7-3 Logarithmic Functions

Lesson Objective

Write equivalent forms for exponential and logarithmic functions.

Write, evaluate, and graph logarithmic functions.

Apr 4-1:51 PM

How many times would you have to double $1 before you had $8? You could use an exponential equation to model this situation. $1(2^x) = 8$. You may be able to solve this equation by using mental math if you know $2^3 = 8$. So you would have to double the dollar 3 times to have $8.

How many times would you have to double $1 before you had $512? You could solve this problem if you could solve $2^x = 8$ by using an inverse operation that undoes raising a base to an exponent equation to model this situation. This operation is called finding the logarithm. A **logarithm** is the exponent to which a specified base is raised to obtain a given value.

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You can write an exponential equation as a logarithmic equation and vice versa.

$$b^x = a \qquad \log_b a = x$$

$$b > 0,\ b \neq 1$$

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$$b^x = a \qquad \log_b a = x$$

Reading Math

Read $\log_b a = x$, as "the log base b of a is x."
Notice that the log is the exponent.

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Write each exponential equation in logarithmic form. State where the red highlighted number goes.

I DO/We Do

$3^5 = 243$

$25^{\frac{1}{2}} = 5$

$10^4 = 10{,}000$

$6^{-1} = \frac{1}{6}$

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Write each exponential equation in logarithmic form. State where the red highlighted number goes.

YOU DO

$9^2 = 81$

$3^3 = 27$

$x^0 = 1 (x \neq 0)$

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Write each logarithmic form in exponential equation.
State where the red highlighted number goes.

I DO/We Do

$\log_9 9 = 1$

$\log_2 512 = 9$

$\log_8 2 = \frac{1}{3}$

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Write each logarithmic form in exponential equation.
State where the red highlighted number goes.

YOU DO

$\log_{10} 10 = 1$

$\log_{12} 144 = 2$

$\log_{\frac{1}{2}} 8 = -3$

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A logarithm is an exponent, so the rules for exponents also apply to logarithms. You may have noticed the following properties in the last example.

Special Properties of Logarithms

For any base b such that $b > 0$ and $b \neq 1$,

LOGARITHMIC FORM	EXPONENTIAL FORM	EXAMPLE
Logarithm of Base b		
$\log_b b = 1$	$b^1 = b$	$\log_{10} 10 = 1$ $10^1 = 10$
Logarithm of 1		
$\log_b 1 = 0$	$b^0 = 1$	$\log_{10} 1 = 0$ $10^0 = 1$

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A logarithm with base 10 is called a **common logarithm**. If no base is written for a logarithm, the base is assumed to be 10. For example, $\log 5 = \log_{10} 5$.

You can use mental math to evaluate some logarithms.

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Evaluate by using mental math.

I DO

log 0.01

$10^? = 0.01$

log 0.00001

$10^? = 0.00001$

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Evaluate by using mental math.

We Do

$\log_5 125$

$\log_{25} 0.04$

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Evaluate by using mental math. **YOU DO**

$\log_5 \frac{1}{5}$

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END OF DAY 1

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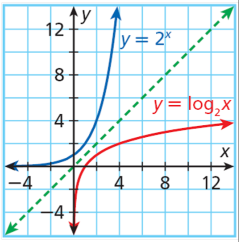
Because logarithms are the inverses of exponents, the inverse of an exponential function, such as $y = 2^x$, is a **logarithmic function**, such as $y = \log_2 x$.
You may notice that the domain and range of each function are switched.

The domain of $y = 2^x$ is all real numbers (R), and the range is $\{y|y>0\}$. The domain of $y = \log_2 x$ is $\{x|x > 0\}$, and the range is all real numbers (R).



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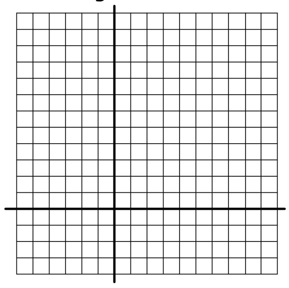
Use the *x*-values {−2, −1, 0, 1, 2}. Graph the function and its inverse. Describe the domain and range of the inverse function.

***f*(*x*) = 1.25^x**

Graph $f(x) = 1.25^x$ by using a table of values

x	y
-2	
-1	
0	
1	
2	



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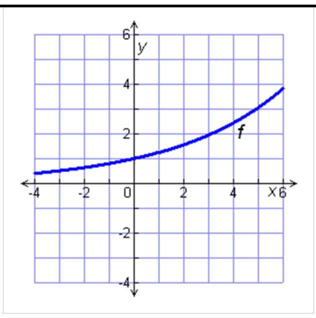
graph the inverse
$f^{-1}(x) = \log_{1.25} x$

x	y
	-2
	-1
	0
	1
	2



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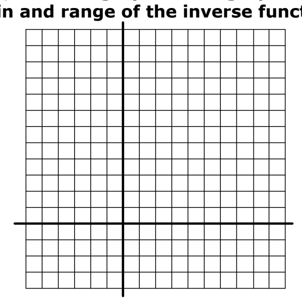
Use *x* = −2, −1, 1, 2, and 3 to graph. Then graph its inverse. Describe the domain and range of the inverse function.

$f(x) = \left(\frac{3}{4}\right)^x$

x	y
-2	
-1	
1	
2	
3	



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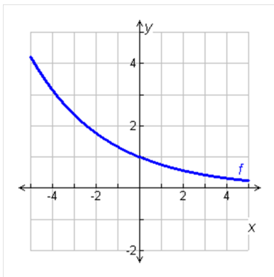
graph the inverse, $f^{-1}(x) = \log_{\frac{3}{4}} x$

x	y
	-2
	-1
	1
	2
	3



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The table lists the hydrogen ion concentrations for a number of food items. Find the pH of each

Substance	H+ conc. (mol/L)
Milk	0.00000025
Tomatoes	0.0000316
Lemon juice	0.0063

Milk

The hydrogen ion concentration is 0.00000025 moles per liter.

pH = $-\log[H^+]$

pH = $-\log(0.00000025)$

Tomatoes

Lemon juice

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