MEA 2013-2014
Teacher: Claudia Valle
Start Date:

Course: Geometry B
Student: ________________
Completed Date:

Unit 6: Transformational Geometry

Objectives: Students will understand how execute various types of transformations. Students will understand how to identify and describe different types of transformations. Students will understand how to represent magnitude and direction with a vector and use them to solve real life problems.

Essential Questions: What is a transformation, and what are the 3 basic types of transformations? How can vectors be used to describe translations? How can a figure be transformed multiple times?

TEKS Standards: G.2.A, G.2.B, G.5.C, G.7.A, G.10.A, G.11.A, G.11.B, G.11.D

Geometry

(2) Geometric structure. The student analyzes geometric relationships in order to make and verify conjectures. The student is expected to:

(A) use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships; and

(B) make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

(5) Geometric patterns. The student uses a variety of representations to describe geometric relationships and solve problems. The student is expected to:

(C) use properties of transformations and their compositions to make connections between mathematics and the real world, such as tessellations; and

(7) Dimensionality and the geometry of location. The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly. The student is expected to:

(A) use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures;

(10) Congruence and the geometry of size. The student applies the concept of congruence to justify properties of figures and solve problems. The student is expected to:

(A) use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane; and

(11) Similarity and the geometry of shape. The student applies the concepts of similarity to justify properties of figures and solve problems. The student is expected to:

(A) use and extend similarity properties and transformations to explore and justify conjectures about geometric figures;

(B) use ratios to solve problems involving similar figures;

(D) describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems.

Turn In:

Assignment #	Activity	TEKS
39	Reflections	G.2.A, G.2.B, G.7.A, G.10.A
40	Translations	G.2.A, G.2.B, G.7.A, G.10.A
41	Rotations	G.2.A, G.2.B, G.7.A, G.10.A
42	Compositions of Transformations	G.5.C, G.10.A
43	Symmetry	G.2.B, G.5.C, G.10.A
44	Tessellations	G.5.C
45	Dilations	G.2.A, 11.A, G.11.B, G.11.D
46	Unit 6 Test	G.2.A, G.2.B, G.5.C, G.7.A, G.10.A, G.11.A, G.11.B, G.11.D

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LESSON **12-1**

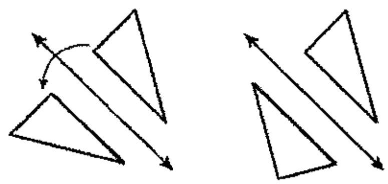
Reteach

Reflections

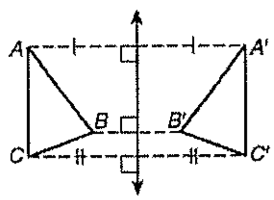
An **isometry** is a transformation that does not change the shape or size of a figure. Reflections, translations, and rotations are all isometries.

A reflection is a transformation that flips a figure across a line.

Reflection **Not a Reflection**

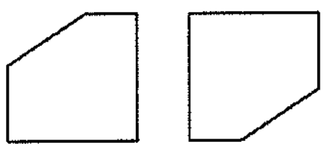


The line of reflection is the perpendicular bisector of each segment joining each point and its image.

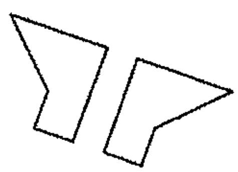


Tell whether each transformation appears to be a reflection.

1.



2.



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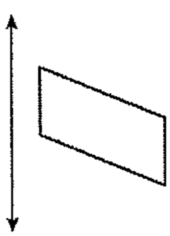
Copy each figure and the line of reflection. Draw the reflection of the figure across the line.

3.



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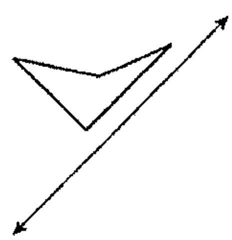
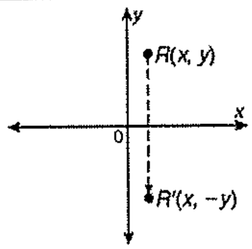
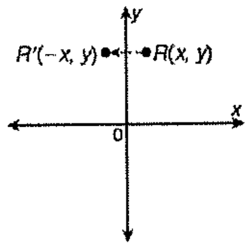
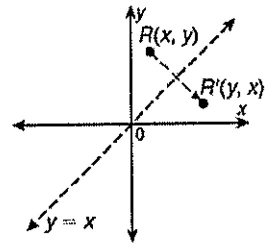


Name ______________________ Date ______________ Class ______________

LESSON 12-1

Reteach

Reflections continued

Reflections in the Coordinate Plane		
Across the *x*-axis	**Across the *y*-axis**	**Across the line $y = x$**
		
$(x, y) \rightarrow (x, -y)$	$(x, y) \rightarrow (-x, y)$	$(x, y) \rightarrow (y, x)$

Reflect $\triangle FGH$ with vertices $F(-1, 4)$, $G(2, 4)$, and $H(4, 1)$ across the *x*-axis.

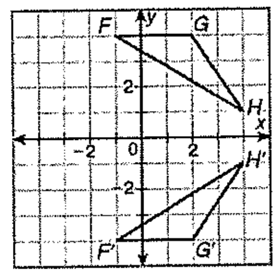
The reflection of (x, y) is $(x, -y)$.

$F(-1, 4) \rightarrow F'(-1, -4)$

$G(2, 4) \rightarrow G'(2, -4)$

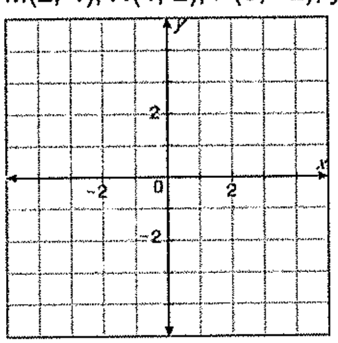
$H(4, 1) \rightarrow H'(4, -1)$

Graph the preimage and image.

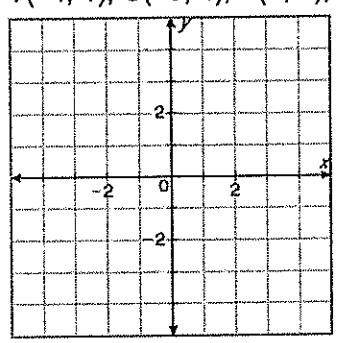


Reflect the figure with the given vertices across the line.

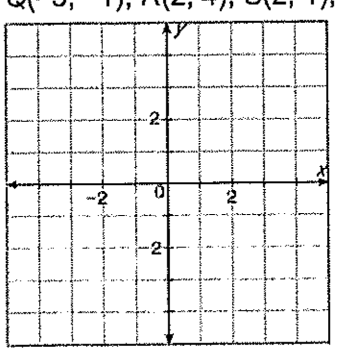
5. $M(2, 4)$, $N(4, 2)$, $P(3, -2)$; *y*-axis



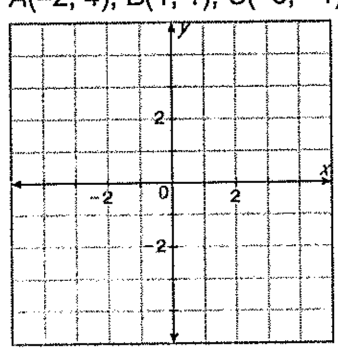
6. $T(-4, 1)$, $U(-3, 4)$, $V(2, 3)$, $W(0, 1)$; *x*-axis



7. $Q(-3, -1)$, $R(2, 4)$, $S(2, 1)$; *x*-axis



8. $A(-2, 4)$, $B(1, 1)$, $C(-5, -1)$; $y = x$



Name ______________________ Date ______________ Class ______________

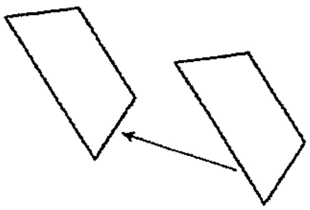
LESSON 12-2

Reteach

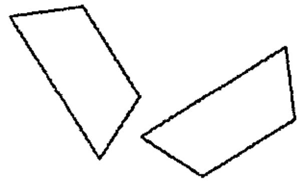
Translations

A translation is a transformation in which all the points of a figure are moved the same distance in the same direction.

Translation

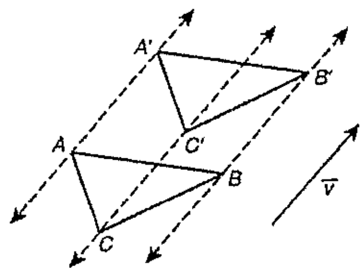


Not a Translation



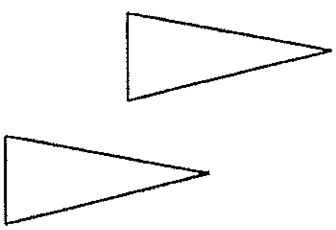
A translation is a transformation along a vector such that each segment joining a point and its image has the same length as the vector and is parallel to the vector.

$\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$ have the same length as $\vec{v}$ and are parallel to $\vec{v}$.



Tell whether each transformation appears to be a translation.

1.



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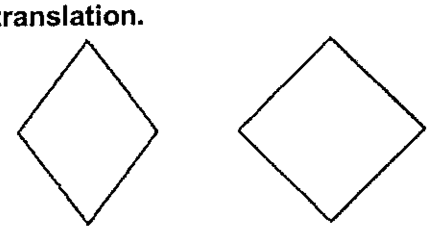
2.



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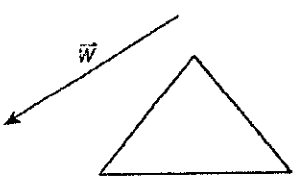
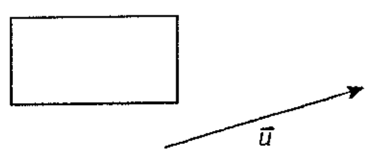
Copy each figure and the translation vector. Draw the translation of the figure along the given vector.

3.



4.

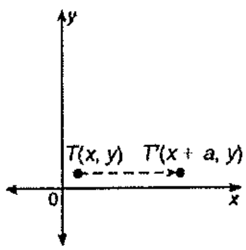
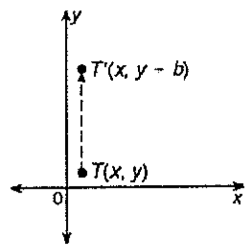
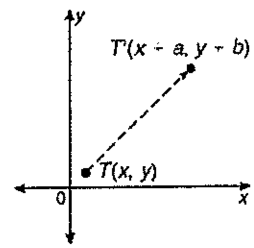


Name ______________________ Date ______________ Class ______________

LESSON 12-2

Reteach

Translations *continued*

Translations in the Coordinate Plane		
Horizontal Translation Along Vector ⟨*a*, 0⟩	**Horizontal Translation Along Vector ⟨0, *b*⟩**	**Horizontal Translation Along Vector ⟨*a*, *b*⟩**
		
$(x, y) \rightarrow (x + a, y)$	$(x, y) \rightarrow (x, y + b)$	$(x, y) \rightarrow (x + a, y + b)$

Translate △*JKL* with vertices *J*(0, 1), *K*(4, 2), and *L*(3, –1) along the vector ⟨–4, 2⟩.

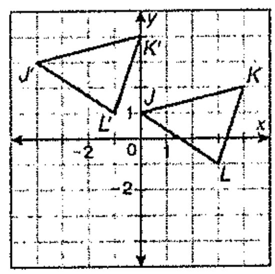
The image of (x, y) is $(x - 4, y + 2)$.

$J(0, 1) \rightarrow J'(0 - 4, 1 + 2) = J'(-4, 3)$

$K(4, 2) \rightarrow K'(4 - 4, 2 + 2) = K'(0, 4)$

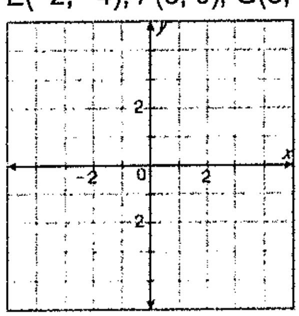
$L(3, -1) \rightarrow L'(3 - 4, -1 + 2) = L'(-1, 1)$

Graph the preimage and image.

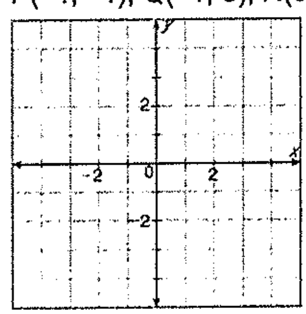


Translate the figure with the given vertices along the given vector.

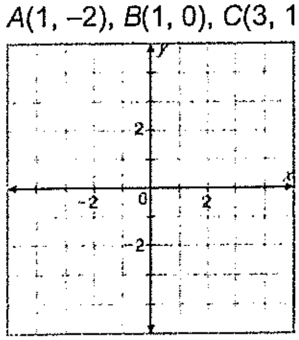
5. *E*(–2, –4), *F*(3, 0), *G*(3, –4); ⟨0, 3⟩



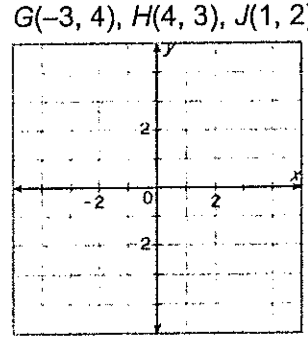
6. *P*(–4, –1), *Q*(–1, 3), *R*(0, –4); ⟨4, 1⟩



7. *A*(1, –2), *B*(1, 0), *C*(3, 1), *D*(4, –3); ⟨–5, 3⟩



8. *G*(–3, 4), *H*(4, 3), *J*(1, 2); ⟨–1, –6⟩



Name ______________________ Date ______________ Class______________

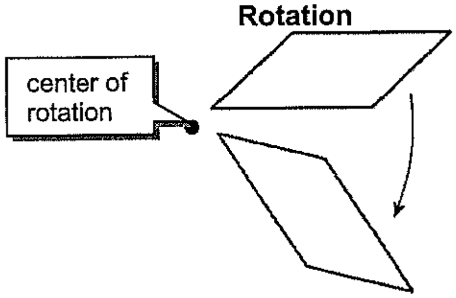
LESSON 12-3

Reteach

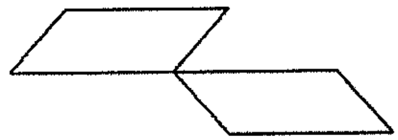
Rotations

A rotation is a transformation that turns a figure around a fixed point, called the center of rotation.

Rotation



Not a Rotation

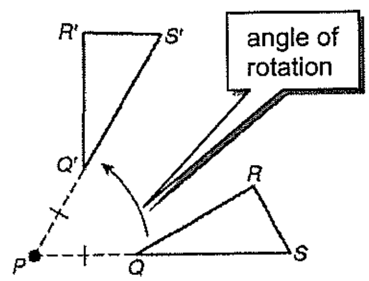


A rotation is a transformation about a point *P* such that each point and its image are the same distance from *P*.

$PQ = PQ'$

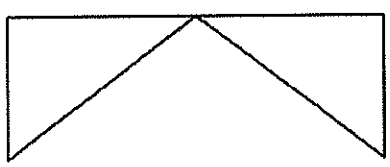
$PR = PR'$

$PS = PS'$



Tell whether each transformation appears to be a rotation.

1.



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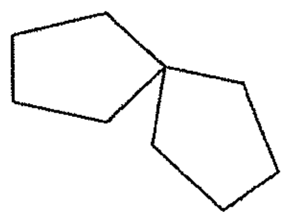
2.



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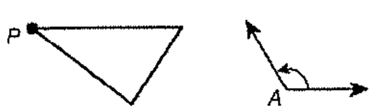
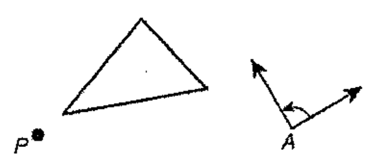
Copy each figure and the angle of rotation. Draw the rotation of the figure about point *P* by m∠*A*.

3.



4.

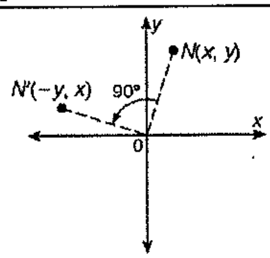
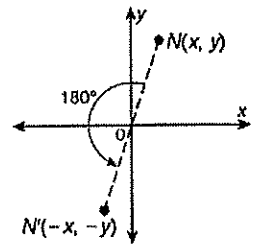


Name ______________________ Date ______________ Class ______________

LESSON 12-3

Reteach

Rotations *continued*

Rotations in the Coordinate Plane	
By 90° About the Origin	**By 180° About the Origin**
 $(x, y) \rightarrow (-y, x)$	 $(x, y) \rightarrow (-x, -y)$

Rotate △*MNP* with vertices *M*(1, 1), *N*(2, 4), and *P*(4, 3) by 180° about the origin.

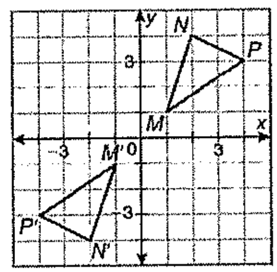
The image of (x, y) is $(-x, -y)$.

$M(1, 1) \rightarrow M'(-1, -1)$

$N(2, 4) \rightarrow N'(-2, -4)$

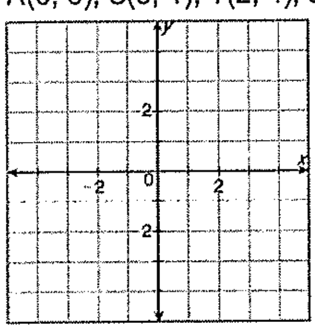
$P(4, 3) \rightarrow P'(-4, -3)$

Graph the preimage and image.

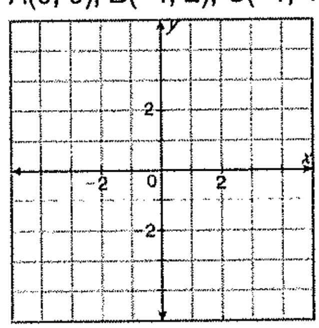


Rotate the figure with the given vertices about the origin using the given angle.

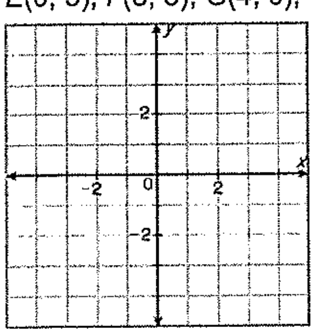
5. *R*(0, 0), *S*(3, 1), *T*(2, 4); 90°



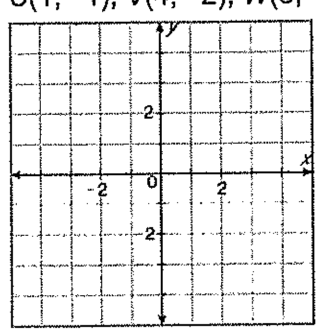
6. *A*(0, 0), *B*(−4, 2), *C*(−1, 4); 180°



7. *E*(0, 3), *F*(3, 5), *G*(4, 0); 180°



8. *U*(1, −1), *V*(4, −2), *W*(3, −4); 90°



Name ______________________ Date ______________ Class ______________

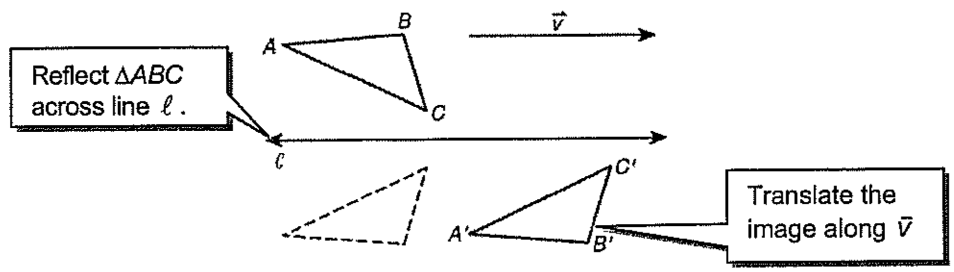
LESSON 12-4

Reteach

Compositions of Transformations

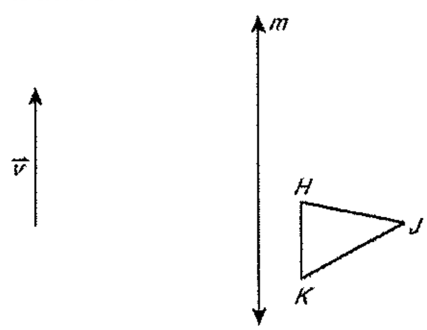
A **composition of transformations** is one transformation followed by another. A **glide reflection** is the composition of a translation and a reflection across a line parallel to the vector of the translation.

Reflect △*ABC* across line ℓ along $\vec{v}$ and then translate it parallel to $\vec{v}$.

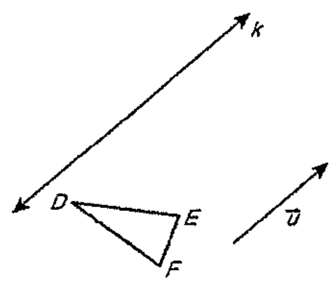


Draw the result of each composition of transformations.

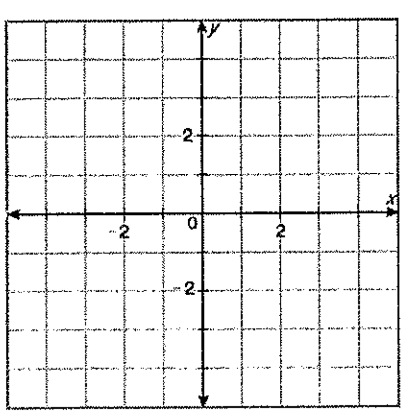
1. Translate △*HJK* along $\vec{v}$ and then reflect it across line *m*.



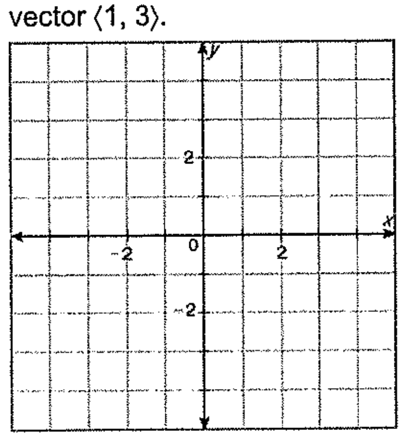
2. Reflect △*DEF* across line *k* and then translate it along $\vec{u}$.



3. △*ABC* has vertices *A*(0, –1), *B*(3, 4), and *C*(3, 1). Rotate △*ABC* 180° about the origin and then reflect it across the *x*-axis.



4. △*QRS* has vertices *Q*(2, 1), *R*(4, –2), and *S*(1, –3). Reflect △*QRS* across the *y*-axis and then translate it along the vector ⟨1, 3⟩.



Name ______________________ Date ____________ Class ____________

LESSON 12-4

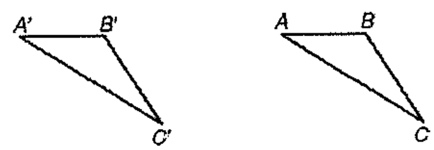
Reteach

Compositions of Transformations continued

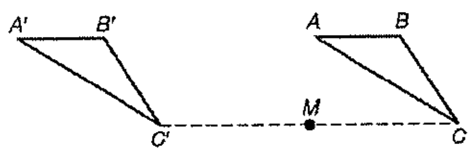
Any translation or rotation is equivalent to a composition of two reflections.

Composition of Two Reflections
To draw two parallel lines of reflection that produce a translation: • Draw $\overline{PP'}$, a segment connecting a preimage point P and its corresponding image point P'. Draw the midpoint M of $\overline{PP'}$. • Draw the perpendicular bisectors of $\overline{PM}$ and $\overline{P'M}$.
To draw two intersecting lines that produce a rotation with center C: • Draw $\angle PCP'$, where P is a preimage point and P' is its corresponding image point. Draw $\overrightarrow{CX}$, the angle bisector of $\angle PCP'$. • Draw the angle bisectors of $\angle PCX$ and $\angle P'CX$.

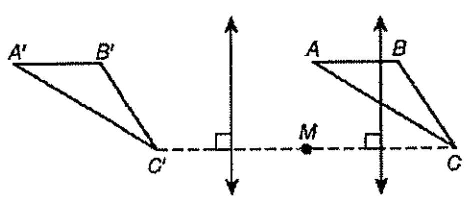
Copy $\triangle ABC$ and draw two lines of reflection that produce the translation $\triangle ABC \rightarrow \triangle A'B'C'$.



Step 1 Draw $\overline{CC'}$ and the midpoint M of $\overline{CC'}$.

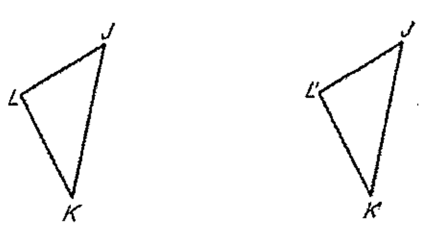


Step 2 Draw the perpendicular bisectors of $\overline{CM}$ and $\overline{C'M}$.

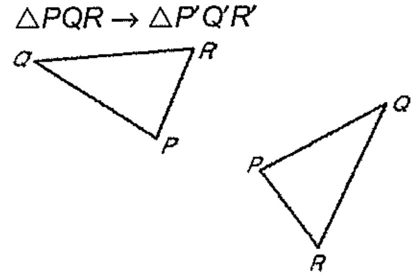


Copy each figure and draw two lines of reflection that produce an equivalent transformation.

5. translation:
$\triangle JKL \rightarrow \triangle J'K'L'$



6. rotation with center C:
$\triangle PQR \rightarrow \triangle P'Q'R'$



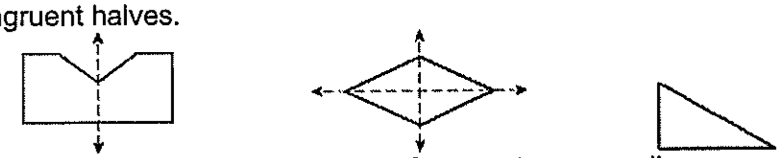
Name ______________________ Date ______________ Class ______________

LESSON **12-5**

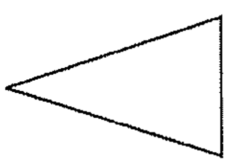
Reteach

Symmetry

A figure has **symmetry** if there is a transformation of the figure such that the image and preimage are identical. There are two kinds of symmetry.

Line Symmetry	The figure has a **line of symmetry** that divides the figure into two congruent halves.  one line of symmetry two lines of symmetry no line symmetry
Rotational Symmetry	When a figure is rotated between 0° and 360°, the resulting figure coincides with the original. • The smallest angle through which the figure is rotated to coincide with itself is called the *angle of rotational symmetry.* • The number of times that you can get an identical figure when repeating the degree of rotation is called the *order* of the rotational symmetry.  angle: 180° 120° no rotational symmetry order: 2 3

Tell whether each figure has line symmetry. If so, draw all lines of symmetry.

1. 

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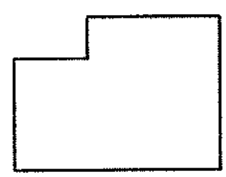
2. 

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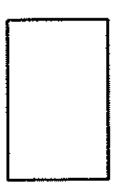
Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

3. 

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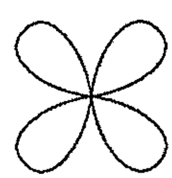
4. 

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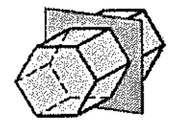
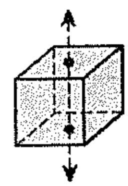
LESSON 12-5

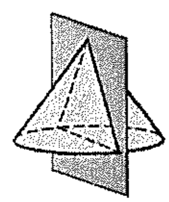
Reteach

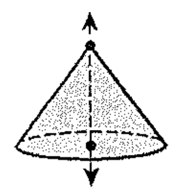
Symmetry *continued*

Three-dimensional figures can also have symmetry.

Symmetry in Three Dimensions	Description	Example
Plane Symmetry	A plane can divide a figure into two congruent halves.	
Symmetry About an Axis	There is a line about which a figure can be rotated so that the image and preimage are identical.	

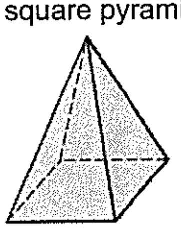
A cone has both plane symmetry and symmetry about an axis.





Tell whether each figure has plane symmetry, symmetry about an axis, both, or neither.

5. square pyramid



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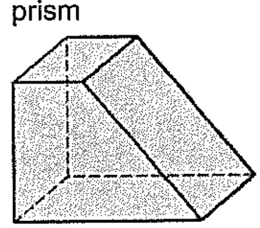
6. prism



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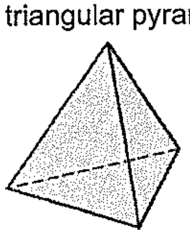
7. triangular pyramid



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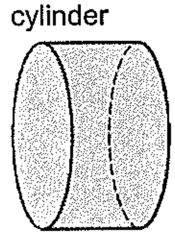
8. cylinder



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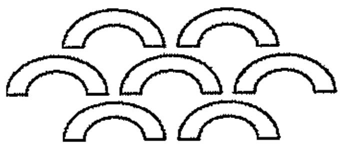
LESSON 12-6

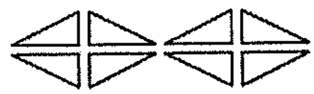
Reteach

Tessellations

A pattern has **translation symmetry** if it can be translated along a vector so that the image coincides with the preimage. A pattern with **glide reflection symmetry** coincides with its image after a glide reflection.



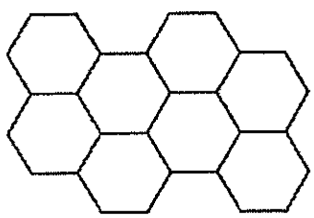
Translation Symmetry



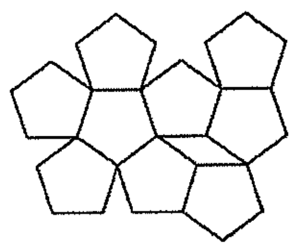
Translation Symmetry and Glide Reflection Symmetry

A **tessellation** is a repeating pattern that completely covers a plane with no gaps or overlaps.

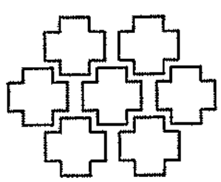
Tessellation



Not a Tessellation



Identify the symmetry in each pattern.

1. 

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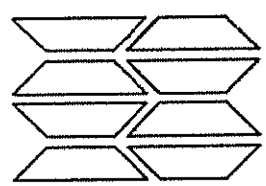
2. 

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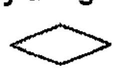
Copy the given figure and use it to create a tessellation.

3. 

4. 

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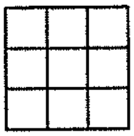
LESSON 12-6

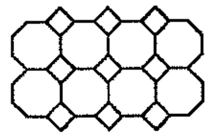
Reteach

Tessellations *continued*

A **regular tessellation** is formed by congruent regular polygons. A **semiregular tessellation** is formed by two or more different regular polygons.

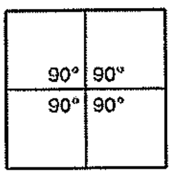


Regular Tessellation

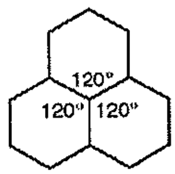


Semiregular Tessellation

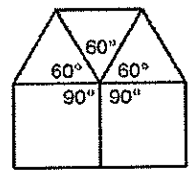
In a tessellation, the measures of the angles that meet at each vertex must have a sum of 360°.



$90° + 90° + 90° + 90° = 360°$

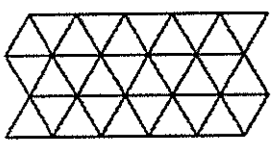


$120° + 120° + 120° = 360°$



$3(60°) + 2(90°) = 360°$

Classify each tessellation as regular, semiregular, or neither.

5. 

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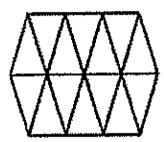
6. 

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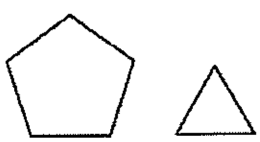
Determine whether the given regular polygon(s) can be used to form a tessellation. If so, draw the tessellation.

7. 

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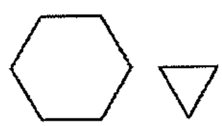
8. 

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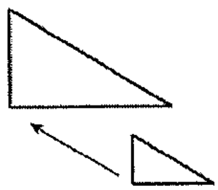
LESSON 12-7

Reteach

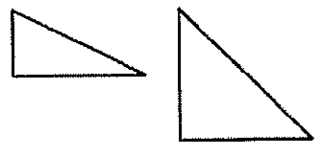
Dilations

A dilation is a transformation that changes the size of a figure but not the shape.

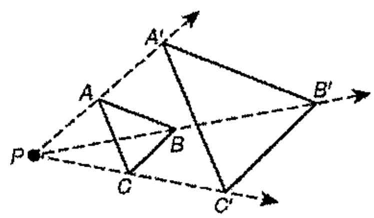
Dilation



Not a Dilation

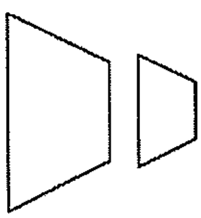


A dilation is a transformation in which the lines connecting every point *A* with its image *A′* all intersect at point *P*, called the **center of dilation.**

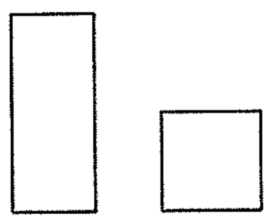


Tell whether each transformation appears to be a dilation.

1.

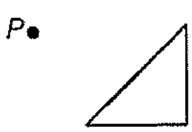


2.

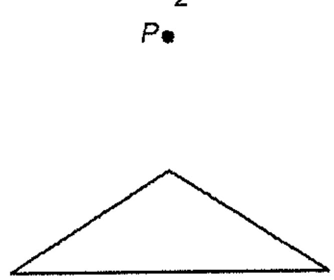


Copy teach triangle and center of dilation. Draw the image of the triangle under a dilation with the given scale factor.

3. scale factor: 2



4. scale factor: $\frac{1}{2}$



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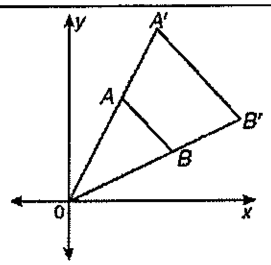
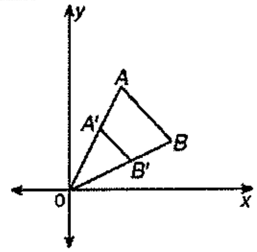
LESSON 12-7

Reteach

Dilations continued

Dilations in the Coordinate Plane	
For $k > 1$	**For $0 < k < 1$**
 $(x, y) \rightarrow (kx, ky)$	 $(x, y) \rightarrow (kx, ky)$

If k has a negative value, the preimage is rotated by 180°.

Draw the image of $\triangle EFG$ with vertices $E(0, 0)$, $F(0, 1)$, and $G(2, 1)$ under a dilation with a scale factor of -3 and centered at the origin.

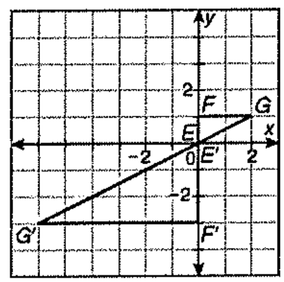
The image of (x, y) is $(-3x, -3y)$.

$E(0, 0) \rightarrow E'(0(-3), 0(-3)) \rightarrow E'(0, 0)$

$F(0, 1) \rightarrow F'(0(-3), 1(-3)) \rightarrow F'(0, -3)$

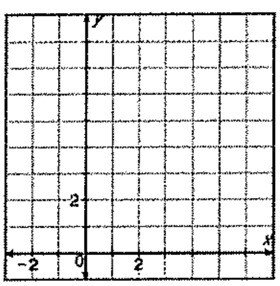
$G(2, 1) \rightarrow G'(2(-3), 1(-3)) \rightarrow G'(-6, -3)$

Graph the preimage and image.

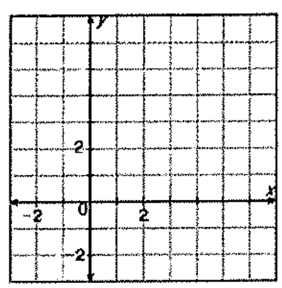


Draw the image of the figure with the given vertices under a dilation with the given scale factor and centered at the origin.

5. $J(0, 0)$, $K(-1, 2)$, $L(3, 4)$; scale factor: 2

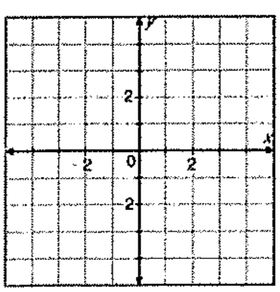


6. $A(0, 0)$, $B(0, 6)$, $C(6, 3)$; scale factor: $\frac{1}{3}$



7. $R(1, 0)$, $S(1, -2)$, $T(-1, -2)$;

scale factor: -2



8. $G(2, 0)$, $H(0, 4)$, $I(4, 2)$; scale factor: $-\frac{1}{2}$