MEA 2013-2014
Teacher: Claudia Valle
Start Date:

Course: Math Models A
Student: ______________
Completed Date:

Unit 3: Modeling Situations in Music, Art, and Architecture

Objectives: Students explore mathematical models in music, art, and architecture.

Essential Questions: How does knowing how to apply mathematical models help to better understand real-world situations?

TEKS Standards: M.1.A, M1.B, M.1.C, M.9.A, M.9.B

Mathematical Models with Applications

(1) The student uses a variety of strategies and approaches to solve both routine and non-routine problems. The student is expected to:

(A) compare and analyze various methods for solving a real-life problem;

(B) use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines; and

(C) select a method to solve a problem, defend the method, and justify the reasonableness of the results.

(9) The student uses algebraic and geometric models to represent patterns and structures. The student is expected to:

(A) use geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and architecture; and

(B) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music.

Turn In:

Assignment #	Activity	TEKS
20	Sound Waves	M.1.A, M1.B, M.1.C, M.9.B
21	Music of Mathematical Patterns	M.1.A, M1.B, M.1.C, M.9.B
22	Rounding Out the Sound	M.1.A, M1.B, M.1.C, M.9.B
23	Tesselations and Transformations	M.1.A, M1.B, M.1.C, M.9.A
24	Dilation and Similarity	M.1.A, M1.B, M.1.C, M.9.A
25	Perspective in Art and Architecture	M.1.A, M1.B, M.1.C, M.9.A
26	Creative Castles	M.1.A, M1.B, M.1.C, M.9.A
27	Analyzing Models in Art and Architecture	M.1.A, M1.B, M.1.C, M.9.A
28	Unit 3 Test	M.1.A, M1.B, M.1.C, M.9.A, M.9.B

Notes

Sound Waves KEY

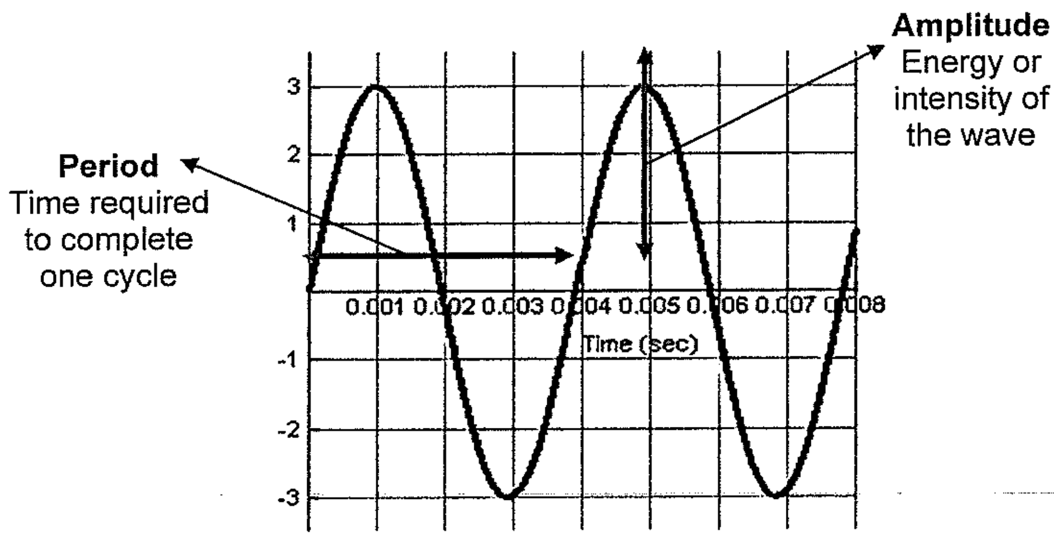
Sound is a longitudinal wave that is created by vibrating objects and travels through a medium. The vibrating objects create pressure on the air molecules which in turn interact with their neighbors. In some regions the air molecules are compressed together (compression) and in other regions the air molecules are spread apart (rarefactions).

1. What are some objects that vibrate to create sound? *Answers may vary.*
   vocal chords, guitar strings, piano wires, bells

2. Through what medium does sound generally travel?
   Sound usually travels through air, but it can travel through other mediums like water and glass.

3. How do you think sound travels through the medium?
   It travels like a longitudinal wave that compresses the medium in one place and stretches it in another. (Example: Slinky®)

The motion of the disturbance can be collected by a data collection device, and the characteristics of the disturbance can be analyzed. Below is a graph of the pressure fluctuation of a sound wave over time.



While the period (T) of a sound wave is the time in seconds required to complete one cycle, the frequency (f) of a sound wave is the number of cycles that can be completed in one second called Hertz (Hz). Frequency and period are reciprocals of one another.

4. What is the approximate period (T) of the wave in the above graph? *0.004 seconds per cycle*

5. What would be the approximate frequency (f) of the wave? *250 cycles per second (Hertz)*

$1/T = F$

$1/0.004 = 250$

Notes

Sound Waves KEY

Frequency is interpreted by the human ear as pitch. The higher the frequency of the wave, the higher the pitch. The lower the frequency of the wave, the lower the pitch. Amplitude is the level of energy, or intensity, of the wave. For sound waves, this intensity is measured in decibels.

6. How do you think wave intensity is interpreted by the human ear?
   Higher intensity would have higher energy and therefore be louder. Lower intensity would be softer.

7. Use the CBL and microphone probe to collect data from a tuning fork. Analyze the data to determine the following: *Answers may vary depending on data collected.*
   - Period
   - Frequency
   - Amplitude—remember, the amplitude is $\frac{1}{2}$(maximum value – minimum value).

Notes

Music of Mathematical Patterns KEY

Patterns in Sound

The first person to make the connection between math and music was Pythagoras of Samos, a famous philosopher who lived around the 5th century BC. He is mainly known for the oldest proof of what we call the "Pythagorean Theorem." Pythagoras believed ratios were everything and every value could be expressed as a fraction. Although that particular assumption was wrong, his work with ratios made the connections between musical instruments of his day and mathematics. These ratios still hold today.

Some sounds when played together are pleasing to the human ear. These sounds are said to be <u>consonant</u>. These intervals in music can be determined by looking at the ratios of the frequencies of the two sounds.

Intervals	Frequency Ratio	Examples
Octave	2:1	512 Hz and 256 Hz
Third	5:4	320 Hz and 256 Hz
Fourth	4:3	342 Hz and 256 Hz
Fifth	3:2	384 Hz and 256 Hz

1. The note A has a frequency of 440 Hz.
   a. What is the period for A? *0.00227273*
   b. Find a frequency that represents an octave below and above A. *220 Hz, 880 Hz*
   c. Find a frequency that represents a third interval below and above A. *352 Hz, 550 Hz*
   d. Find a frequency that represents a fourth interval below and above A. *330 Hz, 587 Hz*
   e. Find a frequency that represents a fifth interval below and above A. *293 Hz, 660 Hz*

2. Is the following statement true or false? Explain your reasoning.

> Doubling the frequency creates a note an octave higher. Conversely, dividing the frequency in half creates a note an octave lower.

It is true. The ratio for an octave is 2:1. The next highest octave would be multiplied by a scale factor of $\frac{2}{1}$ *which would double the frequency. The next lowest octave would be multiplied by a scale factor of* $\frac{1}{2}$ *which would half the frequency.*

Notes

Music of Mathematical Patterns KEY

3. The following table gives the notes in the C Major scale. It also compares the relationship of each to C, which has a frequency of 261.6 Hz. Complete the table and determine which of the following ratios could best be used to compare the given note to middle C.
{1:1 4:3 5:3 17:9 9:8 5:4 3:2}

Note	Frequency	Relation to Middle C (round to nearest ten thousandth)	Approximated Ratio
C	261.6	$\frac{261.6}{261.6} \approx 1.0000$	*1:1*
D	293.7	$\frac{293.7}{261.6} \approx 1.1227$	*9:8*
E	329.6	$\frac{329.6}{261.6} \approx 1.2599$	*5:4*
F	349.2	$\frac{349.2}{261.6} \approx 1.3349$	*4:3*
G	392.0	$\frac{392.0}{261.6} \approx 1.4985$	*3:2*
A	440.0	$\frac{440.0}{261.6} \approx 1.6820$	*5:3*
B	493.9	$\frac{493.9}{261.6} \approx 1.8880$	*17:9*

4. One of the notes in the table is dissonant (not pleasing to the ear) with middle C. Which note do you think this is? Explain your reasoning. *The dissonant note with middle C would be B. Answers for the reasoning may vary. Sample: The ratio values are farther apart.*

Patterns in Notes

Musical scales consist of notes that are ordered in frequency or pitch, with their ordering providing a measure of musical distance. The pitch chosen as the primary sets the mode for the particular scale. The distance between two successive notes in the scale is called the scale step. Composers often transform musical patterns by moving every note in the pattern by a constant number of scale steps. This process is called scalar transposition.

5. How is scalar transposition seen in the following song from *The Sound of Music*?

> "Do" a deer a female deer; "Re" a drop of golden sun.
> "Mi" a name I call myself; "Fa" a long, long way to run.

Answers may vary. Sample answer given.
The first phrase, "Do a deer," starts at a certain pitch. When you go to the next phrase, all of the notes move up a step. The melody stays the same, but a step higher. When you go to the next phrase, all the notes move up another step. The melody again stays the same only now it is two steps higher than the beginning phrase.

Notes

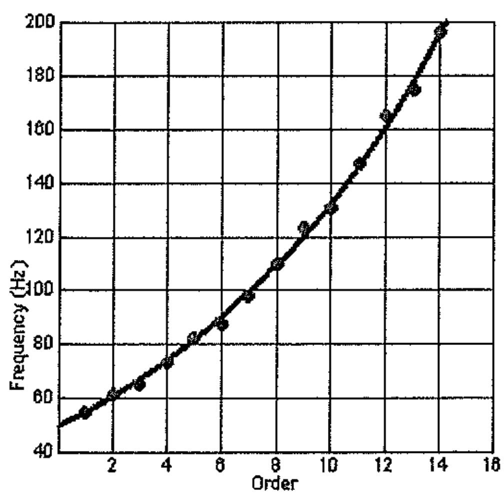
Music of Mathematical Patterns KEY

6. Any <u>primary note</u> has octaves that move up or down by a ratio of 2:1. In other words, if A4 has a frequency of 440 Hz and represents the fourth octave of the primary note A, then A3 would have half of that frequency or 220 Hz. A5, on the other hand, goes up, so it would have double that frequency or 880 Hz. Complete the table below to find the frequencies of five octaves of each of the given primary notes.

Primary Note	Octaves				
	1	2	3	4	5
A	55.0 Hz	110.0 Hz	220.0 Hz	440.0 Hz	880.0 Hz
B	61.7 Hz	123.5 Hz	247.0 Hz	493.9 Hz	987.8 Hz
C	65.4 Hz	130.8 Hz	261.6 Hz	523.2 Hz	1046.4 Hz
D	73.4 Hz	146.9 Hz	293.7 Hz	587.4 Hz	1174.8 Hz
E	82.4 Hz	164.8 Hz	329.6 Hz	659.2 Hz	1318.4 Hz
F	87.3 Hz	174.6 Hz	349.2 Hz	698.4 Hz	1396.8 Hz
G	98.0 Hz	196.0 Hz	392.0 Hz	784.0 Hz	1568.0 Hz

7. Is there a constant rate of change in the frequencies of notes in a scale? Create a graph of the first and second octaves of the notes in the table above. For the x-axis, let the first octave be represented by A →1, B→2, C→3, and so forth. For the second octave, let A→9, B→10, C→11, and so forth. Determine a regression model for the function that best represents the situation. Use these models to justify the answer to the question.

Regression model: $y = 49.77(1.103)^x$



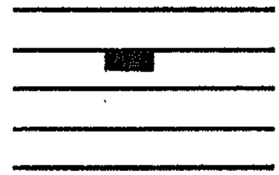
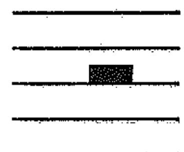
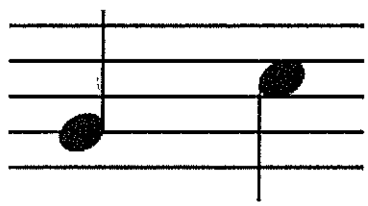
The table and the graph both show that there is not a constant rate of change. The graph indicates an exponential function. When the exponential regression was calculated, the correlation was 0.999182, meaning it would be a very strong function for making predictions.

Notes

Music of Mathematical Patterns KEY

atterns in Beat

Symbols used to represent musical notes also indicate the duration of time the note is played or sung. Music is composed so that there is a fixed number of beats per measure, and musicians must count notes and rests. All the notes and rests must add up to the set number of beats per measure.

Name	Note Symbol	Rest Symbol
Whole note		
Half note		
Quarter note		
Eighth note		
Sixteenth note		

A note value does not stand for an absolute duration, but is understood in relation to other notes. In the table above, the duration of each note is twice as long as the note below it.

8. If a quarter note has a duration of 1 beat, what would be the duration in beats for each of the other notes in the table? Whole – 4 beats, half – 2 beats, quarter – 1 beat, eighth – $\frac{1}{2}$ beat, and sixteenth – $\frac{1}{4}$ beat

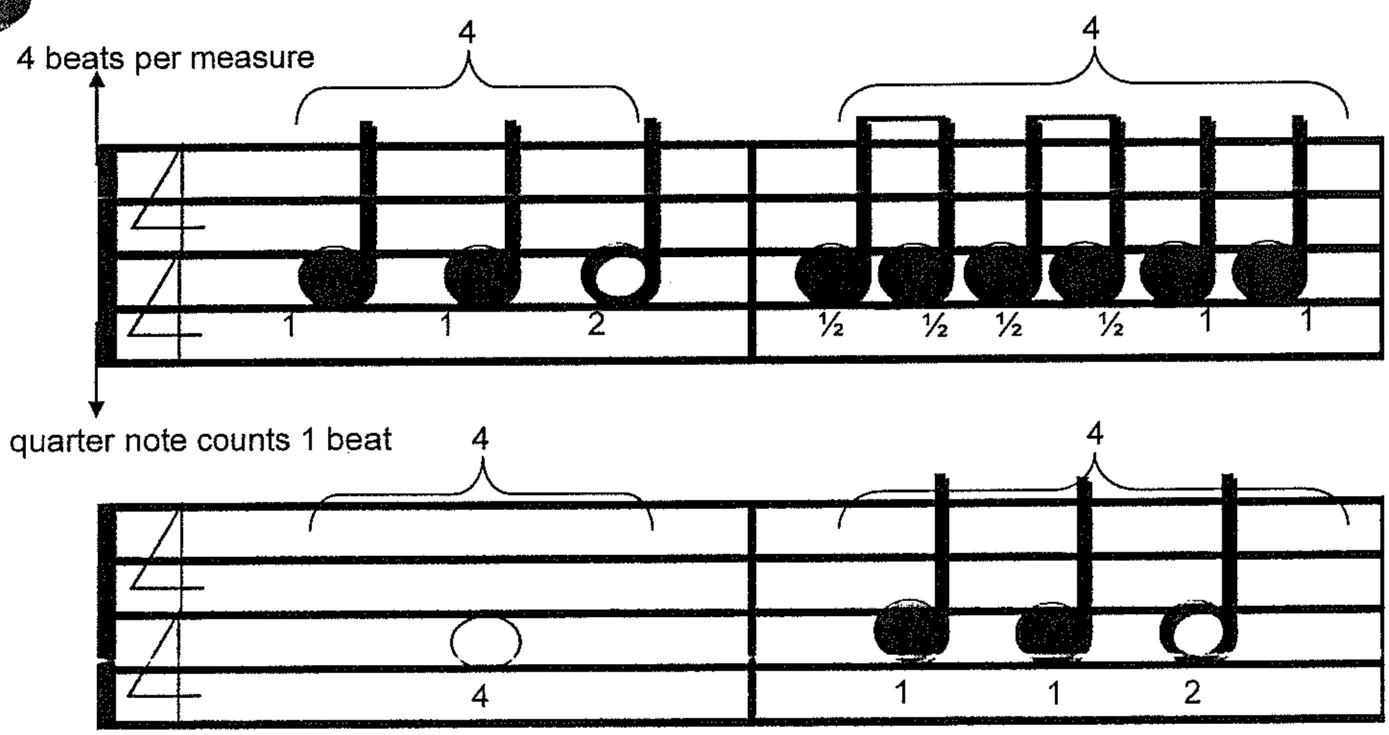
Notes

Music of Mathematical Patterns KEY

The actual time durations are set by the time signature of the music. The following are examples of time signatures or meter of music.

> The **top** number indicates the number of beats per measure.
>
> The **bottom** number indicates the rhythmic note value that receives the beat.

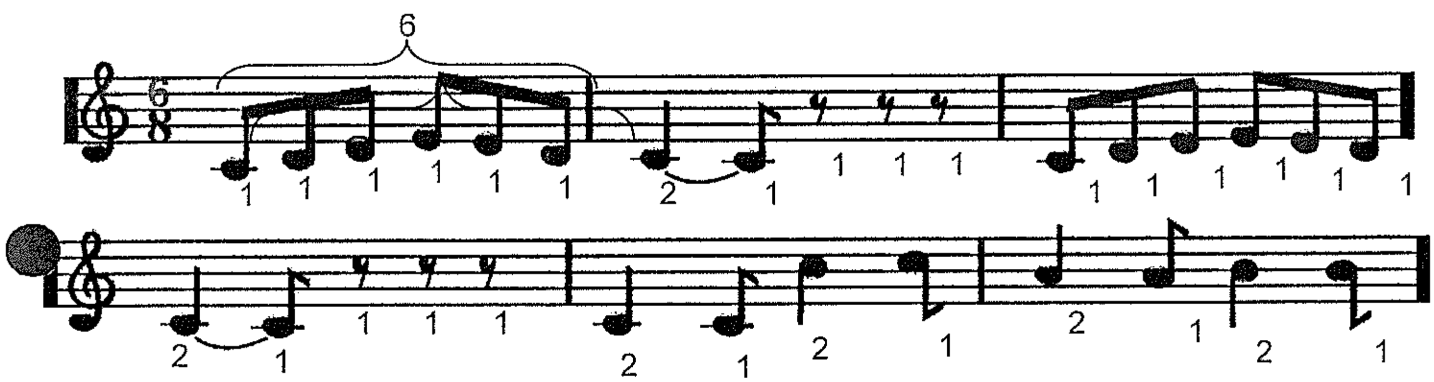
2 4	2 beats per measure Quarter note represents one full beat
3 4	3 beats per measure Quarter note represents one full beat
C or 4 4	4 beats per measure Quarter note represents one full beat This is the most common tempo, so it is sometimes represented with C.



Notes/Do #10

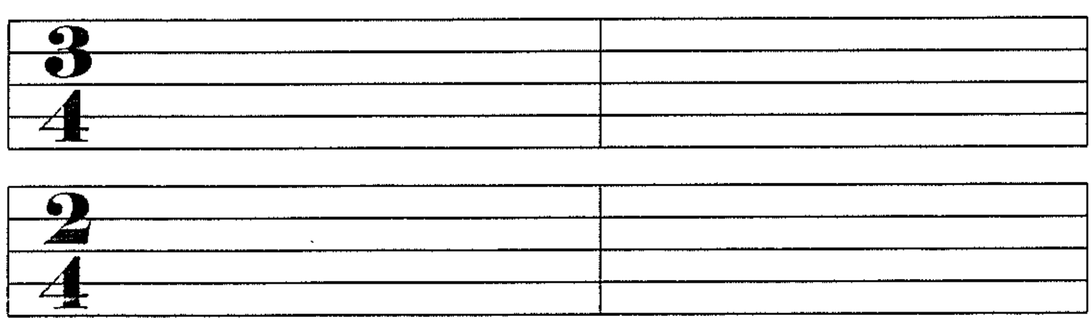
Music of Mathematical Patterns KEY

9. In each of the following pieces of music, write the count below each note. Use this to determine the top value of the time signature. Put this in the box at the beginning of the first measure.



10. In each of the following bars, draw a combination of notes and rests that correctly correspond to the given time signature. (It doesn't matter which pitches you use for notes.) Then write in the counts below each bar.

Do:



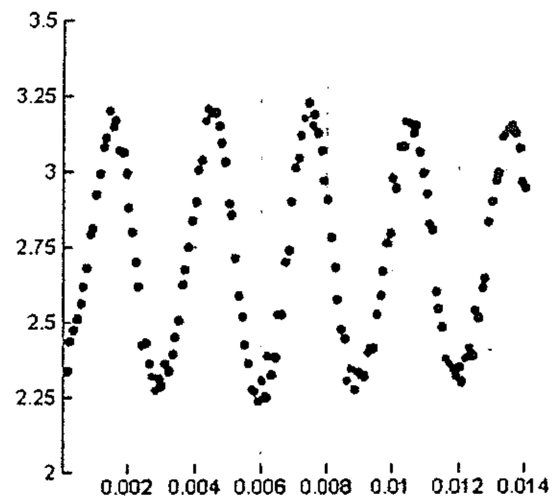
Rounding Out the Sound

1. The following data was collected using a CBL 2 and a graphing calculator using the Data Mate program. Use the scatterplot of the data to approximate the following.

   a. Period

   b. Frequency

   c. Amplitude



2. Alice is looking for frequencies that are consonant to 240 Hz. Complete the table below to determine these frequencies. Explain why Alice would want consonant frequencies.

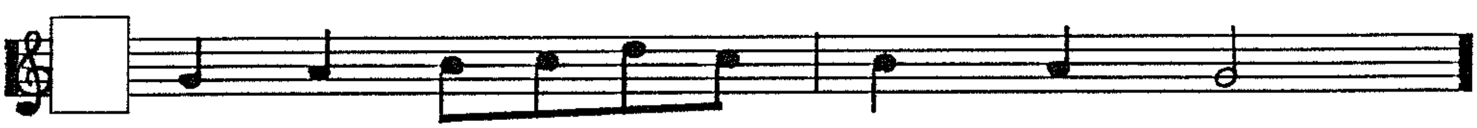
Interval	Above	Below
Octave (2:1)		
Third (5:4)		
Fourth (4:3)		
Fifth (3:2)		

Rounding Out the Sound PI

3. The frequencies below represent an excerpt from *Are You Sleeping, Brother John?* Use transposition to find the frequencies, when the music is moved up a third. Explain how this would affect the music when it is played.

	C	D	E	C
Original frequency	262 Hz	294 Hz	330 Hz	262 Hz
Up a third frequency				

4. In the following piece of music, write in the count for each note. Use this to determine the time signature. Put this in the box at the beginning of the first measure. Explain how the beat and count of the notes affects the time signature of a piece of music.



Rounding Out the Sound PI

5. The following table represents the periods and frequencies for equal-tempered scales or chromatic scale.

Note	Order from Middle C	Period	Frequency
C4 (Middle C)	0	0.0038222	
C#4/Db4	1	0.0036078	
D	2		293.66
D#4/Eb4	3		311.13
E4	4	0.0030337	
F4	5	0.0028634	
F#4/Gb4	6		369.99
G4	7		392.00
G#4/Ab4	8	0.0024079	
A4	9	0.0022727	
A#4/Bb4	10		466.16
B4	11		493.88
C5	12	0.0019111	

a. Complete the table.

b. Graph frequency in terms of increasing order from middle C. What are the independent and dependent variables? What conclusions can be drawn from the graph about frequency? Determine a regression equation for the data.

c. Graph frequency in terms of period. What are the independent and dependent variables? What conclusions can be made about the relationship between frequency and period? Determine a regression equation for the data.

Tessellations and Transformations

A tessellation, or tiling, of a plane is a collection of repeating patterns or shapes that fill the plane with no overlaps or gaps. Tessellations have been used throughout history in art and architecture and are still used today.

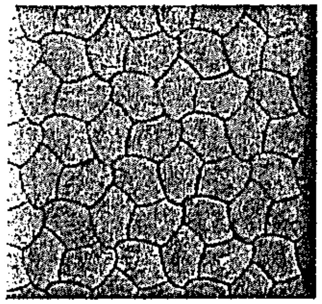


Image Credit: D M. Harvey. (2005). *Cobbled Street Pavement* [Web Photo]. Retrieved from http://en.wikipedia.org/wiki/File:Wallpaper_group-p3-1.jpg

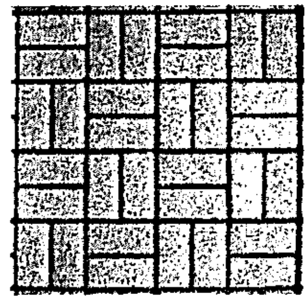


Image Credit: D M. Harvey. (2005). *Cobbled Street Pavement* [Web Photo]. Retrieved from http://en.wikipedia.org/wiki/File:Wallpaper_group-p4g-1.jpg

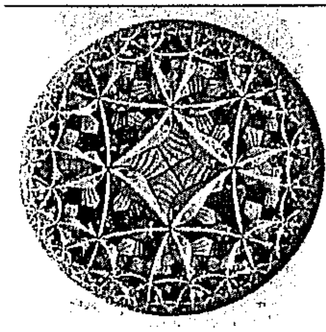


Image Credit: M. C. Escher. (1959). *M.C. Escher Circle Limit III* [Web Photo]. Retrieved from http://upload.wikimedia.org/wikipedia/en/thumb/5/55/Escher_Circle_Limit_III.jpg/250px-Escher_Circle_Limit_III.jpg

Tessellations and Regular Polygons Use Templates at end of packet.

The simplest form of tessellation uses a single regular polygon to tessellate a plane. Only three of the regular polygons will work: equilateral triangles, squares, and regular hexagons.

Triangles

- Use a protractor to measure the angles on one of the equilateral triangles on the Regular Polygon Templates page. What do you observe about all the angles of the equilateral triangle?

- Cut out the equilateral triangles and arrange them to tessellate a plane. Sketch below.

- Locate a point in the center of the tessellation that is surrounded by triangles. This is called the vertex. How many triangles surround the vertex?

- Determine the sum of the angles at the vertex. What is this sum?

Tessellations and Transformations

Squares

- Use a protractor to measure the angles on one of the squares on the **Regular Polygon Templates** page. What do you observe about all the angles of the square?
- Cut out the squares and arrange them to tessellate a plane. Sketch below.
- Locate a point in the center of the tessellation that is surrounded by squares. How many squares surround the vertex?
- Determine the sum of the angles at the vertex. What is this sum?

Regular Hexagons

- Use a protractor to measure the angles on one of the regular hexagons on the **Regular Polygon Templates** page. What do you observe about all the angles of the hexagon?
- Cut out the hexagons and arrange them to tessellate a plane. Sketch below.
- Locate a point in the center of the tessellation that is surrounded by hexagons. How many hexagons surround the vertex?
- Determine the sum of the angles at the vertex. What is this sum?

1. When regular polygons are used as the tessellating pieces, what can you conjecture about the sum of the angles at the vertex?

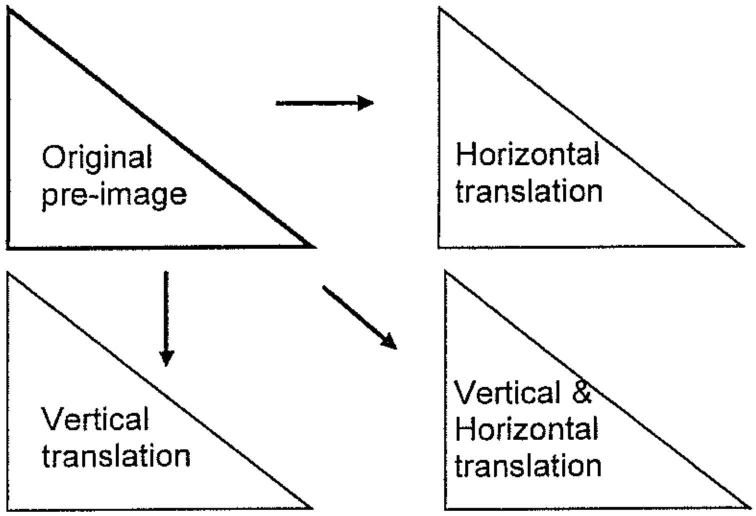
2. Measure the angles of the regular pentagon on the **Regular Polygon Templates** page. Cut out the pentagons and try to tessellate the pieces. Do they tessellate a plane? Explain why or why not.

Tessellations and Transformations

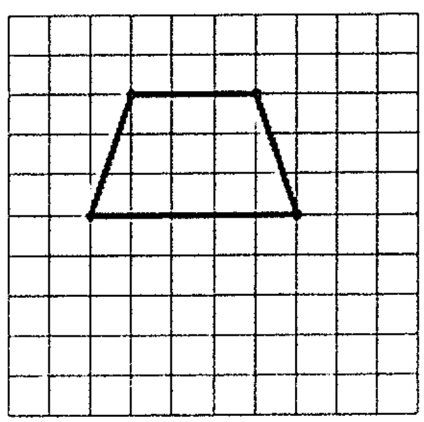
Transformations

A tessellation is an application of a geometric transformation. Some geometric transformations are isometric which means neither shape nor size is changed in the transformation. These transformations include translation, reflection, and rotation.

- Translation – In a translation the object is moved vertically, horizontally, or both.



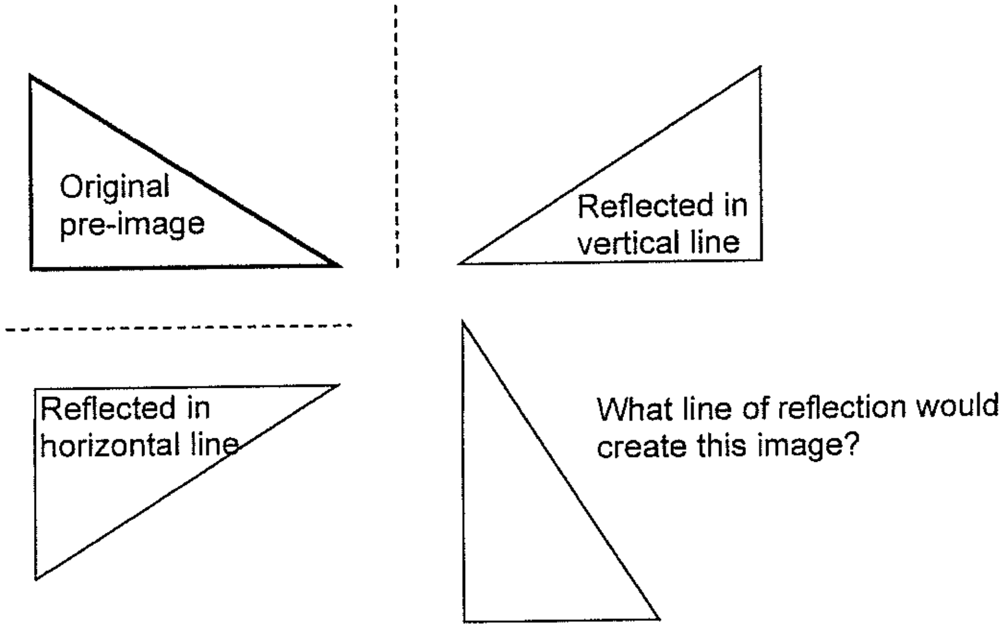
3. Translate the trapezoid four units down and three units right.



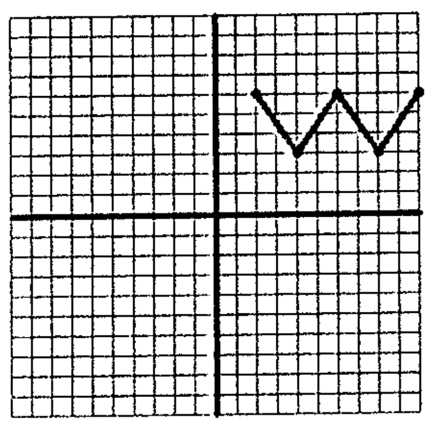
4. Which regular polygon tessellations show translation? Draw a sketch to justify your reasoning.

Tessellations and Transformations

- Reflection – In a reflection the object is reflected across a specified line. The reflected figure is called a mirror image.



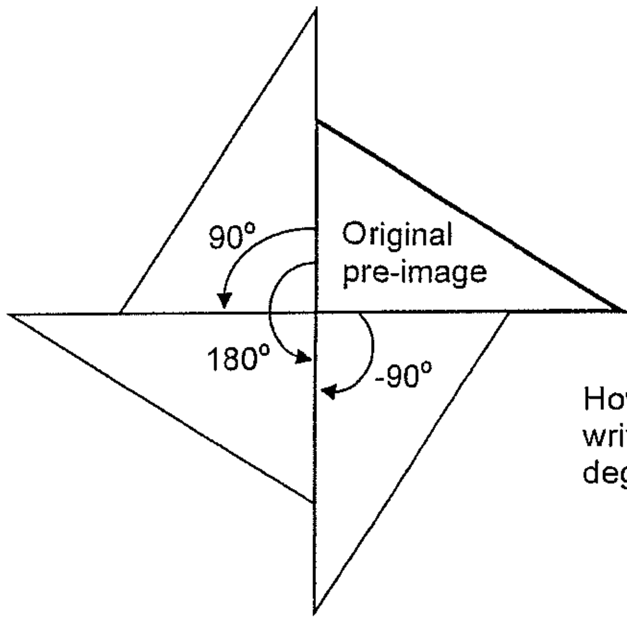
5. Reflect the figure in the **y-axis (blue)**, **x-axis (red)**, and **y = x (green)** line. Make each mirror image a different color.



6. Which regular polygon tessellations show reflection? Draw a sketch to justify your reasoning.

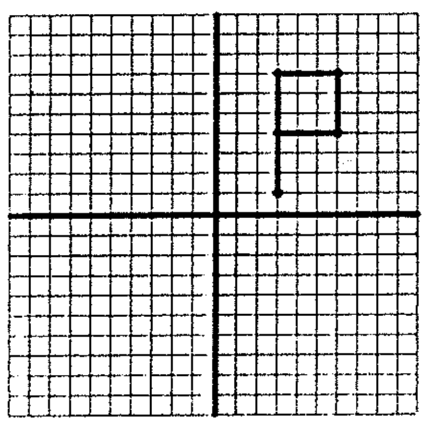
Tessellations and Transformations

- Rotation – In a rotation the object is turned about a specific point called the point of rotation by a specific amount called the angle of rotation. Counterclockwise rotations are measured in positive degrees and clockwise rotations are measured in negative degrees.



How could -90° be written in positive degrees?

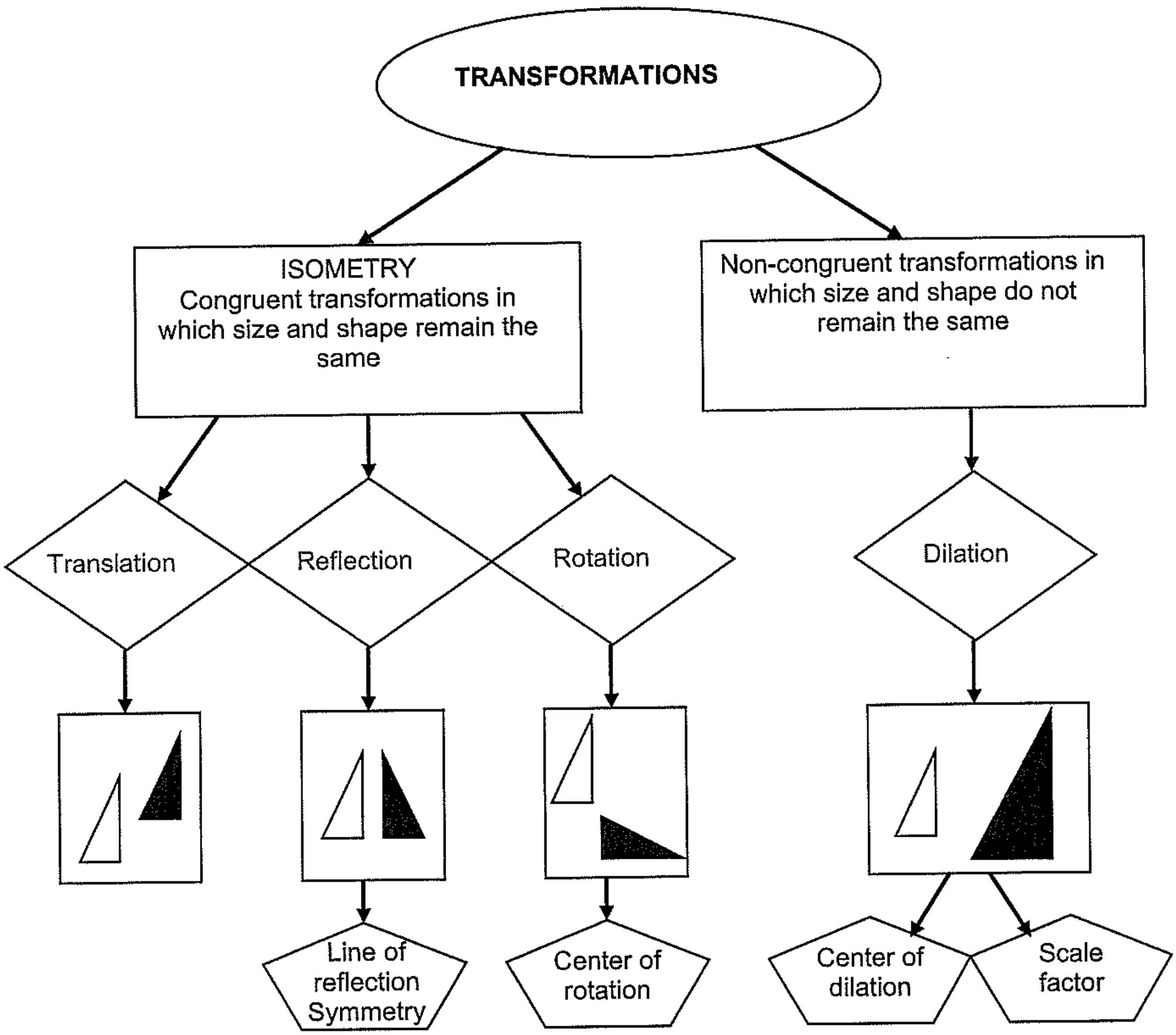
7. Using the origin as the center of rotation, rotate the figure **<u>90° (blue)</u>**, **<u>180° (red)</u>**, and **<u>270° (green)</u>**. Make each image a different color.



8. Which regular polygon tessellations show rotation? Draw a sketch to justify your reasoning.

Notes.

Transformations

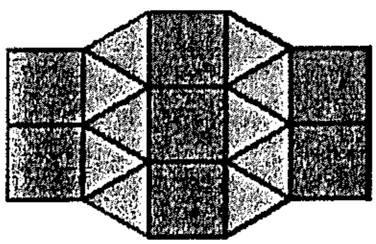
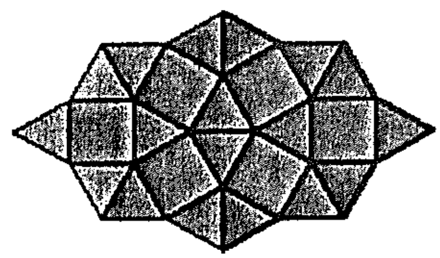


Tessellations and Transformations

Symmetry is also evident in art and architecture. Symmetry implies a balance. Two types of symmetry can be seen in many drawings and pictures. They are reflection symmetry and rotational symmetry. Reflection symmetry reflects across an axis of symmetry. Rotational symmetry rotates around a specific point a certain number of degrees.

9. Which transformations illustrate symmetry? Explain your reasoning with a sketch of each.

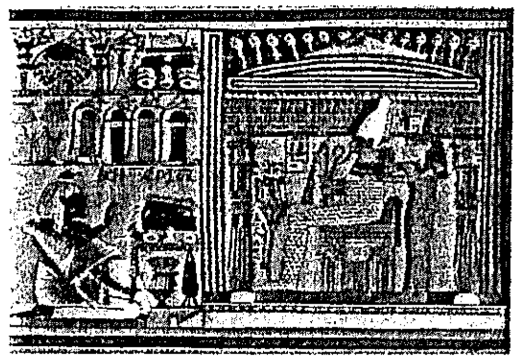
10. What transformations and symmetries can be seen in the tessellations below?

	
Transformations Observed in Pattern	**Transformations Observed in Pattern**
Symmetries Observed in Pattern	**Symmetries Observed in Pattern**

It's a Matter of Perspective

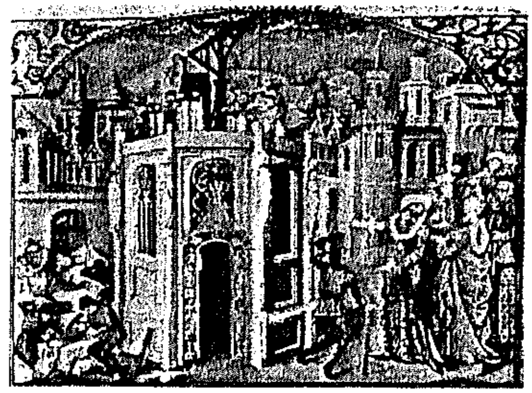
In ancient art, paintings and drawings were meant to be viewed as a group of symbols rather than coherent pictures. Study the ancient Egyptian painting. What observations can you make about the picture?

Image Credit: Wasserman, J. (Producer). (1994). *Ani before osiris.* [Web Photo]. Retrieved from http://commons.wikimedia.org/wiki/File:BD_Ani_before_Osiris.jpg



Even in medieval art the only method used to show distance was by overlapping characters. This made for poor architectural drawings and paintings of the medieval era that have lines of view going in all directions. Study the illustration from Guillaume de Tyr's *Histoire d'Outremer.* What observations can you make about the illustration?

Image Credit: Tyre, W. O. (Artist). (2005). *Reconstructon of the temple of jerusalem.* [Web Photo]. Retrieved from http://commons.wikimedia.org/wiki/File:Reconstruction



One of the first uses of mathematics to achieve perspective in pictures was by Giotto di Bondone in the 1300's using a linear algebraic method. The problem was that the true regression is a sine function, which was not discovered until the 20th century. Still his method did give the illusion of depth and was a large step forward. Study Giotto's painting. What observations can you make about the painting?

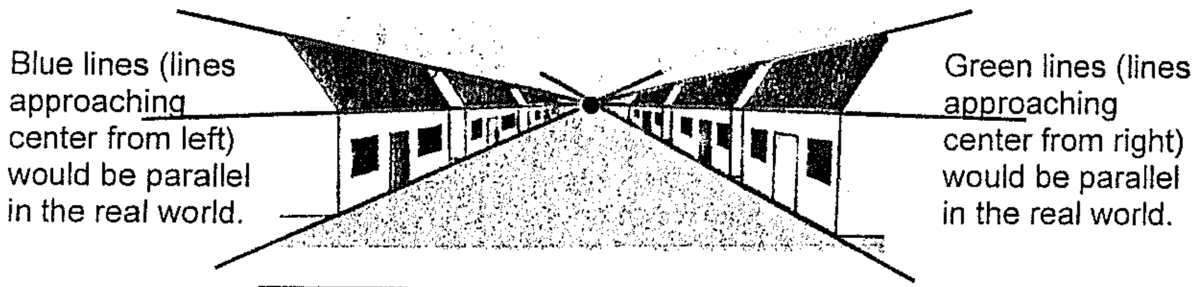
Image Credit: Stom, M. (Artist). (1630). *Christ before caiaphas.* [Web Photo]. Retrieved from http://commons.wikimedia.org/wiki/File:Mattias_Stom

In the early 1400's, Brunelleschi was the first to demonstrate the geometrical method of perspective used today. All seemingly parallel lines converge on the horizon line. Soon most artists began using geometric perspective in their paintings. Some even began using checkerboard floors to show perspective. Study Donatello's painting. What observations can you make about the painting?

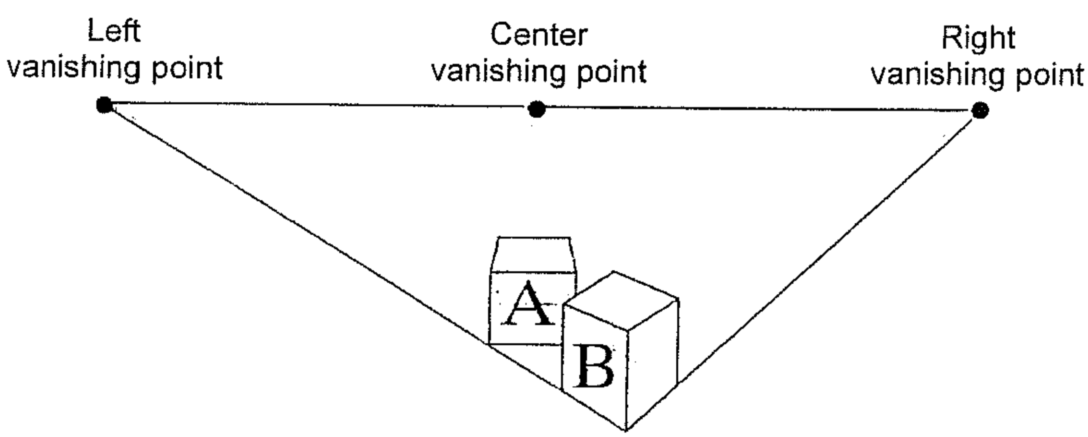
Image Credit: Botticelli, S. (Artist). (1489). *Annunciation.* [Web Photo]. Retrieved from http://commons.wikimedia.org/wiki/File:Sandro_Botticelli_080.jpg

Perspective in Art and Architecture

The concept of dilation is used to incorporate perspective into art and architecture. Although perspective has many meanings, in art and architecture it is a geometric way of allowing an artist or architect to incorporate a sense of depth and 3-dimensions into planar drawings or paintings. Lines that are parallel in the real world are not parallel in the perspective model. They instead converge at a vanishing point or center of dilation. The vanishing points are located on the horizon line or eye level. Objects farther away appear smaller than those closer to the viewer.



One point perspective occurs when all parallel lines moving away from the viewer, appear to converge in a single vanishing point on the horizon line.



Multiple point perspective occurs when parallel lines moving away from the viewer appear to converge at more than one vanishing point on the horizon line.

1. What can you conclude about determining whether a drawing illustrates perspective?

Perspective in Art and Architecture

2. Study the pictures below. Determine whether the drawings illustrate perspective. Explain your reasoning.

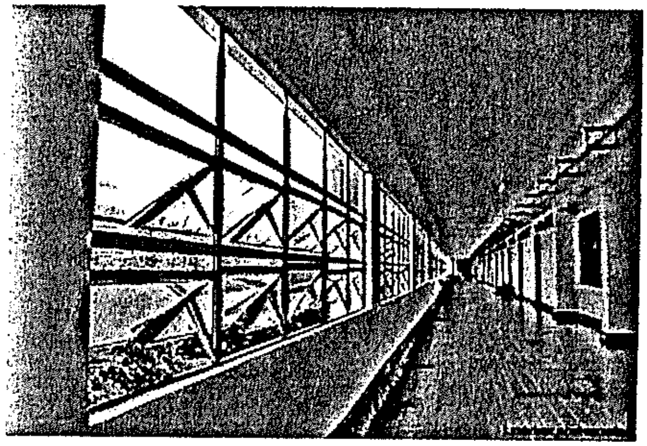


Image Credit: Ruivivar, J. (Photographer). (2004). *Perspective photo.* [Web Photo]. Retrieved from https://en.wikipedia.org/wiki/File:Perspectivephoto.jpg

Image Credit: Perugino, P. (Artist). (1481). *Christ handing the keys to st. peter.* [Print Photo]. Retrieved from http://commons.wikimedia.org/wiki/File:Christ_ Handing_the_Keys_to_St._Peter_by_Pietro_Perugino

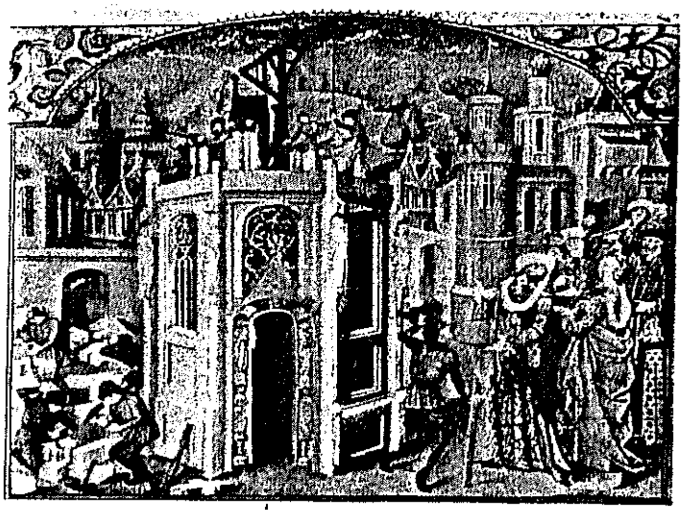


Image Credit: Tyre, W. O. (Artist). (2005). *Reconstructon of the temple of jerusalem.* [Web Photo]. Retrieved from http://commons.wikimedia.org/wiki/File:Reconstruction

3. Imagine you are standing on a long, straight highway in the flat desert. The highway is lined on both sides with telephone poles. Using perspective, sketch what you would visualize looking down the highway. Label the principal vanishing point.

Perspective in Art and Architecture

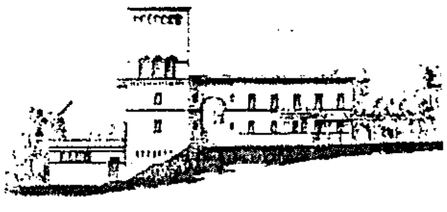
Perspective is impacted by the point of view of an object. How an object looks will depend on the side from which it is viewed. Objects closer to the viewer will at times overlap or conceal objects further from the viewer.

Perspective View Activity:

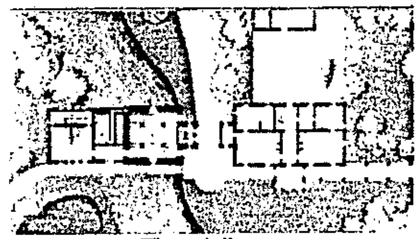
- Get in a group of four.
- Each group member is to build a structure out of 12 linking cubes or 12 one-inch cubes. Structures should be no more than 6 stories high, and must be able to stand.
- Label a sheet of grid paper with North on top, South on bottom, East on the right, and West on the left.
- Place the four buildings on the sheet of grid paper.
- Each member of the group is to label a sheet of grid paper of their own with North, South, East, and West, and draw in the location of each building as it would look if seen from above.
- Assign one member to be North, one to be South, one to be East, and one to be West.
- Each group member will get on the assigned side, get eye level to the buildings, and draw a sketch of what they see from their perspective. This can be done on the back of the grid paper.
- Discuss and compare the North, South, East, and West sketches.
    - How are the sketches alike?
    - How are the sketches different?
    - What do overlapping objects tell us about position and depth?
    - How is size affected by position and depth?
    - What conclusions can be drawn about using perspective to make drawings and paintings more realistic?

Perspective in Art and Architecture

When preparing blueprints architects will include elevations of the structure. Architectural drawings as far back as the middle ages showed different view of structures called elevations. These types of elevations are still seen in architectural plans today.



Front View
of Chateau



Top View
of Chateau

Image Credit: Persius, L. (Artist). (1842). *Gardener house in glienicke(berlin)*. [Web Photo]. Retrieved from http://commons.wikimedia.org/wiki/File:Gaertnerhaus_und_Maschinenhaus_Glienicke_AA.jpg

4. Using the structure you built for the Perspective View Activity, sketch a three dimensional view of the structure. Then sketch a top view, front view, left side view, right side view, and back view.

Sketch in next page.

What transformations are seen in the castle? Explain each.

What axes of symmetry are found in the castle? Explain each.

What solids did you use to build the castle and how many of each?

Back view

Left view

Right view

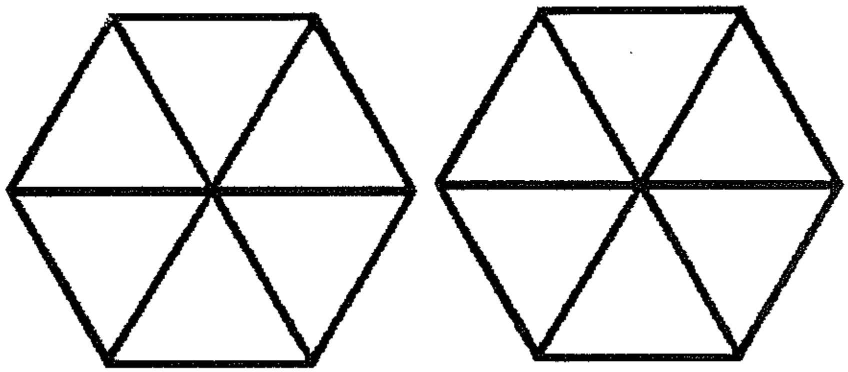
Top view

In the box provided, sketch a one point perspective view of your castle.

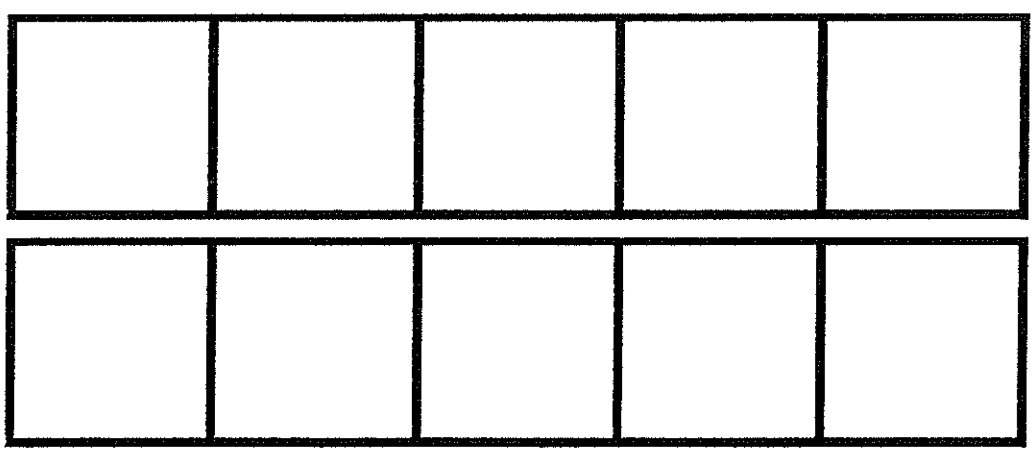
One point perspective view

Regular Polygon Templates

Equilateral Triangles

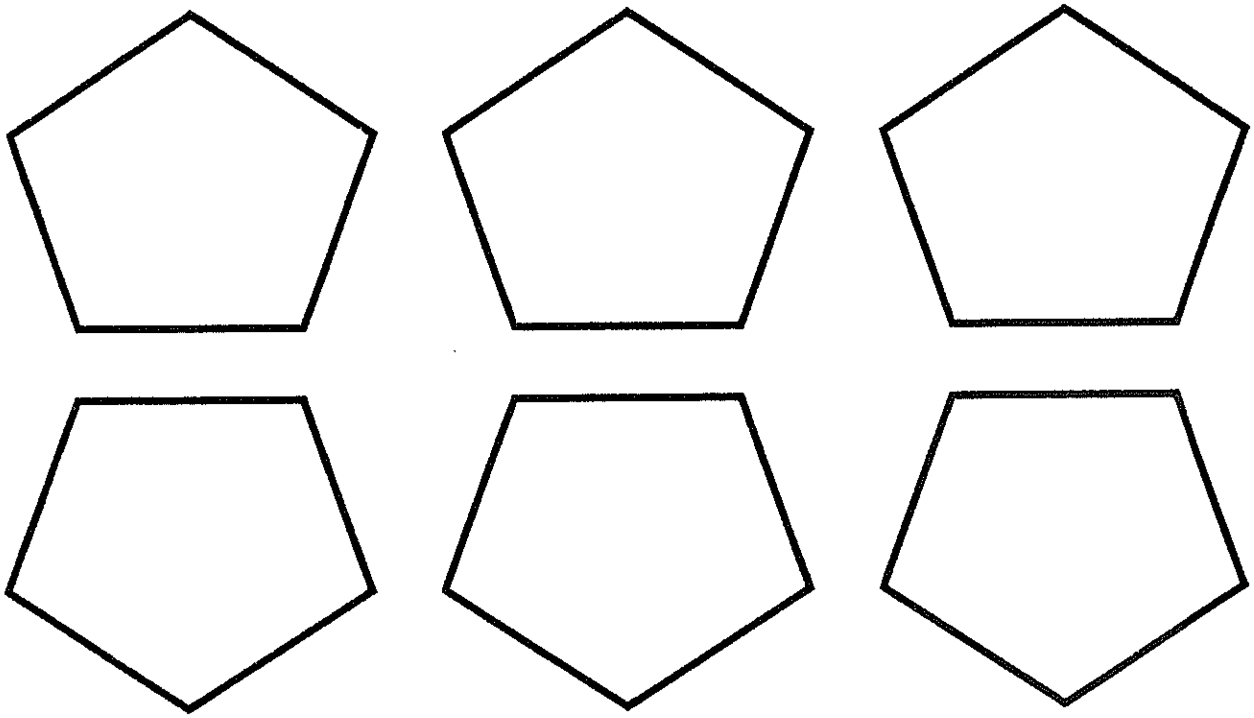


Squares



Regular Polygon Templates

Regular Pentagons



Regular Hexagons