CHAPTER 3

Parallel and Perpendicular Lines

3A Lines with Transversals

3-1 Lines and Angles

Lab Explore Parallel Lines and Transversals

3-2 Angles Formed by Parallel Lines and Transversals

3-3 Proving Lines Parallel

Lab Construct Parallel Lines

3-4 Perpendicular Lines

Lab Construct Perpendicular Lines

CONCEPT CONNECTION

3B Coordinate Geometry

3-5 Slopes of Lines

Lab Explore Parallel and Perpendicular Lines

3-6 Lines in the Coordinate Plane

CONCEPT CONNECTION

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KEYWORD: MG7 ChProj

In the satellite image of the Port of San Diego the piers appear to be parallel.

Port of San Diego
San Diego, CA

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

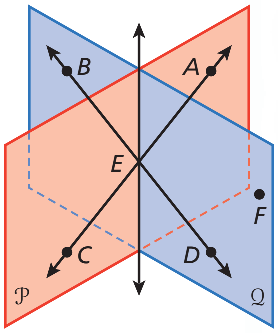
1. acute angle
2. congruent angles
3. obtuse angle
4. collinear
5. congruent segments

A. segments that have the same length
B. an angle that measures greater than 90° and less than 180°
C. points that lie in the same plane
D. angles that have the same measure
E. points that lie on the same line
F. an angle that measures greater than 0° and less than 90°

Conditional Statements

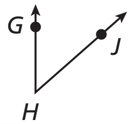
Identify the hypothesis and conclusion of each conditional.

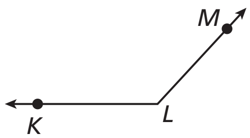
6. If E is on $\overleftrightarrow{AC}$, then E lies in plane $\mathcal{P}$.
7. If A is not in plane $\mathcal{Q}$, then A is not on $\overleftrightarrow{BD}$.
8. If plane $\mathcal{P}$ and plane $\mathcal{Q}$ intersect, then they intersect in a line.

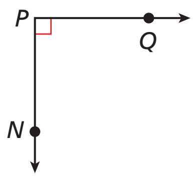


Name and Classify Angles

Name and classify each angle.

9. 

10. 

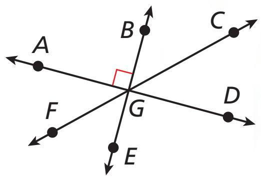
11. 

12. 

Angle Relationships

Give an example of each angle pair.

13. vertical angles
14. adjacent angles
15. complementary angles
16. supplementary angles



Evaluate Expressions

Evaluate each expression for the given value of the variable.

17. $4x + 9$ for $x = 31$
18. $6x - 16$ for $x = 43$
19. $97 - 3x$ for $x = 20$
20. $5x + 3x + 12$ for $x = 17$

Solve Multi-Step Equations

Solve each equation for *x*.

21. $4x + 8 = 24$
22. $2 = 2x - 8$
23. $4x + 3x + 6 = 90$
24. $21x + 13 + 14x - 8 = 180$

CHAPTER 3

Unpacking the Standards

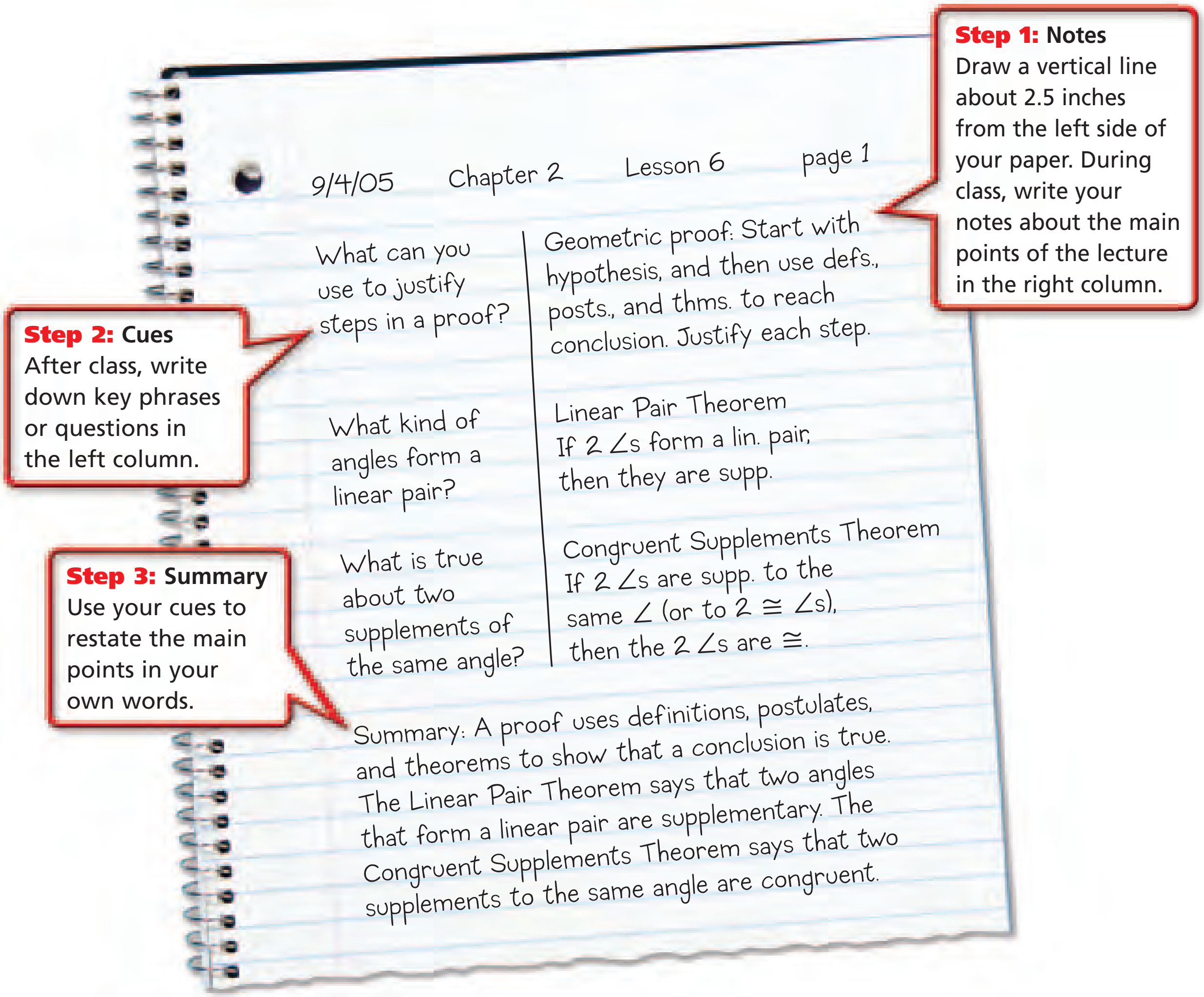
The information below "unpacks" the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

California Standard	Academic Vocabulary	Chapter Concept
1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and **inductive** and deductive **reasoning.** (Lab 3-2)	**demonstrate** show **identifying** seeing and being able to name what something is	You use Geometry software to explore angles that are formed when a transversal intersects a pair of parallel lines. Then you make conjectures about what you think is true.
2.0 Students write geometric proofs, including proofs by contradiction. (Lesson 3-4)	**geometric** relating to the laws and methods of geometry	You use a compass and straightedge to construct the perpendicular bisector of a segment. You also learn theorems so you can prove results that relate to perpendicular lines.
7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles. (Lessons 3-2, 3-3)	**properties** unique features **cut** to go across or through something	You use parallel lines and a transversal to prove that angles they form are congruent and/or supplementary. You use congruent angles to prove that lines are parallel.
16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, **perpendicular bisectors, and the line parallel to a given line through a point off the line.** (Lessons 3-3, 3-4) (Labs 3-3, 3-4)	**basic** most important or fundamental; used as a starting point **bisector(s)** a line that divides an angle or another line into two equal parts	You use a compass and straightedge to construct parallel lines and the perpendicular bisector of a segment. You also learn theorems and their converses so you can apply what you've learned about parallel and perpendicular lines.

Reading and Writing Math

Study Strategy: Take Effective Notes

Taking effective notes is an important study strategy. The Cornell system of note taking is a good way to organize and review main ideas. In the Cornell system, the paper is divided into three main sections. The note-taking column is where you take notes during lecture. The cue column is where you write questions and key phrases as you review your notes. The summary area is where you write a brief summary of the lecture.



Try This

1. Research and write a paragraph describing the Cornell system of note taking. Describe how you can benefit from using this type of system.
2. In your next class, use the Cornell system of note taking. Compare these notes to your notes from a previous lecture.

3-1 Lines and Angles

Objectives

Identify parallel, perpendicular, and skew lines.

Identify the angles formed by two lines and a transversal.

Vocabulary

parallel lines
perpendicular lines
skew lines
parallel planes
transversal
corresponding angles
alternate interior angles
alternate exterior angles
same-side interior angles

Who uses this?

Card architects use playing cards to build structures that contain parallel and perpendicular planes.

Bryan Berg uses cards to build structures like the one at right. In 1992, he broke the Guinness World Record for card structures by building a tower 14 feet 6 inches tall. Since then, he has built structures more than 25 feet tall.

Know it! Note

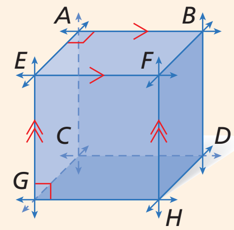
Parallel, Perpendicular, and Skew Lines

Parallel lines ($\parallel$) are coplanar and do not intersect. In the figure, $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$, and $\overleftrightarrow{EG} \parallel \overleftrightarrow{FH}$.

Perpendicular lines ($\perp$) intersect at 90° angles. In the figure, $\overleftrightarrow{AB} \perp \overleftrightarrow{AE}$, and $\overleftrightarrow{EG} \perp \overleftrightarrow{GH}$.

Skew lines are not coplanar. Skew lines are not parallel and do not intersect. In the figure, $\overleftrightarrow{AB}$ and $\overleftrightarrow{EG}$ are skew.

Parallel planes are planes that do not intersect. In the figure, plane *ABE* $\parallel$ plane *CDG*.



Arrows are used to show that $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$ and $\overleftrightarrow{EG} \parallel \overleftrightarrow{FH}$.

EXAMPLE 1 Identifying Types of Lines and Planes

Identify each of the following.

A a pair of parallel segments

$\overline{KN} \parallel \overline{PS}$

B a pair of skew segments

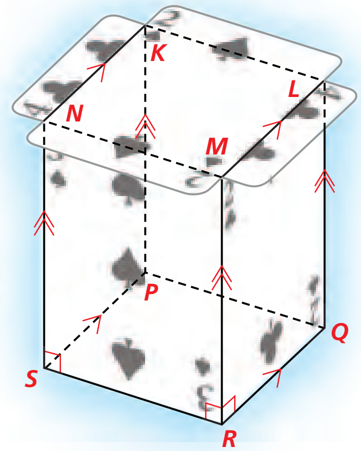
$\overline{LM}$ and $\overline{RS}$ are skew.

C a pair of perpendicular segments

$\overline{MR} \perp \overline{RS}$

D a pair of parallel planes

plane *KPS* $\parallel$ plane *LQR*



Helpful Hint

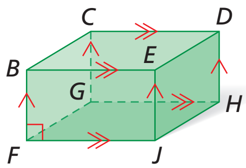
Segments or rays are parallel, perpendicular, or skew if the lines that contain them are parallel, perpendicular, or skew.

California Standards

Preparation for 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.

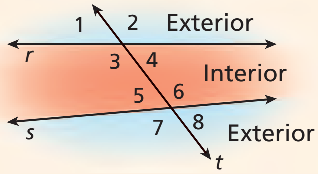
CHECK IT OUT! Identify each of the following.

1a. a pair of parallel segments
1b. a pair of skew segments
1c. a pair of perpendicular segments
1d. a pair of parallel planes



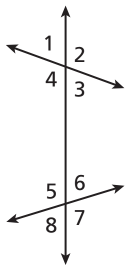
Know it! Note

Angle Pairs Formed by a Transversal

TERM	EXAMPLE
A **transversal** is a line that intersects two coplanar lines at two different points. The transversal *t* and the other two lines *r* and *s* form eight angles.	
Corresponding angles lie on the same side of the transversal *t*, on the same sides of lines *r* and *s*.	∠1 and ∠5
Alternate interior angles are nonadjacent angles that lie on opposite sides of the transversal *t*, between lines *r* and *s*.	∠3 and ∠6
Alternate exterior angles lie on opposite sides of the transversal *t*, outside lines *r* and *s*.	∠1 and ∠8
Same-side interior angles or *consecutive interior angles* lie on the same side of the transversal *t*, between lines *r* and *s*.	∠3 and ∠5

EXAMPLE 2

Classifying Pairs of Angles



Give an example of each angle pair.

A corresponding angles
∠4 and ∠8

B alternate interior angles
∠4 and ∠6

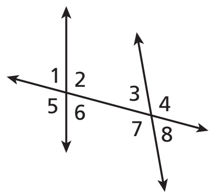
C alternate exterior angles
∠2 and ∠8

D same-side interior angles
∠4 and ∠5

CHECK IT OUT!

Give an example of each angle pair.

2a. corresponding angles
2b. alternate interior angles
2c. alternate exterior angles
2d. same-side interior angles



EXAMPLE 3

Identifying Angle Pairs and Transversals

Helpful Hint

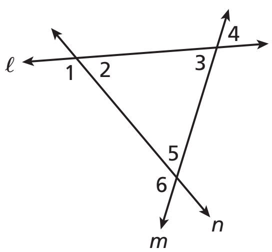
To determine which line is the transversal for a given angle pair, locate the line that connects the vertices.

Identify the transversal and classify each angle pair.

A ∠1 and ∠5
transversal: *n*; alternate interior angles

B ∠3 and ∠6
transversal: *m*; corresponding angles

C ∠1 and ∠4
transversal: ℓ; alternate exterior angles



CHECK IT OUT!

3. Identify the transversal and classify the angle pair ∠2 and ∠5 in the diagram above.

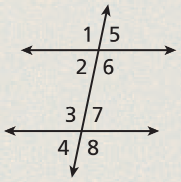
THINK AND DISCUSS

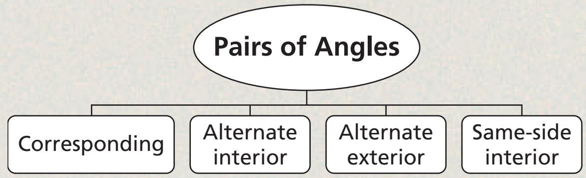
1. Compare perpendicular and intersecting lines.

2. Describe the positions of two alternate exterior angles formed by lines *m* and *n* with transversal *p*.

Know it! Note

3. GET ORGANIZED Copy the diagram and graphic organizer. In each box, list all the angle pairs of each type in the diagram.





3-1 Exercises

California Standards
Preparation for 7.0; 8.0, 7NS2.0

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KEYWORD: MG7 3-1
Parent Resources Online
KEYWORD: MG7 Parent

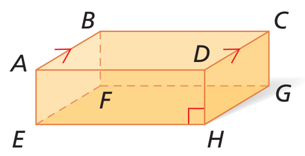
GUIDED PRACTICE

1. **Vocabulary** __?__ are located on opposite sides of a transversal, between the two lines that intersect the transversal. (*corresponding angles, alternate interior angles, alternate exterior angles,* or *same-side interior angles*)

SEE EXAMPLE 1
p. 146

Identify each of the following.

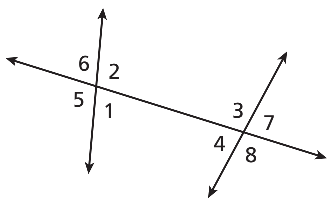
2. one pair of perpendicular segments
3. one pair of skew segments
4. one pair of parallel segments
5. one pair of parallel planes



SEE EXAMPLE 2
p. 147

Give an example of each angle pair.

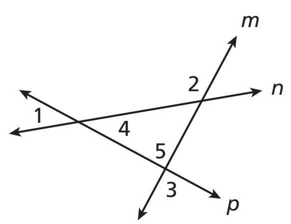
6. alternate interior angles
7. alternate exterior angles
8. corresponding angles
9. same-side interior angles



SEE EXAMPLE 3
p. 147

Identify the transversal and classify each angle pair.

10. ∠1 and ∠2
11. ∠2 and ∠3
12. ∠2 and ∠4
13. ∠4 and ∠5



PRACTICE AND PROBLEM SOLVING

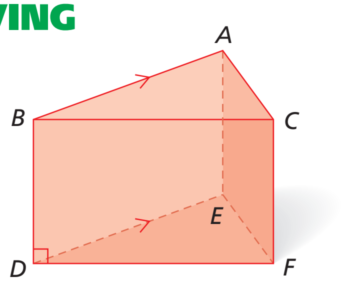
Independent Practice

For Exercises	See Example
14–17	1
18–21	2
22–25	3

Extra Practice

Skills Practice p. S8
Application Practice p. S30

Identify each of the following.



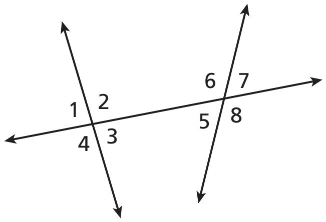
14. one pair of parallel segments

15. one pair of skew segments

16. one pair of perpendicular segments

17. one pair of parallel planes

Give an example of each angle pair.



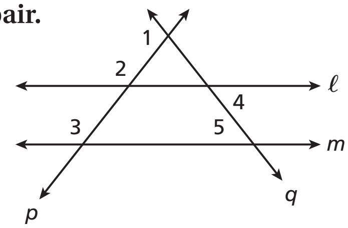
18. same-side interior angles

19. alternate exterior angles

20. corresponding angles

21. alternate interior angles

Identify the transversal and classify each angle pair.



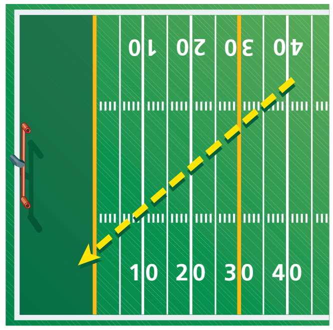
22. ∠2 and ∠3

23. ∠4 and ∠5

24. ∠2 and ∠4

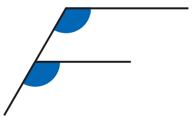
25. ∠1 and ∠2

26. Sports A football player runs across the 30-yard line at an angle. He continues in a straight line and crosses the goal line at the same angle. Describe two parallel lines and a transversal in the diagram.

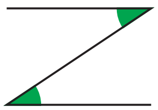


Name the type of angle pair shown in each letter.

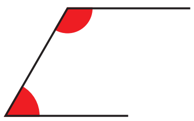
27. F



28. Z



29. C



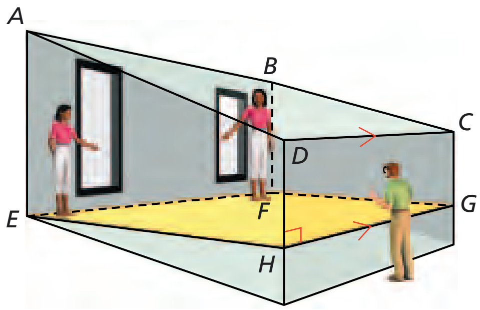
LINK

Entertainment

In an Ames room, two people of the same height that are standing in different parts of the room appear to be different sizes.

Entertainment **Use the following information for Exercises 30–32.**

In an Ames room, the floor is tilted and the back wall is closer to the front wall on one side.

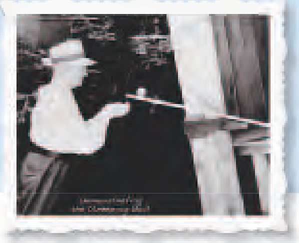


30. Name a pair of parallel segments in the diagram.

31. Name a pair of skew segments in the diagram.

32. Name a pair of perpendicular segments in the diagram.

Concept Connection

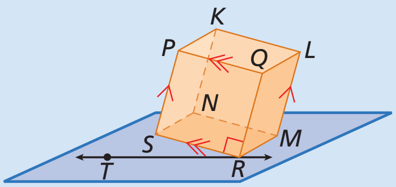


33. This problem will prepare you for the Concept Connection on p 180.

Buildings that are tilted like the one shown are sometimes called mystery spots.

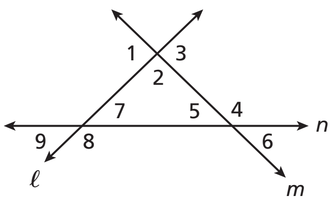
a. Name a plane parallel to plane *KLP*, a plane parallel to plane *KNP*, and a plane parallel to *KLM*.

b. In the diagram, $\overline{QR}$ is a transversal to $\overline{PQ}$ and $\overline{RS}$. What type of angle pair is $\angle PQR$ and $\angle QRS$?



34. **Critical Thinking** Line ℓ is contained in plane *P* and line *m* is contained in plane *Q*. If *P* and *Q* are parallel, what are the possible classifications of ℓ and *m*? Include diagrams to support your answer.

Use the diagram for Exercises 35–40.



35. Name a pair of alternate interior angles with transversal *n*.

36. Name a pair of same-side interior angles with transversal ℓ.

37. Name a pair of corresponding angles with transversal *m*.

38. Identify the transversal and classify the angle pair for $\angle 3$ and $\angle 7$.

39. Identify the transversal and classify the angle pair for $\angle 5$ and $\angle 8$.

40. Identify the transversal and classify the angle pair for $\angle 1$ and $\angle 6$.

41. **Aviation** Describe the type of lines formed by two planes when flight 1449 is flying from San Francisco to Atlanta at 32,000 feet and flight 2390 is flying from Dallas to Chicago at 28,000 feet.

42. **Multi-Step** Draw line *p*, then draw two lines *m* and *n* that are both perpendicular to *p*. Make a conjecture about the relationship between lines *m* and *n*.

43. **Write About It** Discuss a real-world example of skew lines. Include a sketch.

Standardized Test Prep

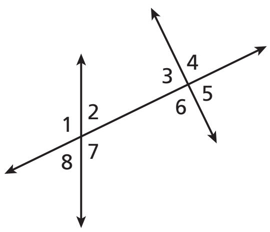
44. Which pair of angles in the diagram are alternate interior angles?

(A) $\angle 1$ and $\angle 5$

(B) $\angle 2$ and $\angle 6$

(C) $\angle 7$ and $\angle 5$

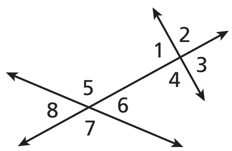
(D) $\angle 2$ and $\angle 3$



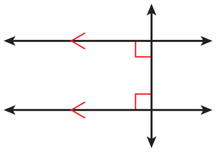
45. How many pairs of corresponding angles are in the diagram?

(F) 2 (H) 8

(G) 4 (J) 16



46. Which type of lines are NOT represented in the diagram?



Ⓐ Parallel lines Ⓒ Skew lines

Ⓑ Intersecting lines Ⓓ Perpendicular lines

47. For two lines and a transversal, ∠1 and ∠8 are alternate exterior angles, and ∠1 and ∠5 are corresponding angles. Classify the angle pair ∠5 and ∠8.

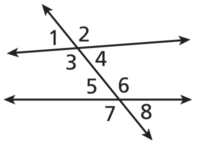
Ⓕ Vertical angles

Ⓖ Alternate interior angles

Ⓗ Adjacent angles

Ⓙ Same-side interior angles

48. Which angles in the diagram are NOT corresponding angles?

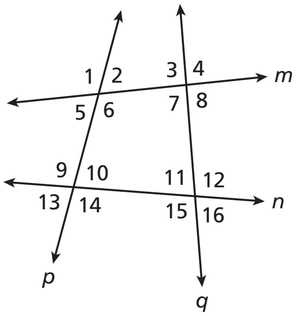


Ⓐ ∠1 and ∠5 Ⓒ ∠4 and ∠8

Ⓑ ∠2 and ∠6 Ⓓ ∠2 and ∠7

CHALLENGE AND EXTEND

Name all the angle pairs of each type in the diagram. Identify the transversal for each pair.

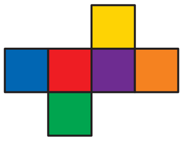


49. corresponding **50.** alternate interior

51. alternate exterior **52.** same-side interior

53. **Multi-Step** Draw two lines and a transversal such that ∠1 and ∠3 are corresponding angles, ∠1 and ∠2 are alternate interior angles, and ∠3 and ∠4 are alternate exterior angles. What type of angle pair is ∠2 and ∠4?

54. If the figure shown is folded to form a cube, which faces of the cube will be parallel?



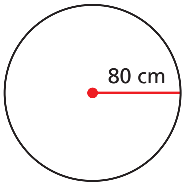
SPIRAL REVIEW

Evaluate each function for $x = -1, 0, 1, 2,$ and 3. *(Previous course)*

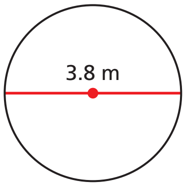
55. $y = 4x^2 - 7$ **56.** $y = -2x^2 + 5$ **57.** $y = (x + 3)(x - 3)$

Find the circumference and area of each circle. Use the π key on your calculator and round to the nearest tenth. *(Lesson 1-5)*

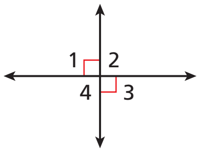
58.



59.



Write a justification for each statement, given that ∠1 and ∠3 are right angles. *(Lesson 2-6)*



60. ∠1 ≅ ∠3

61. m∠1 + m∠2 = 180°

62. ∠2 ≅ ∠4

Connecting Geometry to **Algebra**

See Skills Bank page S67

Systems of Equations

Sometimes angle measures are given as algebraic expressions. When you know the relationship between two angles, you can write and solve a system of equations to find angle measures.

California Standards

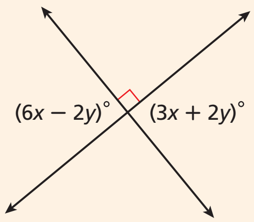
Review of 1A9.0 Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

Solving Systems of Equations by Using Elimination

Step 1 Write the system so that like terms are under one another.
Step 2 Eliminate one of the variables.
Step 3 Substitute that value into one of the original equations and solve.
Step 4 Write the answers as an ordered pair, (x, y).
Step 5 Check your solution.

Example 1

Solve for x and y.



Since the lines are perpendicular, all of the angles are right angles. To write two equations, you can set each expression equal to 90°.

$$(3x + 2y)^\circ = 90^\circ,\ (6x - 2y)^\circ = 90^\circ$$

Step 1 $3x + 2y = 90$, $\underline{6x - 2y = 90}$ — *Write the system so that like terms are under one another.*

Step 2 $9x + 0 = 180$ — *Add like terms on each side of the equations. The y-term has been eliminated.*

$x = 20$ — *Divide both sides by 9 to solve for x.*

Step 3 $3x + 2y = 90$ — *Write one of the original equations.*

$3(20) + 2y = 90$ — *Substitute 20 for x.*

$60 + 2y = 90$ — *Simplify.*

$2y = 30$ — *Subtract 60 from both sides.*

$y = 15$ — *Divide by 2 on both sides.*

Step 4 $(20, 15)$ — *Write the solution as an ordered pair.*

Step 5 Check the solution by substituting 20 for x and 15 for y in the original equations.

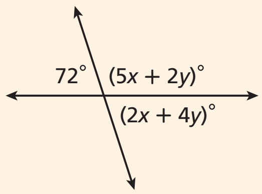
$3x + 2y$	$= 90$
$3(20) + 2(15)$	90
$60 + 30$	90
90	90 ✓

$6x - 2y$	$= 90$
$6(20) - 2(15)$	90
$120 - 30$	90
90	90 ✓

In some cases, before you can do Step 1 you will need to multiply one or both of the equations by a number so that you can eliminate a variable.

Example 2

Solve for x and y.



$(2x + 4y)^\circ = 72^\circ$ *Vertical Angles Theorem*

$(5x + 2y)^\circ = 108^\circ$ *Linear Pair Theorem*

The equations cannot be added or subtracted to eliminate a variable. Multiply the second equation by -2 to get opposite y-coefficients.

$$5x + 2y = 108 \rightarrow -2(5x + 2y) = -2(108) \rightarrow -10x - 4y = -216$$

Step 1 $\begin{aligned} 2x + 4y &= 72 \\ -10x - 4y &= -216 \end{aligned}$ *Write the system so that like terms are under one another.*

Step 2 $-8x = -144$ *Add like terms on both sides of the equations. The y-term has been eliminated.*

$x = 18$ *Divide both sides by −8 to solve for x.*

Step 3 $2x + 4y = 72$ *Write one of the original equations.*

$2(18) + 4y = 72$ *Substitute 18 for x.*

$36 + 4y = 72$ *Simplify.*

$4y = 36$ *Subtract 36 from both sides.*

$y = 9$ *Divide by 4 on both sides.*

Step 4 $(18, 9)$ *Write the solution as an ordered pair.*

Step 5 Check the solution by substituting 18 for x and 9 for y in the original equations.

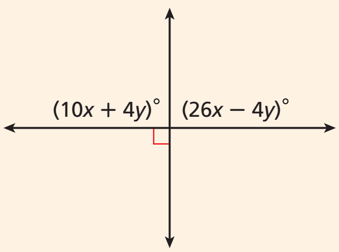
$2x + 4y$	$= 72$
$3(18) + 4(9)$	72
$36 + 36$	72
72	72 ✓

$5x + 2y$	$= 108$
$5(18) + 2(9)$	108
$90 + 18$	108
108	108 ✓

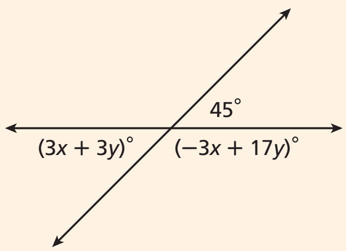
Try This

Solve for x and y.

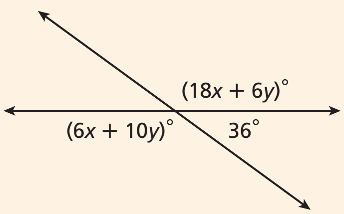
1.



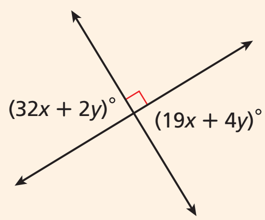
2.



3.



4.



3-2 Technology LAB

Use with Lesson 3-2

Explore Parallel Lines and Transversals

Geometry software can help you explore angles that are formed when a transversal intersects a pair of parallel lines.

California Standards

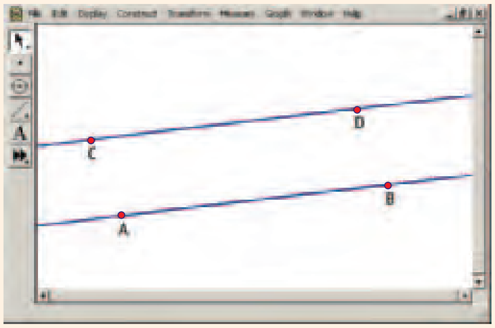
Preparation for 🔑 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.

Also covered: 🔑 **1.0**

go.hrw.com
Lab Resources Online
KEYWORD: MG7 Lab3

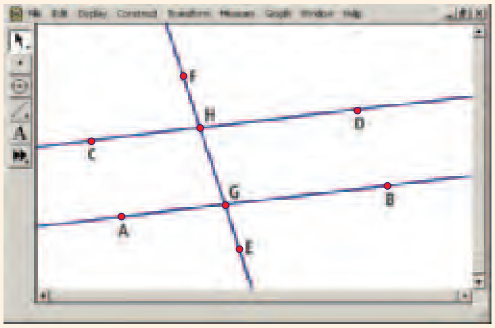
Activity

1. Construct a line and label two points on the line *A* and *B*.



2. Create point *C* not on $\overleftrightarrow{AB}$. Construct a line parallel to $\overleftrightarrow{AB}$ through point *C*. Create another point on this line and label it *D*.

3. Create two points outside the two parallel lines and label them *E* and *F*. Construct transversal $\overleftrightarrow{EF}$. Label the points of intersection *G* and *H*.



4. Measure the angles formed by the parallel lines and the transversal. Write the angle measures in a chart like the one below. Drag point *E* or *F* and chart with the new angle measures. What relationships do you notice about the angle measures? What conjectures can you make?

Angle	∠AGE	∠BGE	∠AGH	∠BGH	∠CHG	∠DHG	∠CHF	∠DHF
Measure								
Measure								

Try This

1. Identify the pairs of corresponding angles in the diagram. What conjecture can you make about their angle measures? Drag a point in the figure to confirm your conjecture.
2. Repeat steps in the previous problem for alternate interior angles, alternate exterior angles, and same-side interior angles.
3. Try dragging point *C* to change the distance between the parallel lines. What happens to the angle measures in the figure? Why do you think this happens?

3-2 Angles Formed by Parallel Lines and Transversals

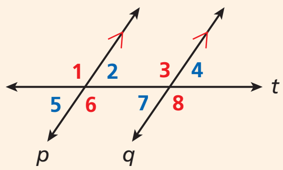
Objective
Prove and use theorems about the angles formed by parallel lines and a transversal.

Who uses this?
Piano makers use parallel strings for the higher notes. The longer strings used to produce the lower notes can be viewed as transversals. (See Example 3.)

When parallel lines are cut by a transversal, the angle pairs formed are either congruent or supplementary.

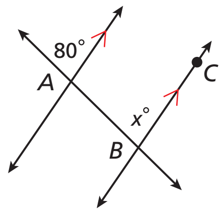
Know it! Note

Postulate 3-2-1 Corresponding Angles Postulate

POSTULATE	HYPOTHESIS	CONCLUSION
If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.		$\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$ $\angle 5 \cong \angle 7$ $\angle 6 \cong \angle 8$

EXAMPLE 1 Using the Corresponding Angles Postulate

Find each angle measure.



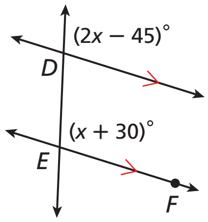
A $m\angle ABC$

$x = 80$ *Corr. ∠s Post.*

$m\angle ABC = 80°$

Algebra

B $m\angle DEF$



$(2x - 45)° = (x + 30)°$ *Corr. ∠s Post.*

$x - 45 = 30$ *Subtract x from both sides.*

$x = 75$ *Add 45 to both sides.*

$m\angle DEF = x + 30$

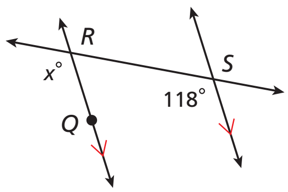
$= 75 + 30$ *Substitute 75 for x.*

$= 105°$

California Standards
7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.

CHECK IT OUT!

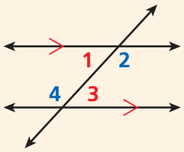
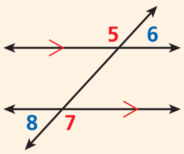
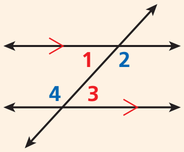
1. Find $m\angle QRS$.



Remember that postulates are statements that are accepted without proof. Since the Corresponding Angles Postulate is given as a postulate, it can be used to prove the next three theorems.

Know it! Note

Theorems — Parallel Lines and Angle Pairs

THEOREM	HYPOTHESIS	CONCLUSION
3-2-2 **Alternate Interior Angles Theorem** If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.		$\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$
3-2-3 **Alternate Exterior Angles Theorem** If two parallel lines are cut by a transversal, then the two pairs of alternate exterior angles are congruent.		$\angle 5 \cong \angle 7$ $\angle 6 \cong \angle 8$
3-2-4 **Same-Side Interior Angles Theorem** If two parallel lines are cut by a transversal, then the two pairs of same-side interior angles are supplementary.		$m\angle 1 + m\angle 4 = 180°$ $m\angle 2 + m\angle 3 = 180°$

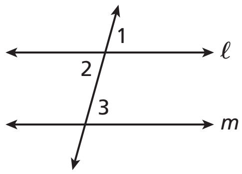
You will prove Theorems 3-2-3 and 3-2-4 in Exercises 25 and 26.

Helpful Hint

If a transversal is perpendicular to two parallel lines, all eight angles are congruent.

PROOF

Alternate Interior Angles Theorem



Given: $\ell \parallel m$

Prove: $\angle 2 \cong \angle 3$

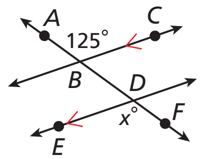
Proof:

- $\ell \parallel m$ (Given) → $\angle 1 \cong \angle 3$ (Corr. ∠s Post.)
- $\angle 2 \cong \angle 1$ (Vert. ∠s Thm.)
- $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 1$ → $\angle 2 \cong \angle 3$ (Trans. Prop. of ≅)

EXAMPLE 2

Finding Angle Measures

Find each angle measure.



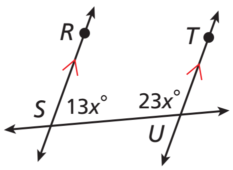
A $m\angle EDF$

$x = 125$

$m\angle EDF = 125°$ *Alt. Ext. ∠s Thm.*

Algebra

B $m\angle TUS$



$13x° + 23x° = 180°$ *Same-Side Int. ∠s Thm.*

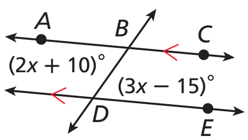
$36x = 180$ *Combine like terms.*

$x = 5$ *Divide both sides by 36.*

$m\angle TUS = 23(5) = 115°$ *Substitute 5 for x.*

CHECK IT OUT!

2. Find $m\angle ABD$.

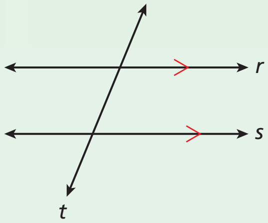


Student to Student

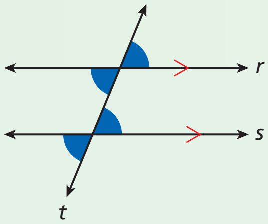
Parallel Lines and Transversals

Nancy Martin
East Branch
High School

When I solve problems with parallel lines and transversals, I remind myself that every pair of angles is either congruent or supplementary.



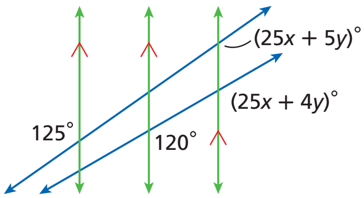
If r ∥ s, all the acute angles are congruent and all the obtuse angles are congruent. The acute angles are supplementary to the obtuse angles.



EXAMPLE 3 Algebra

Music Application

The treble strings of a grand piano are parallel. Viewed from above, the bass strings form transversals to the treble strings. Find x and y in the diagram.



By the Alternate Exterior Angles Theorem, $(25x + 5y)^\circ = 125°$.

By the Corresponding Angles Postulate, $(25x + 4y)^\circ = 120°$.

$$25x + 5y = 125$$

$$-(25x + 4y = 120)$$ *Subtract the second equation from the first equation.*

$$y = 5$$

$$25x + 5(5) = 125$$ *Substitute 5 for y in 25x + 5y = 125. Simplify and solve for x.*

$$x = 4,\ y = 5$$

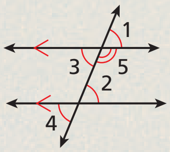
CHECK IT OUT! **3.** Find the measures of the acute angles in the diagram.

THINK AND DISCUSS

1. Explain why a transversal that is perpendicular to two parallel lines forms eight congruent angles.

Know it! Note

2. GET ORGANIZED Copy the diagram and graphic organizer. Complete the graphic organizer by explaining why each of the three theorems is true.



Corr. ∠s Post.
- Alt. Int. ∠s Thm.
- Alt. Ext. ∠s Thm.
- Same-Side Int. ∠s Thm.

3-2 Exercises

California Standards
1.0, 7.0, 7AF4.1, 7MR1.1, 1A5.0

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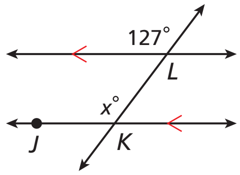
Homework Help Online
KEYWORD: MG7 3-2
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

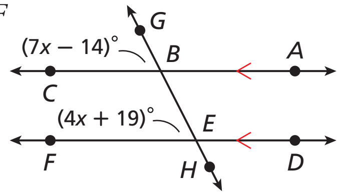
SEE EXAMPLE 1 p. 155

Find each angle measure.

1. m∠*JKL*

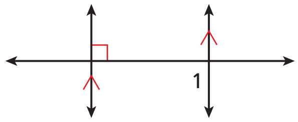


2. m∠*BEF*

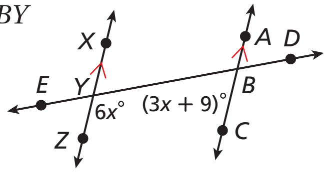


SEE EXAMPLE 2 p. 156

3. m∠1

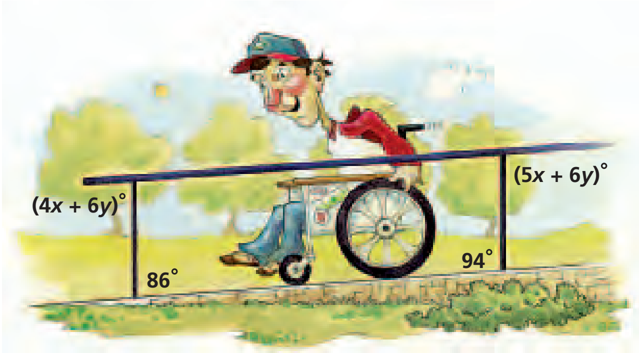


4. m∠*CBY*



SEE EXAMPLE 3 p. 157

5. **Safety** The railing of a wheelchair ramp is parallel to the ramp. Find *x* and *y* in the diagram.



PRACTICE AND PROBLEM SOLVING

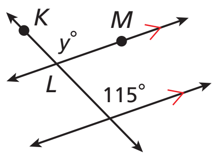
Independent Practice

For Exercises	See Example
6–7	1
8–11	2
12	3

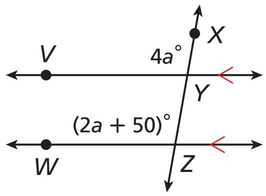
Extra Practice
Skills Practice p. S8
Application Practice p. S30

Find each angle measure.

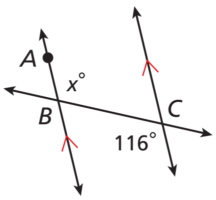
6. m∠*KLM*



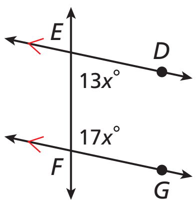
7. m∠*VYX*



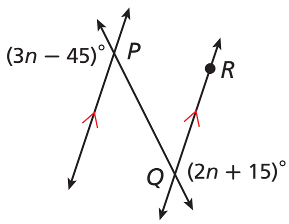
8. m∠*ABC*



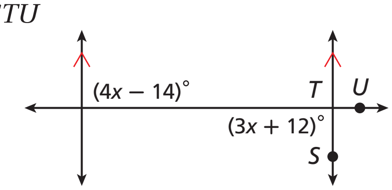
9. m∠*EFG*



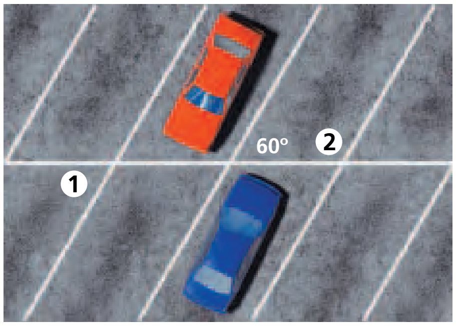
10. m∠*PQR*



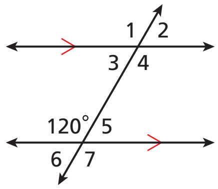
11. m∠*STU*



12. **Parking** In the parking lot shown, the lines that mark the width of each space are parallel.
$m\angle 1 = (2x - 3y)°$
$m\angle 2 = (x + 3y)°$
Find x and y.



Find each angle measure. Justify each answer with a postulate or theorem.

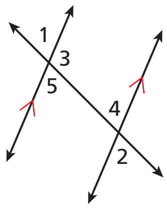


13. $m\angle 1$ **14.** $m\angle 2$ **15.** $m\angle 3$

16. $m\angle 4$ **17.** $m\angle 5$ **18.** $m\angle 6$

19. $m\angle 7$

Algebra **State the theorem or postulate that is related to the measures of the angles in each pair. Then find the angle measures.**



20. $m\angle 1 = (7x + 15)°$, $m\angle 2 = (10x - 9)°$

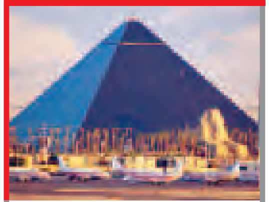
21. $m\angle 3 = (23x + 11)°$, $m\angle 4 = (14x + 21)°$

22. $m\angle 4 = (37x - 15)°$, $m\angle 5 = (44x - 29)°$

23. $m\angle 1 = (6x + 24)°$, $m\angle 4 = (17x - 9)°$

LINK

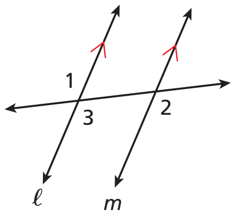
Architecture



The Luxor hotel is 600 feet wide, 600 feet long, and 350 feet high. The atrium in the hotel measures 29 million cubic feet.

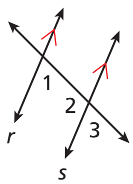
24. **Architecture** The Luxor Hotel in Las Vegas, Nevada, is a 30-story pyramid. The hotel uses an elevator called an inclinator to take people up the side of the pyramid. The inclinator travels at a 39° angle. Which theorem or postulate best illustrates the angles formed by the path of the inclinator and each parallel floor? (*Hint:* Draw a picture.)

25. **Complete the two-column proof of the Alternate Exterior Angles Theorem.**
Given: $\ell \parallel m$
Prove: $\angle 1 \cong \angle 2$
Proof:



Statements	Reasons
1. $\ell \parallel m$	**1.** Given
2. a. ___?___	**2.** Vert. ∠s Thm.
3. $\angle 3 \cong \angle 2$	**3. b.** ___?___
4. c. ___?___	**4. d.** ___?___

26. **Write a paragraph proof of the Same-Side Interior Angles Theorem.**
Given: $r \parallel s$
Prove: $m\angle 1 + m\angle 2 = 180°$



Draw the given situation or tell why it is impossible.

27. Two parallel lines are intersected by a transversal so that the corresponding angles are supplementary.

28. Two parallel lines are intersected by a transversal so that the same-side interior angles are complementary.

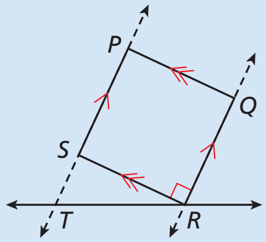
CONCEPT CONNECTION

29. This problem will prepare you for the Concept Connection on page 180.

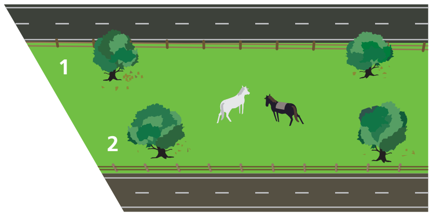
In the diagram, which represents the side view of a mystery spot, m∠*SRT* = 25°. $\overleftrightarrow{RT}$ is a transversal to $\overleftrightarrow{PS}$ and $\overleftrightarrow{QR}$.

a. What type of angle pair is ∠*QRT* and ∠*STR*?

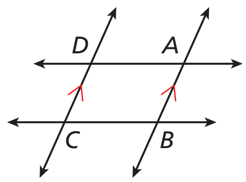
b. Find m∠*STR*. Use a theorem or postulate to justify your answer.



30. Land Development A piece of property lies between two parallel streets as shown. $m\angle 1 = (2x + 6)^\circ$, and $m\angle 2 = (3x + 9)^\circ$. What is the relationship between the angles? What are their measures?



31. ERROR ANALYSIS In the figure, $m\angle ABC = (15x + 5)^\circ$, and $m\angle BCD = (10x + 25)^\circ$. Which value of *m∠BCD* is incorrect? Explain.



A

$15x + 5 = 10x + 25$
$-10x \quad -10x$
$5x + 5 = 25$
$-5 \quad -5$
$5x = 20$
$x = 4$

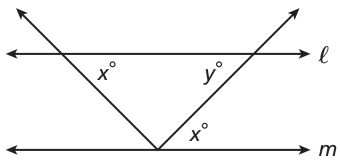
$m\angle BCD = 10(4) + 25 = 65^\circ$

B

$(15x + 5) + (10x + 25) = 180$
$25x + 30 = 180$
$-30 \quad -30$
$25x = 150$
$x = 6$

$m\angle BCD = 10(6) + 25 = 85^\circ$

32. Critical Thinking In the diagram, $\ell \parallel m$. Explain why $\frac{x}{y} = 1$.



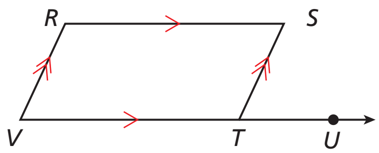
33. Write About It Suppose that lines ℓ and *m* are intersected by transversal *p*. One of the angles formed by ℓ and *p* is congruent to every angle formed by *m* and *p*. Draw a diagram showing lines ℓ, *m*, and *p*, mark any congruent angles that are formed, and explain what you know is true.

STANDARDIZED TEST PREP

34. $m\angle RST = (x + 50)^\circ$, and $m\angle STU = (3x + 20)^\circ$. Find m∠*RVT*.

Ⓐ 15° Ⓒ 65°

Ⓑ 27.5° Ⓓ 77.5°



35. For two parallel lines and a transversal, m∠1 = 83°. For which pair of angle measures is the sum the least?

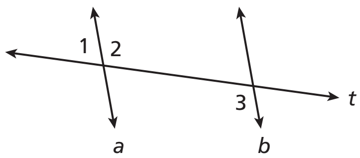
Ⓕ ∠1 and a corresponding angle

Ⓖ ∠1 and a same-side interior angle

Ⓗ ∠1 and its supplement

Ⓙ ∠1 and its complement

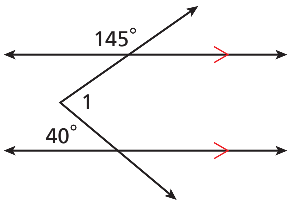
36. **Short Response** Given $a \parallel b$ with transversal t, explain why ∠1 and ∠3 are supplementary.



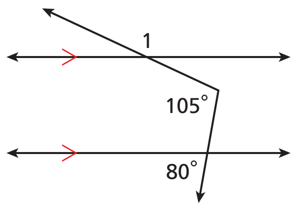
CHALLENGE AND EXTEND

Multi-Step **Find m∠1 in each diagram. (*Hint:* Draw a line parallel to the given parallel lines.)**

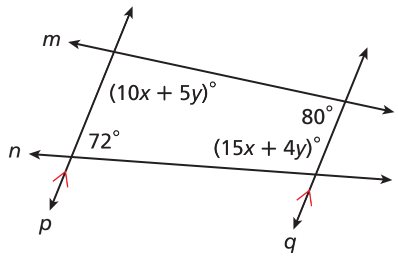
37.



38.



39. Find x and y in the diagram. Justify your answer.



40. Two lines are parallel. The measures of two corresponding angles are $a°$ and $2b°$, and the measures of two same-side interior angles are $a°$ and $b°$. Find the value of a.

SPIRAL REVIEW

If the first quantity increases, tell whether the second quantity is likely to increase, decrease, or stay the same. *(Previous course)*

41. time in years and average cost of a new car

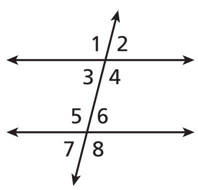
42. age of a student and length of time needed to read 500 words

Use the Law of Syllogism to draw a conclusion from the given information. *(Lesson 2-3)*

43. If two angles form a linear pair, then they are supplementary. If two angles are supplementary, then their measures add to 180°. ∠1 and ∠2 form a linear pair.

44. If a figure is a square, then it is a rectangle. If a figure is a rectangle, then its sides are perpendicular. Figure *ABCD* is a square.

Give an example of each angle pair. *(Lesson 3-1)*



45. alternate interior angles

46. alternate exterior angles

47. same-side interior angles

3-3 Proving Lines Parallel

Objective
Use the angles formed by a transversal to prove two lines are parallel.

Who uses this?
Rowers have to keep the oars on each side parallel in order to travel in a straight line. (See Example 4.)

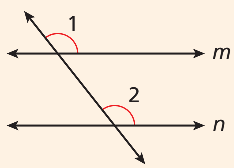
California Standards

7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.
Also covered: **16.0**

Recall that the converse of a theorem is found by exchanging the hypothesis and conclusion. The converse of a theorem is not automatically true. If it is true, it must be stated as a postulate or proved as a separate theorem.

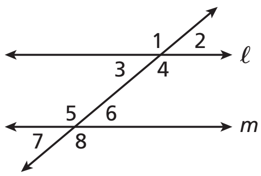
Know it! Note

Postulate 3-3-1 Converse of the Corresponding Angles Postulate

POSTULATE	HYPOTHESIS	CONCLUSION
If two coplanar lines are cut by a transversal so that a pair of corresponding angles are congruent, then the two lines are parallel.	$\angle 1 \cong \angle 2$ 	$m \parallel n$

EXAMPLE 1 Using the Converse of the Corresponding Angles Postulate

Use the Converse of the Corresponding Angles Postulate and the given information to show that $\ell \parallel m$.



A $\angle 1 \cong \angle 5$

$\angle 1 \cong \angle 5$ — *∠1 and ∠5 are corresponding angles.*
$\ell \parallel m$ — *Conv. of Corr. ∠s Post.*

Algebra

B $m\angle 4 = (2x + 10)^\circ, m\angle 8 = (3x - 55)^\circ, x = 65$

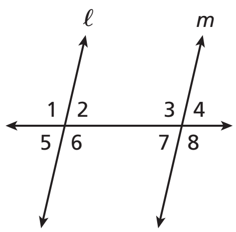
$m\angle 4 = 2(65) + 10 = 140$ — *Substitute 65 for x.*
$m\angle 8 = 3(65) - 55 = 140$ — *Substitute 65 for x.*
$m\angle 4 = m\angle 8$ — *Trans. Prop. of Equality*
$\angle 4 \cong \angle 8$ — *Def. of ≅ ∠s*
$\ell \parallel m$ — *Conv. of Corr. ∠s Post.*

CHECK IT OUT!

Use the Converse of the Corresponding Angles Postulate and the given information to show that $\ell \parallel m$.

1a. $m\angle 1 = m\angle 3$

1b. $m\angle 7 = (4x + 25)^\circ$,
$m\angle 5 = (5x + 12)^\circ, x = 13$



Know it! Note

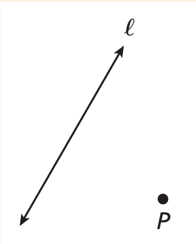
Postulate 3-3-2 Parallel Postulate

Through a point P not on line ℓ, there is exactly one line parallel to ℓ.

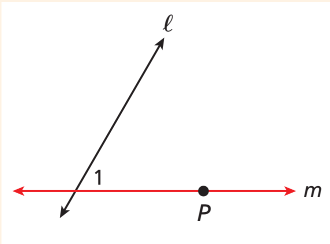
The Converse of the Corresponding Angles Postulate is used to construct parallel lines. The Parallel Postulate guarantees that for any line ℓ, you can always construct a parallel line through a point that is not on ℓ.

Construction Parallel Lines

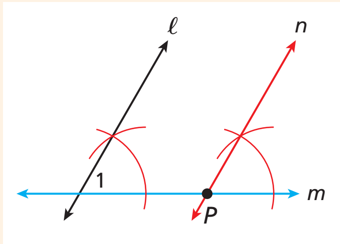
1. Draw a line ℓ and a point P that is not on ℓ.



2. Draw a line m through P that intersects ℓ. Label the angle 1.

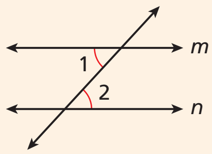
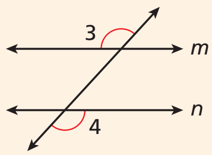
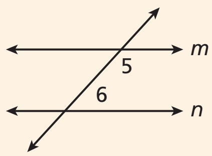


3. Construct an angle congruent to $\angle 1$ at P. By the converse of the Corresponding Angles Postulate, $\ell \parallel n$.



Know it! Note

Theorems Proving Lines Parallel

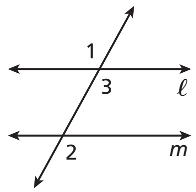
THEOREM	HYPOTHESIS	CONCLUSION
3-3-3 **Converse of the Alternate Interior Angles Theorem** If two coplanar lines are cut by a transversal so that a pair of alternate interior angles are congruent, then the two lines are parallel.	$\angle 1 \cong \angle 2$ 	$m \parallel n$
3-3-4 **Converse of the Alternate Exterior Angles Theorem** If two coplanar lines are cut by a transversal so that a pair of alternate exterior angles are congruent, then the two lines are parallel.	$\angle 3 \cong \angle 4$ 	$m \parallel n$
3-3-5 **Converse of the Same-Side Interior Angles Theorem** If two coplanar lines are cut by a transversal so that a pair of same-side interior angles are supplementary, then the two lines are parallel.	$m\angle 5 + m\angle 6 = 180°$ 	$m \parallel n$

You will prove Theorems 3-3-3 and 3-3-5 in Exercises 38–39.

PROOF

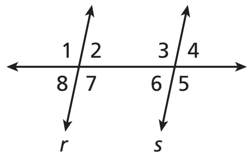
Converse of the Alternate Exterior Angles Theorem

Given: $\angle 1 \cong \angle 2$
Prove: $\ell \parallel m$
Proof: It is given that $\angle 1 \cong \angle 2$. Vertical angles are congruent, so $\angle 1 \cong \angle 3$. By the Transitive Property of Congruence, $\angle 2 \cong \angle 3$. So $\ell \parallel m$ by the Converse of the Corresponding Angles Postulate.



EXAMPLE 2 Determining Whether Lines are Parallel

Use the given information and the theorems you have learned to show that $r \parallel s$.



A $\angle 2 \cong \angle 6$

$\angle 2 \cong \angle 6$	*∠2 and ∠6 are alternate interior angles.*
$r \parallel s$	*Conv. of Alt. Int. ∡ Thm.*

Algebra

B $m\angle 6 = (6x + 18)^\circ$, $m\angle 7 = (9x + 12)^\circ$, $x = 10$

$m\angle 6 = 6x + 18$	
$\quad = 6(10) + 18 = 78^\circ$	*Substitute 10 for x.*
$m\angle 7 = 9x + 12$	
$\quad = 9(10) + 12 = 102^\circ$	*Substitute 10 for x.*
$m\angle 6 + m\angle 7 = 78^\circ + 102^\circ$	
$\quad = 180^\circ$	*∠6 and ∠7 are same-side interior angles.*
$r \parallel s$	*Conv. of Same-Side Int. ∡ Thm.*

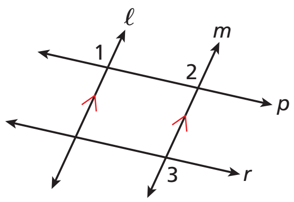
CHECK IT OUT!

Refer to the diagram above. Use the given information and the theorems you have learned to show that $r \parallel s$.

2a. $m\angle 4 = m\angle 8$ **2b.** $m\angle 3 = 2x^\circ$, $m\angle 7 = (x + 50)^\circ$, $x = 50$

EXAMPLE 3 Proving Lines Parallel

Given: $\ell \parallel m$, $\angle 1 \cong \angle 3$
Prove: $r \parallel p$

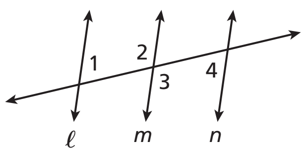


Proof:

Statements	Reasons
1. $\ell \parallel m$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Corr. ∡ Post.
3. $\angle 1 \cong \angle 3$	**3.** Given
4. $\angle 2 \cong \angle 3$	**4.** Trans. Prop. of ≅
5. $r \parallel p$	**5.** Conv. of Alt. Ext. ∡ Thm.

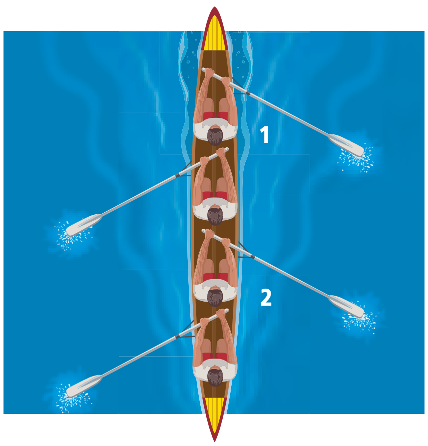
CHECK IT OUT!

3. Given: $\angle 1 \cong \angle 4$, $\angle 3$ and $\angle 4$ are supplementary.
Prove: $\ell \parallel m$

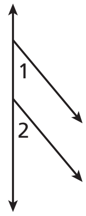


EXAMPLE 4 *Sports Application*

During a race, all members of a rowing team should keep the oars parallel on each side. If $m\angle 1 = (3x + 13)^\circ$, $m\angle 2 = (5x - 5)^\circ$, and $x = 9$, show that the oars are parallel.



A line through the center of the boat forms a transversal to the two oars on each side of the boat.



$\angle 1$ and $\angle 2$ are corresponding angles. If $\angle 1 \cong \angle 2$, then the oars are parallel.

Substitute 9 for x in each expression:

$m\angle 1 = 3x + 13$

$= 3(9) + 13 = 40^\circ$ *Substitute 9 for x in each expression.*

$m\angle 2 = 5x - 5$

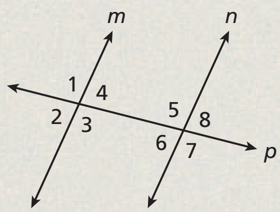
$= 5(9) - 5 = 40^\circ$ *$m\angle 1 = m\angle 2$, so $\angle 1 \cong \angle 2$.*

The corresponding angles are congruent, so the oars are parallel by the Converse of the Corresponding Angles Postulate.

CHECK IT OUT!

4. **What if...?** Suppose the corresponding angles on the opposite side of the boat measure $(4y - 2)^\circ$ and $(3y + 6)^\circ$, where $y = 8$. Show that the oars are parallel.

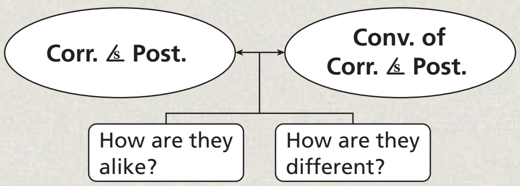
THINK AND DISCUSS



1. Explain three ways of proving that two lines are parallel.
2. If you know $m\angle 1$, how could you use the measures of $\angle 5$, $\angle 6$, $\angle 7$, or $\angle 8$ to prove $m \parallel n$?

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. Use it to compare the Corresponding Angles Postulate with the Converse of the Corresponding Angles Postulate.



3-3 Exercises

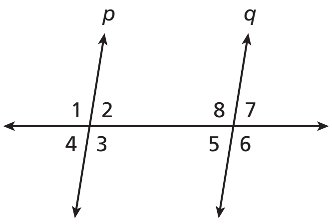
California Standards
3.0, 7.0

go.hrw.com
Homework Help Online
KEYWORD: MG7 3-3
Parent Resources Online
KEYWORD: MG7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1
p. 162

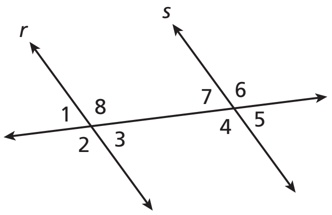
Use the Converse of the Corresponding Angles Postulate and the given information to show that $p \parallel q$.



1. $\angle 4 \cong \angle 5$
2. $m\angle 1 = (4x + 16)^\circ$, $m\angle 8 = (5x - 12)^\circ$, $x = 28$
3. $m\angle 4 = (6x - 19)^\circ$, $m\angle 5 = (3x + 14)^\circ$, $x = 11$

SEE EXAMPLE 2
p. 164

Use the theorems and given information to show that $r \parallel s$.

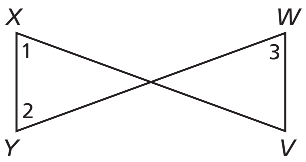


4. $\angle 1 \cong \angle 5$
5. $m\angle 3 + m\angle 4 = 180°$
6. $\angle 3 \cong \angle 7$
7. $m\angle 4 = (13x - 4)^\circ$, $m\angle 8 = (9x + 16)^\circ$, $x = 5$
8. $m\angle 8 = (17x + 37)^\circ$, $m\angle 7 = (9x - 13)^\circ$, $x = 6$
9. $m\angle 2 = (25x + 7)^\circ$, $m\angle 6 = (24x + 12)^\circ$, $x = 5$

SEE EXAMPLE 3
p. 164

10. Complete the following two-column proof.

Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 1$
Prove: $XY \parallel WV$



Proof:

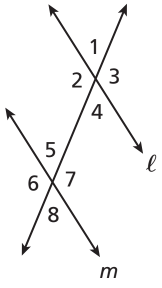
Statements	Reasons
1. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 1$	**1.** Given
2. $\angle 2 \cong \angle 3$	**2. a.** ___?___
3. b. ___?___	**3. c.** ___?___

SEE EXAMPLE 4
p. 165

11. **Architecture** In the fire escape, $m\angle 1 = (17x + 9)^\circ$, $m\angle 2 = (14x + 18)^\circ$, and $x = 3$. Show that the two landings are parallel.

PRACTICE AND PROBLEM SOLVING

Use the Converse of the Corresponding Angles Postulate and the given information to show that $\ell \parallel m$.



12. $\angle 3 \cong 7$
13. $m\angle 4 = 54°$, $m\angle 8 = (7x + 5)^\circ$, $x = 7$
14. $m\angle 2 = (8x + 4)^\circ$, $m\angle 6 = (11x - 41)^\circ$, $x = 15$
15. $m\angle 1 = (3x + 19)^\circ$, $m\angle 5 = (4x + 7)^\circ$, $x = 12$

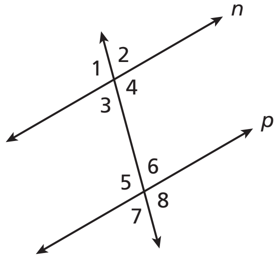
Independent Practice

For Exercises	See Example
12–15	1
16–21	2
22	3
23	4

Extra Practice

Skills Practice p. S8
Application Practice p. S30

Use the theorems and given information to show that $n \parallel p$.



16. $\angle 3 \cong \angle 6$

17. $\angle 2 \cong \angle 7$

18. $m\angle 4 + m\angle 6 = 180°$

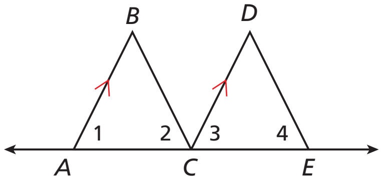
19. $m\angle 1 = (8x - 7)°$, $m\angle 8 = (6x + 21)°$, $x = 14$

20. $m\angle 4 = (4x + 3)°$, $m\angle 5 = (5x - 22)°$, $x = 25$

21. $m\angle 3 = (2x + 15)°$, $m\angle 5 = (3x + 15)°$, $x = 30$

22. Complete the following two-column proof.

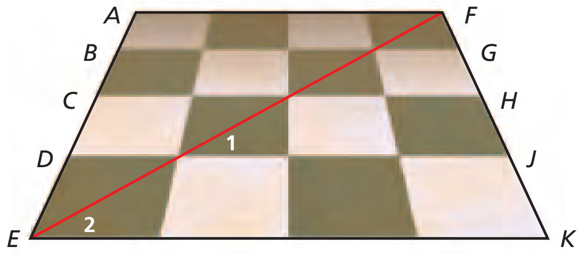
Given: $\overline{AB} \parallel \overline{CD}$, $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$
Prove: $\overline{BC} \parallel \overline{DE}$



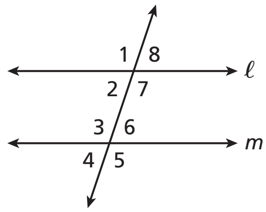
Proof:

Statements	Reasons
1. $\overline{AB} \parallel \overline{CD}$	**1.** Given
2. $\angle 1 \cong \angle 3$	**2. a.** ___?___
3. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	**3. b.** ___?___
4. $\angle 2 \cong \angle 4$	**4. c.** ___?___
5. d. ___?___	**5. e.** ___?___

23. Art Edmund Dulac used perspective when drawing the floor titles in this illustration for *The Wind's Tale* by Hans Christian Andersen. Show that $DJ \parallel EK$ if $m\angle 1 = (3x + 2)°$, $m\angle 2 = (5x - 10)°$, and $x = 6$.



Name the postulate or theorem that proves that $\ell \parallel m$.



24. $\angle 8 \cong \angle 6$

25. $\angle 8 \cong \angle 4$

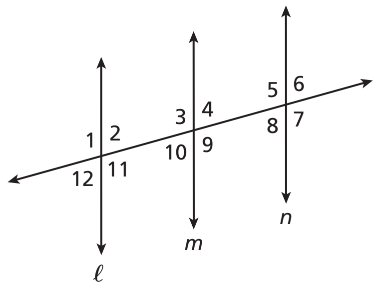
26. $\angle 2 \cong \angle 6$

27. $\angle 7 \cong \angle 5$

28. $\angle 3 \cong \angle 7$

29. $m\angle 2 + m\angle 3 = 180°$

For the given information, tell which pair of lines must be parallel. Name the postulate or theorem that supports your answer.



30. $m\angle 2 = m\angle 10$

31. $m\angle 8 + m\angle 9 = 180°$

32. $\angle 1 \cong \angle 7$

33. $m\angle 10 = m\angle 6$

34. $\angle 11 \cong \angle 5$

35. $m\angle 2 + m\angle 5 = 180°$

36. Multi-Step Two lines are intersected by a transversal so that $\angle 1$ and $\angle 2$ are corresponding angles, $\angle 1$ and $\angle 3$ are alternate exterior angles, and $\angle 3$ and $\angle 4$ are corresponding angles. If $\angle 2 \cong \angle 4$, what theorem or postulate can be used to prove the lines parallel?

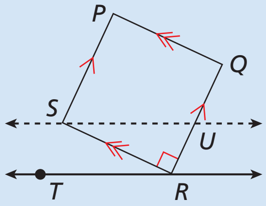
CONCEPT CONNECTION

37. This problem will prepare you for the Concept Connection on page 180.

In the diagram, which represents the side view of a mystery spot, $m\angle SRT = 25°$, and $m\angle SUR = 65°$.

a. Name a same-side interior angle of $\angle SUR$ for lines $\overleftrightarrow{SU}$ and $\overleftrightarrow{RT}$ with transversal $\overline{RU}$. What is its measure? Explain your reasoning.

b. Prove that $\overleftrightarrow{SU}$ and $\overleftrightarrow{RT}$ are parallel.

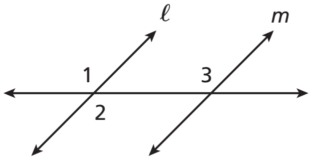


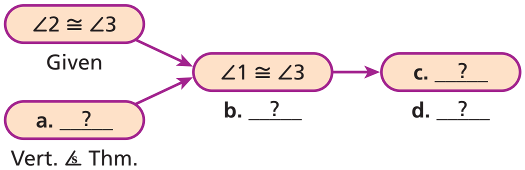
38. Complete the flowchart proof of the Converse of the Alternate Interior Angles Theorem.

Given: $\angle 2 \cong \angle 3$

Prove: $\ell \parallel m$

Proof:

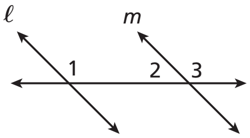




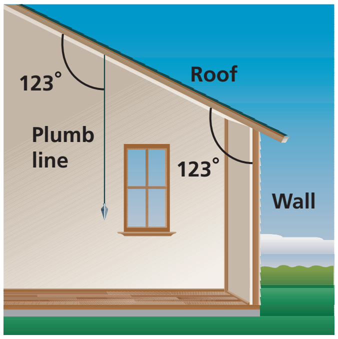
39. Use the diagram to write a paragraph proof of the Converse of the Same-Side Interior Angles Theorem.

Given: $\angle 1$ and $\angle 2$ are supplementary.

Prove: $\ell \parallel m$



40. **Carpentry** A *plumb bob* is a weight hung at the end of a string, called a *plumb line*. The weight pulls the string down so that the plumb line is perfectly vertical. Suppose that the angle formed by the wall and the roof is 123° and the angle formed by the plumb line and the roof is 123°. How does this show that the wall is perfectly vertical?



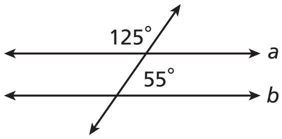
41. **Critical Thinking** Are the Reflexive, Symmetric, and Transitive Properties true for parallel lines? Explain why or why not.

Reflexive: $\ell \parallel \ell$

Symmetric: If $\ell \parallel m$, then $m \parallel \ell$.

Transitive: If $\ell \parallel m$ and $m \parallel n$, then $\ell \parallel n$.

42. **Write About It** Does the information given in the diagram allow you to conclude that $a \parallel b$? Explain.



STANDARDIZED TEST PREP

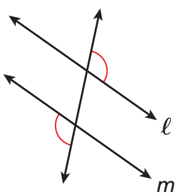
43. Which postulate or theorem can be used to prove $\ell \parallel m$?

Ⓐ Converse of the Corresponding Angles Postulate

Ⓑ Converse of the Alternate Interior Angles Theorem

Ⓒ Converse of the Alternate Exterior Angles Theorem

Ⓓ Converse of the Same-Side Interior Angles Theorem



44. Two coplanar lines are cut by a transversal. Which condition does NOT guarantee that the two lines are parallel?

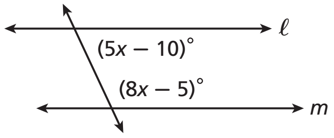
Ⓐ A pair of alternate interior angles are congruent.

Ⓑ A pair of same-side interior angles are supplementary.

Ⓒ A pair of corresponding angles are congruent.

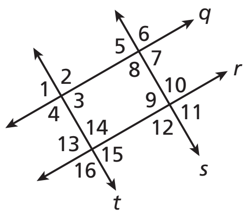
Ⓓ A pair of alternate exterior angles are complementary.

45. **Gridded Response** Find the value of *x* so that $\ell \parallel m$.



CHALLENGE AND EXTEND

Determine which lines, if any, can be proven parallel using the given information. Justify your answers.



46. $\angle 1 \cong \angle 15$

47. $\angle 8 \cong \angle 14$

48. $\angle 3 \cong \angle 7$

49. $\angle 8 \cong \angle 10$

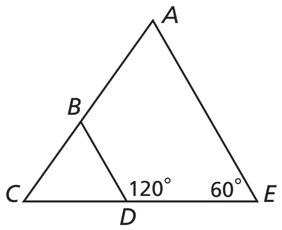
50. $\angle 6 \cong \angle 8$

51. $\angle 13 \cong \angle 11$

52. $m\angle 12 + m\angle 15 = 180°$

53. $m\angle 5 + m\angle 8 = 180°$

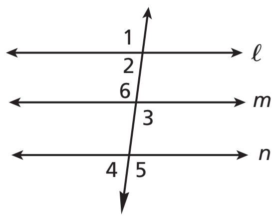
54. Write a paragraph proof that $\overline{AE} \parallel \overline{BD}$.



Use the diagram for Exercises 55 and 56.

55. **Given:** $m\angle 2 + m\angle 3 = 180°$
Prove: $\ell \parallel m$

56. **Given:** $m\angle 2 + m\angle 5 = 180°$
Prove: $\ell \parallel n$



SPIRAL REVIEW

Solve each equation for the indicated variable. *(Previous course)*

57. $a - b = -c$, for a

58. $y = \frac{1}{2}x - 10$, for x

59. $4y + 6x = 12$, for y

Write the converse, inverse, and contrapositive of each conditional statement. Find the truth value of each. *(Lesson 2-2)*

60. If an animal is a bat, then it has wings.

61. If a polygon is a triangle, then it has exactly three sides.

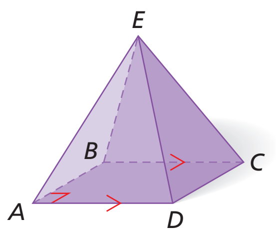
62. If the digit in the ones place of a whole number is 2, then the number is even.

Identify each of the following. *(Lesson 3-1)*

63. one pair of parallel segments

64. one pair of skew segments

65. one pair of perpendicular segments



3-3 Geometry LAB

Use with Lesson 3-3

Construct Parallel Lines

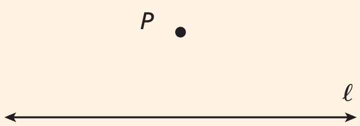
In Lesson 3-3, you learned one method of constructing parallel lines using a compass and straightedge. Another method, called the rhombus method, uses a property of a figure called a *rhombus*, which you will study in Chapter 6. The rhombus method is shown below.

California Standards

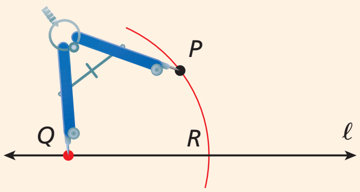
16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and **the line parallel to a given line through a point off the line.**

Activity 1

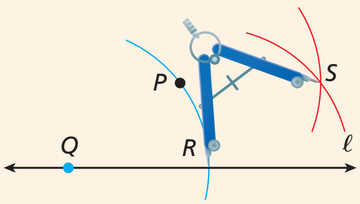
1 Draw a line ℓ and a point P not on the line.



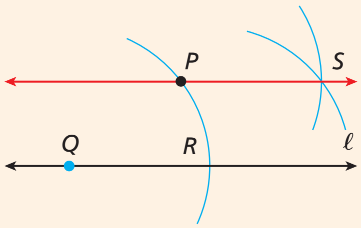
2 Choose a point Q on the line. Place your compass point at Q and draw an arc through P that intersects ℓ. Label the intersection R.



3 Using the same compass setting as the first arc, draw two more arcs: one from P, the other from R. Label the intersection of the two arcs S.



4 Draw $\overleftrightarrow{PS} \parallel \ell$.



Try This

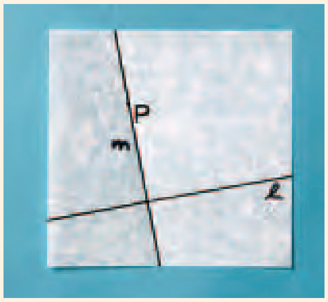
1. Repeat Activity 1 using a different point not on the line. Are your results the same?
2. Using the lines you constructed in Problem 1, draw transversal $\overleftrightarrow{PQ}$. Verify that the lines are parallel by using a protractor to measure alternate interior angles.
3. What postulate ensures that this construction is always possible?
4. A *rhombus* is a quadrilateral with four congruent sides. Explain why this method is called the rhombus method.

Activity 2

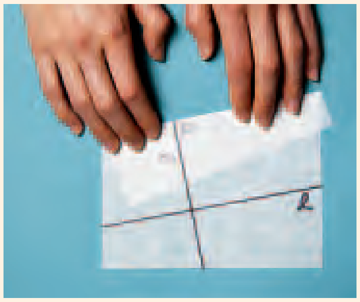
1. Draw a line ℓ and point P on a piece of patty paper.

2. Fold the paper through P so that both sides of line ℓ match up

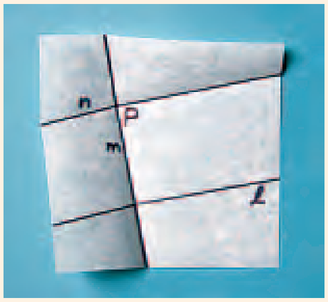
3. Crease the paper to form line m. P should be on line m.



4. Fold the paper again through P so that both sides of line m match up.



5. Crease the paper to form line n. Line n is parallel to line ℓ through P.



Try This

5. Repeat Activity 2 using a point in a different place not on the line. Are your results the same?

6. Use a protractor to measure corresponding angles. How can you tell that the lines are parallel?

7. Draw a triangle and construct a line parallel to one side through the vertex that is not on that side.

8. Line m is perpendicular to both ℓ and n. Use this statement to complete the following conjecture: If two lines in a plane are perpendicular to the same line, then ____?____.