MEA 2013-2014
Teacher: Claudia Valle
Start Date:

Course: Geometry B
Student: ________________
Completed Date:

Unit 4: Solid Geometry

Objectives: Students will understand how to identify various types of quadrilaterals. Students will understand how to use properties of special quadrilaterals to solve real-life problems.

Essential Questions: Students will understand how to use and apply the surface area and volume formulas for various solids. Students will understand how solve real life problems using the surface area and volume formulas.

TEKS Standards: G.6.A, G.6.B, G.6.C, G.7.C, G.8.D, G.9.D, G.11.D

Geometry

(6) Dimensionality and the geometry of location. The student analyzes the relationship between three-dimensional geometric figures and related two-dimensional representations and uses these representations to solve problems. The student is expected to:

(A) describe and draw the intersection of a given plane with various three-dimensional geometric figures;

(B) use nets to represent and construct three-dimensional geometric figures; and

(C) use orthographic and isometric views of three-dimensional geometric figures to represent and construct three-dimensional geometric figures and solve problems.

(7) Dimensionality and the geometry of location. The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly. The student is expected to:

(C) derive and use formulas involving length, slope, and midpoint.

(8) Congruence and the geometry of size. The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations. The student is expected to:

(D) find surface areas and volumes of prisms, pyramids, spheres, cones, cylinders, and composites of these figures in problem situations;

(9) Congruence and the geometry of size. The student analyzes properties and describes relationships in geometric figures. The student is expected to:

(D) analyze the characteristics of polyhedra and other three-dimensional figures and their component parts based on explorations and concrete models.

(11) Similarity and the geometry of shape. The student applies the concepts of similarity to justify properties of figures and solve problems. The student is expected to:

(D) describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems.

Turn In:

Assignment #	Activity	TEKS
22	Solid Geometry	G.6.A, G.6.B, G.9.D
23	Representations of Three-Dimensional Figures	G.6.C, G.9.D
24	Formulas in Three Dimensions	G.6.A, G.7.C, G.9.D
25	Surface Area of Prisms and Cylinders	G.6.B, G.8.D, G.11.D
26	Surface Area of Pyramids and Cones	G.6.B, G.8.D, G.11.D
27	Volume of Prisms and Cylinders	G.6.B, G.8.D, G.11.D
28	Volume of Pyramids and Cones	G.8.D, G.11.D
29	Spheres	G.8.D, G.11.D
30	Unit 4 Test	G.6.A, G.6.B, G.6.C, G.7.C, G.8.D, G.9.D, G.11.D

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LESSON **10-1**

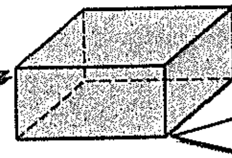
Reteach

Solid Geometry

Three-dimensional figures, or *solids*, can have flat or curved surfaces.

Prisms and pyramids are named by the shapes of their *bases*.

Each flat surface is called a **face**.

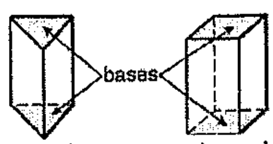


An **edge** is the segment where two faces intersect.

A **vertex** is the point where three or more faces intersect In a cone, it is where the curved surface comes to a point.

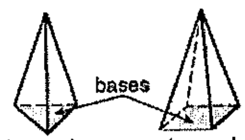
Solids

Prisms



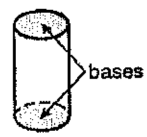
triangular prism — rectangular prism

Pyramids

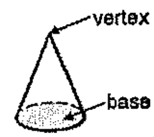


triangular pyramid — rectangular pyramid

Cylinder



Cone



Neither cylinders nor cones have edges.

Classify each figure. Name the vertices, edges, and bases.

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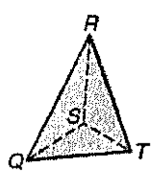

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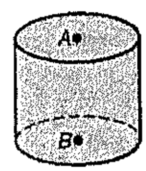

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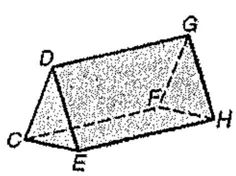

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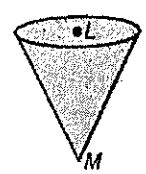

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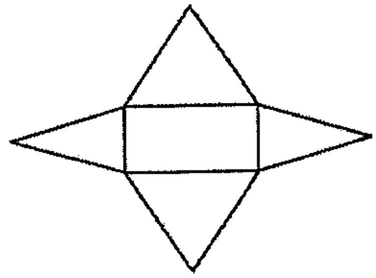
LESSON 10-1

Reteach

Solid Geometry *continued*

A **net** is a diagram of the surfaces of a three-dimensional figure. It can be folded to form the three-dimensional figure.

The net at right has one rectangular face. The remaining faces are triangles, and so the net forms a rectangular pyramid.



net of rectangular pyramid

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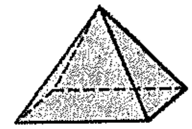
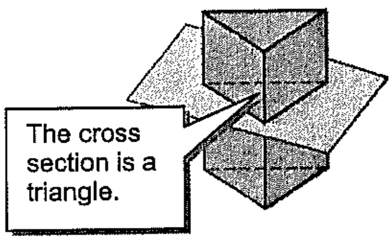


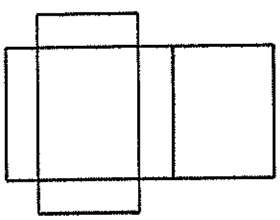
rectangular pyramid

A **cross section** is the intersection of a three-dimensional figure and a plane.



Describe the three-dimensional figure that can be made from the given net.

5.



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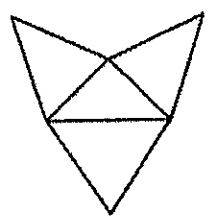
6.



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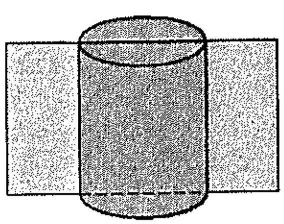
Describe each cross section.

7.



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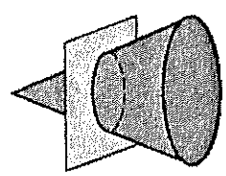

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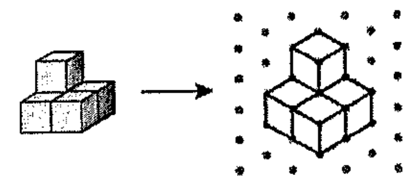
LESSON 10-2

Reteach

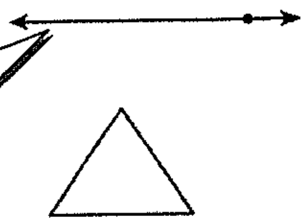
Representations of Three-Dimensional Figures continued

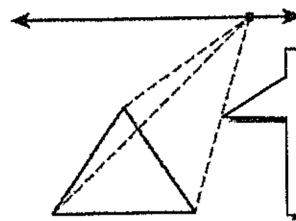
An **isometric drawing** is drawn on isometric dot paper and shows three sides of a figure from a corner view. A solid and an isometric drawing of the solid are shown.



In a **one-point perspective drawing,** nonvertical lines are drawn so that they meet at a **vanishing point.** You can make a one-point perspective drawing of a triangular prism.

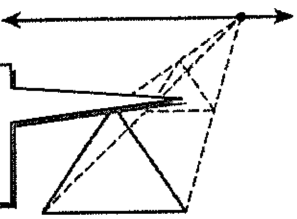
Step 1 Draw a horizontal line and a vanishing point on the line. Draw a triangle below the line.

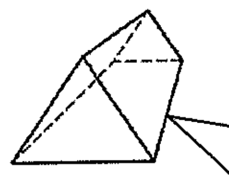




Step 2 From each vertex of the triangle, draw dashed segments to the vanishing point.

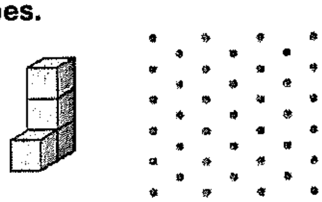
Step 3 Draw a smaller triangle with vertices on the dashed segments.

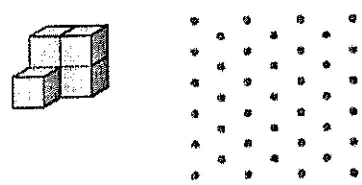




Step 4 Draw the edges of the prism. Use dashed lines for hidden edges. Erase segments that are not part of the prism.

Draw an isometric view of each object. Assume there are no hidden cubes.

3. 

4. 

Draw each object in one-point perspective.

5. a triangular prism with bases that are obtuse triangles

6. a rectangular prism

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LESSON 10-2

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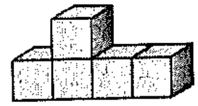
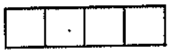
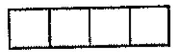
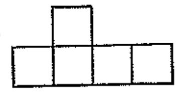
Representations of Three-Dimensional Figures

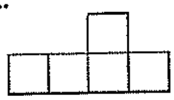
An **orthographic drawing** of a three-dimensional object shows six different views of the object. The six views of the figure at right are shown below.



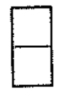
Top: 

Bottom: 

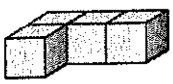
Front: 

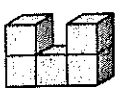
Back: 

Left: 

Right: 

Draw all six orthographic views of each object. Assume there are no hidden cubes.

1. 

2. 

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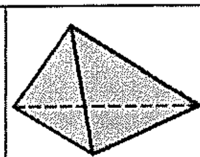
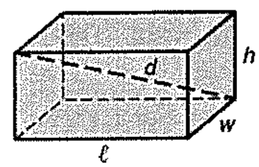
LESSON 10-3

Reteach

Formulas in Three Dimensions

A **polyhedron** is a solid formed by four or more polygons that intersect only at their edges. Prisms and pyramids are polyhedrons. Cylinders and cones are not.

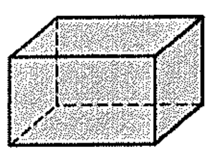
Euler's Formula	
For any polyhedron with V vertices, E edges, and F faces, $V - E + F = 2$.	 **Example** $V - E + F = 2$ Euler's Formula; $4 - 6 + 4 = 2$ $V = 4, E = 6, F = 4$; $2 = 2$; 4 vertices, 6 edges, 4 faces

Diagonal of a Right Rectangular Prism	
The length of a diagonal d of a right rectangular prism with length ℓ, width w, and height h is $d = \sqrt{\ell^2 + w^2 + h^2}$.	

Find the height of a rectangular prism with a 4 cm by 3 cm base and a 7 cm diagonal.

$d = \sqrt{\ell^2 + w^2 + h^2}$	Formula for the diagonal of a right rectangular prism
$7 = \sqrt{4^2 + 3^2 + h^2}$	Substitute 7 for d, 4 for ℓ, and 3 for w.
$49 = 4^2 + 3^2 + h^2$	Square both sides of the equation.
$24 = h^2$	Simplify.
$4.9 \text{ cm} \approx h$	Take the square root of each side.

Find the number of vertices, edges, and faces of each polyhedron. Use your results to verify Euler's Formula.

1. 

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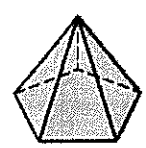
2. 

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Find the unknown dimension in each figure. Round to the nearest tenth if necessary.

3. the length of the diagonal of a 6 cm by 8 cm by 11 cm rectangular prism

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4. the height of a rectangular prism with a 4 in. by 5 in. base and a 9 in. diagonal

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LESSON 10-3

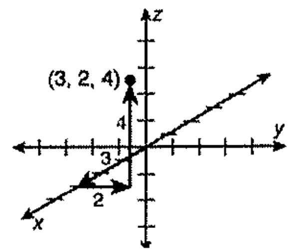
Reteach

Formulas in Three Dimensions *continued*

A three-dimensional coordinate system has three perpendicular axes:

- *x*-axis
- *y*-axis
- *z*-axis

An *ordered triple* (x, y, z) is used to locate a point. The point at (3, 2, 4) is graphed at right.



Formulas in Three Dimensions	
Distance Formula	The distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
Midpoint Formula	The midpoint of the segment with endpoints (x_1, y_1, z_1) and (x_2, y_2, z_2) is $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$.

Find the distance between the points (4, 0, 1) and (2, 3, 0). Find the midpoint of the segment with the given endpoints.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ — Distance Formula

$= \sqrt{(2-4)^2 + (3-0)^2 + (0-1)^2}$ — $(x_1, y_1, z_1) = (4, 0, 1)$, $(x_2, y_2, z_2) = (2, 3, 0)$

$= \sqrt{4+9+1}$ — Simplify.

$= \sqrt{14} \approx 3.7$ units — Simplify.

The distance between the points (4, 0, 1) and (2, 3, 0) is about 3.7 units.

$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) = M\left(\frac{4+2}{2}, \frac{0+3}{2}, \frac{1+0}{2}\right)$ — Midpoint Formula

$= M(3, 1.5, 0.5)$ — Simplify.

The midpoint of the segment with endpoints (4, 0, 1) and (2, 3, 0) is *M*(3, 1.5, 0.5).

Find the distance between the given points. Find the midpoint of the segment with the given endpoints. Round to the nearest tenth if necessary.

5. (0, 0, 0) and (6, 8, 2)

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6. (0, 6, 0) and (4, 8, 0)

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7. (9, 1, 4) and (7, 0, 7)

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8. (2, 4, 1) and (3, 3, 5)

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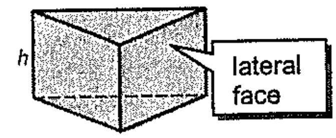
LESSON 10-4

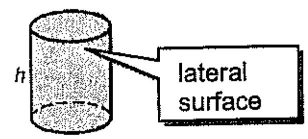
Reteach

Surface Area of Prisms and Cylinders

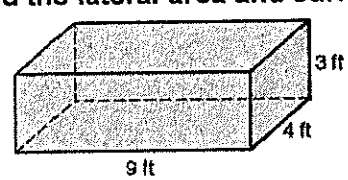
The *lateral area* of a prism is the sum of the areas of all the *lateral faces*. A lateral face is not a base. The **surface area** is the total area of all faces.

Lateral and Surface Area of a Right Prism		
Lateral Area	The lateral area of a right prism with base perimeter P and height h is $L = Ph$.	
Surface Area	The surface area of a right prism with lateral area L and base area B is $S = L + 2B$, or $S = Ph + 2B$.	

The lateral area of a right cylinder is the curved surface that connects the two bases. The **surface area** is the total area of the curved surface and the bases.

Lateral and Surface Area of a Right Cylinder		
Lateral Area	The lateral area of a right cylinder with radius r and height h is $L = 2\pi rh$.	
Surface Area	The surface area of a right cylinder with lateral area L and base area B is $S = L + 2B$, or $S = 2\pi rh + 2\pi r^2$.	

Find the lateral area and surface area of each right prism.

1. 

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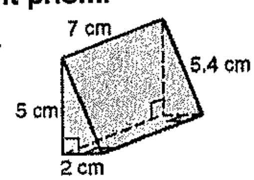
2. 

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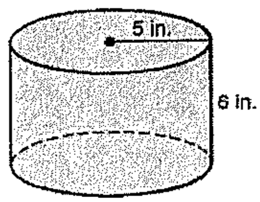
Find the lateral area and surface area of each right cylinder. Give your answers in terms of π.

3. 

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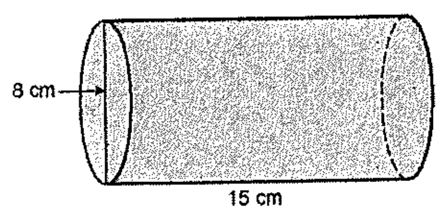
4. 

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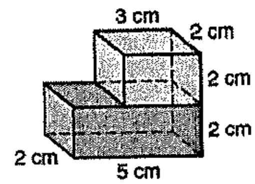
Name ______________________ Date ______________ Class ______________

LESSON 10-4

Reteach

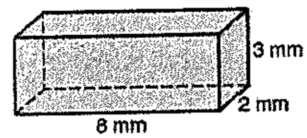
Surface Area of Prisms and Cylinders continued

You can find the surface area of a composite three-dimensional figure like the one shown at right.



surface area of small prism + surface area of large prism − hidden surfaces

The dimensions are multiplied by 3. Describe the effect on the surface area.



original surface area:

$S = Ph + 2B$

$= 20(3) + 2(16)$ $P = 20, h = 3, B = 16$

$= 92\text{ mm}^2$ Simplify.

new surface area, dimensions multiplied by 3:

$S = Ph + 2B$

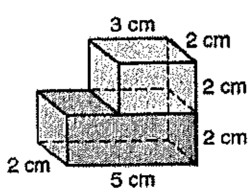
$= 60(9) + 2(144)$ $P = 60, h = 9, B = 144$

$= 828\text{ mm}^2$ Simplify.

Notice that $92 \cdot 9 = 828$. If the dimensions are multiplied by 3, the surface area is multiplied by 3^2, or 9.

Find the surface area of each composite figure. Be sure to subtract the hidden surfaces of each part of the composite solid. Round to the nearest tenth.

5.



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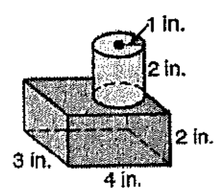
6.



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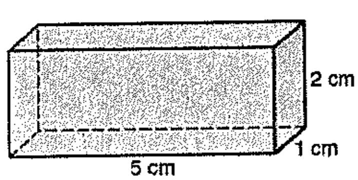
Describe the effect of each change on the surface area of the given figure.

7. The length, width, and height are multiplied by 2.



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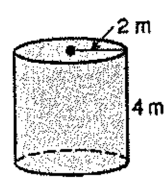
8. The height and radius are multiplied by $\frac{1}{2}$.



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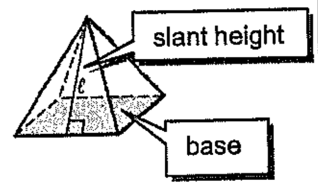
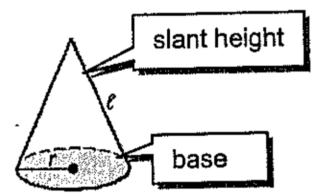
Name ______________________ Date ______________ Class ______________

LESSON 10-5

Reteach

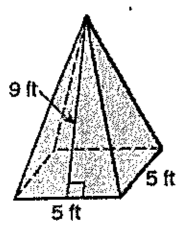
Surface Area of Pyramids and Cones

Lateral and Surface Area of a Regular Pyramid		
Lateral Area	The lateral area of a regular pyramid with perimeter P and slant height ℓ is $L = \frac{1}{2}P\ell$.	
Surface Area	The surface area of a regular pyramid with lateral area L and base area B is $S = L + B$, or $S = \frac{1}{2}P\ell + B$.	

Lateral and Surface Area of a Right Cone		
Lateral Area	The lateral area of a right cone with radius r and slant height ℓ is $L = \pi r\ell$.	
Surface Area	The surface area of a right cone with lateral area L and base area B is $S = L + B$, or $S = \pi r\ell + \pi r^2$.	

Find the lateral area and surface area of each regular pyramid. Round to the nearest tenth.

1.



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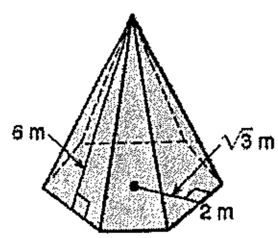
2.



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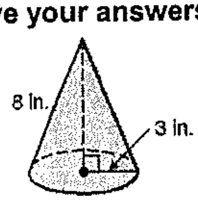
Find the lateral area and surface area of each right cone. Give your answers in terms of π.

3.



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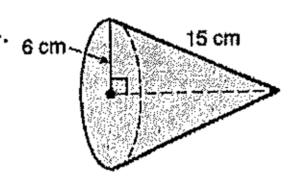
4.



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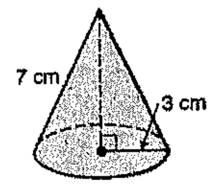
Name ______________________________ Date ______________ Class______________

LESSON 10-5

Reteach

Surface Area of Pyramids and Cones *continued*

The radius and slant height of the cone at right are doubled. Describe the effect on the surface area.



original surface area:

$S = \pi r\ell + \pi r^2$

$= \pi(3)(7) + \pi(3)^2$ $\quad r = 3,\ \ell = 7$

$= 30\pi\ \text{cm}^2$ $\quad$ Simplify.

new surface area, dimensions doubled:

$S = \pi r\ell + \pi r^2$

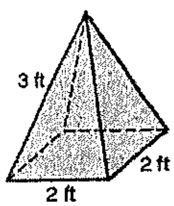
$= \pi(6)(14) + \pi(6)^2$ $\quad r = 6,\ \ell = 14$

$= 120\pi\ \text{cm}^2$ $\quad$ Simplify.

If the dimensions are doubled, then the surface area is multiplied by 2^2, or 4.

Describe the effect of each change on the surface area of the given figure.

5. The dimensions are tripled.



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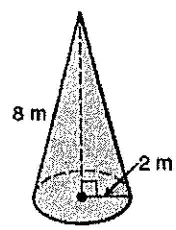
6. The dimensions are multiplied by $\frac{1}{2}$.



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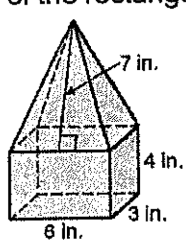
Find the surface area of each composite figure.

7. *Hint:* Do not include the base area of the pyramid or the upper surface area of the rectangular prism.



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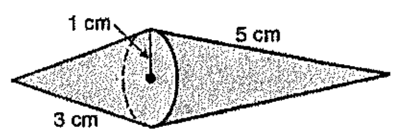
8. *Hint:* Add the lateral areas of the cones.



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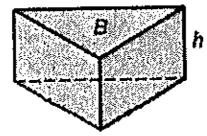
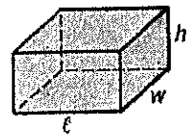
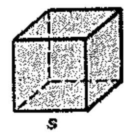
Name ____________________ Date ______________ Class ______________

LESSON 10-6

Reteach

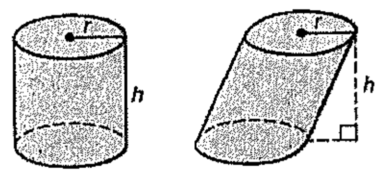
Volume of Prisms and Cylinders

Volume of Prisms		
Prism	The volume of a prism with base area B and height h is $V = Bh$.	
Right Rectangular Prism	The volume of a right rectangular prism with length ℓ, width w, and height h is $V = \ell wh$.	
Cube	The volume of a cube with edge length s is $V = s^3$.	

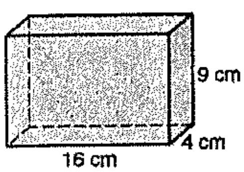
Volume of a Cylinder

The volume of a cylinder with base area B, radius r, and height h is

$$V = Bh, \text{ or } V = \pi r^2 h.$$



Find the volume of each prism.

1. 

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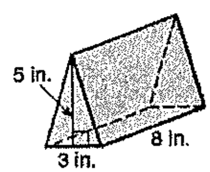
2. 

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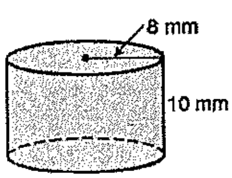
Find the volume of each cylinder. Give your answers both in terms of π and rounded to the nearest tenth.

3. 

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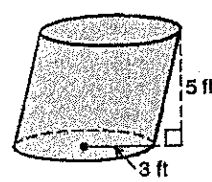
4. 

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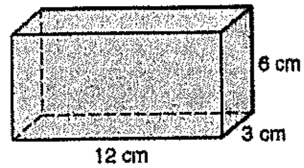
Name ______________________ Date ______________ Class ______________

LESSON 10-6

Reteach

Volume of Prisms and Cylinders *continued*

The dimensions of the prism are multiplied by $\frac{1}{3}$. Describe the effect on the volume.



original volume:

$V = \ell wh$

$= (12)(3)(6)$ $\ell = 12, w = 3, h = 6$

$= 216\text{ cm}^3$ Simplify.

new volume, dimensions multiplied by $\frac{1}{3}$:

$V = \ell wh$

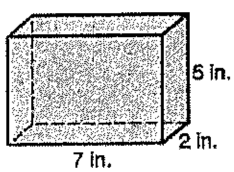
$= (4)(1)(2)$ $\ell = 4, w = 1, h = 2$

$= 8\text{ cm}^3$ Simplify.

Notice that $216 \cdot \frac{1}{27} = 8$. If the dimensions are multiplied by $\frac{1}{3}$, the volume is multiplied by $\left(\frac{1}{3}\right)^3$, or $\frac{1}{27}$.

Describe the effect of each change on the volume of the given figure.

5. The dimensions are multiplied by 2.



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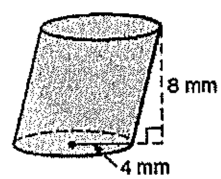
6. The dimensions are multiplied by $\frac{1}{4}$.



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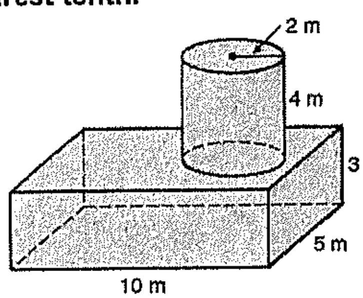
Find the volume of each composite figure. Round to the nearest tenth.

7.



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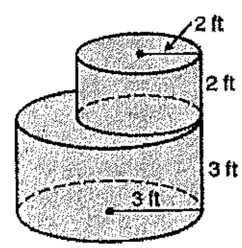
8.



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Name ______________________ Date ______________ Class ______________

LESSON 10-7

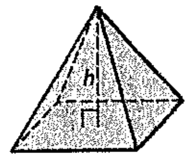
Reteach

Volume of Pyramids and Cones

Volume of a Pyramid

The volume of a pyramid with base area B and height h is

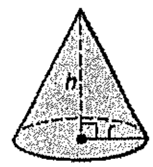
$$V = \frac{1}{3}Bh.$$



Volume of a Cone

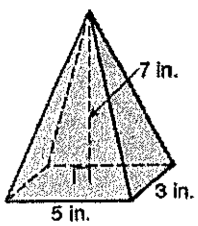
The volume of a cone with base area B, radius r, and height h is

$$V = \frac{1}{3}Bh, \text{ or } V = \frac{1}{3}\pi r^2 h.$$



Find the volume of each pyramid. Round to the nearest tenth if necessary.

1.



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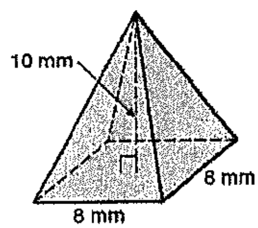
2.



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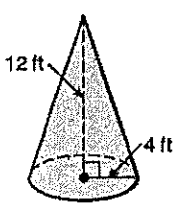
Find the volume of each cone. Give your answers both in terms of π and rounded to the nearest tenth.

3.



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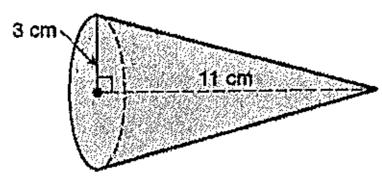
4.



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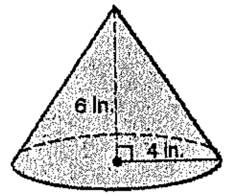
Name ______________________ Date ______________ Class ______________

LESSON 10-7

Reteach

Volume of Pyramids and Cones *continued*

The radius and height of the cone are multiplied by $\frac{1}{2}$. Describe the effect on the volume.



original volume:

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(4)^2(6)$ $r = 4, h = 6$

$= 32\pi \text{ in}^3$ Simplify.

new volume, dimensions multiplied by $\frac{1}{2}$:

$V = \frac{1}{3}\pi r^2 h$

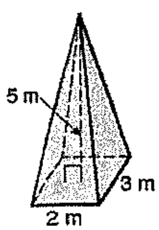
$= \frac{1}{3}\pi(2)^2(3)$ $r = 2, h = 3$

$= 4\pi \text{ in}^3$ Simplify.

If the dimensions are multiplied by $\frac{1}{2}$, then the volume is multiplied by $\left(\frac{1}{2}\right)^3$, or $\frac{1}{8}$.

Describe the effect of each change on the volume of the given figure.

5. The dimensions are doubled.



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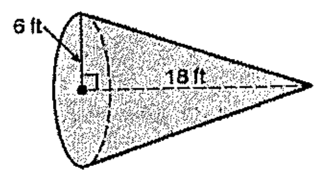
6. The radius and height are multiplied by $\frac{1}{3}$.



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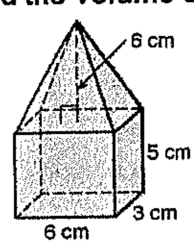
Find the volume of each composite figure. Round to the nearest tenth if necessary.

7.



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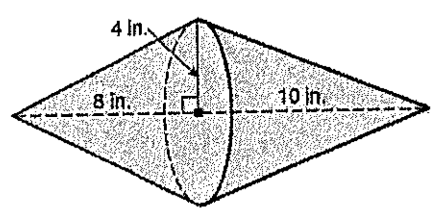
8.



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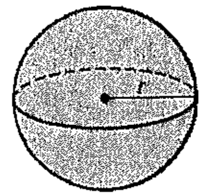
Name ______________________ Date ______________ Class ______________

LESSON 10-8

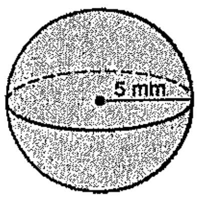
Reteach

Spheres

Volume and Surface Area of a Sphere		
Volume	The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.	
Surface Area	The surface area of a sphere with radius r is $S = 4\pi r^2$.	

Find each measurement. Give your answer in terms of π.

1. the volume of the sphere



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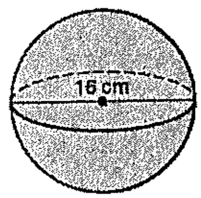
2. the volume of the sphere



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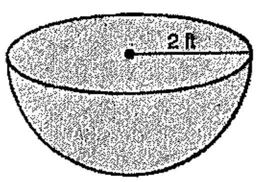
3. the volume of the hemisphere



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4. the radius of a sphere with volume 7776π in^3

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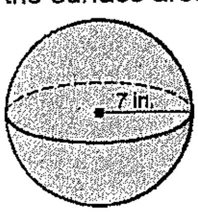
5. the surface area of the sphere



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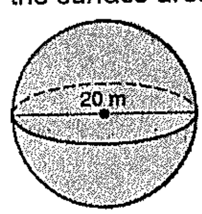
6. the surface area of the sphere



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Name ______________________ Date ______________ Class ______________

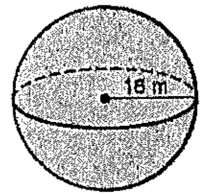
LESSON 10-8

Reteach

Spheres *continued*

The radius of the sphere is multiplied by $\frac{1}{4}$.

Describe the effect on the surface area.



original surface area:

$S = 4\pi r^2$

$= 4\pi(16)^2 \quad r = 16$

$= 1024\pi \text{ m}^2$ Simplify

new surface area, radius multiplied by $\frac{1}{4}$:

$S = 4\pi r^2$

$= 4\pi(4)^2 \quad r = 4$

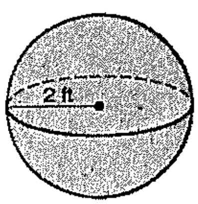
$= 64\pi \text{ m}^2$ Simplify.

Notice that $1024 \cdot \frac{1}{16} = 64$. If the dimensions are multiplied by $\frac{1}{4}$, the surface area is multiplied by $\left(\frac{1}{4}\right)^2$, or $\frac{1}{16}$.

Describe the effect of each change on the given measurement of the figure.

7. surface area

   The radius is multiplied by 4.



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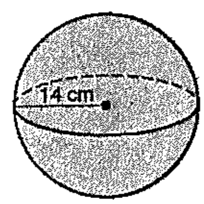
8. volume

   The dimensions are multiplied by $\frac{1}{2}$.



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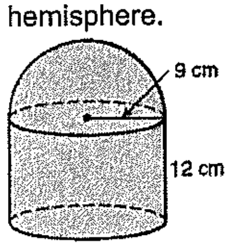
Find the surface area and volume of each composite figure. Round to the nearest tenth.

9. *Hint:* To find the surface area, add the lateral area of the cylinder, the area of one base, and the surface area of the hemisphere.



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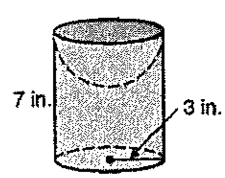
10. *Hint:* To find the volume, subtract the volume of the hemisphere from the volume of the cylinder.



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