MEA 2013-2014
Teacher: Claudia Valle
Start Date:

Course: Geometry A
Student: ________________
Completed Date:

Unit 4: Triangle Congruence

Objectives: Students will understand how to use different postulates and theorems to prove two triangles are congruent. Students will understand how use properties triangles to solve problems.

Essential Questions: How do you decide which theorem or postulate to use to show congruence among triangles? How can you decide if enough information is given to determine if two triangles are congruent?

TEKS Standards: G.1.A, G.2.A, G.2.B, G.7.A, G.9.B, G.10.B

Geometry

(1) Geometric structure. The student understands the structure of, and relationships within, an axiomatic system. The student is expected to:

(A) develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems;

(2) Geometric structure. The student analyzes geometric relationships in order to make and verify conjectures. The student is expected to:

(A) use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships; and

(B) make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

(7) Dimensionality and the geometry of location. The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly. The student is expected to:

(A) use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures;

(9) Congruence and the geometry of size. The student analyzes properties and describes relationships in geometric figures. The student is expected to:

(B) formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models;

(10) Congruence and the geometry of size. The student applies the concept of congruence to justify properties of figures and solve problems. The student is expected to:

(B) justify and apply triangle congruence relationships.

Turn In:

Assignment #	Activity	TEKS
22	Classifying Triangles	G.1.A
23	Angle Relationships in Triangles	G.1.A, G.2.B
24	Congruent Triangles	G.2.B, G.10.B
25	Triangle Congruence: SSS and SAS	G.2.A, G.10.B
26	Triangle Congruence: ASA, AAS, and HL	G.1.A, G.2.A, G.9.B, G.10.B
27	Triangle Congruence: CPCTC	G.1.A, G.7.A, G.10.B
28	Isosceles and Equilateral Triangles	G.2.B, G.10.B
29	Unit 4 Test	G.1.A, G.2.A, G.2.B, G.7.A, G.9.B, G.10.B

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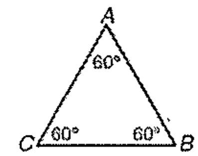
LESSON 4-1

Reteach

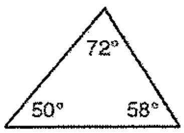
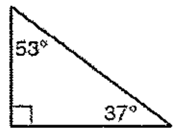
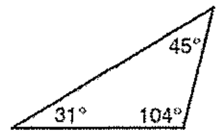
Classifying Triangles

You can classify triangles by their angle measures. An **equiangular triangle,** for example, is a triangle with three congruent angles.

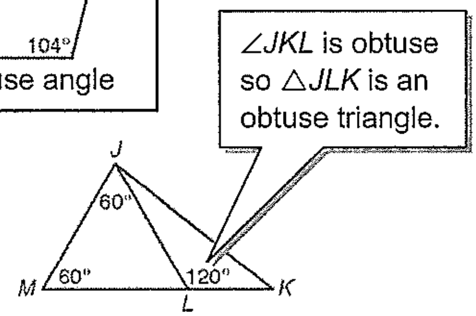
Examples of three other triangle classifications are shown in the table.



$\angle A \cong \angle B \cong \angle C$
$\triangle ABC$ is equiangular.

Acute Triangle	Right Triangle	Obtuse Triangle
 all acute angles	 one right angle	 one obtuse angle

You can use angle measures to classify $\triangle JML$ at right.

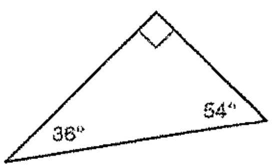


$\angle JLM$ and $\angle JLK$ form a linear pair, so they are supplementary.

$m\angle JLM + m\angle JLK = 180°$	Def. of supp. ∡
$m\angle JLM + 120° = 180°$	Substitution
$m\angle JLM = 60°$	Subtract.

Since all the angles in $\triangle JLM$ are congruent, $\triangle JLM$ is an equiangular triangle.

Classify each triangle by its angle measures.

1. 

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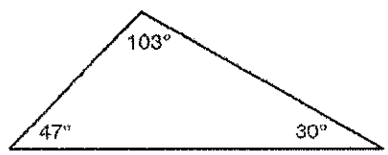
2. 

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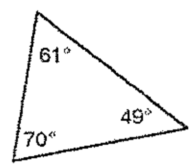
3. 

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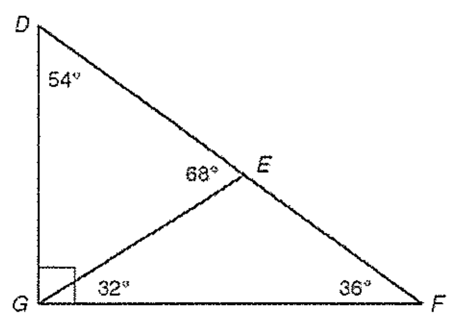
Use the figure to classify each triangle by its angle measures.



4. $\triangle DFG$

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5. $\triangle DEG$

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6. $\triangle EFG$

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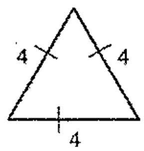
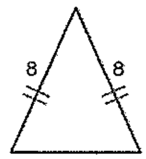
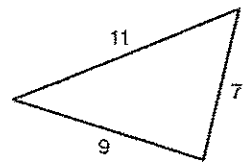
LESSON 4-1

Reteach

Classifying Triangles continued

You can also classify triangles by their side lengths.

Equilateral Triangle	Isosceles Triangle	Scalene Triangle
 all sides congruent	 at least two sides congruent	 no sides congruent

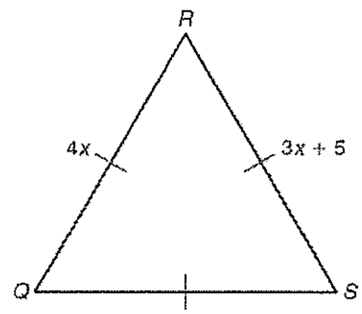
You can use triangle classification to find the side lengths of a triangle.

Step 1 Find the value of x.

$QR = RS$	Def. of $\cong$ segs.
$4x = 3x + 5$	Substitution
$x = 5$	Simplify.

Step 2 Use substitution to find the length of a side.

$4x = 4(5)$	Substitute 5 for x.
$= 20$	Simplify.



Each side length of $\triangle QRS$ is 20.

Classify each triangle by its side lengths.

7. $\triangle EGF$

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8. $\triangle DEF$

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9. $\triangle DFG$

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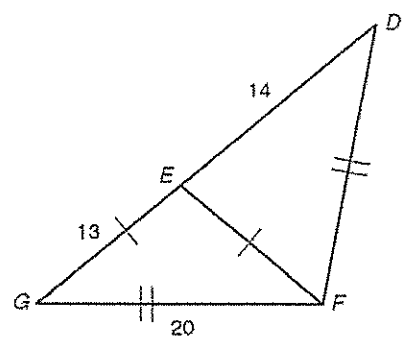
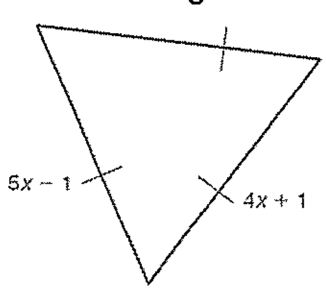


Find the side lengths of each triangle.

10.



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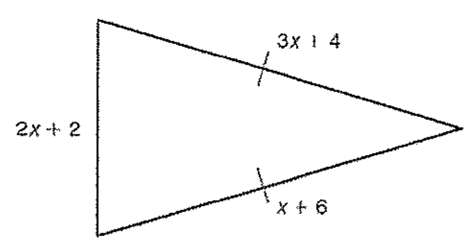
11.



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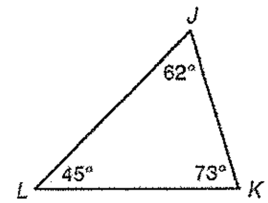
Name ______________________________ Date ______________ Class ______________

LESSON 4-2

Reteach

Angle Relationships in Triangles

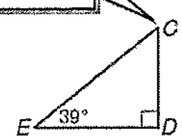


According to the **Triangle Sum Theorem,** the sum of the angle measures of a triangle is 180°.

$m\angle J + m\angle K + m\angle L = 62 + 73 + 45$

$= 180°$

The **corollary** below follows directly from the Triangle Sum Theorem.

Corollary	Example
The acute angles of a right triangle are complementary.	$m\angle C = 90 - 39$ $= 51°$  $m\angle C + m\angle E = 90°$

Use the figure for Exercises 1 and 2.

1. Find m∠*ABC*.

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2. Find m∠*CAD*.

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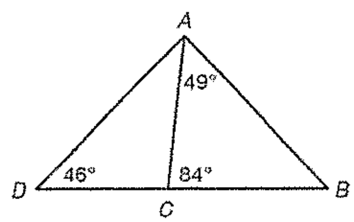

Use △*RST* for Exercises 3 and 4.

3. What is the value of *x*?

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4. What is the measure of each angle?

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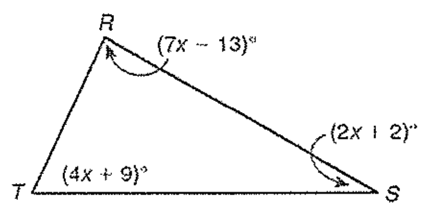
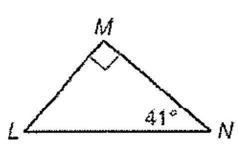
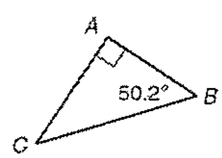
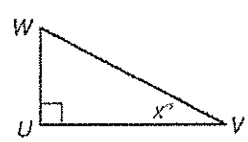


What is the measure of each angle?







5. ∠*L* _______________

6. ∠*C* _______________

7. ∠*W* _______________

Name ______________________ Date ______________ Class ______________

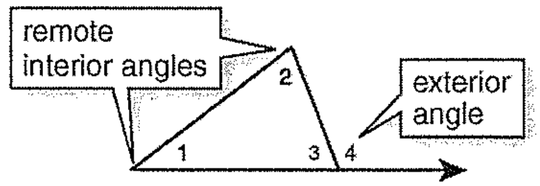
LESSON 4-2

Reteach

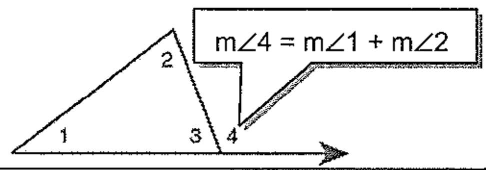
Angle Relationships in Triangles continued

An **exterior angle** of a triangle is formed by one side of the triangle and the extension of an adjacent side.

∠1 and ∠2 are the remote interior angles of ∠4 because they are not adjacent to ∠4.



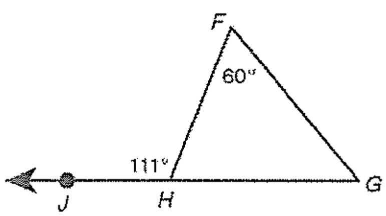
Exterior Angle Theorem
The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.



Third Angles Theorem
If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.

∠1 ≅ ∠4

Find each angle measure.



8. m∠*G*

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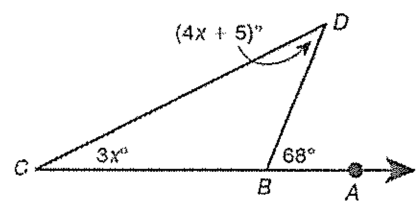

9. m∠*D*

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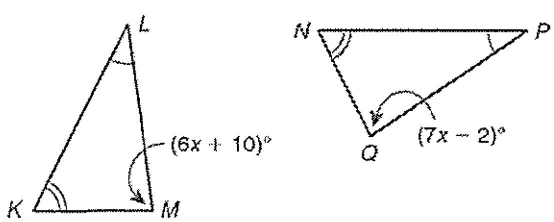
Find each angle measure.



10. m∠*M* and m∠*Q*

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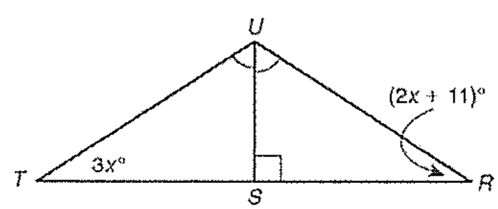

11. m∠*T* and m∠*R*

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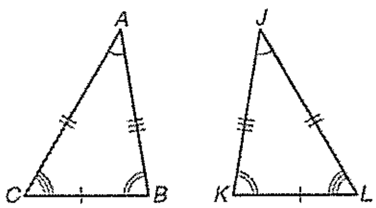
LESSON 4-3

Reteach

Congruent Triangles

Triangles are **congruent** if they have the same size and shape. Their **corresponding parts,** the angles and sides that are in the same positions, are congruent.

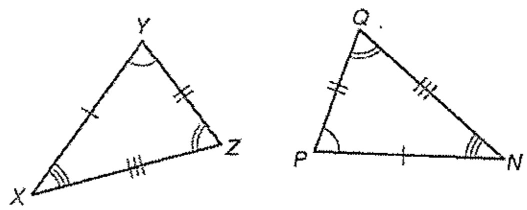


$\triangle ABC \cong \triangle JKL$

Corresponding Parts	
Congruent Angles	**Congruent Sides**
$\angle A \cong \angle J$ $\angle B \cong \angle K$ $\angle C \cong \angle L$	$\overline{AB} \cong \overline{JK}$ $\overline{BC} \cong \overline{KL}$ $\overline{CA} \cong \overline{LJ}$

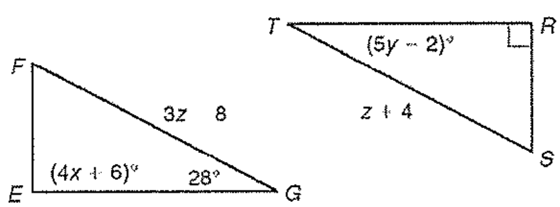
To identify corresponding parts of congruent triangles, look at the order of the vertices in the congruence statement such as $\triangle ABC \cong \triangle JKL$.

Given: $\triangle XYZ \cong \triangle NPQ$. Identify the congruent corresponding parts.



1. $\angle Z \cong$ ______________________
2. $\overline{YZ} \cong$ ______________________
3. $\angle P \cong$ ______________________
4. $\angle X \cong$ ______________________
5. $\overline{NQ} \cong$ ______________________
6. $\overline{PN} \cong$ ______________________

Given: $\triangle EFG \cong \triangle RST$. Find each value below.



7. $x =$ ______________________
8. $y =$ ______________________
9. $m\angle F =$ ______________________
10. $ST =$ ______________________

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LESSON 4-3

Reteach

Congruent Triangles continued

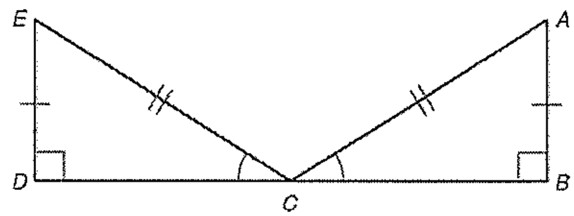
You can prove triangles congruent by using the definition of congruence.

Given: $\angle D$ and $\angle B$ are right angles.

$\angle DCE \cong \angle BCA$

C is the midpoint of $\overline{DB}$.

$\overline{ED} \cong \overline{AB}$, $\overline{EC} \cong \overline{AC}$



Prove: $\triangle EDC \cong \triangle ABC$

Proof:

Statements	Reasons
1. $\angle D$ and $\angle B$ are rt. $\angle$s.	1. Given
2. $\angle D \cong \angle B$	2. Rt. $\angle \cong$ Thm.
3. $\angle DCE \cong \angle BCA$	3. Given
4. $\angle E \cong \angle A$	4. Third $\angle$s Thm.
5. C is the midpoint of $\overline{DB}$.	5. Given
6. $\overline{DC} \cong \overline{BC}$	6. Def. of mdpt.
7. $\overline{ED} \cong \overline{AB}$, $\overline{EC} \cong \overline{AC}$	7. Given
8. $\triangle EDC \cong \triangle ABC$	8. Def. of $\cong \triangle$s

11. Complete the proof.

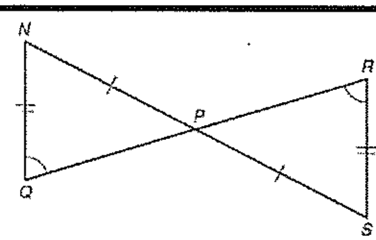
Given: $\angle Q \cong \angle R$

P is the midpoint of $\overline{QR}$.

$\overline{NQ} \cong \overline{SR}$, $\overline{NP} \cong \overline{SP}$

Prove: $\triangle NPQ \cong \triangle SPR$

Proof:



Statements	Reasons
1. $\angle Q \cong \angle$R	1. Given
2. $\angle NPQ \cong \angle SPR$	2. a. ______________________
3. $\angle N \cong \angle S$	3. b. ______________________
4. P is the midpoint of $\overline{QR}$.	4. c. ______________________
5. d. ______________________	5. Def. of mdpt.
6. $\overline{NQ} \cong \overline{SR}$, $\overline{NP} \cong \overline{SP}$	6. e. ______________________
7. $\triangle NPQ \cong \triangle SPR$	7. f. ______________________

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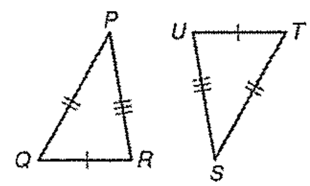
LESSON 4-4

Reteach

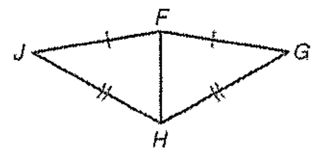
Triangle Congruence: SSS and SAS

Side-Side-Side (SSS) Congruence Postulate

If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.
$\overline{QR} \cong \overline{TU}$, $\overline{RP} \cong \overline{US}$, and $\overline{PQ} \cong \overline{ST}$, so $\triangle PQR \cong \triangle STU$.



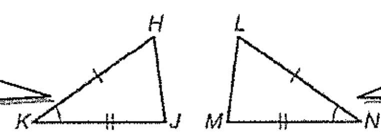
You can use SSS to explain why $\triangle FJH \cong \triangle FGH$.
It is given that $\overline{FJ} \cong \overline{FG}$ and that $\overline{JH} \cong \overline{GH}$. By the Reflex. Prop. of $\cong$, $\overline{FH} \cong \overline{FH}$. So $\triangle FJH \cong \triangle FGH$ by SSS.



Side-Angle-Side (SAS) Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

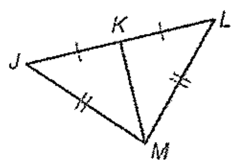
$\angle K$ is the included angle of $\overline{HK}$ and $\overline{KJ}$.



$\triangle HJK \cong \triangle LMN$

$\angle N$ is the included angle of $\overline{LN}$ and $\overline{NM}$.

Use SSS to explain why the triangles in each pair are congruent.



1. $\triangle JKM \cong \triangle LKM$

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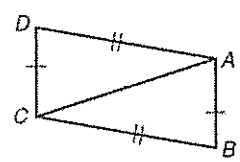

2. $\triangle ABC \cong \triangle CDA$

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3. Use SAS to explain why $\triangle WXY \cong \triangle WZY$.

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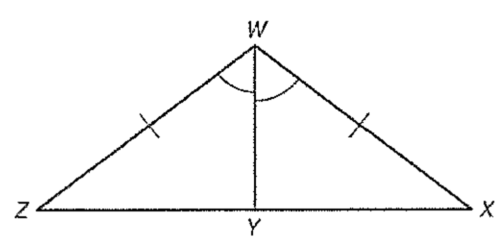


Name ______________________ Date ______________ Class ______________

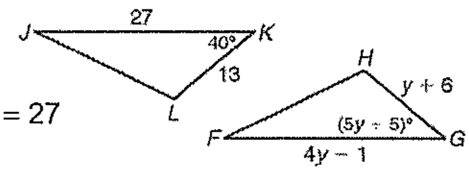
LESSON 4-4

Reteach

Triangle Congruence: SSS and SAS continued

You can show that two triangles are congruent by using SSS and SAS.

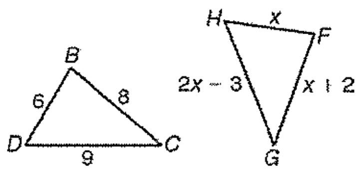
Show that $\triangle JKL \cong \triangle FGH$ for $y = 7$.



$HG = y + 6$ $= 7 + 6 = 13$ $m\angle G = 5y + 5$ $= 5(7) + 5 = 40°$ $FG = 4y - 1$ $= 4(7) - 1 = 27$

$HG = LK = 13$, so $\overline{HG} \cong \overline{LK}$ by def. of ≅ segs. $m\angle G = 40°$, so $\angle G \cong \angle K$ by def. of ≅ ∡. $FG = JK = 27$, so $\overline{FG} \cong \overline{JK}$ by def. of ≅ segs. Therefore $\triangle JKL \cong \triangle FGH$ by SAS.

Show that the triangles are congruent for the given value of the variable.



4. $\triangle BCD \cong \triangle FGH$, $x = 6$

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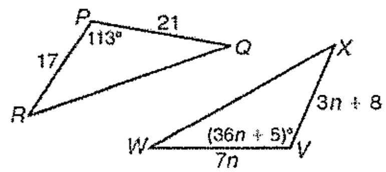

5. $\triangle PQR \cong \triangle VWX$, $n = 3$

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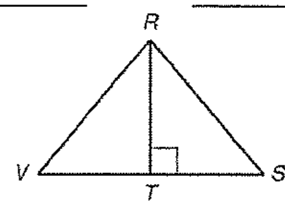
6. Complete the proof.

Given: T is the midpoint of $\overline{VS}$.

$\overline{RT} \perp \overline{VS}$

Prove: $\triangle RST \cong \triangle RVT$



Statements	Reasons
1. T is the midpoint of $\overline{VS}$.	1. Given
2. a. ______________	2. Def. of mdpt.
3. $\overline{RT} \perp \overline{VS}$	3. b. ______________
4. ______________	4. c. ______________
5. d. ______________	5. Rt. ∠ ≅ Thm.
6. $\overline{RT} \cong \overline{RT}$	6. e. ______________
7. $\triangle RST \cong \triangle RVT$	7. f. ______________

Name ______________________ Date ______________ Class ______________

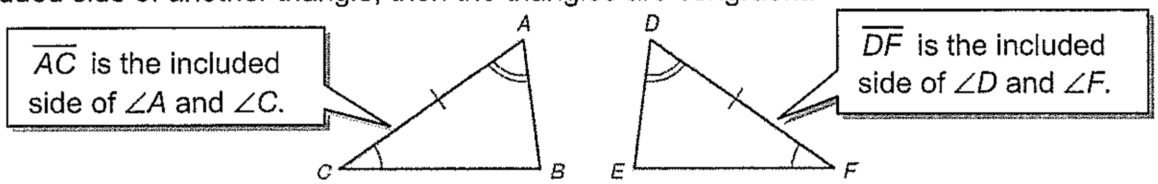
LESSON 4-5

Reteach

Triangle Congruence: ASA, AAS, and HL

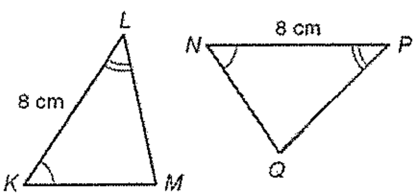
Angle-Side-Angle (ASA) Congruence Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.



$\triangle ABC \cong \triangle DEF$

Determine whether you can use ASA to prove the triangles congruent. Explain.



1. $\triangle KLM$ and $\triangle NPQ$

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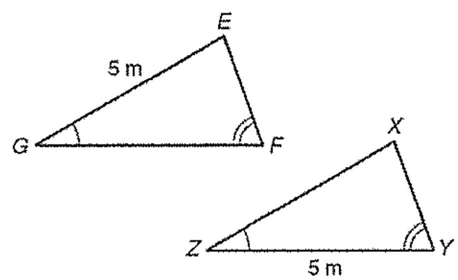

2. $\triangle EFG$ and $\triangle XYZ$

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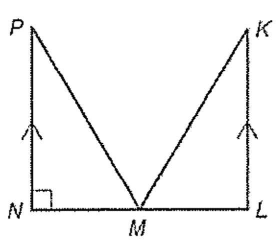

3. $\triangle KLM$ and $\triangle PNM$, given that M is the midpoint of $\overline{NL}$

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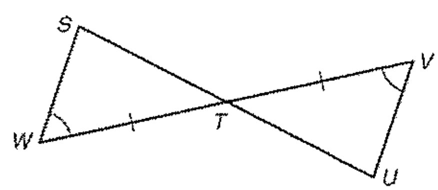

4. $\triangle STW$ and $\triangle UTV$

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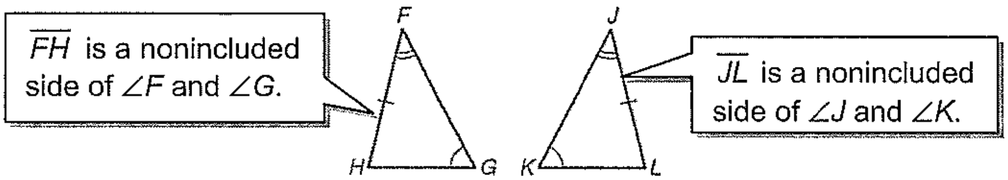
Name ______________________ Date ______________ Class______________

LESSON 4-5

Reteach

Triangle Congruence: ASA, AAS, and HL *continued*

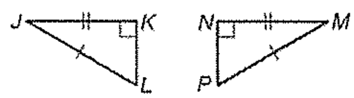
Angle-Angle-Side (AAS) Congruence Theorem
If two angles and a nonincluded side of one triangle are congruent to the corresponding angles and nonincluded side of another triangle, then the triangles are congruent.



$\triangle FGH \cong \triangle JKL$

Special theorems can be used to prove right triangles congruent.

Hypotenuse-Leg (HL) Congruence Theorem
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

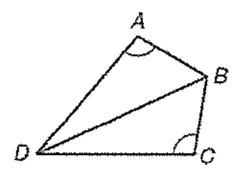


$\triangle JKL \cong \triangle MNP$

5. Describe the corresponding parts and the justifications for using them to prove the triangles congruent by AAS.

   Given: $\overline{BD}$ is the angle bisector of $\angle ADC$.

   Prove: $\triangle ABD \cong \triangle CBD$



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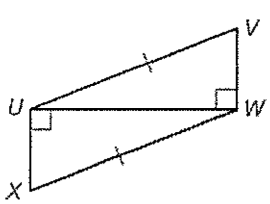
Determine whether you can use the HL Congruence Theorem to prove the triangles congruent. If yes, explain. If not, tell what else you need to know.



6. $\triangle UVW \cong \triangle WXU$

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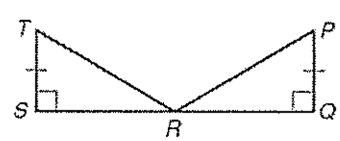

7. $\triangle TSR \cong \triangle PQR$

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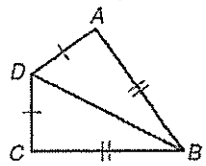
Name ______________________________ Date ______________ Class______________

LESSON 4-6

Reteach

Triangle Congruence: CPCTC

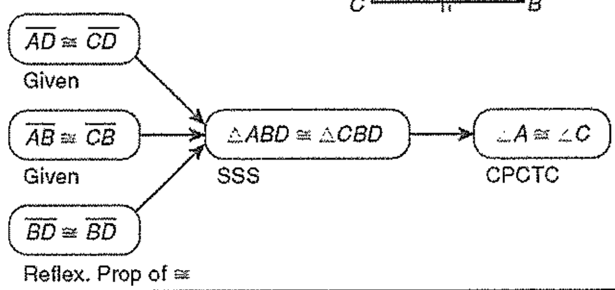
Corresponding Parts of Congruent Triangles are Congruent **(CPCTC)** is useful in proofs. If you prove that two triangles are congruent, then you can use CPCTC as a justification for proving corresponding parts congruent.



Given: $\overline{AD} \cong \overline{CD}, \overline{AB} \cong \overline{CB}$

Prove: $\angle A \cong \angle C$

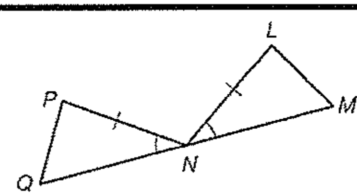
Proof:



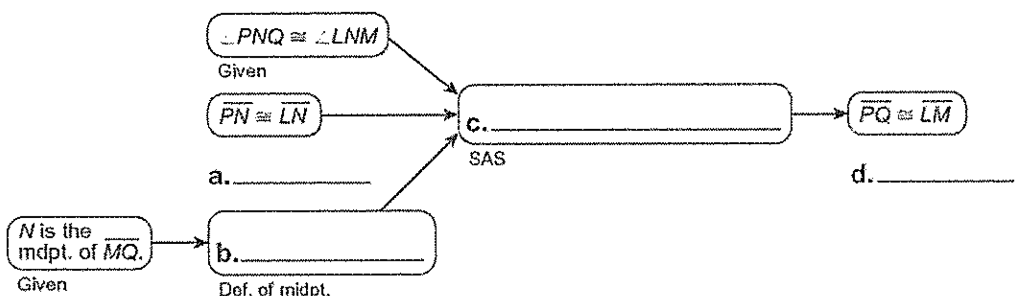
Complete each proof.

1. **Given:** $\angle PNQ \cong \angle LNM$, $\overline{PN} \cong \overline{LN}$,
   N is the midpoint of $\overline{QM}$.

   Prove: $\overline{PQ} \cong \overline{LM}$

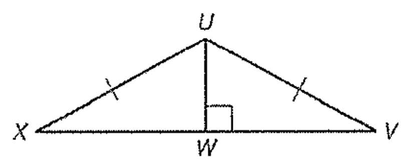


   Proof:



2. **Given:** $\triangle UXW$ and $\triangle UVW$ are right $\triangle$s.
   $\overline{UX} \cong \overline{UV}$

   Prove: $\angle X \cong \angle V$



   Proof:

Statements	Reasons
1. $\triangle UXW$ and $\triangle UVW$ are rt. $\triangle$s.	1. Given
2. $\overline{UX} \cong \overline{UV}$	2. a. ______________________
3. $\overline{UW} \cong \overline{UW}$	3. b. ______________________
4. c. ______________________	4. d. ______________________
5. $\angle X \cong \angle V$	5. e. ______________________

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LESSON 4-6

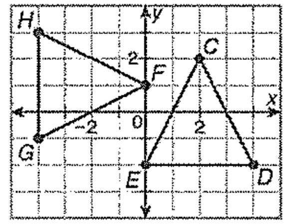
Reteach

Triangle Congruence: CPCTC continued

You can also use CPCTC when triangles are on the coordinate plane.

Given: $C(2, 2)$, $D(4, -2)$, $E(0, -2)$, $F(0, 1)$, $G(-4, -1)$, $H(-4, 3)$

Prove: $\angle CED \cong \angle FHG$



Step 1 Plot the points on a coordinate plane.

Step 2 Find the lengths of the sides of each triangle. Use the Distance Formula if necessary.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$CD = \sqrt{(4-2)^2 + (-2-2)^2}$ $\quad$ $FG = \sqrt{(-4-0)^2 + (-1-1)^2}$

$= \sqrt{4+16} = 2\sqrt{5}$ $\quad$ $= \sqrt{16+4} = 2\sqrt{5}$

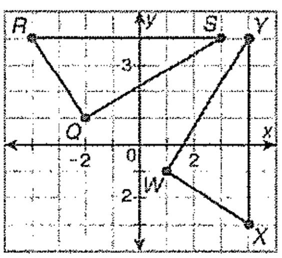
$DE = 4$ $\quad$ $GH = 4$

$EC = \sqrt{(2-0)^2 + [2-(-2)]^2}$ $\quad$ $HF = \sqrt{[0-(-4)]^2 + (1-3)^2}$

$= \sqrt{4+16} = 2\sqrt{5}$ $\quad$ $= \sqrt{16+4} = 2\sqrt{5}$

So, $\overline{CD} \cong \overline{FG}$, $\overline{DE} \cong \overline{GH}$, and $\overline{EC} \cong \overline{HF}$. Therefore $\triangle CDE \cong \triangle FGH$ by SSS, and $\angle CED \cong \angle FHG$ by CPCTC.

Use the graph to prove each congruence statement.



3. $\angle RSQ \cong \angle XYW$

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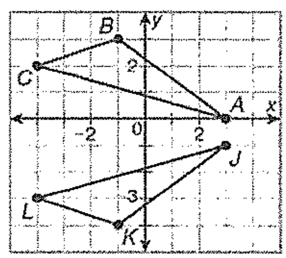

4. $\angle CAB \cong \angle LJK$

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5. Use the given set of points to prove $\angle PMN \cong \angle VTU$.
$M(-2, 4)$, $N(1, -2)$, $P(-3, -4)$, $T(-4, 1)$, $U(2, 4)$, $V(4, 0)$

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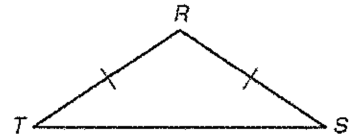
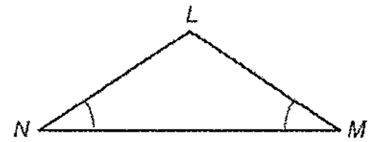
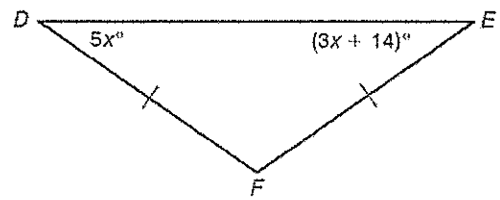
LESSON 4-8

Reteach

Isosceles and Equilateral Triangles

Theorem	Examples
Isosceles Triangle Theorem If two sides of a triangle are congruent, then the angles opposite the sides are congruent.	 If $\overline{RT} \cong \overline{RS}$, then $\angle T \cong \angle S$.
Converse of Isosceles Triangle Theorem If two angles of a triangle are congruent, then the sides opposite those angles are congruent.	 If $\angle N \cong \angle M$, then $\overline{LN} \cong \overline{LM}$.

You can use these theorems to find angle measures in isosceles triangles.

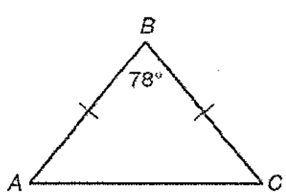


Find m∠*E* in △*DEF*.

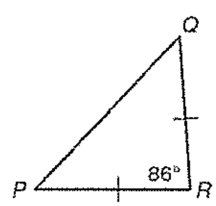
$m\angle D = m\angle E$	Isosc. △ Thm.
$5x° = (3x + 14)°$	Substitute the given values.
$2x = 14$	Subtract 3*x* from both sides.
$x = 7$	Divide both sides by 2.

Thus $m\angle E = 3(7) + 14 = 35°$.

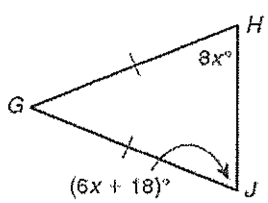
Find each angle measure.



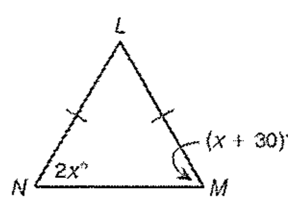
1. m∠*C* = ____________________



2. m∠*Q* = ____________________



3. m∠*H* = ____________________



4. m∠*M* = ____________________

Name ______________________________ Date _______________ Class_______________

LESSON 4-8

Reteach

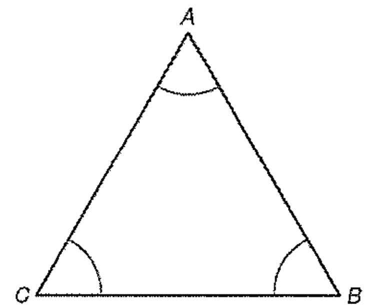
Isosceles and Equilateral Triangles continued

Equilateral Triangle Corollary
If a triangle is equilateral, then it is equiangular.

(equilateral △ → equiangular △)

Equiangular Triangle Corollary
If a triangle is equiangular, then it is equilateral.

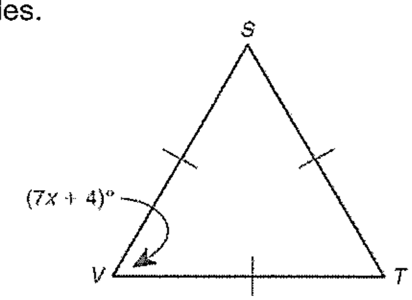
(equiangular △ → equilateral △)



If $\angle A \cong \angle B \cong \angle C$, then $\overline{AB} \cong \overline{BC} \cong \overline{CA}$.

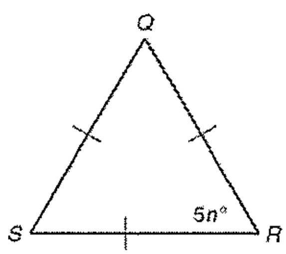
You can use these theorems to find values in equilateral triangles.

Find *x* in △*STV*.

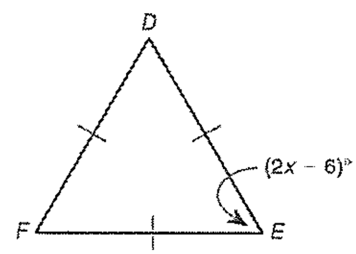


△*STV* is equiangular.	Equilateral △ → equiangular △
$(7x + 4)° = 60°$	The measure of each ∠ of an equiangular △ is 60°.
$7x = 56$	Subtract 4 from both sides.
$x = 8$	Divide both sides by 7.

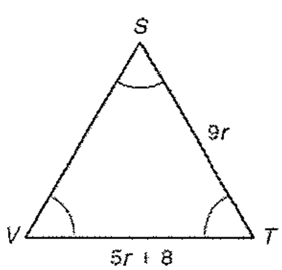
Find each value.



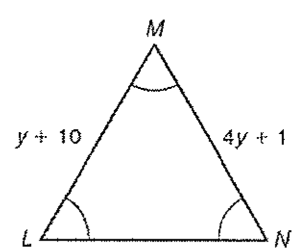
5. $n =$ ____________________



6. $x =$ ____________________



7. $VT =$ ____________________



8. $MN =$ ____________________