



MEA 2013-2014  
Teacher: Claudia Valle  
Start Date:

Course: Geometry A  
Student: \_\_\_\_\_  
Completed Date:

## Unit 5: Properties and Attributes of Triangles

**Objectives:** Students will understand how to identify and apply perpendicular and angle bisectors, medians, and altitudes to find segment lengths in triangles. Students will understand how to use the Triangle Inequality Theorems.

**Essential Questions:** What is the difference between medians and altitudes?

How can you identify which side of a triangle is the longest and which angle in a triangle is the largest?

**TEKS Standards: GG.2.A, G.3.B, G.5.D, G.7.B, G.8.C, G.9.B, G.11.C**

Geometry

(2) Geometric structure. The student analyzes geometric relationships in order to make and verify conjectures. The student is expected to:

(A) use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships; and

(3) Geometric structure. The student applies logical reasoning to justify and prove mathematical statements. The student is expected to:

(B) construct and justify statements about geometric figures and their properties;

(5) Geometric patterns. The student uses a variety of representations to describe geometric relationships and solve problems. The student is expected to:

(D) identify and apply patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.

(7) Dimensionality and the geometry of location. The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly. The student is expected to:

(B) use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons; and

(8) Congruence and the geometry of size. The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations. The student is expected to:

(C) derive, extend, and use the Pythagorean Theorem;

(9) Congruence and the geometry of size. The student analyzes properties and describes relationships in geometric figures. The student is expected to:

(B) formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models;

(11) Similarity and the geometry of shape. The student applies the concepts of similarity to justify properties of figures and solve problems. The student is expected to:

(C) develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods; and

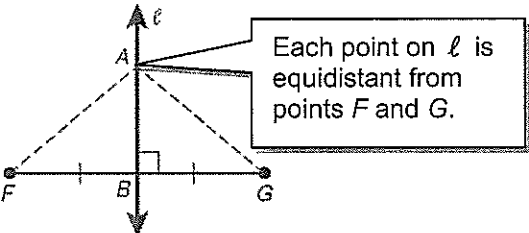
### Turn In:

Assignment #	Activity	TEKS
30	Perpendicular and Angle Bisectors	G.3.B, G.7.B
31	Bisectors of Triangles	G.2.A, G.3.B, G.7.B
32	Medians and Altitudes of Triangles	G.2.A, G.3.B, G.7.B
33	The Triangle Midsegment Theorem	G.2.A, G.3.B, G.7.B, G.9.B
34	Indirect Proof and Inequalities in One Triangle	G.3.B
35	Inequalities in Two Triangles	G.3.B
36	The Pythagorean Theorem	G.5.D, G.8.C, G.11.C
37	Applying Special Right Triangles	G.5.D, G.7.B
38	Unit 5 Test	G.2.A, G.3.B, G.5.D, G.7.B, G.8.C, G.9.B, G.11.C

**LESSON**  
**5-1**

**Reteach**

**Perpendicular and Angle Bisectors**

Theorem	Example
<p><b>Perpendicular Bisector Theorem</b> If a point is on the perpendicular bisector of a segment, then it is <b>equidistant</b>, or the same distance, from the endpoints of the segment.</p>	 <p><b>Given:</b> <math>l</math> is the perpendicular bisector of <math>\overline{FG}</math>. <b>Conclusion:</b> <math>AF = AG</math></p>

The **Converse of the Perpendicular Bisector Theorem** is also true. If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

You can write an equation for the perpendicular bisector of a segment. Consider the segment with endpoints  $Q(-5, 6)$  and  $R(1, 2)$ .

**Step 1** Find the midpoint of  $\overline{QR}$ .

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-5 + 1}{2}, \frac{6 + 2}{2} \right)$$

$$= (-2, 4)$$

**Step 2** Find the slope of the  $\perp$  bisector of  $\overline{QR}$ .

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 6}{1 - (-5)} \text{ Slope of } \overline{QR}$$

$$= -\frac{2}{3}$$

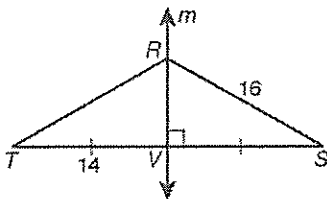
So the slope of the  $\perp$  bisector of  $\overline{QR}$  is  $\frac{3}{2}$ .

**Step 3** Use the point-slope form to write an equation.

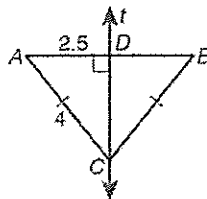
$y - y_1 = m(x - x_1)$  Point-slope form

$y - 4 = \frac{3}{2}(x + 2)$  Slope =  $\frac{3}{2}$ ; line passes through  $(-2, 4)$ , the midpoint of  $\overline{QR}$ .

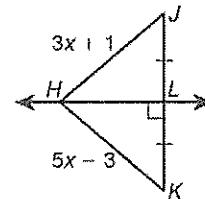
**Find each measure.**



1.  $RT =$  \_\_\_\_\_



2.  $AB =$  \_\_\_\_\_



3.  $HJ =$  \_\_\_\_\_

**Write an equation in point-slope form for the perpendicular bisector of the segment with the given endpoints.**

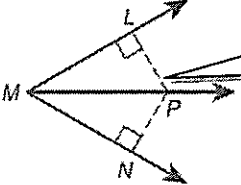
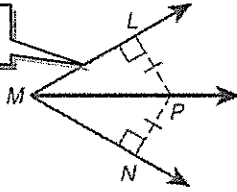
4.  $A(6, -3), B(0, 5)$

5.  $W(2, 7), X(-4, 3)$

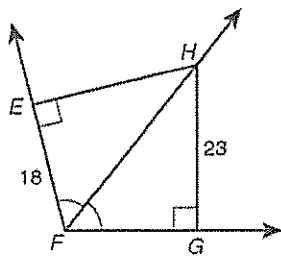
**LESSON**  
**5-1**

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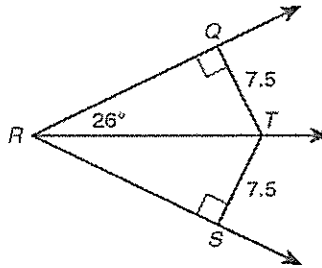
**Perpendicular and Angle Bisectors** *continued*

Theorem	Example
<p><b>Angle Bisector Theorem</b> If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.</p>	 <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;">             Point <math>P</math> is equidistant from sides <math>\overline{ML}</math> and <math>\overline{MN}</math>.         </div> <p><b>Given:</b> <math>\overline{MP}</math> is the angle bisector of <math>\angle LMN</math>. <b>Conclusion:</b> <math>LP = NP</math></p>
<p><b>Converse of the Angle Bisector Theorem</b> If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.</p>	<div style="border: 1px solid black; padding: 5px; display: inline-block; margin-bottom: 10px;"> <math>\angle LMP \cong \angle NMP</math> </div>  <p><b>Given:</b> <math>LP = NP</math> <b>Conclusion:</b> <math>\overline{MP}</math> is the angle bisector of <math>\angle LMN</math>.</p>

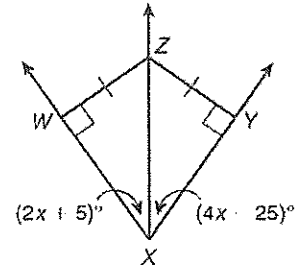
Find each measure.



6.  $EH$



7.  $m\angle QRS$



8.  $m\angle WXZ$

Use the figure for Exercises 9–11.

9. Given that  $\overline{JL}$  bisects  $\angle HJK$  and  $LK = 11.4$ , find  $LH$ .

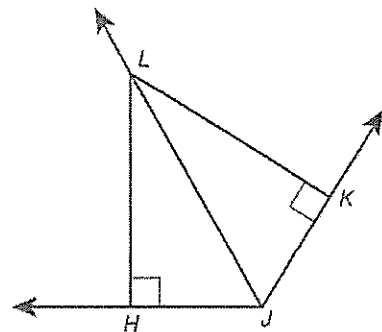
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10. Given that  $LH = 26$ ,  $LK = 26$ , and  $m\angle HJK = 122^\circ$ , find  $m\angle LJK$ .

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11. Given that  $LH = LK$ ,  $m\angle HJL = (3y + 19)^\circ$ , and  $m\angle LJK = (4y + 5)^\circ$ , find the value of  $y$ .

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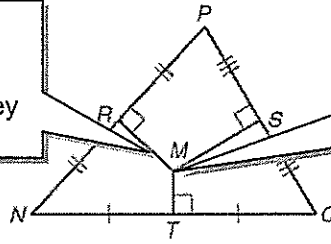


**LESSON**  
**5-2**

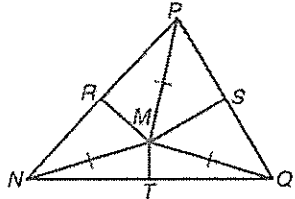
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## Bisectors of Triangles

Perpendicular bisectors  $\overline{MR}$ ,  $\overline{MS}$ , and  $\overline{MT}$  are **concurrent** because they intersect at one point.



The point of intersection of  $\overline{MR}$ ,  $\overline{MS}$ , and  $\overline{MT}$  is called the **circumcenter** of  $\triangle NPQ$ .

Theorem	Example
<p><b>Circumcenter Theorem</b> The circumcenter of a triangle is equidistant from the vertices of the triangle.</p>	<p><b>Given:</b> <math>\overline{MR}</math>, <math>\overline{MS}</math>, and <math>\overline{MT}</math> are the perpendicular bisectors of <math>\triangle NPQ</math>.</p> <p><b>Conclusion:</b> <math>MN = MP = MQ</math></p> 

If a triangle on a coordinate plane has two sides that lie along the axes, you can easily find the circumcenter. Find the equations for the perpendicular bisectors of those two sides. The intersection of their graphs is the circumcenter.

$\overline{HD}$ ,  $\overline{JD}$ , and  $\overline{KD}$  are the perpendicular bisectors of  $\triangle EFG$ . Find each length.

1.  $DG$

2.  $EK$

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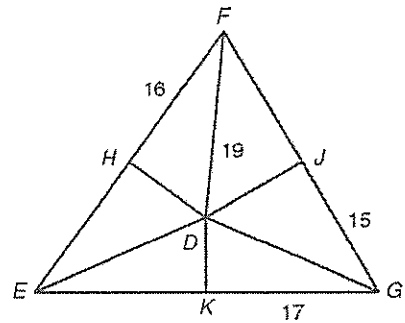
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3.  $FJ$

4.  $DE$

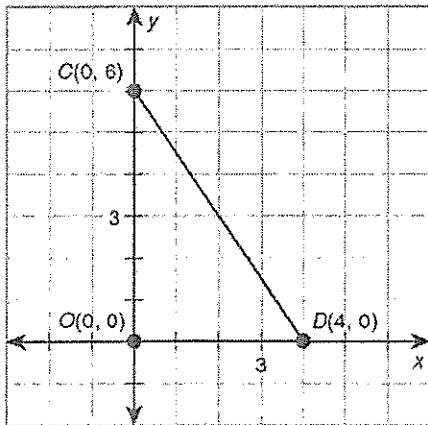
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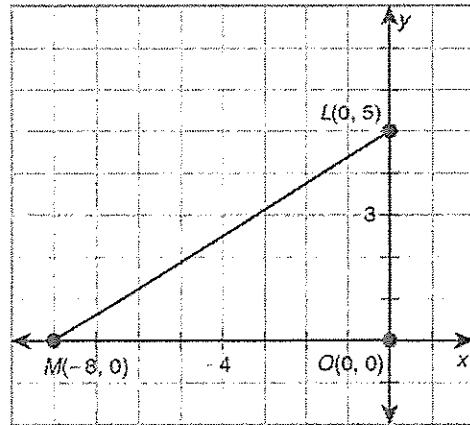


Find the circumcenter of each triangle.

5.



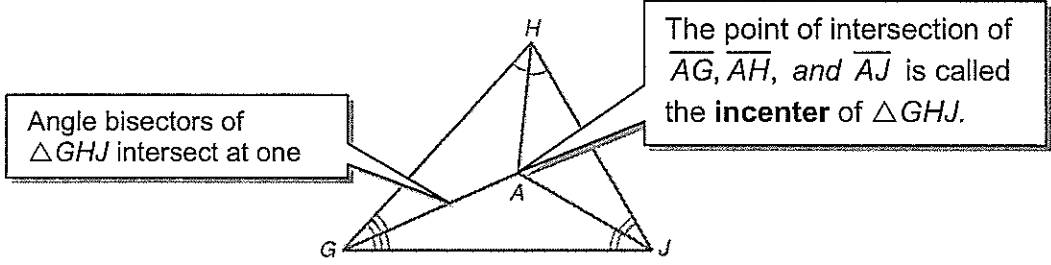
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**LESSON**  
**5-2**

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**Bisectors of Triangles** *continued*

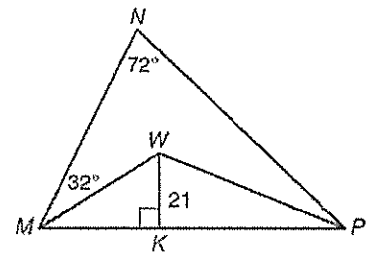


Theorem	Example
<p><b>Incenter Theorem</b> The incenter of a triangle is equidistant from the sides of the triangle.</p>	<p><b>Given:</b> <math>\overline{AG}</math>, <math>\overline{AH}</math>, and <math>\overline{AJ}</math> are the angle bisectors of <math>\triangle GHJ</math>.</p> <p><b>Conclusion:</b> <math>AB = AC = AD</math></p>

$\overline{WM}$  and  $\overline{WP}$  are angle bisectors of  $\triangle MNP$ , and  $WK = 21$ .

Find  $m\angle WPN$  and the distance from  $W$  to  $\overline{MN}$  and  $\overline{NP}$ .

- $m\angle NMP = 2m\angle NMW$  Def. of  $\angle$  bisector
- $m\angle NMP = 2(32^\circ) = 64^\circ$  Substitute.
- $m\angle NMP + m\angle N + m\angle NPM = 180^\circ$   $\triangle$  Sum Thm.
- $64^\circ + 72^\circ + m\angle NPM = 180^\circ$  Substitute.
- $m\angle NPM = 44^\circ$  Subtract  $136^\circ$  from each side.
- $m\angle WPN = \frac{1}{2}m\angle NPM$  Def. of  $\angle$  bisector
- $m\angle WPN = \frac{1}{2}(44^\circ) = 22^\circ$  Substitute.



The distance from  $W$  to  $\overline{MN}$  and  $\overline{NP}$  is 21 by the Incenter Theorem.

$\overline{PC}$  and  $\overline{PD}$  are angle bisectors of  $\triangle CDE$ . Find each measure.

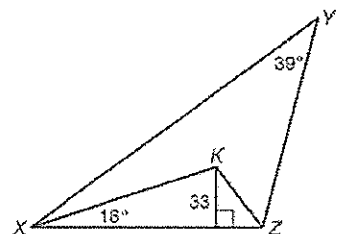
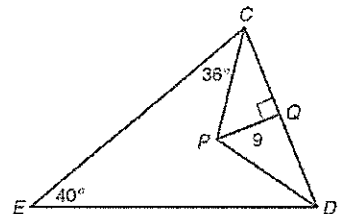
7. the distance from  $P$  to  $\overline{CE}$       8.  $m\angle PDE$

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$\overline{KX}$  and  $\overline{KZ}$  are angle bisectors of  $\triangle XYZ$ . Find each measure.

9. the distance from  $K$  to  $\overline{YZ}$       10.  $m\angle KZY$

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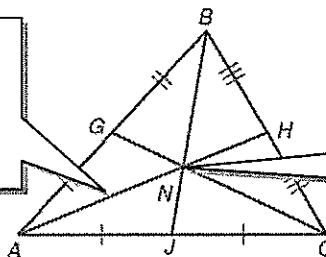


**LESSON**  
**5-3**

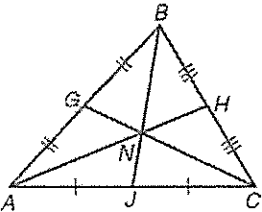
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## Medians and Altitudes of Triangles

$\overline{AH}$ ,  $\overline{BJ}$ , and  $\overline{CG}$  are medians of a triangle. They each join a vertex and the midpoint of the opposite side.



The point of intersection of the medians is called the **centroid** of  $\triangle ABC$ .

Theorem	Example
<p><b>Centroid Theorem</b> The centroid of a triangle is located <math>\frac{2}{3}</math> of the distance from each vertex to the midpoint of the opposite side.</p>	 <p><b>Given:</b> <math>\overline{AH}</math>, <math>\overline{CG}</math>, and <math>\overline{BJ}</math> are medians of <math>\triangle ABC</math>. <b>Conclusion:</b> <math>AN = \frac{2}{3}AH</math>, <math>CN = \frac{2}{3}CG</math>, <math>BN = \frac{2}{3}BJ</math></p>

In  $\triangle ABC$  above, suppose  $AH = 18$  and  $BN = 10$ . You can use the Centroid Theorem to find  $AN$  and  $BJ$ .

$$AN = \frac{2}{3}AH \quad \text{Centroid Thm.}$$

$$BN = \frac{2}{3}BJ \quad \text{Centroid Thm.}$$

$$AN = \frac{2}{3}(18) \quad \text{Substitute 18 for } AH.$$

$$10 = \frac{2}{3}BJ \quad \text{Substitute 10 for } BN.$$

$$AN = 12 \quad \text{Simplify.}$$

$$15 = BJ \quad \text{Simplify.}$$

In  $\triangle QRS$ ,  $RX = 48$  and  $QW = 30$ . Find each length.

1.  $RW$

2.  $WX$

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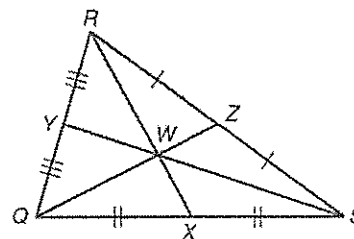
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3.  $QZ$

4.  $WZ$

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In  $\triangle HJK$ ,  $HD = 21$  and  $BK = 18$ . Find each length.

5.  $HB$

6.  $BD$

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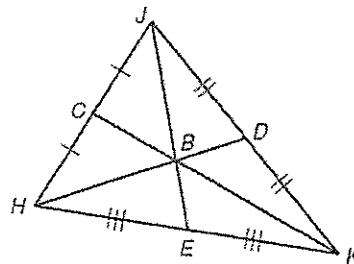
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7.  $CK$

8.  $CB$

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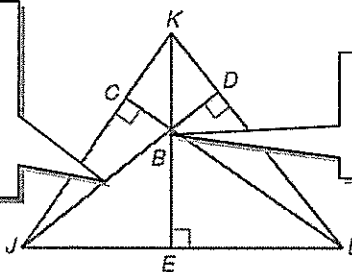


**LESSON**  
**5-3**

**Reteach**

**Medians and Altitudes of Triangles** *continued*

$\overline{JD}$ ,  $\overline{KE}$ , and  $\overline{LC}$  are altitudes of a triangle. They are perpendicular segments that join a vertex and the line containing the side opposite the vertex.



The point of intersection of the altitudes is called the **orthocenter** of  $\triangle JKL$ .

Find the orthocenter of  $\triangle ABC$  with vertices  $A(-3, 3)$ ,  $B(3, 7)$ , and  $C(3, 0)$ .

**Step 1** Graph the triangle.

**Step 2** Find equations of the lines containing two altitudes.

The altitude from  $A$  to  $\overline{BC}$  is the horizontal line  $y = 3$ .

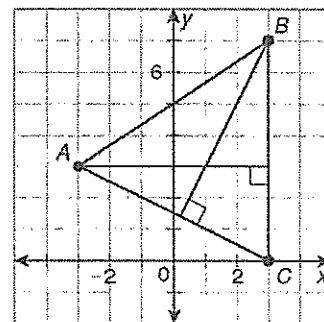
The slope of  $\overline{AC} = \frac{0 - 3}{3 - (-3)} = -\frac{1}{2}$ , so the slope of the altitude

from  $B$  to  $\overline{AC}$  is 2. The altitude must pass through  $B(3, 7)$ .

$y - y_1 = m(x - x_1)$  Point-slope form

$y - 7 = 2(x - 3)$  Substitute 2 for  $m$  and the coordinates of  $B(3, 7)$  for  $(x_1, y_1)$ .

$y = 2x + 1$  Simplify.



**Step 3** Solving the system of equations  $y = 3$  and  $y = 2x + 1$ , you find that the coordinates of the orthocenter are  $(1, 3)$ .

Triangle  $FGH$  has coordinates  $F(-3, 1)$ ,  $G(2, 6)$ , and  $H(4, 1)$ .

9. Find an equation of the line containing the altitude from  $G$  to  $\overline{FH}$ .

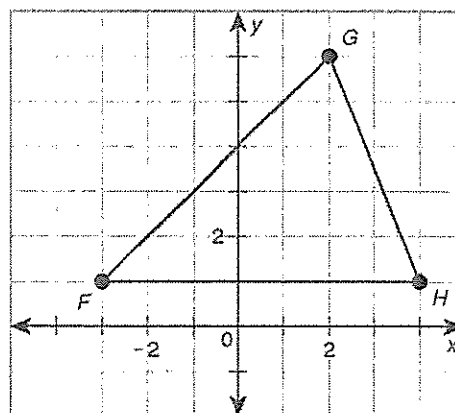
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10. Find an equation of the line containing the altitude from  $H$  to  $\overline{FG}$ .

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11. Solve the system of equations from Exercises 9 and 10 to find the coordinates of the orthocenter.

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Find the orthocenter of the triangle with the given vertices.

12.  $N(-1, 0)$ ,  $P(1, 8)$ ,  $Q(5, 0)$

13.  $R(-1, 4)$ ,  $S(5, -2)$ ,  $T(-1, -6)$

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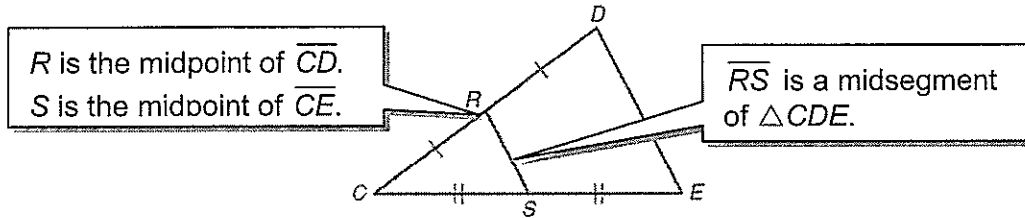
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**LESSON**  
**5-4**

**Reteach**  
**The Triangle Midsegment Theorem**

A **midsegment** of a triangle joins the midpoints of two sides of the triangle. Every triangle has three midsegments.



Use the figure for Exercises 1–4.  $\overline{AB}$  is a midsegment of  $\triangle RST$ .

1. What is the slope of midsegment  $\overline{AB}$  and the slope of side  $\overline{ST}$ ?

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2. What can you conclude about  $\overline{AB}$  and  $\overline{ST}$ ?

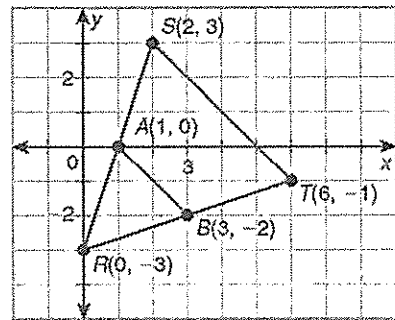
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3. Find  $AB$  and  $ST$ .

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4. Compare the lengths of  $\overline{AB}$  and  $\overline{ST}$ .

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Use  $\triangle MNP$  for Exercises 5–7.

5.  $\overline{UV}$  is a midsegment of  $\triangle MNP$ . Find the coordinates of  $U$  and  $V$ .

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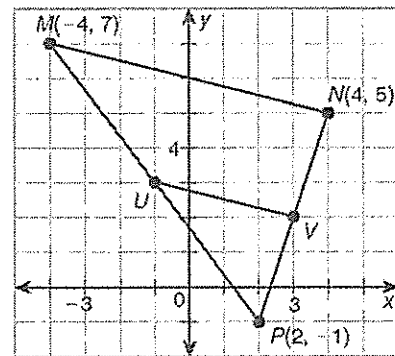
6. Show that  $\overline{UV} \parallel \overline{MN}$ .

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7. Show that  $UV = \frac{1}{2}MN$ .

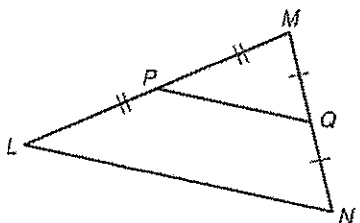
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**LESSON**  
**5-4**

**Reteach**

**The Triangle Midsegment Theorem** *continued*

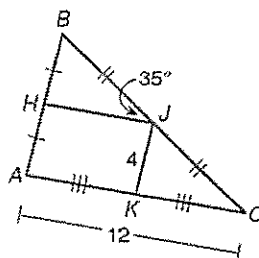
Theorem	Example
<p><b>Triangle Midsegment Theorem</b> A midsegment of a triangle is parallel to a side of the triangle, and its length is half the length of that side.</p>	 <p><b>Given:</b> <math>\overline{PQ}</math> is a midsegment of <math>\triangle LMN</math>. <b>Conclusion:</b> <math>\overline{PQ} \parallel \overline{LN}</math>, <math>PQ = \frac{1}{2}LN</math></p>

You can use the Triangle Midsegment Theorem to find various measures in  $\triangle ABC$ .

$HJ = \frac{1}{2}AC$   $\triangle$  Midsegment Thm.

$HJ = \frac{1}{2}(12)$  Substitute 12 for AC.

$HJ = 6$  Simplify.



$JK = \frac{1}{2}AB$   $\triangle$  Midsegment Thm.

$4 = \frac{1}{2}AB$  Substitute 4 for JK.

$8 = AB$  Simplify.

$\overline{HJ} \parallel \overline{AC}$

Midsegment Thm.

$m\angle BCA = m\angle BJH$

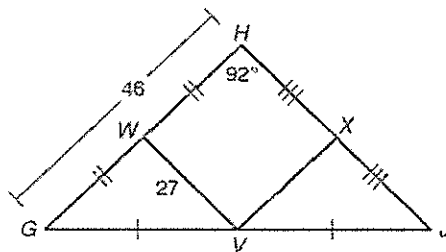
Corr.  $\angle$  Thm.

$m\angle BCA = 35^\circ$

Substitute  $35^\circ$  for  $m\angle BJH$ .

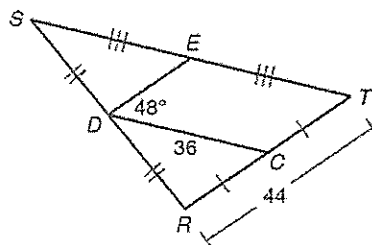
Find each measure.

8.  $VX =$  \_\_\_\_\_
9.  $HJ =$  \_\_\_\_\_
10.  $m\angle VXJ =$  \_\_\_\_\_
11.  $XJ =$  \_\_\_\_\_



Find each measure.

12.  $ST =$  \_\_\_\_\_
13.  $DE =$  \_\_\_\_\_
14.  $m\angle DES =$  \_\_\_\_\_
15.  $m\angle RCD =$  \_\_\_\_\_



**LESSON**  
**5-5**

**Reteach**  
**Indirect Proof and Inequalities in One Triangle**

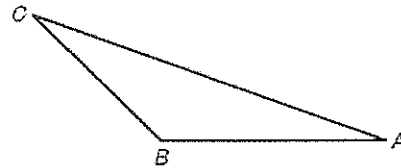
In a direct proof, you begin with a true hypothesis and prove that a conclusion is true. In an **indirect proof**, you begin by assuming that the conclusion is false (that is, that the opposite of the conclusion is true). You then show that this assumption leads to a contradiction.

Consider the statement "Two acute angles do not form a linear pair."

Writing an Indirect Proof	
Steps	Example
1. Identify the conjecture to be proven.	<b>Given:</b> $\angle 1$ and $\angle 2$ are acute angles. <b>Prove:</b> $\angle 1$ and $\angle 2$ do not form a linear pair.
2. Assume the opposite of the conclusion is true.	Assume $\angle 1$ and $\angle 2$ form a linear pair.
3. Use direct reasoning to show that the assumption leads to a contradiction.	$m\angle 1 + m\angle 2 = 180^\circ$ by def. of linear pair. Since $m\angle 1 < 90^\circ$ and $m\angle 2 < 90^\circ$ , $m\angle 1 + m\angle 2 < 180^\circ$ . This is a contradiction.
4. Conclude that the assumption is false and hence that the original conjecture must be true.	The assumption that $\angle 1$ and $\angle 2$ form a linear pair is false. Therefore $\angle 1$ and $\angle 2$ do not form a linear pair.

Use the following statement for Exercises 1–4.

An obtuse triangle cannot have a right angle.



1. Identify the conjecture to be proven.

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2. Assume the opposite of the conclusion. Write this assumption.

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3. Use direct reasoning to arrive at a contradiction.

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4. What can you conclude?

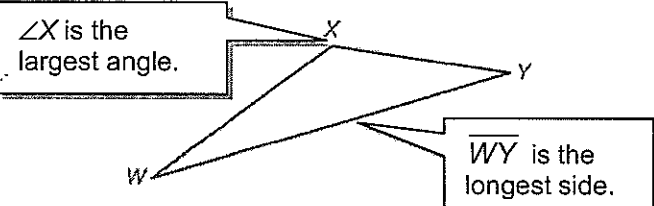
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**LESSON**  
**5-5**

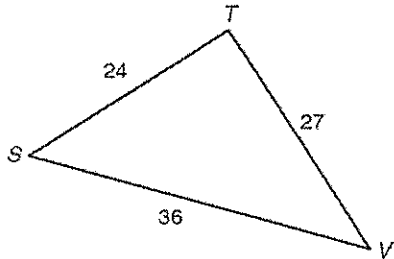
**Reteach**

**Indirect Proof and Inequalities in One Triangle** *continued*

Theorem	Example
<p>If two sides of a triangle are not congruent, then the larger angle is opposite the longer side.</p>	 <p>If <math>WY &gt; XY</math>, then <math>m\angle X &gt; m\angle W</math>.</p>

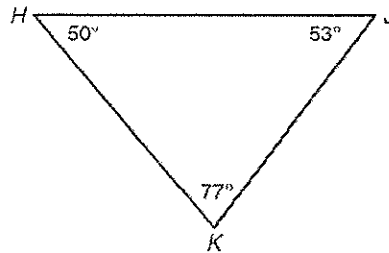
Another similar theorem says that if two angles of a triangle are not congruent, then the longer side is opposite the larger angle.

Write the correct answer.



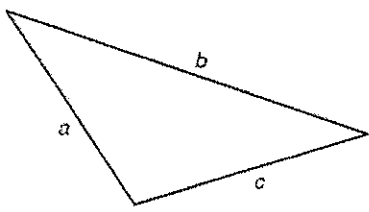
5. Write the angles in order from smallest to largest.

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6. Write the sides in order from shortest to longest.

\_\_\_\_\_

Theorem	Example
<p><b>Triangle Inequality Theorem</b> The sum of any two side lengths of a triangle is greater than the third side length.</p>	 <p> <math>a + b &gt; c</math>  <math>b + c &gt; a</math>  <math>c + a &gt; b</math> </p>

Tell whether a triangle can have sides with the given lengths. Explain.

7. 3, 5, 8

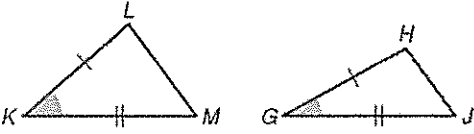
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8. 11, 15, 21

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**LESSON**  
**5-6**

**Reteach**  
**Inequalities in Two Triangles**

Theorem	Example
<p><b>Hinge Theorem</b> If two sides of one triangle are congruent to two sides of another triangle and the included angles are not congruent, then the included angle that is larger has the longer third side across from it.</p>	 <p>If <math>\angle K</math> is larger than <math>\angle G</math>, then side <math>\overline{LM}</math> is longer than side <math>\overline{HJ}</math>.</p>

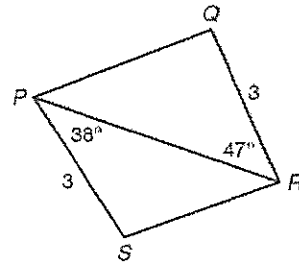
The **Converse of the Hinge Theorem** is also true. In the example above, if side  $\overline{LM}$  is longer than side  $\overline{HJ}$ , then you can conclude that  $\angle K$  is larger than  $\angle G$ . You can use both of these theorems to compare various measures of triangles.

Compare  $NR$  and  $PQ$  in the figure at right.

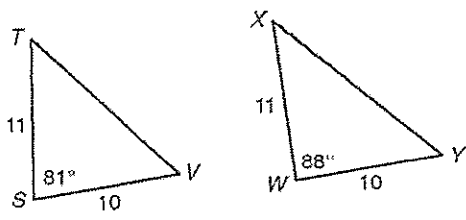
$PN = QR$      $PR = PR$      $m\angle NPR < m\angle QRP$

Since two sides are congruent and  $\angle NPR$  is smaller than  $\angle QRP$ , the side across from it is shorter than the side across from  $\angle QRP$ .

So  $NR < PQ$  by the Hinge Theorem.

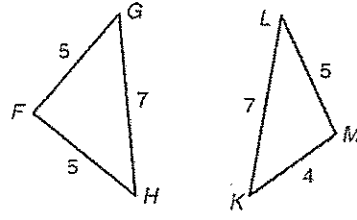


Compare the given measures.



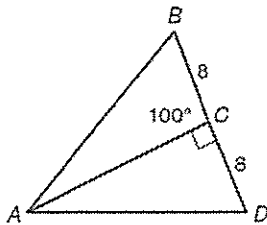
1.  $TV$  and  $XY$

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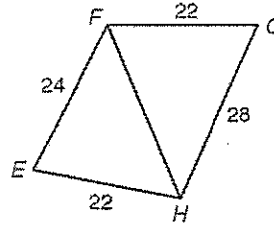
2.  $m\angle G$  and  $m\angle L$

\_\_\_\_\_



3.  $AB$  and  $AD$

\_\_\_\_\_



4.  $m\angle FHE$  and  $m\angle HFG$

\_\_\_\_\_

**LESSON**  
**5-6**

**Reteach**

**Inequalities in Two Triangles** *continued*

You can use the Hinge Theorem and its converse to find a range of values in triangles.

Use  $\triangle MNP$  and  $\triangle QRS$  to find the range of values for  $x$ .

**Step 1** Compare the side lengths in the triangles.

$$NM = SR \quad NP = SQ \quad m\angle N < m\angle S$$

Since two sides of  $\triangle MNP$  are congruent to two sides of  $\triangle QRS$  and  $m\angle N < m\angle S$ , then  $MP < QR$  by the Hinge Theorem.

$MP < QR$	Hinge Thm.
$3x - 6 < 24$	Substitute the given values.
$3x < 30$	Add 6 to each side.
$x < 10$	Divide each side by 3.

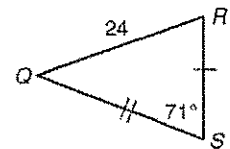
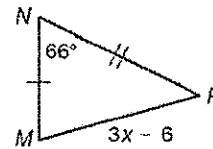
**Step 2** Check that the measures are possible for a triangle.

Since  $\overline{MP}$  is in a triangle, its length must be greater than 0.

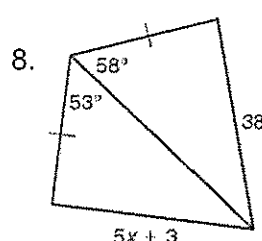
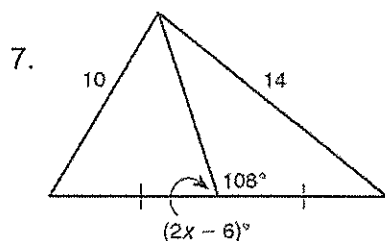
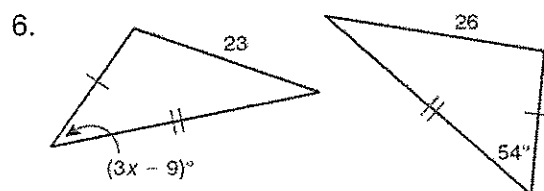
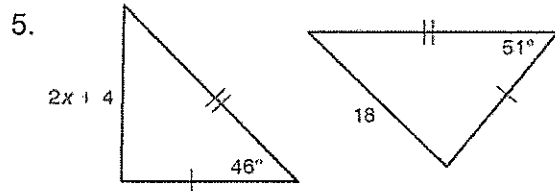
$MP > 0$	Def. of $\triangle$
$3x - 6 > 0$	Substitute $3x - 6$ for $MP$ .
$x > 2$	Simplify.

**Step 3** Combine the inequalities.

A range of values for  $x$  is  $2 < x < 10$ .



Find a range of values for  $x$ .

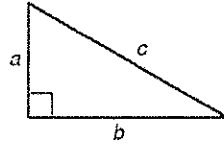


**LESSON**  
**5-7**

**Reteach**

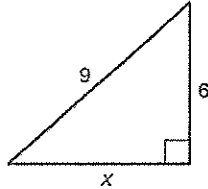
**The Pythagorean Theorem**

The **Pythagorean Theorem** states that the following relationship exists among the lengths of the legs,  $a$  and  $b$ , and the length of the hypotenuse,  $c$ , of any right triangle.



$$a^2 + b^2 = c^2$$

Use the Pythagorean Theorem to find the value of  $x$  in each triangle.



$$a^2 + b^2 = c^2$$

$$x^2 + 6^2 = 9^2$$

$$x^2 + 36 = 81$$

$$x^2 = 45$$

$$x = \sqrt{45}$$

$$x = 3\sqrt{5}$$

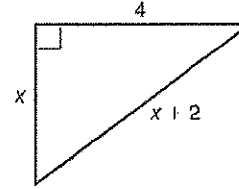
Pythagorean Theorem

Substitute.

Take the squares.

Simplify.

Take the positive square root and simplify.



$$a^2 + b^2 = c^2$$

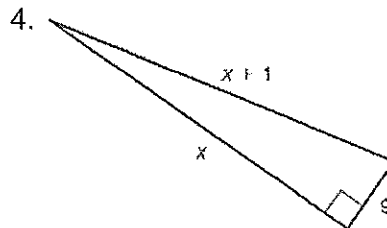
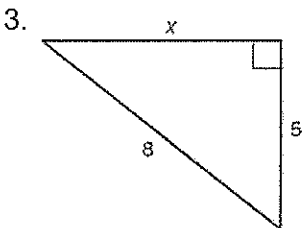
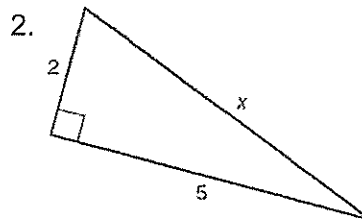
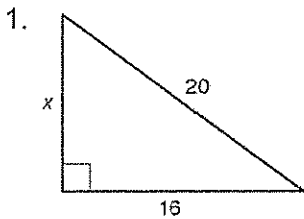
$$x^2 + 4^2 = (x + 2)^2$$

$$x^2 + 16 = x^2 + 4x + 4$$

$$4x = 12$$

$$x = 3$$

Find the value of  $x$ . Give your answer in simplest radical form.



**LESSON**  
**5-7**

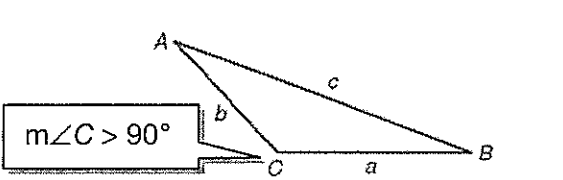
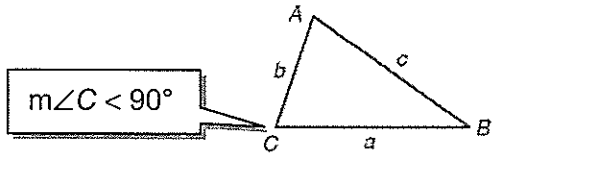
**Reteach**

**The Pythagorean Theorem** *continued*

A **Pythagorean triple** is a set of three nonzero whole numbers  $a$ ,  $b$ , and  $c$  that satisfy the equation  $a^2 + b^2 = c^2$ .

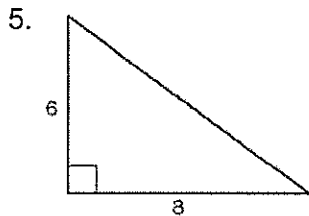
Pythagorean Triples	Not Pythagorean Triples
3, 4, 5, 5, 12, 13	2, 3, 4 6, 9, $\sqrt{117}$

You can use the following theorem to classify triangles by their angles if you know their side lengths. Always use the length of the longest side for  $c$ .

Pythagorean Inequalities Theorem	
 <p>If <math>c^2 &gt; a^2 + b^2</math>, then <math>\triangle ABC</math> is obtuse.</p>	 <p>If <math>c^2 &lt; a^2 + b^2</math>, then <math>\triangle ABC</math> is acute.</p>

Consider the measures 2, 5, and 6. They can be the side lengths of a triangle since  $2 + 5 > 6$ ,  $2 + 6 > 5$ , and  $5 + 6 > 2$ . If you substitute the values into  $c^2 \stackrel{?}{=} a^2 + b^2$ , you get  $36 > 29$ . Since  $c^2 > a^2 + b^2$ , a triangle with side lengths 2, 5, and 6 must be obtuse.

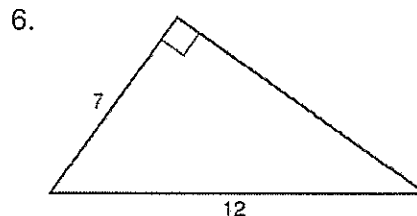
**Find the missing side length. Tell whether the side lengths form a Pythagorean triple. Explain.**



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**Tell whether the measures can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.**

7. 4, 7, 9

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8. 10, 13, 16

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9. 8, 8, 11

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10. 9, 12, 15

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11. 5, 14, 20

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12. 4.5, 6, 10.2

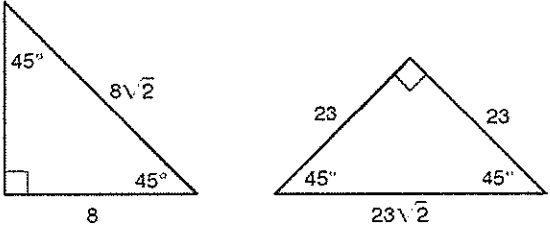
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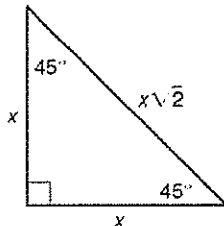
**LESSON**  
**5-8**

# Reteach

## Applying Special Right Triangles

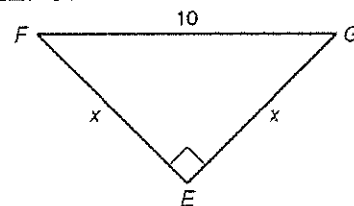
Theorem	Example
<p><b>45°-45°-90° Triangle Theorem</b> In a 45°-45°-90° triangle, both legs are congruent and the length of the hypotenuse is <math>\sqrt{2}</math> times the length of a leg.</p>	

In a 45°-45°-90° triangle, if a leg length is  $x$ , then the hypotenuse length is  $x\sqrt{2}$ .



Use the 45°-45°-90° Triangle Theorem to find the value of  $x$  in  $\triangle EFG$ .

Every isosceles right triangle is a 45°-45°-90° triangle. Triangle  $EFG$  is a 45°-45°-90° triangle with a hypotenuse of length 10.



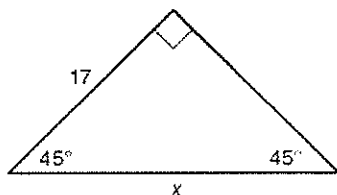
$$10 = x\sqrt{2} \quad \text{Hypotenuse is } \sqrt{2} \text{ times the length of a leg.}$$

$$\frac{10}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}} \quad \text{Divide both sides by } \sqrt{2}.$$

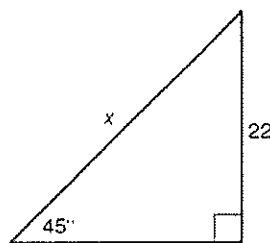
$$5\sqrt{2} = x \quad \text{Rationalize the denominator.}$$

Find the value of  $x$ . Give your answers in simplest radical form.

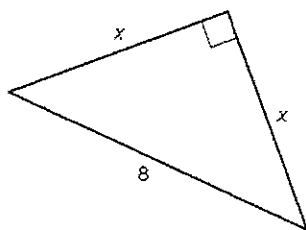
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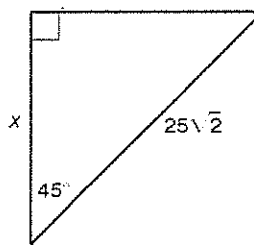
2.



3.



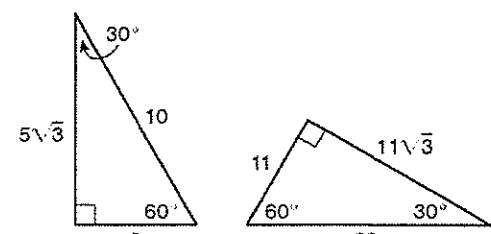
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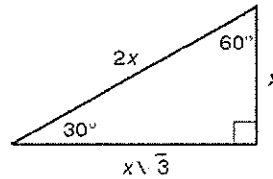
**LESSON**  
**5-8**

**Reteach**

**Applying Special Right Triangles** *continued*

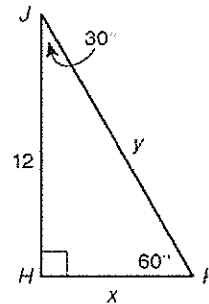
Theorem	Examples
<p><b>30°-60°-90° Triangle Theorem</b> In a 30°-60°-90° triangle, the length of the hypotenuse is 2 multiplied by the length of the shorter leg, and the longer leg is <math>\sqrt{3}</math> multiplied by the length of the shorter leg.</p>	

In a 30°-60°-90° triangle, if the shorter leg length is  $x$ , then the hypotenuse length is  $2x$  and the longer leg length is  $x\sqrt{3}$ .



Use the 30°-60°-90° Triangle Theorem to find the values of  $x$  and  $y$  in  $\triangle HJK$ .

- $12 = x\sqrt{3}$  Longer leg = shorter leg multiplied by  $\sqrt{3}$ .
- $\frac{12}{\sqrt{3}} = x$  Divide both sides by  $\sqrt{3}$ .
- $4\sqrt{3} = x$  Rationalize the denominator.
- $y = 2x$  Hypotenuse = 2 multiplied by shorter leg.
- $y = 2(4\sqrt{3})$  Substitute  $4\sqrt{3}$  for  $x$ .
- $y = 8\sqrt{3}$  Simplify.



Find the values of  $x$  and  $y$ . Give your answers in simplest radical form.

