CHAPTER 7

Similarity

go.hrw.com/Geo/TX
Chapter Project Online
KEYWORD: MG7 ChProj

The Lighthouse Rock is located in Palo Duro Canyon or “the Grand Canyon of Texas.”

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. side of a polygon
2. denominator
3. numerator
4. vertex of a polygon
5. vertical angles

A. two nonadjacent angles formed by two intersecting lines

B. the top number of a fraction, which tells how many parts of a whole are being considered

C. a point that corresponds to one and only one number

D. the intersection of two sides of a polygon

E. one of the segments that form a polygon

F. the bottom number of a fraction, which tells how many equal parts are in the whole

Simplify Fractions

Write each fraction in simplest form.

6. $\frac{16}{20}$ **7.** $\frac{14}{21}$ **8.** $\frac{33}{121}$ **9.** $\frac{56}{80}$

Ratios

Use the table to write each ratio in simplest form.

10. jazz CDs to country CDs

11. hip-hop CDs to jazz CDs

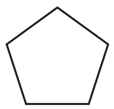
12. rock CDs to total CDs

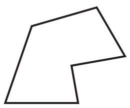
13. total CDs to country CDs

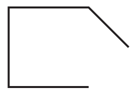
Ryan's CD Collection	
Rock	36
Jazz	18
Hip-hop	34
Country	24

Identify Polygons

Determine whether each figure is a polygon. If so, name it by the number of sides.

14. 

15. 

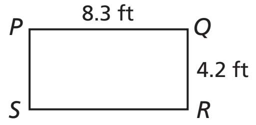
16. 

17. 

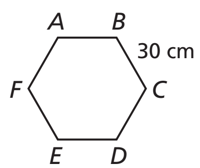
Find Perimeter

Find the perimeter of each figure.

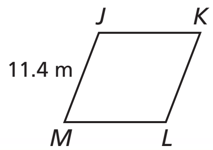
18. rectangle *PQRS*



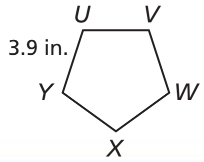
19. regular hexagon *ABCDEF*



20. rhombus *JKLM*



21. regular pentagon *UVWXY*



CHAPTER 7

Study Guide: Preview

Key Vocabulary/Vocabulario

dilation	dilatación
proportion	proporción
ratio	razón
scale	escala
scale drawing	dibujo a escala
scale factor	factor de escala
similar	semejante
similar polygons	polígonos semejantes
similarity ratio	razón de semejanza

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. When an eye doctor dilates your eyes, the pupils become enlarged. What might it mean for one geometric figure to be a **dilation** of another figure?
2. A blueprint is a scale drawing of a building. What do you think is the definition of a **scale drawing**?
3. What does the word *similar* mean in everyday language? What do you think the term **similar polygons** means?
4. Bike riders often talk about gear ratios. Give examples of situations where the word **ratio** is used. What do these examples have in common?

Geometry TEKS	Les. 7-1	7-2 Tech. Lab	Les. 7-2	7-3 Tech. Lab	Les. 7-3	7-4 Tech. Lab	Les. 7-4	Les. 7-5	Les. 7-6
G.1.B Geometric structure* recognize the historical development of geometric systems and know mathematics is developed for a variety of purposes								★	
G.2.A Geometric structure* use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships				★		★	★		
G.3.B Geometric structure* construct and justify statements about geometric figures and their properties				★		★	★		
G.5.B Geometric patterns* use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures ...	★	★	★		★	★	★		
G.9.B Congruence and the geometry of size* formulate and test conjectures about the properties and attributes of polygons ... based on explorations and concrete models				★		★	★		★
G.11.A Similarity and the geometry of shape* use and extend similarity properties and transformations to explore and justify conjectures about geometric figures			★	★	★		★	★	★
G.11.B Similarity and the geometry of shape* use ratios to solve problems involving similar figures	★		★		★		★	★	
G.11.D Similarity and the geometry of shape* describe the effect on perimeter, area ... when one or more dimensions of a figure are changed and apply this idea in solving problems								★	

* *Knowledge and skills* are written out completely on pages TX28–TX35.

2ND PRINT

Reading and Writing Math

Reading Strategy: Read and Understand the Problem

Many of the concepts you are learning are used in real-world situations. Throughout the text, there are examples and exercises that are real-world word problems. Listed below are strategies for solving word problems.

Problem Solving Strategies

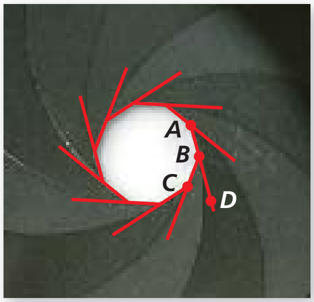
- Read slowly and carefully. Determine what information is given and what you are asked to find.
- If a diagram is provided, read the labels and make sure that you understand the information. If you do not, resketch and relabel the diagram so it makes sense to you. If a diagram is not provided, make a quick sketch and label it.
- Use the given information to set up and solve the problem.
- Decide whether your answer makes sense.

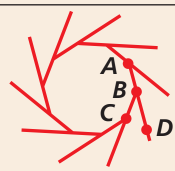
From Lesson 6-1: Look at how the Polygon Exterior Angle Theorem is used in photography.

PROBLEM SOLVING

Photography Application

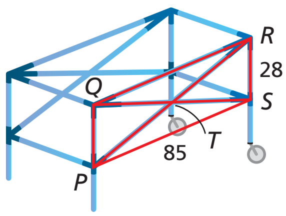
The aperture of the camera shown is formed by ten blades. The blades overlap to form a regular decagon. What is the measure of $\angle CBD$?



Step	Procedure	Result
Understand the Problem	• List the **important information**. • The **answer** will be the measure of $\angle CBD$.	$\angle CBD$ is one of the exterior angles of the regular decagon formed by the apeture.
Make a Plan	• A **diagram** is provided, and it is labeled accurately.	
Solve	• You can use the **Polygon Exterior Angle Theorem**. Then divide to find the measure of **one** of the exterior angles.	$m\angle CBD = \frac{360°}{10} = 36°$
Look Back	• The **answer** is reasonable since a decagon has 10 angles.	$10(36°) = 360°$

Try This

Use the problem-solving strategies for the following problem.



1. A painter's scaffold is constructed so that the braces lie along the diagonals of rectangle $PQRS$. Given $RS = 28$ and $QS = 85$, find QT.

2ND PRINT

7-1 Ratio and Proportion

TEKS G.11.B Similarity and the geometry of shape: use ratios to solve problems involving similar figures. Also G.5.B, G.7.B, G.7.C

Objectives

Write and simplify ratios.

Use proportions to solve problems.

Vocabulary

ratio
proportion
extremes
means
cross products

Who uses this?

Filmmakers use ratios and proportions when creating special effects. (See Example 5.)

The *Lord of the Rings* movies transport viewers to the fantasy world of Middle Earth. Many scenes feature vast fortresses, sprawling cities, and bottomless mines. To film these images, the moviemakers used *ratios* to help them build highly detailed miniature models.

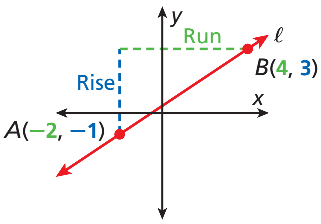
A **ratio** compares two numbers by division. The ratio of two numbers a and b can be written as a to b, $a:b$, or $\frac{a}{b}$, where $b \neq 0$. For example, the ratios 1 to 2, 1:2, and $\frac{1}{2}$ all represent the same comparison.

EXAMPLE 1 Writing Ratios

Remember!

In a ratio, the denominator of the fraction cannot be zero because division by zero is undefined.

Write a ratio expressing the slope of ℓ.



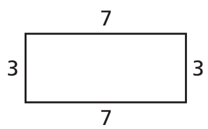
$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3 - (-1)}{4 - (-2)} \quad \textit{Substitute the given values.}$$

$$= \frac{4}{6} = \frac{2}{3} \quad \textit{Simplify.}$$

CHECK IT OUT! 1. Given that two points on m are $C(-2, 3)$ and $D(6, 5)$, write a ratio expressing the slope of m.

A ratio can involve more than two numbers. For the rectangle, the ratio of the side lengths may be written as 3:7:3:7.



EXAMPLE 2 Using Ratios

The ratio of the side lengths of a quadrilateral is 2:3:5:7, and its perimeter is 85 ft. What is the length of the longest side?

Let the side lengths be $2x$, $3x$, $5x$, and $7x$. Then $2x + 3x + 5x + 7x = 85$. After like terms are combined, $17x = 85$. So $x = 5$. The length of the longest side is $7x = 7(5) = 35$ ft.

CHECK IT OUT! 2. The ratio of the angle measures in a triangle is 1:6:13. What is the measure of each angle?

A **proportion** is an equation stating that two ratios are equal. In the proportion $\frac{a}{b} = \frac{c}{d}$, the values a and d are the **extremes**. The values b and c are the **means**. When the proportion is written as $a:b = c:d$, the extremes are in the first and last positions. The means are in the two middle positions.

Reading Math

The Cross Products Property can also be stated as, "In a proportion, the product of the extremes is equal to the product of the means."

In Algebra 1 you learned the Cross Products Property. The product of the extremes ad and the product of the means bc are called the **cross products**.

Cross Products Property

In a proportion, if $\frac{a}{b} = \frac{c}{d}$ and b and $d \neq 0$, then $ad = bc$.

$\frac{a}{b} = \frac{c}{d}$

$ad = bc$

EXAMPLE 3 Solving Proportions

Solve each proportion.

A $\frac{5}{y} = \frac{45}{63}$

$5(63) = y(45)$	*Cross Products Prop.*
$315 = 45y$	*Simplify.*
$y = 7$	*Divide both sides by 45.*

B $\frac{x+2}{6} = \frac{24}{x+2}$

$(x+2)^2 = 6(24)$	*Cross Products Prop.*
$(x+2)^2 = 144$	*Simplify.*
$x + 2 = \pm 12$	*Find the square root of both sides.*
$x + 2 = 12$ or $x + 2 = -12$	*Rewrite as two eqns.*
$x = 10$ or $x = -14$	*Subtract 2 from both sides.*

CHECK IT OUT! **Solve each proportion.**

3a. $\frac{3}{8} = \frac{x}{56}$ **3b.** $\frac{2y}{9} = \frac{8}{4y}$

3c. $\frac{d}{3} = \frac{6}{2}$ **3d.** $\frac{x+3}{4} = \frac{9}{x+3}$

The following table shows equivalent forms of the Cross Products Property.

Know it! Note

Properties of Proportions

ALGEBRA	NUMBERS
The proportion $\frac{a}{b} = \frac{c}{d}$ is equivalent to the following:	The proportion $\frac{1}{3} = \frac{2}{6}$ is equivalent to the following:
$ad = bc$	$1(6) = 3(2)$
$\frac{b}{a} = \frac{d}{c}$	$\frac{3}{1} = \frac{6}{2}$
$\frac{a}{c} = \frac{b}{d}$	$\frac{1}{2} = \frac{3}{6}$

EXAMPLE 4 Using Properties of Proportions

Reading Math

Since x comes before y in the sentence, x will be in the numerator of the fraction.

Given that $4x = 10y$, find the ratio of x to y in simplest form.

$$4x = 10y$$

$$\frac{x}{y} = \frac{10}{4} \qquad \textit{Divide both sides by } 4y.$$

$$\frac{x}{y} = \frac{5}{2} \qquad \textit{Simplify.}$$

CHECK IT OUT! **4.** Given that $16s = 20t$, find the ratio $t:s$ in simplest form.

EXAMPLE 5 *Problem-Solving Application*

PROBLEM SOLVING

During the filming of *The Lord of the Rings*, the special-effects team built a model of Sauron's tower with a height of 8 m and a width of 6 m. If the width of the full-size tower is 996 m, what is its height?

1 Understand the Problem

The **answer** will be the height of the tower.

2 Make a Plan

Let x be the height of the tower. Write a proportion that compares the ratios of the height to the width.

$$\frac{\text{height of model tower}}{\text{width of model tower}} = \frac{\text{height of full-size tower}}{\text{width of full-size tower}}$$

$$\frac{8}{6} = \frac{x}{996}$$

3 Solve

$$\frac{8}{6} = \frac{x}{996}$$

$$6x = 8(996) \qquad \textit{Cross Products Prop.}$$

$$6x = 7968 \qquad \textit{Simplify.}$$

$$x = 1328 \qquad \textit{Divide both sides by 6.}$$

The height of the full-size tower is 1328 m.

4 Look Back

Check the answer in the original problem. The ratio of the height to the width of the model is 8 : 6, or 4 : 3. The ratio of the height to the width of the tower is 1328 : 996. In simplest form, this ratio is also 4 : 3. So the ratios are equal, and the answer is correct.

CHECK IT OUT! **5.** **What if...?** Suppose the special-effects team made a different model with a height of 9.2 m and a width of 6 m. What is the height of the actual tower?

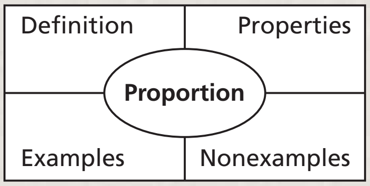
2ND PRINT

THINK AND DISCUSS

1. Is the ratio 6:7 the same ratio as 7:6? Why or why not?
2. Susan wants to know if the fractions $\frac{3}{7}$ and $\frac{12}{28}$ are equivalent. Explain how she can use the properties of proportions to find out.

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. In the boxes, write the definition of a proportion, the properties of proportions, and examples and nonexamples of a proportion.



7-1 Exercises

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Homework Help Online
KEYWORD: MG7 7-1
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

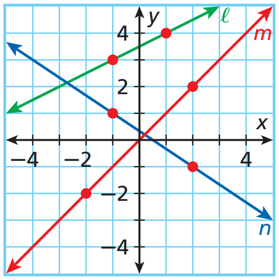
Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. Name the means and extremes in the proportion $\frac{1}{3} = \frac{2}{6}$.
2. Write the cross products for the proportion $\frac{s}{t} = \frac{u}{v}$.

SEE EXAMPLE 1 p. 454

Write a ratio expressing the slope of each line.

3. ℓ
4. m
5. n



SEE EXAMPLE 2 p. 454

6. The ratio of the side lengths of a quadrilateral is 2:4:5:7, and its perimeter is 36 m. What is the length of the shortest side?
7. The ratio of the angle measures in a triangle is 5:12:19. What is the measure of the largest angle?

SEE EXAMPLE 3 p. 455

Solve each proportion.

8. $\frac{x}{2} = \frac{40}{16}$
9. $\frac{7}{y} = \frac{21}{27}$
10. $\frac{6}{58} = \frac{t}{29}$
11. $\frac{y}{3} = \frac{27}{y}$
12. $\frac{16}{x-1} = \frac{x-1}{4}$
13. $\frac{x^2}{18} = \frac{x}{6}$

SEE EXAMPLE 4 p. 456

14. Given that $2a = 8b$, find the ratio of a to b in simplest form.
15. Given that $6x = 27y$, find the ratio $y:x$ in simplest form.

SEE EXAMPLE 5 p. 456

16. **Architecture** The Arkansas State Capitol Building is a smaller version of the U.S. Capitol Building. The U.S. Capitol is 752 ft long and 288 ft tall. The Arkansas State Capitol is 564 ft long. What is the height of the Arkansas State Capitol?

PRACTICE AND PROBLEM SOLVING

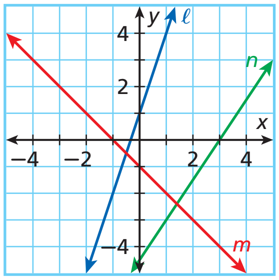
Independent Practice

For Exercises	See Example
17–19	1
20–21	2
22–27	3
28–29	4
30	5

TEKS TAKS

Skills Practice p. S16
Application Practice p. S34

Write a ratio expressing the slope of each line.



17. ℓ **18.** m **19.** n

20. The ratio of the side lengths of an isosceles triangle is 4 : 4 : 7, and its perimeter is 52.5 cm. What is the length of the base of the triangle?

21. The ratio of the angle measures in a parallelogram is 2 : 3 : 2 : 3. What is the measure of each angle?

Solve each proportion.

22. $\frac{6}{8} = \frac{9}{y}$ **23.** $\frac{x}{14} = \frac{50}{35}$ **24.** $\frac{z}{12} = \frac{3}{8}$

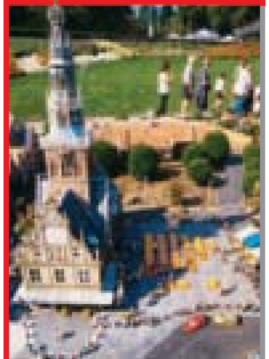
25. $\frac{2m+2}{3} = \frac{12}{2m+2}$ **26.** $\frac{5y}{16} = \frac{125}{y}$ **27.** $\frac{x+2}{12} = \frac{5}{x-2}$

28. Given that $5y = 25x$, find the ratio of x to y in simplest form.

29. Given that $35b = 21c$, find the ratio $b:c$ in simplest form.

LINK

Travel



For more than 50 years, Madurodam has been Holland's smallest city. The canal houses, market, airplanes, and windmills are all replicated on a 1 : 25 scale.

Source: madurodam.nl

30. Travel Madurodam is a park in the Netherlands that contains a complete Dutch city built entirely of miniature models. One of the models of a windmill is 1.2 m tall and 0.8 m wide. The width of the actual windmill is 20 m. What is its height?

Given that $\frac{a}{b} = \frac{5}{7}$, complete each of the following equations.

31. $7a = \blacksquare$ **32.** $\frac{b}{a} = \blacksquare$ **33.** $\frac{a}{5} = \blacksquare$

34. Sports During the 2003 NFL season, the Dallas Cowboys won 10 of their 16 regular-season games. What is their ratio of wins to losses in simplest form?

Write a ratio expressing the slope of the line through each pair of points.

35. $(-6, -4)$ and $(21, 5)$ **36.** $(16, -5)$ and $(6, 1)$

37. $\left(6\frac{1}{2}, -2\right)$ and $\left(4, 5\frac{1}{2}\right)$ **38.** $(-6, 1)$ and $(-2, 0)$

MULTI-STEP TAKS PREP

39. This problem will prepare you for the Multi-Step TAKS Prep on page 478.

A claymation film is shot on a set that is a scale model of an actual city. On the set, a skyscraper is 1.25 in. wide and 15 in. tall. The actual skyscraper is 800 ft tall.

a. Write a proportion that you can use to find the width of the actual skyscraper.

b. Solve the proportion from part **a**. What is the width of the actual skyscraper?

2ND PRINT

40. **Critical Thinking** The ratio of the lengths of a quadrilateral's consecutive sides is 2:5:2:5. The ratio of the lengths of the quadrilateral's diagonals is 1:1. What type of quadrilateral is this? Explain.

41. **Multi-Step** One square has sides 6 cm long. Another has sides 9 cm long. Find the ratio of the areas of the squares.

42. **Photography** A photo shop makes prints of photographs in a variety of sizes. Every print has a length-to-width ratio of 5:3.5 regardless of its size. A customer wants a print that is 20 in. long. What is the width of this print?

43. **Write About It** What is the difference between a ratio and a proportion?

TEST PREP

44. An 18-inch stick breaks into three pieces. The ratio of the lengths of the pieces is 1:4:5. Which of these is NOT a length of one of the pieces?

Ⓐ 1.8 inches Ⓑ 3.6 inches Ⓒ 7.2 inches Ⓓ 9 inches

45. Which of the following is equivalent to $\frac{3}{5} = \frac{x}{y}$?

Ⓕ $\frac{3}{y} = \frac{5}{x}$ Ⓖ $3x = 5y$ Ⓗ $\frac{x}{3} = \frac{y}{5}$ Ⓙ $3(5) = xy$

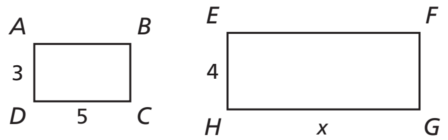
46. A recipe for salad dressing calls for oil and vinegar in a ratio of 5 parts oil to 2 parts vinegar. If you use $1\frac{1}{4}$ cups of oil, how many cups of vinegar will you need?

Ⓐ $\frac{1}{2}$ Ⓑ $\frac{5}{8}$ Ⓒ $2\frac{1}{2}$ Ⓓ $6\frac{1}{4}$

47. **Short Response** Explain how to solve the proportion $\frac{36}{72} = \frac{15}{x}$ for x. Tell what you must assume about x in order to solve the proportion.

CHALLENGE AND EXTEND

48. The ratio of the perimeter of rectangle *ABCD* to the perimeter of rectangle *EFGH* is 4:7. Find x.



49. Explain why $\frac{a}{b} = \frac{c}{d}$ and $\frac{a+b}{b} = \frac{c+d}{d}$ are equivalent proportions.

50. **Probability** The numbers 1, 2, 3, and 6 are randomly placed in these four boxes: $\frac{\square}{\square} \; ? \; \frac{\square}{\square}$. What is the probability that the two ratios will form a proportion?

51. Express the ratio $\frac{x^2 + 9x + 18}{x^2 - 36}$ in simplest form.

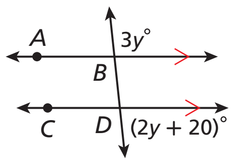
SPIRAL REVIEW

Complete each ordered pair so that it is a solution to $y - 6x = -3$. *(Previous course)*

52. $(0, \square)$ **53.** $(\square, 3)$ **54.** $(-4, \square)$

Find each angle measure. *(Lesson 3-2)*

55. $m\angle ABD$ **56.** $m\angle CDB$



Each set of numbers represents the side lengths of a triangle. Classify each triangle as acute, right, or obtuse. *(Lesson 5-7)*

57. 5, 8, 9 **58.** 8, 15, 20 **59.** 7, 24, 25

2ND PRINT

7-2 Technology LAB

Use with Lesson 7-2

Explore the Golden Ratio

In about 300 B.C.E., Euclid showed in his book *Elements* how to calculate the *golden ratio*. It is claimed that this ratio was used in many works of art and architecture to produce rectangles of pleasing proportions. The *golden ratio* also appears in the natural world and it is said even in the human face. If the ratio of a rectangle's length to its width is equal to the golden ratio, it is called a *golden rectangle*.

TEKS G.5.B Geometric patterns: use numeric and geometric patterns to make generalizations about geometric properties ... ratios in similar figures .... Also G.1.A, G.2.B, G.5.A

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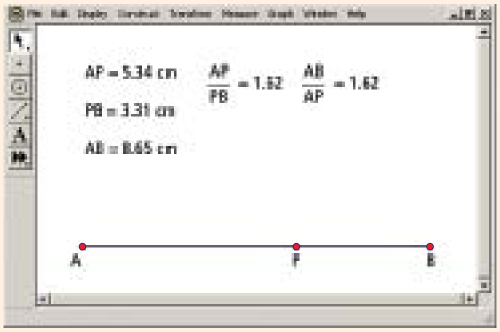
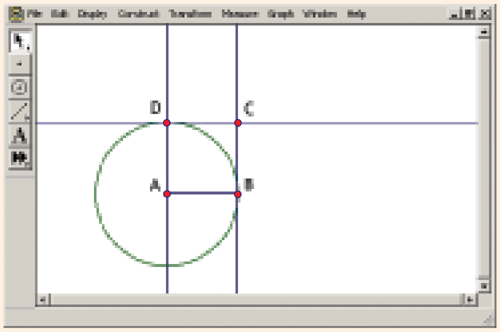
Lab Resources Online
KEYWORD: MG7 Lab7

Activity 1

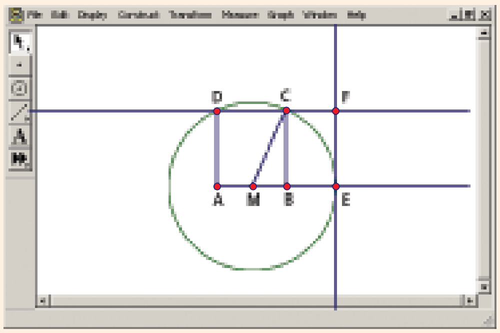
1. Construct a segment and label its endpoints A and B. Place P on the segment so that $\overline{AP}$ is longer than $\overline{PB}$. What are AP, PB, and AB? What is the ratio of AP to PB and the ratio of AB to AP? Drag P along the segment until the ratios are equal. What is the value of the equal ratios to the nearest hundredth?



2. Construct a *golden rectangle* beginning with a square. Create $\overline{AB}$. Then construct a circle with its center at A and a radius of $\overline{AB}$. Construct a line perpendicular to $\overline{AB}$ through A. Where the circle and the perpendicular line intersect, label the point D. Construct perpendicular lines through B and D and label their intersection C. Hide the lines and the circle, leaving only the segments to complete the square.



3. Find the midpoint of $\overline{AB}$ and label it M. Create a segment from M to C. Construct a circle with its center at M and radius of $\overline{MC}$. Construct a ray with endpoint A through B. Where the circle and the ray intersect, label the point E. Create a line through E that is perpendicular to $\overrightarrow{AB}$. Show the previously hidden line through D and C. Label the point of intersection of these two lines F. Hide the lines and circle and create segments to complete golden rectangle $AEFD$.



4. Measure $\overline{AE}$, $\overline{EF}$,and $\overline{BE}$. Find the ratio of AE to EF and the ratio of EF to BE. Compare these ratios to those found in Step 1. What do you notice?

Try This

1. Adjust your construction from Step 2 so that the side of the original square is 2 units long. Use the Pythagorean Theorem to find the length of $\overline{MC}$. Calculate the length of $\overline{AE}$. Write the ratio of AE to EF as a fraction and as a decimal rounded to the nearest thousandth.
2. Find the length of $\overline{BE}$ in your construction from Step 3. Write the ratio of EF to BE as a fraction and as a decimal rounded to the nearest thousandth. Compare your results to those from Try This Problem 1. What do you notice?
3. Each number in the Fibonacci sequence $(1, 1, 2, 3, 5, 8, 13 \ldots)$ is created by adding the two preceding numbers together. That is, $1 + 1 = 2$, $1 + 2 = 3$, $2 + 3 = 5$, and so on. Investigate the ratios of the numbers in the sequence by finding the quotients. $\frac{1}{1} = 1$, $\frac{2}{1} = 2$, $\frac{3}{2} = 1.5$, $\frac{5}{3} = 1.\overline{666}$, $\frac{8}{5} = 1.6$, and so on. What do you notice as you continue to find the quotients?

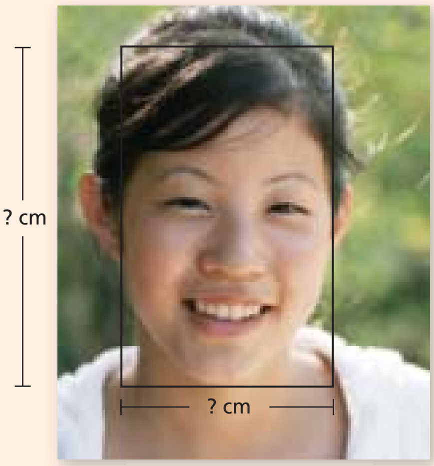
Tell why each of the following is an example of the appearance of the Fibonacci sequence in nature.

4. 

5. 

Determine whether each picture is an example of an application of the golden rectangle. Measure the length and the width of each and decide whether the ratio of the length to the width is approximately the golden ratio.

6. 

7. 

7-2 Ratios in Similar Polygons

TEKS G.5.B Geometric patterns: use ... geometric patterns to make generalizations about ratios in similar figures ...

Objectives

Identify similar polygons.

Apply properties of similar polygons to solve problems.

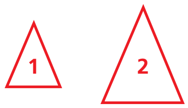
Vocabulary

similar
similar polygons
similarity ratio

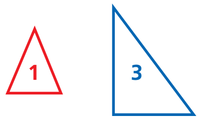
Why learn this?

Similar polygons are used to build models of actual objects. (See Example 3.)

Figures that are **similar** ($\sim$) have the same shape but not necessarily the same size.



$\triangle 1$ is similar to $\triangle 2$ ($\triangle 1 \sim \triangle 2$).

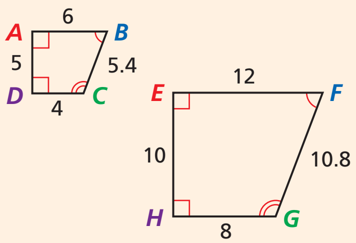


$\triangle 1$ is not similar to $\triangle 3$ ($\triangle 1 \nsim \triangle 3$).

Also G.11.A, G.11.B

Know it! Note

Similar Polygons

DEFINITION	DIAGRAM	STATEMENTS
Two polygons are **similar polygons** if and only if their corresponding angles are congruent and their corresponding side lengths are proportional.	 $ABCD \sim EFGH$	$\angle A \cong \angle E$ $\angle B \cong \angle F$ $\angle C \cong \angle G$ $\angle D \cong \angle H$ $\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE} = \frac{1}{2}$

EXAMPLE 1

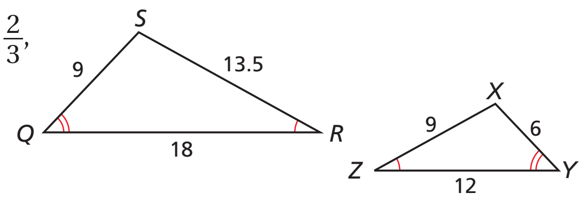
Describing Similar Polygons

Identify the pairs of congruent angles and corresponding sides.

$\angle Z \cong \angle R$ and $\angle Y \cong \angle Q$. By the Third Angles Theorem, $\angle X \cong \angle S$.

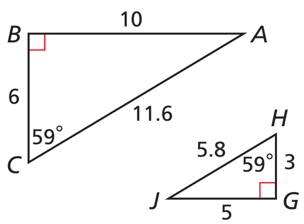
$\frac{XY}{SQ} = \frac{6}{9} = \frac{2}{3}$, $\frac{YZ}{QR} = \frac{12}{18} = \frac{2}{3}$,

$\frac{XZ}{SR} = \frac{9}{13.5} = \frac{2}{3}$

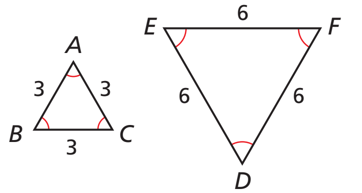


CHECK IT OUT!

1. Identify the pairs of congruent angles and corresponding sides.



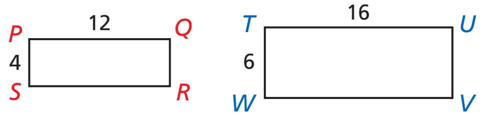
A **similarity ratio** is the ratio of the lengths of the corresponding sides of two similar polygons. The similarity ratio of $\triangle ABC$ to $\triangle DEF$ is $\frac{3}{6}$, or $\frac{1}{2}$. The similarity ratio of $\triangle DEF$ to $\triangle ABC$ is $\frac{6}{3}$, or 2.



EXAMPLE 2 Identifying Similar Polygons

Determine whether the polygons are similar. If so, write the similarity ratio and a similarity statement.

A rectangles *PQRS* and *TUVW*



Step 1 Identify pairs of congruent angles.

$\angle P \cong \angle T$, $\angle Q \cong \angle U$,

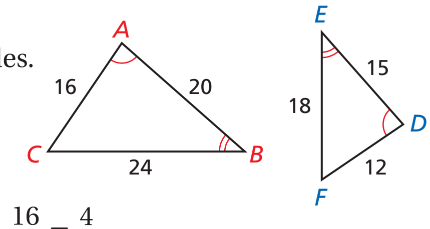
$\angle R \cong \angle V$, and $\angle S \cong \angle W$ *All ∠s of a rect. are rt. ∠s and are ≅.*

Step 2 Compare corresponding sides.

$$\frac{PQ}{TU} = \frac{12}{16} = \frac{3}{4}, \quad \frac{PS}{TW} = \frac{4}{6} = \frac{2}{3}$$

Since corresponding sides are not proportional, the rectangles are not similar.

B $\triangle ABC$ and $\triangle DEF$



Step 1 Identify pairs of congruent angles.

$\angle A \cong \angle D$, $\angle B \cong \angle E$ *Given*

$\angle C \cong \angle F$ *Third ∠s Thm.*

Step 2 Compare corresponding sides.

$$\frac{AB}{DE} = \frac{20}{15} = \frac{4}{3}, \quad \frac{BC}{EF} = \frac{24}{18} = \frac{4}{3}, \quad \frac{AC}{DF} = \frac{16}{12} = \frac{4}{3}$$

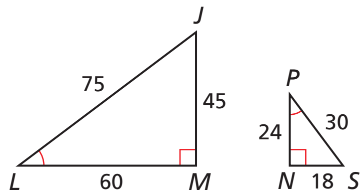
Thus the similarity ratio is $\frac{4}{3}$, and $\triangle ABC \sim \triangle DEF$.

Writing Math

Writing a similarity statement is like writing a congruence statement—be sure to list corresponding vertices in the same order.

CHECK IT OUT!

2. Determine if $\triangle JLM \sim \triangle NPS$. If so, write the similarity ratio and a similarity statement.

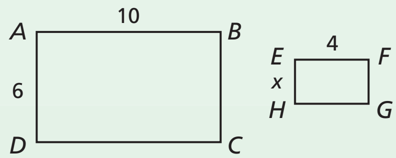


Student to Student

Proportions with Similar Figures

Anna Woods
Westwood High School

When I set up a proportion, I make sure each ratio compares the figures in the same order. To find x, I wrote $\frac{10}{4} = \frac{6}{x}$. This will work because the first ratio compares the lengths starting with rectangle ABCD. The second ratio compares the widths, also starting with rectangle ABCD.

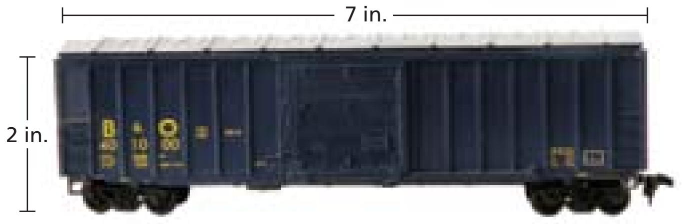


ABCD ~ EFGH

EXAMPLE 3 *Hobby Application*

A Railbox boxcar can be used to transport auto parts. If the length of the actual boxcar is 50 ft, find the width of the actual boxcar to the nearest tenth of a foot.

Let x be the width of the actual boxcar in feet. The rectangular model of a boxcar is similar to the rectangular boxcar, so the corresponding lengths are proportional.



$$\frac{\text{length of boxcar}}{\text{length of model}} = \frac{\text{width of boxcar}}{\text{width of model}}$$

$$\frac{50}{7} = \frac{x}{2}$$

$$7x = (50)(2) \quad \textit{Cross Products Prop.}$$

$$7x = 100 \quad \textit{Simplify.}$$

$$x \approx 14.3 \quad \textit{Divide both sides by 7.}$$

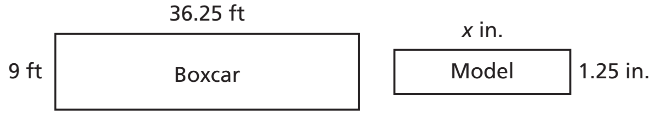
The width of the model is approximately 14.3 ft.

Helpful Hint

When you work with proportions, be sure the ratios compare corresponding measures.

CHECK IT OUT!

3. A boxcar has the dimensions shown. A model of the boxcar is 1.25 in. wide. Find the length of the model to the nearest inch.



THINK AND DISCUSS

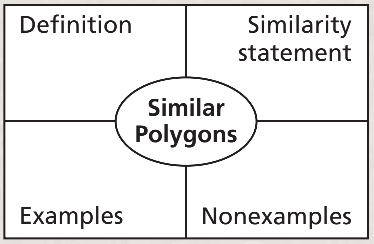
1. If you combine the symbol for similarity with the equal sign, what symbol is formed?

2. The similarity ratio of rectangle $ABCD$ to rectangle $EFGH$ is $\frac{1}{9}$. How do the side lengths of rectangle $ABCD$ compare to the corresponding side lengths of rectangle $EFGH$?

3. What shape(s) are always similar?

Know it! Note

4. GET ORGANIZED Copy and complete the graphic organizer. Write the definition of similar polygons, and a similarity statement. Then draw examples and nonexamples of similar polygons.



7-2 Exercises

go.hrw.com/Geo/TX
Homework Help Online
KEYWORD: MG7 7-2
Parent Resources Online
KEYWORD: MG7 Parent

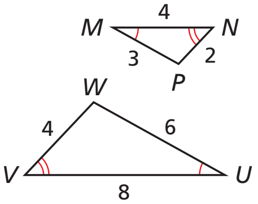
GUIDED PRACTICE

1. **Vocabulary** Give an example of similar figures in your classroom.

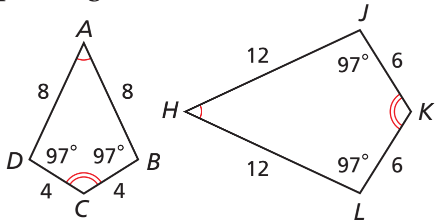
SEE EXAMPLE 1
p. 462

Identify the pairs of congruent angles and corresponding sides.

2.



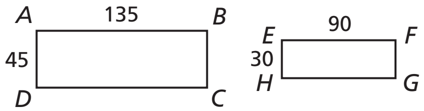
3.



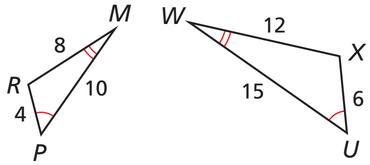
SEE EXAMPLE 2
p. 463

Multi-Step **Determine whether the polygons are similar. If so, write the similarity ratio and a similarity statement.**

4. rectangles *ABCD* and *EFGH*



5. $\triangle RMP$ and $\triangle UWX$



SEE EXAMPLE 3
p. 464

6. **Art** The town of Goodland, Kansas, claims that it has one of the world's largest easels. It holds an enlargement of a van Gogh painting that is 24 ft wide. The original painting is 58 cm wide and 73 cm tall. If the reproduction is similar to the original, what is the height of the reproduction to the nearest foot?

PRACTICE AND PROBLEM SOLVING

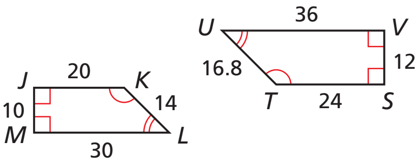
Independent Practice

For Exercises	See Example
7–8	1
9–10	2
11	3

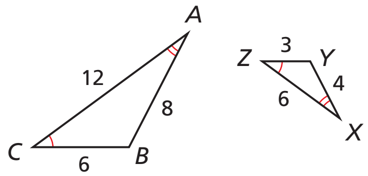
TEKS TAKS
Skills Practice p. S16
Application Practice p. S34

Identify the pairs of congruent angles and corresponding sides.

7.

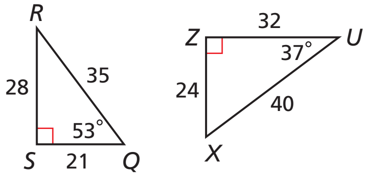


8.

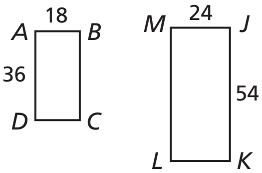


Multi-Step **Determine whether the polygons are similar. If so, write the similarity ratio and a similarity statement.**

9. $\triangle RSQ$ and $\triangle UXZ$



10. rectangles *ABCD* and *JKLM*



11. **Hobbies** The ratio of the model car's dimensions to the actual car's dimensions is $\frac{1}{56}$. The model has a length of 3 in. What is the length of the actual car?

12. Square *ABCD* has an area of 4 m^2. Square *PQRS* has an area of 36 m^2. What is the similarity ratio of square *ABCD* to square *PQRS*? What is the similarity ratio of square *PQRS* to square *ABCD*?

Tell whether each statement is sometimes, always, or never true.

13. Two right triangles are similar.

14. Two squares are similar.

15. A parallelogram and a trapezoid are similar.

16. If two polygons are congruent, they are also similar.

17. If two polygons are similar, they are also congruent.

18. **Critical Thinking** Explain why any two regular polygons having the same number of sides are similar.

LINK

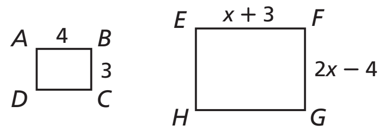
Monument

The height of the Statue of Liberty from the foundation of the pedestal to the torch is 305 ft. Her index finger measures 8 ft, and the fingernail is 13 in. by 10 in.

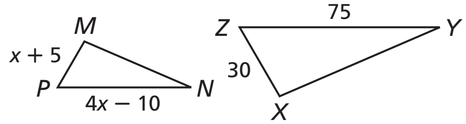
Source: libertystatepark.org

Find the value of *x*.

19. $ABCD \sim EFGH$



20. $\triangle MNP \sim \triangle XYZ$



21. **Estimation** The Statue of Liberty's hand is 16.4 ft long. Assume that your own body is similar to that of the Statue of Liberty and estimate the length of the Statue of Liberty's nose. (*Hint:* Use a ruler to measure your own hand and nose. Then set up a proportion.)

22. Write the definition of similar polygons as two conditional statements.

23. $\square JKLM \sim \square NOPQ$. If $m\angle K = 75°$, name two 75° angles in $\square NOPQ$.

24. A dining room is 18 ft long and 14 ft wide. On a blueprint for the house, the dining room is 3.5 in. long. To the nearest tenth of an inch, what is the width of the dining room on the blueprint?

25. **Write About It** Two similar polygons have a similarity ratio of 1 : 1. What can you say about the two polygons? Explain.

MULTI-STEP TAKS PREP

26. This problem will prepare you for the Multi-Step TAKS Prep on page 478.

A stage set consists of a painted backdrop with some wooden flats in front of it. One of the flats shows a tree that has a similarity ratio of $\frac{1}{2}$ to an actual tree. To give an illusion of distance, the backdrop includes a small painted tree that has a similarity ratio of $\frac{1}{10}$ to the tree on the flat.

a. The tree on the backdrop is 0.9 ft tall. What is the height of the tree on the flat?

b. What is the height of the actual tree?

c. Find the similarity ratio of the tree on the backdrop to the actual tree.

2ND PRINT

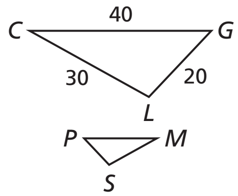
TEST PREP

27. Which value of y makes the two rectangles similar?

Ⓐ 3 Ⓒ 25.2

Ⓑ 8.2 Ⓓ 28.8

28. $\triangle CGL \sim \triangle MPS$. The similarity ratio of $\triangle CGL$ to $\triangle MPS$ is $\frac{5}{2}$. What is the length of $\overline{PS}$?



Ⓕ 8 Ⓗ 50

Ⓖ 12 Ⓙ 75

29. **Short Response** Explain why 1.5, 2.5, 3.5 and 6, 10, 12 cannot be corresponding sides of similar triangles.

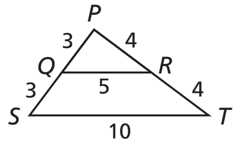
CHALLENGE AND EXTEND

30. **Architecture** An architect is designing a building that is 200 ft long and 140 ft wide. She builds a model so that the similarity ratio of the model to the building is $\frac{1}{500}$. What is the length and width of the model in inches?

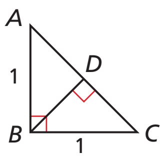
31. Write a paragraph proof.

Given: $\overline{QR} \parallel \overline{ST}$

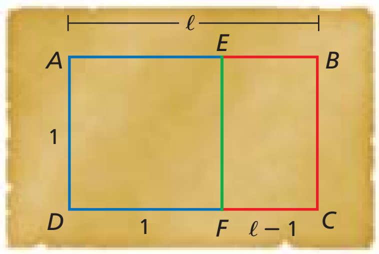
Prove: $\triangle PQR \sim \triangle PST$



32. In the figure, D is the midpoint of $\overline{AC}$.
   a. Find AC, DC, and DB.
   b. Use your results from part **a** to help you explain why $\triangle ABC \sim \triangle CDB$.



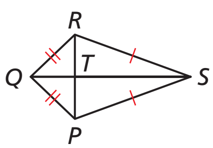
33. A golden rectangle has the following property: If a square is cut from one end of the rectangle, the rectangle that remains is similar to the original rectangle.
   a. Rectangle $ABCD$ is a golden rectangle. Write a similarity statement for rectangle $ABCD$ and rectangle $BCFE$.
   b. Write a proportion using the corresponding sides of these rectangles.
   c. Solve the proportion for ℓ. (*Hint:* Use the Quadratic Formula.)
   d. The value of ℓ is known as the golden ratio. Use a calculator to find ℓ to the nearest tenth.



SPIRAL REVIEW

34. There are four runners in a 200-meter race. Assuming there are no ties, in how many different orders can the runners finish the race? *(Previous course)*

In kite $PQRS$, $\overline{PS} \cong \overline{RS}$, $\overline{QR} \cong \overline{QP}$, $m\angle QPT = 45°$, and $m\angle RST = 20°$. Find each angle measure. *(Lesson 6-6)*



35. $m\angle QTR$ 36. $m\angle PST$ 37. $m\angle TPS$

Complete each of the following equations, given that $\frac{x}{4} = \frac{y}{10}$. *(Lesson 7-1)*

38. $10x = \square$ 39. $\frac{10}{y} = \square$ 40. $\frac{x}{y} = \square$

2ND PRINT

7-3 Technology LAB

Use with Lesson 7-3

Predict Triangle Similarity Relationships

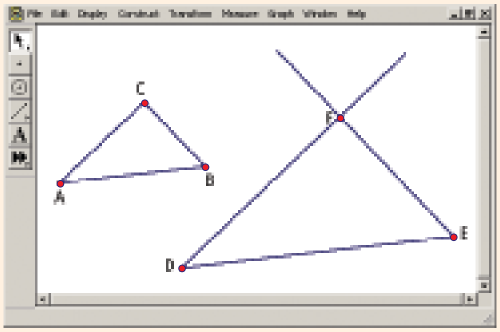
In Chapter 4, you found shortcuts for determining that two triangles are congruent. Now you will use geometry software to find ways to determine that triangles are similar.

TEKS G.11.A Similarity and the geometry of shape: use and extend similarity properties and transformations to explore and justify conjectures about geometric figures. Also G.2.A, G.3.B, G.9.B

go.hrw.com/Geo/TX
Lab Resources Online
KEYWORD: MG7 Lab7

Activity 1

1. Construct $\triangle ABC$. Construct $\overline{DE}$ longer than any of the sides of $\triangle ABC$. Rotate $\overline{DE}$ around D by rotation $\angle BAC$. Rotate $\overline{DE}$ around E by rotation $\angle ABC$. Label the intersection point of the two rotated segments as F.



2. Measure angles to confirm that $\angle BAC \cong \angle EDF$ and $\angle ABC \cong \angle DEF$. Drag a vertex of $\triangle ABC$ or an endpoint of $\overline{DE}$ to show that the two triangles have two pairs of congruent angles.

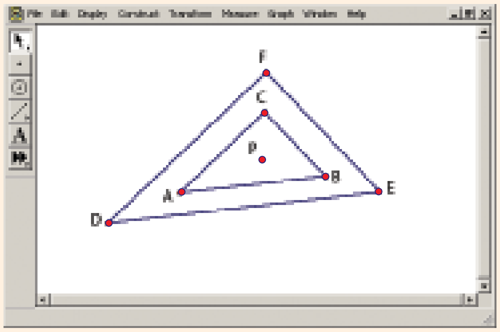
3. Measure the side lengths of both triangles. Divide each side length of $\triangle ABC$ by the corresponding side length of $\triangle DEF$. Compare the resulting ratios. What do you notice?

Try This

1. What theorem guarantees that the third pair of angles in the triangles are also congruent?
2. Will the ratios of corresponding sides found in Step 3 always be equal? Drag a vertex of $\triangle ABC$ or an endpoint of $\overline{DE}$ to investigate this question. State a conjecture based on your results.

Activity 2

1. Construct a new $\triangle ABC$. Create P in the interior of the triangle. Create $\triangle DEF$ by enlarging $\triangle ABC$ around P by a multiple of 2 using the Dilation command. Drag P outside of $\triangle ABC$ to separate the triangles.



2. Measure the side lengths of $\triangle DEF$ to confirm that each side is twice as long as the corresponding side of $\triangle ABC$. Drag a vertex of $\triangle ABC$ to verify that this relationship is true.

3. Measure the angles of both triangles. What do you notice?

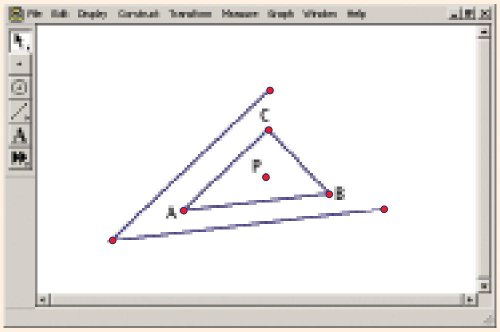
Try This

3. Did the construction of the triangles with three pairs of sides in the same ratio guarantee that the corresponding angles would be congruent? State a conjecture based on these results.

4. Compare your conjecture to the SSS Congruence Theorem from Chapter 4. How are they similar and how are they different?

Activity 3

1. Construct a different $\triangle ABC$. Create P in the interior of the triangle. Expand $\overline{AB}$ and $\overline{AC}$ around P by a multiple of 2 using the Dilation command. Create an angle congruent to $\angle BAC$ with sides that are each twice as long as $\overline{AB}$ and $\overline{AC}$.



2. Use a segment to create the third side of a new triangle and label it $\triangle DEF$. Drag P outside of $\triangle ABC$ to separate the triangles.

3. Measure each side length and determine the relationship between corresponding sides of $\triangle ABC$ and $\triangle DEF$.

4. Measure the angles of both triangles. What do you notice?

Try This

5. Tell whether $\triangle ABC$ is similar to $\triangle DEF$. Explain your reasoning.

6. Write a conjecture based on the activity. What congruency theorem is related to your conjecture?

7-3 Triangle Similarity: AA, SSS, and SAS

TEKS G.11.B Similarity and the geometry of shape: use ratios to solve problems involving similar figures. Also G.5.B, G.11.A

Objectives

Prove certain triangles are similar by using AA, SSS, and SAS.

Use triangle similarity to solve problems.

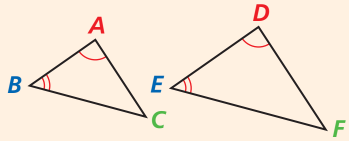
Who uses this?

Engineers use similar triangles when designing buildings, such as the Pyramid Building in San Diego, California. (See Example 5.)

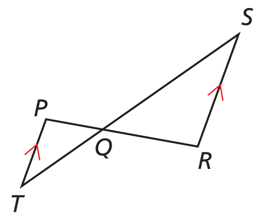
There are several ways to prove certain triangles are similar. The following postulate, as well as the SSS and SAS Similarity Theorems, will be used in proofs just as SSS, SAS, ASA, HL, and AAS were used to prove triangles congruent.

Know it! Note

Postulate 7-3-1 Angle-Angle (AA) Similarity

POSTULATE	HYPOTHESIS	CONCLUSION
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.		$\triangle ABC \sim \triangle DEF$

EXAMPLE 1 Using the AA Similarity Postulate

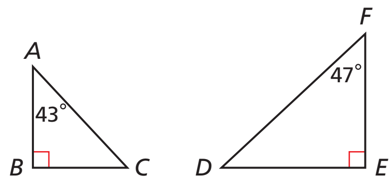


Explain why the triangles are similar and write a similarity statement.

Since $\overline{PT} \parallel \overline{SR}$, $\angle P \cong \angle R$, and $\angle T \cong \angle S$ by the Alternate Interior Angles Theorem. Therefore $\triangle PQT \sim \triangle RQS$ by AA $\sim$.

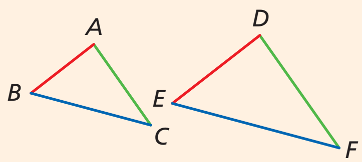
CHECK IT OUT!

1. Explain why the triangles are similar and write a similarity statement.



Know it! Note

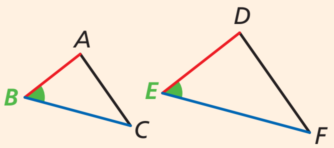
Theorem 7-3-2 Side-Side-Side (SSS) Similarity

THEOREM	HYPOTHESIS	CONCLUSION
If the three sides of one triangle are proportional to the three corresponding sides of another triangle, then the triangles are similar.		$\triangle ABC \sim \triangle DEF$

You will prove Theorem 7-3-2 in Exercise 38.

Know it! Note

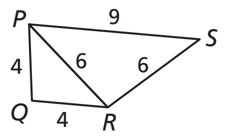
Theorem 7-3-3 Side-Angle-Side (SAS) Similarity

THEOREM	HYPOTHESIS	CONCLUSION
If two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.	 $\angle B \cong \angle E$	$\triangle ABC \sim \triangle DEF$

You will prove Theorem 7-3-3 in Exercise 39.

EXAMPLE 2 Verifying Triangle Similarity

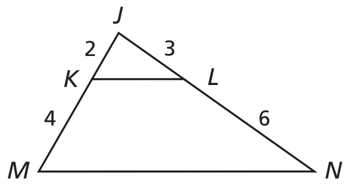
Verify that the triangles are similar.



A **$\triangle PQR$ and $\triangle PRS$**

$\frac{PQ}{PR} = \frac{4}{6} = \frac{2}{3}$, $\frac{QR}{RS} = \frac{4}{6} = \frac{2}{3}$, $\frac{PR}{PS} = \frac{6}{9} = \frac{2}{3}$

Therefore $\triangle PQR \sim \triangle PRS$ by SSS $\sim$.



B **$\triangle JKL$ and $\triangle JMN$**

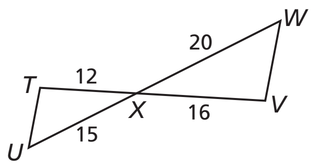
$\angle J \cong \angle J$ by the Reflexive Property of $\cong$.

$\frac{JK}{JM} = \frac{2}{6} = \frac{1}{3}$, $\frac{JL}{JN} = \frac{3}{9} = \frac{1}{3}$

Therefore $\triangle JKL \sim \triangle JMN$ by SAS $\sim$.

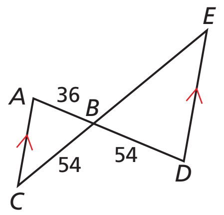
CHECK IT OUT!

2. Verify that $\triangle TXU \sim \triangle VXW$.



EXAMPLE 3 Finding Lengths in Similar Triangles

Explain why $\triangle ABC \sim \triangle DBE$ and then find BE.



Step 1 Prove triangles are similar.

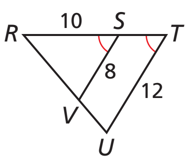
As shown $\overline{AC} \parallel \overline{ED}$, $\angle A \cong \angle D$, and $\angle C \cong \angle E$ by the Alternate Interior Angles Theorem. Therefore $\triangle ABC \sim \triangle DBE$ by AA $\sim$.

Step 2 Find BE.

$\frac{AB}{DB} = \frac{BC}{BE}$	*Corr. sides are proportional.*
$\frac{36}{54} = \frac{54}{BE}$	*Substitute 36 for AB, 54 for DB, and 54 for BC.*
$36(BE) = 54^2$	*Cross Products Prop.*
$36(BE) = 2916$	*Simplify.*
$BE = 81$	*Divide both sides by 36.*

CHECK IT OUT!

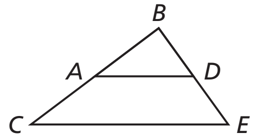
3. Explain why $\triangle RSV \sim \triangle RTU$ and then find RT.



EXAMPLE 4 Writing Proofs with Similar Triangles

Given: A is the midpoint of $\overline{BC}$.
D is the midpoint of $\overline{BE}$.

Prove: $\triangle BDA \sim \triangle BEC$



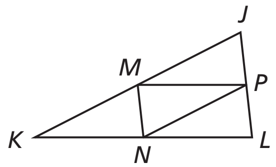
Proof:

Statements	Reasons
1. A is the mdpt. of $\overline{BC}$. D is the mdpt. of $\overline{BE}$.	**1.** Given
2. $\overline{BA} \cong \overline{AC}$, $\overline{BD} \cong \overline{DE}$	**2.** Def. of mdpt.
3. $BA = AC$, $BD = DE$	**3.** Def. of $\cong$ seg.
4. $BC = BA + AC$, $BE = BD + DE$	**4.** Seg. Add. Post.
5. $BC = BA + BA$, $BE = BD + BD$	**5.** Subst. Prop.
6. $BC = 2BA$, $BE = 2BD$	**6.** Simplify.
7. $\frac{BC}{BA} = 2$, $\frac{BE}{BD} = 2$	**7.** Div. Prop. of $=$
8. $\frac{BC}{BA} = \frac{BE}{BD}$	**8.** Trans. Prop. of $=$
9. $\angle B \cong \angle B$	**9.** Reflex. Prop. of $\cong$
10. $\triangle BDA \sim \triangle BEC$	**10.** SAS $\sim$ *Steps 8, 9*

CHECK IT OUT!

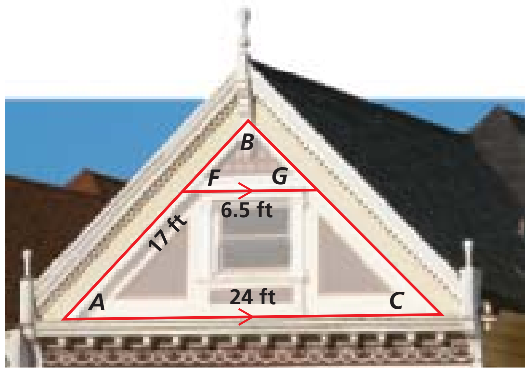
4. Given: M is the midpoint of $\overline{JK}$.
N is the midpoint of $\overline{KL}$,
and P is the midpoint of $\overline{JL}$.

Prove: $\triangle JKL \sim \triangle NPM$
(*Hint:* Use the Triangle Midsegment Theorem and SSS $\sim$.)



EXAMPLE 5 *Engineering Application*

The photo shows a gable roof. $\overline{AC} \parallel \overline{FG}$. Use similar triangles to prove $\triangle ABC \sim \triangle FBG$ and then find BF to the nearest tenth of a foot.



Step 1 Prove the triangles are similar.

$\overline{AC} \parallel \overline{FG}$ — *Given*
$\angle BFG \cong \angle BAC$ — *Corr. ∠s Thm.*
$\angle B \cong \angle B$ — *Reflex. Prop. of $\cong$*

Therefore $\triangle ABC \sim \triangle FBG$ by AA $\sim$.

Step 2 Find BF.

$$\begin{aligned} \frac{BA}{AC} &= \frac{BF}{FG} && \textit{Corr. sides are proportional.} \\ \frac{x+17}{24} &= \frac{x}{6.5} && \textit{Substitute the given values.} \\ 6.5(x+17) &= 24x && \textit{Cross Products Prop.} \\ 6.5x + 110.5 &= 24x && \textit{Distrib. Prop.} \\ 110.5 &= 17.5x && \textit{Subtract 6.5x from both sides.} \\ 6.3 &\approx x \text{ or } BF && \textit{Divide both sides by 17.5.} \end{aligned}$$

CHECK IT OUT!

5. What if...? If $AB = 4x$, $AC = 5x$, and $BF = 4$, find FG. 5

You learned in Chapter 2 that the Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence. These properties also hold true for similarity of triangles.

Know it! Note

Properties of Similarity

Reflexive Property of Similarity

$\triangle ABC \sim \triangle ABC$ (Reflex. Prop. of $\sim$)

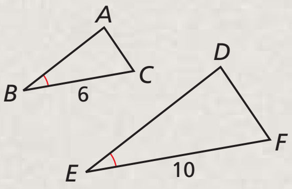
Symmetric Property of Similarity

If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$. (Sym. Prop. of $\sim$)

Transitive Property of Similarity

If $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle XYZ$, then $\triangle ABC \sim \triangle XYZ$. (Trans. Prop. of $\sim$)

THINK AND DISCUSS



1. What additional information, if any, would you you need in order to show that $\triangle ABC \sim \triangle DEF$ by the AA Similarity Postulate?

2. What additional information, if any, would you need in order to show that $\triangle ABC \sim \triangle DEF$ by the SAS Similarity Theorem?

3. Do corresponding sides of similar triangles need to be proportional and congruent? Explain.

Know it! Note

4. GET ORGANIZED Copy and complete the graphic organizer. If possible, write a congruence or similarity theorem or postulate in each section of the table. Include a marked diagram for each.

	Congruence	Similarity
SSS		
SAS		
AA		

2ND PRINT

7-3 Exercises

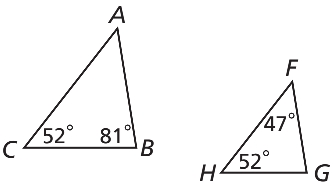
go.hrw.com/Geo/TX
Homework Help Online
KEYWORD: MG7 7-3
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

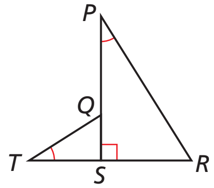
SEE EXAMPLE 1
p. 470

Explain why the triangles are similar and write a similarity statement.

1.



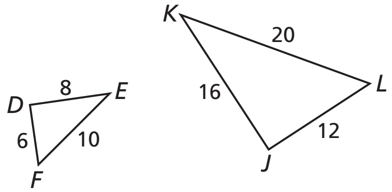
2.



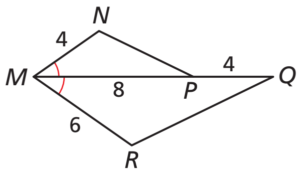
SEE EXAMPLE 2
p. 471

Verify that the triangles are similar.

3. $\triangle DEF$ and $\triangle JKL$



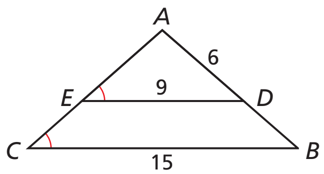
4. $\triangle MNP$ and $\triangle MRQ$



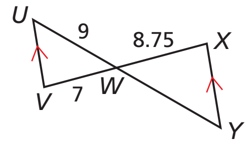
SEE EXAMPLE 3
p. 471

Multi-Step **Explain why the triangles are similar and then find each length.**

5. AB



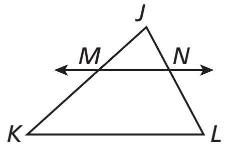
6. WY



SEE EXAMPLE 4
p. 472

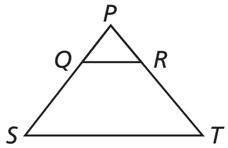
7. Given: $\overleftrightarrow{MN} \parallel \overline{KL}$

Prove: $\triangle JMN \sim \triangle JKL$



8. Given: $SQ = 2QP$, $TR = 2RP$

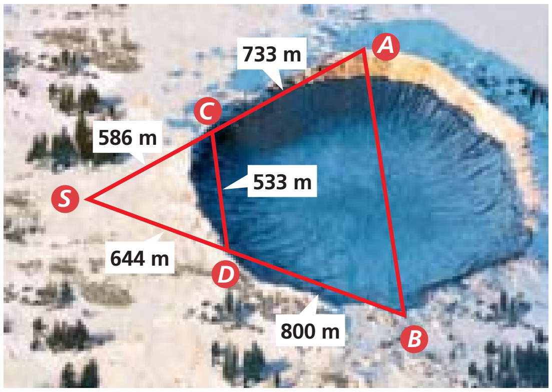
Prove: $\triangle PQR \sim \triangle PST$



9. The coordinates of A, B, and C are $A(0, 0)$, $B(2, 6)$, and $C(8, -2)$. What theorem or postulate justifies the statement $\triangle ABC \sim \triangle ADE$, if the coordinates of D and E are twice the coordinates of B and C?

SEE EXAMPLE 5
p. 472

10. **Surveying** In order to measure the distance AB across the meteorite crater, a surveyor at S locates points A, B, C, and D as shown. What is AB to the nearest meter? nearest kilometer?



PRACTICE AND PROBLEM SOLVING

Independent Practice

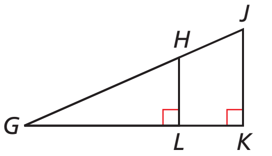
For Exercises	See Example
11–12	1
13–14	2
15–16	3
17–18	4
19	5

TEKS TAKS

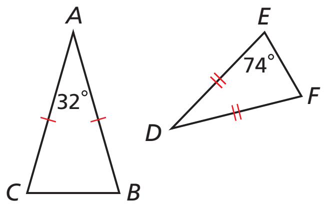
Skills Practice p. S16
Application Practice p. S34

Explain why the triangles are similar and write a similarity statement.

11.

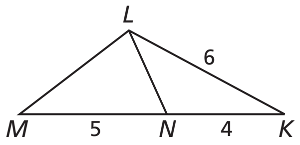


12.

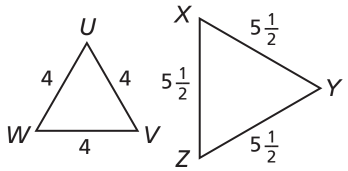


Verify that the given triangles are similar.

13. $\triangle KLM$ and $\triangle KNL$

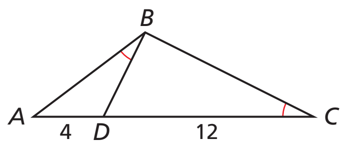


14. $\triangle UVW$ and $\triangle XYZ$

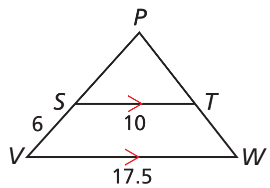


Multi-Step **Explain why the triangles are similar and then find each length.**

15. AB

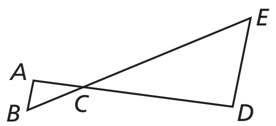


16. PS



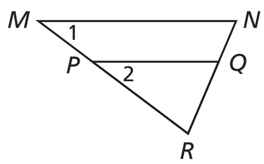
17. Given: $CD = 3AC$, $CE = 3BC$

Prove: $\triangle ABC \sim \triangle DEC$

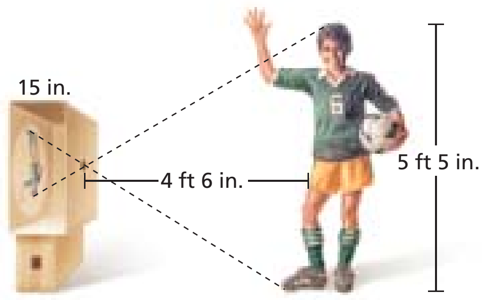


18. Given: $\frac{PR}{MR} = \frac{QR}{NR}$

Prove: $\angle 1 \cong \angle 2$



19. Photography The picture shows a person taking a pinhole photograph of himself. Light entering the opening reflects his image on the wall, forming similar triangles. What is the height of the image to the nearest tenth of an inch?



Draw $\triangle JKL$ and $\triangle MNP$. Determine if you can conclude that $\triangle JKL \sim \triangle MNP$ based on the given information. If so, which postulate or theorem justifies your response?

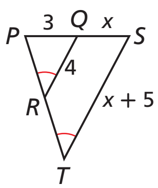
20. $\angle K \cong \angle N$, $\frac{JK}{MN} = \frac{KL}{NP}$

21. $\frac{JK}{MN} = \frac{KL}{NP} = \frac{JL}{MP}$

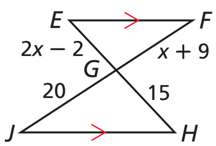
22. $\angle J \cong \angle M$, $\frac{JL}{MP} = \frac{KL}{NP}$

Find the value of x.

23.



24.



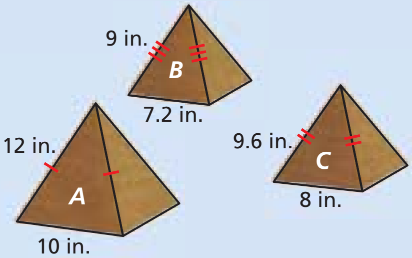
MULTI-STEP TAKS PREP

25. This problem will prepare you for the Multi-Step TAKS Prep on page 478.

The set for an animated film includes three small triangles that represent pyramids.

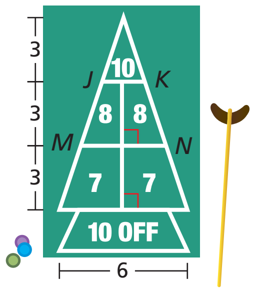
a. Which pyramids are similar? Why?

b. What is the similarity ratio of the similar pyramids?



26. **Critical Thinking** $\triangle ABC$ is not similar to $\triangle DEF$, and $\triangle DEF$ is not similar to $\triangle XYZ$. Could $\triangle ABC$ be similar to $\triangle XYZ$? Why or why not? Make a sketch to support your answer.

27. **Recreation** To play shuffleboard, two teams take turns sliding disks on a court. The dimensions of the scoring area for a standard shuffleboard court are shown. What are JK and MN?



28. Prove the Transitive Property of Similarity.

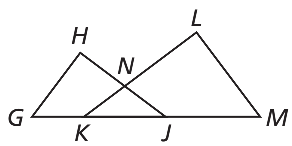
Given: $\triangle ABC \sim \triangle DEF$, $\triangle DEF \sim \triangle XYZ$

Prove: $\triangle ABC \sim \triangle XYZ$

29. Draw and label $\triangle PQR$ and $\triangle STU$ such that $\frac{PQ}{ST} = \frac{QR}{TU}$ but $\triangle PQR$ is NOT similar to $\triangle STU$.

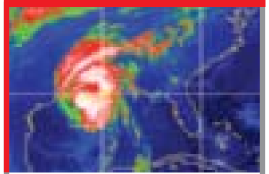
30. Given: $\triangle KNJ$ is isosceles with $\angle N$ as the vertex angle. $\angle H \cong \angle L$

Prove: $\triangle GHJ \sim \triangle MLK$



TEXAS LINK

Meteorology



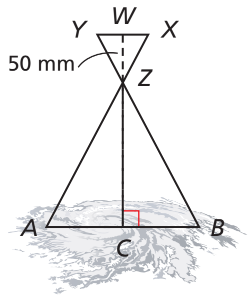
This satellite image shows Hurricane Lili as it moves across the Gulf of Mexico. In October 2002, an estimated 500,000 people evacuated in advance of Lili's hitting Texas.

31. **Meteorology** Satellite photography makes it possible to measure the diameter of a hurricane. The figure shows that a camera's aperture YX is 35 mm and its focal length WZ is 50 mm. The satellite W holding the camera is 150 mi above the hurricane, centered at C.

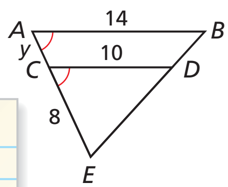
a. Why is $\triangle XYZ \sim \triangle ABZ$? What assumption must you make about the position of the camera in order to make this conclusion?

b. What other triangles in the figure must be similar? Why?

c. Find the diameter AB of the hurricane.



32. **ERROR ANALYSIS** Which solution for the value of y is incorrect? Explain the error.



A

$\triangle ABE \sim \triangle CDE$ by AA $\sim$,

so $\frac{14}{8+y} = \frac{10}{8}$. Then

$10(8 + y) = 8(14)$, or

$80 + 10y = 112$. So

$10y = 32$ and $y = 3.2$.

B

$\triangle ABE \sim \triangle CDE$ by AA $\sim$,

so $\frac{8}{10} = \frac{y}{14}$. Therefore

$8(14) = 10y$, which

means $10y = 112$ and

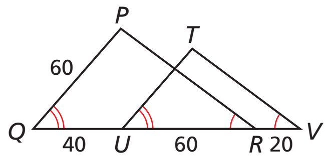
$y = 11.2$.

33. **Write About It** Two isosceles triangles have congruent vertex angles. Explain why the two triangles must be similar.

TEST PREP

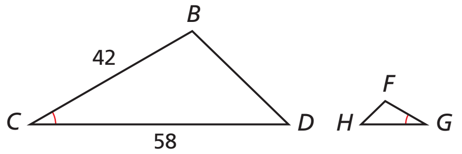
34. What is the length of $\overline{TU}$?

Ⓐ 36 Ⓒ 48
Ⓑ 40 Ⓓ 90



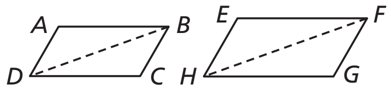
35. Which dimensions guarantee that $\triangle BCD \sim \triangle FGH$?

Ⓕ $FG = 11.6$, $GH = 8.4$
Ⓖ $FG = 12$, $GH = 14$
Ⓗ $FG = 11.4$, $GH = 11.4$
Ⓙ $FG = 10.5$, $GH = 14.5$



36. $\square ABCD \sim \square EFGH$. Which similarity postulate or theorem lets you conclude that $\triangle BCD \sim \triangle FGH$?

Ⓐ AA Ⓒ SAS
Ⓑ SSS Ⓓ None of these



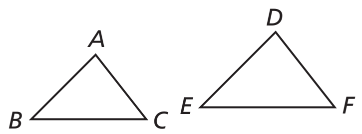
37. **Gridded Response** If 6, 8, and 12 and 15, 20, and x are the lengths of the corresponding sides of two similar triangles, what is the value of x?

CHALLENGE AND EXTEND

38. Prove the SSS Similarity Theorem.

Given: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Prove: $\triangle ABC \sim \triangle DEF$

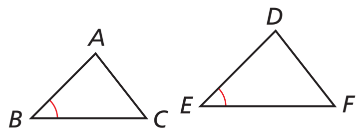


(*Hint:* Assume that $AB < DE$ and choose point X on $\overline{DE}$ so that $\overline{AB} \cong \overline{DX}$. Then choose point Y on $\overline{DF}$ so that $\overleftrightarrow{XY} \parallel \overline{EF}$. Show that $\triangle DXY \sim \triangle DEF$ and that $\triangle ABC \cong \triangle DXY$.)

39. Prove the SAS Similarity Theorem.

Given: $\angle B \cong \angle E$, $\frac{AB}{DE} = \frac{BC}{EF}$

Prove: $\triangle ABC \sim \triangle DEF$



(*Hint:* Assume that $AB < DE$ and choose point X on $\overline{DE}$ so that $\overline{EX} \cong \overline{BA}$. Then choose point Y on $\overline{EF}$ so that $\angle EXY \cong \angle EDF$. Show that $\triangle XEY \sim \triangle DEF$ and that $\triangle ABC \cong \triangle XEF$.)

40. Given $\triangle ABC \sim \triangle XYZ$, $m\angle A = 50°$, $m\angle X = (2x + 5y)°$, $m\angle Z = (5x + y)°$, and that $m\angle B = (102 - x)°$, find $m\angle Z$.

SPIRAL REVIEW

41. Jessika's scores in her last six rounds of golf were 96, 99, 105, 105, 94, and 107. What score must Jessika make on her next round to make her mean score 100? *(Previous course)*

Position each figure in the coordinate plane and give possible coordinates of each vertex. *(Lesson 4-7)*

42. a right triangle with leg lengths of 4 units and 2 units

43. a rectangle with length $2k$ and width k

Solve each proportion. Check your answer. *(Lesson 7-1)*

44. $\frac{2x}{10} = \frac{35}{25}$ **45.** $\frac{5y}{450} = \frac{25}{10y}$ **46.** $\frac{b-5}{28} = \frac{7}{b-5}$

2ND PRINT

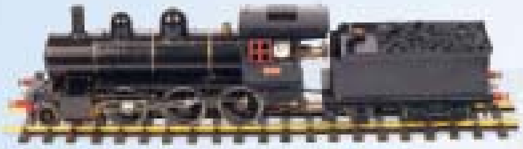
CHAPTER 7

MULTI-STEP TAKS PREP

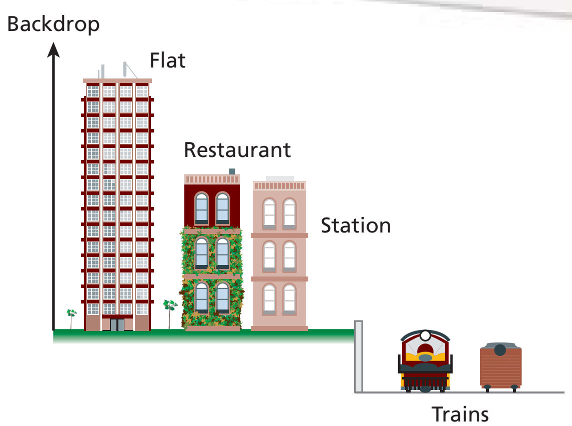
SECTION 7A

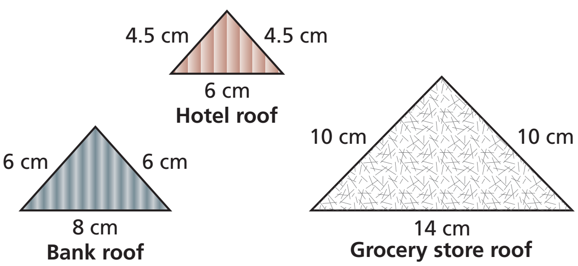
Similarity Relationships

Lights! Camera! Action! Lorenzo, Maria, Sam, and Tia are working on a video project for their history class. They decide to film a scene where the characters in the scene are on a train arriving at a town. Since Lorenzo collects model trains, they decide to use one of his trains and to build a set behind it. To create the set, they use a film technique called forced perspective. They want to use small objects to create an illusion of great distance in a very small space.



1. Lorenzo's model train is $\frac{1}{87}$ the size of the original train. He measures the engine of the model train and finds that it is $2\frac{1}{2}$ in. tall. What is the height of the real engine to the nearest foot?
2. The closest building to the train needs to be made using the same scale as the train. Maria and Sam estimate that the height of an actual station is 20 ft. How tall would they need to build their model of the train station to the nearest $\frac{1}{4}$ in.?
3. To give depth to their scene, they want to construct partial buildings behind the train station. Lorenzo decided to build a restaurant. If the height of the restaurant is actually 24 ft, how tall would they need to build their model of the restaurant to the nearest inch?
4. The other buildings on the set will have triangular roofs. Which of the roofs are similar to each other? Why?





READY TO GO ON?

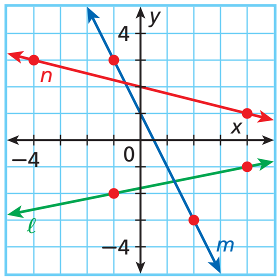
CHAPTER 7

SECTION 7A

Quiz for Lessons 7-1 Through 7-3

7-1 Ratio and Proportion

Write a ratio expressing the slope of each line.



1. ℓ
2. m
3. n
4. x-axis

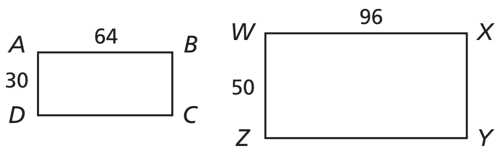
Solve each proportion.

5. $\dfrac{y}{6} = \dfrac{12}{9}$
6. $\dfrac{16}{24} = \dfrac{20}{t}$
7. $\dfrac{x-2}{4} = \dfrac{9}{x-2}$
8. $\dfrac{2}{3y} = \dfrac{y}{24}$
9. An architect's model for a building is 1.4 m long and 0.8 m wide. The actual building is 240 m wide. What is the length of the building?

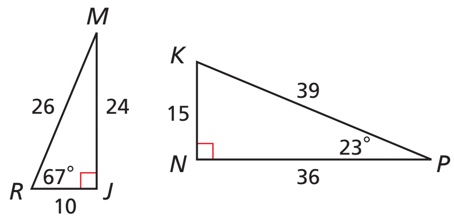
7-2 Ratios in Similar Polygons

Determine whether the two polygons are similar. If so, write the similarity ratio and a similarity statement.

10. rectangles $ABCD$ and $WXYZ$



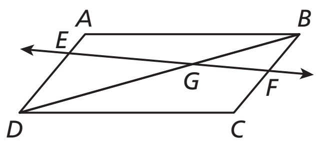
11. $\triangle JMR$ and $\triangle KNP$



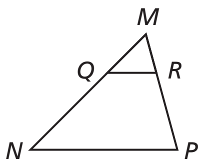
12. Leonardo da Vinci's famous portrait the *Mona Lisa* is 30 in. long and 21 in. wide. Janelle has a refrigerator magnet of the painting that is 3.5 cm wide. What is the length of the magnet?

7-3 Triangle Similarity: AA, SSS, and SAS

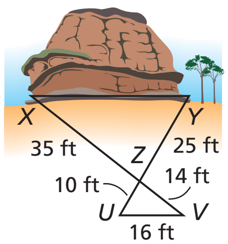
13. **Given:** $\square ABCD$
    Prove: $\triangle EDG \sim \triangle FBG$



14. **Given:** $MQ = \frac{1}{3}MN$, $MR = \frac{1}{3}MP$
    Prove: $\triangle MQR \sim \triangle MNP$



15. A geologist wants to measure the length XY of a rock formation. To do so, she locates points U, V, X, Y, and Z as shown. What is XY?



2ND PRINT

7-4 Technology LAB

Use with Lesson 7-4

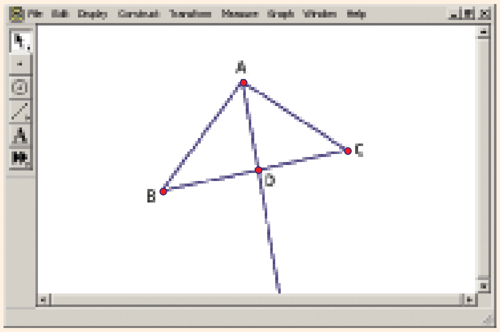
Investigate Angle Bisectors of a Triangle

In a triangle, an angle bisector divides the opposite side into two segments. You will use geometry software to explore the relationships between these segments.

TEKS G.5.B Geometric patterns: use... geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures... Also G.2.A, G.3.B, G.9.B

go.hrw.com/Geo/TX
Lab Resources Online
KEYWORD: MG7 Lab7

Activity 1



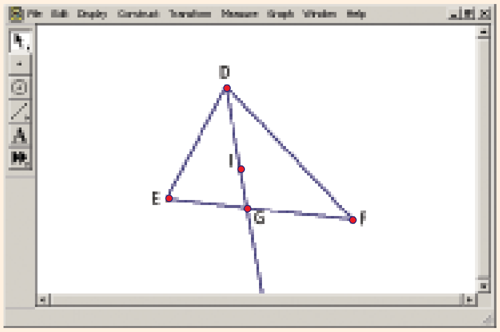
1. Construct $\triangle ABC$. Bisect $\angle BAC$ and create the point of intersection of the angle bisector and $\overline{BC}$. Label the intersection D.

2. Measure $\overline{AB}$, $\overline{AC}$, $\overline{BD}$, and $\overline{CD}$. Use these measurements to write ratios. What are the results? Drag a vertex of $\triangle ABC$ and examine the ratios again. What do you notice?

Try This

1. Choose Tabulate and create a table using the four lengths and the ratios from Step 2. Drag a vertex of $\triangle ABC$ and add the new measurements to the table. What conjecture can you make about the segments created by an angle bisector?
2. Write a proportion based on your conjecture.

Activity 2



1. Construct $\triangle DEF$. Create the *incenter* of the triangle and label it I. Hide the angle bisectors of $\angle E$ and $\angle F$. Find the point of intersection of $\overline{EF}$ and the bisector of $\angle D$. Label the intersection G.

2. Find DI, DG, and the perimeter of $\triangle DEF$.

3. Divide the length of $\overline{DI}$ by the length of DG. Add the lengths of $\overline{DE}$ and $\overline{DF}$. Then divide this sum by the perimeter of $\triangle DEF$. Compare the two quotients. Drag a vertex of $\triangle DEF$ and examine the quotients again. What do you notice?

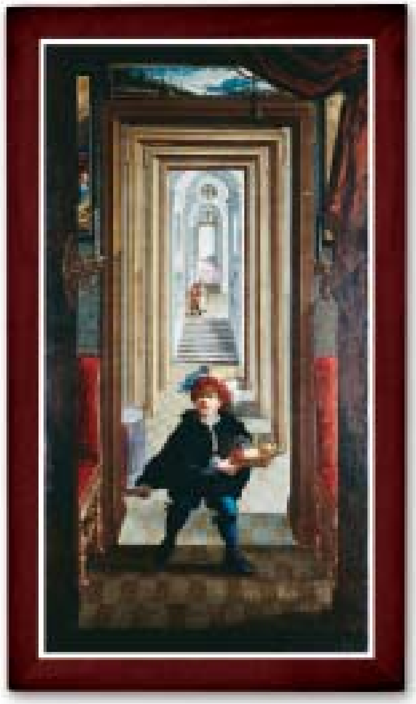
4. Write a proportion based on your quotients. What conjecture can you make about this relationship?

Try This

3. Show the hidden angle bisector of $\angle E$ or $\angle F$. Confirm that your conjecture is true for this bisector. Drag a vertex of $\triangle DEF$ and observe the results.
4. Choose Tabulate and create a table with the measurements you used in your proportion in Step 4.

2ND PRINT

7-4 Applying Properties of Similar Triangles



TEKS G.11.B Similarity and the geometry of shape: use ratios to solve problems involving similar figures. Also G.2.A, G.3.B, G.5.B, G.9.B, G.11.A

Objectives

Use properties of similar triangles to find segment lengths.

Apply proportionality and triangle angle bisector theorems.

Who uses this?

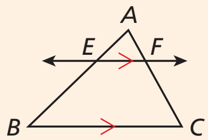
Artists use similarity and proportionality to give paintings an illusion of depth. (See Example 3.)

Artists use mathematical techniques to make two-dimensional paintings appear three-dimensional. The invention of *perspective* was based on the observation that far away objects look smaller and closer objects look larger.

Mathematical theorems like the Triangle Proportionality Theorem are important in making perspective drawings.

Know it! Note

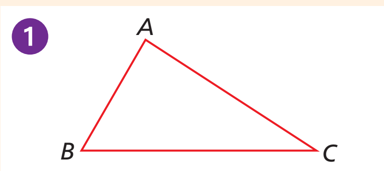
Theorem 7-4-1 Triangle Proportionality Theorem

THEOREM	HYPOTHESIS	CONCLUSION
If a line parallel to a side of a triangle intersects the other two sides, then it divides those sides proportionally.	 $\overline{EF} \parallel \overline{BC}$	$\frac{AE}{EB} = \frac{AF}{FC}$

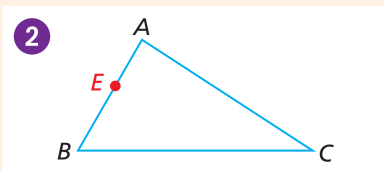
You can use a compass-and-straightedge construction to verify this theorem. Although the construction is not a proof, it should help convince you that the theorem is true. After you have completed the construction, use a ruler to measure $\overline{AE}$, $\overline{EB}$, $\overline{AF}$, and $\overline{FC}$ to see that $\frac{AE}{EB} = \frac{AF}{FC}$.

Construction Triangle Proportionality Theorem

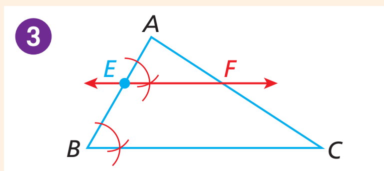
Construct a line parallel to a side of a triangle.



Use a straightedge to draw $\triangle ABC$.



Label E on $\overline{AB}$.

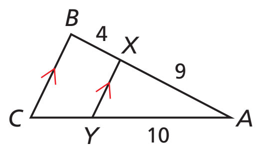


Construct $\angle E \cong \angle B$. Label the intersection of $\overleftrightarrow{EF}$ and $\overline{AC}$ as F. $\overleftrightarrow{EF} \parallel \overline{BC}$ by the Converse of the Corresponding Angles Postulate.

2ND PRINT

EXAMPLE 1 Finding the Length of a Segment

Find CY.



It is given that $\overline{XY} \parallel \overline{BC}$, so $\frac{AX}{XB} = \frac{AY}{YC}$ by the Triangle Proportionality Theorem.

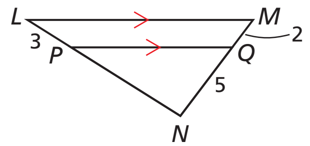
$\frac{9}{4} = \frac{10}{CY}$ — *Substitute 9 for AX, 4 for XB, and 10 for AY.*

$9(CY) = 40$ — *Cross Products Prop.*

$CY = \frac{40}{9}$, or $4\frac{4}{9}$ — *Divide both sides by 9.*

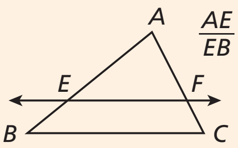
CHECK IT OUT!

1. Find PN.



Know It! Note

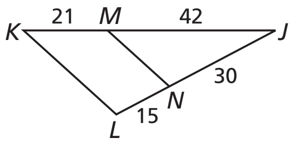
Theorem 7-4-2 Converse of the Triangle Proportionality Theorem

THEOREM	HYPOTHESIS	CONCLUSION
If a line divides two sides of a triangle proportionally, then it is parallel to the third side.	 $\frac{AE}{EB} = \frac{AF}{FC}$	$\overleftrightarrow{EF} \parallel \overline{BC}$

You will prove Theorem 7-4-2 in Exercise 23.

EXAMPLE 2 Verifying Segments are Parallel

Verify that $\overline{MN} \parallel \overline{KL}$.



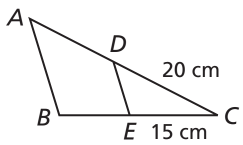
$\frac{JM}{MK} = \frac{42}{21} = 2$

$\frac{JN}{NL} = \frac{30}{15} = 2$

Since $\frac{JM}{MK} = \frac{JN}{NL}$, $\overline{MN} \parallel \overline{KL}$ by the Converse of the Triangle Proportionality Theorem.

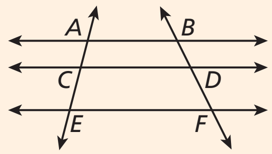
CHECK IT OUT!

2. $AC = 36$ cm, and $BC = 27$ cm. Verify that $\overline{DE} \parallel \overline{AB}$.



Know It! Note

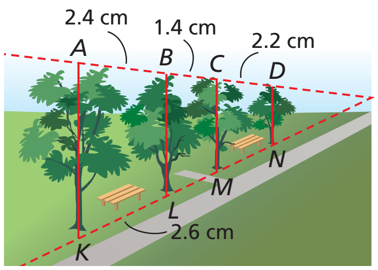
Corollary 7-4-3 Two-Transversal Proportionality

COROLLARY	HYPOTHESIS	CONCLUSION
If three or more parallel lines intersect two transversals, then they divide the transversals proportionally.		$\frac{AC}{CE} = \frac{BD}{DF}$

You will prove Corollary 7-4-3 in Exercise 24.

EXAMPLE 3 Art Application

An artist used perspective to draw guidelines to help her sketch a row of parallel trees. She then checked the drawing by measuring the distances between the trees. What is LN?



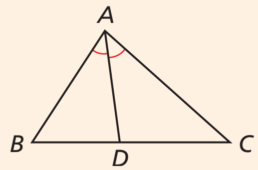
$\overline{AK} \parallel \overline{BL} \parallel \overline{CM} \parallel \overline{DN}$	*Given*
$\frac{KL}{LN} = \frac{AB}{BD}$	*2-Transv. Proportionality Corollary*
$BD = BC + CD$	*Seg. Add. Post.*
$BD = 1.4 + 2.2 = 3.6$ cm	*Substitute 1.4 for BC and 2.2 for CD.*
$\frac{2.6}{LN} = \frac{2.4}{3.6}$	*Substitute the given values.*
$2.4(LN) = 3.6(2.6)$	*Cross Products Prop.*
$LN = 3.9$ cm	*Divide both sides by 2.4.*

CHECK IT OUT! **3.** Use the diagram to find LM and MN to the nearest tenth.

The previous theorems and corollary lead to the following conclusion.

Know it! Note

Theorem 7-4-4 **Triangle Angle Bisector Theorem**

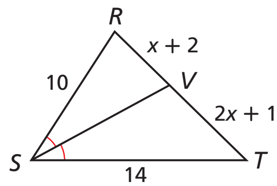
THEOREM	HYPOTHESIS	CONCLUSION
An angle bisector of a triangle divides the opposite side into two segments whose lengths are proportional to the lengths of the other two sides. ($\triangle \angle$ Bisector Thm.)		$\frac{BD}{DC} = \frac{AB}{AC}$

You will prove Theorem 7-4-4 in Exercise 38.

EXAMPLE 4 Using the Triangle Angle Bisector Theorem

Algebra

Find RV and VT.



$\frac{RV}{VT} = \frac{SR}{ST}$ by the $\triangle \angle$ Bisector Thm.

$\frac{x+2}{2x+1} = \frac{10}{14}$	*Substitute the given values.*
$14(x+2) = 10(2x+1)$	*Cross Products Prop.*
$14x + 28 = 20x + 10$	*Dist. Prop.*
$18 = 6x$	*Simplify.*
$x = 3$	*Divide both sides by 6.*

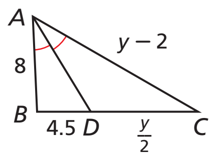
$RV = x + 2 = 3 + 2 = 5$ $\quad$ $VT = 2x + 1 = 2(3) + 1 = 7$ $\quad$ *Substitute 3 for x.*

Helpful Hint

You can check your answer by substituting the values into the proportion.

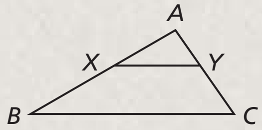
$$\frac{RV}{VT} = \frac{SR}{ST}$$
$$\frac{5}{7} = \frac{10}{14}$$
$$\frac{5}{7} = \frac{5}{7}$$

CHECK IT OUT! **4.** Find AC and DC.



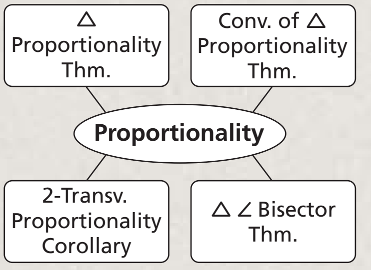
THINK AND DISCUSS

1. $\overline{XY} \parallel \overline{BC}$. Use what you know about similarity and proportionality to state as many different proportions as possible.



Know it! Note

2. **GET ORGANIZED** Copy and complete the graphic organizer. Draw a figure for each proportionality theorem or corollary and then measure it. Use your measurements to write an if-then statement about each figure.



7-4 Exercises

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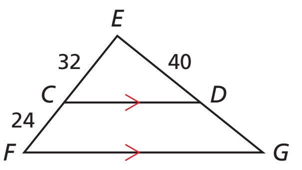
KEYWORD: MG7 7-4
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

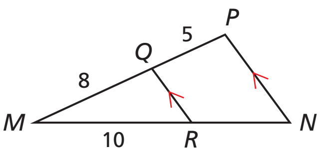
SEE EXAMPLE 1 p. 482

Find the length of each segment.

1. $\overline{DG}$



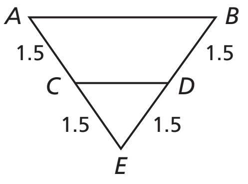
2. $\overline{RN}$



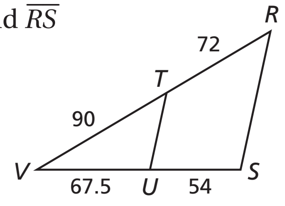
SEE EXAMPLE 2 p. 482

Verify that the given segments are parallel.

3. $\overline{AB}$ and $\overline{CD}$



4. $\overline{TU}$ and $\overline{RS}$



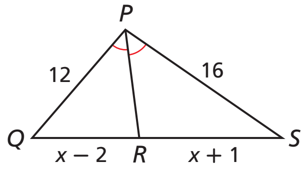
SEE EXAMPLE 3 p. 483

5. **Travel** The map shows the area around Herald Square in Manhattan, New York, and the approximate length of several streets. If the numbered streets are parallel, what is the length of Broadway between 34th St. and 35th St. to the nearest foot?

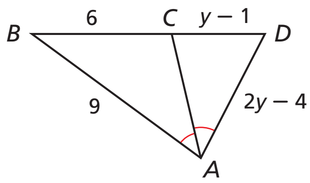
SEE EXAMPLE 4
p. 483

Find the length of each segment.

6. $\overline{QR}$ and $\overline{RS}$



7. $\overline{CD}$ and $\overline{AD}$



PRACTICE AND PROBLEM SOLVING

Independent Practice

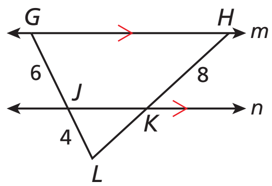
For Exercises	See Example
8–9	1
10–11	2
12	3
13–14	4

TEKS TAKS

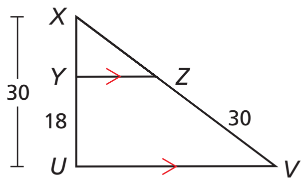
Skills Practice p. S17
Application Practice p. S34

Find the length of each segment.

8. $\overline{KL}$

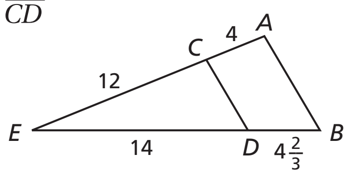


9. $\overline{XZ}$

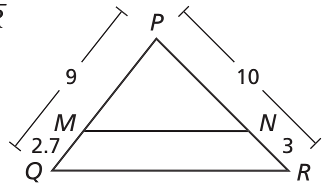


Verify that the given segments are parallel.

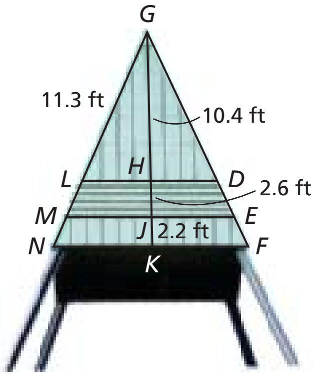
10. $\overline{AB}$ and $\overline{CD}$



11. $\overline{MN}$ and $\overline{QR}$

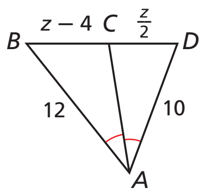


12. **Architecture** The wooden treehouse has horizontal siding that is parallel to the base. What are *LM* and *MN* to the nearest hundredth?

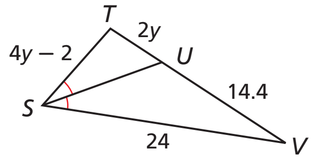


Find the length of each segment.

13. $\overline{BC}$ and $\overline{CD}$



14. $\overline{ST}$ and $\overline{TU}$



In the figure, $\overleftrightarrow{BC} \parallel \overleftrightarrow{DE} \parallel \overleftrightarrow{FG}$. Complete each proportion.

15. $\frac{AB}{BD} = \frac{AC}{\square}$

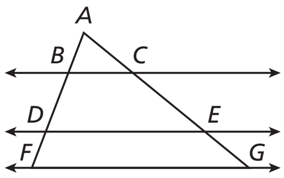
16. $\frac{\square}{DF} = \frac{AE}{EG}$

17. $\frac{DF}{\square} = \frac{EG}{CE}$

18. $\frac{AF}{AB} = \frac{\square}{AC}$

19. $\frac{BD}{CE} = \frac{\square}{EG}$

20. $\frac{AB}{AC} = \frac{BF}{\square}$



21. The bisector of an angle of a triangle divides the opposite side of the triangle into segments that are 12 in. and 16 in. long. Another side of the triangle is 20 in. long. What are two possible lengths for the third side?

Multi-Step TAKS Prep

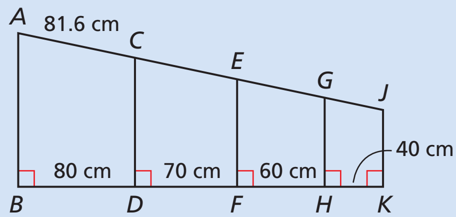
22. This problem will prepare you for the Multi-Step TAKS Prep on page 502.

Jaclyn is building a slide rail, the narrow, slanted beam found in skateboard parks.

a. Write a proportion that Jaclyn can use to calculate the length of $\overline{CE}$.

b. Find *CE*.

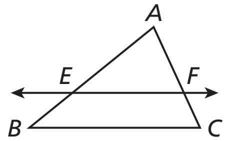
c. What is the overall length of the slide rail *AJ*?



23. Prove the Converse of the Triangle Proportionality Theorem.

Given: $\frac{AE}{EB} = \frac{AF}{FC}$

Prove: $\overleftrightarrow{EF} \parallel \overline{BC}$

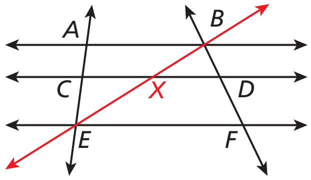


24. Prove the Two-Transversal Proportionality Corollary.

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

Prove: $\frac{AC}{CE} = \frac{BD}{DF}$

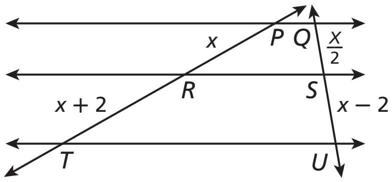
(*Hint:* Draw $\overleftrightarrow{BE}$ through *X*.)



25. Given that $\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS} \parallel \overleftrightarrow{TU}$

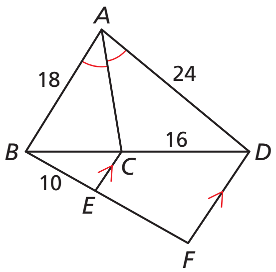
a. Find *PR*, *RT*, *QS*, and *SU*.

b. Use your results from part **b** to write a proportion relating the segment lengths.

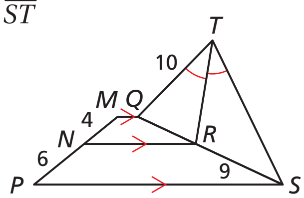


Find the length of each segment.

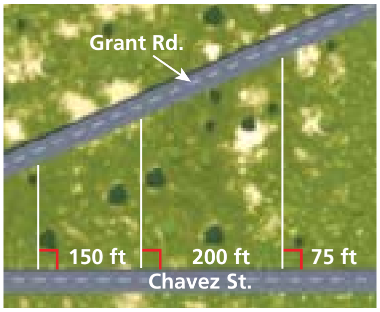
26. $\overline{EF}$



27. $\overline{ST}$

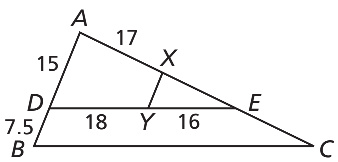


28. **Real Estate** A developer is laying out lots along Grant Rd. whose total width is 500 ft. Given the width of each lot along Chavez St., what is the width of each of the lots along Grant Rd. to the nearest foot?



29. **Critical Thinking** Explain how to use a sheet of lined notebook paper to divide a segment into five congruent segments. Which theorem or corollary do you use?

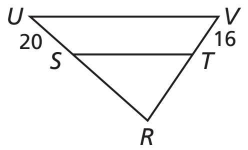
30. Given that $\overline{DE} \parallel \overline{BC}$, $\overline{XY} \parallel \overline{AD}$ Find *EC*.



31. **Write About It** In $\triangle ABC$, $\overrightarrow{AD}$ bisects $\angle BAC$. Write a proportionality statement for the triangle. What theorem supports your conclusion?

TEST PREP

32. Which dimensions let you conclude that $\overline{UV} \parallel \overline{ST}$?



Ⓐ $SR = 12$, $TR = 9$
Ⓑ $SR = 16$, $TR = 20$
Ⓒ $SR = 35$, $TR = 28$
Ⓓ $SR = 50$, $TR = 48$

33. In $\triangle ABC$, the bisector of $\angle A$ divides $\overline{BC}$ into segments with lengths 16 and 20. $AC = 25$. Which of these could be the length of $\overline{AB}$?

Ⓕ 12.8　Ⓖ 16　Ⓗ 18.75　Ⓙ 20

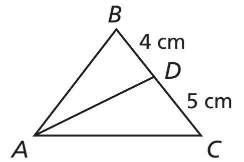
34. On the map, 1st St. and 2nd St. are parallel. What is the distance from City Hall to 2nd St. along Cedar Rd.?

Ⓐ 1.8 mi
Ⓑ 3.2 mi
Ⓒ 4.2 mi
Ⓓ 5.6 mi

35. **Extended Response** Two segments are divided proportionally. The first segment is divided into lengths 20, 15, and x. The corresponding lengths in the second segment are 16, y, and 24. Find the value of x and y. Use these values and write six proportions.

CHALLENGE AND EXTEND

36. The perimeter of $\triangle ABC$ is 29 m. $\overline{AD}$ bisects $\angle A$. Find AB and AC.



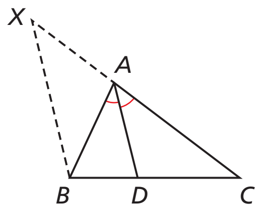
37. Prove that if two triangles are similar, then the ratio of their corresponding angle bisectors is the same as the ratio of their corresponding sides.

38. Prove the Triangle Angle Bisector Theorem.

Given: In $\triangle ABC$, $\overline{AD}$ bisects $\angle A$.

Prove: $\dfrac{BD}{DC} = \dfrac{AB}{AC}$

Plan: Draw $\overleftrightarrow{BX} \parallel \overline{AD}$ and extend $\overline{AC}$ to X. Use properties of parallel lines and the Converse of the Isosceles Triangle Theorem to show that $\overline{AX} \cong \overline{AB}$. Then apply the Triangle Proportionality Theorem.



39. **Construction** Draw $\overline{AB}$ any length. Use parallel lines and the properties of similarity to divide $\overline{AB}$ into three congruent parts.

SPIRAL REVIEW

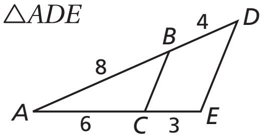
Write an algebraic expression that can be used to find the *n*th term of each sequence. *(Previous course)*

40. 5, 6, 7, 8,…　**41.** 3, 6, 9, 12,…　**42.** 1, 4, 9, 16,…

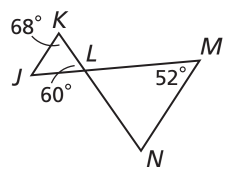
43. B is the midpoint of $\overline{AC}$. A has coordinates $(1, 4)$, and B has coordinates $(3, -7)$. Find the coordinates of C. *(Lesson 1-6)*

Verify that the given triangles are similar. *(Lesson 7-3)*

44. $\triangle ABC$ and $\triangle ADE$



45. $\triangle JKL$ and $\triangle MLN$



7-5 Using Proportional Relationships

Eiffel Tower replica in Paris, Texas

TEKS G.11.D Similarity and the geometry of shape: describe the effect on perimeter, area ... when ... dimensions of a figure are changed .... Also G.1.B, G.5.A, G.11.A, G.11.B

Objectives

Use ratios to make indirect measurements.

Use scale drawings to solve problems.

Vocabulary

indirect measurement
scale drawing
scale

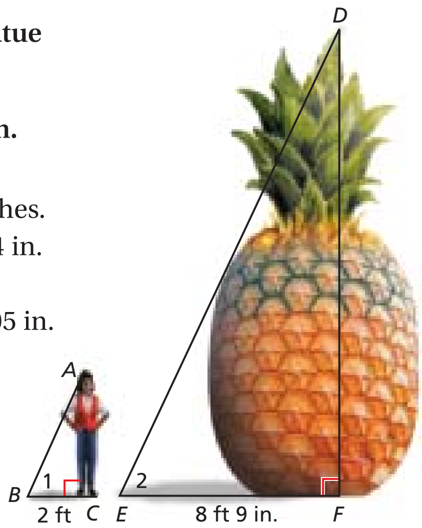
Why learn this?

Proportional relationships help you find distances that cannot be measured directly.

Indirect measurement is any method that uses formulas, similar figures, and/or proportions to measure an object. The following example shows one indirect measurement technique.

EXAMPLE 1 *Measurement Application*

A student wanted to find the height of a statue of a pineapple in Nambour, Australia. She measured the pineapple's shadow and her own shadow. The student's height is 5 ft 4 in. What is the height of the pineapple?



Step 1 Convert the measurements to inches.

$AC = 5 \text{ ft } 4 \text{ in.} = (5 \cdot 12) \text{ in.} + 4 \text{ in.} = 64 \text{ in.}$

$BC = 2 \text{ ft} = (2 \cdot 12) \text{ in.} = 24 \text{ in.}$

$EF = 8 \text{ ft } 9 \text{ in.} = (8 \cdot 12) \text{ in.} + 9 \text{ in.} = 105 \text{ in.}$

Step 2 Find similar triangles.

Because the sun's rays are parallel, $\angle 1 \cong \angle 2$. Therefore $\triangle ABC \sim \triangle DEF$ by $AA \sim$.

Step 3 Find DF.

$\frac{AC}{DF} = \frac{BC}{EF}$ *Corr. sides are proportional.*

$\frac{64}{DF} = \frac{24}{105}$ *Substitute 64 for AC, 24 for BC, and 105 for EF.*

$24(DF) = 64 \cdot 105$ *Cross Products Prop.*

$DF = 280$ *Divide both sides by 24.*

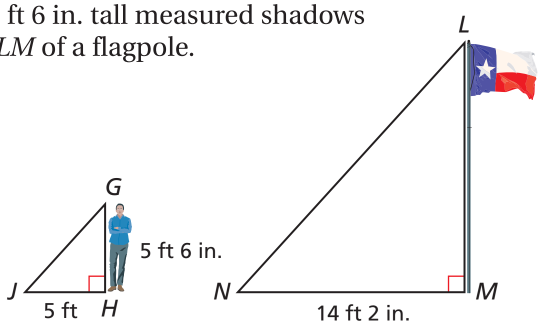
The height of the pineapple is 280 in., or 23 ft 4 in.

Helpful Hint

Whenever dimensions are given in both feet and inches, you must convert them to either feet or inches before doing any calculations.

CHECK IT OUT!

1. A student who is 5 ft 6 in. tall measured shadows to find the height LM of a flagpole. What is LM?



2ND PRINT

A **scale drawing** represents an object as smaller than or larger than its actual size. The drawing's **scale** is the ratio of any length in the drawing to the corresponding actual length. For example, on a map with a scale of 1 cm : 1500 m, one centimeter on the map represents 1500 m in actual distance.

EXAMPLE 2 Solving for a Dimension

The scale of this map of downtown Dallas is 1.5 cm : 300 m. Find the actual distance between Union Station and the Dallas Public Library.

Use a ruler to measure the distance between Union Station and the Dallas Public Library. The distance is 6 cm.

To find the actual distance x write a proportion comparing the map distance to the actual distance.

$$\frac{6}{x} = \frac{1.5}{300}$$

$1.5x = 6(300)$ *Cross Products Prop.*

$1.5x = 1800$ *Simplify.*

$x = 1200$ *Divide both sides by 1.5.*

The actual distance is 1200 m, or 1.2 km.

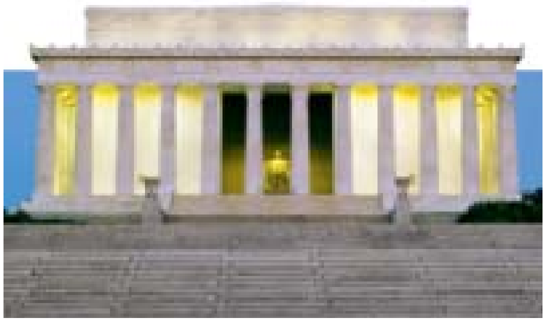
Remember!

A proportion may compare measurements that have different units.

CHECK IT OUT!

2. Find the actual distance between City Hall and El Centro College.

EXAMPLE 3 Making a Scale Drawing



The Lincoln Memorial in Washington, D.C., is approximately 57 m long and 36 m wide. Make a scale drawing of the base of the building using a scale of 1 cm : 15 m.

Step 1 Set up proportions to find the length ℓ and width w of the scale drawing.

$$\frac{\ell}{57} = \frac{1}{15} \qquad \frac{w}{36} = \frac{1}{15}$$

$$15\ell = 57 \qquad 15w = 36$$

$$\ell = 3.8 \text{ m} \qquad w = 2.4 \text{ cm}$$

Step 2 Use a ruler to draw a rectangle with these dimensions.

2.4 cm

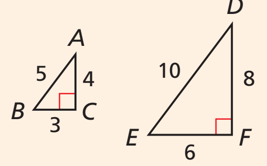
3.8 cm

CHECK IT OUT!

3. The rectangular central chamber of the Lincoln Memorial is 74 ft long and 60 ft wide. Make a scale drawing of the floor of the chamber using a scale of 1 in. : 20 ft.

2ND PRINT

Similar Triangles — Similarity, Perimeter, and Area Ratios

STATEMENT	RATIO
$\triangle ABC \sim \triangle DEF$ 	**Similarity ratio:** $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{1}{2}$ **Perimeter ratio:** $\frac{\text{perimeter } \triangle ABC}{\text{perimeter } \triangle DEF} = \frac{12}{24} = \frac{1}{2}$ **Area ratio:** $\frac{\text{area } \triangle ABC}{\text{area } \triangle DEF} = \frac{6}{24} = \frac{1}{4} = \left(\frac{1}{2}\right)^2$

The comparison of the similarity ratio and the ratio of perimeters and areas of similar triangles leads to the following theorem.

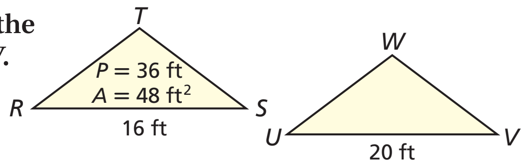
Theorem 7-5-1 Proportional Perimeters and Areas Theorem

If the similarity ratio of two similar figures is $\frac{a}{b}$, then the ratio of their perimeters is $\frac{a}{b}$, and the ratio of their areas is $\frac{a^2}{b^2}$, or $\left(\frac{a}{b}\right)^2$.

You will prove Theorem 7-5-1 in Exercises 44 and 45.

EXAMPLE 4 Using Ratios to Find Perimeters and Areas

Given that $\triangle RST \sim \triangle UVW$, find the perimeter P and area A of $\triangle UVW$.



The similarity ratio of $\triangle RST$ to $\triangle UVW$ is $\frac{16}{20}$, or $\frac{4}{5}$.

By the Proportional Perimeters and Areas Theorem, the ratio of the triangles' perimeters is also $\frac{4}{5}$, and the ratio of the triangles' areas is $\left(\frac{4}{5}\right)^2$, or $\frac{16}{25}$.

Perimeter	Area
$\frac{36}{P} = \frac{4}{5}$	$\frac{48}{A} = \frac{16}{25}$
$4P = 5(36)$	$16A = 25 \cdot 48$
$P = 45$ ft	$A = 75 \text{ ft}^2$

The perimeter of $\triangle UVW$ is 45 ft, and the area is 75 ft^2.

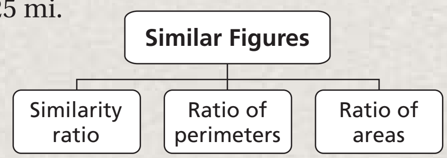
CHECK IT OUT!

4. $\triangle ABC \sim \triangle DEF$, $BC = 4$ mm, and $EF = 12$ mm. If $P = 42$ mm and $A = 96 \text{ mm}^2$ for $\triangle DEF$, find the perimeter and area of $\triangle ABC$.

THINK AND DISCUSS

1. Explain how to find the actual distance between two cities 5.5 in. apart on a map that has a scale of 1 in. : 25 mi.

2. GET ORGANIZED Copy and complete the graphic organizer. Draw and measure two similar figures. Then write their ratios.



2ND PRINT

7-5 Exercises

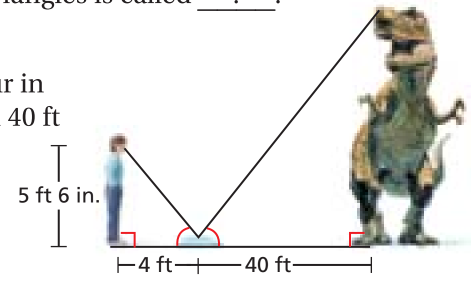
go.hrw.com/Geo/TX
Homework Help Online
KEYWORD: MG7 7-5
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

1. **Vocabulary** Finding distances using similar triangles is called ___?___. (*indirect measurement* or *scale drawing*)

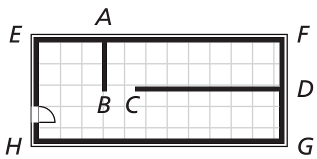
SEE EXAMPLE 1
p. 488

2. **Measurement** To find the height of a dinosaur in a museum, Amir placed a mirror on the ground 40 ft from its base. Then he stepped back 4 ft so that he could see the top of the dinosaur in the mirror. Amir's eyes were approximately 5 ft 6 in. above the ground. What is the height of the dinosaur?



SEE EXAMPLE 2
p. 489

The scale of this blueprint of an art gallery is 1 in.:48 ft. Find the actual lengths of the following walls.



3. $\overline{AB}$
4. $\overline{CD}$
5. $\overline{EF}$
6. $\overline{FG}$

SEE EXAMPLE 3
p. 489

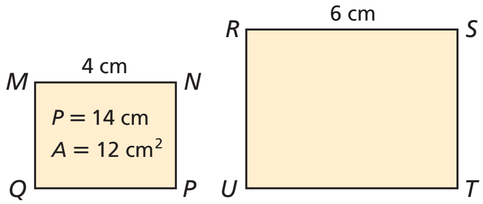
Multi-Step **A rectangular classroom is 10 m long and 4.6 m wide. Make a scale drawing of the classroom using the following scales.**

7. 1 cm : 1 m
8. 1 cm : 2 m
9. 1 cm : 2.3 m

SEE EXAMPLE 4
p. 490

Given: rectangle $MNPQ \sim$ rectangle $RSTU$

10. Find the perimeter of rectangle $RSTU$.
11. Find the area of rectangle $RSTU$.



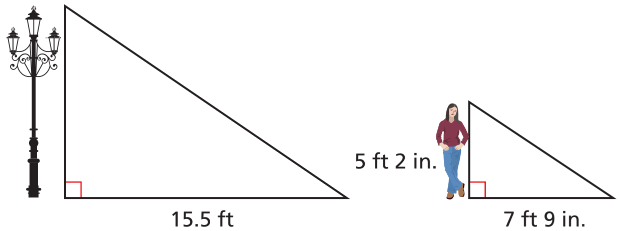
PRACTICE AND PROBLEM SOLVING

Independent Practice

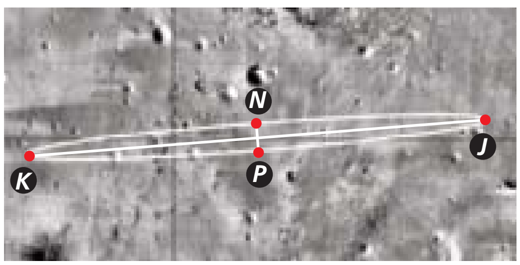
For Exercises	See Example
12	1
13–14	2
15–17	3
18–19	4

TEKS TAKS
Skills Practice p. S17
Application Practice p. S34

12. **Measurement** Jenny is 5 ft 2 in. tall. To find the height of a light pole, she measured her shadow and the pole's shadow. What is the height of the pole?



Space Exploration **Use the following information for Exercises 13 and 14.**
This is a map of the Mars Exploration Rover *Opportunity's* predicted landing site on Mars. The scale is 1 cm : 9.4 km. What are the approximate measures of the actual length and width of the ellipse?



13. KJ
14. NP

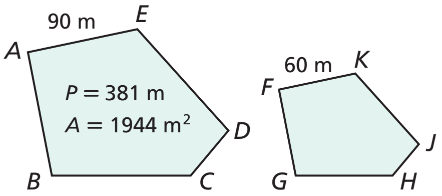
Multi-Step **A park at the end of a city block is a right triangle with legs 150 ft and 200 ft long. Make a scale drawing of the park using the following scales.**

15. 1.5 in. : 100 ft
16. 1 in. : 300 ft
17. 1 in. : 150 ft

Given that pentagon $ABCDE \sim$ pentagon $FGHJK$, find each of the following.

18. perimeter of pentagon $FGHJK$

19. area of pentagon $FGHJK$



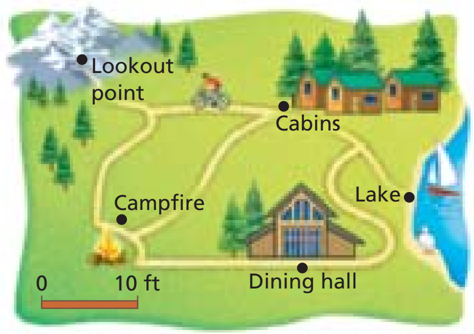
Estimation **Use the scale on the map for Exercises 20–23. Give the approximate distance of the shortest route between each pair of sites.**

20. campfire and the lake

21. lookout point and the campfire

22. cabins and the dining hall

23. lookout point and the lake



Given: $\triangle ABC \sim \triangle DEF$

24. The ratio of the perimeter of $\triangle ABC$ to the perimeter of $\triangle DEF$ is $\frac{8}{9}$. What is the similarity ratio of $\triangle ABC$ to $\triangle DEF$?

25. The ratio of the area of $\triangle ABC$ to the area of $\triangle DEF$ is $\frac{16}{25}$. What is the similarity ratio of $\triangle ABC$ to $\triangle DEF$?

26. The ratio of the area of $\triangle ABC$ to the area of $\triangle DEF$ is $\frac{4}{81}$. What is the ratio of the perimeter of $\triangle ABC$ to the perimeter of $\triangle DEF$?

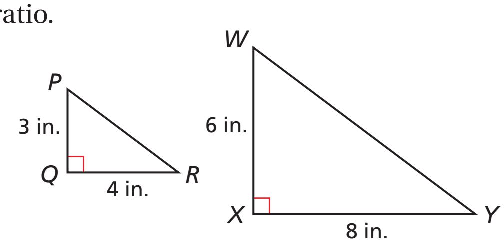
27. **Space Exploration** The scale of this model of the space shuttle is 1 ft : 50 ft. In the actual space shuttle, the main cargo bay measures 15 ft wide by 60 ft long. What are the dimensions of the cargo bay in the model?

28. Given that $\triangle PQR \sim \triangle WXY$, find each ratio.

a. $\dfrac{\text{perimeter of } \triangle PQR}{\text{perimeter of } \triangle WXY}$

b. $\dfrac{\text{area of } \triangle PQR}{\text{area of } \triangle WXY}$

c. How does the result in part **a** compare with the result in part **b**?



29. Given that rectangle $ABCD \sim EFGH$. The area of rectangle $ABCD$ is 135 in^2. The area of rectangle $EFGH$ is 240 in^2. If the width of rectangle $ABCD$ is 9 in., what is the length and width of rectangle $EFGH$?

30. **Sports** An NBA basketball court is 94 ft long and 50 ft wide. Make a scale drawing of a court using a scale of $\frac{1}{4}$ in. : 10 ft.

MULTI-STEP TAKS PREP

31. This problem will prepare you for the Multi-Step TAKS Prep on page 502.
A blueprint for a skateboard ramp has a scale of 1 in. : 2 ft. On the blueprint, the rectangular piece of wood that forms the ramp measures 2 in. by 3 in.

a. What is the similarity ratio of the blueprint to the actual ramp?

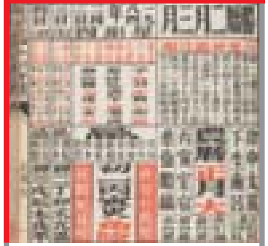
b. What is the ratio of the area of the ramp on the blueprint to its actual area?

c. Find the area of the actual ramp.

2ND PRINT

32. Estimation The photo shows a person who is 5 ft 1 in. tall standing by a statue in Jamestown, North Dakota. Estimate the actual height of the statue by using a ruler to measure her height and the height of the statue in the photo.

Math History

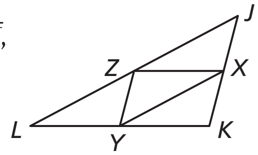


In 1075 C.E., Shen Kua created a calendar for the emperor by measuring the positions of the moon and planets. He plotted exact coordinates three times a night for five years.

Source: history.mcs.st-andrews.ac.uk

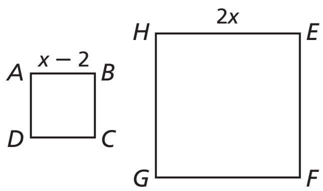
33. Math History In A.D. 1076, the mathematician Shen Kua was asked by the emperor of China to produce maps of all Chinese territories. Shen created 23 maps, each drawn with a scale of 1 cm : 900,000 cm. How many centimeters long would a 1 km road be on such a map?

34. Points X, Y, and Z are the midpoints of $\overline{JK}$, $\overline{KL}$, and $\overline{LJ}$, respectively. What is the ratio of the area of $\triangle JKL$ to the area of $\triangle XYZ$?



35. Critical Thinking Keisha is making two scale drawings of her school. In one drawing, she uses a scale of 1 cm : 1 m. In the other drawing, she uses a scale of 1 cm : 5 m. Which of these scales will produce a smaller drawing? Explain.

36. The ratio of the perimeter of square $ABCD$ to the perimeter of square $EFGH$ is $\frac{4}{9}$. Find the side lengths of each square.



37. Write About It Explain what it would mean to make a scale drawing with a scale of 1 : 1.

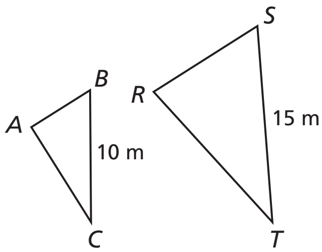
38. Write About It One square has twice the area of another square. Explain why it is impossible for both squares to have side lengths that are whole numbers.

TEST PREP

39. $\triangle ABC \sim \triangle RST$, and the area of $\triangle ABC$ is 24 m^2. What is the area of $\triangle RST$?

Ⓐ 16 m^2 Ⓒ 36 m^2

Ⓑ 29 m^2 Ⓓ 54 m^2



40. A blueprint for a museum uses a scale of $\frac{1}{4}$ in. : 1 ft. One of the rooms on the blueprint is $3\frac{3}{4}$ in. long. How long is the actual room?

Ⓕ 4 ft Ⓖ 15 ft Ⓗ 45 ft Ⓙ 180 ft

41. The similarity ratio of two similar pentagons is $\frac{9}{4}$. What is the ratio of the perimeters of the pentagons?

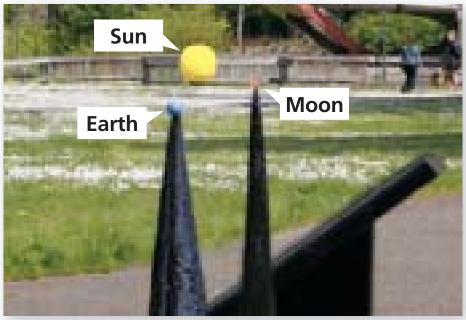
Ⓐ $\frac{2}{3}$ Ⓑ $\frac{3}{2}$ Ⓒ $\frac{9}{4}$ Ⓓ $\frac{81}{16}$

42. Of two similar triangles, the second triangle has sides half the length of the first. Given that the area of the first triangle is 16 ft^2, find the area of the second.

Ⓕ 4 ft^2 Ⓖ 8 ft^2 Ⓗ 16 ft^2 Ⓙ 32 ft^2

2ND PRINT

CHALLENGE AND EXTEND



43. **Astronomy** The city of Eugene, Oregon, has a scale model of the solar system nearly 6 km long. The model's scale is 1 km : 1 billion km.

a. Earth is 150,000,000 km from the Sun. How many meters apart are Earth and the Sun in the model?

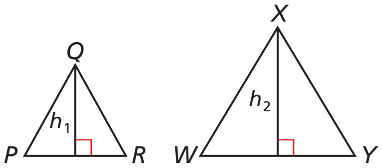
b. The diameter of Earth is 12,800 km. What is the diameter, in centimeters, of Earth in the model?

44. **Given:** $\triangle ABC \sim \triangle DEF$

Prove: $\dfrac{AB + BC + AC}{DE + EF + DF} = \dfrac{AB}{DE}$

45. **Given:** $\triangle PQR \sim \triangle WXY$

Prove: $\dfrac{\text{Area } \triangle PQR}{\text{Area } \triangle WXY} = \dfrac{PR^2}{WY^2}$



46. Quadrilateral $PQRS$ has side lengths of 6 m, 7 m, 10 m, and 12 m. The similarity ratio of quadrilateral $PQRS$ to quadrilateral $WXYZ$ is 1 : 2.

a. Find the lengths of the sides of quadrilateral $WXYZ$.

b. Make a table of the lengths of the sides of both figures.

c. Graph the data in the table.

d. Determine an equation that relates the lengths of the sides of quadrilateral $PQRS$ to the lengths of the sides of quadrilateral $WXYZ$.

SPIRAL REVIEW

Solve each equation. Round to the nearest hundredth if necessary. *(Previous course)*

47. $(x - 3)^2 = 49$ **48.** $(x + 1)^2 - 4 = 0$ **49.** $4(x + 2)^2 - 28 = 0$

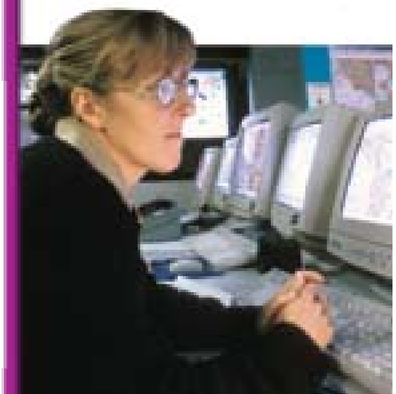
Show that the quadrilateral with the given vertices is a parallelogram. *(Lesson 6-3)*

50. $A(-2, -2), B(1, 0), C(5, 0), D(2, -2)$ **51.** $J(1, 3), K(3, 5), L(6, 2), M(4, 0)$

52. Given that $58x = 26y$, find the ratio $y : x$ in simplest form. *(Lesson 7-1)*

Career Path

go.hrw.com/Geo/TX
Career Resources Online
KEYWORD: MG7 Career



Elaine Koch
Photogrammetrist

Q: What math classes did you take in high school?
A: Algebra, Geometry, and Probability and Statistics

Q: What math-related classes did you take in college?
A: Trigonometry, Precalculus, Drafting, and System Design

Q: How do photogrammetrists use math?
A: Photogrammetrists use aerial photographs to make detailed maps. To prepare maps, I use computers and perform a lot of scale measures to make sure the maps are accurate.

Q: What are your future plans?
A: My favorite part of making maps is designing scale drawings. Someday I'd like to apply these skills toward architectural work.

2ND PRINT

7-6 Dilations and Similarity in the Coordinate Plane

TEKS G.11.A Similarity and the geometry of shape: use ... properties and transformations to ... justify conjectures ....

Objectives

Apply similarity properties in the coordinate plane.

Use coordinate proof to prove figures similar.

Vocabulary

dilation

scale factor

Also G.2.B, G.9.B

Who uses this?

Computer programmers use coordinates to enlarge or reduce images.

Many photographs on the Web are in JPEG format, which is short for Joint Photographic Experts Group. When you drag a corner of a JPEG image in order to enlarge it or reduce it, the underlying program uses coordinates and similarity to change the image's size.

A **dilation** is a transformation that changes the size of a figure but not its shape. The preimage and the image are always similar. A **scale factor** describes how much the figure is enlarged or reduced. For a dilation with scale factor k, you can find the image of a point by multiplying each coordinate by k: $(a, b) \rightarrow (ka, kb)$.

EXAMPLE 1 *Computer Graphics Application*

The figure shows the position of a JPEG photo. Draw the border of the photo after a dilation with scale factor $\frac{3}{2}$.

Step 1 Multiply the vertices of the photo $A(0, 0)$, $B(0, 4)$, $C(3, 4)$, and $D(3, 0)$ by $\frac{3}{2}$.

Rectangle $ABCD$	Rectangle $A'B'C'D'$

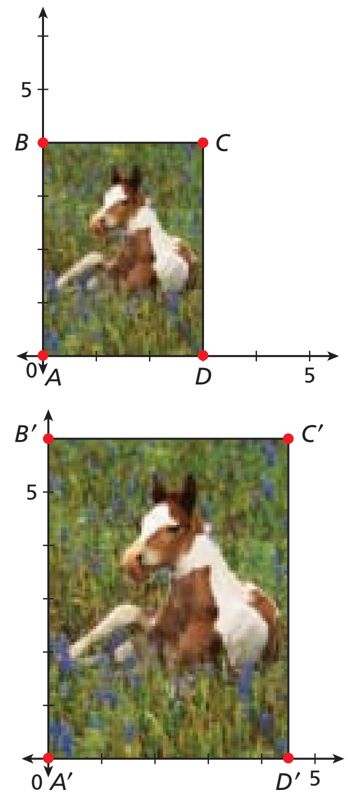
$$A(0, 0) \rightarrow A'\left(0 \cdot \frac{3}{2}, 0 \cdot \frac{3}{2}\right) \rightarrow A'(0, 0)$$

$$B(0, 4) \rightarrow B'\left(0 \cdot \frac{3}{2}, 4 \cdot \frac{3}{2}\right) \rightarrow B'(0, 6)$$

$$C(3, 4) \rightarrow C'\left(3 \cdot \frac{3}{2}, 4 \cdot \frac{3}{2}\right) \rightarrow C'(4.5, 6)$$

$$D(3, 0) \rightarrow D'\left(3 \cdot \frac{3}{2}, 0 \cdot \frac{3}{2}\right) \rightarrow D'(4.5, 0)$$

Step 2 Plot points $A'(0, 0)$, $B'(0, 6)$, $C'(4.5, 6)$, and $D'(4.5, 0)$. Draw the rectangle.



Helpful Hint

If the scale factor of a dilation is greater than 1 $(k > 1)$, it is an *enlargement*. If the scale factor is less than 1 $(k < 1)$, it is a *reduction*.

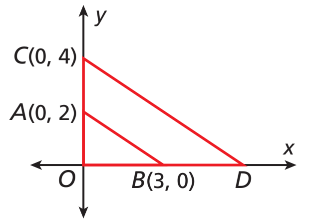
CHECK IT OUT!

1. **What if...?** Draw the border of the original photo after a dilation with scale factor $\frac{1}{2}$.

2ND PRINT

EXAMPLE 2 Finding Coordinates of Similar Triangles

Given that $\triangle AOB \sim \triangle COD$, find the coordinates of D and the scale factor.



Since $\triangle AOB \sim \triangle COD$,

$$\frac{AO}{CO} = \frac{OB}{OD}$$

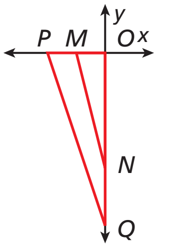
$$\frac{2}{4} = \frac{3}{OD}$$ *Substitute 2 for AO, 4 for CO, and 3 for OB.*

$2OD = 12$ *Cross Products Prop.*

$OD = 6$ *Divide both sides by 2.*

D lies on the x-axis, so its y-coordinate is 0. Since $OD = 6$, its x-coordinate must be 6. The coordinates of D are $(6, 0)$. $(3, 0) \rightarrow (3 \cdot 2, 0 \cdot 2) \rightarrow (6, 0)$, so the scale factor is 2.

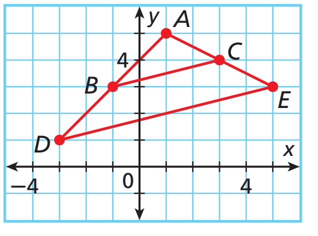
CHECK IT OUT! **2.** Given that $\triangle MON \sim \triangle POQ$ and coordinates $P(-15, 0)$, $M(-10, 0)$, and $Q(0, -30)$, find the coordinates of N and the scale factor.



EXAMPLE 3 Proving Triangles Are Similar

Given: $A(1, 5)$, $B(-1, 3)$, $C(3, 4)$, $D(-3, 1)$, and $E(5, 3)$

Prove: $\triangle ABC \sim \triangle ADE$



Step 1 Plot the points and draw the triangles.

Step 2 Use the Distance Formula to find the side lengths.

$$AB = \sqrt{(-1-1)^2 + (3-5)^2} = \sqrt{8} = 2\sqrt{2}$$

$$AC = \sqrt{(3-1)^2 + (4-5)^2} = \sqrt{5}$$

$$AD = \sqrt{(-3-1)^2 + (1-5)^2} = \sqrt{32} = 4\sqrt{2}$$

$$AE = \sqrt{(5-1)^2 + (3-5)^2} = \sqrt{20} = 2\sqrt{5}$$

Step 3 Find the similarity ratio.

$$\frac{AB}{AD} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{AC}{AE} = \frac{\sqrt{5}}{2\sqrt{5}} = \frac{1}{2}$$

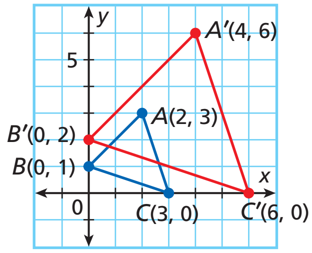
Since $\frac{AB}{AD} = \frac{AC}{AE}$ and $\angle A \cong \angle A$ by the Reflexive Property, $\triangle ABC \sim \triangle ADE$ by SAS $\sim$.

CHECK IT OUT! **3.** **Given:** $R(-2, 0)$, $S(-3, 1)$, $T(0, 1)$, $U(-5, 3)$, and $V(4, 3)$
Prove: $\triangle RST \sim \triangle RUV$

2ND PRINT

EXAMPLE 4 Using the SSS Similarity Theorem

Graph the image of $\triangle ABC$ after a dilation with scale factor 2. Verify that $\triangle A'B'C' \sim \triangle ABC$.



Step 1 Multiply each coordinate by 2 to find the coordinates of the vertices of $\triangle A'B'C'$.

$$A(2, 3) \rightarrow A'(2 \cdot 2, 3 \cdot 2) = A'(4, 6)$$
$$B(0, 1) \rightarrow B'(0 \cdot 2, 1 \cdot 2) = B'(0, 2)$$
$$C(3, 0) \rightarrow C'(3 \cdot 2, 0 \cdot 2) = C'(6, 0)$$

Step 2 Graph $\triangle A'B'C'$.

Step 3 Use the Distance Formula to find the side lengths.

$$AB = \sqrt{(2-0)^2 + (3-1)^2} = \sqrt{8} = 2\sqrt{2} \qquad A'B' = \sqrt{(4-0)^2 + (6-2)^2} = \sqrt{32} = 4\sqrt{2}$$

$$BC = \sqrt{(3-0)^2 + (0-1)^2} = \sqrt{10} \qquad B'C' = \sqrt{(6-0)^2 + (0-2)^2} = \sqrt{40} = 2\sqrt{10}$$

$$AC = \sqrt{(3-2)^2 + (0-3)^2} = \sqrt{10} \qquad A'C' = \sqrt{(6-4)^2 + (0-6)^2} = \sqrt{40} = 2\sqrt{10}$$

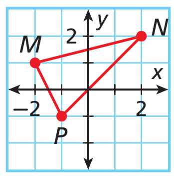
Step 4 Find the similarity ratio.

$$\frac{A'B'}{AB} = \frac{4\sqrt{2}}{2\sqrt{2}} = 2, \quad \frac{B'C'}{BC} = \frac{2\sqrt{10}}{\sqrt{10}} = 2, \quad \frac{A'C'}{AC} = \frac{2\sqrt{10}}{\sqrt{10}} = 2$$

Since $\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC}$, $\triangle ABC \sim \triangle A'B'C'$ by SSS $\sim$.

CHECK IT OUT!

4. Graph the image of $\triangle MNP$ after a dilation with scale factor 3. Verify that $\triangle M'N'P' \sim \triangle MNP$.

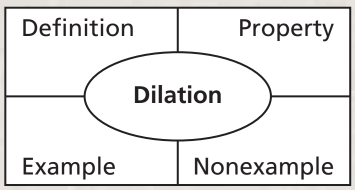


THINK AND DISCUSS

1. $\triangle JKL$ has coordinates $J(0, 0)$, $K(0, 2)$, and $L(3, 0)$. Its image after a dilation has coordinates $J'(0, 0)$, $K'(0, 8)$, and $L'(12, 0)$. Explain how to find the scale factor of the dilation.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. Write the definition of a dilation, a property of dilations, and an example and nonexample of a dilation.



7-6 Exercises

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Homework Help Online
KEYWORD: MG7 7-6
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

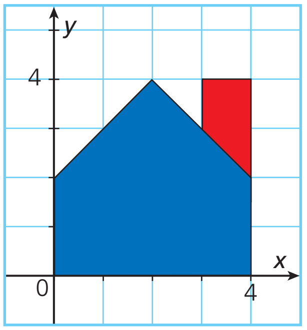
Vocabulary Apply the vocabulary from this lesson to answer each question.

1. A __?__ is a transformation that proportionally reduces or enlarges a figure, such as the pupil of an eye. (*dilation* or *scale factor*)

2. A ratio that describes or determines the dimensional relationship of a figure to that which it represents, such as a map scale of 1 in. : 45 ft, is called a __?__ . (*dilation* or *scale factor*)

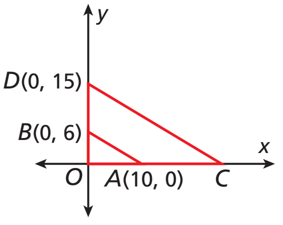
SEE EXAMPLE 1 p. 495

3. **Graphic Design** A designer created this logo for a real estate agent but needs to make the logo twice as large for use on a sign. Draw the logo after a dilation with scale factor 2.

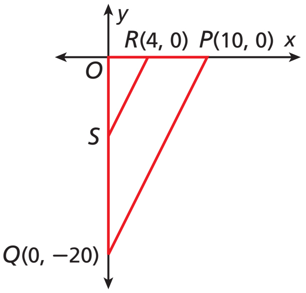


SEE EXAMPLE 2 p. 496

4. Given that $\triangle AOB \sim \triangle COD$, find the coordinates of C and the scale factor.



5. Given that $\triangle ROS \sim \triangle POQ$, find the coordinates of S and the scale factor.



SEE EXAMPLE 3 p. 496

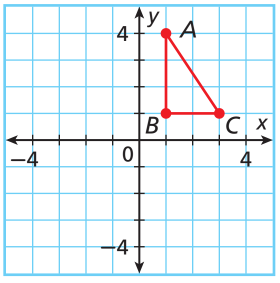
6. **Given:** $A(0, 0)$, $B(-1, 1)$, $C(3, 2)$, $D(-2, 2)$, and $E(6, 4)$
Prove: $\triangle ABC \sim \triangle ADE$

7. **Given:** $J(-1, 0)$, $K(-3, -4)$, $L(3, -2)$, $M(-4, -6)$, and $N(5, -3)$
Prove: $\triangle JKL \sim \triangle JMN$

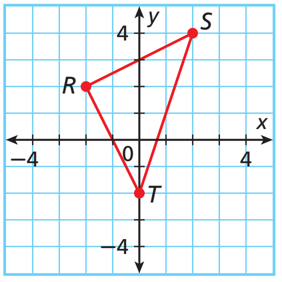
SEE EXAMPLE 4 p. 497

Multi-Step Graph the image of each triangle after a dilation with the given scale factor. Then verify that the image is similar to the given triangle.

8. scale factor 2



9. scale factor $\frac{3}{2}$



PRACTICE AND PROBLEM SOLVING

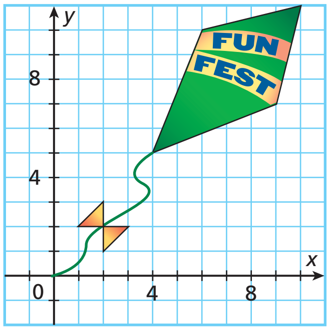
Independent Practice

For Exercises	See Example
10	1
11–12	2
13–14	3
15–16	4

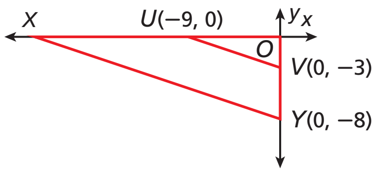
TEKS TAKS

Skills Practice p. S17
Application Practice p. S34

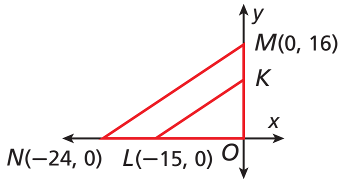
10. Advertising A promoter produced this design for a street festival. She now wants to make the design smaller to use on postcards. Sketch the design after a dilation with scale factor $\frac{1}{2}$.



11. Given that $\triangle UOV \sim \triangle XOY$, find the coordinates of X and the scale factor.



12. Given that $\triangle MON \sim \triangle KOL$, find the coordinates of K and the scale factor.



13. Given: $D(-1, 3)$, $E(-3, -1)$, $F(3, -1)$, $G(-4, -3)$, and $H(5, -3)$
Prove: $\triangle DEF \sim \triangle DGH$

14. Given: $M(0, 10)$, $N(5, 0)$, $P(15, 15)$, $Q(10, -10)$, and $R(30, 20)$
Prove: $\triangle MNP \sim \triangle MQR$

Multi-Step Graph the image of each triangle after a dilation with the given scale factor. Then verify that the image is similar to the given triangle.

15. $J(-2, 0)$ and $K(-1, -1)$, and $L(-3, -2)$ with scale factor 3

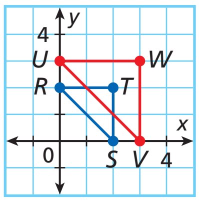
16. $M(0, 4)$, $N(4, 2)$, and $P(2, -2)$ with scale factor $\frac{1}{2}$

17. Critical Thinking Consider the transformation given by the mapping $(x, y) \rightarrow (2x, 4y)$. Is this transformation a dilation? Why or why not?

18. ERROR ANALYSIS Which solution to find the scale factor of the dilation that maps $\triangle RST$ to $\triangle UVW$ is incorrect? Explain the error.

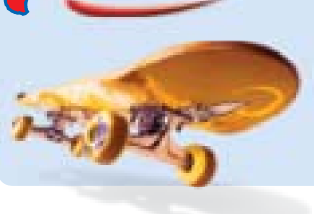
A
To go from $\triangle RST$ to $\triangle UVW$, the coordinates of each point of $\triangle RST$ are multiplied by $\frac{3}{2}$, so the scale factor is $\frac{3}{2}$.

B
The scale factor is the ratio of corresponding measures. Since $\frac{RT}{UW} = \frac{2}{3}$, the scale factor is $\frac{2}{3}$.



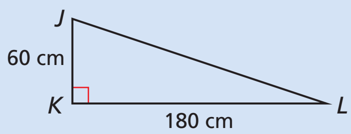
19. Write About It A dilation maps $\triangle ABC$ to $\triangle A'B'C'$. How is the scale factor of the dilation related to the similarity ratio of $\triangle ABC$ to $\triangle A'B'C'$? Explain.

MULTI-STEP TAKS PREP



20. This problem will prepare you for the Multi-Step TAKS Prep on page 502.

a. In order to build a skateboard ramp, Miles draws $\triangle JKL$ on a coordinate plane. One unit on the drawing represents 60 cm of actual distance. Explain how he should assign coordinates for the vertices of $\triangle JKL$.

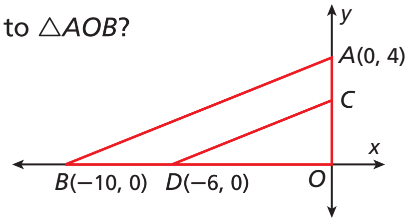


b. Graph the image of $\triangle JKL$ after a dilation with scale factor 3.

TEST PREP

21. Which coordinates for C make $\triangle COD$ similar to $\triangle AOB$?

Ⓐ (0, 2.4) Ⓒ (0, 3)
Ⓑ (0, 2.5) Ⓓ (0, 3.6)

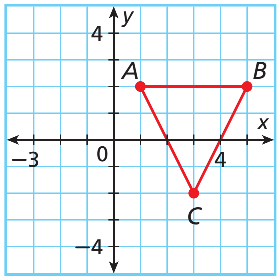


22. A dilation with scale factor 2 maps $\triangle RST$ to $\triangle R'S'T'$. The perimeter of $\triangle RST$ is 60. What is the perimeter of $\triangle R'S'T'$?

Ⓕ 30 Ⓖ 60 Ⓗ 120 Ⓙ 240

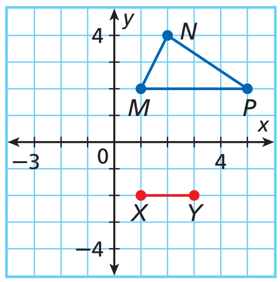
23. Which triangle with vertices D, E, and F is similar to $\triangle ABC$?

Ⓐ $D(1, 2)$, $E(3, 2)$, $F(2, 0)$
Ⓑ $D(-1, -2)$, $E(2, -2)$, $F(1, -5)$
Ⓒ $D(1, 2)$, $E(5, 2)$, $F(3, 0)$
Ⓓ $D(-2, -2)$, $E(0, 2)$, $F(-1, 0)$



24. **Gridded Resonse** $\overline{AB}$ with endpoints $A(3, 2)$ and $B(7, 5)$ is dilated by a scale factor of 3. Find the length of $\overline{A'B'}$.

CHALLENGE AND EXTEND



25. How many different triangles having $\overline{XY}$ as a side are similar to $\triangle MNP$?

26. $\triangle XYZ \sim \triangle MPN$. Find the coordinates of Z.

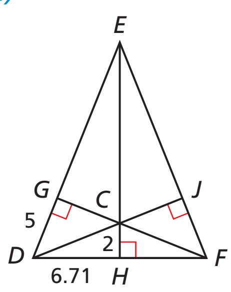
27. A rectangle has two of its sides on the x- and y-axes, a vertex at the origin, and a vertex on the line $y = 2x$. Prove that any two such rectangles are similar.

28. $\triangle ABC$ has vertices $A(0, 1)$, $B(3, 1)$, and $C(1, 3)$. $\triangle DEF$ has vertices $D(1, -1)$ and $E(7, -1)$. Find two different locations for vertex F so that $\triangle ABC \sim \triangle DEF$.

SPIRAL REVIEW

Write an inequality to represent the situation. *(Previous course)*

29. A weight lifter must lift at least 250 pounds. There are two 50-pound weights on a bar that weighs 5 pounds. Let w represent the additional weight that must be added to the bar.

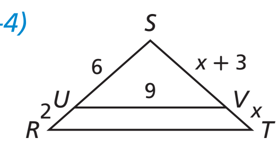


Find the length of each segment, given that $\overline{DE} \cong \overline{FE}$. *(Lesson 5-2)*

30. $\overline{HF}$ **31.** $\overline{JF}$ **32.** $\overline{CF}$

$\triangle SUV \sim \triangle SRT$. Find the length of each segment. *(Lesson 7-4)*

33. $\overline{RT}$ **34.** $\overline{VT}$ **35.** $\overline{ST}$



2ND PRINT

Direct Variation

Algebra

In Lesson 7-6 you learned that for two similar figures, the measure of each point was multiplied by the same scale factor. Is the relationship between the scale factor and the perimeter of the figure a direct variation?

Recall from algebra that if y varies directly as x, then $y = kx$, or $\frac{y}{x} = k$, where k is the constant of variation.

See Skills Bank page S62

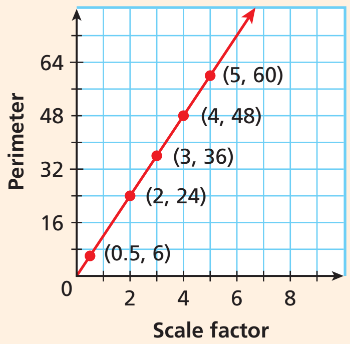
Example

A rectangle has a length of 4 ft and a width of 2 ft. Find the relationship between the scale factors of similar rectangles and their corresponding perimeters. If the relationship is a direct variation, find the constant of variation.

Step 1 Make a table to record data.

Scale Factor x	Length $\ell = x(4)$	Width $w = x(2)$	Perimeter $P = 2\ell + 2w$
$\frac{1}{2}$	$\ell = \frac{1}{2}(4) = 2$	$w = \frac{1}{2}(2) = 1$	$2(2) + 2(1) = 6$
2	8	4	24
3	12	6	36
4	16	8	48
5	20	10	60

Step 2 Graph the points $\left(\frac{1}{2}, 6\right)$, $(2, 24)$, $(3, 36)$, $(4, 48)$, and $(5, 60)$.



Since the points are collinear and the line that contains them includes the origin, the relationship is a direct variation.

Step 3 Find the equation of direct variation.

$y = kx$

$60 = k(5)$ *Substitute 60 for y and 5 for x.*

$12 = k$ *Divide both sides by 5.*

$y = 12x$ *Substitute 12 for k.*

Thus the constant of variation is 12.

Try This

TAKS Grades 9–11 Obj. 3, 8, 10

Use the scale factors given in the above table. Find the relationship between the scale factors of similar figures and their corresponding perimeters. If the relationship is a direct variation, find the constant of variation.

1. regular hexagon with side length 6
2. triangle with side lengths 3, 6, and 7
3. square with side length 3

CHAPTER 7

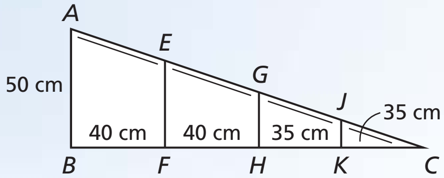
MULTI-STEP TAKS PREP

SECTION 7B

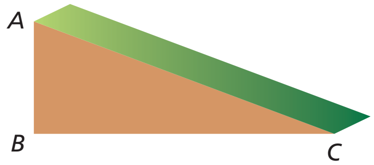
Applying Similarity

Ramp It Up Many companies sell plans for build-it-yourself skateboard ramps. The figures below show a ramp and the plan for the triangular support structure at the side of the ramp. In the plan, $\overline{AB}$, $\overline{EF}$, $\overline{GH}$, and $\overline{JK}$ are perpendicular to the base $\overline{BC}$.

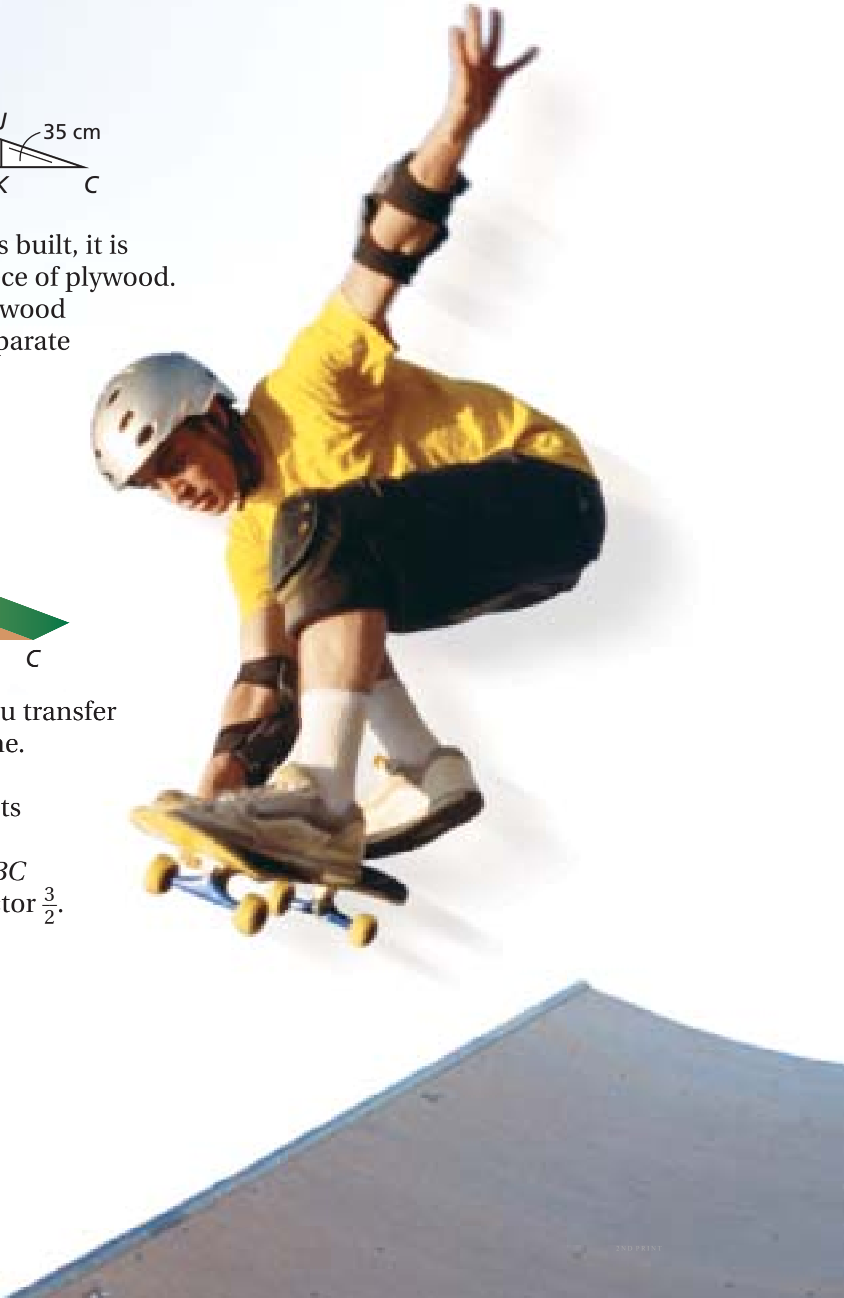
1. The instructions call for extra pieces of wood to reinforce $\overline{AE}$, $\overline{EG}$, $\overline{GJ}$, and $\overline{JC}$. Given $AE = 42.2$ cm, find EG, GJ, and JC to the nearest tenth.



2. Once the support structure is built, it is covered with a triangular piece of plywood. Find the area of the piece of wood needed to cover $\triangle ABC$. A separate blueprint for the ramp uses a scale of 1 cm : 25 cm. What is the area of $\triangle ABC$ in the blueprint?



3. Before building the ramp, you transfer the plan to a coordinate plane. Draw $\triangle ABC$ on a coordinate plane so that 1 unit represents 25 cm and B is at the origin. Then draw the image of $\triangle ABC$ after a dilation with scale factor $\frac{3}{2}$.



READY TO GO ON?

CHAPTER 7

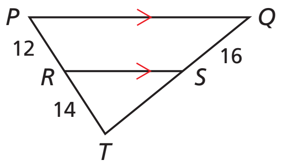
SECTION 7B

Quiz for Lessons 7-4 Through 7-6

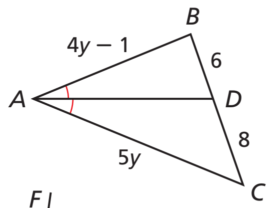
7-4 Applying Properties of Similar Triangles

Find the length of each segment.

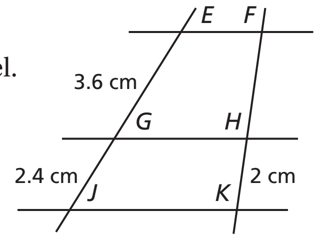
1. $\overline{ST}$



2. $\overline{AB}$ and $\overline{AC}$



3. An artist drew a picture of railroad tracks such that the ties $\overline{EF}$, $\overline{GH}$, and $\overline{JK}$ are parallel. What is the length of $\overline{FH}$?



7-5 Using Proportional Relationships

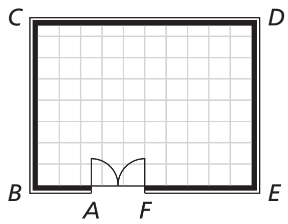
The plan for a restaurant uses the scale of 1.5 in.:60 ft. Find the actual length of the following walls.

4. $\overline{AB}$

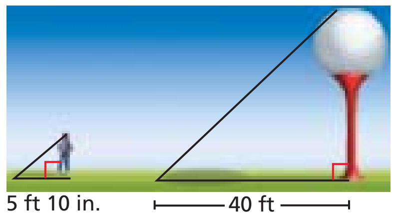
5. $\overline{BC}$

6. $\overline{CD}$

7. $\overline{EF}$



8. A student who is 5 ft 3 in. tall measured her shadow and the shadow cast by a water tower shaped like a golf ball. What is the height of the tower?



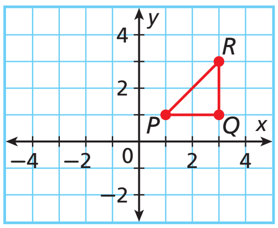
7-6 Dilations and Similarity in the Coordinate Plane

9. Given: $A(-1, 2)$, $B(-3, -2)$, $C(3, 0)$, $D(-2, 0)$, and $E(1, 1)$
Prove: $\triangle ADE \sim \triangle ABC$

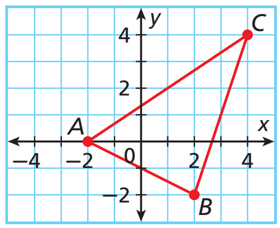
10. Given: $R(0, 0)$, $S(-2, -1)$, $T(0, -3)$, $U(4, 2)$, and $V(0, 6)$
Prove: $\triangle RST \sim \triangle RUV$

Graph the image of each triangle after a dilation with the given scale factor. Then verify that the image is similar to the given triangle.

11. scale factor 3



12. scale factor 1.5



CHAPTER 7 Study Guide: Review

Know It! Note: For a complete list of the postulates and theorems in this chapter, see p. S82.

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. An equation stating that two ratios are equal is called a(n) __?__ .

2. A(n) __?__ is a transformation that changes the size of a figure but not its shape.

3. In the proportion $\frac{u}{v} = \frac{x}{y}$, the __?__ are v and x.

4. A(n) __?__ compares two numbers by division.

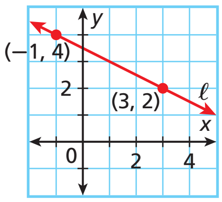
7-1 Ratio and Proportion *(pp. 454–459)*

TEKS G.5.B, G.7.B, G.7.C, G.11.B

EXAMPLES

■ **Write a ratio expressing the slope of ℓ.**

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{-1 - 3} = \frac{2}{-4} = -\frac{1}{2}$$



■ **Solve the proportion.**

$$\frac{2}{4(x-3)} = \frac{x-3}{50}$$

$4(x-3)^2 = 2(50)$	*Cross Products Prop.*
$4(x-3)^2 = 100$	*Simplify.*
$(x-3)^2 = 25$	*Divide both sides by 4.*
$x - 3 = \pm 5$	*Find the square root of both sides.*
$x - 3 = 5$ or $x - 3 = -5$	*Rewrite as two eqns.*
$x = 8$ or $x = -2$	*Add 3 to both sides.*

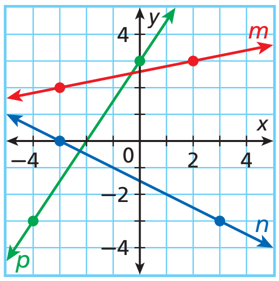
EXERCISES

Write a ratio expressing the slope of each line.

5. m

6. n

7. p



8. If 84 is divided into three parts in the ratio 3 : 5 : 6, what is the sum of the smallest and the largest part?

9. The ratio of the measures of a pair of sides of a rectangle is 7 : 12. If the perimeter of the rectangle is 95, what is the length of each side?

Solve each proportion.

10. $\frac{y}{7} = \frac{9}{3}$

11. $\frac{10}{4} = \frac{25}{s}$

12. $\frac{x}{4} = \frac{9}{x}$

13. $\frac{4}{z-1} = \frac{z-1}{36}$

14. $\frac{12}{2x} = \frac{3x}{32}$

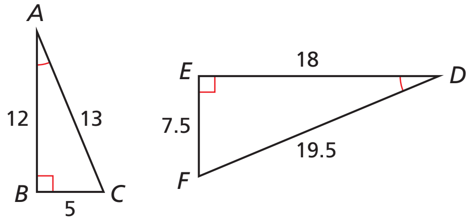
15. $\frac{y+1}{24} = \frac{2}{3(y+1)}$

7-2 Ratios in Similar Polygons *(pp. 462–467)*

TEKS G.5.B, G.11.A, G.11.B

EXAMPLE

- **Determine whether $\triangle ABC$ and $\triangle DEF$ are similar. If so, write the similarity ratio and a similarity statement.**

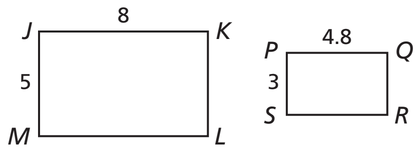


It is given that $\angle A \cong \angle D$ and $\angle B \cong \angle E$. $\angle C \cong \angle F$ by the Third Angles Theorem. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{2}{3}$. Thus the similarity ratio is $\frac{2}{3}$, and $\triangle ABC \sim \triangle DEF$.

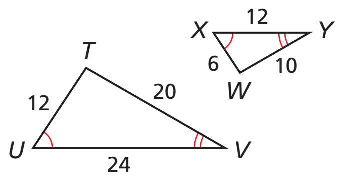
EXERCISES

Determine whether the polygons are similar. If so, write the similarity ratio and a similarity statement.

16. rectangles $JKLM$ and $PQRS$



17. $\triangle TUV$ and $\triangle WXY$

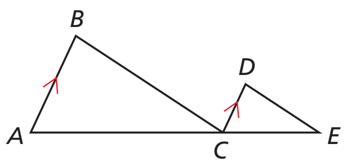


7-3 Triangle Similarity: AA, SSS, and SAS *(pp. 470–477)*

TEKS G.5.B, G.11.A, G.11.B

EXAMPLE

- **Given:** $\overline{AB} \parallel \overline{CD}$, $AB = 2CD$, $AC = 2CE$
  Prove: $\triangle ABC \sim \triangle CDE$

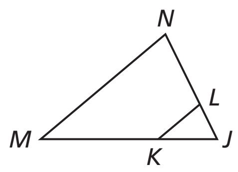


Proof:

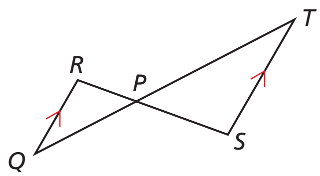
Statements	Reasons
1. $\overline{AB} \parallel \overline{CD}$	1. Given
2. $\angle BAC \cong \angle DCE$	2. Corr. $\measuredangle$ Post.
3. $AB = 2CD$, $AC = 2CE$	3. Given
4. $\frac{AB}{CD} = 2$, $\frac{AC}{CE} = 2$	4. Division Prop.
5. $\frac{AB}{CD} = \frac{AC}{CE}$	5. Trans. Prop. of =
6. $\triangle ABC \sim \triangle CDE$	6. SAS ~ *(Steps 2, 5)*

EXERCISES

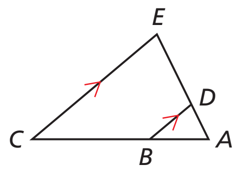
18. Given: $JL = \frac{1}{3}JN$, $JK = \frac{1}{3}JM$
Prove: $\triangle JKL \sim \triangle JMN$



19. Given: $\overline{QR} \parallel \overline{ST}$
Prove: $\triangle PQR \sim \triangle PTS$



20. Given: $\overline{BD} \parallel \overline{CE}$
Prove: $AB(CE) = AC(BD)$



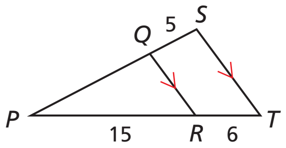
(*Hint:* After you have proved the triangles similar, look for a proportion using AB, AC, CE, and BD, the lengths of corresponding sides.)

7-4 Applying Properties of Similar Triangles *(pp. 481–487)*

TEKS G.2.A, G.3.B, G.5.B, G.9.B, G.11.A, G.11.B

EXAMPLES

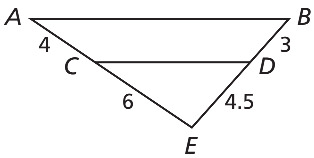
■ **Find PQ.**



It is given that $\overline{QR} \parallel \overline{ST}$, so $\frac{PQ}{QS} = \frac{PR}{RT}$ by the Triangle Proportionality Theorem.

$\frac{PQ}{5} = \frac{15}{6}$	*Substitute 5 for QS, 15 for PR, and 6 for RT.*
$6(PQ) = 75$	*Cross Products Prop.*
$PQ = 12.5$	*Divide both sides by 6.*

■ **Verify that $\overline{AB} \parallel \overline{CD}$.**

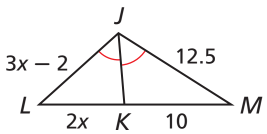


$\frac{EC}{CA} = \frac{6}{4} = 1.5$

$\frac{ED}{DB} = \frac{4.5}{3} = 1.5$

Since $\frac{EC}{CA} = \frac{ED}{DB}$, $\overline{AB} \parallel \overline{CD}$ by the Converse of the Triangle Proportionality Theorem.

■ **Find JL and LK.**



Since $\overrightarrow{JK}$ bisects $\angle LJM$, $\frac{JL}{LK} = \frac{JM}{MK}$ by the Triangle Angle Bisector Theorem.

$\frac{3x - 2}{2x} = \frac{12.5}{10}$	*Substitute the given values.*
$10(3x - 2) = 12.5(2x)$	*Cross Products Prop.*
$30x - 20 = 25x$	*Simplify.*
$30x = 25x + 20$	*Add 20 to both sides.*
$5x = 20$	*Subtract 25x from both sides.*
$x = 4$	*Divide both sides by 5.*

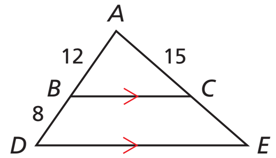
$JL = 3x - 2$
$= 3(4) - 2 = 10$

$LK = 2x$
$= 2(4) = 8$

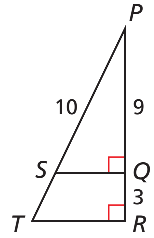
EXERCISES

Find each length.

21. CE

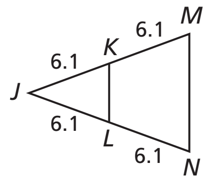


22. ST

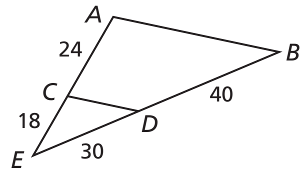


Verify that the given segments are parallel.

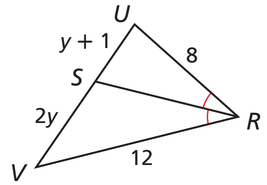
23. $\overline{KL}$ and $\overline{MN}$



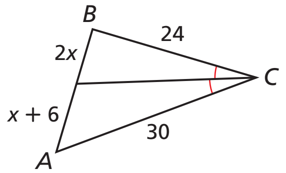
24. $\overline{AB}$ and $\overline{CD}$



25. Find SU and SV.



26. Find the length of the third side of $\triangle ABC$.



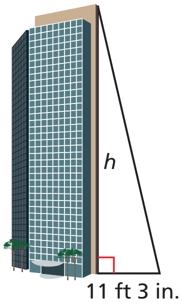
27. One side of a triangle is x inches longer than another side. The ray bisecting the angle formed by these sides divides the opposite side into 3-inch and 5-inch segments. Find the perimeter of the triangle in terms of x.

7-5 Using Proportional Relationships *(pp. 488–494)* TEKS G.1.B, G.5.A, G.11.A, G.11.B, G.11.D

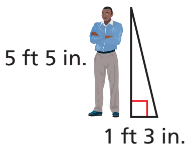
EXAMPLE

- **Use the dimensions in the diagram to find the height h of the tower.**

A student who is 5 ft 5 in. tall measured his shadow and a tower's shadow to find the height of the tower.



5 ft 5 in. = 65 in.
1 ft 3 in. = 15 in.
11 ft 3 in. = 135 in.



$\frac{h}{135} = \frac{65}{15}$ *Corr. sides are proportional.*

$15h = 65(135)$ *Cross Products Prop.*

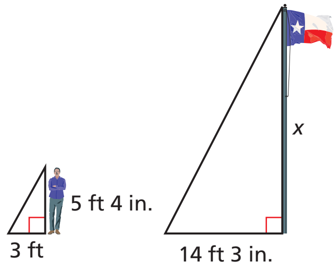
$15h = 8775$ *Simplify.*

$h = 585$ in. *Divide both sides by 15.*

The height of the tower is 48 ft 9 in.

EXERCISES

28. To find the height of a flagpole, Casey measured her own shadow and the flagpole's shadow. Given that Casey's height is 5 ft 4 in., what is the height x of the flagpole?



29. Jonathan is 3 ft from a lamppost that is 12 ft high. The lamppost and its shadow form the legs of a right triangle. Jonathan is 6 ft tall and is standing parallel to the lamppost. How long is Jonathan's shadow?

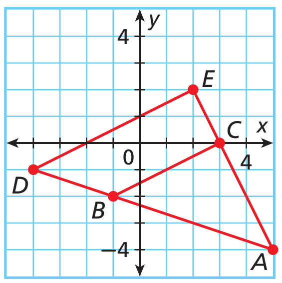
7-6 Dilations and Similarity in the Coordinate Plane *(pp. 495–500)* TEKS G.2.B, G.9.B, G.11.A

EXAMPLE

- **Given:** $A(5, -4), B(-1, -2), C(3, 0), D(-4, -1)$ and $E(2, 2)$

Prove: $\triangle ABC \sim \triangle ADE$

Proof: Plot the points and draw the triangles.



Use the Distance Formula to find the side lengths.

$AC = 2\sqrt{5}, AE = 3\sqrt{5}$

$AB = 2\sqrt{10}, AD = 3\sqrt{10}$

Therefore $\frac{AB}{AD} = \frac{AC}{AE} = \frac{2}{3}$.

Since corresponding sides are proportional and $\angle A \cong \angle A$ by the Reflexive Property, $\triangle ABC \sim \triangle ADE$ by SAS $\sim$.

EXERCISES

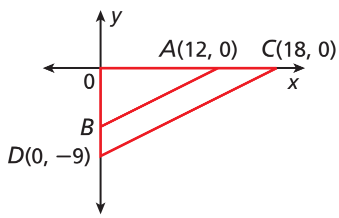
30. **Given:** $R(1, -3), S(-1, -1), T(2, 0), U(-3, 1)$, and $V(3, 3)$

Prove: $\triangle RST \sim \triangle RUV$

31. **Given:** $J(4, 4), K(2, 3), L(4, 2), M(-4, 0)$, and $N(4, -4)$

Prove: $\triangle JKL \sim \triangle JMN$

32. Given that $\triangle AOB \sim \triangle COD$, find the coordinates of B and the scale factor.



33. Graph the image of the triangle after a dilation with the given scale factor. Then verify that the image is similar to the given triangle. $K(0, 3), L(0, 0)$, and $M(4, 0)$ with scale factor 3.

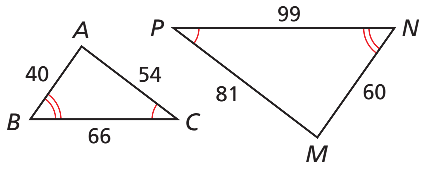
CHAPTER TEST

1. Two points on ℓ are $A(-6, 4)$ and $B(10, -6)$. Write a ratio expressing the slope of ℓ.

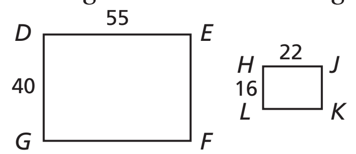
2. Alana has a photograph that is 5 in. long and 3.5 in. wide. She enlarges it so that its length is 8 in. What is the width of the enlarged photograph?

Determine whether the polygons are similar. If so, write the similarity ratio and a similarity statement.

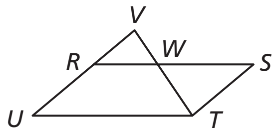
3. $\triangle ABC$ and $\triangle MNP$



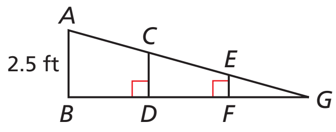
4. rectangle $DEFG$ and rectangle $HJKL$



5. Given: $\square RSTU$
Prove: $\triangle RWV \sim \triangle SWT$

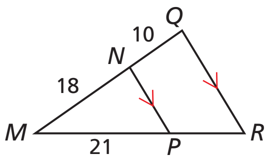


6. Derrick is building a skateboard ramp as shown. Given that $BD = DF = FG = 3$ ft, find CD and EF to the nearest tenth.

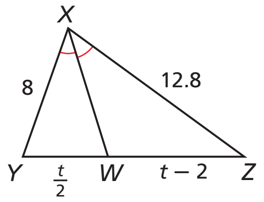


Find the length of each segment.

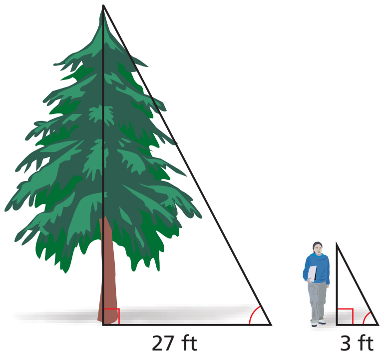
7. $\overline{PR}$



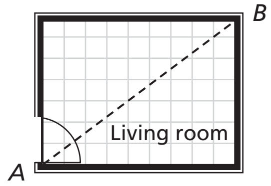
8. $\overline{YW}$ and $\overline{WZ}$



9. To find the height of a tree, a student measured the tree's shadow and her own shadow. If the student's height is 5 ft 8 in., what is the height of the tree?

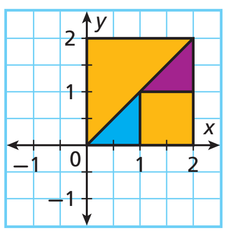


10. The plan for a living room uses the scale of 1.5 in. : 30 ft. Use a ruler and find the length of the actual room's diagonal $\overline{AB}$.



11. Given: $A(6, 5)$, $B(3, 4)$, $C(6, 3)$, $D(-3, 2)$, and $E(6, -1)$
Prove: $\triangle ABC \sim \triangle ADE$

12. A quilter designed this patch for a quilt but needs a larger version for a different project. Draw the quilt patch after a dilation with scale factor $\frac{3}{2}$.



COLLEGE ENTRANCE EXAM PRACTICE

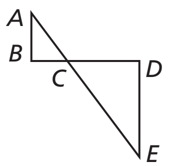
FOCUS ON SAT

The SAT consists of seven test sections: three verbal, three math, and one more verbal or math section not used to compute your final score. The "extra" section is used to try out questions for future tests and to compare your score to previous tests.

HOT TIP! Read each question carefully and make sure you answer the question being asked. Check that your answer makes sense in the context of the problem. If you have time, check your work.

You may want to time yourself as you take this practice test. It should take you about 8 minutes to complete.

1. In the figure below, the coordinates of the vertices are $A(1, 5)$, $B(1, 1)$, $D(10, 1)$, and $E(10, -7)$. If the length of $\overline{CE}$ is 10, what are the coordinates of C?



Note: Figure not drawn to scale.

(A) $(4, 1)$

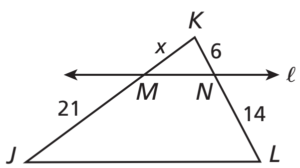
(B) $(1, 4)$

(C) $(7, 1)$

(D) $(1, 7)$

(E) $(6, 1)$

2. In the figure below, triangles JKL and MKN are similar, and ℓ is parallel to segment JL. What is the length of $\overline{KM}$?



Note: Figure not drawn to scale.

(A) 4

(B) 8

(C) 9

(D) 13

(E) 18

3. Three siblings are to share an inheritance of \$750,000 in the ratio 4 : 5 : 6. What is the amount of the greatest share?

(A) \$125,000

(B) \$187,500

(C) \$250,000

(D) \$300,000

(E) \$450,000

4. A 35-foot flagpole casts a 9-foot shadow at the same time that a girl casts a 1.2-foot shadow. How tall is the girl?

(A) 3 feet 8 inches

(B) 4 feet 6 inches

(C) 4 feet 7 inches

(D) 4 feet 8 inches

(E) 5 feet 6 inches

5. What polygon is similar to every other polygon of the same name?

(A) Triangle

(B) Parallelogram

(C) Rectangle

(D) Square

(E) Trapezoid

TAKS TACKLER

Standardized Test Strategies

Any Question Type: Interpret A Diagram

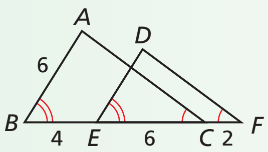
When a diagram is included as part of a test question, do not make any assumptions about the diagram. Diagrams are not always drawn to scale and can be misleading if you are not careful.

EXAMPLE 1

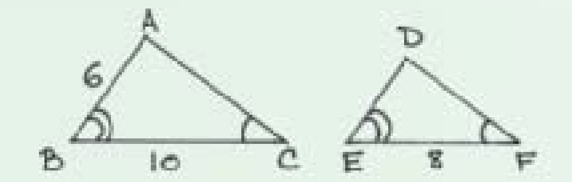
Multiple Choice What is DE?

Ⓐ 3.6 Ⓒ 4.8

Ⓑ 4 Ⓓ 9



Make your own sketch of the diagram. Separate the two triangles so that you are able to find the side length measures.



By redrawing the diagram, it is clear that the two triangles are similar. Set up a proportion to find DE.

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{6}{10} = \frac{DE}{8}$$

$$\frac{48}{10} = DE$$

$$DE = 4.8$$

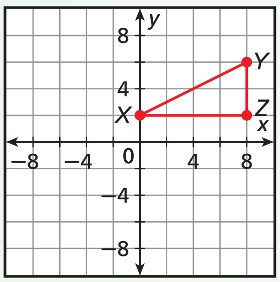
The correct choice is C.

EXAMPLE 2

Gridded Response $\triangle X'Y'Z'$ is the image of $\triangle XYZ$ after a dilation with scale factor $\frac{1}{2}$. Find $X'Z'$.

Before you begin, look at the scale of both the x-axis and the y-axis. Do not assume that the scale is always 1.

At first glance, you might assume that XZ is 4. But by looking closely at the x-axis, notice that each increment represents 2 units. So XZ is actually 8.



When $\triangle XYZ$ is dilated by a factor of $\frac{1}{2}$, $X'Z'$ will be half of XZ.

$$X'Z' = \frac{1}{2}XZ = \frac{1}{2}(8) = 4$$

TAKS Tip If the diagram does not match the given information, draw one that is more accurate.

Read each test item and answer the questions that follow.

Item A

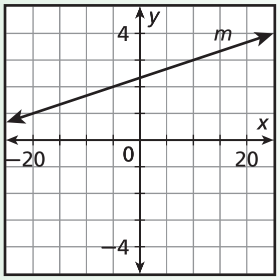
Multiple Choice Which ratio is the slope of m?

Ⓐ $\frac{1}{15}$

Ⓑ $\frac{1}{3}$

Ⓒ 3

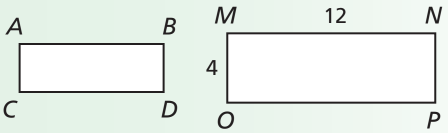
Ⓓ 15



1. What is the scale of the y-axis? Use this scale to determine the rise of the slope.
2. What is the scale of the x-axis? Use this scale to determine the run of the slope.
3. Write the ratio that represents the slope of m.
4. Anna selected choice B as her answer. Is she correct? If not, what do you think she did wrong?

Item B

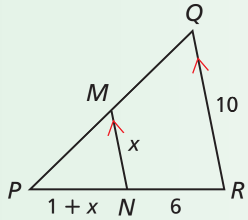
Gridded Response If $ABDC \sim MNPO$ and AC is 6, what is AB?



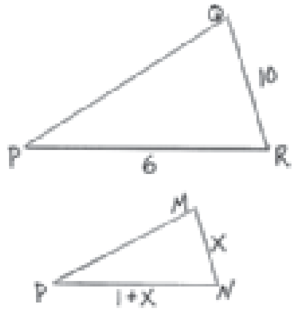
5. Examine the figures. Do you think $\overline{AB}$ is longer or shorter than $\overline{MN}$?
6. Do you think the drawings actually represent the given information? If not, explain why.
7. Create your own sketch of the figures to more accurately match the given information.

Item C

Gridded Response What is the measure of MN?



8. Describe how redrawing the figure can help you better understand the given information.
9. After reading this test question, a student redrew the figure as shown below. Explain if it is a correct interpretation of the original figure. If it is not, redraw and/or relabel it so that it is correct.



Item D

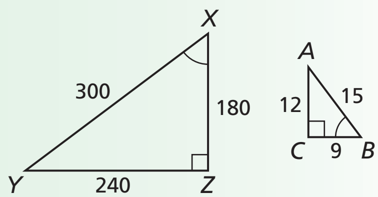
Multiple Choice Which is a similarity ratio for the triangles shown?

Ⓐ $\frac{20}{1}$

Ⓑ $\frac{10}{1}$

Ⓒ $\frac{2}{1}$

Ⓓ $\frac{15}{1}$



10. Chad determined that choice D was correct. Do you agree? If not, what do you think he did wrong?
11. Redraw the figures so that they are easier to understand. Write three statements that describe which vertices correspond to each other and three statements that describe which sides correspond to each other.

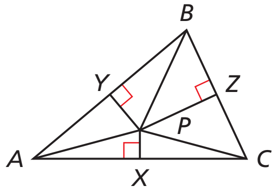
CUMULATIVE ASSESSMENT, CHAPTERS 1–7

Multiple Choice

1. Which similarity statement is true for rectangles *ABCD* and *MNPQ*, given that $AB = 3$, $AD = 4$, $MN = 6$, and $NP = 4.5$?

- Ⓐ Rectangle *ABCD* ~ rectangle *MNPQ*
- Ⓑ Rectangle *ABCD* ~ rectangle *PQMN*
- Ⓒ Rectangle *ABCD* ~ rectangle *MPNQ*
- Ⓓ Rectangle *ABCD* ~ rectangle *QMNP*

2. $\triangle ABC$ has perpendicular bisectors $\overline{XP}$, $\overline{YP}$, and $\overline{ZP}$. If $AP = 6$ and $ZP = 4.5$, what is the length of $\overline{BC}$ to the nearest tenth?



- Ⓕ 4.0
- Ⓖ 7.9
- Ⓗ 9.0
- Ⓙ 12.7

3. What is the converse of the statement "If a quadrilateral has 4 congruent sides, then it is a rhombus"?

- Ⓐ If a quadrilateral is a rhombus, then it has 4 congruent sides.
- Ⓑ If a quadrilateral does not have 4 congruent sides, then it is not a rhombus.
- Ⓒ If a quadrilateral is not a rhombus, then it does not have 4 congruent sides.
- Ⓓ If a rhombus has 4 congruent sides, then it is a quadrilateral.

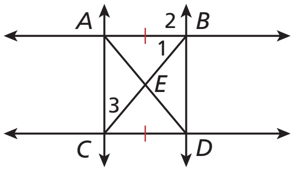
4. A blueprint for a hotel uses a scale of 3 in. : 100 ft. On the blueprint, the lobby has a width of 1.5 in. and a length of 2.25 in. If the carpeting for the lobby costs \$1.25 per square foot, how much will the carpeting for the entire lobby cost?

- Ⓕ \$312.50
- Ⓖ \$1406.25
- Ⓗ \$3000.00
- Ⓙ \$4687.50

5. If $12x = 16y$, what is the ratio of x to y in simplest form?

- Ⓐ $\frac{1}{4}$
- Ⓑ $\frac{3}{4}$
- Ⓒ $\frac{4}{3}$
- Ⓓ $\frac{4}{1}$

Use the diagram for Items 6 and 7.



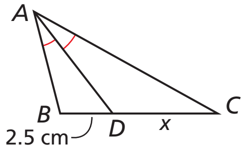
6. Given that $\overline{AB} \cong \overline{CD}$, which additional information would be sufficient to prove that *ABCD* is a parallelogram?

- Ⓕ $\overline{AB} \parallel \overline{CD}$
- Ⓖ $\overline{AC} \parallel \overline{BD}$
- Ⓗ $\angle CAB \cong \angle CDB$
- Ⓙ *E* is the midpoint of $\overline{AD}$.

7. If $\overleftrightarrow{AC}$ is parallel to $\overleftrightarrow{BD}$ and $m\angle 1 + m\angle 2 = 140°$, what is the measure of $\angle 3$?

- Ⓐ 20°
- Ⓑ 40°
- Ⓒ 50°
- Ⓓ 70°

8. If $\overline{AC}$ is twice as long as $\overline{AB}$, what is the length of $\overline{DC}$?



- Ⓕ 2.5 centimeters
- Ⓖ 3.75 centimeters
- Ⓗ 5 centimeters
- Ⓙ 15 centimeters

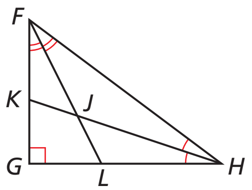
2ND PRINT

TAKS Tip When writing proportions for similar figures, make sure that each ratio compares corresponding side lengths in each figure.

9. What type of triangle has angles that measure $(2x)°$, $(3x - 9)°$, and $(x + 27)°$?

- (A) Isosceles acute triangle
- (B) Isosceles right triangle
- (C) Scalene acute triangle
- (D) Scalene obtuse triangle

Use the diagram for Items 10 and 11.



10. Which of these points is the orthocenter of $\triangle FGH$?

- (F) F
- (H) H
- (G) G
- (J) J

11. Which of the following could be the side lengths of $\triangle FGH$?

- (A) $FG = 2$, $GH = 3$, and $FH = 4$
- (B) $FG = 4$, $GH = 5$, and $FH = 6$
- (C) $FG = 5$, $GH = 4$, and $FH = 3$
- (D) $FG = 6$, $GH = 8$, and $FH = 10$

12. The measure of one of the exterior angles of a right triangle is 120°. What are the measures of the acute interior angles of the triangle?

- (F) 30° and 60°
- (H) 40° and 80°
- (G) 40° and 50°
- (J) 60° and 60°

Gridded Response

13. The ratio of a football field's length to its width is 9:4. If the length of the field is 360 ft, what is the width of the field in feet?

14. The sum of the measures of the interior angles of a convex polygon is 1260°. How many sides does the polygon have?

15. In kite *PQRS*, $\angle P$ and $\angle R$ are opposite angles. If $m\angle P = 25°$ and $m\angle R = 75°$, what is the measure of $\angle Q$ in degrees?

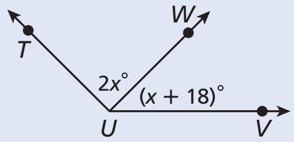
16. Heather is 1.6 m tall and casts a shadow of 3.5 m. At the same time, a barn casts a shadow of 17.5 m. Find the height of the barn in meters.

STANDARDIZED TEST PREP

Short Response

17. $\triangle ABC$ has vertices $A(-2, 0)$, $B(2, 2)$, and $C(2, -2)$. $\triangle DEC$ has vertices $D(0, -1)$, $E(2, 0)$, and $C(2, -2)$. Prove that $\triangle ABC \sim \triangle DEC$.

18. $\angle TUV$ in the diagram below is an obtuse angle.



Write an inequality showing the range of possible measurements for $\angle TUW$. Show your work or explain your answer.

19. $\triangle ABC$ and $\triangle ABD$ share side $\overline{AB}$. Given that $\triangle ABC \sim \triangle ABD$, use AAS to explain why these two triangles must also be congruent.

20. Rectangle *ABCD* has a length of 2.6 cm and a width of 1.8 cm. Rectangle *WXYZ* has a length of 7.8 cm and a width of 5.4 cm. Determine whether rectangle *ABCD* is similar to rectangle *WXYZ*. Explain your reasoning.

21. If $\triangle ABC$ and $\triangle XYZ$ are similar triangles, there are six possible similarity statements.

a. What is the probability that $\triangle ABC \sim \triangle XYZ$ is correct?

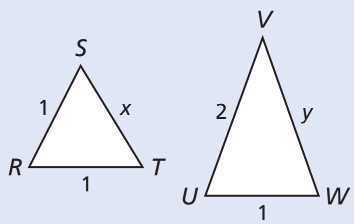
b. If $\triangle ABC$ and $\triangle XYZ$ are isosceles, what is the probability that $\triangle ABC \sim \triangle XYZ$?

c. If $\triangle ABC$ and $\triangle XYZ$ are equilateral, what is the probability that $\triangle ABC \sim \triangle XYZ$? Explain.

Extended Response

22.a. Given: $\triangle SRT \sim \triangle VUW$ and $\overline{SR} \cong \overline{ST}$
Prove: $\overline{VU} \cong \overline{VW}$

b. Explain in words how you determine the possible values for *x* and *y* that would make the two triangles below similar.



<u>Note</u>: Triangles not drawn to scale.

c. Explain why *x* cannot have a value of 1 if the two triangles in the diagram above are similar.

2ND PRINT