Name ______________________________________ Date ___________ Class ____________

LESSON 5-3

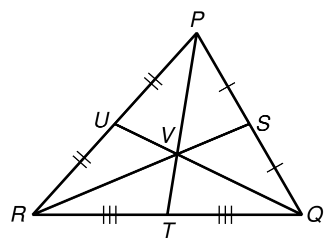
Practice A

Medians and Altitudes of Triangles

Fill in the blanks to complete each definition.

1. A median of a triangle is a segment whose endpoints are a vertex of the triangle and the ____________________ of the opposite side.
2. An altitude of a triangle is a ____________________ segment from a vertex to the line containing the opposite side.
3. The centroid of a triangle is the point where the three ____________________ are concurrent.
4. The orthocenter of a triangle is the point where the three ____________________ are concurrent.

Use the Centroid Theorem and the figure for Exercises 5–8. $\overline{QU}$, $\overline{RS}$, and $\overline{PT}$ are medians of $\triangle PQR$. $RS = 21$ and $VT = 5$. Find each length.



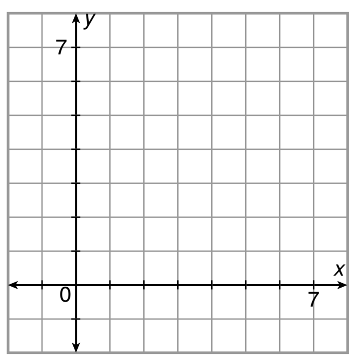
5. *RV* ______________ 6. *SV* ______________

7. *TP* ______________ 8. *VP* ______________

The *centroid* is also called the center of gravity because it is the balance point of the triangle. By holding a tray at the center of gravity, a waiter can carry with one hand a large triangular tray loaded with several dishes.

9. If the vertices of the tray have coordinates *A*(0, 0), *B*(9, 0), and *C*(0, 6), find the coordinates of the balance point (centroid) of the tray. (*Hint:* The *x*-coordinate of the centroid is the average of the *x*-coordinates of the three vertices, and the *y*-coordinate of the centroid is the average of the *y*-coordinates of the three vertices.) (________, ________)
10. If the waiter's hand is at the balance point and the distance from his hand to *A* is 16 inches, find the distance from his hand to $\overline{BC}$. ________________

Complete Exercises 11–15 to find the coordinates of the orthocenter of $\triangle DEF$ with vertices *D*(0, 0), *E*(3, 6), and *F*(4, 0).



11. Plot *D*, *E*, and *F* and draw $\triangle DEF$.
12. Find the equation of a line perpendicular to $\overline{DF}$ through *E*. (*Hint:* A vertical line always takes the form *x* = _____.)

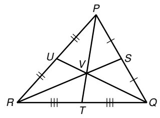
    ____
13. Find the slope of $\overline{ED}$. ______________
14. Find the slope of a line perpendicular to $\overline{ED}$. ______________
15. Find the equation of a line perpendicular to $\overline{ED}$ through *F*. ____________________

LESSON 5-3
Practice A
Medians and Altitudes of Triangles

Fill in the blanks to complete each definition.

1. A median of a triangle is a segment whose endpoints are a vertex of the triangle and the **midpoint** of the opposite side.
2. An altitude of a triangle is a **perpendicular** segment from a vertex to the line containing the opposite side.
3. The centroid of a triangle is the point where the three **medians** are concurrent.
4. The orthocenter of a triangle is the point where the three **altitudes** are concurrent.

Use the Centroid Theorem and the figure for Exercises 5–8. $\overline{QU}$, $\overline{RS}$, and $\overline{PT}$ are medians of $\triangle PQR$. $RS = 21$ and $VT = 5$. Find each length.

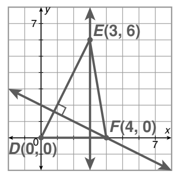


5. RV **14**
6. SV **7**
7. TP **15**
8. VP **10**

The *centroid* is also called the center of gravity because it is the balance point of the triangle. By holding a tray at the center of gravity, a waiter can carry with one hand a large triangular tray loaded with several dishes.

9. If the vertices of the tray have coordinates $A(0, 0)$, $B(9, 0)$, and $C(0, 6)$, find the coordinates of the balance point (centroid) of the tray. (*Hint:* The *x*-coordinate of the centroid is the average of the *x*-coordinates of the three vertices, and the *y*-coordinate of the centroid is the average of the *y*-coordinates of the three vertices.) (**3**, **2**)
10. If the waiter's hand is at the balance point and the distance from his hand to A is 16 inches, find the distance from his hand to $\overline{BC}$. **8 in.**

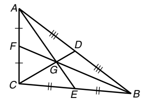
Complete Exercises 11–15 to find the coordinates of the orthocenter of $\triangle DEF$ with vertices $D(0, 0)$, $E(3, 6)$, and $F(4, 0)$.



11. Plot D, E, and F and draw $\triangle DEF$.
12. Find the equation of a line perpendicular to $\overline{DF}$ through E. (*Hint:* A vertical line always takes the form $x =$ ____.) $x = 3$
13. Find the slope of $\overline{ED}$. **2**
14. Find the slope of a line perpendicular to $\overline{ED}$. $-\frac{1}{2}$
15. Find the equation of a line perpendicular to $\overline{ED}$ through F. $y = -\frac{1}{2}x + 2$

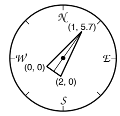
LESSON 5-3
Practice B
Medians and Altitudes of Triangles

Use the figure for Exercises 1–4. $GB = 12\frac{2}{3}$ and $CD = 10$. Find each length.



1. FG $6\frac{1}{3}$
2. BF **19**
3. GD $3\frac{1}{3}$
4. CG $6\frac{2}{3}$

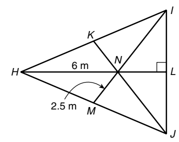
5. A triangular compass needle will turn most easily if it is attached to the compass face through its centroid. Find the coordinates of the centroid. (**1**, **1.9**)



Find the orthocenter of the triangle with the given vertices.

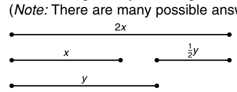
6. $X(-5, 4)$, $Y(2, -3)$, $Z(1, 4)$ (**2**, **5**)
7. $A(0, -1)$, $B(2, -3)$, $C(4, -1)$ (**2**, **−3**)

Use the figure for Exercises 8 and 9. $\overline{HL}$, $\overline{IM}$, and $\overline{JK}$ are medians of $\triangle HIJ$.

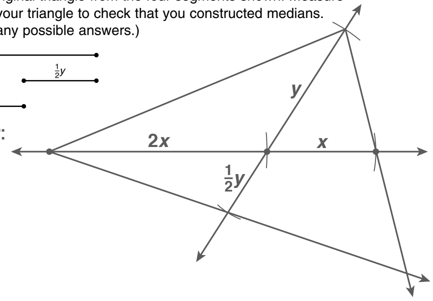


8. Find the area of the triangle. **36 m²**
9. If the perimeter of the triangle is 49 meters, then find the length of $\overline{MH}$. (*Hint:* What kind of a triangle is it?) **10.25 m**

10. Two medians of a triangle were cut apart at the centroid to make the four segments shown below. Use what you know about the Centroid Theorem to reconstruct the original triangle from the four segments shown. Measure the side lengths of your triangle to check that you constructed medians. (*Note:* There are many possible answers.)



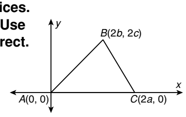
Possible answer:



LESSON 5-3
Practice C
Medians and Altitudes of Triangles

1. In a right triangle, what kind of line connects the orthocenter and the circumcenter? **a median**

After noticing a pattern with several triangles, Regina declares to her class that in any triangle, the *x*-coordinate of the centroid is the average of the *x*-coordinates of the vertices and the *y*-coordinate of the centroid is the average of the *y*-coordinates of the vertices. Regina used inductive reasoning to come to her conclusion. Use deductive reasoning to prove that Regina's conclusion is correct.



2. **Given:** $\triangle ABC$ with $A(0, 0)$, $B(2b, 2c)$, $C(2a, 0)$
   Prove: The coordinates of the centroid are $\left(\frac{2a + 2b}{3}, \frac{2c}{3}\right)$.

The midpoint of $\overline{AC}$ is $(a, 0)$. Name this point D.

The midpoint of $\overline{AB}$ is (b, c). Name this point E.

The slope of $\overline{BD}$ is $\frac{2c}{2b - a}$. Using $(a, 0)$ as a point on $\overline{BD}$ gives the equation $y = \frac{2c}{2b - a}(x - a)$. The slope of $\overline{CE}$ is $\frac{-c}{2a - b}$.

Using $(2a, 0)$ as a point on $\overline{CE}$ gives the equation $y = \frac{-c}{2a - b}(x - 2a)$.

The centroid will be the intersection point of $\overline{BD}$ and $\overline{CE}$, so set the equations equal and simplify:

$$\frac{2c}{2b - a}(x - a) = \frac{-c}{2a - b}(x - 2a)$$

$$(4ac - 2bc)(x - a) = (ac - 2bc)(x - 2a)$$

$$4acx - 2bcx - 4a^2c + 2abc = acx - 2bcx - 2a^2c + 4abc$$

$$3acx = 2a^2c + 2abc$$

$$x = \frac{2a + 2b}{3}$$

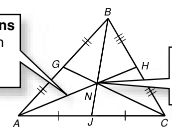
Substituting x into the equation of $\overline{BD}$ yields: $y = \frac{2c}{2b - a}\left(\frac{2a + 2b}{3} - a\right)$

$$= \frac{2c}{2b - a}\left(\frac{2b - a}{3}\right)$$

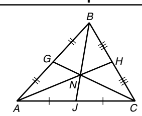
$$= \frac{2c}{3}$$

LESSON 5-3
Reteach
Medians and Altitudes of Triangles

$\overline{AH}$, $\overline{BJ}$, and $\overline{CG}$ are **medians of a triangle.** They each join a vertex and the midpoint of the opposite side.



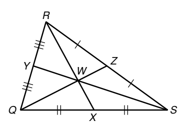
The point of intersection of the medians is called the **centroid** of $\triangle ABC$.

Theorem	Example
Centroid Theorem The centroid of a triangle is located $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.	 **Given:** $\overline{AH}$, $\overline{CG}$, and $\overline{BJ}$ are medians of $\triangle ABC$. **Conclusion:** $AN = \frac{2}{3}AH$, $CN = \frac{2}{3}CG$, $BN = \frac{2}{3}BJ$

In $\triangle ABC$ above, suppose $AH = 18$ and $BN = 10$. You can use the Centroid Theorem to find AN and BJ.

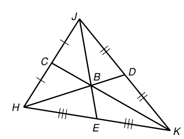
$AN = \frac{2}{3}AH$	Centroid Thm.	$BN = \frac{2}{3}BJ$	Centroid Thm.
$AN = \frac{2}{3}(18)$	Substitute 18 for AH.	$10 = \frac{2}{3}BJ$	Substitute 10 for BN.
$AN = 12$	Simplify.	$15 = BJ$	Simplify.

In $\triangle QRS$, $RX = 48$ and $QW = 30$. Find each length.



1. RW **32**
2. WX **16**
3. QZ **45**
4. WZ **15**

In $\triangle HJK$, $HD = 21$ and $BK = 18$. Find each length.



5. HB **14**
6. BD **7**
7. CK **27**
8. CB **9**