MEA 2013-2014
Teacher: Claudia Valle
Start Date:

Course: Math Models A
Student: ______________
Completed Date:

Unit 2: Modeling Real-World Situations using Functions

Objectives: Students will explore the characteristics of exponential, inverse, and trigonometric functions and investigate how they are used to model real-world situations.

Essential Questions: How can you find a function to model real-world data and predict future situations?

TEKS Standards: M.1.A, M1.B, M.1.C, M.2.D, M.8.A, M.8.B, M.8.C

Mathematical Models with Applications

(1) The student uses a variety of strategies and approaches to solve both routine and non-routine problems. The student is expected to:

(A) compare and analyze various methods for solving a real-life problem;

(B) use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines; and

(C) select a method to solve a problem, defend the method, and justify the reasonableness of the results.

(2) The student uses graphical and numerical techniques to study patterns and analyze data. The student is expected to:

(D) use regression methods available through technology to describe various models for data such as linear, quadratic, exponential, etc., select the most appropriate model, and use the model to interpret information.

(8) The student uses algebraic and geometric models to describe situations and solve problems. The student is expected to:

(A) use geometric models available through technology to model growth and decay in areas such as population, biology, and ecology;

(B) use trigonometric ratios and functions available through technology to calculate distances and model periodic motion; and

(C) use direct and inverse variation to describe physical laws such as Hook's, Newton's, and Boyle's laws.

Turn In:

Assignment #	Activity	TEKS
8	Scatter Plots and Trend Lines	M.1.A, M1.B, M.1.C, M.2.D
9	Variables in Variation	M.1.A, M1.B, M.1.C, M.8.C
10	Applications of Variation	M.1.A, M1.B, M.1.C, M.8.C
11	Population Explosion	M.1.A, M1.B, M.1.C, M.8.A
12	Rebounding Ball	M.1.A, M1.B, M.1.C, M.8.C
13	Exponential Models	M.1.A, M1.B, M.1.C, M.2.D
14	Modeling the Spread of Diseases	M.1.A, M1.B, M.1.C, M.8.A
15	Modeling Radioactive Decay	M.1.A, M1.B, M.1.C, M.8.A
16	Modeling Periodic Motion	M.1.A, M1.B, M.1.C, M.8.B
17	Applications of Trig Functions	M.1.A, M1.B, M.1.C, M.8.B
18	Regression: Modeling Real-World Data	M.1.A, M1.B, M.1.C, M.2.D
19	Unit 2 Test	M.1.A, M1.B, M.1.C, M.2.D, M.8.A, M.8.B, M.8.C

Name ______________________________ Date ______________ Class ______________

LESSON 4-5

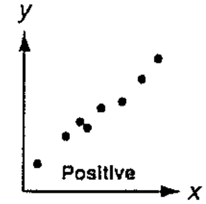
Review for Mastery

Scatter Plots and Trend Lines

Correlation is one way to describe the relationship between two sets of data.

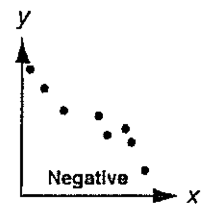
Positive Correlation

Data: As one set **increases,** the other set **increases.**
Graph: The graph **goes up** from left to right.



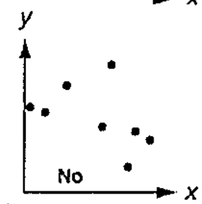
Negative Correlation

Data: As one set **increases,** the other set **decreases.**
Graph: The graph **goes down** from left to right.



No Correlation

Data: There is **no relationship** between the sets.
Graph: The graph has **no pattern.**



Identify the correlation you would expect to see between the number of grams of fat and the number of calories in different kinds of pizzas.

When you *increase* the amount of fat in a food, you also *increase* calories. So you would expect to see a positive correlation.

Identify the correlation you would expect to see between each pair of data sets. Explain.

1. the number of knots tied in a rope and the length of the rope

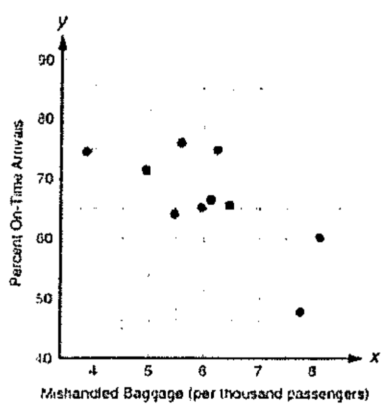
____

2. the height of a woman and her score on an algebra test

____

Describe the correlation illustrated by each scatter plot.

3.



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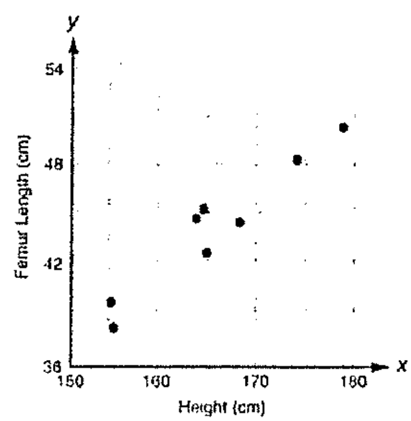
4.



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Name ______________________ Date ______________ Class ______________

LESSON 4-5

Review for Mastery

Scatter Plots and Trend Lines continued

By drawing a **trend line** over a graph of data, you can make predictions.

The scatter plot shows a relationship between a man's height and the length of his femur (thigh bone). Based on this relationship, predict the length of a man's femur if his height is 160 cm.

Step 1: Draw a trend line through the points.

Step 2: Go from 160 cm on the *x*-axis up to the line.

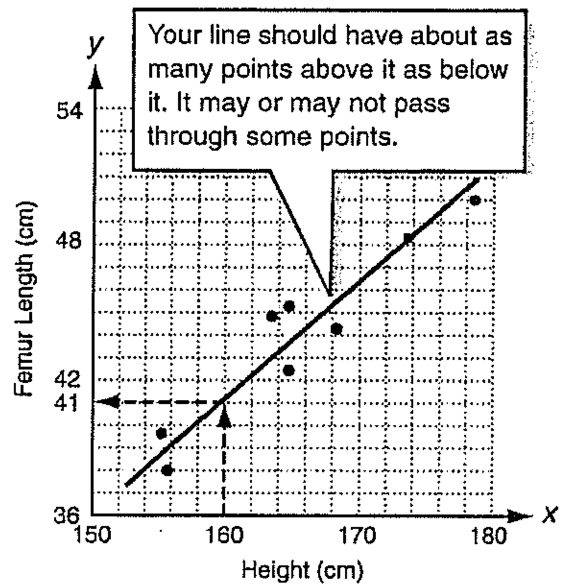
Step 3: Go from the line left to the *y*-axis.

The point (160, 41) is on the line.

Height Femur

A man that is 160 cm tall would have a femur about 41 cm long.

To find an *x*-value, go right from the *y*-value, and then down to the *x*-value. So, a man with a 42 cm femur would be about 162 cm tall.



The scatter plot shows a relationship between engine size and city fuel economy for ten automobiles.

5. Draw a trend line on the graph.
6. Based on the relationship, predict...

   a. the city fuel economy of an automobile with an engine size of 5 L.

   ____

   b. the city fuel economy of an automobile with an engine size of 2.8 L.

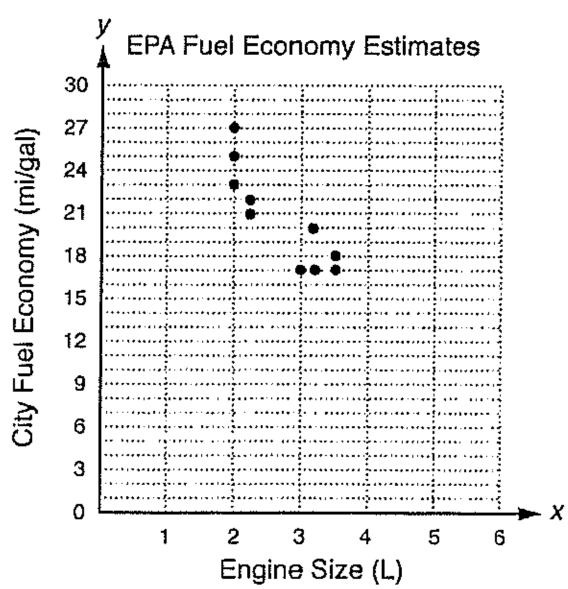
   ____

   c. the engine size of an automobile with a city fuel economy of 11 mi/gal.

   ____

   d. the engine size of an automobile with a city fuel economy of 28 mi/gal.

   ____



Name ______________________ Date ______________ Class ______________

LESSON 4-5

Problem Solving

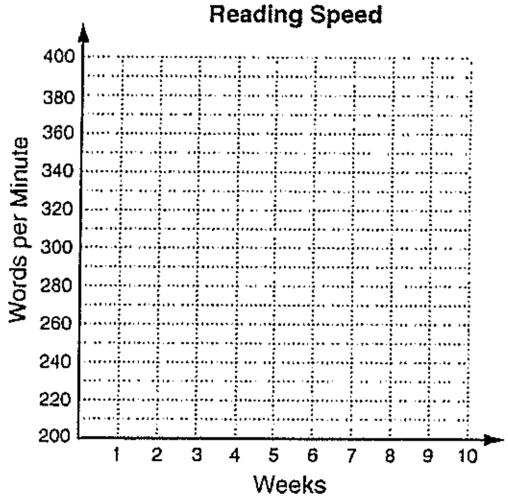
Scatter Plots and Trend Lines

Fawn is trying to improve her reading skills by taking a speed-reading class. She is measuring how many words per minute (wpm) she can read after each week of the class.

1. Graph a scatter plot using the given data.

Weeks	1	2	3	4	5
wpm	220	230	260	260	280

Reading Speed



2. Describe the correlation illustrated by the scatter plot.

____

____

3. Draw a trend line and use it to predict the number of words per minute that Fawn will read after 8 weeks of this class.

____

4. Fawn is paying for this class each week out of her savings account. Identify the correlation between the number of classes and Fawn's account balance.

____

____

Choose the scatter plot that best represents the described relationship.

5. the distance a person runs and how physically tired that person is

   A Graph 1 C Graph 3

   B Graph 2 D Graph 4

6. the price of a new car and the number of hours in a day

   F Graph 1 H Graph 3

   G Graph 2 J Graph 4

7. a person's age and the amount of broccoli the person eats

   A Graph 1 C Graph 3

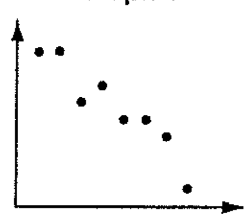
   B Graph 2 D Graph 4

8. the number of cats in a barn and the number of mice in that barn

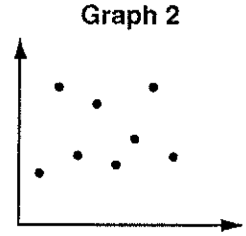
   F Graph 1 H Graph 3

   G Graph 2 J Graph 4

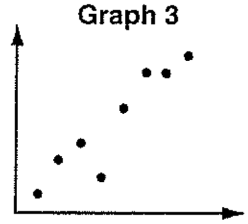
Graph 1



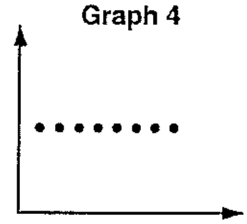
Graph 2



Graph 3



Graph 4



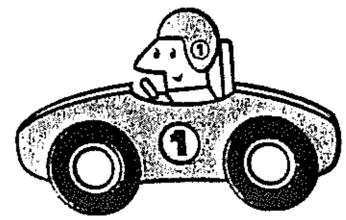
Notes

Variables in Variation KEY

Two very useful mathematical models are the variation functions.

- In one, the dependent variable is found by multiplying a constant by a power of x.
- In the other, the dependent variable is found by dividing a constant by a power of x.

Let us examine the variation functions by exploring the relationships between distance, rate, and time.



Rate is equal to distance traveled divided by time. (speed = miles/hour)

Formula: $r = \frac{d}{t}$

Time is equal to distance traveled divided by rate. (time = miles/speed)

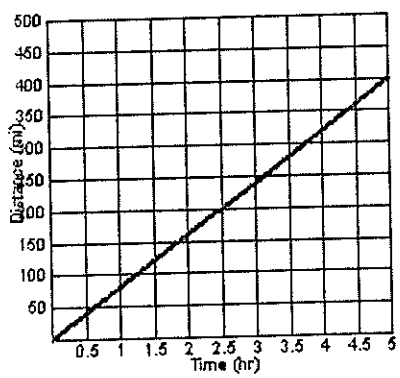
Formula: $t = \frac{d}{r}$

Miles traveled is equal to rate times time. (miles = speed • hours)

Formula: $d = rt$

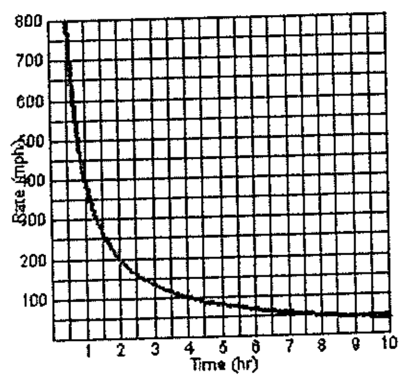
Race Car 1 is going an average of 80 mph.
Distance traveled depends on time.

t	d
1	80
2	160
3	240
4	320
5	400



Race Car 1 must cover a distance of 400 miles.
Rate of car depends on time.

t	r
1	400
2	200
4	100
5	80
8	50



1. Which represents a direct variation? Explain your reasoning.
   Race Car 1 is going an average of 80 mph. Distance is found by multiplying time by the constant rate of 80 mph.
2. Which represents an inverse variation? Explain your reasoning.
   Race Car 1 must cover a distance of 400 miles. Rate is found by dividing the constant distance of 400 miles by time.
3. Compare the graphs of direct and inverse variation.
   Answers may vary. Sample: The direct variation graph is linear and increasing. The inverse variation graph is curved and decreasing. It approaches the x-axis and y-axis, but does not cross them.
4. What are the asymptotes of the inverse variation?
   The x-axis and y-axis are the asymptotes because they are approached, but not crossed.

Notes

Variables in Variation KEY

Sample Equations for Variation Functions	
Verbal Description	**General Equation**
y varies directly as x	$y = kx$
y varies directly as the square of x	$y = kx^2$
y varies directly as the square root of x	$y = k\sqrt{x}$
y varies inversely as x	$y = \frac{k}{x}$
y varies inversely as the square of x	$y = \frac{k}{x^2}$
y varies inversely as the square root of x	$y = \frac{k}{\sqrt{x}}$

The "k" in each general equation is called the constant of variation, and can be determined for the particular equation that represents a problem as long as one point is given.

- Determine the general equation.
- Plug in the x and y values.
- Solve for k.
- Rewrite a particular equation for the problem using variables x and y and the value found for k.

Example 1
Given that $y = 8$ when $x = 4$, find a particular equation to represent each of the above sample equations.

Verbal Description	General Equation	Plug in x and y values.	Solve for k.	Particular Equation
a. y varies directly as x	$y = kx$	$8 = k(4)$	$k = 2$	$y = 2x$
b. y varies directly as the square of x	$y = kx^2$	$8 = k(4)^2$	$k = \frac{1}{2}$	$y = \frac{1}{2}x^2$
c. y varies directly as the square root of x	$y = k\sqrt{x}$	$8 = k\sqrt{4}$	$k = 4$	$y = 4\sqrt{x}$
d. y varies inversely as x	$y = \frac{k}{x}$	$8 = \frac{k}{4}$	$k = 32$	$y = \frac{32}{x}$
e. y varies inversely as the square of x	$y = \frac{k}{x^2}$	$8 = \frac{k}{4^2}$	$k = 128$	$y = \frac{128}{x^2}$
f. y varies inversely as the square root of x	$y = \frac{k}{\sqrt{x}}$	$8 = \frac{k}{\sqrt{4}}$	$k = 16$	$y = \frac{16}{\sqrt{x}}$

Notes

Variables in Variation KEY

Guided Practice

1. Using the particular equation found to represent each of the previous sample equations, predict the *y* value if *x* is 16.

Verbal Description	Particular Equation	y value at x = 16
a. *y* varies directly as *x*	$y = 2x$	32
b. *y* varies directly as the square of *x*	$y = \frac{1}{2}x^2$	128
c. *y* varies directly as the square root of x	$y = 4\sqrt{x}$	16
d. *y* varies inversely as *x*	$y = \frac{32}{x}$	2
e. *y* varies inversely as the square of *x*	$y = \frac{128}{x^2}$	0.5
f. *y* varies inversely as the square root of *x*	$y = \frac{16}{\sqrt{x}}$	4

2. Using the particular equation found to represent each of the above sample equations, predict the *x* value if *y* is 16. Use the graphing calculator if necessary and find the intersection point. Window: Domain ($0 \le x \le 20$, by 2), Range ($0 \le y \le 20$, by 2)

Verbal Description	Particular Equation	x value if y = 16
a. *y* varies directly as *x*	$y = 2x$	8
b. *y* varies directly as the square of *x*	$y = \frac{1}{2}x^2$	5.7
c. *y* varies directly as the square root of *x*	$y = 4\sqrt{x}$	16
d. *y* varies inversely as *x*	$y = \frac{32}{x}$	2
e. *y* varies inversely as the square of *x*	$y = \frac{128}{x^2}$	2.8
f. *y* varies inversely as the square root of *x*	$y = \frac{16}{\sqrt{x}}$	1

Variables in Variation

Practice Problems

1. y varies directly as x. When $x = 6$, $y = 120$.
   a. Find the constant of variation.
   b. Determine the particular equation.
   c. Find y, when $x = 100$.
   d. Find x, when $y = 400$.

2. y varies directly as the cube of x. When $x = 2$, $y = 24$.
   a. Find the constant of variation.
   b. Determine the particular equation.
   c. Find y, when $x = 3$.
   d. Find x, when $y = 375$.

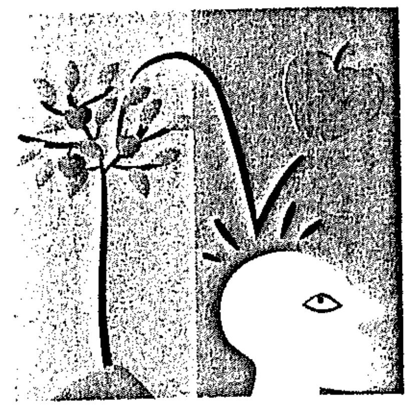
3. y varies inversely as x. When $x = 9$, $y = 6$.
   a. Find the constant of variation.
   b. Determine the particular equation.
   c. Find y, when $x = 27$.
   d. Find x, when $y = -6$.

4. y varies inversely as the cube of x. When $x = 2$, $y = 50$.
   a. Find the constant of variation.
   b. Determine the particular equation.
   c. Find y, when $x = 4$.
   d. Find x, when $y = 3{,}200$.

5. y varies directly as the square root of x. When $x = 9$, $y = 36$.
   a. Find the constant of variation.
   b. Determine the particular equation.
   c. Find y, when $x = \frac{1}{4}$.
   d. Find x, when $y = 72$.

6. y varies inversely as the square root of x. When $x = \frac{1}{9}$, $y = 6$.
   a. Find the constant of variation.
   b. Determine the particular equation.
   c. Find y, when $x = 9$.
   d. Find x, when $y = 30$.

Applications of Variation



1. If an object is dropped, Newton's Second Law of Motion says the applied force (F) with which it will hit the ground is directly related to its mass (m). If a 5 kilogram mass is dropped, it will strike the ground with a force of 49 Newtons.

   a. What is the particular equation that represents the problem situation?

   b. Construct a table of at least four points that represent the problem situation.

   c. Construct a graph to represent the problem situation. Label and scale the axes over an appropriate domain and range.

   d. If the mass was 8 kg, what would be the predicted force at which it would strike the ground?

   e. If the force at which it struck the ground was 117.6 Newtons, what was the mass of the object?

   f. Newton's Second Law is written mathematically as $F = ma$, where "F" is the force exerted, "m" is the mass of the object in kilograms, and "a" is the acceleration due to gravity. What is the acceleration due to gravity, according to your function?

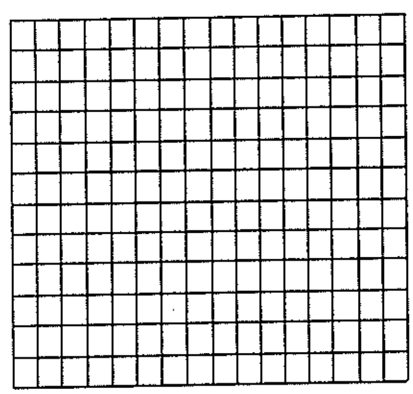
Applications of Variation

2. Boyle's Law states that the volume of a fixed amount of gas (at constant temperature) is inversely proportional to the pressure of the gas. A pressure of 46 pounds per square inch (psi) has a volume of 360 cubic feet.

   a. What is the particular equation that represents the problem situation?

   b. Construct a table of at least four points that represent the problem situation.

   c. Construct a graph to represent the problem situation. Label and scale the axes over a domain from $0 < x < 100$ psi.

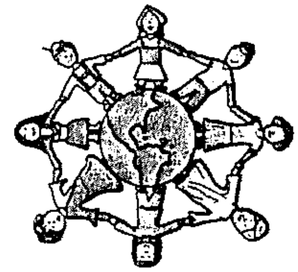


   d. If the pressure was 75 psi, what would be the predicted volume?

   e. What pressure would be necessary to compress the gas to a volume of 276 cubic feet?

   f. According to the model, could the gas be compressed to a volume of zero cubic feet? Explain your reasoning.

Population Explosion



The population of the world is growing rapidly. Resources for food and water are limited. How will this "population explosion" impact future world population predictions? The table below is the 1998 revision of the official United Nations world population estimates and projections for 1980 through 1989.

Year	Years Since 1950	Population in billions	Common Ratio
1950	0	2.56	$\frac{2.78}{2.56} = 1.0859$
1955		2.78	
1960		3.04	
1965		3.35	
1970		3.71	
1975		4.09	
1980		4.46	
1985		4.85	
1990		5.28	
1995		5.89	
2000		6.65	Average: ______

1. Complete the table above.
2. Assume that population depends on the years since 1950. What are the independent and dependent variables?
3. Enter the independent variable into the graphing calculator under L1 and the dependent variable into the graphing calculator under L2.

Population Explosion

4. Find a linear function to model the population growth using linear regression. What is the correlation (*r* value)?

5. Use the linear function model to predict the population in 2010.

6. Find a quadratic function to model the population growth using quadratic regression. What is the correlation (*r* value)?

7. Use the quadratic function model to predict the population in 2010.

8. Find an exponential function to model the population growth using exponential regression. What is the correlation (r value)? How does the common ratio value given in the regression equation compare to the one found as the average in the table?

Exponential Model

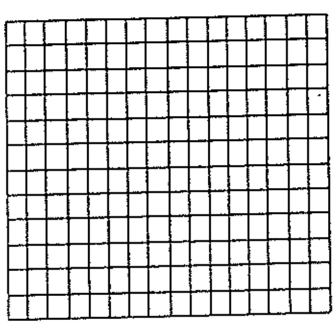
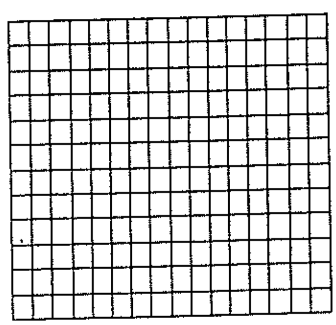
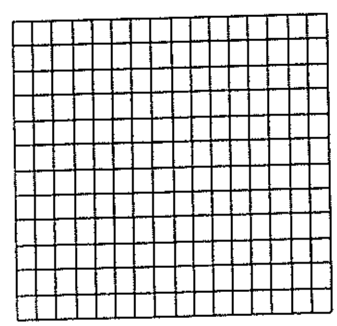
$$y = a(b)^x$$

a = population at year zero
b = common ratio

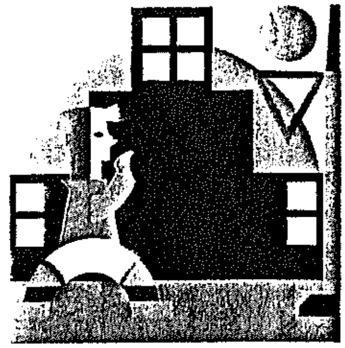
9. Use the exponential function model to predict the population in 2010.

10. Which of the three models is the best to predict the population in 2010? Explain your reasoning.

11. Create a separate scatterplot for each algebraic model of the data. Label and scale axes over an appropriate domain and range.

Rebounding Ball

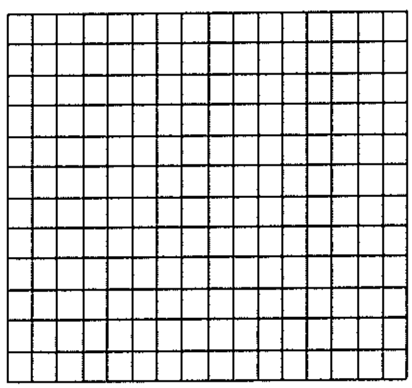


Jackson is shooting hoops on a Saturday afternoon. As he watches the basketball drop from the basketball hoop and bounce on the court, he notices that when the ball bounces it does not reach the original height. He decides that he would like to develop a mathematical model to predict the height of the ball on successive bounces. The basketball falls from the rim of a basketball hoop that is 10 feet above the court. It bounces back $\frac{4}{5}$ of its original height on each bounce of the ball.

1. Complete the table to represent the height of successive bounces.

Bounce	Height (feet)	Common Ratio
0	10	
1		
2		
3		
4		

2. Create a scatterplot of the relationship. Label and scale axes over an appropriate domain and range.



3. Enter the data into the graphing calculator and determine an exponential function to model the situation. Graph the function on the scatterplot.

Rebounding Ball

4. How high does the ball rebound after the tenth bounce?

5. After what bounce will the ball rebound to a height of approximately 3.28 feet? Give two ways to find this value.

6. According to the function model, after what bounce will the rebound height be zero?

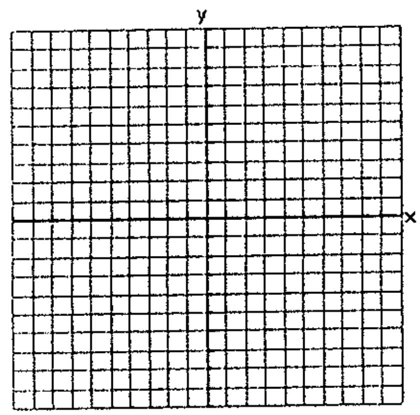
7. Explain how the function model and the real-world situation of the bouncing ball would differ.

Exponential Models

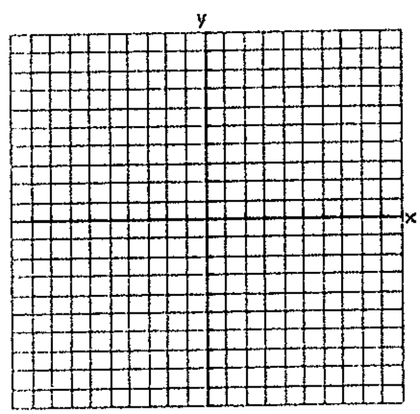
Draw a sketch of the following exponential functions using the graphing calculator.

- What is the common ratio?
- Does it represent exponential growth or decay? Be careful with problem 3.
- Find the values at f(3), f(-1), and f(0).

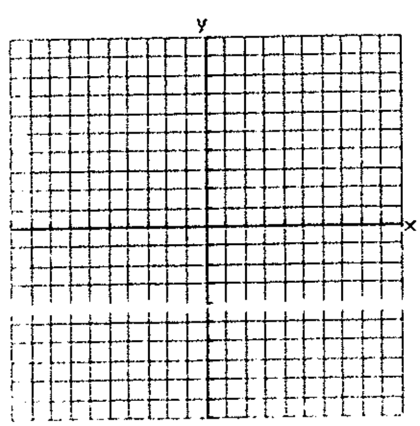
1. $f(x) = 3(2)^x$



2. $f(x) = (\frac{1}{2})^x - 1$



3. $f(x) = -(2)^x + 3$



Exponential Models

Solve each of the following problems using exponential functions.

4. Suppose the future population of Smallville can be modeled by the formula $f(x) = 12000(1.015)^x$, where x is the number of years since 2000. What is the initial population of Smallville in 2000? Use the function to predict the population in Smallville in the year 2020.

5. The future value of an investment at 6% compounded monthly can be modeled by the formula $f(x) = 7500(1.005)^x$, where x represents the number of months that the money is invested. What is the initial amount invested according to the formula? What would be the value of the investment after 10 years?

6. Australia can be used for a study of population and effects of predators. In the 1800s, rabbits were brought into Australia which had no natural predators to keep the rabbit population in check. Assume the number of rabbits increased exponentially by the function, $f(x) = 60(6.32)^x$, where x represents the number of years elapsed since 1865. How many rabbits were in Australia at time zero when the function model began in 1865? How many rabbits would be predicted at the end of 1875? Use the model to predict when the first pair of rabbits was introduced into Australia.

7. The number of hours milk can stay fresh decreases as temperature increases. A model for this situation is a decreasing exponential function, $f(x) = 192(0.9)^x$, where x represents the temperature in °C. According to the formula, how long does milk stay fresh at 0°C? Predict how long the milk will stay fresh at 30°C and 60°C.

LESSON 8-2

Reteach

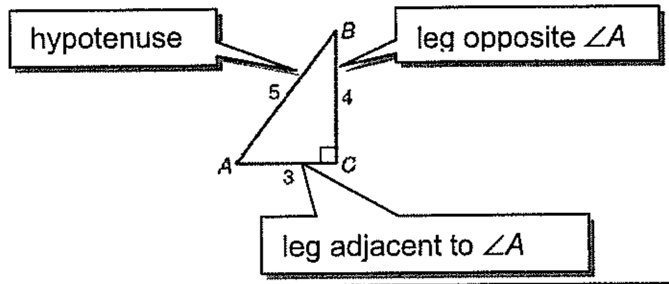
Trigonometric Ratios

Trigonometric Ratios

$$\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} = \frac{4}{5} = 0.8$$

$$\cos A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}} = \frac{3}{5} = 0.6$$

$$\tan A = \frac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A} = \frac{4}{3} \approx 1.33$$

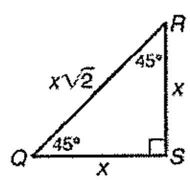


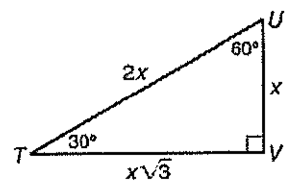
You can use special right triangles to write trigonometric ratios as fractions.

$$\sin 45° = \sin Q = \frac{\text{leg opposite } \angle Q}{\text{hypotenuse}}$$

$$= \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$$

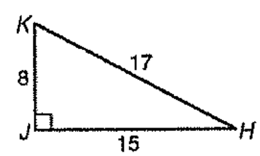
$$= \frac{\sqrt{2}}{2}$$





So $\sin 45° = \frac{\sqrt{2}}{2}$.

Write each trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



1. sin K ______________________

2. cos H ______________________

3. cos K ______________________

4. tan H ______________________

Use a special right triangle to write each trigonometric ratio as a fraction.

5. cos 45° ______________________

6. tan 45° ______________________

7. sin 60° ______________________

8. tan 30° ______________________

Name ______________________ Date ______________ Class ______________

LESSON 8-2

Reteach

Trigonometric Ratios continued

You can use a calculator to find the value of trigonometric ratios.

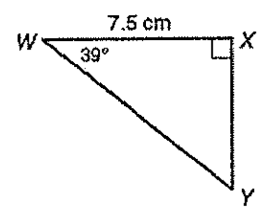
$\cos 38° \approx 0.7880107536$ or about 0.79

You can use trigonometric ratios to find side lengths of triangles.

Find *WY*.

$\cos W = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$ Write a trigonometric ratio that involves *WY*.

$\cos 39° = \dfrac{7.5 \text{ cm}}{WY}$ Substitute the given values.



$WY = \dfrac{7.5}{\cos 39°}$ Solve for *WY*.

$WY \approx 9.65$ cm Simplify the expression.

Use your calculator to find each trigonometric ratio. Round to the nearest hundredth.

9. sin 42° ______________________

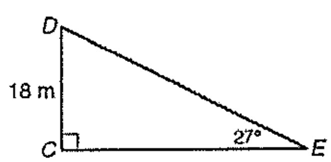
10. cos 89° ______________________

11. tan 55° ______________________

12. sin 6° ______________________

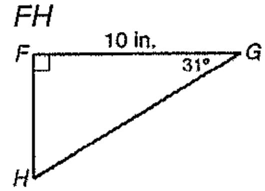
Find each length. Round to the nearest hundredth.

13. *DE*



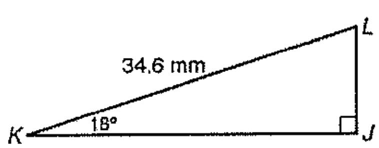
____

14. *FH*



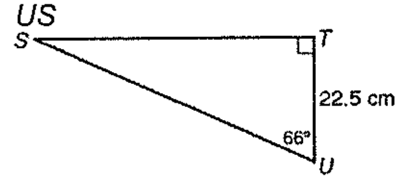
____

15. *JK*



____

16. *US*



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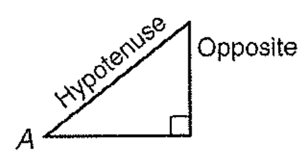
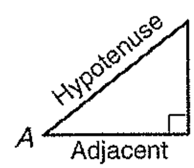
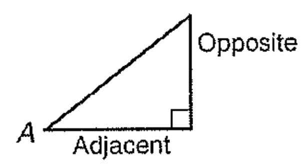
Notes

6. Get Organized In each cell, write the meaning of each abbreviation and draw a diagram for each. (p. 528).

ABBREVIATION	WORDS	DIAGRAM
$\sin = \frac{\text{opp. leg}}{\text{hyp.}}$	**The sine of an angle is the ratio of the length of the leg opposite the angle to the length of the hypotenuse**	
$\cos = \frac{\text{adj. leg}}{\text{hyp.}}$	**The cosine of an angle is the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse**	
$\tan = \frac{\text{opp. leg}}{\text{adj. leg}}$	**The tangent of an angle is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle.**	

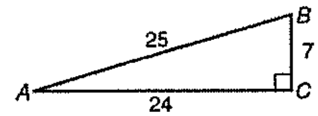
Name ______________________ Date ______________ Class______________

LESSON 8-2

Practice B

Trigonometric Ratios

Use the figure for Exercises 1–6. Write each trigonometric ratio as a simplified fraction and as a decimal rounded to the nearest hundredth.



1. sin *A* ______________
2. cos *B* ______________
3. tan *B* ______________
4. sin *B* ______________
5. cos *A* ______________
6. tan *A* ______________

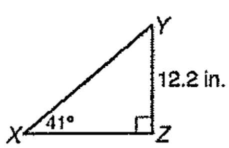
Use special right triangles to write each trigonometric ratio as a simplified fraction.

7. sin 30° ________
8. cos 30° ________
9. tan 45° ________
10. tan 30° ________
11. cos 45° ________
12. tan 60° ________

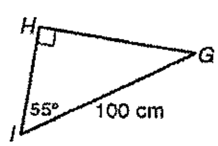
Use a calculator to find each trigonometric ratio. Round to the nearest hundredth.

13. sin 64° ________
14. cos 58° ________
15. tan 15° ________

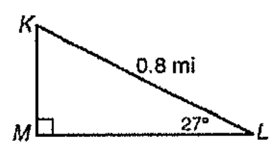
Find each length. Round to the nearest hundredth.

16.


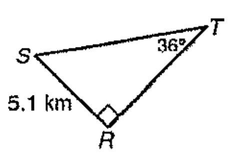
XZ ______________

17.


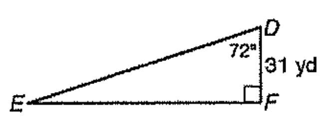
HI ______________

18.


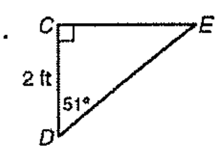
KM ______________

19.


ST ______________

20.


EF ______________

21.


DE ______________

Applications of Trig Functions

Trigonometric ratios have many real-world applications, from surveying to navigation where calculation of distances and angles might be required. Round answers to the nearest hundredth.

Trigonometric ratios can be applied in any situation involving a right triangle where only one acute angle and one side are known.

- Draw a diagram of the right triangle.
- Determine which trig ratio will work best with the given data.
- Solve the problem.
- Justify the solution in the problem situation.

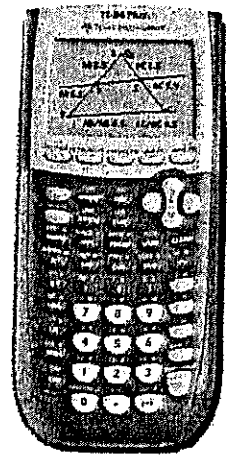
1. A 20-foot ladder is used to put up Christmas lights. The angle the ladder forms with the ground is 50°. How far up the house does the ladder reach?

2. A flag pole creates a shadow of 38 feet when the sun is at an angle of elevation of 36°. How tall is the flag pole?

3. The angle of depression from a person standing on the 300-foot harbor bridge to a ship coming in to the harbor is 4°. To the nearest foot, how far out is the ship from the harbor bridge?

Regression (Curve Fitting) with the TI-83/84 Calculator

Contents



Have Your Calculator Ready and Follow Along!

An Overview

The ordered pairs (*t*, *N*) of data values are obtained from some measuring process. Using these data, lets find a formula between *t* and *N* by a linear regression.

Regression is a statistical method that is used to do just that. The purpose of regression is to construct a rule or formula (function) that "best" explains the observed data. It is always possible to reproduce the data exactly by choosing a sufficiently complicated formula, e.g., an $(n\text{-}1)^{th}$ degree polynomial. In practice, we try to find a simpler relationship that "fits" most of the data.

The TI-83/84 can be used to fit various situations: linear (using least squares or median-median regression), polynomial (quadratic, cubic, and quartic), exponential, logarithmic, power, logistic, and sinusoidal. The examples below show how we fit linear and polynomial models to data and plot the results.

Data for Regression Example

t	*N*	*t*	*N*
5	119.94	**30**	424.72
10	166.65	**35**	591.15
15	213.32	**40**	757.96
20	256.01	**45**	963.36
25	406.44	**50**	1226.58

Simple Linear Regression

Entering the Data

Fig. 1 Choose Edit to Enter Data

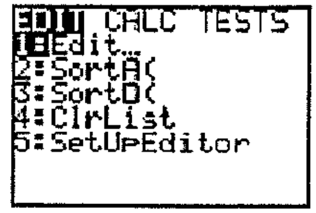


Fig. 2 The Edit Window

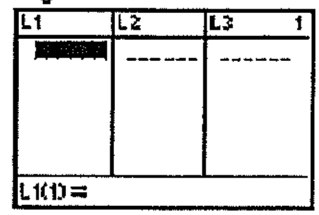
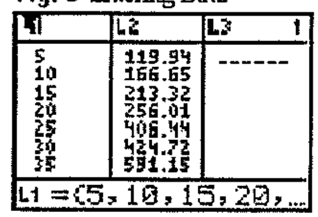


Fig. 3 Entering Data



1. Press the STAT key and choose [EDIT] then [1:Edit] ... (Fig. 1).
    - This brings up the edit window (Fig. 2).
    - On the TI-83, data are entered into "list" variables.
    - There are six list variables available, called L1, L2, L3, L4, L5, and L6.
        - Their names appear as the 2nd functions of the numeric keys 1, 2, 3, 4, 5, and 6.
        - To use these list variables, you must choose them from the keyboard; they cannot be typed using ordinary alphanumeric characters.
        - You may also choose your own names for the lists by inserting a column and then typing the name at the Name= prompt.
        - **Be careful with lists. Do not accidentally delete a list column!** If you do, you can go to the MEM reset option and reset the calculator to get all of the columns back, but the data is lost!
2. Enter the data values one column at a time pressing ENTER after each data value.
    - Place the t-values in L1 and N-values in L2.
    - Fig. 3 shows the first seven observations.
    - Make sure you have as many t's as N's. If you don't you will get a "Dimension Error."

Plotting the Data (Scatter Plots)

Fig. 4 Stat Plots Menu

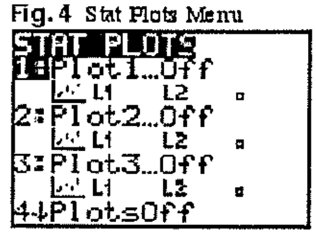


Fig. 5 Stat Plot Settings

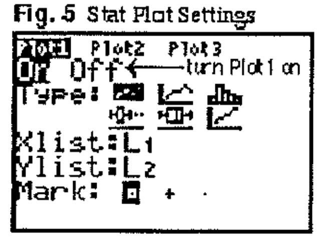
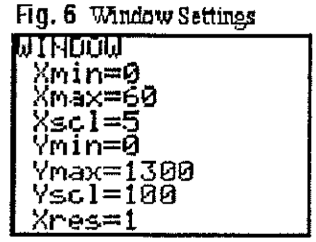


Fig. 6 Window Settings

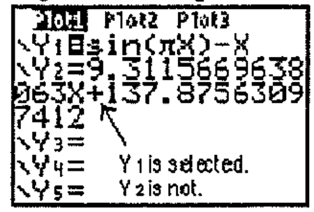


To decide with which model to fit, create a scatter plot of the data.

1. Begin by turning **Stat Plots** ***on***. Press STAT PLOT using 2nd Y= to bring up the display shown in Fig. 4.
    - With the first plot selected, press ENTER to bring up the settings for this plot (Fig. 5).
    - Change the first option to On.
    - The default settings for the remaining options will produce a scatter plot using L1 as the x-variable and L2 as the y-variable. Mark defines the symbol type used in the plot, a box, or plus or dot are the options. Typically we just leave it as the box since it is easy to see.
2. Define the **data range** for the plot.
    - Press WINDOW and enter the settings shown in Fig. 6.
    - The nice thing about working with stats, you know the domain and range that best fits the data by looking for the extremes in ***input*** and ***output*** in the data set.
3. Next press Y=. If there are any functions displayed already, be sure they are **deselected.** They do not need to be deleted!
    1. A function is **selected** if the "=" sign is displayed in reverse video ■. (In Fig. 7, Y_1 is selected while Y_2 is not.)

2. To **deselect** a function, use the arrow keys to move the blinking cursor directly on top of the = sign of the selected function. Press ENTER to **deselect** the function. (Pressing ENTER again will reselect it.)
4. Finally, press GRAPH to produce the scatter plot (Fig. 8).

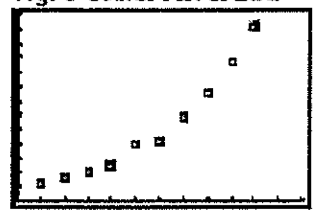
Fig. 7 Deselecting Functions



Fitting a Linear Function

The scatter plot in Fig. 8 suggests that a straight line relationship is not too unreasonable. Our next step is to get the formula that would help us in approximating values of N given t. The process is tedious, but beats hand plotting and calculation!

Fig. 8 Scatter Plot of Data



1. Press the STAT key and choose [CALC] then [4:LinReg(ax+b)] as shown in Fig. 9.
   - (Note: choosing option [8: LinReg(a+bx)] will produce the same results as choosing option 4 except that the labels for the intercept and slope will be reversed. It doesn't matter which of these two options you choose.)
   - This puts LinReg(ax+b) on the home screen.
2. By default it is assumed that the x-variable is in L_1 and the y-variable is in L_2, so pressing ENTER at this point will produce the correct results. A better choice is to list L_1 and L_2 as a part of the command.
   - The order of arguments is *x-variable*, *y-variable, storage location* (Fig. 10).
   - So the command LinReg(ax+b) L_1,L_2,Y_1 will use L_1 as the x-list, L_2 as the y-list, and overwrite the contents of Y_1 with the regression formula and automatically selects Y_1 for plotting! Wow!
   - Notice that the arguments are separated by commas.
3. Also remember, L_1, L_2, and Y_1 are special characters and cannot be entered by typing ordinary letters and numbers.
   - L_1 is the 2^{nd} function of numeric key 1,
   - L_2 is the 2^{nd} function of numeric key 2
   - Y_1 is obtained by pressing VARS, choosing [Y-VARS] then [1:Function], and then [FUNCTION] then [1:Y_1]. (See Fig. 11 and 12.)
4. Once the command and its arguments are pasted to the screen, press ENTER to produce the regression results shown in Fig. 13.
   - The values of **a** and **b** are displayed on the screen along with the from of the model that was chosen.
   - Based on the output the fitted model is $N(t) = -130.17 + 23.374t$.
   - Since the regression function is now stored in Y_1 and is selected, pressing GRAPH will produce a scatter plot with the regression line superimposed (Fig. 14).

Fig. 9 Linear Regression Option

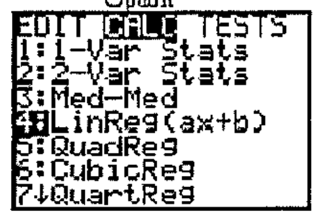


Fig. 10 Linear Regression Arguments

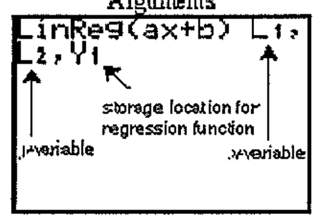


Fig. 11 Locating the Yn variables

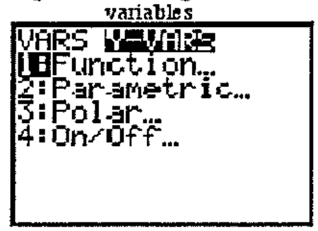


Fig. 12 Function variables

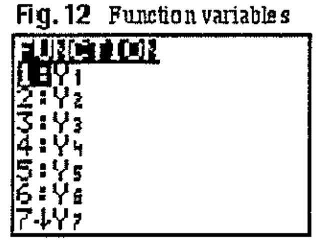


Fig. 14 Regression Line and Scatter Plot

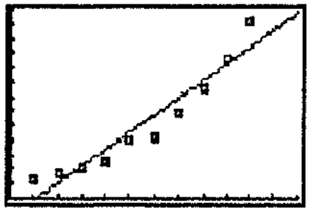
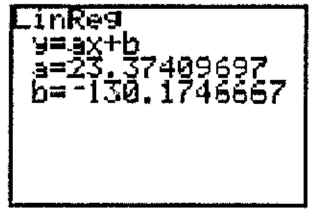


Fig. 13 Regression Results



Interpreting the Results - How Good is the "Fit?"

Qualitatively it would appear from the graph in Fig. 14 that a linear function is a "reasonable" model. The standard ***quantitative*** measure of the usefulness of the regression model is R^2, the ***coefficient of determination (also, correlation coefficient)*** . R^2 can take on values between 0 and 1. The higher the value the better the fit. So a 1 would be perfect. The TI-83 calculates this quantity automatically.

Fig. 15 Locating Statistics Variables

1. Confusingly, for simple straight line models such as this one, the TI-83 stores the coefficient of determination in a variable it calls r^2. For more complicated models, it stores the coefficient of determination in the variable R^2.
2. To access either one press VARS and then select from the [VARS] submenu option [5:Statistics] (Fig. 15).
3. In the Statistics window that appears move the cursor to the third column to display the [EQ] menu. The eighth option, [8:r^2], is where the coefficient of determination is stored for this model (Fig. 16).
4. Press ENTER to paste the value to the screen and then ENTER again to see its contents (Fig. 17).

Fig. 16 r^2 and R^2

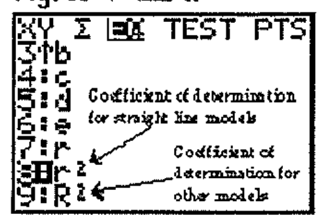
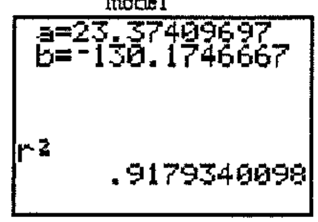


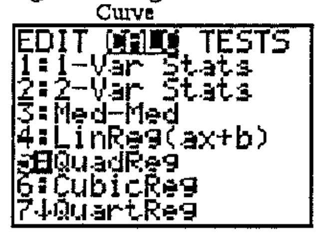
Fig. 17 r^2 for the linear model



We can conclude that the fit from this linear regression has an r^2 value of about 92%. Without reference to a different model, we really don't know if this is the best fit we can achieve short of a a 9^{th} degree polynomial. As you will see a little later, it isn't all that great!

Fitting a Quadratic Function

Fig. 18 Fitting a Parabolic Curve



The scatter plot in Fig. 8 or 14 reveals a slight curve in the data trend. A higher degree polynomial model might be appropriate. The nice thing about calculators, they allow exploration without doing a lot of extra work.

1. Press the STAT key and choose CALC 5:QuadReg (Fig. 18). This puts QuadReg on the home screen. (QuadReg fits a second degree polynomial. Third and fourth degree polynomials can be fit by choosing CubicReg and QuartReg respectively.)
2. Let's modify the command in the same way as before to create the

command QuadReg L_1,L_2,Y_2 will use L_1 as the *x*-list, L_2 as the *y*-list, and overwrite the contents of Y_2 with the regression function and automatically select Y_2 for plotting.

3. As was described for the linear model, L_1, L_2, and Y_2 must be pasted in by making the appropriate keyboard and menu choices. Press [ENTER] to produce the regression results shown in Fig. 20.
4. The values of *a, b,* and *c* are displayed on the screen along with model that was fit. Based on the output, the fitted model is $N(t) = 0.531t^2 - 5.807t + 161.637$ *using some rounding.*
5. Since the quadratic regression function is now stored in Y_2 and is selected, and the straight-line regression function is still stored in Y_1, pressing [GRAPH] will produce a scatter plot with the quadratic regression function and linear regression function superimposed on a scatter plot of the data (Fig. 21).
6. Now go to the [VARS] menu again and find the values for R^2. The ***coefficient of determination*** has jumped to about 99%! Also, the graph of the Y_2 formula is significantly better is a fit to the data.

Fig. 19 Quadratic Regression Arguments

Fig. 20 Quadratic Regression Results

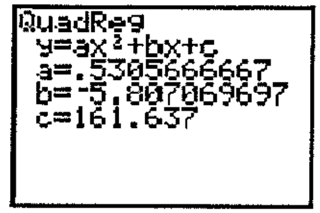


Fig. 21 Graph of Linear and Quadratic Regressions

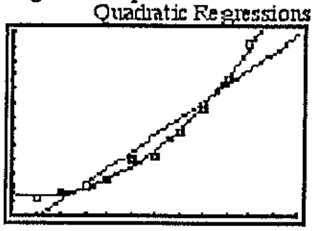


Fig. 22 Choosing Quadratic Model R^2

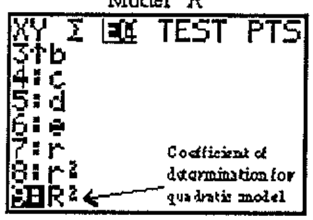
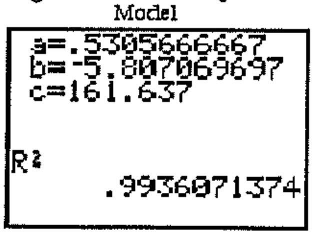


Fig. 23 R^2 for the Quadratic Model



Try other choices in the Regression menu. While the quadratic is a pretty good, one of the other choices may do even better. The data has the appearance of an exponential and some characteristics of a logistic model.

Name ______________________ Date ______________ Class ______________

LESSON 9-6

Reteach

Modeling Real-World Data (continued)

After determining a parent function to model a data set, use the regression feature on a graphing calculator to find a function that models the data.

Write a function that models the data.

x	4	5	6	7	8
y	71	93	121	157	204

Make sure the *x*-values are evenly spaced.

Step 1 Find first differences.

First differences: 22 28 36 47

$204 - 157 = 47$

Step 2 Since first differences are not constant, find second differences.

Second differences: 6 8 11

Step 3 Since second differences are not constant, analyze ratios.

$$\frac{93}{71} = 1.310, \frac{121}{93} = 1.301, \frac{157}{121} = 1.298, \frac{204}{157} = 1.299$$

Ratios are all close to 1.3.

Step 4 An exponential model best fits the data since the ratios are almost constant. Use a graphing calculator. Perform exponential regression. Select ExpReg from the STAT CALC menu.

ExpReg
$y = a*b^x$
$a = 24.8379125$
$b = 1.301415677$
$r^2 = .999953961$
$r = .9999769803$

An exponential model that fits the data is $f(x) = 24.8(1.3^x)$.

Complete to write a function that models the given data.

x	3	4	5	6	7
y	33	56	86	123	167

3. Are the *x*-values evenly spaced? ______________
4. Are the first differences constant? ______________
5. Are the second differences constant? ______________
6. What is an appropriate model for the data? ______________
7. Find a function that models the data. ______________

Name ______________________ Date ______________ Class ______________

LESSON 9-6

Practice B

Modeling Real-World Data

Use constant differences or ratios to determine which parent function would best model the given data set.

1.

x	12	16	20	24	28
y	0.8	3.6	16.2	72.9	328.05

____

2.

x	13	19	25	31	37	43
y	–1	17	35	53	71	89

____

3.

x	2	7	12	17	22
y	–100	–55	40	185	380

____

4.

x	0.10	0.37	0.82	1.45	2.26
y	0.3	0.6	0.9	1.2	1.5

____

Write a function that models the data set.

5.

x	2.2	2.6	3.0	3.4	3.8
y	0.68	4.52	9.0	14.12	19.88

____

6.

x	–5	0	5	10	15	20
y	8	6	4	2	0	–2

____

7.

x	0.3	0.7	1.1	1.5	1.9
y	2.5	3	3.6	4.32	5.184

____

8.

x	0.06	0.375	0.96	1.815	2.94
y	0.2	0.5	0.8	1.1	1.4

____

9.

x	–6	1	8	15	22
y	15	1	30.12	102.36	217.72

____

10.

x	0.32	2.07	4.8	8.51	13.2
y	0.9	1.6	2.3	3.0	3.7

____

Solve.

11. The table shows the population growth of a small town.

Years after 1974	1	6	11	16	21	26	31
Population	662	740	825	908	1003	1095	1200

a. Write a function that models the data. ______________________

b. Use your model to predict the population in 2020. ______________________