75 in dividends for the quarter.

Notes

Stocks, Bonds, and Mutual Funds KEY

g. PE represents the price to earnings ratio. What would be the earnings per share for Disney stock?
15.67 = 32.49/earnings Earnings = $2.07

h. How can the PE values of the three stock options be used to recommend stock options?
The lower the PE value, the better the investment option.
Stock T: the PE value means an investment of $19.56 earns the company $1 or for every investment of $1 the company earns $0.051.
Stock ETE: the PE value means an investment of $20.97 earns the company $1 or for every investment of $1 the company earns $0.048.
Stock DIS: the PE value means and investment of $15.67 earns the company $1 or for every investment of $1, the company earns $0.064.

i. Suppose you purchased 50 shares of Energy Transfer Equity (ETE) at its lowest price the last 52-Week period and sold it on this day. You paid $9.95 for an online fee for each transaction. Determine if you made or lost money. Show calculations to back up your conclusion.
Purchase price = 50(26.99) = $1,349.50
Total fees = 2(9.95) = $19.90
Sale price = 50(32.00) = $1600.00
1600 – 1349.50 – 19.90 = $230.60 profit was realized

Mutual Funds
For some, an easier way to invest in the stock market is to invest in a group of stocks managed by a professional who develops a portfolio of various stocks and bonds called a mutual fund. A fee is paid to the manager of the portfolio. Invested funds from several investors are pooled and the fund manager buys and sells stock to try and achieve the best return for the investors.

Bonds
Three types of bonds are available to the public: treasury bonds (issued by federal government), municipal bonds (issued by state and local governments), and corporate bonds (issued by corporations). All bonds are a type of loan to governments or corporations that they agree to pay back to investors with interest. Bonds earn simple interest which is paid in addition to the face value of the bond to investors. Bonds are considered to be safer investments than the stock market, but they usually have a lower return.

Sample 2
Why would a person want to include several investment options in their financial planning for the future?
Answers will vary. The answers should address safe investments versus those that are riskier. They should also address higher and lower returns.

Stocks, Bonds, and Mutual Funds

Practice Problems

52-WEEK		STOCK	DIV	YLD%	PE	VOL 100s	CLOSE	NET CHG
HIGH	LOW							
42.97	33.32	T	1.60	4.20	19.56	228,944	37.88	-0.04

Use the table above to answer the following questions.

1. How many shares of AT&T (T on NYSE) stock were traded on this day?

2. What was the closing price of a share of AT&T stock?

3. Is the closing price of AT&T stock higher or lower than the previous day?

4. What was the price of a share of AT&T stock on the previous day?

5. What was the percent increase between the high and low of AT&T stock over the past 52-week period?

6. If you owned 50 shares of AT&T stock, how much will you receive in dividends for the quarter?

7. Explain the meaning of the PE ratio for this company.

8. Suppose you purchased 150 shares of AT&T at its highest price in the last 52-week period and sold it on this day. You paid $10.50 for an online fee for each transaction. Determine if you made or lost money. Show calculations to back up your conclusion.

Solve the following problem using a system of equations.

9. Mike received a bonus of $6,000. He wants to invest the money in stocks that have historically had an annual yield of 10% and in a mutual fund that had a 5% annual yield. Mike does not want to earn more than $500 or it will impact his taxes adversely. What amount should he invest in stocks and what amount should he invest in the mutual fund?