7-6

The Natural Base, *e*

Lesson Objective:

Use the number *e* to write and graph exponential functions representing real-world situations.

Solve equations and problems involving *e* or natural logarithms.

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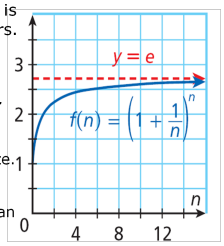
The compound interest formula $A = P(1 + \frac{r}{n})^{nt}$,

A is the amount, *P* is the principal, *r* is the annual interest, *n* is the number of times the interest is compounded per year and *t* is the time in years.

Suppose that $1 is invested at 100% interest ($r = 1$) compounded *n* times for one year as represented by the function $f(n) = P(1 + \frac{1}{n})^n$.

As *n* gets very large, interest is *continuously compounded*. Examine the graph of $f(n) = (1 + \frac{1}{n})^n$. The function has a horizontal asymptote. As *n* becomes infinitely large, the value of the function approaches approximately 2.7182818.... This number is called *e*. Like π, the constant *e* is an irrational number.



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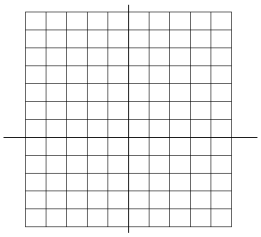
Exponential functions with *e* as a base have the same properties as the functions you have studied.

Graph $f(x) = e^x$

x	e^x

The domain of $f(x) = e^x$ is all real numbers. The range is $\{y|y > 0\}$.



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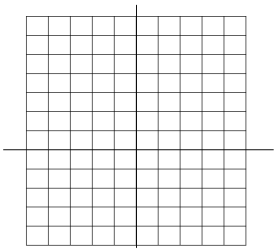
Graph $f(x) = e^x + 1$.

x	–2	–1	0	1	2	3	4
$f(x) = e^x + 1$							



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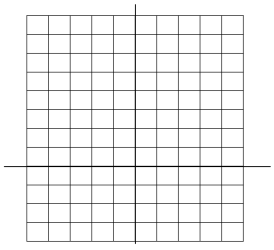
Graph $f(x) = e^{x-2} + 1$.

x	–2	–1	0	1	2	3	4
$f(x) = e^{x-2} + 1$							



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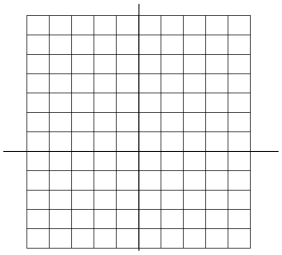
Graph $f(x) = e^{x+1} - 3$.

x	–4	–3	–2	–1	0	1	2
$f(x) = e^{x+1} - 3$							



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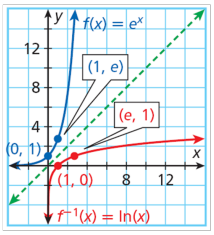
A logarithm with a base of e is called a **natural logarithm** and is abbreviated as "ln" (rather than as $\log_e$). Natural logarithms have the same properties as log base 10 and logarithms with other bases.

The **natural logarithmic function** $f(x) = \ln x$ is the inverse of the natural exponential function $f(x) = e^x$.



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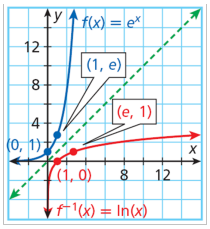
The domain of $f(x) = \ln x$ is $\{x|x > 0\}$.

The range of $f(x) = \ln x$ is all real numbers.

All of the properties of logarithms from Lesson 7-4 also apply to natural logarithms.



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Simplify.

A. $\ln e^{0.15t}$

B. $e^{3\ln(x+1)}$

C. $\ln e^{2x} + \ln e^{x}$

D. $\ln e^{3x} - \ln e^{x}$

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Simplify.

a. $\ln e^{3.2}$

b. $e^{2\ln x}$

c. $\ln e^{x+4y}$

d. $\ln e^{x-3x}$

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End of Part 1

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