* 1. **Understanding Points, Lines, and Planes**



Points that lie on the same line are **collinear**. *K*, *L*, and *M* are collinear. *K*, *L*, and *N* are *noncollinear*. Points that lie on the same plane are **coplanar**. Otherwise they are *noncoplanar*.

M

***K***

L

***N***



A **postulate**, or *axiom*, is a statement of proof. Postulates are accepted as true WITHOUT proof. Postulates about points, lines, and planes help describe geometric properties.



* 1. **Measuring and Constructing Segments**



The **distance** between any two points is the absolute value of the difference of the coordinates. If the coordinates of points *A* and *B* are *a* and *b*, then the distance between *A* and *B* is |*a* – *b*| or |*b* – *a*|. The distance between *A* and *B* is also called the **length** of *AB*, or *AB*.

 *AB* = *|a* – *b|* or |*b* - *a*|

 A B

**Congruent segments** are segments that have the same length. In the diagram, *PQ* = *RS*, so you can write *PQ* ≅ *RS*. This is read as “segment *PQ* is congruent to segment *RS*.” ***Tick marks*** are used in a figure to show congruent segments.

You can make a sketch or measure and draw a segment. These may not be exact. A **construction** is a way of creating a figure that is more precise. One way to make a geometric construction is to use a compass and straightedge.



In order for you to say that a point *B* is **between** two points *A* and *C*, all three points must lie on the same line, and *AB* + *BC* = *AC*.

* 1. **Measuring and Constructing Angles**

An **angle** is a figure formed by two rays, or sides, with a common endpoint called the **vertex** (plural: *vertices*). You can name an angle several ways: by its vertex, by a point on each ray and the vertex, or by a number.

The set of all points between the sides of the angle is the **interior of an angle**. The **exterior of an angle** is the set of all points outside the angle.




**Congruent angles** are angles that have the same measure. In the diagram, m∠*ABC* = m∠*DEF*, so you can write ∠*ABC* ≅ ∠*DEF*. This is read as “angle ABC is congruent to angle *DEF*.” *Arc marks* are used to show that the two angles are congruent.



An **angle bisector** is a ray that divides an angle into two congruent angles.

* 1. **Pairs of Angles**



 

Another angle pair relationship exists between two angles whose sides form two pairs of opposite rays. **Vertical angles** are two nonadjacent angles formed by two intersecting lines. **∠1** and **∠3** are vertical angles, as are **∠2** and **∠4**.

* 1. **Using Formulas in Geometry**

The **perimeter** *P* of a plane figure is the sum of the side lengths of the figure.

The **area** *A* of a plane figure is the number of non-overlapping square units of a given size that exactly cover the figure.



The **base *b*** can be any side of a triangle. The **height *h*** is a segment from a vertex that forms a right angle with a line containing the base. The height may be a side of the triangle or in the interior or the exterior of the triangle.





* 1. **Mid-Point and Distance in the Coordinate Plane**

A **coordinate plane** is a plane that is divided into four regions by a horizontal line (*x*-axis) and a vertical line (*y*-axis). The location, or coordinates, of a point are given by an ordered pair (*x*, *y*).



The Ruler Postulate can be used to find the distance between two points on a number line. The Distance Formula is used to calculate the distance between two points in a coordinate plane.



You can also use the Pythagorean Theorem to find the distance between two points in a coordinate plane.



**2.1 Using Inductive Reasoning to Make Conjectures**

When several examples form a pattern and you assume the pattern will continue, you are applying *inductive reasoning*. **Inductive reasoning** is the process of reasoning that a rule or statement is true because specific cases are true. You may use inductive reasoning to draw a conclusion from a pattern. A statement you believe to be true based on inductive reasoning is called a **conjecture**.

To show a conjecture is always true, you must prove it. To show that a conjecture is false, you have to find only one example in which the conjecture is not true. This case is called a **counterexample**.

|  |
| --- |
| **Inductive Reasoning** |
| **1.** Look for a pattern. |
| **2.** Make a conjecture. |
| **3.** Prove the conjecture or find a counterexample. |

**2.2 Conditional Statements**



Many sentences without the words *if* and *then* can be written as conditionals. To do so, identify the sentence’s hypothesis and conclusion by figuring out which part of the statement depends on the other.

A conditional statement has a **truth value** of either true (T) or false (F). It is false only when the hypothesis is true and the conclusion is false.To show that a conditional statement is false, you need to find only one counterexample where the hypothesis is true and the conclusion is false.

The **negation** of statement *p* is “not *p*,” written as *~p*. The negation of a true statement is false, and the negation of a false statement is true.



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|  |  |
| --- | --- |
| **Definition** | **Symbols** |
| The **converse** is the statement formed by exchanging the hypothesis and conclusion. | *q* 🡪 *p*  |
| **Definition** | **Symbols** |
| The **inverse** is the statement formed by negating the hypothesis and conclusion. | *~p* 🡪 ~*q*  |



Related conditional statements that have the same truth value are called **logically equivalent statements**. A conditional and its contrapositive are logically equivalent, and so are the converse and inverse.

The logical equivalence of a conditional and its contrapositive is known as the Law of Contrapositive.

**2.3 Using Deductive Reasoning to verify conjectures**

**Deductive reasoning** is the process of using logic to draw conclusions from given facts, definitions, and properties.

In deductive reasoning, if the given facts are true and you apply the correct logic, then the conclusion must be true. The Law of Detachment is one valid form of deductive reasoning.

If *p* 🡪 *q* is a true statement and *p* is true, then *q* is true.

Another valid form of deductive reasoning is the Law of Syllogism. It allows you to draw conclusions from two conditional statements when the conclusion of one is the hypothesis of the other.

Law of Syllogism: If *p* 🡪 *q* and q 🡪 *r* are true statements, then *p 🡪 r* is a true statement.

**2.4 Biconditional Statements and Definitions**

When you combine a conditional statement and its converse, you create a *biconditional statement*.

A **biconditional statement** is a statement that can be written in the form “*p* if and only if *q*.” This means “if *p*, then *q*” and “if *q*, then *p*.”

***p*** ***q*** means ***p***  ***q*** and ***q*** ***p***

The biconditional “*p* if and only if *q*” can also be written as “*p* iff *q*” or *p* ↔ *q*.

For a biconditional statement to be true, both the conditional statement and its converse must be true. If either the conditional or the converse is false, then the biconditional statement is false.

In geometry, biconditional statements are used to write *definitions*. A **definition** is a statement that describes a mathematical object and can be written as a true biconditional.

In the glossary, a **polygon** is defined as a closed plane figure formed by three or more line segments.



A **triangle** is defined as a three-sided polygon, and a **quadrilateral** is a four-sided polygon.



Think of definitions as being reversible. Postulates, however are not necessarily true when reversed.

**2.5 Algebraic Proofs**

A **proof** is an argument that uses logic, definitions, properties, and previously proven statements to show that a conclusion is true. An important part of writing a proof is giving justifications to show that every step is valid.



Like algebra, geometry also uses numbers, variables, and operations. For example, segment lengths and angle measures are numbers. So you can use these same properties of equality to write algebraic proofs in geometry.



Numbers are equal (=) and figures are congruent (≅).

**2.6 Geometric Proofs**

Hypothesis

Definitions,

Theorems,

Postulates,

Properties

 Conclusion

A **theorem** is any statement that you can prove. Once you have proven a theorem, you can use it as a reason in later proofs.





A geometric proof begins with *Given* and *Prove* statements, which restate the hypothesis and conclusion of the conjecture. In a **two-column proof**, you list the steps of the proof in the left column. You write the matching reason for each step in the right column.

