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Chapter 6 Resource Masters

Geometry

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Consumable Workbooks Many of the worksheets contained in the Chapter Resource Masters are available as consumable workbooks in both English and Spanish.

	ISBN10	ISBN13
Study Guide and Intervention Workbook	0-07-890848-5	978-0-07-890848-4
Homework Practice Workbook	0-07-890849-3	978-0-07-890849-1
Spanish Version		
Homework Practice Workbook	0-07-890853-1	978-0-07-890853-8

Answers for Workbooks The answers for Chapter 6 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

StudentWorks Plus™ This CD-ROM includes the entire Student Edition test along with the English workbooks listed above.

TeacherWorks Plus™ All of the materials found in this booklet are included for viewing, printing, and editing in this CD-ROM.

Spanish Assessment Masters (ISBN10: 0-07-890856-6, ISBN13: 978-0-07-890856-9) These masters contain a Spanish version of Chapter 6 Test Form 2A and Form 2C.

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Teacher's Guide to Using the *Chapter 6 Resource Masters*

The *Chapter 6 Resource Masters* includes the core materials needed for Chapter 6. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing on the *TeacherWorks Plus*™ CD-ROM.

Chapter Resources

Student-Built Glossary (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to recording definitions and/or examples for each term. You may suggest that student highlight or star the terms with which they are not familiar. Give to students before beginning Lesson 6-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 3–4) This master presented in both English and Spanish is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. They are written for use with all levels of students.

Graphing Calculator or Spreadsheet Activities These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.

Assessment Options

The assessment masters in the *Chapter 6 Resource Masters* offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Extended-Response Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students' knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests

- *Form 1* contains multiple-choice questions and is intended for use with below grade level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Form 3* is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Samples answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages.
- Full-size answer keys are provided for the assessment masters.

NAME _____ DATE _____ PERIOD _____

6 Student-Built Glossary

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 6. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
base		
diagonal		
isosceles trapezoid		
legs		
midsegment of a trapezoid		

(continued on the next page)

NAME _____ DATE _____ PERIOD _____

6 Student-Built Glossary *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
parallelogram		
rectangle		
rhombus		
square		
trapezoid		

NAME _____ DATE _____ PERIOD _____

6 Anticipation Guide

Quadrilaterals

Step 1 ***Before you begin Chapter 6***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. A triangle has no diagonals.	
	2. A diagonal of a polygon is a segment joining the midpoints of two sides of the polygon.	
	3. The sum of the measures of the angles in a polygon can be determined by subtracting 2 from the number of sides and multiplying the result by 180.	
	4. For a quadrilateral to be a parallelogram it must have two pairs of parallel sides.	
	5. The diagonals of a parallelogram are congruent.	
	6. If you know that one pair of opposite sides of a quadrilateral is both parallel and congruent, then you know the quadrilateral is a parallelogram.	
	7. If a quadrilateral is a rectangle, then all four angles are congruent.	
	8. The diagonals of a rhombus are congruent.	
	9. The properties of a rhombus are not true for a square.	
	10. A trapezoid has only one pair of parallel sides.	
	11. The median of a trapezoid is perpendicular to the bases.	
	12. An isosceles trapezoid has exactly one pair of congruent sides.	

Step 2 ***After you complete Chapter 6***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NOMBRE _____ FECHA _____ PERÍODO _____

6 Ejercicios preparatorios

Cuadriláteros

Paso 1 ***Antes de comenzar el Capítulo 6***

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a).

PASO 1 A, D o NS	Enunciado	PASO 2 A o D
	1. Un triángulo no tiene diagonales.	
	2. La diagonal de un polígono es un segmento que une los puntos medios de dos lados del polígono.	
	3. La suma de las medidas de los ángulos de un polígono puede determinarse restando 2 del número de lados y multiplicando el resultado por 180.	
	4. Para que un cuadrilátero sea un paralelogramo debe tener dos pares de lados paralelos.	
	5. Las diagonales de un paralelogramo son congruentes.	
	6. Si sabes que un par de lados opuestos de un cuadrilátero son paralelos y congruentes, entonces sabes que el cuadrilátero es un paralelogramo.	
	7. Si un cuadrilátero es un rectángulo, entonces todos los cuatro lados son congruentes.	
	8. Las diagonales de un rombo son congruentes.	
	9. Las propiedades de un rombo no son verdaderas para un cuadrado.	
	10. Un trapecio sólo tiene dos lados paralelos.	
	11. La mediana de un trapecio es perpendicular a las bases.	
	12. Un trapecio isósceles tiene exactamente un par de lados congruentes.	

Paso 2 **Después de completar el Capítulo 6**

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.

NAME _____ DATE _____ PERIOD _____

6-1 Study Guide and Intervention

Angles of Polygons

Polygon Interior Angles Sum The segments that connect the nonconsecutive vertices of a polygon are called **diagonals**. Drawing all of the diagonals from one vertex of an ***n*-gon** separates the polygon into $n - 2$ triangles. The sum of the measures of the interior angles of the polygon can be found by adding the measures of the interior angles of those $n - 2$ triangles.

Polygon Interior Angle Sum Theorem	The sum of the interior angle measures of an n-sided convex polygon is $(n - 2) \cdot 180$.

Example 1 **A convex polygon has 13 sides. Find the sum of the measures of the interior angles.**

$$\begin{aligned}(n - 2) \cdot 180 &= (13 - 2) \cdot 180 \\ &= 11 \cdot 180 \\ &= 1980\end{aligned}$$

Example 2 **The measure of an interior angle of a regular polygon is 120. Find the number of sides.**

The number of sides is n, so the sum of the measures of the interior angles is $120n$.

$$\begin{aligned}120n &= (n - 2) \cdot 180 \\ 120n &= 180n - 360 \\ -60n &= -360 \\ n &= 6\end{aligned}$$

Exercises

Find the sum of the measures of the interior angles of each convex polygon.

1. decagon **2.** 16-gon **3.** 30-gon

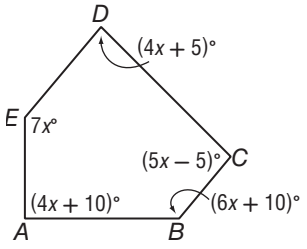
4. octagon **5.** 12-gon **6.** 35-gon

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

7. 150 **8.** 160 **9.** 175

10. 165 **11.** 144 **12.** 135

13. Find the value of x.



NAME _____ DATE _____ PERIOD _____

6-1 Study Guide and Intervention *(continued)*

Angles of Polygons

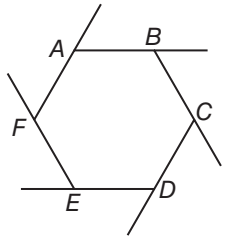
Polygon Exterior Angles Sum There is a simple relationship among the exterior angles of a convex polygon.

Polygon Exterior Angle Sum Theorem	The sum of the exterior angle measures of a convex polygon, one angle at each vertex, is 360.

Example 1 **Find the sum of the measures of the exterior angles, one at each vertex, of a convex 27-gon.**

For *any* convex polygon, the sum of the measures of its exterior angles, one at each vertex, is 360.

Example 2 **Find the measure of each exterior angle of regular hexagon *ABCDEF*.**



The sum of the measures of the exterior angles is 360 and a hexagon has 6 angles. If n is the measure of each exterior angle, then

$6n = 360$

$n = 60$

The measure of each exterior angle of a regular hexagon is 60.

Exercises

Find the sum of the measures of the exterior angles of each convex polygon.

1. decagon **2.** 16-gon **3.** 36-gon

Find the measure of each exterior angle for each regular polygon.

4. 12-gon **5.** hexagon **6.** 20-gon

7. 40-gon **8.** heptagon **9.** 12-gon

10. 24-gon **11.** dodecagon **12.** octagon

NAME _____ DATE _____ PERIOD _____

6-1 Skills Practice

Angles of Polygons

Find the sum of the measures of the interior angles of each convex polygon.

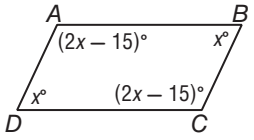
1. nonagon **2.** heptagon **3.** decagon

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

4. 108 **5.** 120 **6.** 150

Find the measure of each interior angle.

7.



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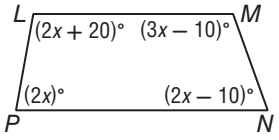
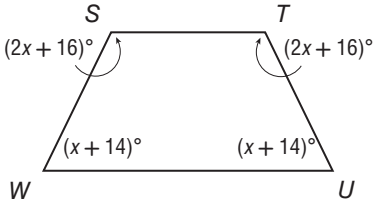
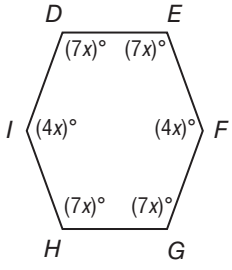


9.



10.



Find the measures of each interior angle of each regular polygon.

11. quadrilateral **12.** pentagon **13.** dodecagon

Find the measures of each exterior angle of each regular polygon.

14. octagon **15.** nonagon **16.** 12-gon

NAME _____ DATE _____ PERIOD _____

6-1 Practice

Angles of Polygons

Find the sum of the measures of the interior angles of each convex polygon.

1. 11-gon

2. 14-gon

3. 17-gon

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

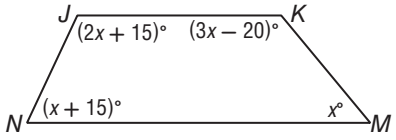
4. 144

5. 156

6. 160

Find the measure of each interior angle.

7.



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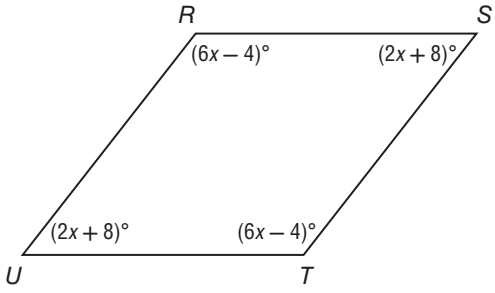


Find the measures of an exterior angle and an interior angle given the number of sides of each regular polygon. Round to the nearest tenth, if necessary.

9. 16

10. 24

11. 30

12. 14

13. 22

14. 40

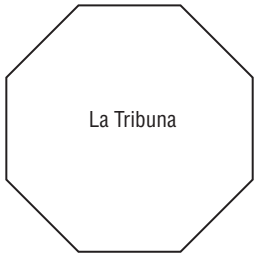
15. CRYSTALLOGRAPHY Crystals are classified according to seven crystal systems. The basis of the classification is the shapes of the faces of the crystal. Turquoise belongs to the triclinic system. Each of the six faces of turquoise is in the shape of a parallelogram. Find the sum of the measures of the interior angles of one such face.

NAME _____ DATE _____ PERIOD _____

6-1 Word Problem Practice

Angles of Polygons

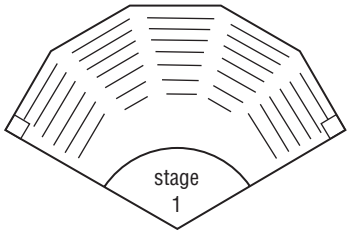
1. ARCHITECTURE In the Uffizi gallery in Florence, Italy, there is a room built by Buontalenti called the Tribune (*La Tribuna* in Italian). This room is shaped like a regular octagon.



What angle do consecutive walls of the Tribune make with each other?

2. BOXES Jasmine is designing boxes she will use to ship her jewelry. She wants to shape the box like a regular polygon. In order for the boxes to pack tightly, she decides to use a regular polygon that has the property that the measure of its interior angles is half the measure of its exterior angles. What regular polygon should she use?

3. THEATER A theater floor plan is shown in the figure. The upper five sides are part of a regular dodecagon.

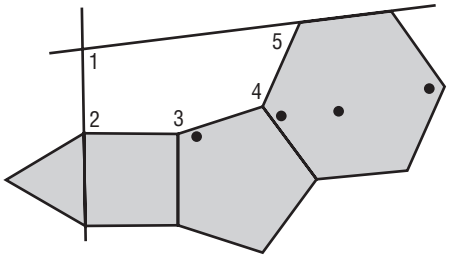


Find $m\angle 1$.

4. ARCHEOLOGY Archeologists unearthed parts of two adjacent walls of an ancient castle.

Before it was unearthed, they knew from ancient texts that the castle was shaped like a regular polygon, but nobody knew how many sides it had. Some said 6, others 8, and some even said 100. From the information in the figure, how many sides did the castle really have?

5. POLYGON PATH In Ms. Rickets' math class, students made a "polygon path" that consists of regular polygons of 3, 4, 5, and 6 sides joined together as shown.



a. Find $m\angle 2$ and $m\angle 5$.

b. Find $m\angle 3$ and $m\angle 4$.

c. What is $m\angle 1$?

Lesson 6-1

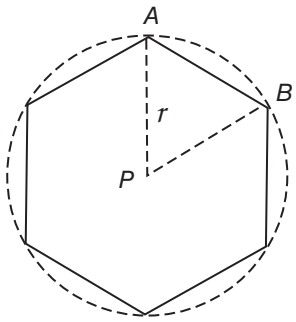
NAME _____ DATE _____ PERIOD _____

6-1 Enrichment

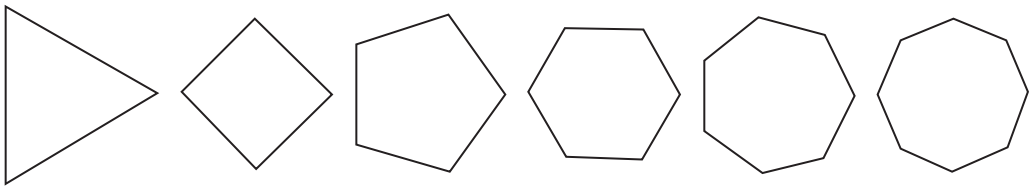
Central Angles of Regular Polygons

You have learned about the interior and exterior angles of a polygon. Regular polygons also have **central angles**. A central angle is measured from the center of the polygon.

The center of a polygon is the point equidistant from all of the vertices of the polygon, just as the center of a circle is the point equidistant from all of the points on the circle. The central angle is the angle drawn with the vertex at the center of the circle and the sides of angle drawn through consecutive vertices of the polygon. One of the central angles that can be drawn in this regular hexagon is $\angle APB$. You may remember from making circle graphs that there are 360° around the center of a circle.



1. By using logic or by drawing sketches, find the measure of the central angle of each regular polygon.



2. Make a conjecture about how the measure of a central angle of a regular polygon relates to the measures of the interior angles and exterior angles of a regular polygon.

3. CHALLENGE In obtuse $\triangle ABC$, $\overline{BC}$ is the longest side. $\overline{AC}$ is also a side of a 21-sided regular polygon. $\overline{AB}$ is also a side of a 28-sided regular polygon. The 21-sided regular polygon and the 28-sided regular polygon have the same center point P. Find n if $\overline{BC}$ is a side of a n-sided regular polygon that has center point P.

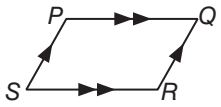
(*Hint:* Sketch a circle with center P and place points A, B, and C on the circle.)

NAME _____ DATE _____ PERIOD _____

6-2 Study Guide and Intervention

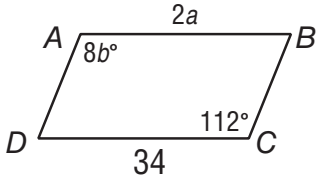
Parallelograms

Sides and Angles of Parallelograms A quadrilateral with both pairs of opposite sides parallel is a **parallelogram**. Here are four important properties of parallelograms.



	If *PQRS* is a parallelogram, then
If a quadrilateral is a parallelogram, then its opposite sides are congruent.	$\overline{PQ} \cong \overline{SR}$ and $\overline{PS} \cong \overline{QR}$
If a quadrilateral is a parallelogram, then its opposite angles are congruent.	$\angle P \cong \angle R$ and $\angle S \cong \angle Q$
If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.	$\angle P$ and $\angle S$ are supplementary; $\angle S$ and $\angle R$ are supplementary; $\angle R$ and $\angle Q$ are supplementary; $\angle Q$ and $\angle P$ are supplementary.
If a parallelogram has one right angle, then it has four right angles.	If $m\angle P = 90$, then $m\angle Q = 90$, $m\angle R = 90$, and $m\angle S = 90$.

Example **If *ABCD* is a parallelogram, find the value of each variable.**



$\overline{AB}$ and $\overline{CD}$ are opposite sides, so $\overline{AB} \cong \overline{CD}$.

$2a = 34$

$a = 17$

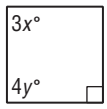
$\angle A$ and $\angle C$ are opposite angles, so $\angle A \cong \angle C$.

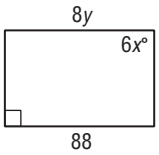
$8b = 112$

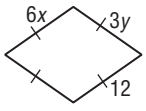
$b = 14$

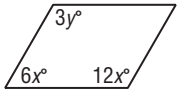
Exercises

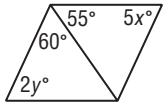
Find the value of each variable.

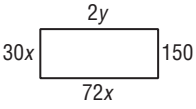
1. 

2. 

3. 

4. 

5. 

6. 

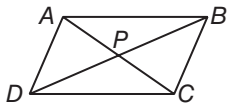
Lesson 6-2

NAME _____ DATE _____ PERIOD _____

6-2 Study Guide and Intervention *(continued)*

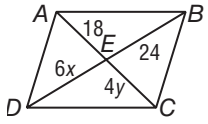
Parallelograms

Diagonals of Parallelograms Two important properties of parallelograms deal with their diagonals.



	If *ABCD* is a parallelogram, then
If a quadrilateral is a parallelogram, then its diagonals bisect each other.	$AP = PC$ and $DP = PB$
If a quadrilateral is a parallelogram, then each diagonal separates the parallelogram into two congruent triangles.	$\triangle ACD \cong \triangle CAB$ and $\triangle ADB \cong \triangle CBD$

Example **Find the value of x and y in parallelogram $ABCD$.**



The diagonals bisect each other, so $AE = CE$ and $DE = BE$.

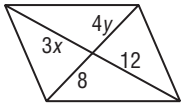
$6x = 24$ $\quad$ $4y = 18$

$x = 4$ $\quad$ $y = 4.5$

Exercises

Find the value of each variable.

1.



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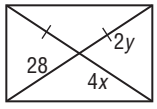

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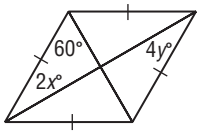
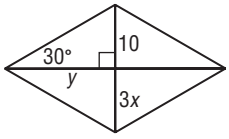


4.



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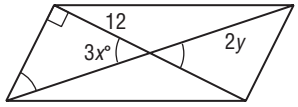

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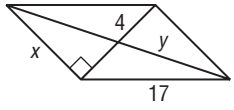


COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of $\square ABCD$ with the given vertices.

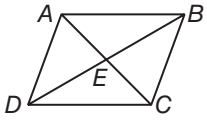
7. $A(3, 6)$, $B(5, 8)$, $C(3, -2)$, and $D(1, -4)$

8. $A(-4, 3)$, $B(2, 3)$, $C(-1, -2)$, and $D(-7, -2)$

9. PROOF Write a paragraph proof of the following.

Given: $\square ABCD$

Prove: $\triangle AED \cong \triangle BEC$



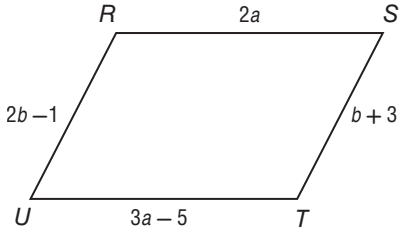
NAME _____ DATE _____ PERIOD _____

6-2 Skills Practice

Parallelograms

ALGEBRA Find the value of each variable.

1.



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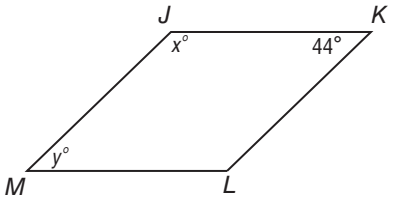

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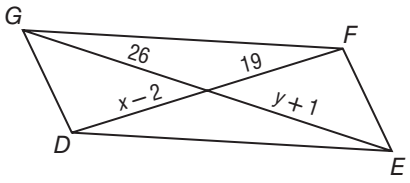

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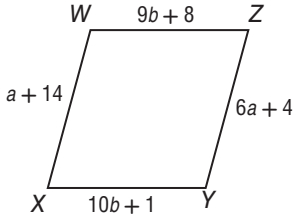

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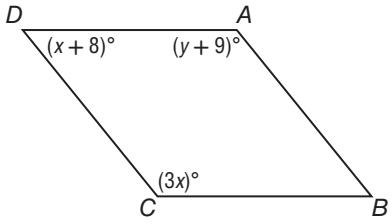
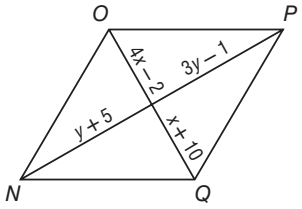


6.



COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of $\square HJKL$ with the given vertices.

7. $H(1, 1)$, $J(2, 3)$, $K(6, 3)$, $L(5, 1)$

8. $H(-1, 4)$, $J(3, 3)$, $K(3, -2)$, $L(-1, -1)$

9. PROOF Write a paragraph proof of the theorem *Consecutive angles in a parallelogram are supplementary*.

Lesson 6-2

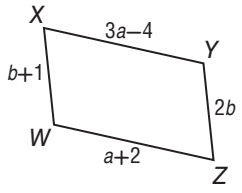
NAME _____ DATE _____ PERIOD _____

6-2 Practice

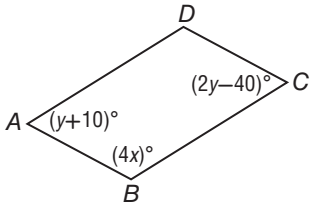
Parallelograms

ALGEBRA Find the value of each variable.

1.



2.



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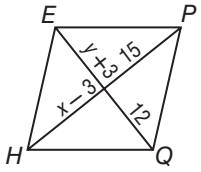
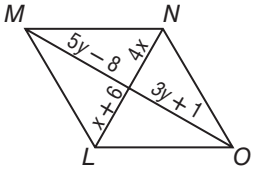
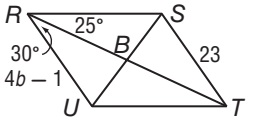


4.



ALGEBRA Use $\square RSTU$ to find each measure or value.



5. $m\angle RST =$ _____

6. $m\angle STU =$ _____

7. $m\angle TUR =$ _____

8. $b =$ _____

COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of $\square PRYZ$ with the given vertices.

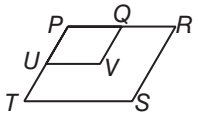
9. $P(2, 5)$, $R(3, 3)$, $Y(-2, -3)$, $Z(-3, -1)$

10. $P(2, 3)$, $R(1, -2)$, $Y(-5, -7)$, $Z(-4, -2)$

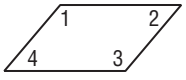
11. PROOF Write a paragraph proof of the following.

Given: $\square PRST$ and $\square PQVU$

Prove: $\angle V \cong \angle S$



12. CONSTRUCTION Mr. Rodriquez used the parallelogram at the right to design a herringbone pattern for a paving stone. He will use the paving stone for a sidewalk. If $m\angle 1$ is 130, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.



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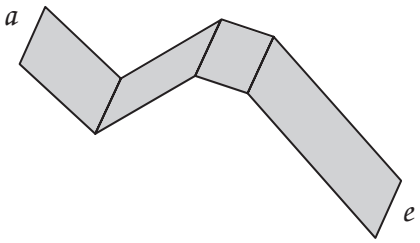
6-2 Word Problem Practice

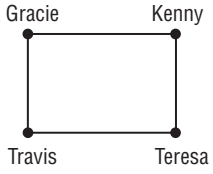
Parallelograms

1. WALKWAY A walkway is made by adjoining four parallelograms as shown.

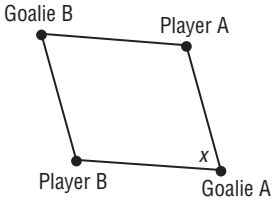


Are the end segments a and e parallel to each other? Explain.

2. DISTANCE Four friends live at the four corners of a block shaped like a parallelogram. Gracie lives 3 miles away from Kenny. How far apart do Teresa and Travis live from each other?



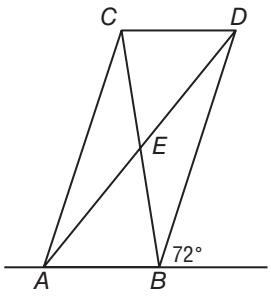
3. SOCCER Four soccer players are located at the corners of a parallelogram. Two of the players in opposite corners are the goalies. In order for goalie A to be able to see the three others, she must be able to see a certain angle x in her field of vision.



What angle does the other goalie have to be able to see in order to keep an eye on the other three players?

4. VENN DIAGRAMS Make a Venn diagram showing the relationship between squares, rectangles, and parallelograms.

5. SKYSCRAPERS On vacation, Tony's family took a helicopter tour of the city. The pilot said the newest building in the city was the building with this top view. He told Tony that the exterior angle by the front entrance is 72°. Tony wanted to know more about the building, so he drew this diagram and used his geometry skills to learn a few more things. The front entrance is next to vertex B.



a. What are the measures of the four angles of the parallelogram?

b. How many pairs of congruent triangles are there in the figure? What are they?

Lesson 6-2

NAME _____ DATE _____ PERIOD _____

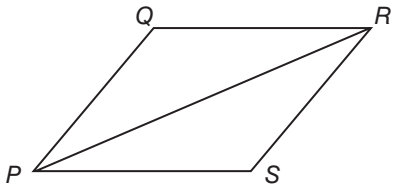
6-2 Enrichment

Diagonals of Parallelograms

In some drawings the diagonal of a parallelogram appears to be the angle bisector of both opposite angles. When might that be true?

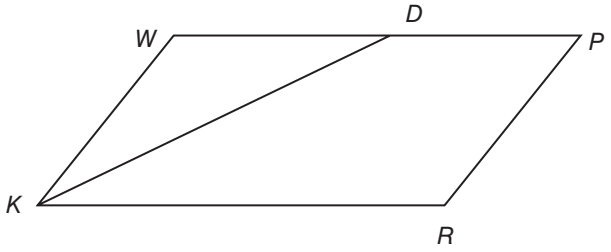
1. Given: Parallelogram $PQRS$ with diagonal $\overline{PR}$. $\overline{PR}$ is an angle bisector of $\angle QPS$ and $\angle QRS$.

What type of parallelogram is $PQRS$? Justify your answer.



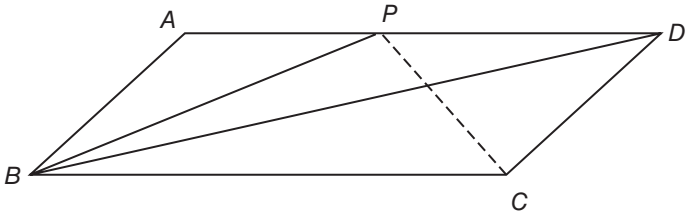
2. Given: Parallelogram $WPRK$ with angle bisector $\overline{KD}$, $DP = 5$, and $WD = 7$.

Find WK and KR.



3. Refer to Exercise 2. Write a statement about parallelogram $WPRK$ and angle bisector $\overline{KD}$.

4. Given: Parallelogram $ABCD$ with diagonal $\overline{BD}$ and angle bisector $\overline{BP}$. $PD = 5$, $BP = 6$, and $CP = 6$. The perimeter of triangle PCD is 15. Find AB and BC.



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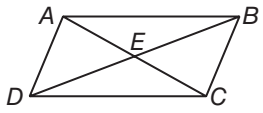
6-3 Study Guide and Intervention

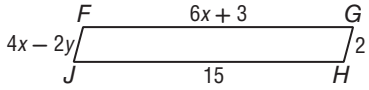
Tests for Parallelograms

Conditions for Parallelograms There are many ways to establish that a quadrilateral is a parallelogram.



If:	If:
both pairs of opposite sides are parallel,	$\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$,
both pairs of opposite sides are congruent,	$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$,
both pairs of opposite angles are congruent,	$\angle ABC \cong \angle ADC$ and $\angle DAB \cong \angle BCD$,
the diagonals bisect each other,	$\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$,
one pair of opposite sides is congruent and parallel,	$\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$, or $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$,
then: the figure is a parallelogram.	**then:** ABCD is a parallelogram.

Example **Find x and y so that $FGHJ$ is a parallelogram.**



$FGHJ$ is a parallelogram if the lengths of the opposite sides are equal.

$6x + 3 = 15$ $\qquad$ $4x - 2y = 2$

$6x = 12$ $\qquad$ $4(2) - 2y = 2$

$x = 2$ $\qquad$ $8 - 2y = 2$

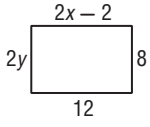
$-2y = -6$

$y = 3$

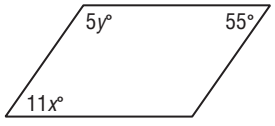
Exercises

Find x and y so that the quadrilateral is a parallelogram.

1.



2.



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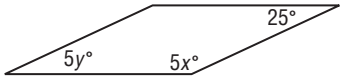

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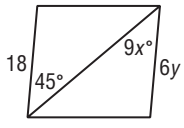

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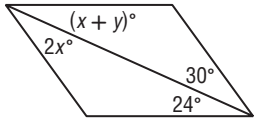
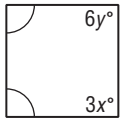


6.



NAME _____ DATE _____ PERIOD _____

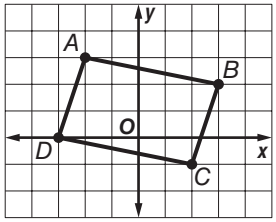
6-3 Study Guide and Intervention *(continued)*

Tests for Parallelograms

Parallelograms on the Coordinate Plane On the coordinate plane, the Distance, Slope, and Midpoint Formulas can be used to test if a quadrilateral is a parallelogram.

Example **Determine whether *ABCD* is a parallelogram.**

The vertices are $A(-2, 3)$, $B(3, 2)$, $C(2, -1)$, and $D(-3, 0)$.



Method 1: Use the Slope Formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

slope of $\overline{AD} = \frac{3 - 0}{-2 - (-3)} = \frac{3}{1} = 3$ slope of $\overline{BC} = \frac{2 - (-1)}{3 - 2} = \frac{3}{1} = 3$

slope of $\overline{AB} = \frac{2 - 3}{3 - (-2)} = -\frac{1}{5}$ slope of $\overline{CD} = \frac{-1 - 0}{2 - (-3)} = -\frac{1}{5}$

Since opposite sides have the same slope, $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$. Therefore, $ABCD$ is a parallelogram by definition.

Method 2: Use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$AB = \sqrt{(-2 - 3)^2 + (3 - 2)^2} = \sqrt{25 + 1}$ or $\sqrt{26}$

$CD = \sqrt{(2 - (-3))^2 + (-1 - 0)^2} = \sqrt{25 + 1}$ or $\sqrt{26}$

$AD = \sqrt{(-2 - (-3))^2 + (3 - 0)^2} = \sqrt{1 + 9}$ or $\sqrt{10}$

$BC = \sqrt{(3 - 2)^2 + (2 - (-1))^2} = \sqrt{1 + 9}$ or $\sqrt{10}$

Since both pairs of opposite sides have the same length, $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$. Therefore, $ABCD$ is a parallelogram by Theorem 6.9.

Exercises

Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

1. $A(0, 0)$, $B(1, 3)$, $C(5, 3)$, $D(4, 0)$; Slope Formula

2. $D(-1, 1)$, $E(2, 4)$, $F(6, 4)$, $G(3, 1)$; Slope Formula

3. $R(-1, 0)$, $S(3, 0)$, $T(2, -3)$, $U(-3, -2)$; Distance Formula

4. $A(-3, 2)$, $B(-1, 4)$, $C(2, 1)$, $D(0, -1)$; Distance and Slope Formulas

5. $S(-2, 4)$, $T(-1, -1)$, $U(3, -4)$, $V(2, 1)$; Distance and Slope Formulas

6. $F(3, 3)$, $G(1, 2)$, $H(-3, 1)$, $I(-1, 4)$; Midpoint Formula

7. A parallelogram has vertices $R(-2, -1)$, $S(2, 1)$, and $T(0, -3)$. Find all possible coordinates for the fourth vertex.

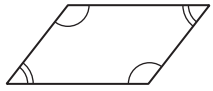
NAME _____ DATE _____ PERIOD _____

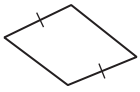
6-3 Skills Practice

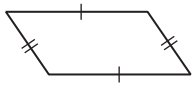
Tests for Parallelograms

Determine whether each quadrilateral is a parallelogram. Justify your answer.

1. 

2. 

3. 

4. 

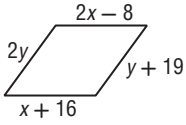
COORDINATE GEOMETRY **Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.**

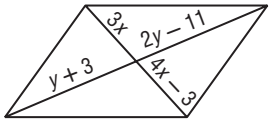
5. $P(0, 0)$, $Q(3, 4)$, $S(7, 4)$, $Y(4, 0)$; Slope Formula

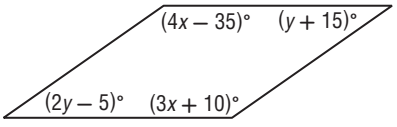
6. $S(-2, 1)$, $R(1, 3)$, $T(2, 0)$, $Z(-1, -2)$; Distance and Slope Formulas

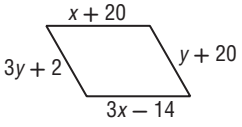
7. $W(2, 5)$, $R(3, 3)$, $Y(-2, -3)$, $N(-3, 1)$; Midpoint Formula

ALGEBRA **Find x and y so that each quadrilateral is a parallelogram.**

8. 

9. 

10. 

11. 

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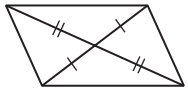
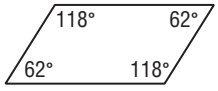
6-3 Practice

Tests for Parallelograms

Determine whether each quadrilateral is a parallelogram. Justify your answer.

1. 

2. 

3. 

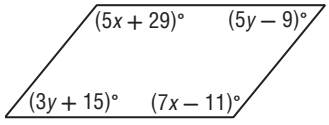
4. 

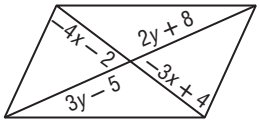
COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

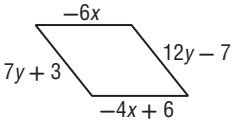
5. $P(-5, 1)$, $S(-2, 2)$, $F(-1, -3)$, $T(2, -2)$; Slope Formula

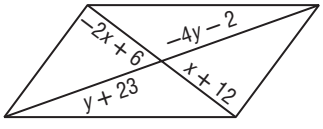
6. $R(-2, 5)$, $O(1, 3)$, $M(-3, -4)$, $Y(-6, -2)$; Distance and Slope Formulas

ALGEBRA Find x and y so that the quadrilateral is a parallelogram.

7. 

8. 

9. 

10. 

11. TILE DESIGN The pattern shown in the figure is to consist of congruent parallelograms. How can the designer be certain that the shapes are parallelograms?

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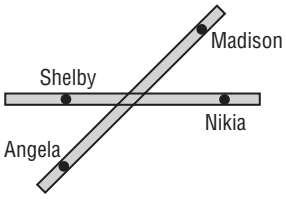
6-3 Word Problem Practice

Tests for Parallelograms

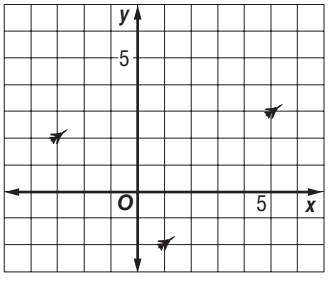
1. BALANCING Nikia, Madison, Angela, and Shelby are balancing themselves on an "X"-shaped floating object. To balance themselves, they want to make themselves the vertices of a parallelogram.



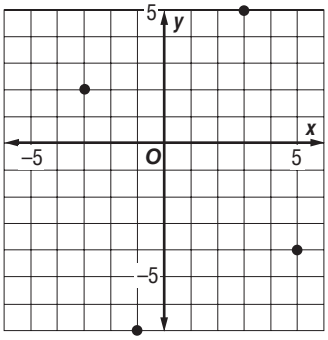
In order to achieve this, do all four of them have to be the same distance from the center of the object? Explain.

2. COMPASSES Two compass needles placed side by side on a table are both 2 inches long and point due north. Do they form the sides of a parallelogram?

3. FORMATION Four jets are flying in formation. Three of the jets are shown in the graph. If the four jets are located at the vertices of a parallelogram, what are the three possible locations of the missing jet?



4. STREET LAMPS When a coordinate plane is placed over the Harrisville town map, the four street lamps in the center are located as shown. Do the four lamps form the vertices of a parallelogram? Explain.



5. PICTURE FRAME Aaron is making a wooden picture frame in the shape of a parallelogram. He has two pieces of wood that are 3 feet long and two that are 4 feet long.

a. If he connects the pieces of wood at their ends to each other, in what order must he connect them to make a parallelogram?

b. How many different parallelograms could he make with these four lengths of wood?

c. Explain something Aaron might do to specify precisely the shape of the parallelogram.

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6-3 Enrichment

Tests for Parallelograms

By definition, a quadrilateral is a parallelogram *if and only if* both pairs of opposite sides are parallel. What conditions other than both pairs of opposite sides parallel will guarantee that a quadrilateral is a parallelogram? In this activity, several possibilities will be investigated by drawing quadrilaterals to satisfy certain conditions. Remember that any test that seems to work is not guaranteed to work unless it can be formally proven.

Complete.

1. Draw a quadrilateral with one pair of opposite sides congruent. Must it be a parallelogram?

2. Draw a quadrilateral with both pairs of opposite sides congruent. Must it be a parallelogram?

3. Draw a quadrilateral with one pair of opposite sides parallel and the other pair of opposite sides congruent. Must it be a parallelogram?

4. Draw a quadrilateral with one pair of opposite sides both parallel and congruent. Must it be a parallelogram?

5. Draw a quadrilateral with one pair of opposite angles congruent. Must it be a parallelogram?

6. Draw a quadrilateral with both pairs of opposite angles congruent. Must it be a parallelogram?

7. Draw a quadrilateral with one pair of opposite sides parallel and one pair of opposite angles congruent. Must it be a parallelogram?

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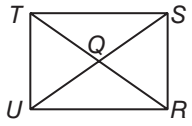
6-4 Study Guide and Intervention

Rectangles

Properties of Rectangles A **rectangle** is a quadrilateral with four right angles. Here are the properties of rectangles.



A rectangle has all the properties of a parallelogram.

- Opposite sides are parallel.
- Opposite angles are congruent.
- Opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Also:

- All four angles are right angles. $\angle UTS$, $\angle TSR$, $\angle SRU$, and $\angle RUT$ are right angles.
- The diagonals are congruent. $\overline{TR} \cong \overline{US}$

Example 1 **Quadrilateral *RUTS* above is a rectangle. If $US = 6x + 3$ and $RT = 7x - 2$, find x.**

The diagonals of a rectangle are congruent, so $US = RT$.

$6x + 3 = 7x - 2$

$3 = x - 2$

$5 = x$

Example 2 **Quadrilateral *RUTS* above is a rectangle. If $m\angle STR = 8x + 3$ and $m\angle UTR = 16x - 9$, find $m\angle STR$.**

$\angle UTS$ is a right angle, so $m\angle STR + m\angle UTR = 90$.

$8x + 3 + 16x - 9 = 90$

$24x - 6 = 90$

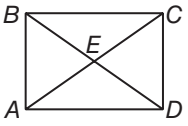
$24x = 96$

$x = 4$

$m\angle STR = 8x + 3 = 8(4) + 3$ or 35

Exercises

Quadrilateral *ABCD* is a rectangle.



1. If $AE = 36$ and $CE = 2x - 4$, find x.

2. If $BE = 6y + 2$ and $CE = 4y + 6$, find y.

3. If $BC = 24$ and $AD = 5y - 1$, find y.

4. If $m\angle BEA = 62$, find $m\angle BAC$.

5. If $m\angle AED = 12x$ and $m\angle BEC = 10x + 20$, find $m\angle AED$.

6. If $BD = 8y - 4$ and $AC = 7y + 3$, find BD.

7. If $m\angle DBC = 10x$ and $m\angle ACB = 4x^2 - 6$, find $m\angle ACB$.

8. If $AB = 6y$ and $BC = 8y$, find BD in terms of y.

NAME _____ DATE _____ PERIOD _____

6-4 Study Guide and Intervention *(continued)*

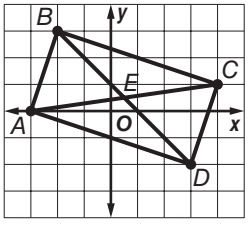
Rectangles

Prove that Parallelograms Are Rectangles The diagonals of a rectangle are congruent, and the converse is also true.

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

In the coordinate plane you can use the Distance Formula, the Slope Formula, and properties of diagonals to show that a figure is a rectangle.

Example **Quadrilateral *ABCD* has vertices *A*(−3, 0), *B*(−2, 3), *C*(4, 1), and *D*(3, −2). Determine whether *ABCD* is a rectangle.**



Method 1: Use the Slope Formula.

slope of $\overline{AB} = \frac{3-0}{-2-(-3)} = \frac{3}{1}$ or 3 slope of $\overline{AD} = \frac{-2-0}{3-(-3)} = \frac{-2}{6}$ or $-\frac{1}{3}$

slope of $\overline{CD} = \frac{-2-1}{3-4} = \frac{-3}{-1}$ or 3 slope of $\overline{BC} = \frac{1-3}{4-(-2)} = \frac{-2}{6}$ or $-\frac{1}{3}$

Opposite sides are parallel, so the figure is a parallelogram. Consecutive sides are perpendicular, so *ABCD* is a rectangle.

Method 2: Use the Distance Formula.

$AB = \sqrt{(-3-(-2))^2 + (0-3)^2}$ or $\sqrt{10}$ $BC = \sqrt{(-2-4)^2 + (3-1)^2}$ or $\sqrt{40}$

$CD = \sqrt{(4-3)^2 + (1-(-2))^2}$ or $\sqrt{10}$ $AD = \sqrt{(-3-3)^2 + (0-(-2))^2}$ or $\sqrt{40}$

Opposite sides are congruent, thus *ABCD* is a parallelogram.

$AC = \sqrt{(-3-4)^2 + (0-1)^2}$ or $\sqrt{50}$ $BD = \sqrt{(-2-3)^2 + (3-(-2))^2}$ or $\sqrt{50}$

ABCD is a parallelogram with congruent diagonals, so *ABCD* is a rectangle.

Exercises

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

1. *A*(−3, 1), *B*(−3, 3), *C*(3, 3), *D*(3, 1); Distance Formula

2. *A*(−3, 0), *B*(−2, 3), *C*(4, 5), *D*(3, 2); Slope Formula

3. *A*(−3, 0), *B*(−2, 2), *C*(3, 0), *D*(2 −2); Distance Formula

4. *A*(−1, 0), *B*(0, 2), *C*(4, 0), *D*(3, −2); Distance Formula

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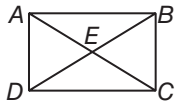
6-4 Skills Practice

Rectangles

ALGEBRA Quadrilateral *ABCD* is a rectangle.



1. If $AC = 2x + 13$ and $DB = 4x - 1$, find DB.

2. If $AC = x + 3$ and $DB = 3x - 19$, find AC.

3. If $AE = 3x + 3$ and $EC = 5x - 15$, find AC.

4. If $DE = 6x - 7$ and $AE = 4x + 9$, find DB.

5. If $m\angle DAC = 2x + 4$ and $m\angle BAC = 3x + 1$, find $m\angle BAC$.

6. If $m\angle BDC = 7x + 1$ and $m\angle ADB = 9x - 7$, find $m\angle BDC$.

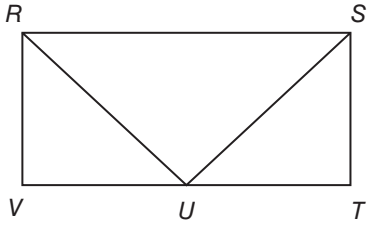
7. If $m\angle ABD = 7x - 31$ and $m\angle CDB = 4x + 5$, find $m\angle ABD$.

8. If $m\angle BAC = x + 3$ and $m\angle CAD = x + 15$, find $m\angle BAC$.

9. PROOF: Write a two-column proof.

Given: $RSTV$ is a rectangle and U is the midpoint of $\overline{VT}$.

Prove: $\triangle RUV \cong \triangle SUT$



Statements	Reasons

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

10. $P(-3, -2)$, $Q(-4, 2)$, $R(2, 4)$, $S(3, 0)$; Slope Formula

11. $J(-6, 3)$, $K(0, 6)$, $L(2, 2)$, $M(-4, -1)$; Distance Formula

12. $T(4, 1)$, $U(3, -1)$, $X(-3, 2)$, $Y(-2, 4)$; Distance Formula

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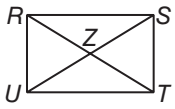
6-4 Practice

Rectangles

ALGEBRA Quadrilateral *RSTU* is a rectangle.



1. If $UZ = x + 21$ and $ZS = 3x - 15$, find US.

2. If $RZ = 3x + 8$ and $ZS = 6x - 28$, find UZ.

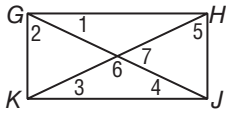
3. If $RT = 5x + 8$ and $RZ = 4x + 1$, find ZT.

4. If $m\angle SUT = 3x + 6$ and $m\angle RUS = 5x - 4$, find $m\angle SUT$.

5. If $m\angle SRT = x + 9$ and $m\angle UTR = 2x - 44$, find $m\angle UTR$.

6. If $m\angle RSU = x + 41$ and $m\angle TUS = 3x + 9$, find $m\angle RSU$.

Quadrilateral *GHJK* is a rectangle. Find each measure if $m\angle 1 = 37$.



7. $m\angle 2$

8. $m\angle 3$

9. $m\angle 4$

10. $m\angle 5$

11. $m\angle 6$

12. $m\angle 7$

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

13. $B(-4, 3)$, $G(-2, 4)$, $H(1, -2)$, $L(-1, -3)$; Slope Formula

14. $N(-4, 5)$, $O(6, 0)$, $P(3, -6)$, $Q(-7, -1)$; Distance Formula

15. $C(0, 5)$, $D(4, 7)$, $E(5, 4)$, $F(1, 2)$; Slope Formula

16. LANDSCAPING Huntington Park officials approved a rectangular plot of land for a Japanese Zen garden. Is it sufficient to know that opposite sides of the garden plot are congruent and parallel to determine that the garden plot is rectangular? Explain.

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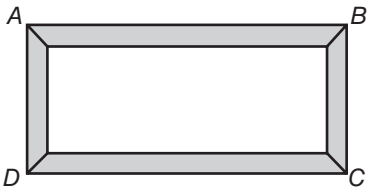
6-4 Word Problem Practice

Rectangles

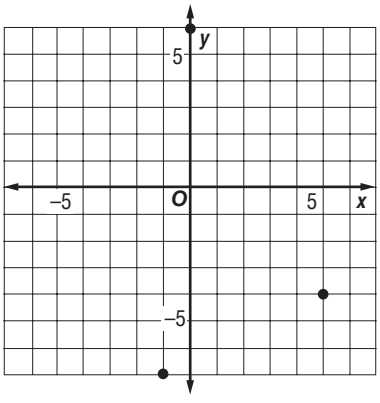
1. FRAMES Jalen makes the rectangular frame shown.



In order to make sure that it is a rectangle, Jalen measures the distances *BD* and *AC*. How should these two distances compare if the frame is a rectangle?

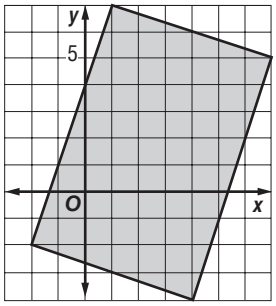
2. BOOKSHELVES A bookshelf consists of two vertical planks with five horizontal shelves. Are each of the four sections for books rectangles? Explain.

3. LANDSCAPING A landscaper is marking off the corners of a rectangular plot of land. Three of the corners are in place as shown.

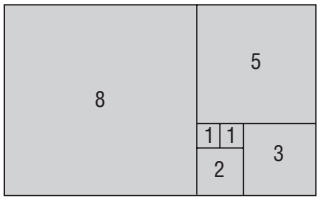


What are the coordinates of the fourth corner?

4. SWIMMING POOLS Antonio is designing a swimming pool on a coordinate grid. Is it a rectangle? Explain.



5. PATTERNS Veronica made the pattern shown out of 7 rectangles with four equal sides. The side length of each rectangle is written inside the rectangle.



a. How many rectangles can be formed using the lines in this figure?

b. If Veronica wanted to extend her pattern by adding another rectangle with 4 equal sides to make a larger rectangle, what are the possible side lengths of rectangles that she can add?

Lesson 6-4

NAME _____ DATE _____ PERIOD _____

6-4 Enrichment

Constant Perimeter

Douglas wants to fence a rectangular region of his back yard for his dog. He bought 200 feet of fence.

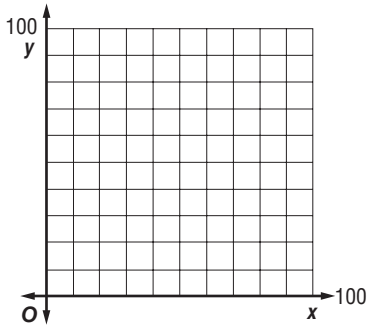
1. Complete the table to show the dimensions of five different rectangular pens that would use the entire 200 feet of fence. Then find the area of each rectangular pen.

2. Do all five of the rectangular pens have the same area? If not, which one has the larger area?

Perimeter	Length	Width	Area
200	80		
200	70		
200	60		
200	50		
200	45		

3. Write a rule for finding the dimensions of a rectangle with the largest possible area for a given perimeter.

4. Let x represent the length of a rectangle and y the width. Write the formula for all rectangles with a perimeter of 200. Then graph this relationship on the coordinate plane at the right.



Julio read that a dog the size of his new pet, Bennie, should have at least 100 square feet in his pen. Before going to the store to buy fence, Julio made a table to determine the dimensions for Bennie's rectangular pen.

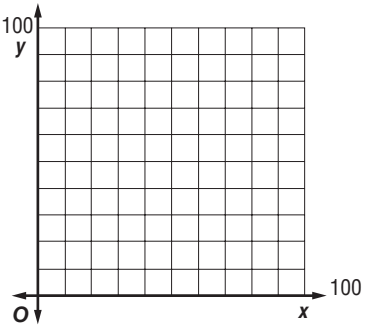
5. Complete the table to find five possible dimensions of a rectangular fenced area of 100 square feet.

6. Julio wants to save money by purchasing the least number feet of fencing to enclose the 100 square feet. What will be the dimensions of the completed pen?

Area	Length	Width	How much fence to buy
100			
100			
100			
100			
100			

7. Write a rule for finding the dimensions of a rectangle with the least possible perimeter for a given area.

8. For length x and width y, write a formula for the area of a rectangle with an area of 100 square feet. Then graph the formula.



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6-4 Graphing Calculator Activity

TI-Nspire: Exploring Rectangles

A quadrilateral with four right angles is a **rectangle.** The TI-Nspire can be used to explore some of the characteristics of a rectangle. Use the following steps to draw a rectangle.

Step 1 Set up the calculator in the correct mode.

- Choose **Graphs & Geometry** from the Home Menu.
- From the View menu, choose **4: Hide Axis**

Step 2 Draw the rectangle

- From the **8: Shapes** menu choose **3: Rectangle.**
- Click once to define the corner of the rectangle. Then move and click again. The side of the rectangle is now defined. Move perpendicularly to draw the rectangle. Click to anchor the shape.

Step 3 Measure the lengths of the sides of the rectangle.

- From the **7: Measurement** menu choose **1: Length** (Note that when you scroll over the rectangle, the value now shown is the perimeter of the rectangle.)
- Select each endpoint of a segment of the rectangle. Then click or press Enter to anchor the length of the segment in the work area.
- Repeat for the other sides of the rectangle.

Exercises

1. What appears to be true about the opposite sides of the rectangle?

2. Draw the diagonals of the rectangle using **5: Segment** from the **6: Points and Lines** Menu. Click on two opposite vertices to draw the diagonal. Repeat to draw the other diagonal.

a. Measure each diagonal using the measurement tool. What do you observe?

b. What is true about the triangles formed by the sides of the rectangle and a diagonal? Justify your conclusion.

3. Press **Clear** three times and select **Yes** to clear the screen. Repeat the steps and draw another rectangle. Do the relationships that you found for the first rectangle you drew hold true for this rectangle?

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6-4 Geometer's Sketchpad Activity

Exploring Rectangles

A quadrilateral with four right angles is a **rectangle**. The Geometer's Sketchpad is a useful tool for exploring some of the characteristics of a rectangle. Use the following steps to draw a rectangle.

Step 1 Use the Line tool to draw a line anywhere on the screen.

Step 2 Use the Point tool to draw a point that is not on the line. To draw a line perpendicular to the first line you drew, select the first line and the point. Then choose **Perpendicular Line** from the **Construct** menu.

Step 3 Use the Point tool to draw a point that is not on either of the lines you have drawn. Repeat the procedure in Step 2 to draw lines perpendicular to the two lines you have drawn.

A rectangle is formed by the segments whose endpoints are the points of intersection of the lines.

Exercises

Use the measuring capabilities of The Geometer's Sketchpad to explore the characteristics of a rectangle.

1. What appears to be true about the opposite sides of the rectangle that you drew? Make a conjecture and then measure each side to check your conjecture.

2. Draw the diagonals of the rectangle by using the Selection Arrow tool to choose two opposite vertices. Then choose **Segment** from the **Construct** menu to draw the diagonal. Repeat to draw the other diagonal.

a. Measure each diagonal. What do you observe?

b. What is true about the triangles formed by the sides of the rectangle and a diagonal? Justify your conclusion.

3. Choose **New Sketch** from the **File** menu and follow steps 1–3 to draw another rectangle. Do the relationships you found for the first rectangle you drew hold true for this rectangle also?

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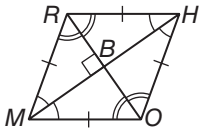
6-5 Study Guide and Intervention

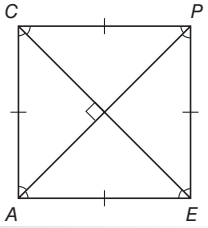
Rhombi and Squares

Properties of Rhombi and Squares A **rhombus** is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.

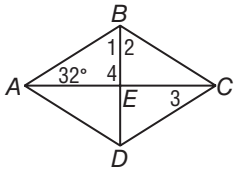


The diagonals are perpendicular.	$\overline{MH} \perp \overline{RO}$
Each diagonal bisects a pair of opposite angles.	$\overline{MH}$ bisects $\angle RMO$ and $\angle RHO$. $\overline{RO}$ bisects $\angle MRH$ and $\angle MOH$.

A **square** is a parallelogram with four congruent sides and four congruent angles. A square is both a rectangle and a rhombus; therefore, all properties of parallelograms, rectangles, and rhombi apply to squares.



Example **In rhombus $ABCD$, $m\angle BAC = 32$. Find the measure of each numbered angle.**

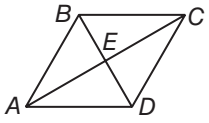


$ABCD$ is a rhombus, so the diagonals are perpendicular and $\triangle ABE$ is a right triangle. Thus $m\angle 4 = 90$ and $m\angle 1 = 90 - 32$ or 58. The diagonals in a rhombus bisect the vertex angles, so $m\angle 1 = m\angle 2$. Thus, $m\angle 2 = 58$.

A rhombus is a parallelogram, so the opposite sides are parallel. $\angle BAC$ and $\angle 3$ are alternate interior angles for parallel lines, so $m\angle 3 = 32$.

Exercises

Quadrilateral $ABCD$ is a rhombus. Find each value or measure.



1. If $m\angle ABD = 60$, find $m\angle BDC$.

2. If $AE = 8$, find AC.

3. If $AB = 26$ and $BD = 20$, find AE.

4. Find $m\angle CEB$.

5. If $m\angle CBD = 58$, find $m\angle ACB$.

6. If $AE = 3x - 1$ and $AC = 16$, find x.

7. If $m\angle CDB = 6y$ and $m\angle ACB = 2y + 10$, find y.

8. If $AD = 2x + 4$ and $CD = 4x - 4$, find x.

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6-5 Study Guide and Intervention *(continued)*

Rhombi and Squares

Conditions for Rhombi and Squares The theorems below can help you prove that a parallelogram is a rectangle, rhombus, or square.

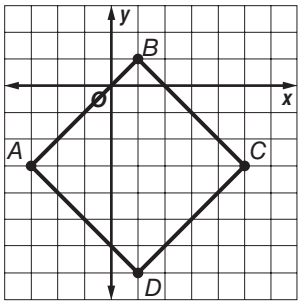
If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.

If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.

If a quadrilateral is both a rectangle and a rhombus, then it is a square.

Example **Determine whether parallelogram *ABCD* with vertices *A*(−3, −3), *B*(1, 1), *C*(5, −3), *D*(1, −7) is a *rhombus*, a *rectangle*, or a *square*.**



$AC = \sqrt{(-3-5)^2 + ((-3-(-3))^2} = \sqrt{64} = 8$

$BD = \sqrt{(1-1)^2 + (-7-1)^2} = \sqrt{64} = 8$

The diagonals are the same length; the figure is a rectangle.

Slope of $\overline{AC} = \frac{-3-(-3)}{-3-5} = \frac{0}{-8} = 8$ The line is horizontal.

Slope of $\overline{BD} = \frac{1-(-7)}{1-1} = \frac{8}{0} =$ *undefined* The line is vertical.

Since a horizontal and vertical line are perpendicular, the diagonals are perpendicular. Parallelogram *ABCD* is a square which is also a rhombus and a rectangle.

Exercises

Given each set of vertices, determine whether $\square ABCD$ is a *rhombus*, *rectangle*, or *square*. List all that apply. Explain.

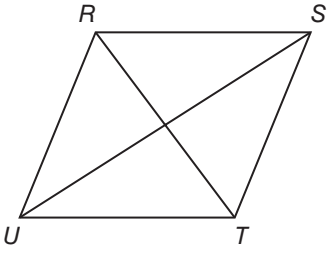
1. *A*(0, 2), *B*(2, 4), *C*(4, 2), *D*(2, 0)

2. *A*(−2, 1), *B*(−1, 3), *C*(3, 1), *D*(2, −1)

3. *A*(−2, −1), *B*(0, 2), *C*(2, −1), *D*(0, −4)

4. *A*(−3, 0), *B*(−1, 3), *C*(5, −1), *D*(3, −4)

5. PROOF Write a two-column proof.
Given: Parallelogram *RSTU*. $\overline{RS} \cong \overline{ST}$
Prove: *RSTU* is a rhombus.



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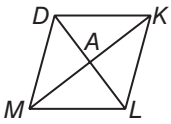
6-5 Skills Practice

Rhombi and Squares

ALGEBRA Quadrilateral *DKLM* is a rhombus.



1. If $DK = 8$, find KL.

2. If $m\angle DML = 82$ find $m\angle DKM$.

3. If $m\angle KAL = 2x - 8$, find x.

4. If $DA = 4x$ and $AL = 5x - 3$, find DL.

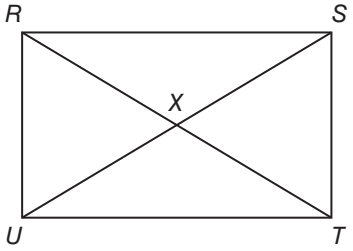
5. If $DA = 4x$ and $AL = 5x - 3$, find AD.

6. If $DM = 5y + 2$ and $DK = 3y + 6$, find KL.

7. PROOF Write a two-column proof.

Given: $RSTU$ is a parallelogram.
$\overline{RX} \cong \overline{TX} \cong \overline{SX} \cong \overline{UX}$

Prove: $RSTU$ is a rectangle.



Statements	Reasons

COORDINATE GEOMETRY Given each set of vertices, determine whether $\square QRST$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.

8. $Q(3, 5)$, $R(3, 1)$, $S(-1, 1)$, $T(-1, 5)$

9. $Q(-5, 12)$, $R(5, 12)$, $S(-1, 4)$, $T(-11, 4)$

10. $Q(-6, -1)$, $R(4, -6)$, $S(2, 5)$, $T(-8, 10)$

11. $Q(2, -4)$, $R(-6, -8)$, $S(-10, 2)$, $T(-2, 6)$

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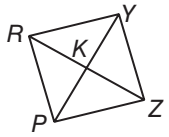
6-5 Practice

Rhombi and Squares

***PRYZ* is a rhombus. If $RK = 5$, $RY = 13$ and $m\angle YRZ = 67$, find each measure.**



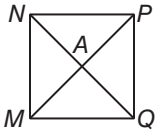
1. KY

2. PK

3. $m\angle YKZ$

4. $m\angle PZR$

***MNPQ* is a rhombus. If $PQ = 3\sqrt{2}$ and $AP = 3$, find each measure.**



5. AQ

6. $m\angle APQ$

7. $m\angle MNP$

8. PM

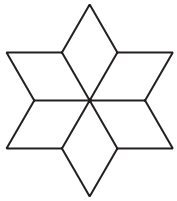
COORDINATE GEOMETRY Given each set of vertices, determine whether $\square BEFG$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.

9. $B(-9, 1)$, $E(2, 3)$, $F(12, -2)$, $G(1, -4)$

10. $B(1, 3)$, $E(7, -3)$, $F(1, -9)$, $G(-5, -3)$

11. $B(-4, -5)$, $E(1, -5)$, $F(-2, -1)$, $G(-7, -1)$

12. TESSELLATIONS The figure is an example of a tessellation. Use a ruler or protractor to measure the shapes and then name the quadrilaterals used to form the figure.



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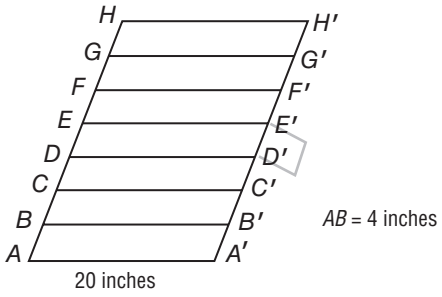
6-5 Word Problem Practice

Rhombi and Squares

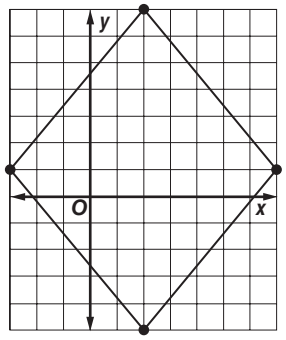
1. TRAY RACKS A tray rack looks like a parallelogram from the side. The levels for the trays are evenly spaced.



What two labeled points form a rhombus with base AA'?

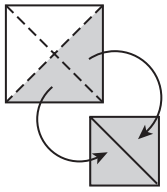
2. SLICING Charles cuts a rhombus along both diagonals. He ends up with four congruent triangles. Classify these triangles as *acute, obtuse,* or *right*.

3. WINDOWS The edges of a window are drawn in the coordinate plane.

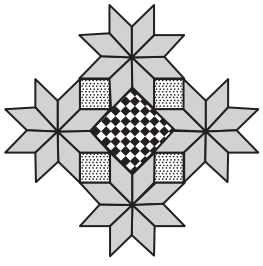


Determine whether the window is a square or a rhombus.

4. SQUARES Mackenzie cut a square along its diagonals to get four congruent right triangles. She then joined two of them along their long sides. Show that the resulting shape is a square.



5. DESIGN Tatianna made the design shown. She used 32 congruent rhombi to create the flower-like design at each corner.



a. What are the angles of the corner rhombi?

b. What kinds of quadrilaterals are the dotted and checkered figures?

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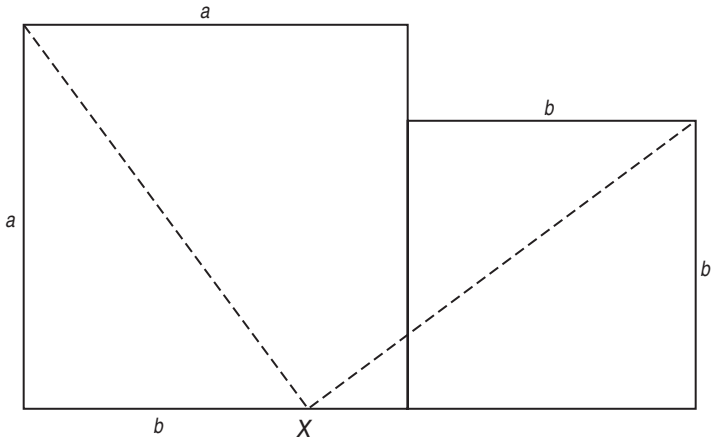
6-5 Enrichment

Creating Pythagorean Puzzles

By drawing two squares and cutting them in a certain way, you can make a puzzle that demonstrates the Pythagorean Theorem. A sample puzzle is shown. You can create your own puzzle by following the instructions below.



1. Carefully construct a square and label the length of a side as a. Then construct a smaller square to the right of it and label the length of a side as b, as shown in the figure above. The bases should be adjacent and collinear.

2. Mark a point X that is b units from the left edge of the larger square. Then draw the segments from the upper left corner of the larger square to point X, and from point X to the upper right corner of the smaller square.

3. Cut out and rearrange your five pieces to form a larger square. Draw a diagram to show your answer.

4. Verify that the length of each side is equal to $\sqrt{a^2 + b^2}$.

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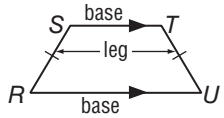
6-6 Study Guide and Intervention

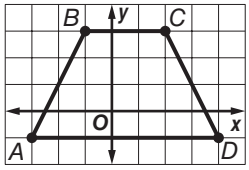
Trapezoids and Kites

Properties of Trapezoids A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The **midsegment** or **median** of a trapezoid is the segment that connects the midpoints of the legs of the trapezoid. Its measure is equal to one-half the sum of the lengths of the bases. If the legs are congruent, the trapezoid is an **isosceles trapezoid**. In an isosceles trapezoid both pairs of **base angles** are congruent and the diagonals are congruent.



STUR is an isosceles trapezoid.
$\overline{SR} \cong \overline{TU}$; $\angle R \cong \angle U$, $\angle S \cong \angle T$

Example **The vertices of *ABCD* are *A*(−3, −1), *B*(−1, 3), *C*(2, 3), and *D*(4, −1). Show that *ABCD* is a trapezoid and determine whether it is an isosceles trapezoid.**



slope of $\overline{AB} = \frac{3-(-1)}{-1-(-3)} = \frac{4}{2} = 2$

slope of $\overline{AD} = \frac{-1-(-1)}{4-(-3)} = \frac{0}{7} = 0$

slope of $\overline{BC} = \frac{3-3}{2-(-1)} = \frac{0}{3} = 0$

slope of $\overline{CD} = \frac{-1-3}{4-2} = \frac{-4}{2} = -2$

$AB = \sqrt{(-3-(-1))^2 + (-1-3)^2}$

$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$CD = \sqrt{(2-4)^2 + (3-(-1))^2}$

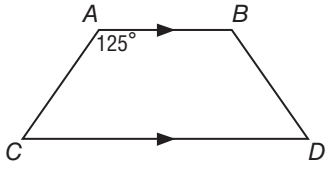
$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

Exactly two sides are parallel, $\overline{AD}$ and $\overline{BC}$, so *ABCD* is a trapezoid. $AB = CD$, so *ABCD* is an isosceles trapezoid.

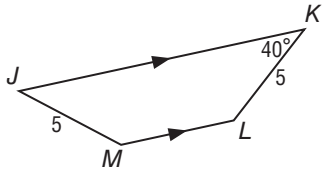
Exercises

Find each measure.

1. $m\angle D$



2. $m\angle L$



COORDINATE GEOMETRY For each quadrilateral with the given vertices, verify that the quadrilateral is a trapezoid and determine whether the figure is an isosceles trapezoid.

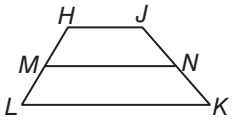
3. *A*(−1, 1), *B*(3, 2), *C*(1,−2), *D*(−2, −1)

4. *J*(1, 3), *K*(3, 1), *L*(3, −2), *M*(−2, 3)

For trapezoid *HJKL*, *M* and *N* are the midpoints of the legs.

5. If $HJ = 32$ and $LK = 60$, find MN.

6. If $HJ = 18$ and $MN = 28$, find LK.



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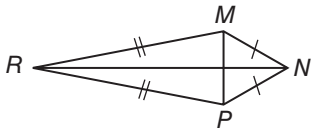
6-6 Study Guide and Intervention *(continued)*

Trapezoids and Kites

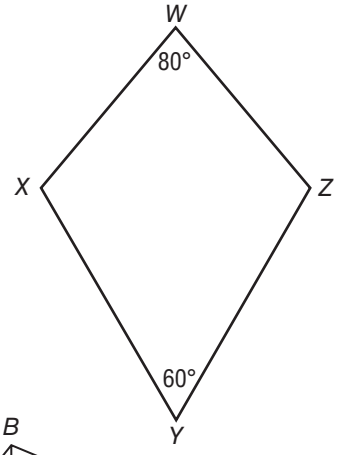
Properties of Kites A **kite** is a quadrilateral with exactly two pairs of consecutive congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.



The diagonals of a kite are perpendicular.
For kite $RMNP$, $\overline{MP} \perp \overline{RN}$

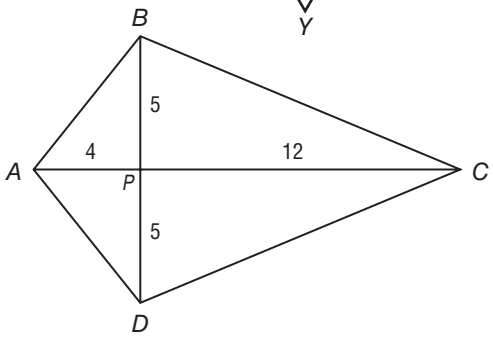
In a kite, exactly one pair of opposite angles is congruent.
For kite $RMNP$, $\angle M \cong \angle P$

Example 1 **If $WXYZ$ is a kite, find $m\angle Z$.**



The measures of $\angle Y$ and $\angle W$ are not congruent, so $\angle X \cong \angle Z$.
$m\angle X + m\angle Y + m\angle Z + m\angle W = 360$
$m\angle X + 60 + m\angle Z + 80 = 360$
$m\angle X + m\angle Z = 220$
$m\angle X = 110$, $m\angle Z = 110$

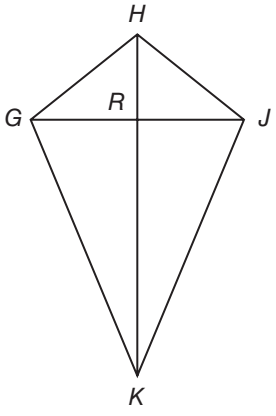
Example 2 **If $ABCD$ is a kite, find BC.**



The diagonals of a kite are perpendicular. Use the Pythagorean Theorem to find the missing length.
$BP^2 + PC^2 = BC^2$
$5^2 + 12^2 = BC^2$
$169 = BC^2$
$13 = BC$

Exercises

If $GHJK$ is a kite, find each measure.



1. Find $m\angle JRK$.

2. If $RJ = 3$ and $RK = 10$, find JK.

3. If $m\angle GHJ = 90$ and $m\angle GKJ = 110$, find $m\angle HGK$.

4. If $HJ = 7$, find HG.

5. If $HG = 7$ and $GR = 5$, find HR.

6. If $m\angle GHJ = 52$ and $m\angle GKJ = 95$, find $m\angle HGK$.

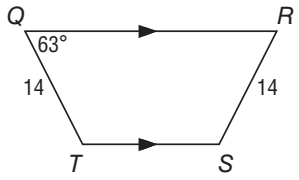
NAME _____ DATE _____ PERIOD _____

6-6 Skills Practice

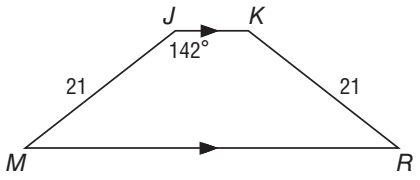
Trapezoids and Kites

ALGEBRA Find each measure.

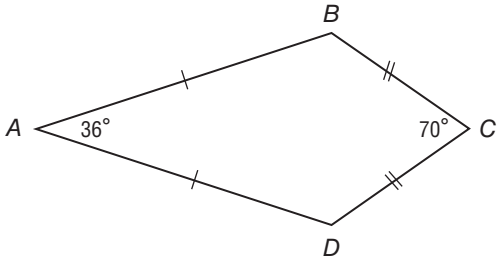
1. $m\angle S$



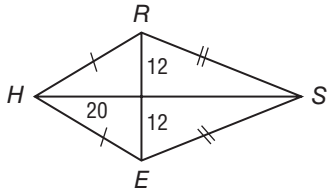
2. $m\angle M$



3. $m\angle D$



4. RH



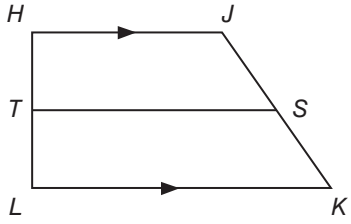
ALGEBRA For trapezoid *HJKL*, *T* and *S* are midpoints of the legs.

5. If $HJ = 14$ and $LK = 42$, find TS.

6. If $LK = 19$ and $TS = 15$, find HJ.

7. If $HJ = 7$ and $TS = 10$, find LK.

8. If $KL = 17$ and $JH = 9$, find ST.



COORDINATE GEOMETRY *EFGH* is a quadrilateral with vertices *E*(1, 3), *F*(5, 0), *G*(8, −5), *H*(−4, 4).

9. Verify that *EFGH* is a trapezoid.

10. Determine whether *EFGH* is an isosceles trapezoid. Explain.

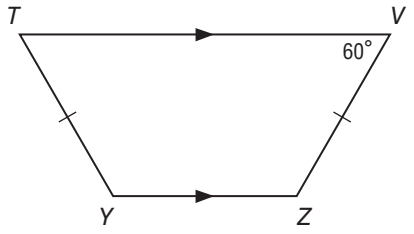
NAME _____ DATE _____ PERIOD _____

6-6 Practice

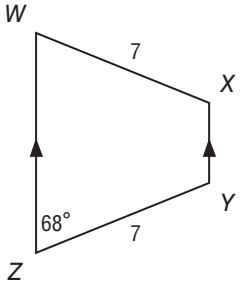
Trapezoids and Kites

Find each measure.

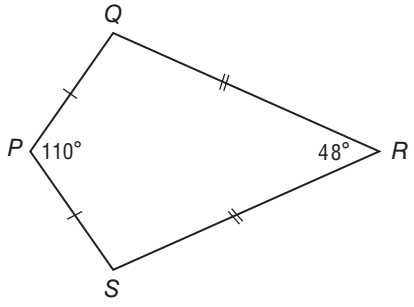
1. $m\angle T$



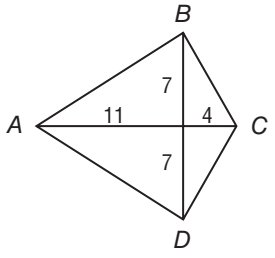
2. $m\angle Y$



3. $m\angle Q$



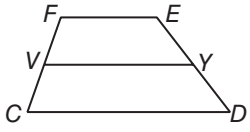
4. BC



ALGEBRA For trapezoid *FEDC*, *V* and *Y* are midpoints of the legs.

5. If $FE = 18$ and $VY = 28$, find CD.

6. If $m\angle F = 140$ and $m\angle E = 125$, find $m\angle D$.



COORDINATE GEOMETRY *RSTU* is a quadrilateral with vertices *R*(−3, −3), *S*(5, 1), *T*(10, −2), *U*(−4, −9).

7. Verify that *RSTU* is a trapezoid.

8. Determine whether *RSTU* is an isosceles trapezoid. Explain.

9. CONSTRUCTION A set of stairs leading to the entrance of a building is designed in the shape of an isosceles trapezoid with the longer base at the bottom of the stairs and the shorter base at the top. If the bottom of the stairs is 21 feet wide and the top is 14 feet wide, find the width of the stairs halfway to the top.

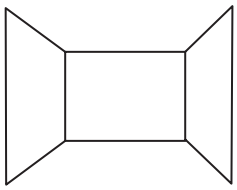
10. DESK TOPS A carpenter needs to replace several trapezoid-shaped desktops in a classroom. The carpenter knows the lengths of both bases of the desktop. What other measurements, if any, does the carpenter need?

NAME _____ DATE _____ PERIOD _____

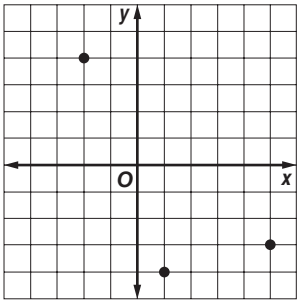
6-6 Word Problem Practice

Trapezoids and Kites

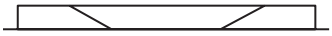
1. PERSPECTIVE Artists use different techniques to make things appear to be 3-dimensional when drawing in two dimensions. Kevin drew the walls of a room. In real life, all of the walls are rectangles. In what shape did he draw the side walls to make them appear 3-dimensional?



2. PLAZA In order to give the feeling of spaciousness, an architect decides to make a plaza in the shape of a kite. Three of the four corners of the plaza are shown on the coordinate plane. If the fourth corner is in the first quadrant, what are its coordinates?

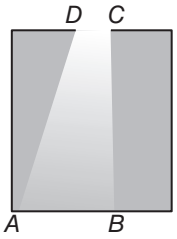


3. AIRPORTS A simplified drawing of the reef runway complex at Honolulu International Airport is shown below.



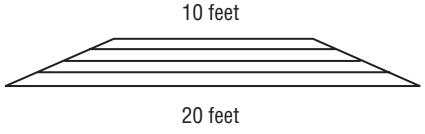
How many trapezoids are there in this image?

4. LIGHTING A light outside a room shines through the door and illuminates a trapezoidal region *ABCD* on the floor.



Under what circumstances would trapezoid *ABCD* be isosceles?

5. RISERS A riser is designed to elevate a speaker. The riser consists of 4 trapezoidal sections that can be stacked one on top of the other to produce trapezoids of varying heights.



All of the stages have the same height. If all four stages are used, the width of the top of the riser is 10 feet.

a. If only the bottom two stages are used, what is the width of the top of the resulting riser?

b. What would be the width of the riser if the bottom three stages are used?

NAME _____ DATE _____ PERIOD _____

6-6 Enrichment

Quadrilaterals in Construction

Quadrilaterals are often used in construction work.

1. The diagram at the right represents a roof frame and shows many quadrilaterals. Find the following shapes in the diagram and shade in their edges.

a. isosceles triangle

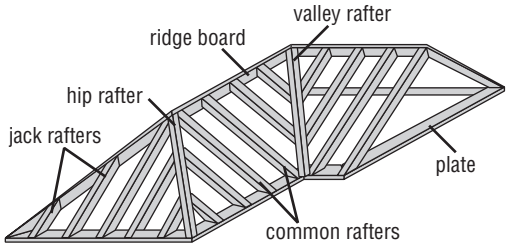
b. scalene triangle

c. rectangle

d. rhombus

e. trapezoid (not isosceles)

f. isosceles trapezoid



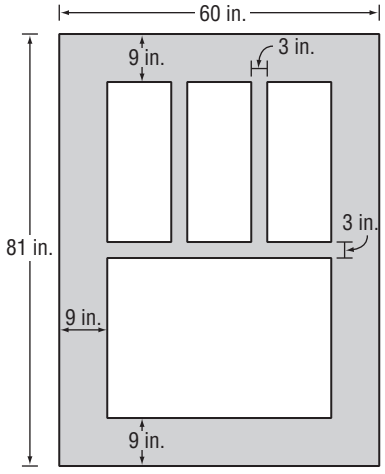
Roof Frame

2. The figure at the right represents a window. The wooden part between the panes of glass is 3 inches wide. The frame around the outer edge is 9 inches wide. The outside measurements of the frame are 60 inches by 81 inches. The height of the top and bottom panes is the same. The top three panes are the same size.

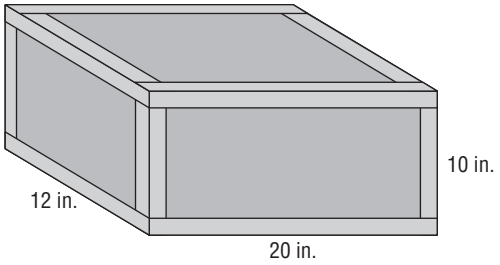
a. How wide is the bottom pane of glass?

b. How wide is each top pane of glass?

c. How high is each pane of glass?



3. Each edge of this box has been reinforced with a piece of tape. The box is 10 inches high, 20 inches wide, and 12 inches deep. What is the length of the tape that has been used?



NAME _____ DATE _____ PERIOD _____

6 Student Recording Sheet

SCORE _____

Use this recording sheet with pages 452–453 of the Student Edition.

Multiple Choice

Read each question. Then fill in the correct answer.

1. Ⓐ Ⓑ Ⓒ Ⓓ	**4.** Ⓕ Ⓖ Ⓗ Ⓙ	**7.** Ⓐ Ⓑ Ⓒ Ⓓ
2. Ⓕ Ⓖ Ⓗ Ⓙ	**5.** Ⓐ Ⓑ Ⓒ Ⓓ	
3. Ⓐ Ⓑ Ⓒ Ⓓ	**6.** Ⓕ Ⓖ Ⓗ Ⓙ	

Short Response/Gridded Response

Record your answer in the blank.

For gridded response questions, also enter your answer in the grid by writing each number or symbol in a box. Then fill in the corresponding circle for that number or symbol.

8. _____ *(grid in)*

9. _____

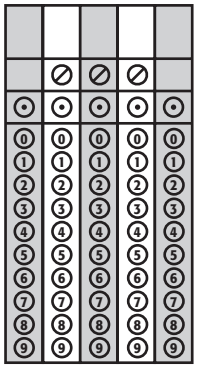
10. _____

11. _____

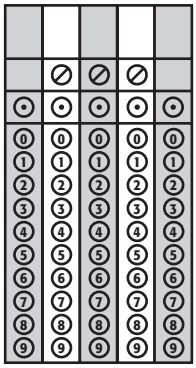
12. _____ *(grid in)*

13. _____

8.



12.



Extended Response

Record your answers for Question 14 on the back of this paper.

Assessment

NAME _____ DATE _____ PERIOD _____

6 Rubric for Scoring Extended Response

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended-response questions require the student to show work.
- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is *not* considered a fully correct response.
- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 14 Rubric

Score	Specific Criteria
4	Students correctly determine the quadrilateral in part **a** is a parallelogram since opposite sides are congruent. Students correctly determine the quadrilateral in part **b** does not contain sufficient information to prove it is a parallelogram. The two horizontal sides must also be congruent. Students correctly determine that the quadrilateral in part **c** is a parallelogram since both pairs of opposite angles are congruent.
3	A generally correct solution, but may contain minor flaws in reasoning or computation.
2	A partially correct interpretation and/or solution to the problem.
1	A correct solution with no evidence or explanation.
0	An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.

NAME _____ DATE _____ PERIOD _____

6 Chapter 6 Quiz 1

SCORE _____

(Lessons 6-1 and 6-2)

1. Find the sum of the measures of the interior angles of a convex 70-gon. **1.** _____

2. The measure of each interior angle of a regular polygon is 172. Find the number of sides in the polygon. **2.** _____

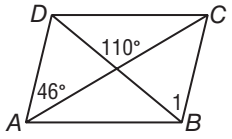
3. The measure of each exterior angle of a regular polygon is 18. Find the number of sides in the polygon. **3.** _____

4. Given parallelogram $ABCD$ with $C(5, 4)$, find the coordinates of A if the diagonals $\overline{AC}$ and $\overline{BD}$ intersect at $(2, 7)$. **4.** _____

5. MULTIPLE CHOICE Find $m\angle 1$ in parallelogram $ABCD$.

A 64 **C** 46

B 58 **D** 36



5. _____

Assessment

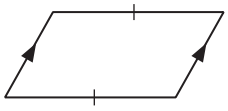
NAME _____ DATE _____ PERIOD _____

6 Chapter 6 Quiz 2

SCORE _____

(Lesson 6-3)

1. Determine whether this quadrilateral is a parallelogram. Justify your answer.



1. _____

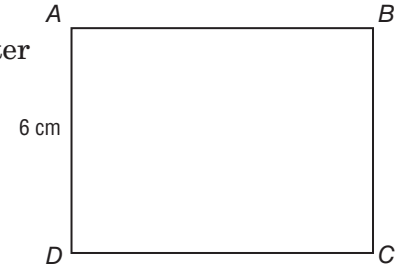
For Questions 2–4, write *true* or *false*.

2. A quadrilateral with two pairs of parallel sides is always a parallelogram. **2.** _____

3. The diagonals of a parallelogram are always perpendicular. **3.** _____

4. The slope of $\overline{AB}$ and $\overline{CD}$ is $\frac{3}{5}$ and the slope of $\overline{BC}$ and $\overline{AD}$ is $-\frac{5}{3}$. $ABCD$ is a parallelogram. **4.** _____

5. Refer to parallelogram $ABCD$. If $AB = 8$ cm, what is the perimeter of the parallelogram?



5. _____

NAME _____ DATE _____ PERIOD _____

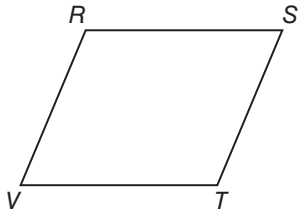
6 Chapter 6 Quiz 3

SCORE _____

(Lessons 6-4 and 6-5)

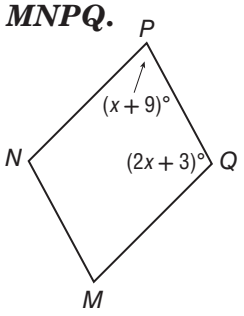
1. MULTIPLE CHOICE *RSTV* is a rhombus. Which of the following statements is NOT true?

A $\overline{RV} \cong \overline{TS}$
B $\overline{RV} \perp \overline{TS}$
C $\overline{RS} \parallel \overline{TV}$
D $\angle R \cong \angle T$



1. _____

For Questions 2 and 3, refer to trapezoid *MNPQ*.



2. Find $m\angle M$.

2. _____

3. Find $m\angle Q$.

3. _____

4. *True* or *false*. A quadrilateral that is a rectangle and a rhombus is a square.

4. _____

5. $\square ABCD$ has vertices $A(4, 0)$, $B(0, 4)$, $C(-4, 0)$, and $D(0, -4)$. Determine whether *ABCD* is a *rectangle*, *rhombus*, or *square*. List all that apply.

5. _____

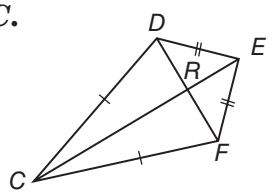
NAME _____ DATE _____ PERIOD _____

6 Chapter 6 Quiz 4

SCORE _____

(Lesson 6-6)

For Questions 1 and 2, refer to kite *DEFC*.



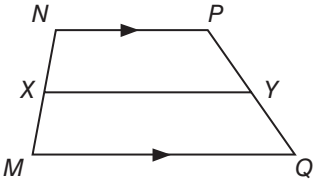
1. If $m\angle DCF = 34$ and $m\angle DEF = 90$, find $m\angle CDE$.

1. _____

2. If $DR = 5$ and $RE = 5$, find FE.

2. _____

For Questions 3 and 4, refer to trapezoid *NPQM* where *X* and *Y* are midpoints of the sides.



3. If $MQ = 15$ and $XY = 10$, find NP.

3. _____

4. If $NP = 13$ and $MQ = 18$, find XY.

4. _____

5. If *CDEF* is a trapezoid with vertices $C(0, 2)$, $D(2, 4)$, $E(7, 3)$, and $F(1, -3)$, how can you prove that it is an isosceles trapezoid?

5. _____

NAME _____ DATE _____ PERIOD _____

6 Chapter 6 Mid-Chapter Test

SCORE _____

(Lessons 6-1 through 6-3)

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

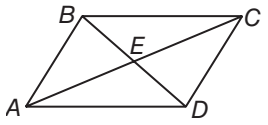
1. Find the measure of each exterior angle of a regular 56-gon. Round to the nearest tenth.

A 3.2 **B** 6.4 **C** 173.6 **D** 9720 **1.** ____

2. Given $BE = 2x + 6$ and $ED = 5x - 12$ in parallelogram $ABCD$, find BD.

F 6 **H** 18

G 12 **J** 36



2. ____

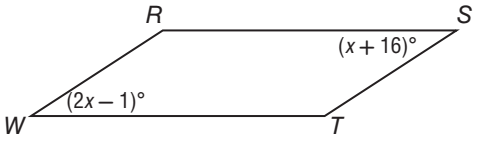
3. If the slope of $\overline{PQ}$ is $\frac{2}{3}$ and the slope of $\overline{QR}$ is $-\frac{1}{2}$, find the slope of $\overline{SR}$ so that $PQRS$ is a parallelogram.

A $\frac{2}{3}$ **B** $\frac{3}{2}$ **C** $-\frac{1}{2}$ **D** 2 **3.** ____

4. Find $m\angle W$ in parallelogram $RSTW$.

F 17 **H** 55

G 33 **J** 125



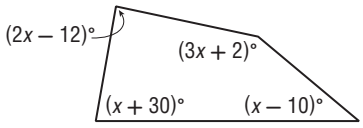
4. ____

5. Find the sum of the measures of the interior angles of a convex 48-gon.

A 172.5 **B** 360 **C** 8280 **D** 8640 **5.** ____

Part II

6. Find x.

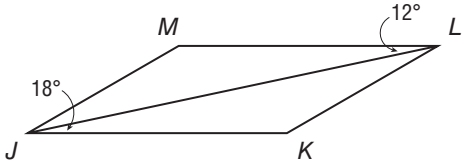


6. _____

7. $ABCD$ is a parallelogram with $m\angle A = 138$. Find $m\angle B$. **7.** _____

8. Determine whether $ABCD$ is a parallelogram if $AB = 6$, $BC = 12$, $CD = 6$, and $DA = 12$. Justify your answer. **8.** _____

9. In parallelogram $MLKJ$, find $m\angle MLK$ and $m\angle LKJ$.



9. _____

10. $XYWZ$ is a quadrilateral with vertices $W(1, -4)$, $X(-4, 2)$, $Y(1, -1)$, and $Z(-2, -3)$. Determine if the quadrilateral is a parallelogram. Use slope to justify your answer. **10.** _____

Assessment

NAME _____ DATE _____ PERIOD _____

6 Chapter 6 Vocabulary Test

SCORE _____

base	legs	rhombus
base angle	midsegment of a trapezoid	square
diagonal	parallelogram	trapezoid
isosceles trapezoid	rectangle	

Choose from the terms above to complete each sentence.

1. A quadrilateral with only one pair of opposite sides parallel and the other pair of opposite sides congruent is a(n) ___?___. **1.** _____

2. A quadrilateral with two pairs of opposite sides parallel is a(n) ___?___. **2.** _____

3. A quadrilateral with only one pair of opposite sides parallel is a(n) ___?___. **3.** _____

4. A quadrilateral that is both a rectangle and a rhombus is a(n) ___?___. **4.** _____

5. A quadrilateral with four congruent sides is a(n) ___?___ . **5.** _____

Write whether each sentence is *true* or *false*. If false, replace the underlined word or number to make a true sentence.

6. A quadrilateral with four right angles is a trapezoid. **6.** _____

7. A quadrilateral with two pairs of congruent consecutive sides is a kite. **7.** _____

Choose the correct term to complete each sentence.

8. Segments that join opposite vertices in a quadrilateral are called (*medians, diagonals*). **8.** _____

9. The segment joining the midpoints of the nonparallel sides of a trapezoid is called the (*median, diagonal*). **9.** _____

Define each term in your own words.

10. base angles of an isosceles trapezoid **10.** _____

11. legs of a trapezoid **11.** _____

NAME _____ DATE _____ PERIOD _____

6 Chapter 6 Test, Form 1

SCORE _____

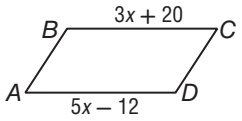
Write the letter for the correct answer in the blank at the right of each question.

1. Find the sum of the measures of the interior angles of a convex 30-gon.
A 5400 **B** 5040 **C** 360 **D** 168 **1.** ____

2. Find the sum of the measures of the exterior angles of a convex 21-gon.
F 21 **G** 180 **H** 360 **J** 3420 **2.** ____

3. If the measure of each interior angle of a regular polygon is 108, find the measure of each exterior angle.
A 18 **B** 72 **C** 90 **D** 108 **3.** ____

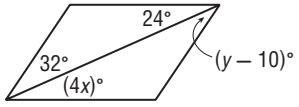
4. For parallelogram $ABCD$, find the value of x.
F 4 **H** 16
G 10.25 **J** 21.5



4. ____

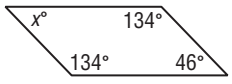
5. Which of the following is a property of a parallelogram?
A The diagonals are congruent. **C** The diagonals are perpendicular.
B The diagonals bisect the angles. **D** The diagonals bisect each other. **5.** ____

6. Find the values of x and y so that $ABCD$ will be a parallelogram.
F $x = 6, y = 42$
G $x = 6, y = 22$
H $x = 20, y = 42$
J $x = 20, y = 22$



6. ____

7. Find the value of x so that this quadrilateral is a parallelogram.
A 44 **C** 90
B 46 **D** 134



7. ____

8. Parallelogram $ABCD$ has vertices $A(0, 0)$, $B(2, 4)$, and $C(10, 4)$. Find the coordinates of D.
F $D(8, 0)$ **G** $D(10, 0)$ **H** $D(0, 4)$ **J** $D(10, 8)$ **8.** ____

9. Which of the following is a property of all rectangles?
A four congruent sides **C** diagonals are perpendicular
B diagonals bisect the angles **D** four right angles **9.** ____

10. $ABCD$ is a rectangle with diagonals $\overline{AC}$ and $\overline{BD}$. If $AC = 2x + 10$ and $BD = 56$, find the value of x.
F 23 **G** 33 **H** 78 **J** 122 **10.** ____

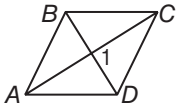
11. $ABCD$ is a rectangle with $B(-5, 0)$, $C(7, 0)$ and $D(7, 3)$. Find the coordinates of A.
A $A(-5, 7)$ **B** $A(3, 5)$ **C** $A(-5, 3)$ **D** $A(7, -3)$ **11.** ____

NAME _____ DATE _____ PERIOD _____

6 Chapter 6 Test, Form 1 *(continued)*

12. For rhombus $ABCD$, find $m\angle 1$.

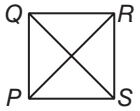
F 45 **H** 90
G 60 **J** 120



12. ____

13. Find $m\angle PRS$ in square $PQRS$.

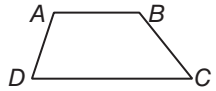
A 30 **C** 60
B 45 **D** 90



13. ____

14. Choose a pair of base angles of trapezoid $ABCD$.

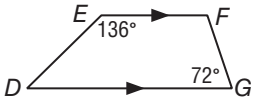
F $\angle A$, $\angle C$ **H** $\angle A$, $\angle D$
G $\angle B$, $\angle D$ **J** $\angle D$, $\angle C$



14. ____

15. In trapezoid $DEFG$, find $m\angle D$.

A 44 **C** 108
B 72 **D** 136



15. ____

16. The hood of Olivia's car is the shape of a trapezoid. The base bordering the windshield measures 30 inches and the base at the front of the car measures 24 inches. What is the width of the median of the hood?

F 25 in. **G** 27 in. **H** 28 in. **J** 29 in.

16. ____

17. The length of one base of a trapezoid is 44, the median is 36, and the other base is $2x + 10$. Find the value of x.

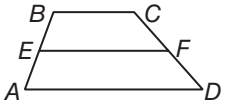
A 9 **B** 17 **C** 21 **D** 40

17. ____

18. Given trapezoid $ABCD$ with median $\overline{EF}$, which of the following is true?

F $EF = \frac{1}{2}AD$ **H** $AB = EF$

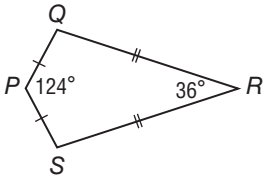
G $AE = FD$ **J** $EF = \frac{BC + AD}{2}$



18. ____

19. $PQRS$ is a kite. Find $m\angle S$.

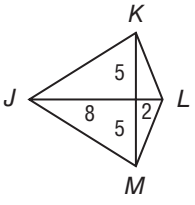
A 100 **C** 200
B 160 **D** 360



19. ____

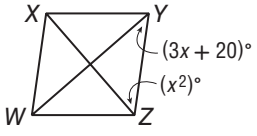
20. $JKLM$ is a kite, find JM.

F $\sqrt{29}$ **H** $\sqrt{13}$
G $\sqrt{89}$ **J** 11



20. ____

Bonus Find x and $m\angle WYZ$ in rhombus $XYZW$.



B: _____

NAME _____ DATE _____ PERIOD _____

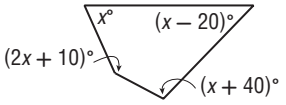
6 Chapter 6 Test, Form 2A

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

1. Find the sum of the measures of the interior angles of a convex 45-gon.
A 8100 **B** 7740 **C** 360 **D** 172 **1.** ____

2. Find the value of x.
F 30 **H** 102
G 66 **J** 138

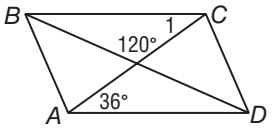


2. ____

3. Find the sum of the measures of the exterior angles of a convex 39-gon.
A 39 **B** 90 **C** 180 **D** 360 **3.** ____

4. Which of the following is a property of a parallelogram?
F Each pair of opposite sides is congruent.
G Only one pair of opposite angles is congruent.
H Each pair of opposite angles is supplementary.
J There are four right angles. **4.** ____

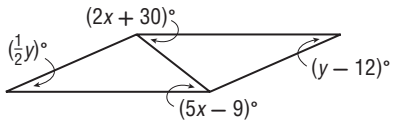
5. For parallelogram $ABCD$, find $m\angle 1$.
A 60 **C** 36
B 54 **D** 18



5. ____

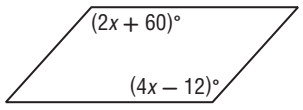
6. $ABCD$ is a parallelogram with diagonals intersecting at E. If $AE = 3x + 12$ and $EC = 27$, find the value of x.
F 5 **G** 17 **H** 27 **J** 47 **6.** ____

7. Find the values of x and y so that this quadrilateral is a parallelogram.
A $x = 13, y = 24$ **C** $x = 7, y = 24$
B $x = 13, y = 6$ **D** $x = 7, y = 6$



7. ____

8. Find the value of x so that this quadrilateral is a parallelogram.
F 12 **H** 36
G 24 **J** 132



8. ____

9. Parallelogram $ABCD$ has vertices $A(8, 2)$, $B(6, -4)$, and $C(-5, -4)$. Find the coordinates of D.
A $D(-5, 2)$ **B** $D(-3, 2)$ **C** $D(-2, 2)$ **D** $D(-4, 8)$ **9.** ____

10. $ABCD$ is a rectangle. If $AC = 5x + 2$ and $BD = x + 22$, find the value of x.
F 5 **G** 6 **H** 11 **J** 26 **10.** ____

11. Which of the following is true for all rectangles?
A The diagonals are perpendicular.
B The diagonals bisect the angles.
C The consecutive sides are congruent.
D The consecutive sides are perpendicular. **11.** ____

Assessment

NAME _____ DATE _____ PERIOD _____

6 Chapter 6 Test, Form 2A *(continued)*

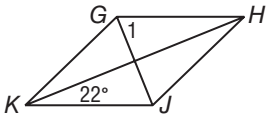
12. *ABCD* is a rectangle with $B(-4, 6)$, $C(-4, 2)$, and $D(10, 2)$. Find the coordinates of A.

F $A(6, 4)$ **G** $A(10, 4)$ **H** $A(2, 6)$ **J** $A(10, 6)$ **12.** ____

13. For rhombus *GHJK*, find $m\angle 1$.

A 22 **C** 68

B 44 **D** 90



13. ____

14. The diagonals of square *ABCD* intersect at E. If $AE = 2x + 6$ and $BD = 6x - 10$, find AC.

F 11 **G** 28 **H** 56 **J** 90 **14.** ____

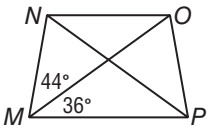
15. *ABCD* is an isosceles trapezoid with $A(10, -1)$, $B(8, 3)$, and $C(-1, 3)$. Find the coordinates of D.

A $D(-3, -1)$ **B** $D(-10, -11)$ **C** $D(-1, 8)$ **D** $D(-3, 3)$ **15.** ____

16. For isosceles trapezoid *MNOP*, find $m\angle MNP$.

F 44 **H** 80

G 64 **J** 116



16. ____

17. The length of one base of a trapezoid is 19 inches and the length of the median is 16 inches. Find the length of the other base.

A 35 in. **B** 19 in. **C** 17.5 in. **D** 13 in. **17.** ____

18. Judith built a fence to surround her property. On a coordinate plane, the four corners of the fence are located at $(-16, 1)$, $(-6, 5)$, $(4, 1)$, and $(-6, -3)$. Which of the following most accurately describes the shape of Judith's fence?

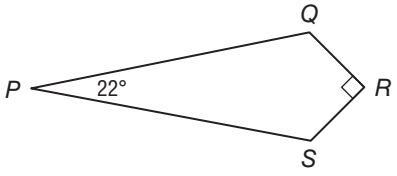
F square **H** rhombus

G rectangle **J** trapezoid **18.** ____

19. For kite *PQRS*, find $m\angle S$

A 248 **C** 112

B 68 **D** 124



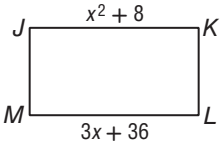
19. ____

20. *ABCD* is a parallelogram with coordinates $A(4, 2)$, $B(4, -1)$, $C(-2, -1)$, and $D(-2, 2)$. To prove that *ABCD* is a rectangle, you would plot the parallelogram on a coordinate plane and then find which of the following?

F measures of the angles **H** slopes of the diagonals

G lengths of the diagonals **J** midpoints of the diagonals **20.** ____

Bonus Find the possible value(s) of x in rectangle *JKLM*.



B: _____

NAME _____ DATE _____ PERIOD _____

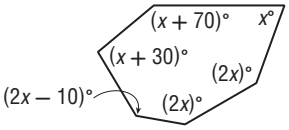
6 Chapter 6 Test, Form 2B

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

1. Find the sum of the measures of the interior angles of a convex 50-gon.
A 9000 **B** 8640 **C** 360 **D** 172.8 **1.** ____

2. Find the value of x.
F 16 **H** 50
G 34 **J** 70

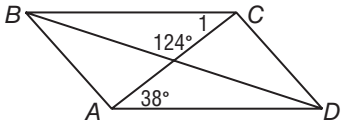


2. ____

3. Find the sum of the measures of the exterior angles of a convex 65-gon.
A 5.54 **B** 90 **C** 180 **D** 360 **3.** ____

4. Which of the following is a property of all parallelograms?
F Each pair of opposite angles is congruent.
G Only one pair of opposite sides is congruent.
H Each pair of opposite angles is supplementary.
J There are four right angles. **4.** ____

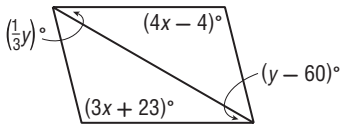
5. For parallelogram $ABCD$, find $m\angle 1$.
A 19 **C** 52
B 38 **D** 56



5. ____

6. $ABCD$ is a parallelogram with diagonals intersecting at E. If $AE = 4x - 8$ and $EC = 36$, find the value of x.
F 7 **G** 11 **H** 15.5 **J** 38 **6.** ____

7. Find the values of the values of x and y so that the quadrilateral is a parallelogram.
A $x = 27, y = 90$ **C** $x = 13, y = 90$
B $x = 27, y = 40$ **D** $x = 13, y = 40$



7. ____

8. Find the value of x so that the quadrilateral is a parallelogram.
F $7\frac{1}{3}$ **H** 12
G 8 **J** 66

8. ____

9. $ABCD$ is a parallelogram with $A(5, 4)$, $B(-1, -2)$, and $C(8, -2)$. Find the coordinates of D.
A $D(-5, 4)$ **B** $D(8, 2)$ **C** $D(14, 4)$ **D** $D(4, 1)$ **9.** ____

10. $ABCD$ is a rectangle. If $AB = 7x - 6$ and $CD = 5x + 30$, find the value of x.
F $5\frac{1}{3}$ **G** 12 **H** 13 **J** 18 **10.** ____

11. Which of the following is true for all rectangles?
A The diagonals are perpendicular.
B The consecutive angles are supplementary.
C The opposite sides are supplementary.
D The opposite angles are complementary. **11.** ____

6 Chapter 6 Test, Form 2B *(continued)*

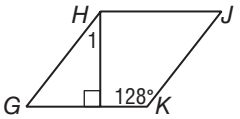
12. *ABCD* is a rectangle with $B(-7, 3)$, $C(5, 3)$, and $D(5, -8)$. Find the coordinates of A.

F $A(-8, -7)$ **G** $A(-7, -8)$ **H** $A(-5, -3)$ **J** $A(-8, -5)$ **12.** ____

13. For rhombus *GHJK*, find $m\angle 1$.

A 90 **C** 52

B 64 **D** 38



13. ____

14. The diagonals of square *ABCD* intersect at E. If $AE = 3x - 4$ and $BD = 10x - 48$, find AC.

F 90 **G** 52 **H** 26 **J** 10 **14.** ____

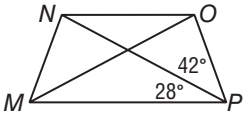
15. *ABCD* is an isosceles trapezoid with $A(0, -1)$, $B(-2, 3)$, and $D(6, -1)$. Find the coordinates of C.

A $C(6, 1)$ **B** $C(9, 4)$ **C** $C(2, 3)$ **D** $C(8, 3)$ **15.** ____

16. For isosceles trapezoid *MNOP*, find $m\angle MNP$.

F 42 **H** 82

G 70 **J** 98



16. ____

17. The length of one base of a trapezoid is 19 meters and the length of the median is 23 meters. Find the length of the other base.

A 15 m **B** 21 m **C** 27 m **D** 42 m **17.** ____

18 On a coordinate plane, the four corners of Ronald's garden are located at (0, 2), (4, 6), (8, 2), and (4, −2). Which of the following most accurately describes the shape of Ronald's garden?

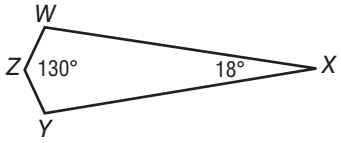
F square **H** rhombus

G rectangle **J** trapezoid **18.** ____

19. For kite *WXYZ*, find $m\angle W$.

A 106 **C** 212

B 148 **D** 360



19. ____

20. *ABCD* is a parallelogram with coordinates $A(4, 2)$, $B(3, -1)$, $C(-1, -1)$, and $D(-1, 2)$. To prove that *ABCD* is a rhombus, you would plot the parallelogram on a coordinate plane and then find which of the following?

F measures of the angles **H** slopes of the diagonals

G lengths of the diagonals **J** midpoints of the diagonals **20.** ____

Bonus The sum of the measures of the interior angles of a convex polygon is ten times the sum of the measures of its exterior angles. Find the number of sides of the polygon.

B: _____

NAME _____ DATE _____ PERIOD _____

6 Chapter 6 Test, Form 2C

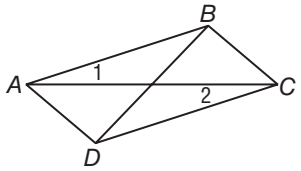
SCORE _____

1. What is the sum of the interior angles of an octagonal box? **1.** _____

2. A convex pentagon has interior angles with measures $(5x - 12)°$, $(2x + 100)°$, $(4x + 16)°$, $(6x + 15)°$, and $(3x + 41)°$. Find the value of x. **2.** _____

3. If the measure of each interior angle of a regular polygon is 171, find the number of sides in the polygon. **3.** _____

4. In parallelogram $ABCD$, $m\angle 1 = x + 12$, and $m\angle 2 = 6x - 18$. Find $m\angle 1$.



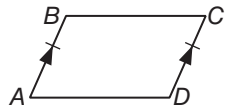
4. _____

5. Find the measure of each exterior angle of a regular 45-gon. **5.** _____

6. In parallelogram $ABCD$, $m\angle A = 58$. Find $m\angle B$. **6.** _____

7. Find the coordinates of the intersection of the diagonals of parallelogram $XYZW$ with vertices $X(2, 2)$, $Y(3, 6)$, $Z(10, 6)$, and $W(9, 2)$. **7.** _____

8. Determine whether $ABCD$ is a parallelogram. Justify your answer.

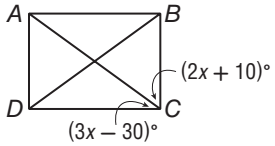


8. _____

9. Determine whether the quadrilateral with vertices $A(5, 7)$, $B(1, -2)$, $C(-6, -3)$, and $D(2, 5)$ is a parallelogram. Use the slope formula. **9.** _____

10. For quadrilateral $ABCD$, the slope of $\overline{AB}$ is $\frac{1}{4}$, the slope of $\overline{BC}$ is $-\frac{2}{3}$, and the slope of $\overline{CD}$ is $\frac{1}{4}$. Find the slope of $\overline{DA}$ so that $ABCD$ will be a parallelogram. **10.** _____

11. Given rectangle $ABCD$, find the value of x.



11. _____

12. $ABCD$ is a parallelogram and $\overline{AC} \cong \overline{BD}$. Determine whether $ABCD$ is a rectangle. Justify your answer. **12.** _____

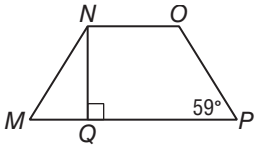
13. $ABCD$ is a rhombus with diagonals intersecting at E. If $m\angle ABC$ is three times $m\angle BAD$, find $m\angle EBC$. **13.** _____

Assessment

6 Chapter 6 Test, Form 2C *(continued)*

14. *TUVW* is a square with $U(10, 2)$, $V(8, 8)$, and $W(2, 6)$. Find the coordinates of T. **14.** _____

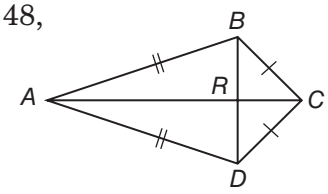
15. For isosceles trapezoid *MNOP*, find $m\angle MNQ$. **15.** _____



16. *ABCD* is a quadrilateral with vertices $A(8, 3)$, $B(6, 7)$, $C(-1, 5)$, and $D(-6, -1)$. Determine whether *ABCD* is a trapezoid. Justify your answer. **16.** _____

17. The length of the median of trapezoid *EFGH* is 13 feet. If the bases have lengths $2x + 4$ and $10x - 50$, find x. **17.** _____

18. *ABCD* is a kite, If $RC = 10$, and $BD = 48$, find *CD*. **18.** _____



For Questions 19–25, write *true* or *false*.

19. A rectangle is always a parallelogram. **19.** _____

20. The diagonals of a rhombus are always perpendicular. **20.** _____

21. The diagonals of a square always bisect each other. **21.** _____

22. A trapezoid always has two congruent sides. **22.** _____

23. The median of a trapezoid is always parallel to the bases. **23.** _____

24. A kite has exactly two congruent angles. **24.** _____

25. If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rectangle. **25.** _____

Bonus In parallelogram *ABCD*, $AB = 2x - 7$, $BC = x + 3y$, $CD = x + y$, and $AD = 2x - y - 1$. Find the values of x and y. **B:** _____

NAME _____ DATE _____ PERIOD _____

6 Chapter 6 Test, Form 2D

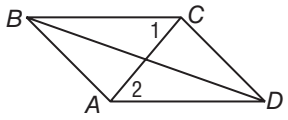
SCORE _____

1. Bruce is building a tabletop in the shape of an octagon. Find the sum of the external angles of the tabletop. **1.** _____

2. A convex octagon has interior angles with measures $(x + 55)°$, $(3x + 20)°$, $4x°$, $(4x - 10)°$, $(6x - 55)°$, $(3x + 52)°$, $3x°$, and $(2x + 30)°$. Find the value of x. **2.** _____

3. If the measure of each interior angle of a regular polygon is 176 find the number of sides in the polygon. **3.** _____

4. In parallelogram $ABCD$, $m\angle 1 = x + 25$, and $m\angle 2 = 2x$. Find $m\angle 2$.



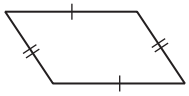
4. _____

5. Find the measure of each exterior angle of a regular 100-gon. **5.** _____

6. In parallelogram $ABCD$, $m\angle A = 63$. Find $m\angle B$. **6.** _____

7. Find the coordinates of the intersection of the diagonals of parallelogram $XYZW$ with vertices $X(3, 0)$, $Y(3, 8)$, $Z(-2, 6)$, and $W(-2, -2)$. **7.** _____

8. Determine whether this quadrilateral is a parallelogram. Justify your answer.

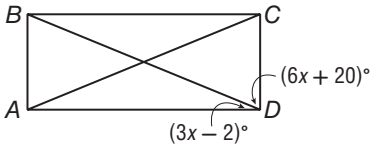


8. _____

9. Determine whether a quadrilateral with vertices $A(5, 7)$, $B(1, -1)$, $C(-6, -3)$, and $D(-2, 5)$ is a parallelogram. Use the slope formula. **9.** _____

10. If the slope of $\overline{AB}$ is $\frac{1}{2}$, the slope of $\overline{BC}$ is -4, and the slope of $\overline{CD}$ is $\frac{1}{2}$, find the slope of $\overline{DA}$ so that $ABCD$ is a parallelogram. **10.** _____

11. For rectangle $ABCD$, find the value of x.



11. _____

12. $ABCD$ is a parallelogram and $m\angle A = 90$. Determine whether $ABCD$ is a rectangle. Justify your answer. **12.** _____

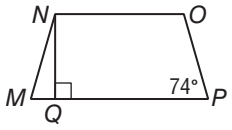
Assessment

6 Chapter 6 Test, Form 2D *(continued)*

13. $ABCD$ is a rhombus with diagonals intersecting at E. If $m\angle ABC$ is four times $m\angle BAD$, find $m\angle EBC$. **13.** _____

14. $PQRS$ is a square with $Q(-2, 8)$, $R(5, 7)$, and $S(4, 0)$. Find the coordinates of P. **14.** _____

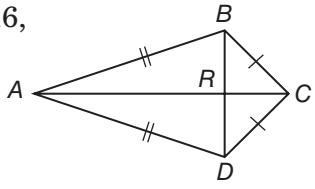
15. For isosceles trapezoid $MNOP$, find $m\angle MNQ$. **15.** _____



16. $ABCD$ is a quadrilateral with $A(8, 21)$, $B(10, 27)$, $C(26, 26)$, and $D(18, 2)$. Determine whether $ABCD$ is a trapezoid. Justify your answer. **16.** _____

17. The length of the median of trapezoid $EFGH$ is 17 centimeters. If the bases have lengths $2x + 4$ and $8x - 50$, find the value of x. **17.** _____

18. For kite $ABCD$, if $RA = 15$, and $BD = 16$, find AD. **18.** _____



For Questions 19–25, write *true* or *false*.

19. A parallelogram always has four right angles. **19.** _____

20. The diagonals of a rhombus always bisect the angles. **20.** _____

21. A rhombus is always a square. **21.** _____

22. A rectangle is always a square. **22.** _____

23. The diagonals of an isosceles trapezoid are always congruent. **23.** _____

24. The median of a trapezoid always bisects the angles. **24.** _____

25. The diagonals of a kite are always perpendicular. **25.** _____

Bonus The measure of each interior angle of a regular polygon is 24 more than 38 times the measure of each exterior angle. Find the number of sides of the polygon. **B:** _____

NAME _____ DATE _____ PERIOD _____

6 Chapter 6 Test, Form 3

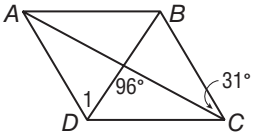
SCORE _____

1. The sum of the interior angles of an animal pen is 900°. How many sides does the pen have? **1.** _____

2. A convex hexagon has interior angles with measures $x°$, $(5x - 103)°$, $(2x + 60)°$, $(7x - 31)°$, $(6x - 6)°$, and $(9x - 100)°$. Find the value of x and the measure of each angle. **2.** _____

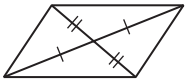
3. Find the measure of each exterior angle of a regular $2x$-gon. **3.** _____

4. For parallelogram $ABCD$, find $m\angle 1$. **4.** _____



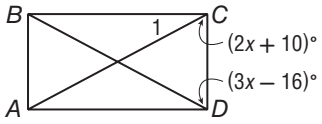
5. $ABCD$ is a parallelogram with diagonals that intersect each other at E. If $AE = x^2$ and $EC = 6x - 8$, find all possible values of AC. **5.** _____

6. Determine whether the quadrilateral is a parallelogram. Justify your answer. **6.** _____



7. For quadrilateral $ABCD$, the slope of $\overline{AB}$ is $\frac{2}{3}$ and the slope of $\overline{BC}$ is -2. Find the slopes of $\overline{CD}$ and $\overline{DA}$ so that $ABCD$ will be a parallelogram. **7.** _____

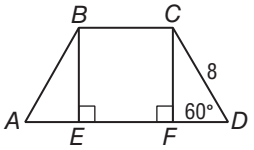
8. In rectangle $ABCD$, find $m\angle 1$. **8.** _____



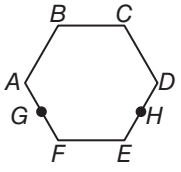
9. The diagonals of rhombus $ABCD$ intersect at E. If $m\angle BAE = \frac{2}{3}(m\angle ABE)$, find $m\angle BCD$. **9.** _____

10. The diagonals of square $ABCD$ intersect at E. If $AE = 2$, find the perimeter of $ABCD$. **10.** _____

11. For isosceles trapezoid $ABCD$, find AE. **11.** _____



12. Points G and H are midpoints of $\overline{AF}$ and $\overline{DE}$ in regular hexagon $ABCDEF$. If $AB = 6$ find GH. **12.** _____



13. The vertices of trapezoid ABCD are $A(10, -1)$, $B(6, 6)$, $C(-2, 6)$, and $D(-8, -1)$. Find the length of the median. **13.** _____

Assessment

NAME _____ DATE _____ PERIOD _____

6 Chapter 6 Test, Form 3 *(continued)*

14. Determine whether the quadrilateral $ABCD$ with vertices $A(0,-1)$, $B(-4,-3)$, $C(-5, 1)$, $D(1, 7)$ is a kite. Justify your answer.

14. _____

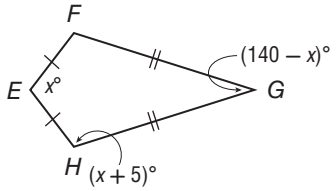
15. Determine whether the quadrilateral $ABCD$ with vertices $A(6, 2)$, $B(2, 10)$, $C(-6, 6)$, and $D(-2, -2)$ is a rectangle. Justify your answer.

15. _____

16. Determine whether quadrilateral $ABCD$ with vertices $A(1, 6)$, $B(7, 6)$, $C(2, -3)$, and $D(-4, -3)$ is a parallelogram. Use the distance formula.

16. _____

17. Find the value of x in kite $EFGH$.

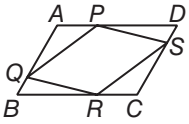


17. _____

For Questions 18 and 19, complete the two-column proof by supplying the missing information for each corresponding location.

Given: $ABCD$ is a parallelogram.
$\overline{BQ} \cong \overline{DS}$, $\overline{PA} \cong \overline{RC}$

Prove: $PQRS$ is a parallelogram.

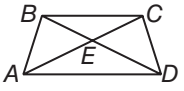


Statements	Reasons
1. $ABCD$ is a $\square$.	**1.** Given
2. $\overline{AD} \cong \overline{CB}$	**2.** (Question 18)
3. $\overline{PA} \cong \overline{RC}$	**3.** Given
4. $\overline{PD} \cong \overline{RB}$	**4.** Seg. Sub. Prop.
5. $\overline{AB} \cong \overline{CD}$	**5.** Opp. sides of a $\square$ are $\cong$.
6. $\overline{BQ} \cong \overline{DS}$	**6.** Given
7. $\overline{AQ} \cong \overline{CS}$	**7.** Seg. Sub. Prop.
8. $\angle B \cong \angle D$, $\angle A \cong \angle C$	**8.** Opp. $\measuredangle$ of a $\square$ are $\cong$.
9. $\triangle QBR \cong \triangle SDP$, $\triangle PAQ \cong \triangle RCS$	**9.** SAS
10. $\overline{QP} \cong \overline{RS}$, $\overline{QR} \cong \overline{PS}$	**10.** CPCTC
11. $PQRS$ is a parallelogram.	**11.** (Question 19)

18. _____

19. _____

20. In isosceles trapezoid $ABCD$, $AE = 2x + 5$, $EC = 3x - 12$, and $BD = 4x + 20$. Find the value of x.



20. _____

Bonus If three of the interior angles of a convex hexagon each measure 140, a fourth angle measures 84, and the measure of the fifth angle is 3 times the measure of the sixth angle, find the measure of the sixth angle.

B: _____

NAME _____ DATE _____ PERIOD _____

6 Chapter 6 Extended-Response Test

SCORE _____

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. a. Draw a regular convex polygon and a convex polygon that is not regular, each with the same number of sides.

b. Label the measures of each exterior angle on your figures.

c. Find the sum of the exterior angles for each figure. What conjecture can be made?

2. Draw a rectangle. Connect the midpoints of the consecutive sides. What type of quadrilateral is formed? How do you know?

3. Draw an example to show why *one pair of opposite sides congruent and the other pair of opposite sides parallel* is not sufficient to form a parallelogram.

4. a. Name a property that is true for a square and not always true for a rectangle.

b. Name a property that is true for a square and not always true for a rhombus.

c. Name a property that is true for a rectangle and not always true for a parallelogram.

NAME _____ DATE _____ PERIOD _____

6 Standardized Test Practice

SCORE _____

(Chapters 1–6)

Part 1: Multiple Choice

Instructions: Fill in the appropriate circle for the best answer.

1. Find the coordinates of X if $V(0.5, 5)$ is the midpoint of $\overline{UX}$ with $U(15, 21)$. (Lesson 1-3)

A $(-14, -11)$ **C** $(0, 0)$

B $(7.75, 22.5)$ **D** $(15.5, -5)$

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Which of the following are possible measures for vertical angles G and H? (Lesson 2-8)

F $m\angle G = 125$ and $m\angle H = 55$

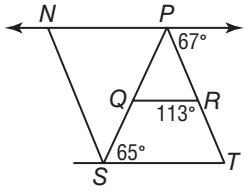
G $m\angle G = 125$ and $m\angle H = 125$

H $m\angle G = 55$ and $m\angle H = 45$

J $m\angle G = 55$ and $m\angle H = 152.5$

2. Ⓕ Ⓖ Ⓗ Ⓙ

3. Determine which lines are parallel. (Lesson 3-5)



A $\overleftrightarrow{NS} \parallel \overleftrightarrow{PT}$ **C** $\overleftrightarrow{QR} \parallel \overleftrightarrow{ST}$

B $\overleftrightarrow{NP} \parallel \overleftrightarrow{ST}$ **D** $\overleftrightarrow{NP} \parallel \overleftrightarrow{QR}$

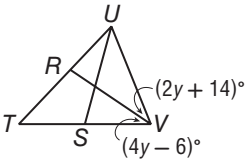
3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Find the coordinates of B, the midpoint of $\overline{AC}$, if $A(2a, b)$ and $C(0, 2b)$. (Lesson 4-8)

F $(2a, 2b)$ **G** (a, b) **H** $\left(a, \frac{3}{2}b\right)$ **J** $\left(\frac{3}{2}a, b\right)$

4. Ⓕ Ⓖ Ⓗ Ⓙ

5. If $\overline{RV}$ is an angle bisector, find $m\angle UVT$. (Lesson 5-1)



A 10 **C** 68

B 34 **D** 136

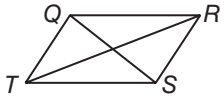
5. Ⓐ Ⓑ Ⓒ Ⓓ

6. Find the slope of the line that passes through points $A(-7, 14)$ and $B(5, -2)$. (Lesson 3-3)

F $-\frac{4}{3}$ **G** $-\frac{3}{4}$ **H** $\frac{3}{4}$ **J** $\frac{4}{3}$

6. Ⓕ Ⓖ Ⓗ Ⓙ

7. Which statement ensures that quadrilateral $QRST$ is a parallelogram? (Lesson 6-3)



A $\angle Q \cong \angle S$ **C** $\overline{QT} \parallel \overline{RS}$

B $\overline{QR} \cong \overline{TS}$ and $\overline{QR} \parallel \overline{TS}$ **D** $m\angle Q + m\angle S = 180$

7. Ⓐ Ⓑ Ⓒ Ⓓ

6 Standardized Test Practice *(continued)*

8. What is the equation of the line that contains $(-12, 9)$ and is perpendicular to the line $y = \frac{2}{3}x + 5$? (Lesson 3-4)

F $y = -\frac{3}{2}x - 9$ **H** $y = -\frac{2}{3}x - 1$

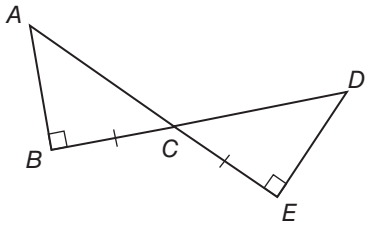
G $y = \frac{3}{2}x - 1$ **J** $y = \frac{2}{3}x + 17$

8. Ⓕ Ⓖ Ⓗ Ⓙ

9. Which of the following theorems can be used to prove $\triangle ABC \cong \triangle DEC$? (Lesson 4-5)

A SSS **C** SAS

B AAS **D** ASA

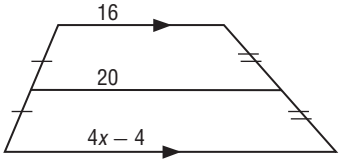


9. Ⓐ Ⓑ Ⓒ Ⓓ

10. What is the value of x? (Lesson 6-6)

F 2 **H** 5.5

G 4 **J** 7



10. Ⓕ Ⓖ Ⓗ Ⓙ

11. For $\triangle ABC$, $AB = 6$ and $BC = 17$. Which of the following is a possible length for $\overline{AC}$? (Lesson 5-3)

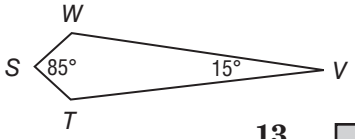
A 5 **B** 9 **C** 13 **D** 24

11. Ⓐ Ⓑ Ⓒ Ⓓ

12. What is $m\angle T$ in kite $STVW$?

F 100 **H** 95

G 130 **J** 260



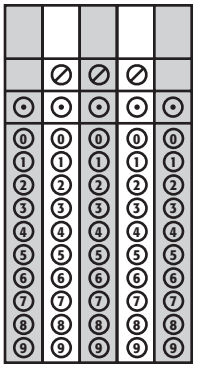
12. Ⓕ Ⓖ Ⓗ Ⓙ

Part 2: Gridded Response

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

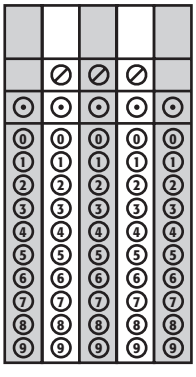
13. If $\triangle UVW$ is an isosceles triangle, $\overline{UV} \cong \overline{WU}$, $UV = 16b - 40$, $VW = 6b$, and $WU = 10b + 2$, find the value of b. (Lesson 4-1)

13.



14. Find the sum of the measures of the interior angles for a convex heptagon. (Lesson 6-1)

14.



Assessment

NAME _____ DATE _____ PERIOD _____

6 Standardized Test Practice *(continued)*

Part 3: Short Response

Instructions: Place your answers in the space provided.

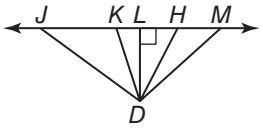
15. A polygon has six congruent sides. Lines containing two of its sides contain points in its interior. Name the polygon by its number of sides, and then classify it as *convex* or *concave* and *regular* or *irregular*. (Lesson 1-6)

15. _____

16. If $\overline{RT} \cong \overline{QM}$ and $RT = 88.9$ centimeters, find QM. (Lesson 2-7)

16. _____

17. Which segment is the shortest segment from D to $\overleftrightarrow{JM}$? (Lesson 5-2)



17. _____

18. If $\triangle ABC \cong \triangle WXY$, $AB = 72$, $BC = 65$, $CA = 13$, $XY = 7x - 12$, and $WX = 19y + 34$, find the values of x and y. (Lesson 4-3)

18. _____

19. Freda bought two bells for just over \$90 before tax. State the assumption you would make to write an indirect proof to show that at least one of the bells costs more than \$45. (Lesson 5-4)

19. _____

20. The area of the base of a cylinder is 5 cm^2 and the height of the cylinder is 8 cm. Find the volume of the cylinder. (Lesson 1-7)

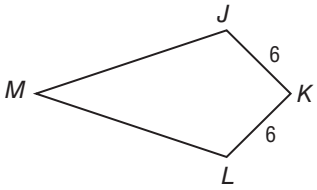
20. _____

21. *JKLM* is a kite. Complete each statement. (Lesson 6-6)

a. $\overline{MJ} \cong$ ____

b. $\overline{MK} \perp$ ____

c. $m\angle L = m\angle$ ____



21. a. _____

b. _____

c. _____

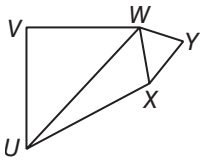
NAME _____ DATE _____ PERIOD _____

6 Unit 2 Test

SCORE _____

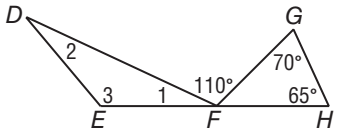
(Chapters 4–6)

1. Use a protractor to classify $\triangle UVW$, $\triangle UWX$, and $\triangle XWY$ as *acute*, *equiangular*, *obtuse*, or *right*.



1. _____

2. In the figure, $\angle 1 \cong \angle 2$. Find the measures of the numbered angles.



2. _____

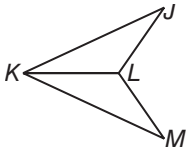
3. Name the corresponding congruent sides for $\triangle AFP \cong \triangle STX$.

3. _____

4. Determine whether $\triangle ABC \cong \triangle PQR$ given $A(2, -7)$, $B(5, 3)$, $C(-4, 6)$, $P(8, -1)$, $Q(11, 9)$, and $R(2, 12)$.

4. _____

5. In the figure, $\overline{LK}$ bisects $\angle JKM$ and $\angle KLJ \cong \angle KLM$. Determine which theorem or postulate can be used to prove that $\triangle JKL \cong \triangle MKL$.

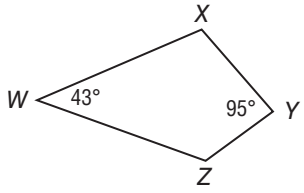


5. _____

6. Triangle ABC is isosceles with $AB = BC$. Name a pair of congruent angles in this triangle.

6. _____

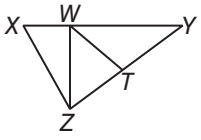
7. For kite $WXYZ$, find $m\angle Z$.



7. _____

For Questions 8 and 9, refer to the figure.

8. Find the value of a and $m\angle ZWT$ if $\overline{ZW}$ is an altitude of $\triangle XYZ$, $m\angle ZWT = 3a + 5$, and $m\angle TWY = 5a + 13$.



8. _____

9. Determine which angle has the greatest measure: $\angle YWZ$, $\angle WZY$, or $\angle ZYW$.

9. _____

10. Mr. Ramirez bought a stove and a dishwasher for just over \$1206. State the assumption you would make to start an indirect proof to show that at least one of the appliances cost more than \$603.

10. _____

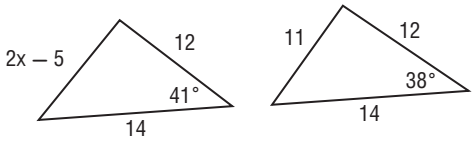
Assessment

NAME _____ DATE _____ PERIOD _____

6 Unit 2 Test *(continued)*

11. Determine whether 128 feet, 136 feet, and 245 feet can be the lengths of the sides of a triangle. **11.** _____

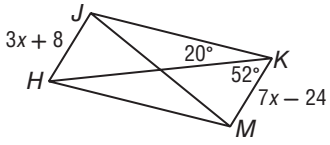
12. Write an inequality to describe the possible values of x.



12. _____

13. The measure of an interior angle of a regular polygon is 140. Find the number of sides in the polygon. **13.** _____

14. For parallelogram $JKMH$, find $m\angle JHK$, $m\angle HMK$, and the value of x.

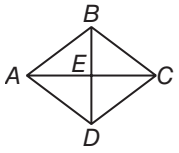


14. _____

15. Determine whether the vertices of quadrilateral $DEFG$ form a parallelogram given $D(-3, 5)$, $E(3, 6)$, $F(-1, 0)$, and $G(6, 1)$. **15.** _____

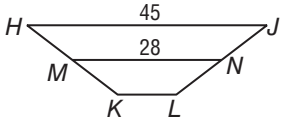
16. For rectangle $WXYZ$ with diagonals $\overline{WY}$ and $\overline{XZ}$, $WY = 3d + 4$ and $XZ = 4d - 1$, find the value of d. **16.** _____

17. If $m\angle BEC = 9z + 45$ in rhombus $ABCD$, find the value of z.



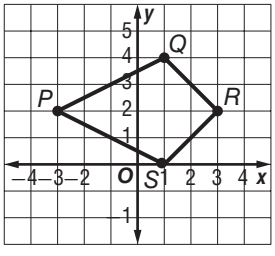
17. _____

18. In trapezoid $HJLK$, M and N are midpoints of the legs. Find KL.



18. _____

19. Prove that quadrilateral $PQRS$ is NOT a parallelogram.



19. _____

NAME _____ DATE _____ PERIOD _____

6 Anticipation Guide

Quadrilaterals

Step 1 ***Before you begin Chapter 6***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. A triangle has no diagonals.	A
	2. A diagonal of a polygon is a segment joining the midpoints of two sides of the polygon.	D
	3. The sum of the measures of the angles in a polygon can be determined by subtracting 2 from the number of sides and multiplying the result by 180.	A
	4. For a quadrilateral to be a parallelogram it must have two pairs of parallel sides.	A
	5. The diagonals of a parallelogram are congruent.	D
	6. If you know that one pair of opposite sides of a quadrilateral is both parallel and congruent, then you know the quadrilateral is a parallelogram.	A
	7. If a quadrilateral is a rectangle, then all four angles are congruent.	A
	8. The diagonals of a rhombus are congruent.	D
	9. The properties of a rhombus are not true for a square.	D
	10. A trapezoid has only one pair of parallel sides.	A
	11. The median of a trapezoid is perpendicular to the bases.	D
	12. An isosceles trapezoid has exactly one pair of congruent sides.	A

Step 2 ***After you complete Chapter 6***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NAME _____ DATE _____ PERIOD _____

6-1 Study Guide and Intervention

Angles of Polygons

Polygon Interior Angles Sum The segments that connect the nonconsecutive vertices of a polygon are called **diagonals**. Drawing all of the diagonals from one vertex of an ***n*-gon** separates the polygon into $n - 2$ triangles. The sum of the measures of the interior angles of the polygon can be found by adding the measures of the interior angles of those $n - 2$ triangles.

Polygon Interior Angle Sum Theorem	The sum of the interior angle measures of an n-sided convex polygon is $(n - 2) \cdot 180$.

Example 1 **A convex polygon has 13 sides. Find the sum of the measures of the interior angles.**

$(n - 2) \cdot 180 = (13 - 2) \cdot 180$
$= 11 \cdot 180$
$= 1980$

Example 2 **The measure of an interior angle of a regular polygon is 120. Find the number of sides.**

The number of sides is n, so the sum of the measures of the interior angles is $120n$.

$120n = (n - 2) \cdot 180$
$120n = 180n - 360$
$-60n = -360$
$n = 6$

Exercises

Find the sum of the measures of the interior angles of each convex polygon.

1. decagon
1440

2. 16-gon
2520

3. 30-gon
5040

4. octagon
1080

5. 12-gon
1800

6. 35-gon
5940

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

7. 150
12

8. 160
18

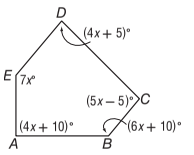
9. 175
72

10. 165
24

11. 144
10

12. 135
8

13. Find the value of x.
20



Answers

NAME _____ DATE _____ PERIOD _____

6-1 Study Guide and Intervention *(continued)*

Angles of Polygons

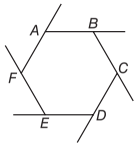
Polygon Exterior Angles Sum There is a simple relationship among the exterior angles of a convex polygon.

Polygon Exterior Angle Sum Theorem	The sum of the exterior angle measures of a convex polygon, one angle at each vertex, is 360.

Example 1 **Find the sum of the measures of the exterior angles, one at each vertex, of a convex 27-gon.**

For *any* convex polygon, the sum of the measures of its exterior angles, one at each vertex, is 360.

Example 2 **Find the measure of each exterior angle of regular hexagon *ABCDEF*.**



The sum of the measures of the exterior angles is 360 and a hexagon has 6 angles. If n is the measure of each exterior angle, then

$6n = 360$

$n = 60$

The measure of each exterior angle of a regular hexagon is 60.

Exercises

Find the sum of the measures of the exterior angles of each convex polygon.

1. decagon
360

2. 16-gon
360

3. 36-gon
360

Find the measure of each exterior angle for each regular polygon.

4. 12-gon
30

5. hexagon
60

6. 20-gon
18

7. 40-gon
9

8. heptagon
51.4

9. 12-gon
30

10. 24-gon
15

11. dodecagon
30

12. octagon
45

NAME _____ DATE _____ PERIOD _____

6-1 Skills Practice

Angles of Polygons

Find the sum of the measures of the interior angles of each convex polygon.

1. nonagon
1260

2. heptagon
900

3. decagon
1440

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

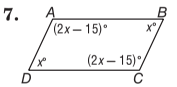
4. 108
5

5. 120
6

6. 150
12

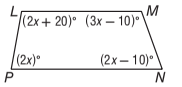
Find the measure of each interior angle.

7.



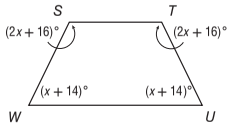
$m\angle A = 115$, $m\angle B = 65$, $m\angle C = 115$, $m\angle D = 65$

8.



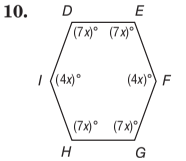
$m\angle L = 100$, $m\angle M = 110$, $m\angle N = 70$, $m\angle P = 80$

9.



$m\angle S = 116$, $m\angle T = 116$, $m\angle W = 64$, $m\angle U = 64$

10.



$m\angle D = 140$, $m\angle E = 140$, $m\angle I = 80$, $m\angle F = 80$, $m\angle H = 140$, $m\angle G = 140$

Find the measures of each interior angle of each regular polygon.

11. quadrilateral
90, 90

12. pentagon
108, 72

13. dodecagon
150, 30

Find the measures of each exterior angle of each regular polygon.

14. octagon
45

15. nonagon
40

16. 12-gon
30

NAME _____ DATE _____ PERIOD _____

6-1 Practice

Angles of Polygons

Find the sum of the measures of the interior angles of each convex polygon.

1. 11-gon
1620

2. 14-gon
2160

3. 17-gon
2700

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

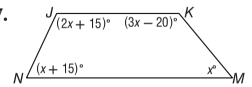
4. 144
10

5. 156
15

6. 160
18

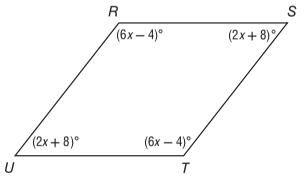
Find the measure of each interior angle.

7.



$m\angle J = 115$, $m\angle K = 130$,
$m\angle M = 50$, $m\angle N = 65$

8.



$m\angle R = 128$, $m\angle S = 52$
$m\angle T = 128$, $m\angle U = 52$

Find the measures of an exterior angle and an interior angle given the number of sides of each regular polygon. Round to the nearest tenth, if necessary.

9. 16
157.5, 22.5

10. 24
165, 15

11. 30
168, 12

12. 14
154.3, 25.7

13. 22
163.6, 16.4

14. 40
171, 9

15. CRYSTALLOGRAPHY Crystals are classified according to seven crystal systems. The basis of the classification is the shapes of the faces of the crystal. Turquoise belongs to the triclinic system. Each of the six faces of turquoise is in the shape of a parallelogram. Find the sum of the measures of the interior angles of one such face.
360

NAME _____ DATE _____ PERIOD _____

6-1 Word Problem Practice

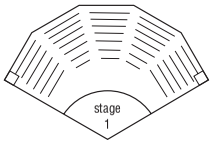
Angles of Polygons

1. ARCHITECTURE In the Uffizi gallery in Florence, Italy, there is a room built by Buontalenti called the Tribune (*La Tribuna* in Italian). This room is shaped like a regular octagon.

What angle do consecutive walls of the Tribune make with each other?
135°

2. BOXES Jasmine is designing boxes she will use to ship her jewelry. She wants to shape the box like a regular polygon. In order for the boxes to pack tightly, she decides to use a regular polygon that has the property that the measure of its interior angles is half the measure of its exterior angles. What regular polygon should she use?
an equilateral triangle

3. THEATER A theater floor plan is shown in the figure. The upper five sides are part of a regular dodecagon.

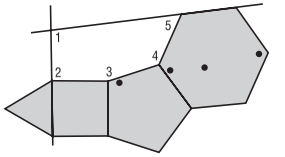


Find $m\angle 1$.
120

4. ARCHEOLOGY Archeologists unearthed parts of two adjacent walls of an ancient castle.

Before it was unearthed, they knew from ancient texts that the castle was shaped like a regular polygon, but nobody knew how many sides it had. Some said 6, others 8, and some even said 100. From the information in the figure, how many sides did the castle really have?
15

5. POLYGON PATH In Ms. Rickets' math class, students made a "polygon path" that consists of regular polygons of 3, 4, 5, and 6 sides joined together as shown.



a. Find $m\angle 2$ and $m\angle 5$.
90 and 60

b. Find $m\angle 3$ and $m\angle 4$.
162 and 132

c. What is $m\angle 1$?
96

Answers

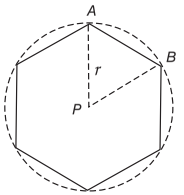
NAME _____ DATE _____ PERIOD _____

6-1 Enrichment

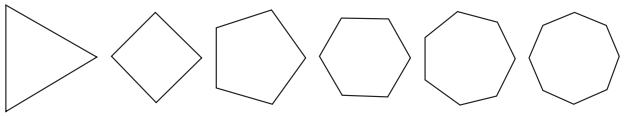
Central Angles of Regular Polygons

You have learned about the interior and exterior angles of a polygon. Regular polygons also have **central angles**. A central angle is measured from the center of the polygon.

The center of a polygon is the point equidistant from all of the vertices of the polygon, just as the center of a circle is the point equidistant from all of the points on the circle. The central angle is the angle drawn with the vertex at the center of the circle and the sides of angle drawn through consecutive vertices of the polygon. One of the central angles that can be drawn in this regular hexagon is $\angle APB$. You may remember from making circle graphs that there are 360° around the center of a circle.



1. By using logic or by drawing sketches, find the measure of the central angle of each regular polygon.



120 90 72 60 about 51.43 45

2. Make a conjecture about how the measure of a central angle of a regular polygon relates to the measures of the interior angles and exterior angles of a regular polygon.

The measure of the exterior angle equals the measure of the central angle. The central angle is supplementary to interior angle.

3. CHALLENGE In obtuse $\triangle ABC$, $\overline{BC}$ is the longest side. $\overline{AC}$ is also a side of a 21-sided regular polygon. $\overline{AB}$ is also a side of a 28-sided regular polygon. The 21-sided regular polygon and the 28-sided regular polygon have the same center point P. Find n if $\overline{BC}$ is a side of a n-sided regular polygon that has center point P.

(*Hint:* Sketch a circle with center P and place points A, B, and C on the circle.)

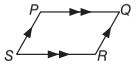
$n = 12$

NAME _____ DATE _____ PERIOD _____

6-2 Study Guide and Intervention

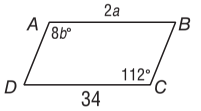
Parallelograms

Sides and Angles of Parallelograms A quadrilateral with both pairs of opposite sides parallel is a **parallelogram**. Here are four important properties of parallelograms.



	If $PQRS$ is a parallelogram, then
If a quadrilateral is a parallelogram, then its opposite sides are congruent.	$\overline{PQ} \cong \overline{SR}$ and $\overline{PS} \cong \overline{QR}$
If a quadrilateral is a parallelogram, then its opposite angles are congruent.	$\angle P \cong \angle R$ and $\angle S \cong \angle Q$
If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.	$\angle P$ and $\angle S$ are supplementary; $\angle S$ and $\angle R$ are supplementary; $\angle R$ and $\angle Q$ are supplementary; $\angle Q$ and $\angle P$ are supplementary.
If a parallelogram has one right angle, then it has four right angles.	If $m\angle P = 90$, then $m\angle Q = 90$, $m\angle R = 90$, and $m\angle S = 90$.

Example **If $ABCD$ is a parallelogram, find the value of each variable.**



$\overline{AB}$ and $\overline{CD}$ are opposite sides, so $\overline{AB} \cong \overline{CD}$.

$2a = 34$

$a = 17$

$\angle A$ and $\angle C$ are opposite angles, so $\angle A \cong \angle C$.

$8b = 112$

$b = 14$

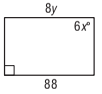
Exercises

Find the value of each variable.

1.

$x = 30$; $y = 22.5$

2.

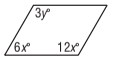


$x = 15$; $y = 11$

3.

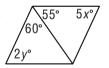
$x = 2$; $y = 4$

4.



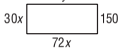
$x = 10$; $y = 40$

5.



$x = 13$; $y = 32.5$

6.



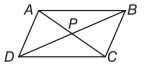
$x = 5$; $y = 180$

NAME _____ DATE _____ PERIOD _____

6-2 Study Guide and Intervention *(continued)*

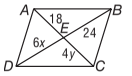
Parallelograms

Diagonals of Parallelograms Two important properties of parallelograms deal with their diagonals.



	If *ABCD* is a parallelogram, then
If a quadrilateral is a parallelogram, then its diagonals bisect each other.	$AP = PC$ and $DP = PB$
If a quadrilateral is a parallelogram, then each diagonal separates the parallelogram into two congruent triangles.	$\triangle ACD \cong \triangle CAB$ and $\triangle ADB \cong \triangle CBD$

Example **Find the value of x and y in parallelogram $ABCD$.**



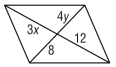
The diagonals bisect each other, so $AE = CE$ and $DE = BE$.

$6x = 24$ $\quad 4y = 18$

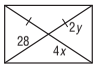
$x = 4$ $\quad y = 4.5$

Exercises

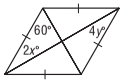
Find the value of each variable.

1. 

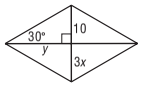
$x = 4$; $y = 2$

2. 

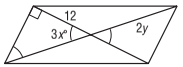
$x = 7$; $y = 14$

3. 

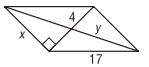
$x = 15$; $y = 7.5$

4. 

$x = 3\frac{1}{3}$; $y = 10\sqrt{3}$

5. 

$x = 15$; $y = 6\sqrt{2}$

6. 

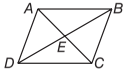
$x = 15$; $y = \sqrt{241}$

COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of $\square ABCD$ with the given vertices.

7. $A(3, 6)$, $B(5, 8)$, $C(3, -2)$, and $D(1, -4)$
(3, 2)

8. $A(-4, 3)$, $B(2, 3)$, $C(-1, -2)$, and $D(-7, -2)$
(−2.5, 0.5)

9. **PROOF** Write a paragraph proof of the following.



Given: $\square ABCD$

Prove: $\triangle AED \cong \triangle BEC$

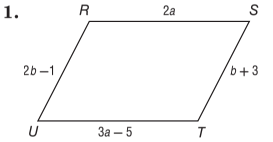
Diagonals of a parallelogram bisect each other, so $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$. Opposite sides of a parallelogram are congruent, therefore $\overline{AD} \cong \overline{BC}$. Because corresponding parts of the two triangles are congruent, the triangles are congruent by SSS.

NAME _____ DATE _____ PERIOD _____

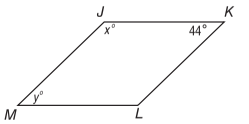
6-2 Skills Practice

Parallelograms

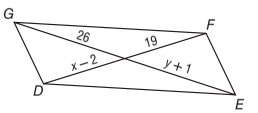
ALGEBRA Find the value of each variable.

1. 

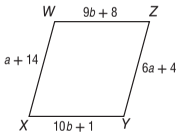
$a = 5$, $b = 4$

2. 

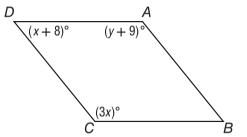
$x = 136$, $y = 44$

3. 

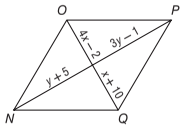
$x = 21$, $y = 25$

4. 

$a = 2$, $b = 7$

5. 

$x = 43$, $y = 120$

6. 

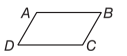
$x = 4$, $y = 3$

COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of $\square HJKL$ with the given vertices.

7. $H(1, 1)$, $J(2, 3)$, $K(6, 3)$, $L(5, 1)$
(3.5, 2)

8. $H(-1, 4)$, $J(3, 3)$, $K(3, -2)$, $L(-1, -1)$
(1, 1)

9. **PROOF** Write a paragraph proof of the theorem *Consecutive angles in a parallelogram are supplementary*.



Given: $\square ABCD$

Prove: $\angle A$ and $\angle B$ are supplementary.
$\angle B$ and $\angle C$ are supplementary.
$\angle C$ and $\angle D$ are supplementary.
$\angle D$ and $\angle A$ are supplementary.

Proof: We are given $\square ABCD$, so we know that $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DA}$ by the definition of a parallelogram. We also know that if two parallel lines are cut by a transversal, then consecutive interior angles are supplementary. So, $\angle A$ and $\angle B$, $\angle B$ and $\angle C$, $\angle C$ and $\angle D$, and $\angle D$ and $\angle A$ are pairs of supplementary angles.

Answers

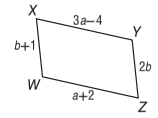
NAME _____ DATE _____ PERIOD _____

6-2 Practice

Parallelograms

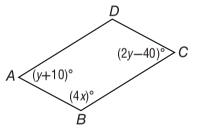
ALGEBRA Find the value of each variable.

1.



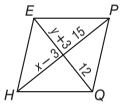
$a = 3, b = 1$

2.



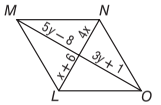
$x = 30°, y = 50°$

3.



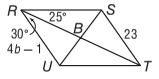
$x = 18, y = 9$

4.



$x = 2, y = 4.5$

ALGEBRA Use $\square RSTU$ to find each measure or value.



5. $m\angle RST =$ 125

6. $m\angle STU =$ 55

7. $m\angle TUR =$ 125

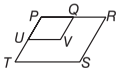
8. $b =$ 6

COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of $\square PRYZ$ with the given vertices.

9. $P(2, 5)$, $R(3, 3)$, $Y(-2, -3)$, $Z(-3, -1)$
(0, 1)

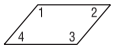
10. $P(2, 3)$, $R(1, -2)$, $Y(-5, -7)$, $Z(-4, -2)$
(−1.5, −2)

11. PROOF Write a paragraph proof of the following.
Given: $\square PRST$ and $\square PQVU$
Prove: $\angle V \cong \angle S$



Proof: We are given $\square PRST$ and $\square PQVU$. Since opposite angles of a parallelogram are congruent, $\angle P \cong \angle V$ and $\angle P \cong \angle S$. Since congruence of angles is transitive, $\angle V \cong \angle S$ by the Transitive Property of congruence.

12. CONSTRUCTION Mr. Rodriquez used the parallelogram at the right to design a herringbone pattern for a paving stone. He will use the paving stone for a sidewalk. If $m\angle 1$ is 130, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.
50, 130, 50

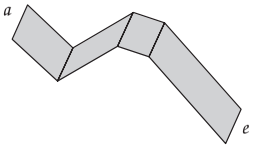


NAME _____ DATE _____ PERIOD _____

6-2 Word Problem Practice

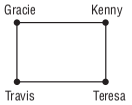
Parallelograms

1. WALKWAY A walkway is made by adjoining four parallelograms as shown.



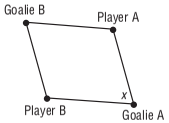
Are the end segments a and e parallel to each other? Explain.
Yes. Opposite sides of a parallelogram are parallel and the parallel property is transitive.

2. DISTANCE Four friends live at the four corners of a block shaped like a parallelogram. Gracie lives 3 miles away from Kenny. How far apart do Teresa and Travis live from each other?



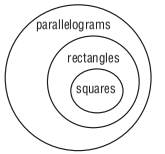
3 mi

3. SOCCER Four soccer players are located at the corners of a parallelogram. Two of the players in opposite corners are the goalies. In order for goalie A to be able to see the three others, she must be able to see a certain angle x in her field of vision.

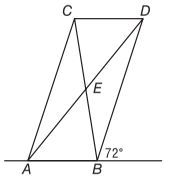


What angle does the other goalie have to be able to see in order to keep an eye on the other three players?
He or she also has to see angle x.

4. VENN DIAGRAMS Make a Venn diagram showing the relationship between squares, rectangles, and parallelograms.



5. SKYSCRAPERS On vacation, Tony's family took a helicopter tour of the city. The pilot said the newest building in the city was the building with this top view. He told Tony that the exterior angle by the front entrance is 72°. Tony wanted to know more about the building, so he drew this diagram and used his geometry skills to learn a few more things. The front entrance is next to vertex B.



a. What are the measures of the four angles of the parallelogram?
72, 72, 108, 108

b. How many pairs of congruent triangles are there in the figure? What are they?
4 pairs;
$\triangle ABE \cong \triangle DCE$,
$\triangle ABC \cong \triangle DCB$,
$\triangle ACE \cong \triangle DBE$, and
$\triangle ABD \cong \triangle DCA$

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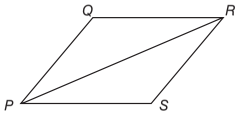
6-2 Enrichment

Diagonals of Parallelograms

In some drawings the diagonal of a parallelogram appears to be the angle bisector of both opposite angles. When might that be true?

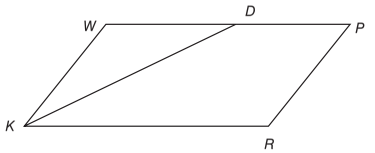
1. Given: Parallelogram $PQRS$ with diagonal $\overline{PR}$. $\overline{PR}$ is an angle bisector of $\angle QPS$ and $\angle QRS$.



What type of parallelogram is $PQRS$? Justify your answer.

$\angle QRS \cong \angle QPS$ opposite angles of a parallelogram
$\angle QPR \cong \angle RPS$ definition of angle bisector
$\angle QRP \cong \angle SRP$ definition of angle bisector
$\angle QPR \cong \angle PRS$ alternate interior angles
$\angle RPS \cong \angle PRS$ Transitive Property
$\overline{SP} \cong \overline{SR}$ Isosceles Triangle Theorem
So all sides are $\cong$ and $QRPS$ is a rhombus

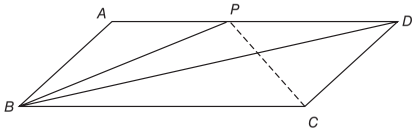
2. Given: Parallelogram $WPRK$ with angle bisector $\overline{KD}$, $DP = 5$, and $WD = 7$.



Find WK and KR.
$WK = 7$, $KR = 12$

3. Refer to Exercise 2. Write a statement about parallelogram $WPRK$ and angle bisector $\overline{KD}$.
Sample answer: Parallelogram $WKRP$ is not a rhombus and $\overline{KD}$ is not a diagonal.

4. Given: Parallelogram $ABCD$ with diagonal $\overline{BD}$ and angle bisector $\overline{BP}$. $PD = 5$, $BP = 6$, and $CP = 6$. The perimeter of triangle PCD is 15. Find AB and BC.



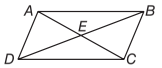
$AB = 4$, $BC = 9$

NAME _____ DATE _____ PERIOD _____

6-3 Study Guide and Intervention

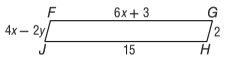
Tests for Parallelograms

Conditions for Parallelograms There are many ways to establish that a quadrilateral is a parallelogram.



If:	If:
both pairs of opposite sides are parallel,	$\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$,
both pairs of opposite sides are congruent,	$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$,
both pairs of opposite angles are congruent,	$\angle ABC \cong \angle ADC$ and $\angle DAB \cong \angle BCD$,
the diagonals bisect each other,	$\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$,
one pair of opposite sides is congruent and parallel,	$\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$, or $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$,
then: the figure is a parallelogram.	**then:** ABCD is a parallelogram.

Example **Find x and y so that $FGHJ$ is a parallelogram.**

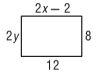


$FGHJ$ is a parallelogram if the lengths of the opposite sides are equal.

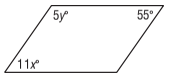
$6x + 3 = 15$ $\quad 4x - 2y = 2$
$6x = 12$ $\quad 4(2) - 2y = 2$
$x = 2$ $\quad 8 - 2y = 2$
$-2y = -6$
$y = 3$

Exercises

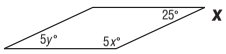
Find x and y so that the quadrilateral is a parallelogram.

1. 

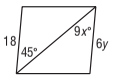
$x = 7$; $y = 4$

2. 

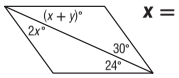
$x = 5$; $y = 25$

3. 

$x = 31$; $y = 5$

4. 

$x = 5$; $y = 3$

5. 

$x = 15$; $y = 9$

6. 

$x = 30$; $y = 15$

Answers

NAME _____ DATE _____ PERIOD _____

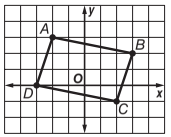
6-3 Study Guide and Intervention *(continued)*

Tests for Parallelograms

Parallelograms on the Coordinate Plane On the coordinate plane, the Distance, Slope, and Midpoint Formulas can be used to test if a quadrilateral is a parallelogram.

Example **Determine whether *ABCD* is a parallelogram.**

The vertices are $A(-2, 3)$, $B(3, 2)$, $C(2, -1)$, and $D(-3, 0)$.



Method 1: Use the Slope Formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

slope of $\overline{AD} = \frac{3-0}{-2-(-3)} = \frac{3}{1} = 3$ slope of $\overline{BC} = \frac{2-(-1)}{3-2} = \frac{3}{1} = 3$

slope of $\overline{AB} = \frac{2-3}{3-(-2)} = -\frac{1}{5}$ slope of $\overline{CD} = \frac{-1-0}{2-(-3)} = -\frac{1}{5}$

Since opposite sides have the same slope, $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$. Therefore, $ABCD$ is a parallelogram by definition.

Method 2: Use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$AB = \sqrt{(-2-3)^2 + (3-2)^2} = \sqrt{25+1}$ or $\sqrt{26}$

$CD = \sqrt{(2-(-3))^2 + (-1-0)^2} = \sqrt{25+1}$ or $\sqrt{26}$

$AD = \sqrt{(-2-(-3))^2 + (3-0)^2} = \sqrt{1+9}$ or $\sqrt{10}$

$BC = \sqrt{(3-2)^2 + (2-(-1))^2} = \sqrt{1+9}$ or $\sqrt{10}$

Since both pairs of opposite sides have the same length, $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$. Therefore, $ABCD$ is a parallelogram by Theorem 6.9.

Exercises

Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

See students' work

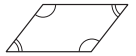
1. $A(0, 0)$, $B(1, 3)$, $C(5, 3)$, $D(4, 0)$; Slope Formula
   yes
2. $D(-1, 1)$, $E(2, 4)$, $F(6, 4)$, $G(3, 1)$; Slope Formula
   yes
3. $R(-1, 0)$, $S(3, 0)$, $T(2, -3)$, $U(-3, -2)$; Distance Formula
   no
4. $A(-3, 2)$, $B(-1, 4)$, $C(2, 1)$, $D(0, -1)$; Distance and Slope Formulas
   yes
5. $S(-2, 4)$, $T(-1, -1)$, $U(3, -4)$, $V(2, 1)$; Distance and Slope Formulas
   yes
6. $F(3, 3)$, $G(1, 2)$, $H(-3, 1)$, $I(-1, 4)$; Midpoint Formula
   no
7. A parallelogram has vertices $R(-2, -1)$, $S(2, 1)$, and $T(0, -3)$. Find all possible coordinates for the fourth vertex.
   (4, −1), (0, 3), or (−4, −5)

NAME _____ DATE _____ PERIOD _____

6-3 Skills Practice

Tests for Parallelograms

Determine whether each quadrilateral is a parallelogram. Justify your answer.

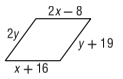
1. 
   Yes; a pair of opposite sides is parallel and congruent.
2. 
   Yes; both pairs of opposite angles are congruent.
3. 
   No; none of the tests for parallelograms is fulfilled.
4. 
   Yes; both pairs of opposite sides are congruent.

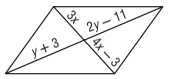
COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

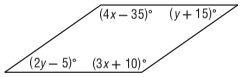
See students' graphs.

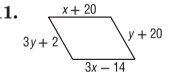
5. $P(0, 0)$, $Q(3, 4)$, $S(7, 4)$, $Y(4, 0)$; Slope Formula
   Yes; the slope of $\overline{PY}$ and $\overline{QS}$ are equal and the slope of $\overline{PQ}$ and $\overline{YS}$ are equal, so $\overline{PY} \parallel \overline{QS}$ and $\overline{PQ} \parallel \overline{YS}$. Opposite sides are parallel.
6. $S(-2, 1)$, $R(1, 3)$, $T(2, 0)$, $Z(-1, -2)$; Distance and Slope Formulas
   Yes; $SR = ZT$ and the slopes of $\overline{SR}$ and $\overline{ZT}$ are equal, so one pair of opposite sides is parallel and cogruent.
7. $W(2, 5)$, $R(3, 3)$, $Y(-2, -3)$, $N(-3, 1)$; Midpoint Formula
   No; the midpoints of the diagonals are not the same point.

ALGEBRA Find x and y so that each quadrilateral is a parallelogram.

8. 

   $x = 24, y = 19$
9. 

   $x = 3, y = 14$
10. 

    $x = 45, y = 20$
11. 

    $x = 17, y = 9$

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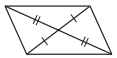
6-3 Practice

Tests for Parallelograms

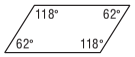
Determine whether each quadrilateral is a parallelogram. Justify your answer.

1. 

Yes; the diagonals bisect each other.

2. 

No; none of the tests for parallelograms is fulfilled.

3. 

Yes; both pairs of opposite angles are congruent.

4. 

No; none of the tests for parallelograms is fulfilled.

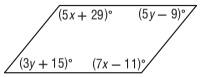
COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

See students' work

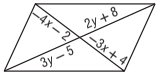
5. $P(-5, 1)$, $S(-2, 2)$, $F(-1, -3)$, $T(2, -2)$; Slope Formula
yes

6. $R(-2, 5)$, $O(1, 3)$, $M(-3, -4)$, $Y(-6, -2)$; Distance and Slope Formulas
yes

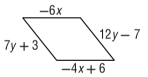
ALGEBRA Find x and y so that the quadrilateral is a parallelogram.

7. 

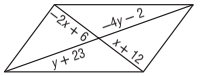
$x = 20$, $y = 12$

8. 

$x = -6$, $y = 13$

9. 

$x = -3$, $y = 2$

10. 

$x = -2$, $y = -5$

11. **TILE DESIGN** The pattern shown in the figure is to consist of congruent parallelograms. How can the designer be certain that the shapes are parallelograms?

Sample answer: Confirm that both pairs of opposite ∡ are ≅.

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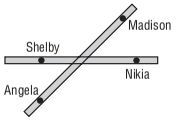
6-3 Word Problem Practice

Tests for Parallelograms

1. **BALANCING** Nikia, Madison, Angela, and Shelby are balancing themselves on an "X"-shaped floating object. To balance themselves, they want to make themselves the vertices of a parallelogram.



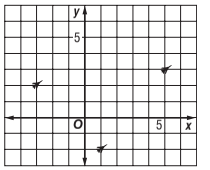
In order to achieve this, do all four of them have to be the same distance from the center of the object? Explain.

No, Madison and Angela have to be the same distance and Nikia and Shelby have to be the same distance but Nikia and Shelby's distance from the center does not have to be equal to Madison and Angela's distance.

2. **COMPASSES** Two compass needles placed side by side on a table are both 2 inches long and point due north. Do they form the sides of a parallelogram?

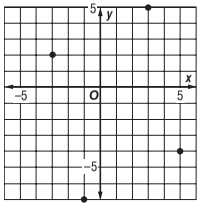
yes

3. **FORMATION** Four jets are flying in formation. Three of the jets are shown in the graph. If the four jets are located at the vertices of a parallelogram, what are the three possible locations of the missing jet?



(1, 7), (9, −1), (−7, −3)

4. **STREET LAMPS** When a coordinate plane is placed over the Harrisville town map, the four street lamps in the center are located as shown. Do the four lamps form the vertices of a parallelogram? Explain.



Yes; the lengths of the opposite sides are congruent.

5. **PICTURE FRAME** Aaron is making a wooden picture frame in the shape of a parallelogram. He has two pieces of wood that are 3 feet long and two that are 4 feet long.

   a. If he connects the pieces of wood at their ends to each other, in what order must he connect them to make a parallelogram?
   He must alternate the lengths 3, 4, 3, 4 or 4, 3, 4, 3.

   b. How many different parallelograms could he make with these four lengths of wood?
   infinitely many

   c. Explain something Aaron might do to specify precisely the shape of the parallelogram.
   Sample answer: He could specify the length of a diagonal.

Answers

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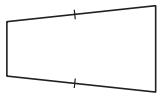
6-3 Enrichment

Tests for Parallelograms

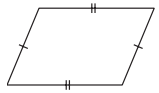
By definition, a quadrilateral is a parallelogram *if and only if* both pairs of opposite sides are parallel. What conditions other than both pairs of opposite sides parallel will guarantee that a quadrilateral is a parallelogram? In this activity, several possibilities will be investigated by drawing quadrilaterals to satisfy certain conditions. Remember that any test that seems to work is not guaranteed to work unless it can be formally proven.

Complete.

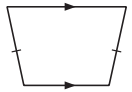
1. Draw a quadrilateral with one pair of opposite sides congruent. Must it be a parallelogram? **no**



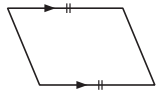
2. Draw a quadrilateral with both pairs of opposite sides congruent. Must it be a parallelogram? **yes**



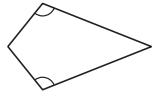
3. Draw a quadrilateral with one pair of opposite sides parallel and the other pair of opposite sides congruent. Must it be a parallelogram? **no**



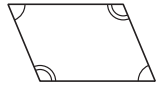
4. Draw a quadrilateral with one pair of opposite sides both parallel and congruent. Must it be a parallelogram? **yes**



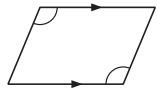
5. Draw a quadrilateral with one pair of opposite angles congruent. Must it be a parallelogram? **no**



6. Draw a quadrilateral with both pairs of opposite angles congruent. Must it be a parallelogram? **yes**



7. Draw a quadrilateral with one pair of opposite sides parallel and one pair of opposite angles congruent. Must it be a parallelogram? **yes**



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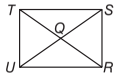
6-4 Study Guide and Intervention

Rectangles

Properties of Rectangles A **rectangle** is a quadrilateral with four right angles. Here are the properties of rectangles.



A rectangle has all the properties of a parallelogram.

- Opposite sides are parallel.
- Opposite angles are congruent.
- Opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Also:

- All four angles are right angles. $\angle UTS$, $\angle TSR$, $\angle SRU$, and $\angle RUT$ are right angles.
- The diagonals are congruent. $\overline{TR} \cong \overline{US}$

Example 1 **Quadrilateral *RUTS* above is a rectangle. If $US = 6x + 3$ and $RT = 7x - 2$, find x.**

The diagonals of a rectangle are congruent, so $US = RT$.

$6x + 3 = 7x - 2$

$3 = x - 2$

$5 = x$

Example 2 **Quadrilateral *RUTS* above is a rectangle. If $m\angle STR = 8x + 3$ and $m\angle UTR = 16x - 9$, find $m\angle STR$.**

$\angle UTS$ is a right angle, so $m\angle STR + m\angle UTR = 90$.

$8x + 3 + 16x - 9 = 90$

$24x - 6 = 90$

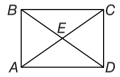
$24x = 96$

$x = 4$

$m\angle STR = 8x + 3 = 8(4) + 3$ or 35

Exercises

Quadrilateral *ABCD* is a rectangle.



1. If $AE = 36$ and $CE = 2x - 4$, find x. **20**
2. If $BE = 6y + 2$ and $CE = 4y + 6$, find y. **2**
3. If $BC = 24$ and $AD = 5y - 1$, find y. **5**
4. If $m\angle BEA = 62$, find $m\angle BAC$. **59**
5. If $m\angle AED = 12x$ and $m\angle BEC = 10x + 20$, find $m\angle AED$. **120**
6. If $BD = 8y - 4$ and $AC = 7y + 3$, find BD. **52**
7. If $m\angle DBC = 10x$ and $m\angle ACB = 4x^2 - 6$, find $m\angle ACB$. **30**
8. If $AB = 6y$ and $BC = 8y$, find BD in terms of y. **10y**

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6-4 Study Guide and Intervention *(continued)*

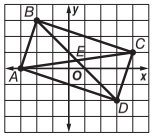
Rectangles

Prove that Parallelograms Are Rectangles The diagonals of a rectangle are congruent, and the converse is also true.

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

In the coordinate plane you can use the Distance Formula, the Slope Formula, and properties of diagonals to show that a figure is a rectangle.

Example **Quadrilateral *ABCD* has vertices *A*(−3, 0), *B*(−2, 3), *C*(4, 1), and *D*(3, −2). Determine whether *ABCD* is a rectangle.**



Method 1: Use the Slope Formula.

slope of $\overline{AB} = \frac{3-0}{-2-(-3)} = \frac{3}{1}$ or 3 slope of $\overline{AD} = \frac{-2-0}{3-(-3)} = \frac{-2}{6}$ or $-\frac{1}{3}$

slope of $\overline{CD} = \frac{-2-1}{3-4} = \frac{-3}{-1}$ or 3 slope of $\overline{BC} = \frac{1-3}{4-(-2)} = \frac{-2}{6}$ or $-\frac{1}{3}$

Opposite sides are parallel, so the figure is a parallelogram. Consecutive sides are perpendicular, so *ABCD* is a rectangle.

Method 2: Use the Distance Formula.

$AB = \sqrt{(-3-(-2))^2 + (0-3)^2}$ or $\sqrt{10}$ $BC = \sqrt{(-2-4)^2 + (3-1)^2}$ or $\sqrt{40}$

$CD = \sqrt{(4-3)^2 + (1-(-2))^2}$ or $\sqrt{10}$ $AD = \sqrt{(-3-3)^2 + (0-(-2))^2}$ or $\sqrt{40}$

Opposite sides are congruent, thus *ABCD* is a parallelogram.

$AC = \sqrt{(-3-4)^2 + (0-1)^2}$ or $\sqrt{50}$ $BD = \sqrt{(-2-3)^2 + (3-(-2))^2}$ or $\sqrt{50}$

ABCD is a parallelogram with congruent diagonals, so *ABCD* is a rectangle.

Exercises

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

See students' work

1. *A*(−3, 1), *B*(−3, 3), *C*(3, 3), *D*(3, 1); Distance Formula
   Yes; *AB* = 2, *BC* = 6, *CD* = 2, *DA* = 6, *AC* = $\sqrt{40}$, *BD* = $\sqrt{40}$, opposite sides and diagonals are congruent.
2. *A*(−3, 0), *B*(−2, 3), *C*(4, 5), *D*(3, 2); Slope Formula
   No; slope of $\overline{AB}$ = 3, slope of $\overline{BC} = \frac{1}{3}$, slopes show that two consecutive sides are not perpendicular.
3. *A*(−3, 0), *B*(−2, 2), *C*(3, 0), *D*(2 −2); Distance Formula
   No; *AC* = 6, *BD* = $\sqrt{32}$, diagonals are not congruent.
4. *A*(−1, 0), *B*(0, 2), *C*(4, 0), *D*(3, −2); Distance Formula **Yes; *AB* = $\sqrt{5}$, *BC* = $\sqrt{20}$, *CD* = $\sqrt{5}$, *DA* = $\sqrt{20}$, *AC* = 5, *BD* = 5, opposite sides and diagonals are congruent.**

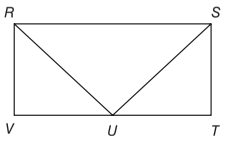
NAME _____ DATE _____ PERIOD _____

6-4 Skills Practice

Rectangles

ALGEBRA Quadrilateral *ABCD* is a rectangle.

1. If *AC* = 2*x* + 13 and *DB* = 4*x* − 1, find *DB*. **27**
2. If *AC* = *x* + 3 and *DB* = 3*x* − 19, find *AC*. **14**
3. If *AE* = 3*x* + 3 and *EC* = 5*x* − 15, find *AC*. **60**
4. If *DE* = 6*x* − 7 and *AE* = 4*x* + 9, find *DB*. **82**
5. If $m\angle DAC = 2x + 4$ and $m\angle BAC = 3x + 1$, find $m\angle BAC$. **52**
6. If $m\angle BDC = 7x + 1$ and $m\angle ADB = 9x - 7$, find $m\angle BDC$. **43**
7. If $m\angle ABD = 7x - 31$ and $m\angle CDB = 4x + 5$, find $m\angle ABD$. **53**
8. If $m\angle BAC = x + 3$ and $m\angle CAD = x + 15$, find $m\angle BAC$. **39**
9. **PROOF:** Write a two-column proof.



Given: *RSTV* is a rectangle and *U* is the midpoint of $\overline{VT}$.
Prove: $\triangle RUV \cong \triangle SUT$
Proof

Statements	Reasons
1. *RSTV* is a rectangle.	1. Given
2. $\angle V$ and $\angle T$ are right angles.	2. Definition of rectangle
3. $\angle V \cong \angle T$	3. All rt ∡ are ≅.
4. *U* is the midpoint of $\overline{VT}$.	4. Given
5. $\overline{VU} \cong \overline{TU}$	5. Definition of midpoint
6. $\overline{VR} \cong \overline{TS}$	6. Opp sides of ▱ are congruent.
7. $\triangle RUV \cong \triangle SUT$	7. SAS

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

See students' graphs.

10. *P*(−3, −2), *Q*(−4, 2), *R*(2, 4), *S*(3, 0); Slope Formula
    No; Sample answer: Angles are not right angles.
11. *J*(−6, 3), *K*(0, 6), *L*(2, 2), *M*(−4, −1); Distance Formula
    Yes; Sample answer: Both pairs of opposite sides are congruent and diagonals are congruent.
12. *T*(4, 1), *U*(3, −1), *X*(−3, 2), *Y*(−2, 4); Distance Formula
    Yes; Sample answer: Both pairs of opposite sides are congruent and the diagonals are congruent.

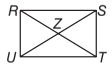
Answers

NAME _____ DATE _____ PERIOD _____

6-4 Practice

Rectangles

ALGEBRA Quadrilateral *RSTU* is a rectangle.



1. If $UZ = x + 21$ and $ZS = 3x - 15$, find US. **78**

2. If $RZ = 3x + 8$ and $ZS = 6x - 28$, find UZ. **44**

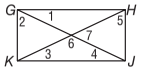
3. If $RT = 5x + 8$ and $RZ = 4x + 1$, find ZT. **9**

4. If $m\angle SUT = 3x + 6$ and $m\angle RUS = 5x - 4$, find $m\angle SUT$. **39**

5. If $m\angle SRT = x + 9$ and $m\angle UTR = 2x - 44$, find $m\angle UTR$. **62**

6. If $m\angle RSU = x + 41$ and $m\angle TUS = 3x + 9$, find $m\angle RSU$. **57**

Quadrilateral *GHJK* is a rectangle. Find each measure if $m\angle 1 = 37$.



7. $m\angle 2$ **53**

8. $m\angle 3$ **37**

9. $m\angle 4$ **37**

10. $m\angle 5$ **53**

11. $m\angle 6$ **106**

12. $m\angle 7$ **74**

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

See students' work

13. $B(-4, 3)$, $G(-2, 4)$, $H(1, -2)$, $L(-1, -3)$; Slope Formula

Yes; sample answer: Opposite sides are parallel and consecutive sides are perpendicular.

14. $N(-4, 5)$, $O(6, 0)$, $P(3, -6)$, $Q(-7, -1)$; Distance Formula

Yes; sample answer: Opposite sides are congruent and diagonals are congruent.

15. $C(0, 5)$, $D(4, 7)$, $E(5, 4)$, $F(1, 2)$; Slope Formula

No; sample answer: Diagonals are not congruent.

16. LANDSCAPING Huntington Park officials approved a rectangular plot of land for a Japanese Zen garden. Is it sufficient to know that opposite sides of the garden plot are congruent and parallel to determine that the garden plot is rectangular? Explain.

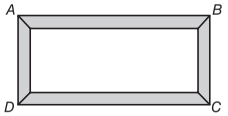
No; if you only know that opposite sides are congruent and parallel, the most you can conclude is that the plot is a parallelogram.

NAME _____ DATE _____ PERIOD _____

6-4 Word Problem Practice

Rectangles

1. FRAMES Jalen makes the rectangular frame shown.



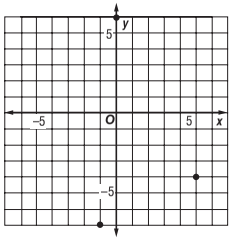
In order to make sure that it is a rectangle, Jalen measures the distances BD and AC. How should these two distances compare if the frame is a rectangle?

They should be equal.

2. BOOKSHELVES A bookshelf consists of two vertical planks with five horizontal shelves. Are each of the four sections for books rectangles? Explain.

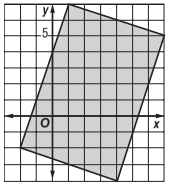
Yes; each of the four angles of each rectangle is created by a straight horizontal line and a straight vertical line, so each has four 90° angles.

3. LANDSCAPING A landscaper is marking off the corners of a rectangular plot of land. Three of the corners are in place as shown.



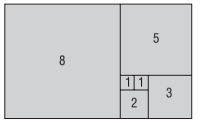
What are the coordinates of the fourth corner? **(−6, 3)**

4. SWIMMING POOLS Antonio is designing a swimming pool on a coordinate grid. Is it a rectangle? Explain.



Yes. Sample answer: The slope of the long sides is 3 and the slope of the short sides is $-\frac{1}{3}$, so each pair of sides is parallel. Also, the long and short sides have slopes that are perpendicular to each other, making the four corners right angles.

5. PATTERNS Veronica made the pattern shown out of 7 rectangles with four equal sides. The side length of each rectangle is written inside the rectangle.



a. How many rectangles can be formed using the lines in this figure? **11**

b. If Veronica wanted to extend her pattern by adding another rectangle with 4 equal sides to make a larger rectangle, what are the possible side lengths of rectangles that she can add?

8 and 13 units

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6-4 Enrichment

Constant Perimeter

Douglas wants to fence a rectangular region of his back yard for his dog. He bought 200 feet of fence.

1. Complete the table to show the dimensions of five different rectangular pens that would use the entire 200 feet of fence. Then find the area of each rectangular pen.

Perimeter	Length	Width	Area
200	80	20	1600
200	70	30	2100
200	60	40	2400
200	50	50	2500
200	45	55	2475

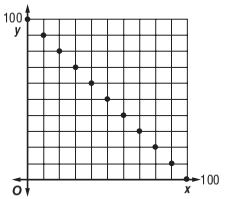
2. Do all five of the rectangular pens have the same area? If not, which one has the larger area?

No, the 50 × 50 pen has the largest area.

3. Write a rule for finding the dimensions of a rectangle with the largest possible area for a given perimeter.

The rectangle with the largest area for a given perimeter is a square.

4. Let x represent the length of a rectangle and y the width. Write the formula for all rectangles with a perimeter of 200. Then graph this relationship on the coordinate plane at the right.



$2x + 2y = 200$ or $x + y = 100$

Julio read that a dog the size of his new pet, Bennie, should have at least 100 square feet in his pen. Before going to the store to buy fence, Julio made a table to determine the dimensions for Bennie's rectangular pen.

5. Complete the table to find five possible dimensions of a rectangular fenced area of 100 square feet.

See table for sample answers.

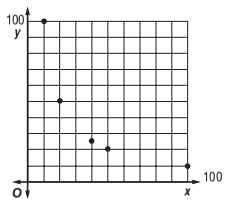
Area	Length	Width	How much fence to buy
100	1	100	202
100	2	50	104
100	4	25	58
100	5	20	50
100	10	10	40

6. Julio wants to save money by purchasing the least number feet of fencing to enclose the 100 square feet. What will be the dimensions of the completed pen?

10 ft × 10 ft

7. Write a rule for finding the dimensions of a rectangle with the least possible perimeter for a given area.

The rectangle with the least possible perimeter for a given area is a square.



8. For length x and width y, write a formula for the area of a rectangle with an area of 100 square feet. Then graph the formula.

$xy = 100$

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6-4 Graphing Calculator Activity

TI-Nspire: Exploring Rectangles

A quadrilateral with four right angles is a **rectangle.** The TI-Nspire can be used to explore some of the characteristics of a rectangle. Use the following steps to draw a rectangle.

Step 1 Set up the calculator in the correct mode.
- Choose **Graphs & Geometry** from the Home Menu.
- From the View menu, choose **4: Hide Axis**

Step 2 Draw the rectangle
- From the **8: Shapes** menu choose **3: Rectangle.**
- Click once to define the corner of the rectangle. Then move and click again. The side of the rectangle is now defined. Move perpendicularly to draw the rectangle. Click to anchor the shape.

Step 3 Measure the lengths of the sides of the rectangle.
- From the **7: Measurement** menu choose **1: Length** (Note that when you scroll over the rectangle, the value now shown is the perimeter of the rectangle.)
- Select each endpoint of a segment of the rectangle. Then click or press Enter to anchor the length of the segment in the work area.
- Repeat for the other sides of the rectangle.

Exercises

1. What appears to be true about the opposite sides of the rectangle?

The opposite sides of a rectangle are congruent.

2. Draw the diagonals of the rectangle using **5: Segment** from the **6: Points and Lines** Menu. Click on two opposite vertices to draw the diagonal. Repeat to draw the other diagonal.
   - **a.** Measure each diagonal using the measurement tool. What do you observe?
   - **b.** What is true about the triangles formed by the sides of the rectangle and a diagonal? Justify your conclusion.

a. The diagonals of a rectangle are congruent.

b. The triangles are congruent by SSS.

3. Press **Clear** three times and select **Yes** to clear the screen. Repeat the steps and draw another rectangle. Do the relationships that you found for the first rectangle you drew hold true for this rectangle?

Yes, the opposite sides are congruent and the diagonals are congruent.

Answers

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6-4 Geometer's Sketchpad Activity

Exploring Rectangles

A quadrilateral with four right angles is a **rectangle**. The Geometer's Sketchpad is a useful tool for exploring some of the characteristics of a rectangle. Use the following steps to draw a rectangle.

Step 1 Use the Line tool to draw a line anywhere on the screen.

Step 2 Use the Point tool to draw a point that is not on the line. To draw a line perpendicular to the first line you drew, select the first line and the point. Then choose **Perpendicular Line** from the **Construct** menu.

Step 3 Use the Point tool to draw a point that is not on either of the lines you have drawn. Repeat the procedure in Step 2 to draw lines perpendicular to the two lines you have drawn.

A rectangle is formed by the segments whose endpoints are the points of intersection of the lines.

Exercises

Use the measuring capabilities of The Geometer's Sketchpad to explore the characteristics of a rectangle.

1. What appears to be true about the opposite sides of the rectangle that you drew? Make a conjecture and then measure each side to check your conjecture.
   The opposite sides of a rectangle are congruent.

2. Draw the diagonals of the rectangle by using the Selection Arrow tool to choose two opposite vertices. Then choose **Segment** from the **Construct** menu to draw the diagonal. Repeat to draw the other diagonal.
   a. Measure each diagonal. What do you observe?
      The diagonals of a rectangle are congruent.
   b. What is true about the triangles formed by the sides of the rectangle and a diagonal? Justify your conclusion.
      The triangles are congruent by SSS.

3. Choose **New Sketch** from the **File** menu and follow steps 1–3 to draw another rectangle. Do the relationships you found for the first rectangle you drew hold true for this rectangle also?
   Yes, the opposite sides are congruent and the diagonals are congruent.

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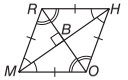
6-5 Study Guide and Intervention

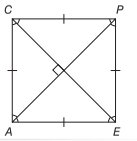
Rhombi and Squares

Properties of Rhombi and Squares A **rhombus** is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.

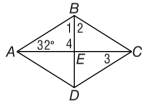


The diagonals are perpendicular.	$\overline{MH} \perp \overline{RO}$
Each diagonal bisects a pair of opposite angles.	$\overline{MH}$ bisects $\angle RMO$ and $\angle RHO$. $\overline{RO}$ bisects $\angle MRH$ and $\angle MOH$.

A **square** is a parallelogram with four congruent sides and four congruent angles. A square is both a rectangle and a rhombus; therefore, all properties of parallelograms, rectangles, and rhombi apply to squares.



Example **In rhombus *ABCD*, $m\angle BAC = 32$. Find the measure of each numbered angle.**

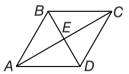


ABCD is a rhombus, so the diagonals are perpendicular and $\triangle ABE$ is a right triangle. Thus $m\angle 4 = 90$ and $m\angle 1 = 90 - 32$ or 58. The diagonals in a rhombus bisect the vertex angles, so $m\angle 1 = m\angle 2$. Thus, $m\angle 2 = 58$.

A rhombus is a parallelogram, so the opposite sides are parallel. $\angle BAC$ and $\angle 3$ are alternate interior angles for parallel lines, so $m\angle 3 = 32$.

Exercises

Quadrilateral *ABCD* is a rhombus. Find each value or measure.



1. If $m\angle ABD = 60$, find $m\angle BDC$. **60**
2. If $AE = 8$, find AC. **16**
3. If $AB = 26$ and $BD = 20$, find AE. **24**
4. Find $m\angle CEB$. **90**
5. If $m\angle CBD = 58$, find $m\angle ACB$. **32**
6. If $AE = 3x - 1$ and $AC = 16$, find x. **3**
7. If $m\angle CDB = 6y$ and $m\angle ACB = 2y + 10$, find y. **10**
8. If $AD = 2x + 4$ and $CD = 4x - 4$, find x. **4**

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6-5 Study Guide and Intervention *(continued)*

Rhombi and Squares

Conditions for Rhombi and Squares The theorems below can help you prove that a parallelogram is a rectangle, rhombus, or square.

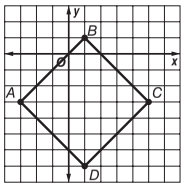
If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.

If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.

If a quadrilateral is both a rectangle and a rhombus, then it is a square.

Example **Determine whether parallelogram *ABCD* with vertices *A*(−3, −3), *B*(1, 1), *C*(5, −3), *D*(1, −7) is a *rhombus*, a *rectangle*, or a *square*.**



$AC = \sqrt{(-3-5)^2 + ((-3-(-3))^2} = \sqrt{64} = 8$

$BD = \sqrt{(1-1)^2 + (-7-1)^2} = \sqrt{64} = 8$

The diagonals are the same length; the figure is a rectangle.

Slope of $\overline{AC} = \frac{-3-(-3)}{-3-5} = \frac{0}{-8} = 8$ The line is horizontal.

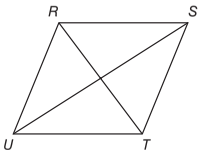
Slope of $\overline{BD} = \frac{1-(-7)}{1-1} = \frac{8}{0} = \textit{undefined}$ The line is vertical.

Since a horizontal and vertical line are perpendicular, the diagonals are perpendicular. Parallelogram *ABCD* is a square which is also a rhombus and a rectangle.

Exercises

Given each set of vertices, determine whether $\square ABCD$ is a *rhombus*, *rectangle*, or *square*. List all that apply. Explain.

1. $A(0, 2)$, $B(2, 4)$, $C(4, 2)$, $D(2, 0)$
   Rectangle, rhombus, square; the four sides are ≅ and consecutive sides are ⊥.
2. $A(-2, 1)$, $B(-1, 3)$, $C(3, 1)$, $D(2, -1)$
   Rectangle; consecutive sides are ⊥.
3. $A(-2, -1)$, $B(0, 2)$, $C(2, -1)$, $D(0, -4)$
   Rhombus; the four sides are ≅ and consecutive sides are not ⊥.
4. $A(-3, 0)$, $B(-1, 3)$, $C(5, -1)$, $D(3, -4)$
   Rectangle; consecutive sides are ⊥.
5. **PROOF** Write a two-column proof.
   Given: Parallelogram *RSTU*. $\overline{RS} \cong \overline{ST}$
   Prove: *RSTU* is a rhombus.



Statements	Reasons
1. *RSTU* is a parallelogram $\overline{RS} \cong \overline{ST}$	1. Given
2. $\overline{RS} \cong \overline{UT}$, $\overline{RU} \cong \overline{ST}$	2. Definition of a parallelogram
3. $\overline{UT} \cong \overline{RS} \cong \overline{ST} \cong \overline{RU}$	3. Substitution
4. *RSTU* is a rhombus	4. Definition of a rhombus

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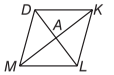
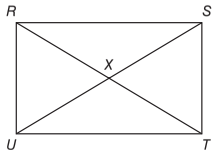
6-5 Skills Practice

Rhombi and Squares

ALGEBRA Quadrilateral *DKLM* is a rhombus.



1. If $DK = 8$, find KL. **8**
2. If $m\angle DML = 82$ find $m\angle DKM$. **41**
3. If $m\angle KAL = 2x - 8$, find x. **49**
4. If $DA = 4x$ and $AL = 5x - 3$, find DL. **24**
5. If $DA = 4x$ and $AL = 5x - 3$, find AD. **12**
6. If $DM = 5y + 2$ and $DK = 3y + 6$, find KL. **12**
7. **PROOF** Write a two-column proof.
   Given: *RSTU* is a parallelogram. $\overline{RX} \cong \overline{TX} \cong \overline{SX} \cong \overline{UX}$
   Prove: *RSTU* is a rectangle.
   Proof



Statements	Reasons
1. *RSTU* is a parallelogram. $\overline{RX} \cong \overline{TX} \cong \overline{SX} \cong \overline{UX}$	1. Given
2. $RX = TX = SX = UX$	2. Definition of congruent segments
3. $RX + XT = RT$, $SX + XU = SU$	3. Seg Add Postulate
4. $RX + XT = SU$	4. Substitution Property
5. $RT = SU$	5. Transitive Property
6. $\overline{RT} \cong \overline{SU}$	6. Definition of ≅ segment
7. *RSTU* is a rectangle.	7. If diagonals ≅, the figure is a rectangle.

COORDINATE GEOMETRY Given each set of vertices, determine whether $\square QRST$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.

8. $Q(3, 5)$, $R(3, 1)$, $S(-1, 1)$, $T(-1, 5)$
   Rhombus, rectangle, square; all sides are congruent and the diagonals are perpendicular and congruent.
9. $Q(-5, 12)$, $R(5, 12)$, $S(-1, 4)$, $T(-11, 4)$
   Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.
10. $Q(-6, -1)$, $R(4, -6)$, $S(2, 5)$, $T(-8, 10)$
    Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.
11. $Q(2, -4)$, $R(-6, -8)$, $S(-10, 2)$, $T(-2, 6)$
    None; opposite sides are congruent, but the diagonals are neither congruent nor perpendicular.

Answers

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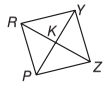
6-5 Practice

Rhombi and Squares

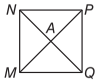
***PRYZ* is a rhombus. If $RK = 5$, $RY = 13$ and $m\angle YRZ = 67$, find each measure.**

1. KY **12**
2. PK **12**
3. $m\angle YKZ$ **90**
4. $m\angle PZR$ **67**



***MNPQ* is a rhombus. If $PQ = 3\sqrt{2}$ and $AP = 3$, find each measure.**

5. AQ **3**
6. $m\angle APQ$ **45**
7. $m\angle MNP$ **90**
8. PM **6**



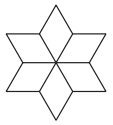
COORDINATE GEOMETRY Given each set of vertices, determine whether $\square BEFG$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.

9. $B(-9, 1)$, $E(2, 3)$, $F(12, -2)$, $G(1, -4)$
Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.

10. $B(1, 3)$, $E(7, -3)$, $F(1, -9)$, $G(-5, -3)$
Rhombus, rectangle, square; all sides are congruent and the diagonals are perpendicular and congruent.

11. $B(-4, -5)$, $E(1, -5)$, $F(-2, -1)$, $G(-7, -1)$
Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.

12. **TESSELLATIONS** The figure is an example of a tessellation. Use a ruler or protractor to measure the shapes and then name the quadrilaterals used to form the figure.
The figure consists of 6 congruent rhombi.



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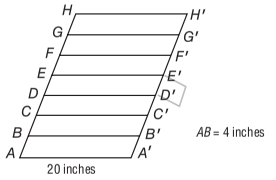
6-5 Word Problem Practice

Rhombi and Squares

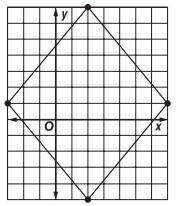
1. **TRAY RACKS** A tray rack looks like a parallelogram from the side. The levels for the trays are evenly spaced.



What two labeled points form a rhombus with base AA'?
F* and *F′

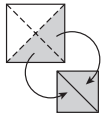
2. **SLICING** Charles cuts a rhombus along both diagonals. He ends up with four congruent triangles. Classify these triangles as *acute, obtuse,* or *right*.
right triangles

3. **WINDOWS** The edges of a window are drawn in the coordinate plane.



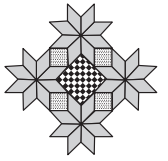
Determine whether the window is a square or a rhombus.
rhombus

4. **SQUARES** Mackenzie cut a square along its diagonals to get four congruent right triangles. She then joined two of them along their long sides. Show that the resulting shape is a square.



Sample answer: The diagonals of a square create four congruent right isosceles triangles. When two congruent right isosceles triangles are joined along their hypotenuses, the result is a quadrilateral with 4 equal sides and 4 right angle corners (because 45 + 45 = 90), making it a square.

5. **DESIGN** Tatianna made the design shown. She used 32 congruent rhombi to create the flower-like design at each corner.



a. What are the angles of the corner rhombi? **45, 135, 45, 135**

b. What kinds of quadrilaterals are the dotted and checkered figures?
They are all squares.

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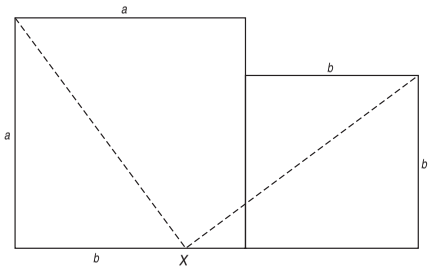
6-5 Enrichment

Creating Pythagorean Puzzles

By drawing two squares and cutting them in a certain way, you can make a puzzle that demonstrates the Pythagorean Theorem. A sample puzzle is shown. You can create your own puzzle by following the instructions below.



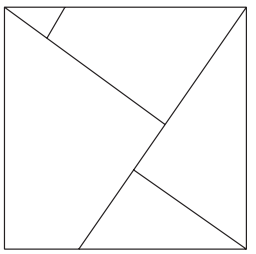
See students' work. A sample answer is shown.

1. Carefully construct a square and label the length of a side as a. Then construct a smaller square to the right of it and label the length of a side as b, as shown in the figure above. The bases should be adjacent and collinear.

2. Mark a point X that is b units from the left edge of the larger square. Then draw the segments from the upper left corner of the larger square to point X, and from point X to the upper right corner of the smaller square.

3. Cut out and rearrange your five pieces to form a larger square. Draw a diagram to show your answer.

4. Verify that the length of each side is equal to $\sqrt{a^2 + b^2}$.

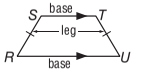


NAME _____ DATE _____ PERIOD _____

6-6 Study Guide and Intervention

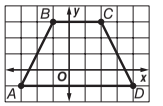
Trapezoids and Kites

Properties of Trapezoids A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The **midsegment** or **median** of a trapezoid is the segment that connects the midpoints of the legs of the trapezoid. Its measure is equal to one-half the sum of the lengths of the bases. If the legs are congruent, the trapezoid is an **isosceles trapezoid**. In an isosceles trapezoid both pairs of **base angles** are congruent and the diagonals are congruent.



STUR is an isosceles trapezoid.
$\overline{SR} \cong \overline{TU}$; $\angle R \cong \angle U$, $\angle S \cong \angle T$

Example **The vertices of *ABCD* are *A*(−3, −1), *B*(−1, 3), *C*(2, 3), and *D*(4, −1). Show that *ABCD* is a trapezoid and determine whether it is an isosceles trapezoid.**



slope of $\overline{AB} = \frac{3-(-1)}{-1-(-3)} = \frac{4}{2} = 2$

slope of $\overline{AD} = \frac{-1-(-1)}{4-(-3)} = \frac{0}{7} = 0$

slope of $\overline{BC} = \frac{3-3}{2-(-1)} = \frac{0}{3} = 0$

slope of $\overline{CD} = \frac{-1-3}{4-2} = \frac{-4}{2} = -2$

$AB = \sqrt{(-3-(-1))^2 + (-1-3)^2}$
$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

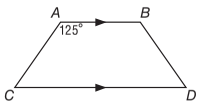
$CD = \sqrt{(2-4)^2 + (3-(-1))^2}$
$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

Exactly two sides are parallel, $\overline{AD}$ and $\overline{BC}$, so *ABCD* is a trapezoid. $AB = CD$, so *ABCD* is an isosceles trapezoid.

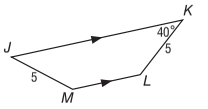
Exercises

Find each measure.

1. $m\angle D$ **55**



2. $m\angle L$ **140**



COORDINATE GEOMETRY For each quadrilateral with the given vertices, verify that the quadrilateral is a trapezoid and determine whether the figure is an isosceles trapezoid.

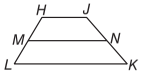
3. *A*(−1, 1), *B*(3, 2), *C*(1,−2), *D*(−2, −1)
$\overline{AD} \parallel \overline{BC}$, $\overline{AB} \nparallel \overline{CD}$, *ABCD* is a trapezoid but not an isosceles trapezoid because $AB = \sqrt{17}$, $CD = \sqrt{10}$.

4. *J*(1, 3), *K*(3, 1), *L*(3, −2), *M*(−2, 3)
$\overline{JK} \parallel \overline{LM}$, $\overline{JM} \nparallel \overline{KL}$, *JKLM* is an isosceles trapezoid because $JM = KL = 3$.

For trapezoid *HJKL*, *M* and *N* are the midpoints of the legs.

5. If $HJ = 32$ and $LK = 60$, find MN. **46**

6. If $HJ = 18$ and $MN = 28$, find LK. **38**



Answers

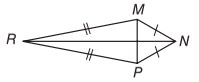
NAME _____ DATE _____ PERIOD _____

6-6 Study Guide and Intervention *(continued)*

Trapezoids and Kites

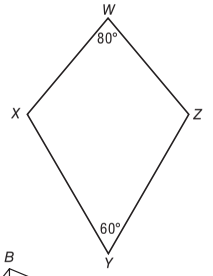
Properties of Kites A **kite** is a quadrilateral with exactly two pairs of consecutive congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.

The diagonals of a kite are perpendicular.
For kite $RMNP$, $\overline{MP} \perp \overline{RN}$



In a kite, exactly one pair of opposite angles is congruent.
For kite $RMNP$, $\angle M \cong \angle P$

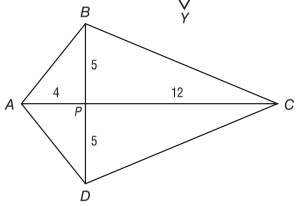
Example 1 **If $WXYZ$ is a kite, find $m\angle Z$.**



The measures of $\angle Y$ and $\angle W$ are not congruent, so $\angle X \cong \angle Z$.
$m\angle X + m\angle Y + m\angle Z + m\angle W = 360$
$m\angle X + 60 + m\angle Z + 80 = 360$
$m\angle X + m\angle Z = 220$
$m\angle X = 110$, $m\angle Z = 110$

Example 2 **If $ABCD$ is a kite, find BC.**

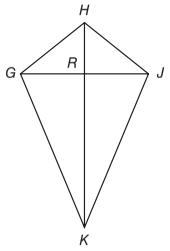
The diagonals of a kite are perpendicular. Use the Pythagorean Theorem to find the missing length.



$BP^2 + PC^2 = BC^2$
$5^2 + 12^2 = BC^2$
$169 = BC^2$
$13 = BC$

Exercises

If $GHJK$ is a kite, find each measure.



1. Find $m\angle JRK$. **90**
2. If $RJ = 3$ and $RK = 10$, find JK. **$\sqrt{109}$**
3. If $m\angle GHJ = 90$ and $m\angle GKJ = 110$, find $m\angle HGK$. **80**
4. If $HJ = 7$, find HG. **7**
5. If $HG = 7$ and $GR = 5$, find HR. **$\sqrt{24} = 2\sqrt{6}$**
6. If $m\angle GHJ = 52$ and $m\angle GKJ = 95$, find $m\angle HGK$. **106.5**

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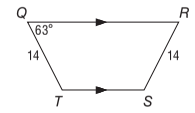
6-6 Skills Practice

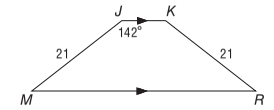
Trapezoids and Kites

ALGEBRA Find each measure.

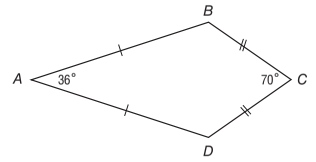
1. $m\angle S$ **117**



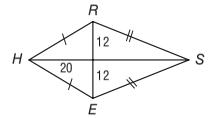
2. $m\angle M$ **38**



3. $m\angle D$ **127**

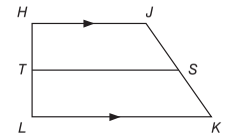


4. RH **$\sqrt{544} = 4\sqrt{34}$**



ALGEBRA For trapezoid $HJKL$, T and S are midpoints of the legs.

5. If $HJ = 14$ and $LK = 42$, find TS. **28**
6. If $LK = 19$ and $TS = 15$, find HJ. **11**
7. If $HJ = 7$ and $TS = 10$, find LK. **13**
8. If $KL = 17$ and $JH = 9$, find ST. **13**



COORDINATE GEOMETRY $EFGH$ is a quadrilateral with vertices $E(1, 3)$, $F(5, 0)$, $G(8, -5)$, $H(-4, 4)$.

9. Verify that $EFGH$ is a trapezoid.
   $\overline{EF} \parallel \overline{GH}$, but $\overline{HE} \nparallel \overline{FG}$
10. Determine whether $EFGH$ is an isosceles trapezoid. Explain.
    not isosceles; $EH = \sqrt{26}$ and $FG = \sqrt{34}$

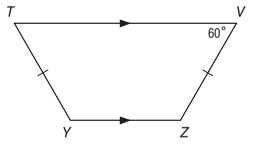
NAME _____ DATE _____ PERIOD _____

6-6 Practice

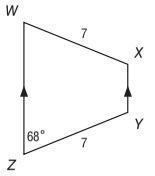
Trapezoids and Kites

Find each measure.

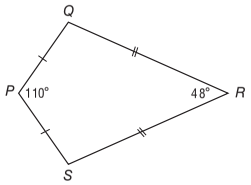
1. $m\angle T$ **60**



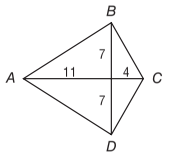
2. $m\angle Y$ **112**



3. $m\angle Q$ **101**



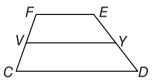
4. BC $\sqrt{65}$



ALGEBRA For trapezoid *FEDC*, *V* and *Y* are midpoints of the legs.

5. If $FE = 18$ and $VY = 28$, find CD. **38**

6. If $m\angle F = 140$ and $m\angle E = 125$, find $m\angle D$. **55**



COORDINATE GEOMETRY *RSTU* is a quadrilateral with vertices *R*(−3, −3), *S*(5, 1), *T*(10, −2), *U*(−4, −9).

7. Verify that *RSTU* is a trapezoid. $\overline{RS} \parallel \overline{TU}$

8. Determine whether *RSTU* is an isosceles trapezoid. Explain.
not isosceles; $RU = \sqrt{37}$ and $ST = \sqrt{34}$

9. **CONSTRUCTION** A set of stairs leading to the entrance of a building is designed in the shape of an isosceles trapezoid with the longer base at the bottom of the stairs and the shorter base at the top. If the bottom of the stairs is 21 feet wide and the top is 14 feet wide, find the width of the stairs halfway to the top. **17.5 ft**

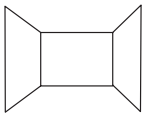
10. **DESK TOPS** A carpenter needs to replace several trapezoid-shaped desktops in a classroom. The carpenter knows the lengths of both bases of the desktop. What other measurements, if any, does the carpenter need?
Sample answer: the measures of the base angles

NAME _____ DATE _____ PERIOD _____

6-6 Word Problem Practice

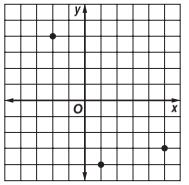
Trapezoids and Kites

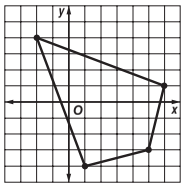
1. **PERSPECTIVE** Artists use different techniques to make things appear to be 3-dimensional when drawing in two dimensions. Kevin drew the walls of a room. In real life, all of the walls are rectangles. In what shape did he draw the side walls to make them appear 3-dimensional?



trapezoids

2. **PLAZA** In order to give the feeling of spaciousness, an architect decides to make a plaza in the shape of a kite. Three of the four corners of the plaza are shown on the coordinate plane. If the fourth corner is in the first quadrant, what are its coordinates?





(6, 1)

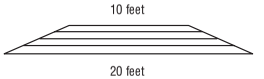
3. **AIRPORTS** A simplified drawing of the reef runway complex at Honolulu International Airport is shown below.

How many trapezoids are there in this image? **5**

4. **LIGHTING** A light outside a room shines through the door and illuminates a trapezoidal region *ABCD* on the floor.

Under what circumstances would trapezoid *ABCD* be isosceles?
When the light source is an equal distance from C and D, shining straight through the door.

5. **RISERS** A riser is designed to elevate a speaker. The riser consists of 4 trapezoidal sections that can be stacked one on top of the other to produce trapezoids of varying heights.



All of the stages have the same height. If all four stages are used, the width of the top of the riser is 10 feet.

a. If only the bottom two stages are used, what is the width of the top of the resulting riser? **15 ft**

b. What would be the width of the riser if the bottom three stages are used? **12.5 ft**

Answers

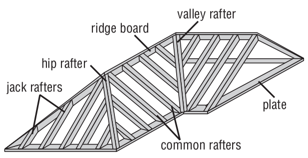
NAME ______________________ DATE ____________ PERIOD ____________

6-6 Enrichment

Quadrilaterals in Construction

Quadrilaterals are often used in construction work.

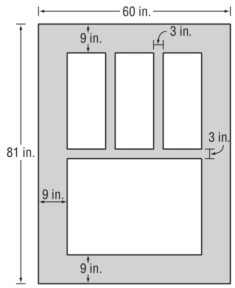
1. The diagram at the right represents a roof frame and shows many quadrilaterals. Find the following shapes in the diagram and shade in their edges. **See students' work.**



Roof Frame

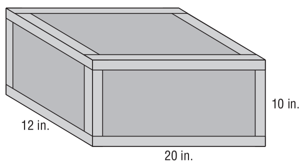
   a. isosceles triangle
   b. scalene triangle
   c. rectangle
   d. rhombus
   e. trapezoid (not isosceles)
   f. isosceles trapezoid

2. The figure at the right represents a window. The wooden part between the panes of glass is 3 inches wide. The frame around the outer edge is 9 inches wide. The outside measurements of the frame are 60 inches by 81 inches. The height of the top and bottom panes is the same. The top three panes are the same size.



   a. How wide is the bottom pane of glass? **42 in.**
   b. How wide is each top pane of glass? **12 in.**
   c. How high is each pane of glass? **30 in.**

3. Each edge of this box has been reinforced with a piece of tape. The box is 10 inches high, 20 inches wide, and 12 inches deep. What is the length of the tape that has been used? **168 in.**



Chapter 6 Assessment Answer Key

Quiz 1 (Lessons 6-1 and 6-2)
Page 45

1. 12,240
2. 45
3. 20
4. (−1, 10)
5. A

Quiz 2 (Lesson 6-3)
Page 45

1. No; none of the tests for ▱s are fulfilled.
2. true
3. false
4. true
5. 28 cm

Quiz 3 (Lessons 6-4 and 6-5)
Page 46

1. B
2. 65
3. 115
4. true
5. rectangle, rhombus, square

Quiz 4 (Lesson 6-6)
Page 46

1. 118
2. $\sqrt{50} = 5\sqrt{2}$
3. 5
4. 15.5
5. Use the distance formula to show $CF = DE$.

Mid-Chapter Test
Page 47

1. B
2. J
3. A
4. G
5. C
6. 50
7. 42
8. yes; opp. sides are ≅ to each other
9. 30, 150
10. No; slope $\overline{XY} = -\frac{3}{5}$ and slope of $\overline{WZ} = -\frac{1}{3}$, so opposite sides are not parallel.

Chapter 6 Assessment Answer Key

Vocabulary Test

Page 48

1. isosceles trapezoid
2. parallelogram
3. trapezoid
4. square
5. rhombus
6. false, rectangle
7. false; rhombus
8. diagonals
9. median
10. Sample answer: angles formed by the base and one of the legs of the trapezoid
11. the nonparallel sides of a trapezoid

Form 1

Page 49

1. B
2. H
3. B
4. H
5. D
6. F
7. B
8. F
9. D
10. F
11. C

Page 50

12. H
13. B
14. J
15. A
16. G
17. A
18. J
19. A
20. G

B: $x = 7$, $m\angle WYZ = 41$

Chapter 6 Assessment Answer Key

Form 2A

Page 51

1. B
2. G
3. D
4. F
5. C
6. F
7. A
8. H
9. B
10. F
11. D

Page 52

12. J
13. C
14. H
15. A
16. G
17. D
18. H
19. D
20. G

B: 7 or −4

Form 2B

Page 53

1. B
2. J
3. D
4. F
5. B
6. G
7. A
8. H
9. C
10. J
11. B

Page 54

12. G
13. D
14. G
15. D
16. H
17. C
18. F
19. A
20. H

B: 22

Chapter 6 Assessment Answer Key

Form 2C

Page 55

1. 1080
2. 19
3. 40
4. 18
5. 8
6. 122
7. (6, 4)
8. Yes; $\overline{AB}$ and $\overline{CD}$ are || and ≅.
9. No; the slopes are $\frac{9}{4}$, $\frac{1}{7}$, 1, and $\frac{2}{3}$. Thus, *ABCD* does not have || sides.
10. $-\frac{2}{3}$
11. 22
12. Yes; if the diagonals of a ▱ are ≅, then the ▱ is a rectangle.
13. 67.5

Page 56

14. (4, 0)
15. 31
16. Yes; *ABCD* has only one pair of opposite sides ||, $\overline{BC}$ and $\overline{AD}$.
17. 6
18. 26
19. true
20. true
21. true
22. false
23. true
24. true
25. false

B: $x = 9, y = 2$

Chapter 6 Assessment Answer Key

Form 2D

Page 57

1. 360
2. 38
3. 90
4. 50
5. 3.6
6. 117
7. $\left(\frac{1}{2}, 3\right)$
8. Yes; Both pairs of opp. sides are $\cong$.
9. *ABCD* has two pairs of || sides, $\overline{AB} \parallel \overline{CD}$ and $\overline{BD} \parallel \overline{DA}$; it is a $\square$.
10. −4
11. 8
12. One rt. $\angle$ means that the other $\measuredangle$s will be rt. $\measuredangle$s. If all 4 $\measuredangle$s are rt. $\measuredangle$s, the $\square$ is a rectangle.

Page 58

13. 72
14. (−3, 1)
15. 16
16. Yes; ABCD has only one pair of opp. sides ||, $\overline{AB}$ and $\overline{BD}$.
17. 8
18. 17
19. false
20. true
21. false
22. false
23. true
24. false
25. true

B: 90

Chapter 6 Assessment Answer Key

Form 3

Page 59

1. 7

2. 30; 30, 47, 120, 179, 174, and 170

3. $\frac{180}{x}$

4. 65

5. 8 or 32

6. Yes; the diagonals bisect each other.

7. slope of $\overline{CD} = \frac{2}{3}$; slope of $\overline{DA} = -2$.

8. 28

9. 72

10. $8\sqrt{2}$

11. 4

12. 9

13. 13

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14. No; two pairs of congruent consecutive sides do not exist. $CD = \sqrt{72}$, $DA = \sqrt{65}$, $AB = \sqrt{20}$, $BC = \sqrt{17}$

15. Yes; $\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{CD}$, $\overline{CD} \perp \overline{AD}$, so opp $\measuredangle$ are $\cong$, and all $\measuredangle$ are rt. $\measuredangle$.

16. *ABCD* has 2 pairs of opp. sides $\cong$, $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, so *ABCD* is a $\square$.

17. 105

18. Opp. sides of a $\square$ are $\cong$.

19. If both pairs of opp. sides of a quad. are $\cong$, then the quad. is a $\square$.

20. 27

B: 54

Chapter 6 Assessment Answer Key

Extended-Response Test, Page 61
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *angles of polygons, properties of parallelograms, rectangles, rhombi, squares, and trapezoids.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Graphs and figures are accurate and appropriate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *angles of polygons, properties of parallelograms, rectangles, rhombi, squares, and trapezoids.* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Graphs and figures are mostly accurate and appropriate. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *angles of polygons, properties of parallelograms, rectangles, rhombi, squares, and trapezoids.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Graphs and figures are mostly accurate. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work shown to substantiate the final computation. • Graphs and figures may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *angles of polygons, properties of parallelograms, rectangles, rhombi, squares, and trapezoids.* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Graphs and figures are inaccurate or inappropriate. • Does not satisfy requirements of problems. • No answer may be given.

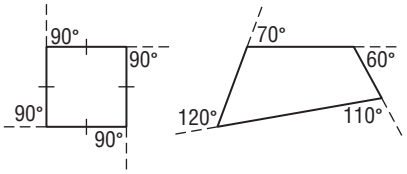
Chapter 6 Assessment Answer Key

Extended-Response Test, Page 61

Sample Answers

In addition to the scoring rubric found on page A30, the following sample answers may be used as guidance in evaluating open-ended assessment items.

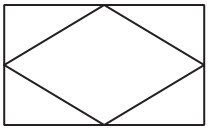
1. a. Any type of convex polygon can be drawn as long as one is regular and one is not regular and both have the same number of sides.



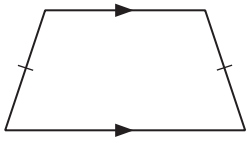
b. Check to be sure that the exterior angles have been properly drawn and accurately measured.

c. $4(90) = 360$; $120 + 70 + 60 + 110 = 360$; The sum of the exterior angles of each figure should be 360°. The sum of the exterior angles of both the regular convex polygon and the irregular convex polygon is 360°.

2. The student should draw a rectangle and join the midpoints of consecutive sides. The figure formed inside is a rhombus. Since all four small triangles can be proved to be congruent by SAS, the four sides of the interior quadrilateral are congruent by CPCTC, making it a rhombus.



3. The student should draw an isosceles trapezoid with one pair of opposite sides parallel and the other pair of opposite sides congruent, as in the figure below.



4. a. Possible properties:
A square has four congruent sides and a rectangle may not.
A square has perpendicular diagonals and a rectangle may not.
The diagonals of a square bisect the angles and those in a rectangle may not.

b. Possible properties:
A square has four right angles and a rhombus may not.
The diagonals of a square are congruent and those of a rhombus may not be.

c. Possible properties:
A rectangle has four right angles and a parallelogram may not.
The diagonals of a rectangle are congruent and those of a parallelogram may not be.

Chapter 6 Assessment Answer Key

Standardized Test Practice

Page 62

1. ● Ⓑ Ⓒ Ⓓ

2. Ⓕ ● Ⓗ Ⓙ

3. Ⓐ Ⓑ Ⓒ ●

4. Ⓕ Ⓖ ● Ⓙ

5. Ⓐ Ⓑ ● Ⓓ

6. ● Ⓖ Ⓗ Ⓙ

7. Ⓐ ● Ⓒ Ⓓ

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8. ● Ⓖ Ⓗ Ⓙ

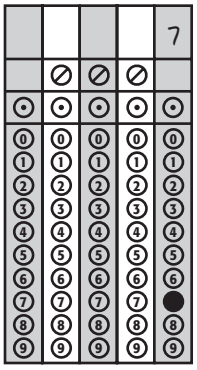
9. Ⓐ Ⓑ Ⓒ ●

10. Ⓕ Ⓖ Ⓗ ●

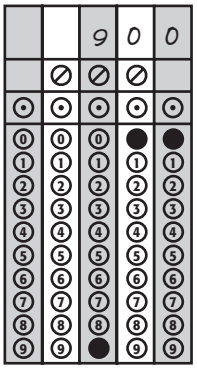
11. Ⓐ Ⓑ ● Ⓓ

12. Ⓕ ● Ⓗ Ⓙ

13.



14.



Answers

Standardized Test Practice *(continued)*

Page 64

15. hexagon; concave; irregular

16. 88.9 cm

17. $\overline{DL}$

18. $x = 11, y = 2$

19. Assume that neither bell cost more than $45.

20. 40 cm^3

21. a. $\overline{ML}$

b. $\overline{JL}$

c. J

Chapter 6 Assessment Answer Key

Unit 2 Test

Page 65

1. right; obtuse; acute

2. $m\angle 1 = 25$; $m\angle 2 = 25$; $m\angle 3 = 130$

3. $\overline{AF} \cong \overline{ST}$, $\overline{FP} \cong \overline{TX}$, $\overline{PA} \cong \overline{XS}$

4. yes

5. ASA Postulate

6. $\angle A \cong \angle C$

7. 111

8. $a = 9$; $m\angle ZWT = 32$

9. $\angle YWZ$

10. Neither appliance cost more than $603.

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11. yes

12. $\frac{7}{2} < x < \frac{31}{2}$

13. 9

14. $m\angle JHK = 52$; $m\angle HMK = 108$, and $x = 8$

15. No; opp. side are not ||.

16. 5

17. 5

18. 11

19. In a parallelogram, opposite sides are congruent. Using the distance formula, $PQ = \sqrt{20}$, $PS = \sqrt{20}$, $QR = \sqrt{8}$, $RS = \sqrt{8}$.