Name ______________________ Date ______________ Class______________

LESSON 6-5

Reteach

Finding Real Roots of Polynomial Equations

To find the roots of a polynomial equation, set the equation equal to zero. Factor the polynomial expression completely. Then set each factor equal to zero to solve for the variable.

Solve the equation: $2x^5 + 6x^4 = 8x^3$.

Step 1 To set the equation equal to 0, rearrange the equation so that all the terms are on one side.

$2x^5 + 6x^4 = 8x^3$

$2x^5 + 6x^4 - 8x^3 = 0$

Step 2 Look for the greatest number and the greatest power of *x* that can be factored from each term.

$2x^5 + 6x^4 - 8x^3 = 0$ — The GCF is $2x^3$.

$2x^3(x^2 + 3x - 4) = 0$

Step 3 Factor the quadratic.

$2x^3(x^2 + 3x - 4) = 0$

$2x^3(x + 4)(x - 1) = 0$

Step 4 Set each factor equal to 0.

$2x^3 = 0 \qquad x + 4 = 0 \qquad x - 1 = 0$

Step 5 Solve each equation.

$2x^3 = 0 \qquad x + 4 = 0 \qquad x - 1 = 0$

$x = 0 \qquad x = -4 \qquad x = 1$

The solutions of the equation are called the roots.

The roots are –4, 0, and 1.

Solve each polynomial equation.

1. $3x^6 - 9x^5 = 30x^4$

   $3x^6 - 9x^5 - 30x^4 = 0$

   $3x^4(x^2 - 3x - 10) = 0$

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2. $x^4 + 6x^2 = 5x^3$

   $x^4 - 5x^3 + 6x^2 = 0$

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3. $2x^3 - 6x^2 - 36x = 0$

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4. $2x^6 - 32x^4 = 0$

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LESSON 6-5

Reteach

Finding Real Roots of Polynomial Equations (continued)

You can use the Rational Root Theorem to find rational roots.

Rational Root Theorem

If a polynomial has integer coefficients, then every rational root can be written in the form $\frac{p}{q}$, where p is a factor of the constant term and q is a factor of the leading coefficient.

Use the Rational Root Theorem. Solve the equation: $x^3 + 3x^2 - 6x - 8 = 0$.

The constant term is –8. The leading coefficient is 1.

p: factors of –8 are ±1, ±2, ±4, ±8

q: factors of 1 are ±1

Possible roots, $\frac{p}{q}$: ±1, ±2, ±4, ±8

Test some possible roots to find an actual root. Use a synthetic substitution table. The first column lists possible roots. The last column represents the remainders. A root has a remainder of 0.

$\frac{p}{q}$	Coefficients of the Equation			
	1	**3**	**–6**	**–8**
1	1	4	–2	–10
2	1	5	4	0
4	1	7	22	80

2 is a root, so $x - 2$ is a factor.
Use the coefficients from the table to write the other factor.

$(x - 2)(x^2 + 5x + 4) = 0$

$(x - 2)(x + 4)(x + 1) = 0$ ← Factor the quadratic to find the other factors.

$x = 2$ or $x = -4$ or $x = -1$

The roots of the equation are –4, –1, and 2.

Use the Rational Root Theorem. Solve $x^3 - 7x^2 + 7x + 15 = 0$.

5. a. Identify possible roots. ____________________________

b. Use the synthetic substitution table to identify an actual root.______________________

$\frac{p}{q}$	Coefficients of the Equation			
	1	**–7**	**7**	**15**

c. Write the factors of the equation.

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d. Identify the roots of the equation.

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b. ±1, ±2, ±4, ±8

c. 2, $\frac{-3 \pm i\sqrt{7}}{2}$; no, 2 of the roots are irrational numbers.

d. 2 m wide, 4 m long, and 1 m deep

Practice C

1. –5, 0, 7
2. 0, 3, 4
3. $x = 2$ with multiplicity 3
4. $x = -4$ with multiplicity 2; $x = -2$ with multiplicity 1
5. –8, 0, 6
6. 3, 6, $2 \pm \sqrt{3}$
7. –3, 0, 1
8. –3, 1, $-3 \pm \sqrt{11}$
9. a. $2x^3 - 4x^2 - 64 = 0$

   b. ±1, ±2, ±4, ±8, ±16, ±32, ±64

   c. 4, $-1 \pm i\sqrt{7}$; no, 2 of the roots are irrational numbers.

   d. 4 in. wide, 8 in. long, and 2 in. deep

Reteach

1. $3x^4(x - 5)(x + 2)$; –2, 0, 5
2. $x^2(x^2 - 5x + 6)$; $x^2(x - 2)(x - 3)$; 0, 2, 3
3. $2x(x^2 - 3x - 18)$; $2x(x - 6)(x + 3)$; –3, 0, 6
4. $2x^4(x^2 - 16)$; $2x^4(x + 4)(x - 4)$; –4, 0, 4
5. a. ±1, ±3, ±5, ±15

   b. 3 or 5

$\frac{p}{q}$	Coefficients of the Equation			
	1	–7	7	15
1	1	6	13	28
3	1	–4	–5	0
5	1	–2	–3	0

   c. $(x - 3)(x^2 - 4x - 5) = 0$; $(x - 3)(x - 5)(x + 1) = 0$

   d. $x = 3$ or $x = 5$ or $x = -1$

Challenge

1. $y = (x + 3)(x)(x - 4)$
2. $y = -(x + 1)^2(x - 1)(x - 3)$
3. $y = (x + 5)(x + 2)^2\left(x - \frac{1}{2}\right)(x - 3)$
4. $y = (x + 6)^2(x)(x - 1)(x - 3)^2(x - 4)^2$
5. $y = (x + 3i)(x - 3i)(x - 1)^2\left(x - \left(1 + \sqrt{3}\right)\right)\left(x + \left(1 + \sqrt{3}\right)\right)$

Problem Solving

1. $V = w(w + 10)(w - 14)$
2. $w^3 - 4w^2 - 140w - 76{,}725 = 0$
3. No; yes; no

   The constant term is 76,725, which is not a multiple or 4 or 10, but is a multiple of 5.
4. Students should test possible roots that are multiples of 5 but not multiples of 10, such as 35, 45, and 55.
5. C
6. A

Reading Strategies

1. Substitute the value of the root in the function and see if it equals 0.
2. $(x - 3)$ and $(x + 2)$
3. a. $(x + 4)$

   b. 3 times
4. $4x$, $(x - 3)$, $(x + 3)$; –3, 0, 3
5. $-x$, $(x - 5)$, $(x - 1)$; 0, 1, 5
6. $(x + 2)$, $(x + 2)$, $(x - 2)$; –2, 2

LESSON 6-6

Practice A

1. 3
2. 5
3. 4
4. a. $P(x) = x(x + 1)(x - 2)$

   b. $P(x) = (x^2 + x)(x - 2)$

   c. $P(x) = x^3 - 2x^2 + x^2 - 2x$

   d. $P(x) = x^3 - x^2 - 2x$
5. $P(x) = x^3 - 3x^2 - 13x + 15$
6. $P(x) = x^3 + 4x^2 - x - 4$
7. a. 2

   b. $2i$, $-2i$