MEA 2013-2014
Teacher: Claudia Valle
Start Date:

Course: Geometry A
Student: ________________
Completed Date:

Unit 2: Geometric Reasoning

Objectives: Students will understand and use the basic undefined terms and defined terms of geometry. Students will understand how to measure and describe angles and segments.

Essential Questions: What are some real life situations where conditional statements are relevant? How can strong communication skills enhance the mathematical experience?

TEKS Standards: G.1.A, G.2.B, G.3.A, G.3.B, G.3.C, G.3.D, G.3.E, G.5.B

(1) Geometric structure. The student understands the structure of, and relationships within, an axiomatic system. The student is expected to:

(A) develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems;

(2) Geometric structure. The student analyzes geometric relationships in order to make and verify conjectures. The student is expected to:

(B) make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

(3) Geometric structure. The student applies logical reasoning to justify and prove mathematical statements. The student is expected to:

(A) determine the validity of a conditional statement, its converse, inverse, and contrapositive;

(B) construct and justify statements about geometric figures and their properties;

(C) use logical reasoning to prove statements are true and find counter examples to disprove statements that are false;

(D) use inductive reasoning to formulate a conjecture; and

(E) use deductive reasoning to prove a statement.

(5) Geometric patterns. The student uses a variety of representations to describe geometric relationships and solve problems. The student is expected to:

(B) use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles;

Turn In:

Assignment #	Activity	TEKS
7	Using Inductive Reasoning to Make Conjectures	G.2.B, G.3.D, G.5.B
8	Conditional Statements	G.3.A, G.3.C
9	Using Deductive Reasoning to Verify Conjectures	G.2.B, G.3.B, G.3.C, G.3.E
10	Biconditional Statements and Definitions	G.3.A, G.3.B
11	Algebraic Proof	G.3.B, G.3.C, G.3.E
12	Geometric Proof	G.1.A, G.3.B, G.3.C, G.3.E
13	Flowcharts and Paragraph Proofs	G.1.A, G.2.B, G.3.C, G.3.E
14	Unit 2 Test	G.1.A, G.2.B, G.3.A, G.3.B, G.3.C, G3.D, G.3.E, G.5.B

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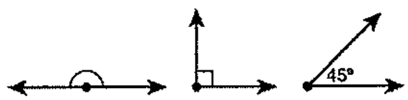
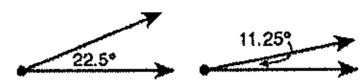
LESSON 2-1

Reteach

Using Inductive Reasoning to Make Conjectures

When you make a general rule or conclusion based on a pattern, you are using **inductive reasoning.** A conclusion based on a pattern is called a **conjecture.**

Pattern	Conjecture	Next Two Items
–8, –3, 2, 7, . . .	Each term is 5 more than the previous term.	7 + 5 = 12 12 + 5 = 17
	The measure of each angle is half the measure of the previous angle.	

Find the next item in each pattern.

1. $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \ldots$

   ____

2. 100, 81, 64, 49, . . .

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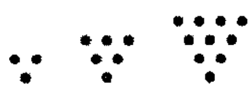
3. 

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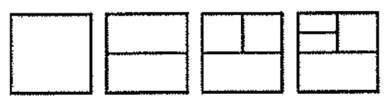
4. 

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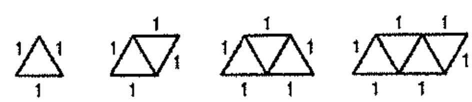
Complete each conjecture.

5. If the side length of a square is doubled, the perimeter of the square is ______________________.

6. The number of nonoverlapping angles formed by n lines intersecting in a point is ______________________.

Use the figure to complete the conjecture in Exercise 7.

7. The perimeter of a figure that has n of these triangles is ______________________.



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LESSON 2-1

Reteach

Using Inductive Reasoning to Make Conjectures *continued*

Since a conjecture is an educated guess, it may be true or false. It takes only one example, or **counterexample,** to prove that a conjecture is false.

Conjecture: For any integer n, $n \leq 4n$.

n	$n \leq 4n$	True or False?
3	$3 \leq 4(3)$ $3 \leq 12$	true
0	$0 \leq 4(0)$ $0 \leq 0$	true
−2	$-2 \leq 4(-2)$ $-2 \leq -8$	false

$n = -2$ is a counterexample, so the conjecture is false.

Show that each conjecture is false by finding a counterexample.

8. If three lines lie in the same plane, then they intersect in at least one point.

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9. Points A, G, and N are collinear. If $AG = 7$ inches and $GN = 5$ inches, then $AN = 12$ inches.

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10. For any real numbers x and y, if $x > y$, then $x^2 > y^2$.

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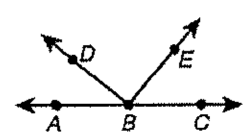
11. The total number of angles in the figure is 3.



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12. If two angles are acute, then the sum of their measures equals the measure of an obtuse angle.

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Determine whether each conjecture is true. If not, write or draw a counterexample.

13. Points Q and R are collinear.

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14. If J is between H and K, then $HJ = JK$.

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LESSON 2-2

Reteach

Conditional Statements

A **conditional statement** is a statement that can be written as an if-then statement, "if *p*, then *q*."

The **hypothesis** comes after the word *if.*

The **conclusion** comes after the word *then.*

If you buy this cell phone, then you will receive 10 free ringtone downloads.

Sometimes it is necessary to rewrite a conditional statement so that it is in if-then form.

Conditional: A person who practices putting will improve her golf game.

If-Then Form: If a person practices putting, then she will improve her golf game.

A conditional statement has a false **truth value** *only* if the hypothesis (H) is true and the conclusion (C) is false.

For each conditional, underline the hypothesis and double-underline the conclusion.

1. If x is an even number, then x is divisible by 2.
2. The circumference of a circle is 5π inches if the diameter of the circle is 5 inches.
3. If a line containing the points J, K, and L lies in plane $\mathcal{P}$, then J, K, and L are coplanar.

For Exercises 4–6, write a conditional statement from each given statement.

4. Congruent segments have equal measures.

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5. On Tuesday, play practice is at 6:00.

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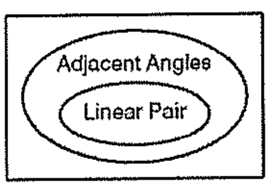
6. 

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Determine whether the following conditional is true. If false, give a counterexample.

7. If two angles are supplementary, then they form a linear pair.

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LESSON 2-2

Reteach

Conditional Statements continued

The **negation** of a statement, "not *p*," has the opposite truth value of the original statement.

If *p* is true, then *not p* is false.

If *p* is false, then *not p* is true.

Statement	Example	Truth Value
Conditional	(H) If [a figure is a square], then (C) [it has four right angles].	True
Converse: Switch H and C.	If a figure has four right angles, then it is a square.	False
Inverse: Negate H and C.	If a figure is not a square, then it does not have four right angles.	False
Contrapositive: Switch and negate H and C.	If a figure does not have four right angles, then it is not a square.	True

Write the converse, inverse, and contrapositive of each conditional statement. Find the truth value of each.

8. If an animal is an armadillo, then it is nocturnal.

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9. If $y = 1$, then $y^2 = 1$.

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10. If an angle has a measure less than 90°, then it is acute.

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LESSON 2-7

Reteach

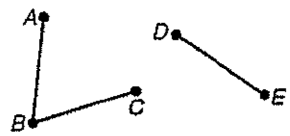
Flowchart and Paragraph Proofs continued

To write a paragraph proof, use sentences to write a paragraph that presents the statements and reasons.

You can use the given two-column proof to write a paragraph proof.

Given: $\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{DE}$

Prove: $\overline{AB} \cong \overline{DE}$



Two-Column Proof:

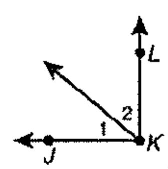
Statements	Reasons
1. $\overline{AB} \cong \overline{BC}$, $\overline{BC} \cong \overline{DE}$	1. Given
2. $AB = BC$, $BC = DE$	2. Definition of congruent segments
3. $AB = DE$	3. Transitive Property of Equality
4. $\overline{AB} \cong \overline{DE}$	4. Definition of congruent segments

Paragraph Proof: It is given that $\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{DE}$, so $AB = BC$ and $BC = DE$ by the definition of congruent segments. By the Transitive Property of Equality, $AB = DE$. Thus, by the definition of congruent segments, $\overline{AB} \cong \overline{DE}$.

2. Use the given two-column proof to write a paragraph proof.

Given: $\angle JKL$ is a right angle.

Prove: $\angle 1$ and $\angle 2$ are complementary angles.



Two-Column Proof:

Statements	Reasons
1. $\angle JKL$ is a right angle.	1. Given
2. $m\angle JKL = 90°$	2. Definition of right angle
3. $m\angle JKL = m\angle 1 + m\angle 2$	3. Angle Addition Postulate
4. $90° = m\angle 1 + m\angle 2$	4. Substitution
5. $\angle 1$ and $\angle 2$ are complementary angles.	5. Definition of complementary angles

Paragraph Proof: ____

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LESSON 2-7

Reteach

Flowchart and Paragraph Proofs

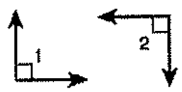
In addition to the two-column proof, there are other types of proofs that you can use to prove conjectures are true.

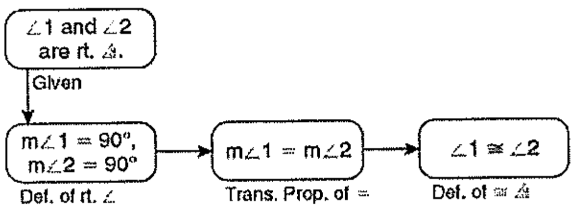
Flowchart Proof	• Uses boxes and arrows. • Steps go left to right or top to bottom, as shown by arrows. • The justification for each step is written below the box.

You can write a flowchart proof of the Right Angle Congruence Theorem.

Given: $\angle 1$ and $\angle 2$ are right angles.

Prove: $\angle 1 \cong \angle 2$

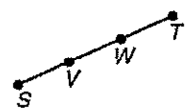




1. Use the given two-column proof to write a flowchart proof.

Given: V is the midpoint of $\overline{SW}$, and W is the midpoint of $\overline{VT}$.

Prove: $\overline{SV} \cong \overline{WT}$



Two-Column Proof:

Statements	Reasons
1. V is the midpoint of $\overline{SW}$.	1. Given
2. W is the midpoint of $\overline{VT}$.	2. Given
3. $\overline{SV} \cong \overline{VW}$, $\overline{VW} \cong \overline{WT}$	3. Definition of midpoint
4. $\overline{SV} \cong \overline{WT}$	4. Transitive Property of Equality

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LESSON 2-6

Reteach

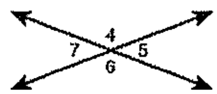
Geometric Proof continued

A **theorem** is any statement that you can prove. You can use **two-column proofs** and deductive reasoning to prove theorems.

Congruent Supplements Theorem	If two angles are supplementary to the same angle (or to two congruent angles), then the two angles are congruent.
Right Angle Congruence Theorem	All right angles are congruent.

Here is a two-column proof of one case of the Congruent Supplements Theorem.

Given: ∠4 and ∠5 are supplementary and ∠5 and ∠6 are supplementary.

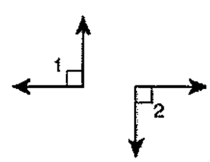


Prove: ∠4 ≅ ∠6

Proof:

Statements	Reasons
1. ∠4 and ∠5 are supplementary.	1. Given
2. ∠5 and ∠6 are supplementary.	2. Given
3. m∠4 + m∠5 = 180°	3. Definition of supplementary angles
4. m∠5 + m∠6 = 180°	4. Definition of supplementary angles
5. m∠4 + m∠5 = m∠5 + m∠6	5. Substitution Property of Equality
6. m∠4 = m∠6	6. Subtraction Property of Equality
7. ∠4 ≅ ∠6	7. Definition of congruent angles

Fill in the blanks to complete the two-column proof of the Right Angle Congruence Theorem.



2. **Given:** ∠1 and ∠2 are right angles.

   Prove: ∠1 ≅ ∠2

   Proof:

Statements	Reasons
1. a. ______________________	1. Given
2. m∠1 = 90°	2. b. ______________________
3. c. ______________________	3. Definition of right angle
4. m∠1 = m∠2	4. d. ______________________
5. e. ______________________	5. Definition of congruent angles

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LESSON 2-6

Reteach

Geometric Proof

To write a geometric proof, start with the hypothesis of a conditional. → **Hypothesis**

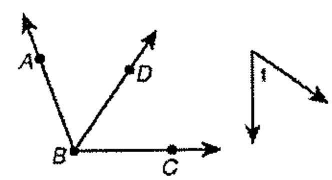
Apply deductive reasoning. → **Deductive Reasoning**
- Definitions
- Properties
- Postulates
- Theorems

Prove that the conclusion of the conditional is true. → **Conclusion**

> **Conditional:** If $\overrightarrow{BD}$ is the angle bisector of $\angle ABC$, and $\angle ABD \cong \angle 1$, then $\angle DBC \cong \angle 1$.

Given: $\overrightarrow{BD}$ is the angle bisector of $\angle ABC$, and $\angle ABD \cong \angle 1$.

Prove: $\angle DBC \cong \angle 1$

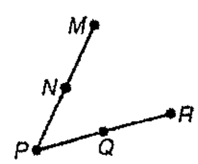


Proof:

1. $\overrightarrow{BD}$ is the angle bisector of $\angle ABC$.	1. Given
2. $\angle ABD \cong \angle DBC$	2. Def. of $\angle$ bisector
3. $\angle ABD \cong \angle 1$	3. Given
4. $\angle DBC \cong \angle 1$	4. Transitive Prop. of $\cong$

1. **Given:** N is the midpoint of $\overline{MP}$, Q is the midpoint of $\overline{RP}$, and $\overline{PQ} \cong \overline{NM}$.

   Prove: $\overline{PN} \cong \overline{QR}$



Write a justification for each step.

Proof:

1. N is the midpoint of $\overline{MP}$.	1. ______________________
2. Q is the midpoint of $\overline{RP}$.	2. ______________________
3. $\overline{PN} \cong \overline{NM}$	3. ______________________
4. $\overline{PQ} \cong \overline{NM}$	4. ______________________
5. $\overline{PN} \cong \overline{PQ}$	5. ______________________
6. $\overline{PQ} \cong \overline{QR}$	6. ______________________
7. $\overline{PN} \cong \overline{QR}$	7. ______________________

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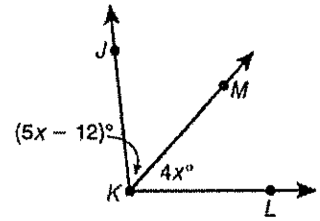
LESSON 2-5

Reteach

Algebraic Proof *continued*

When writing algebraic proofs in geometry, you can also use definitions, postulates, properties, and pieces of given information to justify the steps.

$m\angle JKM = m\angle MKL$	Definition of congruent angles
$(5x - 12)° = 4x°$	Substitution Property of Equality
$x - 12 = 0$	Subtraction Property of Equality
$x = 12$	Addition Property of Equality



Properties of Congruence	Symbols	Examples
Reflexive	figure $A \cong$ figure A	$\angle CDE \cong \angle CDE$
Symmetric	If figure $A \cong$ figure B, then figure $B \cong$ figure A.	If $\overline{JK} \cong \overline{LM}$, then $\overline{LM} \cong \overline{JK}$.
Transitive	If figure $A \cong$ figure B and figure $B \cong$ figure C, then figure $A \cong$ figure C.	If $\angle N \cong \angle P$ and $\angle P \cong \angle Q$, then $\angle N \cong \angle Q$.

Write a justification for each step.

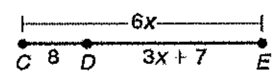
5. $CE = CD + DE$ ______________________

$6x = 8 + (3x + 7)$ ______________________

$6x = 15 + 3x$ ______________________

$3x = 15$ ______________________

$x = 5$ ______________________



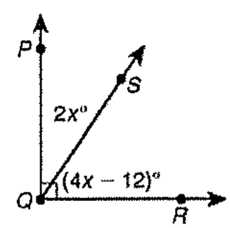
6. $m\angle PQR = m\angle PQS + m\angle SQR$ ______________________

$90° = 2x° + (4x - 12)°$ ______________________

$90 = 6x - 12$ ______________________

$102 = 6x$ ______________________

$17 = x$ ______________________



Identify the property that justifies each statement.

7. If $\angle ABC \cong \angle DEF$, then $\angle DEF \cong \angle ABC$.

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8. $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, so $\angle 1 \cong \angle 3$.

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9. If $FG = HJ$, then $HJ = FG$.

____

10. $\overline{WX} \cong \overline{WX}$

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LESSON 2-5

Reteach

Algebraic Proof

A **proof** is a logical argument that shows a conclusion is true. An algebraic proof uses algebraic properties, including the Distributive Property and the properties of equality.

Properties of Equality	Symbols	Examples
Addition	If $a = b$, then $a + c = b + c$.	If $x = -4$, then $x + 4 = -4 + 4$.
Subtraction	If $a = b$, then $a - c = b - c$.	If $r + 1 = 7$, then $r + 1 - 1 = 7 - 1$.
Multiplication	If $a = b$, then $ac = bc$.	If $\frac{k}{2} = 8$, then $\frac{k}{2}(2) = 8(2)$.
Division	If $a = 2$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.	If $6 = 3t$, then $\frac{6}{3} = \frac{3t}{3}$.
Reflexive	$a = a$	$15 = 15$
Symmetric	If $a = b$, then $b = a$.	If $n = 2$, then $2 = n$.
Transitive	If $a = b$ and $b = c$, then $a = c$.	If $y = 3^2$ and $3^2 = 9$, then $y = 9$.
Substitution	If $a = b$, then b can be substituted for a in any expression.	If $x = 7$, then $2x = 2(7)$.

When solving an algebraic equation, justify each step by using a definition, property, or piece of given information.

$2(a + 1) = -6$	Given equation
$2a + 2 = -6$	Distributive Property
$\underline{-2} \quad \underline{-2}$	Subtraction Property of Equality
$2a = -8$	Simplify.
$\frac{2a}{2} = \frac{-8}{2}$	Division Property of Equality
$a = -4$	Simplify.

Solve each equation. Write a justification for each step.

1. $\frac{n}{6} - 3 = 10$

2. $5 + x = 2x$

3. $\frac{y+4}{7} = 3$

4. $4(t - 3) = -20$

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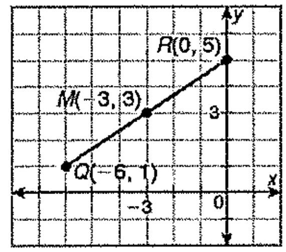
LESSON 2-4

Reteach

Biconditional Statements and Definitions continued

A biconditional statement is false if either the conditional statement is false or its converse is false.

The midpoint of $\overline{QR}$ is $M(-3, 3)$ if and only if the endpoints are $Q(-6, 1)$ and $R(0, 5)$.



Conditional: If the midpoint of $\overline{QR}$ is $M(-3, 3)$, then the endpoints are $Q(-6, 1)$ and $R(0, 5)$. *false*

Converse: If the endpoints of $\overline{QR}$ are $Q(-6, 1)$ and $R(0, 5)$, then the midpoint of $\overline{QR}$ is $M(-3, 3)$. *true*

The conditional is false because the endpoints of $\overline{QR}$ could be $Q(-3, 6)$ and $R(-3, 0)$. So the biconditional statement is false.

Definitions can be written as biconditionals.

Definition: Circumference is the distance around a circle.

Biconditional: A measure is the circumference if and only if it is the distance around a circle.

Determine if each biconditional is true. If false, give a counterexample.

5. Students perform during halftime at the football games if and only if they are in the high school band.

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6. An angle in a triangle measures 90° if and only if the triangle is a right triangle.

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7. $a = 4$ and $b = 3$ if and only if $ab = 12$.

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Write each definition as a biconditional.

8. An isosceles triangle has at least two congruent sides.

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9. Deductive reasoning requires the use of facts, definitions, and properties to draw conclusions.

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LESSON 2-4

Reteach

Biconditional Statements and Definitions

A **biconditional statement** combines a conditional statement, "if *p*, then *q*," with its converse, "if *q*, then *p*."

Conditional: If the sides of a triangle are congruent (*p*), then the angles are congruent (*q*).

Converse: If the angles of a triangle are congruent (*q*), then the sides are congruent (*p*).

Biconditional: The sides of a triangle are congruent (*p*) if and only if the angles are congruent (*q*).

Write the conditional statement and converse within each biconditional.

1. Lindsay will take photos for the yearbook if and only if she doesn't play soccer.

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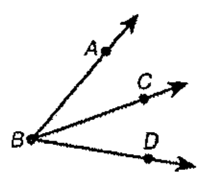
2. $m\angle ABC = m\angle CBD$ if and only if $\overrightarrow{BC}$ is an angle bisector of $\angle ABD$.



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For each conditional, write the converse and a biconditional statement.

3. If you can download 6 songs for $5.94, then each song costs $0.99.

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4. If a figure has 10 sides, then it is a decagon.

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Name ______________________ Date ______________ Class ______________

LESSON 2-3

Reteach

Using Deductive Reasoning to Verify Conjectures continued

Another valid form of deductive reasoning is the **Law of Syllogism.** It is similar to the Transitive Property of Equality.

Transitive Property of Equality	Law of Syllogism
If $y = 10x$ and $10x = 20$, then $y = 20$.	**Given:** If you have a horse, then you have to feed it. If you have to feed a horse, then you have to get up early every morning. **Conjecture:** If you have a horse, then you have to get up early every morning.

Determine whether each conjecture is valid by the Law of Syllogism.

6. Given: If you buy a car, then you can drive to school. If you can drive to school, then you will not ride the bus.

   Conjecture: If you buy a car, then you will not ride the bus. ______________

7. Given: If $\angle K$ is obtuse, then it does not have a measure of 90°. If an angle does not have a measure of 90°, then it is not a right angle.

   Conjecture: If $\angle K$ is obtuse, then it is not a right angle. ______________

8. Given: If two segments are congruent, then they have the same measure. If two segments each have a measure of 6.5 centimeters, then they are congruent.

   Conjecture: If two segments are congruent, then they each have a measure of 6.5 centimeters. ______________

Draw a conclusion from the given information.

9. If $\triangle LMN$ is translated in the coordinate plane, then it has the same size and shape as its preimage. If an image and preimage have the same size and shape, then the figures have equal perimeters. $\triangle LMN$ is translated in the coordinate plane.

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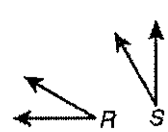
10. If $\angle R$ and $\angle S$ are complementary to the same angle, then the two angles are congruent. If two angles are congruent, then they are supplementary to the same angle. $\angle R$ and $\angle S$ are complementary to the same angle.



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LESSON 2-3

Reteach

Using Deductive Reasoning to Verify Conjectures

With inductive reasoning, you use examples to make a conjecture. With **deductive reasoning,** you use facts, definitions, and properties to draw conclusions and prove that conjectures are true.

Given: If two points lie in a plane, then the line containing those points also lies in the plane. *A* and *B* lie in plane $\mathcal{N}$.

Conjecture: $\overleftrightarrow{AB}$ lies in plane $\mathcal{N}$.

One valid form of deductive reasoning that lets you draw conclusions from true facts is called the **Law of Detachment.**

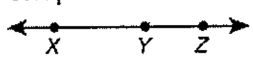
Given	If you have $2, then you can buy a snack. You have $2.	If you have $2, then you can buy a snack. You can buy a snack.
Conjecture	You can buy a snack.	You have $2.
Valid Conjecture?	Yes; the conditional is true and the hypothesis is true.	No; the hypothesis may or may not be true. For example, if you borrowed money, you could also buy a snack.

Tell whether each conclusion uses inductive or deductive reasoning.

1. A sign in the cafeteria says that a car wash is being held on the last Saturday of May. Tomorrow is the last Saturday of May, so Justin concludes that the car wash is tomorrow. ______________

2. So far, at the beginning of every Latin class, the teacher has had students review vocabulary. Latin class is about to start, and Jamilla assumes that they will first review vocabulary. ______________

3. Opposite rays are two rays that have a common endpoint and form a line. $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are opposite rays. ______________



Determine whether each conjecture is valid by the Law of Detachment.

4. Given: If you ride the Titan roller coaster in Arlington, Texas, then you will drop 255 feet.
   Michael rode the Titan roller coaster.
   Conjecture: Michael dropped 255 feet. ______________

5. Given: A segment that is a diameter of a circle has endpoints on the circle.
   $\overline{GH}$ has endpoints on a circle.
   Conjecture: $\overline{GH}$ is a diameter. ______________