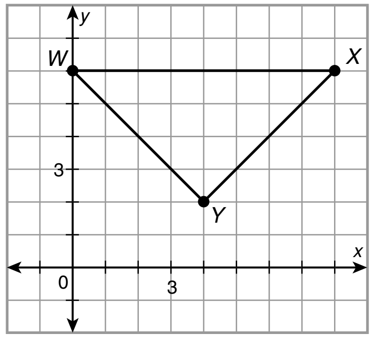
Name ______________________ Date __________ Class __________

LESSON 5-3

Problem Solving

Medians and Altitudes of Triangles

1. The diagram shows the coordinates of the vertices of a triangular patio umbrella. The umbrella will rest on a pole that will support it. Where should the pole be attached so that the umbrella is balanced?



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2. In a plan for a triangular wind chime, the coordinates of the vertices are $J(10, 2)$, $K(7, 6)$, and $L(12, 10)$. At what coordinates should the manufacturer attach the chain from which it will hang in order for the chime to be balanced?

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3. Triangle PQR has vertices at $P(-3, 5)$, $Q(-1, 7)$, and $R(3, 1)$. Find the coordinates of the orthocenter and the centroid.

____

Choose the best answer.

4. A triangle has coordinates at $A(0, 6)$, $B(8, 6)$, and $C(5, 0)$. $\overline{CD}$ is a median of the triangle, and $\overline{CE}$ is an altitude of the triangle. Which is a true statement?

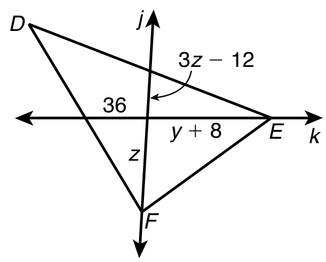
   A The coordinates of D and E are the same.

   B The distance between D and E is 1 unit.

   C The distance between D and E is 2 units.

   D D is on the triangle, and E is outside the triangle.

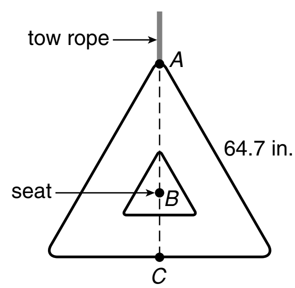
5. Lines j and k contain medians of $\triangle DEF$. Find y and z.



   F $y = 16$; $z = 4$ **H** $y = 64$; $z = 4.8$

   G $y = 32$; $z = 4$ **J** $y = 108$; $z = 8$

6. An inflatable triangular raft is towed behind a boat. The raft is an equilateral triangle. To maintain balance, the seat is at the centroid B of the triangle. What is AB, the distance from the seat to the tow rope? Round to the nearest tenth.



   A 18.7 in.

   B 37.4 in.

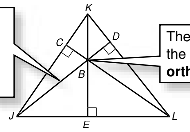
   C 43.1 in.

   D 56.0 in.

LESSON 5-3

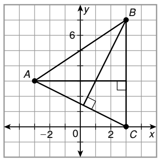
Reteach
Medians and Altitudes of Triangles continued

$\overline{JD}$, $\overline{KE}$, and $\overline{LC}$ are **altitudes of a triangle.** They are perpendicular segments that join a vertex and the line containing the side opposite the vertex.



The point of intersection of the altitudes is called the **orthocenter** of $\triangle JKL$.

Find the orthocenter of $\triangle ABC$ with vertices $A(-3, 3)$, $B(3, 7)$, and $C(3, 0)$.



Step 1 Graph the triangle.

Step 2 Find equations of the lines containing two altitudes.

The altitude from A to $\overline{BC}$ is the horizontal line $y = 3$.

The slope of $\overleftrightarrow{AC} = \frac{0 - 3}{3 - (-3)} = -\frac{1}{2}$, so the slope of the altitude from B to $\overline{AC}$ is 2. The altitude must pass through $B(3, 7)$.

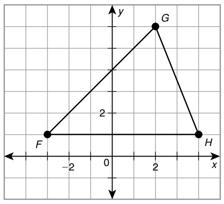
$y - y_1 = m(x - x_1)$ Point-slope form

$y - 7 = 2(x - 3)$ Substitute 2 for m and the coordinates of $B(3, 7)$ for (x_1, y_1).

$y = 2x + 1$ Simplify.

Step 3 Solving the system of equations $y = 3$ and $y = 2x + 1$, you find that the coordinates of the orthocenter are (1, 3).

Triangle *FGH* has coordinates $F(-3, 1)$, $G(2, 6)$, and $H(4, 1)$.



9. Find an equation of the line containing the altitude from G to $\overline{FH}$.

   $x = 2$

10. Find an equation of the line containing the altitude from H to $\overline{FG}$.

    $y = -x + 5$

11. Solve the system of equations from Exercises 9 and 10 to find the coordinates of the orthocenter.

    (2, 3)

Find the orthocenter of the triangle with the given vertices.

12. $N(-1, 0)$, $P(1, 8)$, $Q(5, 0)$

    (1, 1)

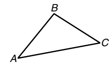
13. $R(-1, 4)$, $S(5, -2)$, $T(-1, -6)$

    (3, −2)

LESSON 5-3

Challenge
Exploring Another Point of Concurrency

The **Fermat point,** named after seventeenth century mathematician Pierre de Fermat, is a special point in an acute triangle that can be found by using the following steps.

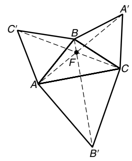


Step 1 Draw an acute triangle ABC.

Step 2 Construct an equilateral triangle on each side of $\triangle ABC$ and label the new vertices A', B', C' as shown.

Step 3 Connect the opposite vertices by drawing $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$.

Step 4 The point of concurrency of these segments is the Fermat point of $\triangle ABC$. Label this point F.



Use the figure above for Exercises 1 and 2.

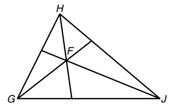
1. Make a conjecture comparing the lengths of $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$. Verify your conjecture by measuring the segments.

   The segments have equal lengths.

2. Find the sum of AF, BF, and CF. Compare this sum to the lengths of the segments that you found in Exercise 1.

   The sum equals one segment length.

Point *F* is the Fermat point in $\triangle GHJ$.



3. Find the sum of the distances from the Fermat point to each vertex of $\triangle GHJ$. Round to the nearest tenth of a centimeter.

   Possible answer: 5.8 cm

4. Draw two more points inside $\triangle GHJ$ and label the points X and Y. Then find each sum: $GX + HX + JX$ and $GY + HY + JY$. Compare the sums to the sum you found by using the Fermat point.

   These sums are both greater than the sum found by using the Fermat point.

5. Make a conjecture about the sum of the distances from the Fermat point to the vertices of a triangle, compared to the sum of the distances from any point to the vertices of a triangle.

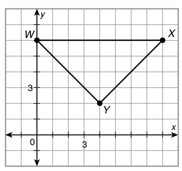
   The sum of the distances from the Fermat point to the vertices is the least sum that is possible from any point in a triangle to each of the three vertices.

6. Draw an acute triangle. Locate the Fermat point. Repeat Exercises 3 and 4. How do the results compare to the conjecture you made in Exercise 5?

   The sums found using *X* and *Y* are both greater than the sum found by using the Fermat point. This verifies the conjecture.

LESSON 5-3

Problem Solving
Medians and Altitudes of Triangles



1. The diagram shows the coordinates of the vertices of a triangular patio umbrella. The umbrella will rest on a pole that will support it. Where should the pole be attached so that the umbrella is balanced?

   $\left(4, \frac{14}{3}\right)$

2. In a plan for a triangular wind chime, the coordinates of the vertices are $J(10, 2)$, $K(7, 6)$, and $L(12, 10)$. At what coordinates should the manufacturer attach the chain from which it will hang in order for the chime to be balanced?

   $\left(9\frac{2}{3}, 6\right)$

3. Triangle PQR has vertices at $P(-3, 5)$, $Q(-1, 7)$, and $R(3, 1)$. Find the coordinates of the orthocenter and the centroid.

   $\left(-1\frac{4}{5}, 5\frac{4}{5}\right)$; $\left(-\frac{1}{3}, 4\frac{1}{3}\right)$

Choose the best answer.

4. A triangle has coordinates at $A(0, 6)$, $B(8, 6)$, and $C(5, 0)$. $\overline{CD}$ is a median of the triangle, and $\overline{CE}$ is an altitude of the triangle. Which is a true statement?

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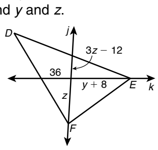

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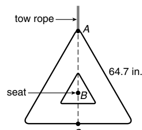

   A 18.7 in.
   (B) 37.4 in.
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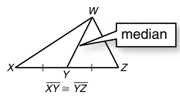
LESSON 5-3

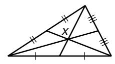
Reading Strategies
Understanding Vocabulary

The following vocabulary terms identify special points and segments for triangles.

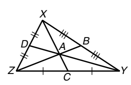
A **median of a triangle** is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side.



The **centroid of a triangle** is the point of concurrency of the medians of a triangle. X is the centroid in this triangle.



Use the figure for Exercises 2–4.



1. How many medians does a triangle have? Explain your answer.

   Three; a triangle has three sides and three vertices, so three segments connect the vertices with their opposite sides.

2. Explain why $\overline{XC}$ is a median of $\triangle XYZ$.

   It starts at one of the vertices and ends at the midpoint of the opposite side.

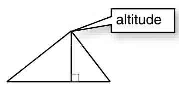
3. What are the other medians of $\triangle XYZ$?

   $\overline{ZB}$ and $\overline{DY}$

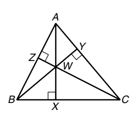
4. What is the centroid of $\triangle XYZ$?

   point *A*

An **altitude of a triangle** is a perpendicular segment from a vertex to the line containing the opposite side.



The **orthocenter of a triangle** is the point of concurrency of the altitudes of a triangle.



5. How many altitudes does a triangle have? Explain.

   One altitude can be drawn from each vertex of a triangle, so every triangle has three altitudes.

6. Name the altitudes of $\triangle ABC$.

   $\overline{AX}$; $\overline{BY}$; $\overline{CZ}$

7. What is the orthocenter of $\triangle ABC$?

   point *W*